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REAL EFFECTS OF ANTICIPATED AND UNANTICIPATED MONEY: SOME PROBLEMS OF ESTIMATION AND HYPOTHESIS TESTING

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Real Effects of Anticipated and Unanticipated Money: Some Problems of Estimation and Hypothesis Testing

ABSTRACT

The paper addresses two issues that arise in estimation of testing of the real effects of anticipated and unanticipated money. First it is shown that identification of the effects of unanticipated (or unperceived) monetary growth on real output is possible only if the a priori restriction is imposed that monetary growth does not depend on unanticipated (or unperceived) output. Second, it is shown that anticipated money can enter "semi-reduced form" output equations of the kind estimated by Barro, through three additional channels not allowed for in existing empirical work. These are 1) past and present anticipations of future monetary growth (the inflation tax channel), 2) expectations of monetary growth in a given period conditioned at various preceding dates (the Fischer-Phelps-Taylor effect) and 3) past and present revisions in forecasts of monetary growth (the Turnovsky-Weiss effect). The presence of the first of these would mean that alternative open-loop monetary growth rules have real effects. The presence of the other two implies that monetary feedback rules can have real effects.

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Real Effects of Anticipated and Unanticipated Money; Some Problems of Estimation and Hypothesis Testing

1) Introduction.

This paper addresses two issues that arise in the estimation of models of the real effects of anticipated and unanticipated changes in the money supply and in tests of hypotheses in such models. The first issue concerns an identification problem. The observational equivalence is established of models in which a real variable (say, output) is a function of unanticipated money growth and models in which money growth is a function of unanticipated output, e.g. via a policy reaction function or through a response of the private banking system. Identification of the effects of current unanticipated (or unperceived) monetary growth on real output is possible only if the *a priori* restriction is imposed that monetary growth does not depend on current unanticipated (or unperceived) output. The restriction that there be no effects of unanticipated output on money growth is quite distinct from the restriction(s) required for the identification of the effects of anticipated money on output (see e.g. Barro [1978, 1979]).

The second issue remains even if reliable time series on anticipated and unanticipated monetary growth are somehow available. It concerns the ways in which anticipated and unanticipated monetary growth can enter a "semi-reduced form" 1/ equation for output or any other real variable. Barro's specification of the output equation includes only current and lagged actual monetary growth and a distributed lag on unperceived contemporaneous monetary growth.2/ It is argued that monetary and macro economic theory suggest at

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1/ Semi-reduced form because endogenous expectations still appear on the r.h.s.

2/ Or, in empirical applications, a distributed lag on one period ahead forecast errors for monetary growth.

least three further channels through which money affects real output. (1) Past and present anticipated future monetary growth. (2) Past and present revisions in forecasts of monetary growth. (3) A more general specification of unanticipated money, with expectations of monetary growth in a given period conditioned at various preceding dates -- a Fischer [1977]-type hypothesis.

If anticipated future monetary growth affects real output, alternative fixed (open-loop) monetary growth paths will be associated with different paths for real output. If one of the other two channels is operative, alternative flexible (closed-loop) money supply rules or feedback rules will be associated with different distribution functions for real output.

The paper is motivated by Barro's seminal empirical work on the role of unanticipated money growth. (Barro [1978, 1979], Barro and Rush [1979]; see also Gordon [1979], Attfield, Demery and Duck [1979 a, b]). The issue of the observational equivalence of natural and unnatural (*sic*) rate theories of macroeconomics was first addressed by Sargent [1979]. McCallum [1979] showed that there were circumstances in which Sargent's general observational equivalence proposition did not rule out the construction of tests capable of discriminating between classical and Keynesian hypotheses. Barro [1979] provides a penetrating discussion of the observational equivalence problem and of the kinds of *a priori* information (generally cross-equation over-identifying restrictions on the reduced form parameters) required for testing classical against Keynesian hypotheses. The innovation in the first part of this paper is the incorporation of unanticipated output (or other "real" innovations) as an argument in the monetary growth equation.

The second part of the paper argues that a variety of not implausible structural models generate "semi-reduced form" equations for output in which anticipated and unanticipated money enter in a number of ways not allowed for in Barro's original contributions and in other empirical work on anticipated and unanticipated money.

2) The identification of the effects of anticipated and unanticipated monetary growth on output.

a) Monetary growth independent of anticipated and unanticipated output.

