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WAGE - EMPLOYMENT CONTRACTS

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Wage - Employment Contracts

ABSTRACT

This paper studies the efficient agreements about the dependence of workers' earnings on employment, when the employment level is controlled by firms. Under plausible assumptions, such agreements will cause employment to diverge from efficiency as a byproduct of their attempt to mitigate risk. However, employment is above rather than below the efficient level when the conditions of profitability are worse than average. Such a one-period implicit contracting model cannot, therefore, be used to "explain" unemployment as it is traditionally conceived.

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1. Introduction

Most labor agreements specify the relationship between total compensation and employment,but leave the latter under the firm's control. Such a provision for contract governance may be necessary because information about the value of the firm's short-run production is not easily perceived and verified by labor. This asymmetry sets up a potential conflict between the goals of risk-sharing and efficiency. In this paper I attempt to analyze the solution to this problem: I look at some properties of the efficient labor contracts that can be agreed upon with the knowledge that the firm will choose the employment level after it ascertains some relevant random parameters.

The results can be roughly characterized as follows, subject of course to assumptions whose innocence and plausibility I will later espouse:

1) There is less employment fluctuation under the optimal contract than would be observed if employment were chosen to maximize profits subject to the constraint that worker's utility be held constant in all situations.

2) There is less employment fluctuation than in the contract that would be implemented if the information-impactedness issue were not relevant.

3) In "bad times" the level of employment corresponds to "involuntary overtime" in that, if workers could at that point recontract with the firm under conditions of symmetric information, the level of employment would be lower. In other words, when firms have chosen to reduce the level of employment, the value of the marginal product of labor is less than workers' marginal valuation of their leisure.

Conversely, when "overtime" is required by firms, they will be keeping the value of the marginal product of labor above laborers' valuation of their time.

4) Finally, workers' utility is lowest in "normal times." They like both overtime and unemployment because they are "overcompensated" in both directions.

These results show that the asymmetry of information that has been suggested as a source of suboptimal employment policies results in the opposite bias. It cannot be used as a foundation for a theory of involuntary unemployment.

Risk sharing between firms and workers has been a central focus of the literature on implicit contracts.* Most of the papers have assumed that the relevant uncertainties, when resolved, could be observed by all parties and therefore could be used as contingencies in the agreement. In this paper I emphasize the asymmetry of information. For tractability, I exclude some other complications that previous writers were able to handle. The results, as summarized above, are nevertheless quite striking; and they point out phenomena that do not arise in models with symmetric information.

Specifically, I will maintain the following highly stylized assumptions.

i) Workers are risk averse and firms are risk neutral.

ii) Firms have complete control of employment, ex post.

iii) The worker's welfare is represented by a single collective utility function, as if a union with well-specified risk preferences were to strike the bargaining agreement. The actual implementation of the agreements within the group of workers -- for example, seniority rules and the wage structure for different categories of workers -- is not addressed.

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* I cannot attempt any reasonable summary of this interesting and rapidly expanding literature here. The most closely related papers to this one are Phelps-Calvo (1977) and Hall-Lillien (1979). More details on these will be discussed below. An excellent survey of this area is Azariadis (1979). He mentions the problem I treat here on pp. 28-30.

iv) All the relevant uncertainty impacts upon the value of the firm's output, labor's preferences being assumed non-stochastic over the life of the contract.

v) Finally, the form of feasible contracts is highly simplified. Compensation can be made to depend only upon the firm's contemporaneous choice of employment.

The model is presented and discussed in the next section. I also compare it to the other papers in this literature and to related papers on "moral hazard" problems. The following section presents the solution, in the nature of an approximation that will be appropriate under circumstances outlined therein. A brief conclusion follows.

2. The Model

The relevant uncertainty is parameterized by θ, and impinges only on the value of the firm's output. If ℓ is the employment level, then $f(\ell,\theta)$ is this value. The contract specifies the wage paid, $w(\ell)$, as a function of employment. The net payoff to the firm is thus

$$(2.1) \qquad f(\ell,\theta) - \ell w(\ell)$$

The firm is assumed to be risk neutral, and therefore the mathematical expectation of (2.1) is its objective. The function f will be assumed concave and increasing in ℓ for each θ.

