NBER WORKING PAPER SERIES

DOES PURCHASING POWER PARITY WORK?

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Working Paper No. 607

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

December 1980

This paper is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

The logarithm of the purchasing power ratio (PPR) is shown for seven countries and three alternative price indices to follow a stationary and invertible process in the first differences. This means that permanent shifts in the parity value accumulate over time. Therefore, as the prediction interval lengthens, the variance of the level of the PPR goes towards infinity while the variance of its average growth rate goes to zero. Since the variance of the permanent shifts is substantial: (1) Harmonized money growth cannot maintain constant exchange rates; reserve flows feedback is required. (2) Economic explanations of the permanent shifts are an important research topic.

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D.4: December 15, 1980

DOES PURCHASING POWER PARITY WORK?

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When an empirical question remains controversial for decades, there is likely a root conceptual problem. This paper argues that the controversy over purchasing power parity (PPP) indeed arises from murky concepts rather than differences in data. A framework is proposed to identify those problems for which PPP "works" and those for which it does not. It is shown that in a stochastic framework, growth rates may converge to the PPP relationships even though the levels of the variables become unpredictable. This occurs because uncorrelated "permanent" shifts in PPP cumulate for the levels but average out for the growth rates. Alternative predictors of the price level and exchange rate are considered and implications for monetary policy under pegged exchange rates are drawn.

I. Concepts of Purchasing Power Parity

People who ask whether purchasing power parity works normally examine either of two distinct concepts:[1] the level concept and the growth concept. The level concept refers to the ability to predict the price level conditional upon the exchange rate and foreign price level or else the exchange rate conditional upon the two price levels. The growth concept refers to predictions of the inflation rate given the growth rates of the exchange rate and the foreign price level or to predictions of the growth rate of the exchange rate given the two inflation rates. It is generally unappreciated that the properties of the prediction errors of these two concepts may differ sharply.

The difference between the two concepts may be seen by reference to a simple discrete stochastic model. Define the purchasing power ratio (PPR) as the ratio of the domestic price level to the product of the exchange rate and the foreign price level, or, measuring the variables in logarithms,

$$\Psi_t \equiv P_t - E_t - P_t^* \tag{1}$$

Suppose further that the stochastic process governing the log PPR is a martingale so that

$$\Psi_t = \Psi_{t-1} + \varepsilon_t \tag{2}$$

where ε_t is white noise with mean 0 and variance σ^2. For the moment the random-walk process (2) is only for illustrative purposes although previous estimates by Frenkel (1980a, 1980b), Roll (1979), and Stockman (1978b) suggest it is not too far from the truth.

In this case the best prediction of Ψ_{t+n} given the information available at time t is simply Ψ_t. The prediction error is

(3) $$\Psi_{t+n} - \Psi_t = \sum_{i=1}^{n} \varepsilon_{t+i}$$

This has mean zero and variance $n\sigma^2$. Thus the variance of the prediction error increases with the forecast period and goes to infinity for long-run predictions $(n\to\infty)$.

But suppose we were interested in the growth concept of PPP. The relevant identity is now

(4) $$\Gamma_n \Psi_t \equiv \Gamma_n P_t - \Gamma_n E_t - \Gamma_n P_t$$

where the growth rate operator Γ_n computes the average growth rate over n periods $(\Gamma_n X_t \equiv (X_t - X_{t-n})/n)$. The average growth rate of Ψ_t from t to t + n is

(5) $$\Gamma_n \Psi_{t+n} = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_{t+i}$$

By inspection, the optimal prediction and the mean of the prediction error (5) are zero. The variance of the prediction error is σ^2/n which decreases with the forecast period and goes to zero for long-run predictions $(n\to\infty)$. Note that only for one-period predictions are the error variances the same for the level and growth concepts of PPP.

It seems paradoxical that the longer the period over which we are predicting, the less accurate are our predictions of the price level and the more accurate are our predictions of the average inflation rate, both conditional upon the behavior of the exchange-rate-converted foreign price level. This occurs because errors shift the price level in a permanent and cumulative fashion in the level case, but these uncorrelated shifts average out in the growth case.