A simplified version of Barro's model of the effect of anticipated and unanticipated money on output is given in equations (1), (2) and (3).

$$(1)\quad Y_t = a_1 X_t + b_1(\dot{m}_t - \hat{\dot{m}}_{t|t}) + c_1 \dot{m}_t + u_{1t}$$

$$(2)\quad \dot{m}_t = a_2 X_t + u_{2t}$$

$$(3)\quad \Sigma = E((u_{1t}, u_{2t})' (u_{1t}, u_{2t})) = \begin{bmatrix} \sigma_{u_1}^2 & 0 \\ 0 & \sigma_{u_2}^2 \end{bmatrix}$$

Y is real output, X a vector of regressors which can include lagged values of the endogenous variables and $\dot{m}$ the actual rate of growth of money. $\hat{\dot{m}}_{t|\tau}$ is the rate of growth of money in period t, anticipated in period τ. $\hat{\dot{m}}_t - \hat{\dot{m}}_{t|t}$ is therefore the currently unperceived part of current period monetary growth. u_1 and u_2 are serially uncorrelated normally distributed random variables with mean zero and a constant variance. These random disturbances are also assumed to be contemporaneously independent. 3/ Equation (1) is the output equation; (2) describes the money supply process. For simplicity only the current innovation in the money supply process and

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3/ $\mathrm{Cov}(u_t, X_{t-s}) = 0$ for all t and all $s \geq 0$, where $u_t' = (u_{1t}', u_{2t}')$. X_t is assumed to be known when expectations of $\dot{m}_t$ are formed in period t but Y_t, $\dot{m}_t$, u_{1t} and u_{2t} are unobserved until period $t + 1$.

current actual monetary growth are assumed to be arguments in the output equation.[4/]

The assumption of rational expectations means that equations (1), (2) and (3) are known to private agents when they infer the growth of the money supply. u_{1t}, u_{2t}, Y_t and $\dot{m}_t$ are assumed unknown when the current expectation of money supply growth is formed. Thus, assuming that anticipations are mathematical expectations conditional on the available information set, I_t, which consists of the model and X_t we have

$$(4) \quad \hat{\dot{m}}_{t|t} = E(\dot{m}_t | I_t) = a_2 X_t$$

The reduced form of the model of (1), (2) and (3) is:

4/ Barro includes a distributed lag function on actual and unanticipated monetary growth in (1). i.e. (1) is replaced by:

$$(1') \quad Y_t = a_1 X_t + \sum_{i=0}^{T} b_{1,i} (\dot{m}_{t-i} - \hat{\dot{m}}_{t-i|t-i}) + \sum_{i=0}^{T} c_{1,i} \dot{m}_{t-i} + u_{1t}$$

It is true that even in a model in which only unanticipated money has real effects, these effects can be distributed over time, e.g. because the monetary surprises are "built into" changes in the capital stock or in inventories. If such were the case and if neither the lagged capital stock nor lagged inventories are included as arguments in the reduced form equation for output, this equation should include an infinite distributed lag on past monetary innovations, not a finite order lag as in Barro.

(5) $Y_t = \beta_1 X_t + v_{1t}$

(6) $\dot{m}_t = \beta_2 X_t + v_{2t}$

(7) $\beta_1 = a_1 + c_1 a_2$

(8) $\beta_2 = a_2$

(9a) $v_{1t} = (b_1+c_1)u_{2t} + u_{1t}$

(9b) $v_{2t} = u_{2t}$

(9c) $\Omega_1 = E((v_{1t},v_{2t})'(v_{1t},v_{2t})) = \begin{pmatrix} \sigma_{v_1}^2 & \sigma_{v_1v_2} \\ \sigma_{v_1v_2} & \sigma_{v_2}^2 \end{pmatrix}$

$$= \begin{pmatrix} (b_1 + c_1)^2\sigma_{u_2}^2 + \sigma_{u_1}^2 & (b_1+c_1)\sigma_{u_2}^2 \\ (b_1 + c_1)\sigma_{u_2}^2 & \sigma_{u_2}^2 \end{pmatrix}$$

The effect of anticipated money on output, $\frac{\partial y_t}{\partial \dot{m}_t} + \frac{\partial y_t}{\partial \hat{\dot{m}}_{t|t}}$, is given by c_1,

the effect of anticipated money on output, $\left.\frac{\partial y_t}{\partial \dot{m}_t}\right|_{\hat{\dot{m}}_{t|t}}$, by $b_1 + c_1$.