Workers' utility depends upon earnings, $\ell w(\ell)$, and employment, negatively with respect to the latter. Because they are risk averse we write their objective as

$$(2.2) \qquad Eu(\ell w(\ell),\ell)$$

where u is a concave function. The expectation in (2.2) is taken with respect to the distribution of ℓ. However, ℓ is chosen by firms. Its distribution will therefore depend on the form of the entire contract and on the distribution of θ.

The result of any contract $w(\ell)$ is the firm's choice of employment for each θ, $\ell(\theta)$. Associated with this the wage $w(\ell(\theta))$ will be paid. It is notationally simpler to work with total compensation than with the wage rate; thus we define

$$(2.3) \qquad r(\theta) = \ell(\theta)\, w(\ell(\theta)) \quad .$$

The problem I analyze is to choose $w(\cdot)$ so as to maximize

(2.4) $\quad Eu(r(\theta),\ell(\theta))$

subject to

(2.5) $\quad E\ f(\ell(\theta),\theta) - r(\theta) \geq c$

where $\ell(\theta)$ is defined by the solution to

(2.6) $\quad \max_{\ell}\ f(\ell,\theta) - \ell w(\ell)$

and $r(\theta)$ is given by (2.3). By varying c parametrically, the family of efficient contracts will be delineated.

The idea of this paper is to compare the solution obtained with solutions to related problems in order to ascertain some of the qualitative impacts of informational asymmetry and differential attitudes towards risk. Specifically I ask whether and to what extent profits, employment and labor compensation are more stable in this problem than when these features are absent.

Before proceeding further, therefore, I present two less restrictive versions of this problem that will be useful as benchmarks. First, consider the maximization of (2.4) subject to (2.5), but where $\ell(\theta)$ can be chosen arbitrarily. This corresponds to the bulk of the implicit contracts literature in which the realization of uncertainties can be verified by both parties and therefore can be used to explicitly condition the outcomes. Second, we can consider the problem as posed but in the special case when utility takes the form

(2.7) $u(r,\ell) = v(r - h(\ell))$

where h is a convex function describing the marginal disutility of labor and v is an arbitrary increasing concave function. This is a problem studied by Hall and Lillien* (2.7).in the case when v is linear. The solution they found applies to the case of concave v as well. It is to set $w(\ell)$ so as to implicitly describe an indifference curve; that is

$$\ell w(\ell) - h(\ell) = \overline{U}$$

The firm then chooses the profit maximizing ℓ keeping workers on the $\overline{U}$ indifference curve for all realizations of θ. Efficiency is achieved for each θ. The firm bears all the risks because, for this form of the utility function, the marginal utility of income is constant when utility itself is constant. Thus although workers' incomes fluctuate, they do not experience any need for income smoothing, and it is in this sense that all the relevant risk can be eliminated in a contract implemented entirely at the firm's discretion.

To put this model in the proper perspective it is useful to compare it with the literature on the "principal-agent problem." Here, the firm is the agent and the workers are playing the principal's role. It is the firm whose information is private. In the usual specification of the principal-agent problem the agent is risk averse. With a risk neutral agent a very simple contract can achieve a first-best outcome: The principal receives a fixed payment and the agent takes the residual; therefore he fully inter-

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* They also allow random effects in the utility function, and they show that a contract administered by firms cannot implement the full-information optimum even in this special case.

nalizes the group's objective function when he decides on how much effort to put forth. What makes the present problem non-trivial, even though the firm is risk-neutral, is that his choice variable "employment", enters directly into the utility function of the workers. This combination of an "externality" problem with the "principal-agent problem" is the essential feature of the model of wage-employment contracting that I present.

3. Solution

The solution to the second-best contracting problem presented above is quite complex in full generality. I will therefore proceed under a further assumption about the nature of the uncertainty facing the firm.

(3.1) $f(\ell,\theta) = \theta g(\ell)$ $\theta \epsilon \mathbb{R}_+$, g concave and monotone increasing.

The interpretation of (3.1) is that θ is a random valuation of the firm's output and that g is its short-run production schedule. This form of uncertainty may usefully characterize or approximate various realistic situations. It would be inappropriate, however, when non-linearities in the valuation of output are important, as for example in the case of a firm facing variable constraints on its sales at an inflexible price. We assume throughout that θ has a positive continuous density function.