II. Need the Purchasing Power Parity Ratio Take a Random Walk?

Some authors -- most notably Roll (1979) -- have asserted that under conditions such that the interest arbitrage relation holds, the logarithm of the purchasing power ratio, Ψ_t, must follow a random walk. There is some evidence that those conditions do not obtain for national interest rates where risks of currency controls and default in the forward contract arise.[2] Nonetheless let us suppose a strong form of the efficient-market-interest-arbitrage approach holds and show that it is not strictly necessary for Ψ_t to take a random walk.

The interest relation for continuously compounded nominal interest rates i_t and i_t^* is

$$i_t = \xi_t(\Delta E_{t+1}) + i_t^* \tag{6}$$

where ξ_t denotes expected values conditional upon information at time t. Also assume a simple Fisher relationship holds.

$$i_t = r_t + \xi_t(\Delta P_{t+1}) \tag{7}$$

$$i_t^* = r_t^* + \xi_t(\Delta P_{t+1}^*) \tag{8}$$

where r_t and r^* are the respective real interest rates.[3] Substituting equations (7) and (8) in (6) and rearranging terms yields

$$\xi_t(\Delta P_{t+1}) - \xi_t(\Delta E_{t+1}) - \xi_t(\Delta P_{t+1}^*) = r_t^* - r_t \tag{9}$$

Note that the left hand side is simply $\xi_t(\Delta\Psi_{t+1})$, the expected change in the log PPR. This can differ from zero if the real interest rates differ. If shocks do occur which affect investment or saving in the countries and if it is costly to adjust the international allocation of capital instantaneously, then such shocks can cause temporary self-reversing movements in the log PPR which do not present any profit opportunities on either the financial or real side.

Thus, the form of the stochastic process governing the evolution of the purchasing power ratio is an empirical question. This process may be a random walk, but there are good theoretical reasons to expect a more complicated process. In particular a random walk with an overlaid self-reversing moving average process is suggested by the above analysis. Doubtless other, possibly stationary processes could be justified by relaxation of some of the assumptions. Let us turn to some estimated processes and see in what, if any, senses, PPP works.

III. Estimated Stochastic Processes for the Purchasing Power Ratio

This section reports estimates of the stochastic process governing the log PPR for the United Kingdom, Canada, France, Germany, Italy, Japan, and the Netherlands. These are the seven nonreserve countries which have been examined by the NBER Project on the International Transmission of Inflation and will adequately serve to illustrate the senses in which PPP does and does not work. We define Ψ_t, the logarithm of the ratio of the domestic price level to the exchange-rate converted foreign price level, in three alternative ways according to whether wholesale price indices, consumer price indices, or implicit price deflators for GNP (sometimes GDP) are used. The post-Bretton-Woods data (July 1971-December 1978) are taken from the International Monetary Fund as detailed in the Data Appendix.

In every case Ψ_t appeared to be nonstationary so that there is no fixed, long-run parity level of the purchasing power ratio. A similar finding was made by Stockman (1978) and is implicit in Frenkel's inability (1980a) to reject the hypothesis that the PPR follows a random walk.[4] The estimated ARIMA processes are reported in Table 1. All are of the form ARIMA(0,1,q) -- that is, moving average processes MA(q) applied to $\Delta\Psi_t$, the first difference in the log PPR:[5]

$$\Delta\Psi_t = \mu + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \qquad (10)$$

The drift term μ is admitted to allow for the possibility of trend movements in relative price levels. These may occur because of trends in movements of the relative prices of individual commodities weighted differently in the two national price levels.[6] Random shifts in relative prices are the most obvious explanation for permanent shifts in the log PPR so trends should be allowed also. If only trends were at work, then Ψ_t would be nonstationary but the ARIMA(0,1,q) processes would be noninvertible due to overdifferencing as discussed further below. The correct model to estimate would have the form

$$\Psi_t = \alpha + \mu t + u_t \tag{11}$$

We shall see that this deterministic but trended parity ($\alpha + \mu t$) does not appear to hold. Besides, with the exceptions of Japan, Netherlands, and perhaps Germany, the estimated values of μ in Table 1 are all insignificant.