The effect of unanticipated money can be obtained from Ω_1, the variance-covariance matrix of the reduced form disturbances: $b_1 + c_1 = \frac{\sigma_{v_1v_2}}{\sigma_{v_2}^2}$. 5/

5/ Consistent and asymptotically efficient estimates of β_1, β_2 and $b_1 + c_1$ can be obtained by estimating (5) and (6) with an unrestricted variance-covariance matrix of the reduced form disturbances.

Note that this requires independence of the structural disturbances u_{1t} and u_{2t}. To identify the effect of anticipated money, however, further *a priori* restrictions are required. Since we can obtain consistent and asymptotically efficient estimates of $\beta_1 = a_1 + c_1 a_2$ and $\beta_2 = a_2$, an exclusion restriction permitting the identification of c_1 is that $a_1 = 0$. Barro's exclusion restriction that government purchases do not affect real output falls in this category (Barro [1979]).[6/]

b) <u>Monetary growth dependent on anticipated and unanticipated output</u>.

The structure of equations (1), (2) and (3) is observationally equivalent to the model of equations (10), (11) and (3).

(10) $Y_t = a_1 X_t + u_{1t}$

(11) $\dot{m}_t = a_2 X_t + b_2(Y_t - \hat{Y}_{t|t}) \; c_2 Y_t + u_{2t}$

Neither anticipated nor unanticipated money affect output but monetary growth responds both to anticipated and unanticipated output. Such a money supply response could reflect either the behaviour of the authorities through a monetary policy reaction function or the response of the private

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6/ It is of course not necessary that the entire vector a_1 equal zero. Let X_t be an N-component vector $X_t = [X_{1t}, \ldots, X_{it}, \ldots X_{Nt}]'$ and let $a_1 = [a_{11}, \ldots, a_{1i}, \ldots, a_{1N}]$ and $a_2 = [a_{21}, \ldots, a_{2i}, \ldots, a_{2N}]$. Given estimates of $\beta_1 = [a_{11} + c_1 a_{21}, \ldots, a_{1i} + c_1 a_{2i}, \ldots, a_{1N} + c_1 a_{2N}]$ and $\beta_2 = [a_{21}, \ldots, a_{2i}, \ldots, a_{2N}]$, c_1 is identified provided $a_{ij} = 0$ for some $j = 1, \ldots, N$.

banking system to changes in the demand for money due to anticipated and unanticipated changes in income and associated changes in interest rates. A positive value of $b_2 + c_2$ or of c_2 can be interpreted as "leaning with the wind" by the monetary authorities: with an unchanged monetary policy stance an (unanticipated) increase in output would tend to raise interest rates. The money supply expands to counteract this. Negative values of $b_2 + c_2$ or of c_2 could indicate a policy of "leaning with the wind". Note that $\hat{Y}_{t|t} \equiv E(Y_t | I_t) = a_1 X_t$. The reduced form of (10), (11) and (3) is:

(12) $Y_t = \alpha_1 X_t + \eta_{1t}$

(13) $\dot{m}_t = \alpha_2 X_t + \eta_{2t}$

(14a) $\alpha_1 = a_1$

(14b) $\alpha_2 = a_2 + c_2 a_1$

(14c) $\eta_{1t} = u_{1t}$

(14d) $\eta_{2t} = (b_2+c_2)u_{1t} + u_{2t}$

(14e) $$\Omega_2 = E\left((\eta_{1t}, \eta_{2t})' (\eta_{1t}, \eta_{2t})\right) = \begin{bmatrix} \sigma_{\eta_1}^2 & \sigma_{\eta_1\eta_2} \\ \sigma_{\eta_1\eta_2} & \sigma_{\eta_2}^2 \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{u_1}^2 & (b_2+c_2)\sigma_{u_1}^2 \\ (b_2+c_2)\sigma_{u_1}^2 & (b_2+c_2)^2\sigma_{u_1}^2 + \sigma_{u_2}^2 \end{bmatrix}$$

The model of (1), (2) and (3) is observationally equivalent to the model of (10), (11) and (3): their reduced forms cannot be distinguished (compare (5-9) with (12-14)) and will yield identical likelihood functions for the endogenous variables.