The method of solution is novel in models of implicit contracting, drawing heavily on some techniques first developed in the literature on incentive compatibility and optimal auction design.* ** ***

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* See Wilson (1977), Riley and Samuelson (1979), for an introduction to the auction design problem. Stochastic auction designs have been treated by Maskin, Riley and Weitzman (1979).

** On incentive compatibility see Green and Laffont (1979) and Laffont and Maskin (1980), where the treatment of the continuous-parameter problem is closest to what will be used here.

*** After a draft of this paper had been written I saw a closely related approach in Azariadis (1980). His specification is, however, different in several important respects, and the solution method he proposes is therefore quite unlike what follows in the next section. In particular, that paper deals with risk averse firms, as well as risk averse workers. However, because there is a constant marginal "disutility" (forgone income) of work, the utility function is of the Hall-Lillien form (2.7). Results are only presented for the case of two states, whereas my solution method relies heavily on θ having a continuous distribution.

The idea is to regard the problem as the choice of two functions of θ, $r(\theta)$ and $\ell(\theta)$, instead of the single relation $w(\ell)$. Thus we have, using (3.1),

$$\text{(3.2)} \qquad \max E\, u(r(\theta), \ell(\theta))$$

subject to

$$\text{(3.3)} \qquad E\theta\, g(\ell(\theta)) - r(\theta) \geqq c$$

and that, for each θ,

$$\text{(3.4)} \qquad \max_{\tilde{\theta}} \quad \theta g(\ell(\tilde{\theta})) - r(\tilde{\theta}) \quad \text{occurs at } \tilde{\theta} = \theta$$

$$\text{(3.5)} \qquad \ell'(\theta) \geqq 0$$

The second set of constraints corresponds to (2.6). The final set, (3.5), is needed to insure that in reparameterizing the firm's problem in terms of θ rather than ℓ we have not spuriously introduced any more flexibility into the solution -- there must still be a unique level of total compensation associated with any one level of employment. By positing that $\ell'(\theta) > 0$, we know that each level of employment is chosen for at most one value of θ, and thus that this potential problem cannot arise.

The next step is to replace (3.4) by the statement that the first and second order conditions for that problem hold as identities in θ at $\tilde{\theta} = \theta$. These are

$$\text{(3.6)} \qquad \theta g'(\ell(\theta))\, \ell'(\theta) - r'(\theta) = 0$$

and

$$\text{(3.7)} \qquad \theta g''(\ell(\theta))\, (\ell'(\theta))^2 + \theta g'(\ell(\theta))\ell''(\theta) - r''(\theta) < 0$$

Since (3.6) is an identity in θ, we can differentiate it to obtain an expression for $r''(\theta)$. Substituting this in (3.7) we can then rewrite the second order conditions as

$$-g'(\ell(\theta))\ell'(\theta) < 0$$

or, by the monotonicity of $g(\cdot)$,

$$\ell'(\theta) > 0$$

Of course, we already have this as a condition, (3.5).

The difficulty with this formalization -- maximize (3.2) subject to (3.3), (3.5) and (3.6) -- is that there are two unknown functions and hence a pair of Euler equations to solve, in principal. To simplify this problem, we integrate (3.6) by parts, obtaining the following expression for $r(\theta)$ in terms of $\ell(\theta)$:

(3.8) $$r(\theta) = \theta g(\ell(\theta)) - \int^{\theta} g(\ell(t))dt + K$$

where K is a constant of integration.

Thus efficient employment contracts can be obtained by solving the following problem for the single unknown function $\ell(\theta)$, and by using (3.8) to recover the required compensation associated with each choice of ℓ:

(3.9) $$\max E\, u(\theta g(\ell(\theta)) - \int^{\theta} g(\ell(t))dt + K, \ell(\theta))$$

subject to

(3.10) $$E[\int^{\theta} g(\ell(t))dt - K] \geqq c$$

and

(3.11) $$\ell'(\theta) \geqq 0 \qquad \text{for all } \theta.$$

As stated, the problem is not in a form suitable for the implementation of control theoretic methods because the objective function is not additively separable across θ in the state ($\ell(\theta)$) and the control ($\ell'(\theta)$). The integral inside the expectation operator in the objective destroys this separability.