Of the 40 processes estimated and reported in Table 1, only 5 are strict random walks ($q = 0$). The rest of the cases may be viewed as a random walk to which one or more moving-average terms has been added. As suggested by Muth (1960), for the $q = 1$ cases we can view $(1 - \theta_1)\varepsilon_t$ as the permanent shift in the parity value of the log PPR and $\theta_1\varepsilon_t$ as the transitory change in log PPR during the (one-period) adjustment process. If $0 < \theta_1 < 1$, then the initial change in Ψ_t is greater than the change in the parity value so that a partially self-reversing correction occurs the following period. If however $-1 < \theta_1 < 0$, then the initial change in the log PPR is less than the change in the parity value and two periods are taken for full adjustment. For $q > 1$, similar albeit more complicated adjustment patterns are indicated for the first $q + 1$ quarters until the permanent shift $(1 - \sum_{i=1}^{q} \theta_i)\varepsilon_t$ in the parity value is reflected in the log PPR.

If there were no permanent shifts in the parity value of Ψ_t, then $\sum_{i=1}^{q} \theta_i$ would be 1. Table 2 reports t-statistics of the form

$$t = \frac{\sum_{i=1}^{q} \theta_i - 1}{\text{s.e. of } \sum_{i=1}^{q} \theta_i} \tag{11}$$

Plosser and Schwert (1977) have pointed out that under the null hypothesis ($\Sigma\theta_i = 1$), the moving-average process is strictly noninvertible. In their examination of the case $q = 1$ and $\theta_1 = 1$, they showed that $\hat{\theta}_1$ was biased downward and their Monte Carlo experiments suggested that for sample sizes such as these a t greater than 3 or even 4 would be required to safely reject the null hypothesis at the 5 percent level of significance. Since this criterion is met in every case except for the Italian deflator definition, the hypothesis that there is a constant or deterministically trended parity value for the log PPR appears to be generally rejected. Note, however, that time-series tests for weak but persistent adjustment processes are prone to reject them; so it is perhaps appropriate to view the estimated processes as casting purchasing power parity in the worst light consistent with the data. It would be possible to impose a deterministically trended parity or force in more positive moving average terms, either of which would reduce estimated prediction errors over substantial lengths of time.

The basic hypothesis of Sections I and II -- that the log purchasing power ratio takes a random walk with perhaps a moving-average adjustment process added -- appears to be consistent with the data. On this hypothesis there is no parity value[7] towards which the log PPR tends in the long run. The further ahead we make predictions of Ψ_t, the greater is the variance. On the other hand, the longer the period over which we predict the average growth rate of Ψ_t, the smaller is the variance. This paradoxical result is illustrated in Table 3 which reports the standard errors of prediction implied for the models of Table 1 for periods ranging from one observation to six years in the future. For ease in interpretation and comparison, the average growth rates per period $\Gamma_n\Psi_t$ have all been converted to annual rates.

Thus the prediction error for the one-year-ahead level and the corresponding annualized growth rate are the same. Over shorter periods, the prediction error of the annualized growth rates is greater than that of the corresponding level. Over longer periods the standard errors of the growth rates fall toward zero while those of the levels rise toward infinity. As was illustrated for the random-walk case in Section I, this progression toward 0 and ∞, respectively, progresses roughly with $1/\sqrt{n}$ and $\sqrt{n}$, respectively.[8] Thus at 24 years the level standard errors will be about double and the growth standard errors about half of those indicated for six years.