For the model of (10), (11) and (13) the effect of unanticipated output on money growth is identified *via* Ω_2 as $b_2 + c_2 = \sigma_{\eta_1\eta_2} / \sigma_{\eta_1}^2$.

The effect of anticipated output on money can then be identified from α_1 and α_2 if e.g. an exclusion restriction is imposed on one of the elements of a_2. If the true model is given by (10) and (11) but the econometrician mistakenly believes the true model to be (1) and (2), what is thought to be an estimate of $b_1 + c_1$, using the estimated variance-covariance matrix of the reduced form $\hat{\sigma}_{\eta_1\eta_2} / \hat{\sigma}_{\eta_2}^2$, is neither that nor an estimate of $b_2 + c_2$, which would be given by $\hat{b}_2 = \hat{\sigma}_{\eta_1\eta_2} / \hat{\sigma}_{\eta_1}^2$.

c) Observational equivalence in a more general model.

Consider the general model that permits, in principle, dependence of output on both anticipated and unanticipated money and dependence of monetary growth on both anticipated and unanticipated output.

(1) $Y_t = a_1 X_t + b_1(\dot{m}_t - \hat{\dot{m}}_{t|t}) + c_1\dot{m}_t + u_{1t}$

(11 $\dot{m}_t = a_2 X_t + b_2(Y_t - \hat{Y}_{t|t}) + c_2 Y_t + u_{2t}$

The disturbance terms are again as in (3).

With rational expectations we have

$$(15a) \quad \hat{Y}_{t|t} = \frac{a_1 + c_1 a_2}{1 - c_1 c_2} X_t$$

$$(15b) \quad \hat{\dot{m}}_{t|t} = \frac{a_2 + c_2 a_1}{1 - c_1 c_2} X_t$$

The reduced form of this model is given by

$$(16) \quad Y_t = \delta_1 X_t + \Psi_{1t}$$

$$(17) \quad \dot{m}_t = \delta_2 X_t + \Psi_{2t}$$

$$(18a) \quad \delta_1 = \frac{a_1 + c_1 a_2}{1 - c_1 c_2}$$

$$(18b) \quad \delta_2 = \frac{a_2 + c_2 a_1}{1 - c_1 c_2}$$

$$(18c) \quad \Psi_{1t} = [1-(b_1+c_1)(b_2+c_2)]^{-1}[u_{1t}+(b_1+c_1)u_{2t}]$$

$$(18d) \quad \Psi_{2t} = [1-(b_1+c_1)(b_2+c_2)]^{-1}[(b_2+c_2)u_{1t}+u_{2t}]$$

$$(18e) \quad \Omega_3 = E((\Psi_{1t},\Psi_{2t})'(\Psi_{1t},\Psi_{2t})) = \begin{pmatrix} \sigma_{\Psi_1}^2 & \sigma_{\Psi_1\Psi_2} \\ \sigma_{\Psi_1\Psi_2} & \sigma_{\Psi_2}^2 \end{pmatrix}$$

$$= [1-(b_1+c_1)(b_2+c_2)]^{-2} \begin{pmatrix} \sigma_{u_1}^2 + (b_1+c_1)^2\sigma_{u_2}^2 & (b_2+c_2)\sigma_{u_1}^2 + (b_1+c_1)\sigma_{u_2}^2 \\ (b_2+c_2)\sigma_{u_1}^2 + (b_1+c_1)\sigma_{u_2}^2 & (b_2+c_2)^2\sigma_{u_1}^2 + \sigma_{u_2}^2 \end{pmatrix}$$

The general structural model of equations (1), (11) and (3) in which both unanticipated and anticipated money affect real output ($b_1 + c_1 \neq 0$ and $c_1 \neq 0$ respectively) is observationally equivalent to a large set of sub-models with widely differing implications for the conduct of monetary policy. While these sub-models are special cases of the general structural model they are not nested either in each other or in the general model. The most interesting cases are considered briefly.