To overcome this difficulty we make another transformation of the problem, by introducing the function $F(\cdot)$ defined by

(3.12) $$F(\theta) = \int^{\theta} g(\ell(t))dt - K$$

The function $F(\cdot)$ is just the firm's profit in state θ, under the contract. We will use F as the state variable of our problem. Note that

(3.13) $$\dot{F}(\theta) \equiv \frac{d}{d\theta} F(\theta) = g(\ell(\theta)).$$

The problem can now be written in terms of $F(\cdot)$ as

(3.14) $$\max E\, u(\theta\dot{F}(\theta) - F(\theta)\ ,\ g^{-1}(\dot{F}(\theta)))$$

subject to

(3.15) $$E\, F(\theta) \geq c$$

and

(3.16) $$\ddot{F}(\theta) > 0$$

Except for the appearance of $\ddot{F}$ in the last constraint, this maximization is a standard "isoperimetric" control problem (see Intrilligator (1971), p.318). What we will do is to drop (3.16), solve (3.14) subject to (3.15), and then show that the solution will obey (3.16) automatically, provided that a certain approximation, to be explained below, is valid.

Write the Lagrangian

(3.17) $$E\{u(\theta\dot{F}(\theta) - F(\theta),\ g^{-1}(\dot{F}(\theta))) + yF(\theta)\}$$

where y is a scalar Lagrange multiplier. The Euler equation is

(3.18) $$0 = -2u_1 + y - \ddot{F}\{\theta^2 u_{11} + 2\theta g^{-1'} u_{12} + (g^{-1'})^2 u_{22} + g^{-1''} u_{22}\}$$

Equation (3.18) is the basis of our analysis of the form of the optimal contract. To proceed further however, we will rely on an approximation that is probably quite valid and is well-representative of the kind of situation we envision. We may suppose that in most circumstances there is little or no variation in θ. The role of this kind of contract is to mitigate the destabilizing forces associated with the relatively rare cases in which θ departs from its normal value. Let us call this normal value $\bar{\theta}$.

The scenario above provides us with two important facts about the nature of the optimal contract at $\bar{\theta}$. First, efficiency should be maintained at $\bar{\theta}$. Any contract allowing inefficiency will have a welfare loss associated with it, with high probability. Thus

(3.19) $$\bar{\theta} g'(\ell(\bar{\theta})) = \frac{-u_2(r(\bar{\theta}),\ell(\bar{\theta}))}{u_1(r(\bar{\theta}),\ell(\bar{\theta}))}$$

From which we know using (3.6) that the optimal contract must have the same slope as the indifference curve through the point $r(\bar{\theta}),\ell(\bar{\theta})$.

Second, we can approximate y by $-u_1(r(\bar{\theta}),\ell(\bar{\theta}))$. We know that $y = -Eu_1$ because one way to maintain incentive compatibility of the employment choice and increase profits is to decrease total compensation equally at every level of ℓ. Since $\bar{\theta}$ is the typical value of θ, we can use the value of u_1 there as a good approximation to its true average.

Let us now return to the optimization problem and to the issue of whether it is appropriate to drop the constraint (3.16), without loss of generality.

The approximation above allows us to rewrite (3.18) as

$$\ddot{F} = \frac{-u_1}{\theta^2 u_{11} + 2\theta g^{-1'} u_{12} + (g^{-1'})^2 u_{22} + g^{-1''} u_2} \tag{3.20}$$

Note that the final term in the denominator is negative because $u_2 < 0$ and g is concave. The first three terms are just the quadratic form defined by the Hessian of $u(\cdot)$ evaluated at $(\theta, g^{-1'})$, and hence are negative by the concavity of u. As $u_1 > 0$ we have $\ddot{F} > 0$ automatically, as required. Thus our dropping of the constraint generated by the second-order condition for firms is, under the approximation discussed above, justified.