To interpret Table 3, note that the growth rates are in decimal form so that the Canadian CPI-PPR six-year-average growth rate has a standard error of 1.06% per annum. On the other hand, the corresponding British standard error is 3.98% per annum, nearly four times greater. These standard errors pretty well bracket the range for growth rates, with the WPI definitions (with the exception of Britain and Canada) having smaller standard errors than the CPI definitions and the ranking of the deflator definitions mixed. The standard errors on the levels refer to the log PPR, so they indicate approximate proportionate errors. That is two thirds of the time after six years, the actual level of the CPI-definition Canadian purchasing power ratio will be within 6.6% ($e^{0.0635} - 1 = 0.0656$) of the predicted level.

In view of Table 3, can we say that purchasing power parity works? In an absolute sense, most of the average growth rate standard errors seem large even at six years, and it would take generation-long averages to halve these. The level standard errors are similarly large and growing. This would seem to suggest that purchasing power parity does not work. This answer appears too easy for a number of reasons: (1) Statistically, the standard errors in Table 3 may be biased upwards by omission of statistically insignificant but cumulatively important weak adjustment factors. Further, some observers would argue that the standard errors may be inflated by greater instability in Ψ_t in the early post-Bretton-Woods years than in recent years. (2) If one country is inflating much more rapidly than the other, the standard error will be a small fraction of the predicted change or average growth rate of the exchange rate. Only when relative price levels are fairly similar are the other factors important by comparison. (3) Finally, the other factors causing permanent movements in the log PPR are unpredictable (see Roll (1979)); so purchasing power parity provides the best -- although perhaps poor -- predictor of differences in real interest rates.[9]

The large standard errors of Table 3 certainly do indicate the importance of efforts aimed at explaining movements in the value of the purchasing power ratio relative to a deterministic purchasing power parity. It is not possible from these results, however, to infer whether these movements imply the absence of effective price arbitrage or merely period-to-period movements in the arbitrage parity values.

IV. Conclusions and Implications for Monetary Policy

The key implication of these results for monetary policy is that it is not possible to maintain a pegged exchange rate or achieve an exchange-rate growth goal by manipulating monetary growth according to relative price levels. The fact that there are permanent shifts in the parity value of the purchasing power ratio implies that a policy targeted on a deterministic parity will result in deviations from the actual parity which become arbitrarily large as time progresses. Therefore, to maintain a pegged rate system, even if short-run sterilization policies are effective, a current and/or lagged balance-of-payments feedback rule must be used so that the domestic price level will fluctuate relative to the foreign price level according to the movements in the parity value. Similarly an exchange-rate growth goal can be achieved only if a feedback rule based on either actual exchange-rate growth or exchange intervention is followed. If two countries both follow either constant inflation-rate or money-growth rules, the level of their exchange rate (although not its growth rate) becomes increasingly unpredictable the further into the future one considers.

Nelson and Plosser (1980) have recently presented evidence that a large number of macroeconomic series are stationary and invertible in the first differences and therefore do not follow deterministic models such as equation (11). For all of those series, we have a result analogous to that in this paper: The variance of the levels increases without limit and the variance of the average growth rate goes to zero as the prediction interval goes to infinity. Thus, for example, given the growth path of nominal money over a long period, we can only predict the future price level with great uncertainty although the average inflation rate is almost perfectly predictable.

Permanent shifts -- whether in the real demand for money or the purchasing power ratio -- imply an opportunity for economists to explain the shifts by underlying real factors.[10] But they do place important restrictions on the evolution of the factors causing the permanent changes. As to purchasing power parity, while it may be the best predictor, it is not a very good one. There is much room for economic explanations of the permanent shifts in the purchasing power ratio. Does purchasing power parity work? For what?