I Only anticipated money affects real output ($c_1 \neq 0$ and $b_1 + c_1 = 0$)

$$(19a) \quad \delta_1' = \frac{a_1 + c_1a_2}{1 - c_1c_2}$$

$$(19b) \quad \delta_2' = \frac{a_2 + c_2a_1}{1 - c_1c_2}$$

$$(19c) \quad \Omega_3' = \begin{pmatrix} \sigma_{u_1}^2 & (b_2+c_2)\sigma_{u_1}^2 \\ (b_2+c_2)\sigma_{u_1}^2 & (b_2+c_2)^2\sigma_{u_1}^2 + \sigma_{u_2}^2 \end{pmatrix}$$

II Anticipated money has the same effect and output as unanticipated money ($c_1 = b_1 + c_1$ or $b_1 = 0$)

(20a) $\delta_1'' = \dfrac{a_1 + c_1 a_2}{1 - c_1 c_2}$

(20b) $\delta_2'' = \dfrac{a_2 + c_2 a_1}{1 - c_1 c_2}$

(20c) $$\Omega_3'' = [1 - c_1(b_2+c_2)]^{-2} \begin{bmatrix} \sigma_{u_1}^2 + c_1^2 \sigma_{u_2}^2 & (b_2+c_2)\sigma_{u_1}^2 + c_1 \sigma_{u_2}^2 \\ (b_2+c_2)\sigma_{u_1}^2 + c_1 \sigma_{u_2}^2 & (b_2+c_2)^2 \sigma_{u_1}^2 + \sigma_{u_2}^2 \end{bmatrix}$$

III Only unanticipated money affects real output ($c_1 = 0$ and $b_1 + c_1 \neq 0$)

(21a) $\delta_1''' = a_1$

(21b) $\delta_2''' = a_2 + c_2 a_1$

(21c) $$\Omega_3''' = [1-b_1(b_2+c_2)]^{-2} \begin{pmatrix} \sigma_{u_1}^2 + b_1^2 \sigma_{u_2}^2 & (b_2+c_2)\sigma_{u_1}^2 + b_1\sigma_{u_2}^2 \\ (b_2+c_2)\sigma_{u_1}^2 + b_1\sigma_{u_2}^2 & (b_2+c_2)^2\sigma_{u_1}^2 + \sigma_{u_2}^2 \end{pmatrix}$$

IV Neither anticipated nor unanticipated money affects real output

$(b_1 = c_1 = 0)$

(22a) $\delta_1''''' = a_1$

(22b) $\delta_2''''' = a_2 + c_2 a_1$

(22c) $\Omega_3''''' = [1-(b_2+c_2)]^{-2} \begin{bmatrix} \sigma_{u_1}^2 & (b_2+c_2)\sigma_{u_1}^2 \\ (b_2+c_2)\sigma_{u_1}^2 & (b_2+c_2)^2\sigma_{u_1}^2 + \sigma_{u_2}^2 \end{bmatrix}$

For simplicity and without loss of generality for the observational equivalence propositions, we assume in what follows that X_t (and therefore a_1 and a_2) is a scalar. If both anticipated and unanticipated output can affect money, a model in which both anticipated and unanticipated money can have (possibly distinct) effects on output (with reduced form (18a, b, e)) is observationally equivalent to a model in which only anticipated money affects output (with reduced form (19a, b, c)) and to a model in which anticipated and unanticipated money have the same effect on output (with reduced form (20a, b, c)).
A model in which only anticipated money affects output (with reduced form (21a, b, c)) is observationally equivalent to a model in which neither anticipated nor unanticipated money have real effects (with reduced form (22a, b, c).

A priori knowledge of the value of a_1 is in some cases sufficient to discriminate between "Both anticipated and unanticipated money matter" or "Only anticipated money matters" on the one hand and "Only unanticipated money matters" or "Neither anticipated nor unanticipated money matter" on the other hand. (Compare (18a), (19a) or (20a) with (21a) or (22a)).

and (20a) can be correct only if either $a_1 = \frac{-a_2}{c_2}$ or $c_1 = 0$. If $c_1 = 0$, anticipated money does not affect real output. If $a_1 = \frac{-a_2}{c_2}$, $\delta_2 = 0$, a hypothesis that can be tested. Thus if δ_1 is not significantly different from a_1 and δ_2 is significantly different from zero, $c_1 = 0$ is accepted. Even if this hypothesis is accepted, we cannot further discriminate between the hypothesis that only unanticipated money affects real output and the hypothesis that neither anticipated nor unanticipated money affect real output unless we impose the *a priori* constraint that unanticipated output does not affect monetary growth $(b_2 + c_2 = 0)$. Given that restriction (22c) will be a diagonal matrix while (21c) is only a diagonal matrix if $b_1 = 0$.