The Euler equation (3.20)describes the optimal contract implicitly. Of course (3.20)is a second order differential equation. To actually find the optimal contract we must use two boundary conditions. Let θ_{min} be the lowest point in the support of the distribution of θ. Then we know that

$$F(\theta_{min}) = -K$$

$$\dot{F}(\theta_{min}) = g(\ell(\theta_{min}))$$

The choice of K and of $\ell(\theta_{min})$ therefore, together with the Euler equation, completely characterize the optimum. By varying K we can vary $EF(\theta)$ so that $EF(\theta) = c$.

We now come to the heart of the paper -- using this Euler equation to tell us something about the nature of the optimal labor contract. The results are quite striking: The optimal labor contract is such that workers are worst off under the typical conditions -- i.e. near $\bar{\theta}$. They like both bad times, low θ, and good times, high θ. Firm's profits are increasing in θ. The level of employment in bad times is above the efficient level in the sense that if workers could choose employment and compensation subject to the firm's short-run profits being constant they would choose to work less. The marginal value of worker's time exceeds the value of their marginal product.

Let us use (3.19) to eliminate $\bar{\theta}$ in (3.20) obtaining*

$$(3.21) \qquad \ell' = \frac{u_1 g'}{-(\frac{u_2}{u_1})^2 u_{11} + 2u_{12}\frac{u_2}{u_1} - u_{22} + \frac{g''}{g'} u_2}$$

This tells us how sensitive employment is to θ, at the mean $\bar{\theta}$. We will first compare this sensitivity to what would happen if the contract implemented were instead such that u were held constant

$$(3.22) \quad \bar{u} \equiv u(r(\bar{\theta}),\ell(\bar{\theta})) = u(r(\theta),\ell(\theta))$$

Then we will use this comparison of sensitivities to relate the form of the optimal contract to the $\bar{u}$ locus.

We know that whenever the firm optimizes over an indifference locus we have efficiency, that is, (3.19) holds for all θ, not only $\bar{\theta}$.

$$-\theta g'(\ell(\theta)) = \frac{u_2}{u_1} \qquad \text{for all } \theta.$$

Totally differentiating (3.19) and (3.22) and eliminating r' we have

$$\ell' = \frac{-g'u_1}{\theta g'' u_1 - \frac{u_{12}u_2}{u_1} + u_{22}}$$

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* We also use $g^{-1'} = \frac{1}{g'}$, $g^{-1''} = \frac{-g''}{(g')^3}$.

Evaluating this at $\bar{\theta}$ using (3.19) to eliminate θ, we obtain

$$(3.23) \qquad \ell' = \frac{g' u_1}{u_{12} \frac{u_2}{u_1} - u_{22} + \frac{g''}{g'} u_2}$$

Therefore $\ell'(\bar{\theta})$ under the contract $\bar{u}$ will be above $\ell'(\bar{\theta})$ under the optimal contract if

$$u_{12} \frac{u_2}{u_1} - u_{22} < -\left(\frac{u_2}{u_1}\right)^2 u_{11} + 2u_{12} \frac{u_2}{u_1} - u_{22}$$

or if

$$(3.24) \qquad u_{12} - \frac{u_2}{u_1} u_{11} < 0$$

Expression (3.24) has a very intuitive meaning that is directly relevant to the risk bearing issue. In a first-best contract -- i.e. with θ observable by both parties, we would have both efficiency of the employment level and constancy of the marginal utility of income, over all θ. It is therefore natural to compare the $\bar{u}$ locus and the locus where u_1 is constant. Expression (3.24) is valid if and only if $dr/d\ell$ is higher along the former than along the latter. In terms of labor supply, it means that leisure is a normal good.

To shed further light on this issue recall the Hall-Lillien form of utility function, as discussed in the introduction:

$$u(r,\ell) = v(r - h(\ell))$$

Here, these two loci coincide. Thus (3.21) and (3.23) are the same expression -- the Euler equation tells us to implement the first-best by taking $\bar{u}$ as the contract. Most other forms of utility that one might think of will satisfy (3.24) with strict inequality. For example, additive separability or any concave transformation thereof satisfies it.

Now that we know that employment will be more sensitive to θ under the second-best than it would under the $\bar{u}$ contract, it is straightforward to prove the main results on the form of the contract. A graphical summary of these results is presented in Figure 1, and discussed at the end of the section.