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DATA APPENDIX

All data series were taken from the International Financial Statistics tape of the International Monetary Fund, dated April 1979. The actual series used are:

EXCHANGE RATES

Country	IMF Code	Description
United Kingdom	112_AF	market rate/par or central rate (£/$US)
Canada	156_AF	market rate/par or central rate ($Can/$US)
France	132_AF	market rate/par or central rate (FF/$US)
Germany	134_AF	market rate/par or central rate (DM/$US)
Italy	136_AF	market rate/par or central rate (L/$US)
Japan	158_AF	market rate/par or central rate (¥/$US)
Netherlands	138_AF	market rate/par or central rate (Dfl/$US)

WHOLESALE PRICE INDICES

Country	IMF Code	Description
United States	111_63	wholesale prices, 1975=100
United Kingdom	112_63	wholesale prices, 1975=100
Canada	156_63	wholesale prices, 1975=100
France	132_63	wholesale prices, 1975=100
Germany	134_63	wholesale prices, 1975=100
Italy	136_63	wholesale prices, 1975=100
Japan	158_63	wholesale prices, 1975=100
Netherlands	138_63	wholesale prices, 1975=100

CONSUMER PRICE INDICES

Country	IMF Code	Description
United States	111_64	consumer prices, 1975 = 100
United Kingdom	112_64	consumer prices, 1975 = 100
Canada	156_64	consumer prices, 1975 = 100
France	132_64	consumer prices, 1975 = 100
Germany	134_64	consumer prices, 1975 = 100
Italy	136_64	consumer prices, 1975 = 100
Japan	158_64	consumer prices, 1975 = 100
Netherlands	138_64	consumer prices, 1975 = 100

IMPLICIT PRICE DEFLATORS

These series are the ratio of the nominal to real product series for each country as follows:

Country	IMF Code	Description
United States	111_99A	GNP (billions of $US)
	111_99A.R	GNP, 1975 prices (billions of $US), seasonally adjusted
United Kingdom*	112_99A	GNP (£)
	112_99B	GDP: 1975 prices (£)
Canada	156_99A	GNP ($Can)
	156_99A.R	GNP: 1975 prices ($Can), SA

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*For the United Kingdom, the IMF tape contains no nominal GDP or real GNP data. Judging from the ARIMA process reported in Table 1, the difference is close enough to proportionate to present no problem in this case.

Country	IMF Code	Description
Germany	134_99A	GNP (DM)
	134_99A.R	GNP: 1975 prices (DM), SA
Italy	136_99B	GDP (L)
	136_99B.R	GDP: 1975 prices (L), SA
Japan	158_99A	GNP (¥)
	158_99A.R	GNP: 1975 prices (¥), SA

TABLE 1

Estimated ARIMA Processes for ψ_t, the Logarithm of the Purchasing Power Ratio

All Fitted Processes Are ARIMA (0,1,q)

Statistics	Price Index Definition		
	W.P.I.	C.P.I.	Deflator
		United Kingdom	
q	1	1	0
μ	0.0020(0.0033)	0.0026(0.0977)	0.0067(0.0087)
MA parameters	θ_1 = -0.4009(0.0997)	θ_1 = -0.4689(0.0977)	-
$\hat{\sigma}$	0.0223	0.0192	0.0459
Q(12) [d.f.]	8.80 [11]	6.50 [11]	11.00 [12]
		Canada	
q	1	9	1
μ	0.0001(0.0014)	-0.0006(0.0008)	0.0002(0.0051)
MA parameters	θ_1 = -0.2075(0.1059) -	θ_1 = -0.1784(0.1001) θ_9 = 0.4086(0.1150)	θ_1 = -0.5187(0.1791) -
$\hat{\sigma}$	0.0107	0.0095	0.0186
Q(12) [d.f.]	7.90 [11]	11.70 [10]	13.0 [11]
		France	
q	6	6	NA
μ	0.0019(0.0019)	0.0043(0.0027)	
MA parameters	θ_2 = 0.1150(0.1071) θ_6 = 0.1694(0.1100) -	θ_1 = -0.2820(0.0894) θ_3 = -0.2249(0.1012) θ_6 = 0.3389(0.1074)	
$\hat{\sigma}$	0.0246	0.0221	
Q(12) [d.f.]	13.90 [10]	7.30 [9]	