Any incentive compatible pair r(θ), ℓ(θ) will satisfy (3.6). Totally differentiating we have

$$\theta g''(\ell')^2 + g'\ell' + \theta g'\ell'' = r'' \tag{3.25}$$

Implicit differentiation of the parametric relation between r(θ) and ℓ(θ) yields

$$\frac{d^2 r}{d\ell^2} = \frac{\ell' r'' - r'\ell''}{(\ell')^3} \tag{3.26}$$

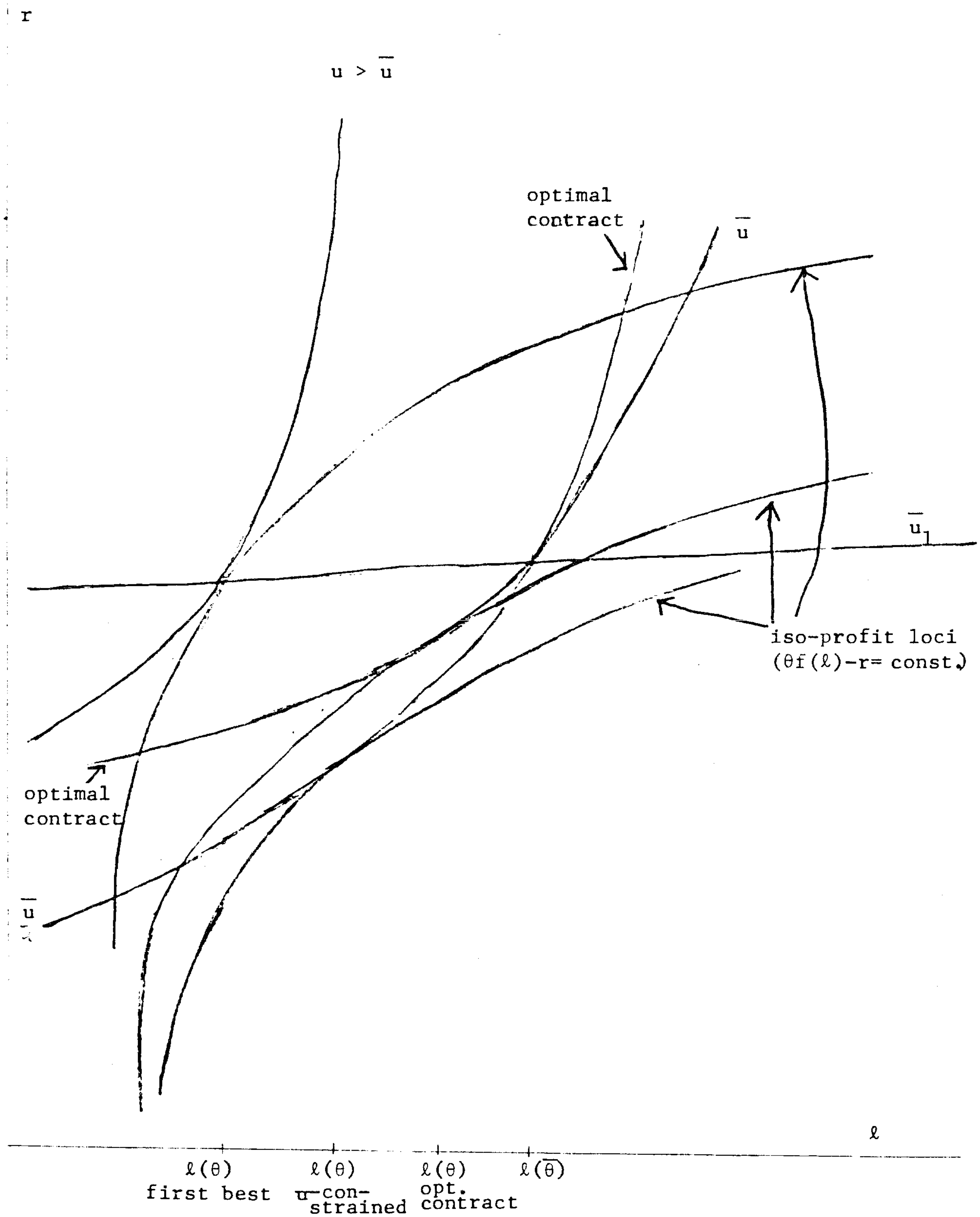
Substituting (3.19) and (3.25) into (3.26) we have

$$\frac{d^2 r}{d\ell^2} = \theta g'' + \frac{g'}{\ell'} \tag{3.27}$$

for any incentive compatible contract. Since we know that $\ell(\bar{\theta})$ is the same (i.e. the efficient level) under the two contracts being compared, (3.27) is evaluated with g" and g' at the same value. Therefore, since ℓ' is lower under the second-best contract than under the $\bar{u}$ contract, (3.27) tells us that its curvature is more than that of $\bar{u}$.