	W.P.I.	C.P.I.	Deflator
		Germany	
q	6	6	0
μ	0.0032(0.0019)	0.0050(0.0025)	0.0163(0.0092)
MA parameters	θ_6 = 0.3757(0.1046)	θ_1 = -0.2706(0.1022)	-
	-	θ_6 = 0.3550(0.1132)	-
$\hat{\sigma}$	0.0275	0.0252	0.0495
Q(12) [d.f.]	9.40 [11]	2.90 [10]	14.9 [12]
		Italy	
q	4	7	4
μ	0.0018(0.0015)	0.0020(0.0027)	0.0040(0.0035)
MA parameters	θ_4 = 0.2718(0.1055)	θ_1 = -0.2725(0.1049)	θ_4 = 0.6427(0.2336)
		θ_7 = -0.2359(0.1093)	-
$\hat{\sigma}$	0.0190	0.0177	0.0349
Q(12) [d.f.]	6.40 [11]	10.70 [10]	11.6 [11]
		Japan	
q	0	0	0
μ	0.0050(0.0025)	0.0088(0.0026)	0.0253(0.0076)
MA parameters	-	-	-
$\hat{\sigma}$	0.0235	0.0248	0.0403
Q(12) [d.f.]	9.00 [12]	11.40 [12]	18.7 [12]
		Netherlands	
q	6	6	NA
μ	0.0033(0.0017)	0.0065(0.0025)	
MA parameters	θ_6 = 0.3770(0.1055)	θ_1 = -0.2880(0.1028)	
	-	θ_6 = 0.2834(0.1153)	
$\hat{\sigma}$	0.0254	0.0233	
Q(12) [d.f.]	9.40 [11]	6.00 [10]	

Notes:
1. Standard errors appear in parentheses
2. W.P.I. and C.P.I. data are monthly. Deflator data are quarterly.
3. All estimations are for the period July 1971-December 1978, except some of the deflators were not yet available for the whole period; for these cases (deflator only) the estimation ends as follows: United Kingdom and Japan in September 1978, Italy in December 1977.
4. The estimated standard error $\hat{\sigma}$ of the white noise process is the single-period standard error of forecast computed by the conditional method.
5. Q(12) is the Box-Ljung variant of the Box-Pierce statistic for the "portmanteau lack of fit test." It is approximately distributed as χ^2(d.f.) where the degrees of freedom are indicated in square brackets following the value of Q. To reject the model at the 5 percent significance level, Q must exceed 21.0 for 12 d.f., 19.7 for 11 d.f., 18.3 for 10 d.f., and 16.9 for 9 d.f.; thus all the models pass this overall lack of fit test.
6. Estimation (backcasting method) was performed using BMDQ2T (a preliminary version of BMDP2T) on the UCLA computer.

TABLE 2

Test t-Statistics for Stationarity of $\Psi_t - \mu t$

Country	t-Statistics		
	WPI	CPI	Deflator
United Kingdom	-14.05	-15.03	ǂ -4.77
Canada	-11.40	-5.03	-8.48
France	-5.02	-5.74	NA
Germany	-5.97	-5.53	ǂ -6.19
Italy	-6.90	-4.43	-1.53
Japan	ǂ -10.60	ǂ -11.18	ǂ -6.47
Netherlands	-5.91	-5.95	NA

Notes: 1. The t-statistics (11) are for the null hypothesis $\sum_i \theta_i = 1$ which would imply that $\Psi_t - \mu t$ is stationary (there is a deterministic parity value of log PPR) and that the moving-average process is noninvertible. These t-ratios are biased downward under the null hypothesis, but judging from Monte Carlo experiments reported in Plosser and Schwert (1977), $t < -4$ should be sufficient to reject the null hypothesis at the 5 percent significance level or better.

2. In those cases in which an ARIMA(0,1,0) process (random walk) was reported as the optimal process in Table 1, an alternative model ARIMA(0,1,1) was fitted and the t-statistic for $\theta_1 = 1$ is reported prefixed with a ǂ.

TABLE 3

Standard Errors of Prediction for ψ_t and Annualized $\Gamma_n\psi_t$

Periods* n		W.P.I. Level ψ_t	W.P.I. Growth $12\Gamma_n\psi_t$	C.P.I. Level ψ_t	C.P.I. Growth $12\Gamma_n\psi_t$	Deflator Level ψ_t	Deflator Growth $4\Gamma_n\psi_t$
				United Kingdom			
1	[-]	0.0223	0.2676	0.0192	0.2304	-	-
3	[1]	0.0494	0.1976	0.0443	0.1772	0.0459	0.1836
12	[4]	0.1058	0.1058	0.0957	0.0957	0.0917	0.0917
24	[8]	0.1512	0.0756	0.1369	0.0685	0.1297	0.0649
48	[16]	0.2149	0.0537	0.1947	0.0487	0.1834	0.0459
72	[24]	0.2636	0.0439	0.2389	0.0398	0.2247	0.0375
				Canada			
1	[-]	0.0107	0.1284	0.0095	0.1140	-	-
3	[1]	0.0212	0.0848	0.0185	0.0740	0.0186	0.0744
12	[4]	0.0443	0.0443	0.0352	0.0352	0.0547	0.0547
24	[8]	0.0630	0.0315	0.0424	0.0212	0.0808	0.0404
48	[16]	0.0894	0.0224	0.0540	0.0135	0.1166	0.0292
72	[24]	0.1096	0.0183	0.0635	0.0106	0.1437	0.0240
				France			
1	[-]	0.0246	0.2952	0.0221	0.2652	NA	NA
3	[1]	0.0411	0.1644	0.0464	0.1856		
12	[4]	0.0701	0.0701	0.0989	0.0989		
24	[8]	0.0923	0.0462	0.1339	0.0670		
48	[16]	0.1254	0.0314	0.1851	0.0463		
72	[24]	0.1515	0.0253	0.2249	0.0375		

Periods* n	W.P.I. Level ψ_t	W.P.I. Growth $12\Gamma_n\psi_t$	C.P.I. Level ψ_t	C.P.I. Growth $12\Gamma_n\psi_t$	Deflator Level ψ_t	Deflator Growth $4\Gamma_n\psi_t$
			Germany			
1 [-]	0.0275	0.3300	0.0252	0.3024	-	-
3 [1]	0.0477	0.1908	0.0518	0.2072	0.0495	0.1980
12 [4]	0.0792	0.0792	0.0942	0.0942	0.0991	0.0991
24 [8]	0.0985	0.0493	0.1230	0.0615	0.1401	0.0701
48 [16]	0.1287	0.0322	0.1662	0.0416	0.1982	0.0486
72 [24]	0.1531	0.0255	0.2003	0.0334	0.2427	0.0405
			Italy			
1 [-]	0.0190	0.2280	0.0177	0.2124	-	-
3 [1]	0.0329	0.1316	0.0360	0.1440	0.0349	0.1396
12 [4]	0.0543	0.0543	0.0820	0.0820	0.0698	0.0698
24 [8]	0.0722	0.0361	0.1225	0.0613	0.0711	0.0356
48 [16]	0.0987	0.0247	0.1778	0.0445	0.0735	0.0184
72 [24]	0.1195	0.0199	0.2195	0.0366	0.0758	0.0126
			Japan			
1 [-]	0.0235	0.2820	0.0248	0.2976	-	-
3 [1]	0.0408	0.1632	0.0430	0.1720	0.0403	0.1612
12 [4]	0.0816	0.0816	0.0860	0.0860	0.0806	0.0806
24 [8]	0.1154	0.0577	0.1216	0.0608	0.1140	0.0570
48 [16]	0.1632	0.0408	0.1719	0.0430	0.1613	0.0403
72 [24]	0.1998	0.0333	0.2105	0.0351	0.1975	0.0329