It is also interesting to compare the extent of employment fluctuation in the second-best to what would be optimal in a world where θ was verifiable by both parties and hence where the incentive compatibility constraint is dropped. The employment contract is now given by the curve $\bar{u}_1$ in Figure 1, but the employment choice along this locus is given by the efficiency criterion (3.19) instead of the firm's optimality condition (3.6).

Figure 1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optimal contract
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$\bar{u}_1$
iso-profit loci
($\theta f(\ell)$-r= const.)
optimal contract
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opt. contract
$\ell(\bar{\theta})$

Differentiating the implicit relationship $\bar{u}_1 = u_1(r(\theta),\ell(\theta))$ and the efficiency condition (3.19) and eliminating r' we obtain

$$(3.28) \qquad \ell' = \frac{g'u_1}{\frac{u_{12}^2}{u_{11}} - u_{22} + \frac{g''}{g'} u_2}$$

Comparing (3.28) with the expression from the second-best Euler equation (3.21) we see that the first-best has a higher labor sensitivity than the second-best if and only if

$$-(\frac{u_2}{u_1})^2 u_{11} + 2u_{12} \frac{u_2}{u_1} > \frac{u_{12}^2}{u_{11}}$$

That this is always true can be verified by rewriting it, noting $u_{11} < 0$, as

$$0 < (u_{12} - u_{11} \frac{u_2}{u_1})^2$$

Thus we have shown that the sensitivity of employment in the second-best is below that of the first-best but above what would be attained if $\bar{u}$ were the contract.

Figure 1 shows the realized employment, utility and profit levels at a value of θ below $\bar{\theta}$, "bad times." The three contracts we have discussed are the optimal contract, the contract given parametrically by the indifference curve $\bar{u}$, and the first-best contract, given by $\bar{u}_1$. Note that $\bar{u}$ cuts $\bar{u}_1$ from above by virtue of our assumption that (3.24) holds. Under the first two contracts we have employment chosen by the firms, shown graphically as a tangency between an iso-profit locus and the contract locus. In the first-best, employment is given by the tangency between an iso-profit locus and an indifference curve.

As one can see, the three employment levels associated with these contracts are such that the first-best is lowest, then the utility-constrained contract and then the optimal contract. Note that, since the iso-profit loci are simply vertical shifts of each other, the employment choice in the optimal contract is above the level that would maximize profit for a given utility level.

In the optimum, paradoxically, workers prefer the outcome at θ to that at $\bar{\theta}$. Profit also decreases, as shown, but this is not theoretically necessary. Workers' ex post resistance to layoffs and the associated lower earnings even when they are covered by a labor contract, is prima facia evidence against the contract being governed under the informationally imperfect environment on which this paper is based.

4. Conclusion

Since its beginnings, the implicit contracts literature has had the explanation of unemployment and wage rigidity as its goal. The intention was to offer a structure under which wage rigidity is optimal, and in which unemployment follows as a result. To some extent these goals were achieved, but, I think it is safe to say, always by introducing some special features in the contracting process that were not obviously an essential part of the model. For example, a common device is a two-period structure in which the contract operates somewhat differently in the second period than in the first.

In this paper I have given what I believe to be the first results using the implicit contracts theme which does not rely on any of these structural conditions. Paradoxically, the interaction of differential risk aversion and incomplete information is precisely the opposite of the original intention. Long term relationships between employers and workers decrease employment variability, resulting in more employment that would be ex post efficient when profitability conditions are adverse. Thus, I believe, the implicit contracts theory may not yield the underpinnings for a theory of macroeconomic fluctuations.

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