Periods* n	W.P.I. Level ψ_t	W.P.I. Growth $12\Gamma_n\psi_t$	C.P.I. Level ψ_t	C.P.I. Growth $12\Gamma_n\psi_t$	Deflator Level ψ_t	Deflator Growth $4\Gamma_n\phi_t$
			Netherlands			
1 [-]	0.0254	0.3048	0.0233	0.2796	NA	NA
3 [1]	0.0439	0.1756	0.0484	0.1936		
12 [4]	0.0726	0.0726	0.0912	0.0912		
24 [8]	0.0900	0.0450	0.1219	0.0610		
48 [16]	0.1172	0.0293	0.1672	0.0418		
72 [24]	0.1392	0.0232	0.2026	0.0338		

*The number of periods for the deflator is in square brackets.

Note: Growth rates are reported on an annualized basis.

FOOTNOTES

*The author acknowledges comments from Deborah Allen, Dan Friedman, Michael Melvin, and Charles Plosser, and the generous support of the National Science Foundation (grants APR78-13072 and DAR-7922874), the Scaife Family Trusts, and the Alex C. Walker Foundation. Michael T. Melvin and Andrew A. Vogel provided able research assistance. Pigott and Sweeney in an unpublished paper (1980) have independently arrived at similar conclusions by a rather more complicated and (I believe) problematic route. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

[1] I treat the level -- as well as growth rate -- concept in relative purchasing power rather than absolute terms. Some writers require that all prices be the same across countries, but this absolute purchasing power parity is generally conceded to fail without necessarily reducing the usefulness of the parity idea. A third concept of purchasing power parity is sometimes assumed in theoretical modeling -- that a parity value exists for any moment (although it might change unpredictably in the next period) so that $\frac{dP}{dX} = \frac{dE}{dX} + \frac{dP^*}{dX}$ holds. Most of the literature (and this paper) is concerned with the unconditional <u>predictive</u> power of PPP, so this concept is not considered further here. No attempt is made here to review the huge literature on purchasing power parity, but see the May 1978 issue of the *Journal of International Economics* for a number of recent perspectives.

[2] Interest arbitrage clearly does apply in the Eurocurrency markets where there is no differential control and default risk, but Eurocurrency

rates do vary relative to the respective domestic interest rates. As discussed in Darby (1980), since central banks seem to exercise monetary control under pegged exchange rates, we may infer that different national assets are not perfect substitutes. This latter finding implies, of course, that interest arbitrage won't hold with respect to the various national interest rates. See Dooley and Isard (1980) for an excellent treatment of these issues.

[3]This abstracts from the effects of income taxes on nominal interest rates discussed in Darby (1975). To consider taxes, we would also have to consider differential taxation of interest and exchange gains which would greatly complicate the analysis.

[4]Frenkel preferred the model AR(1) with an autoregressive parameter in the neighborhood of 0.9 in which case Ψ_t is (barely) stationary and slowly tends toward a long-run value. If this is so, ARIMA(p,1,q) models will be overdifferenced; tests for this are reported below, but it is very difficult to distinguish a very slow auto-regressive adjustment to a long-run parity value from a random walk for Ψ_t.

[5]At most q moving average parameters were fitted, since some series appeared to be MA(0) or MA(1) plus a seasonal component (usually quarterly or semiannual) which was also fitted.

[6]See Stockman (1978a, 1980). Balassa (1964) and Samuelson (1964) provide different explanations for a trend.

[7]This applies to both a constant or a deterministic time trend which could be used for making predictions.

[8] For non-random-walk cases, the early prediction errors are reduced by knowledge of past shocks, but this knowledge soon becomes unimportant. Nonetheless, the square-root rule is only approximate for these cases.

[9] See Section II.

[10] For example, shifts in the real demand for money may be explained by real income taking a random walk and by random occurrence of institutional innovations. Or, permanent changes in purchasing power parity may reflect changes in commercial policy and changes in the relative prices of goods with differing weights in the price indices.