Thus, starting from the general model of equations (1), (11) and (3) we can identify both the effect of anticipated money on output (c_1) and the effect of anticipated output on money (c_2) if we have two independent restrictions on the reduced form coefficient δ_1 and δ_2. To identify just c_1, the exclusion restriction $a_1 = 0$ is sufficient. This is of course the standard identification problem, familiar from partial equilibrium demand and supply analysis, when one does not encounter the additional problem of innovations from one process entering as arguments into another process.[7/] If innovations in either process can enter as an argument in the other, the identification problem is compounded. Evidence other than the time series properties of $\dot{m}_t$, Y_t and X_t is required to establish the validity of ruling out an effect of unanticipated output on monetary growth.

d) Lagged money and output innovations

In empirical applications the output equation given in (1) has been modified to include current and lagged anticipated and unanticipated money as arguments. The rationale for this is inertia in the real output process due to costs of adjustment (capital stock, inventories, quasi-fixed labour etc.), or to lags in the perception of new information (see e.g. Lucas [1975]).[8/] Note that in the presence of such real costs of adjustment the

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7/ The demand and supply analogue is given by:

$$Y_t = a_1 X_t + c_1 \dot{m}_t + u_{1t}$$

$$\dot{m}_t = a_2 X_t + c_2 Y_t + u_{2t}$$

This has the reduced from

$$Y_t = \left(\frac{a_1+a_2 c_1}{1-c_1 c_2}\right) X_t + (1-c_1 c_2)^{-1}(u_{1t}+c_1 u_{2t})$$

$$\dot{m}_t = \left(\frac{a_2+a_1 c_1}{1-c_1 c_2}\right) X_t + (1-c_1 c_2)^{-1}(c_1 u_{1t}+u_{2t})$$

$$\Omega_4 = (1-c_1 c_2)^{-2} \begin{bmatrix} \sigma_{u_1}^2 + c_1^2 \sigma_{u_2}^2 & c_2 \sigma_{u_1}^2 + c_1 \sigma_{u_2}^2 \\ c_2 \sigma_{u_1}^2 + c_1 \sigma_{u_2}^2 & c_2^2 \sigma_{u_1}^2 + \sigma_{u_2}^2 \end{bmatrix}$$

8/ But see footnote 3 for a brief discussion of a problem associated with this specification.

anticipated real rate of return on money balances, which is a function of the anticipated component of the money supply process, should affect the probability density function of real output. The empirical importance of this "inflation tax" argument would be reflected in the coefficients on current and lagged anticipated future monetary growth. (See Section 3 below). Monetary growth can also, in principle, be a function of current and lagged anticipated and unanticipated output. The monetary reaction function can incorporate <u>measurement</u> or <u>perception</u> lags (the time interval between the occurrence of an event and its observation) and <u>realisation</u> lags (the time needed to decide upon and realise a control action) (see Deissenberg [1979]). Equations (23) and (24) are a generalisation of (1) and (11), with output a function of a T-period distributed lag on anticipated and unanticipated money and money a function of a T-period distributed lag on anticipated and unanticipated output. It is assumed that the maximal lag is know *a priori*.

$$(23) \quad Y_t = a_1 X_t + \sum_{i=0}^{T} b_{1,i} (\dot{m}_{t-i} - \hat{\dot{m}}_{t|t-i}) + \sum_{i=0}^{T} c_{1,i} \dot{m}_{t-i} + u_{1t}$$

$$(24) \quad \dot{m}_t = a_2 X_t + \sum_{i=0}^{T} b_{2,i} (Y_{t-i} - \hat{Y}_{t|t-i}) + \sum_{i=0}^{T} c_{2,i} Y_{t-i} + u_{2t}$$

u_{1t} and u_{2t} are as before.

The presence of the distributed lag terms does alter the conditions for identification. This is most easily seen when only a single lag is included, as in (25) and (26).

(25) $Y_t = a_1 X_t + b_1 (\dot{m}_t - \hat{\dot{m}}_{t|t}) + c_1 \dot{m}_t + d_1 (\dot{m}_{t-1} - \hat{\dot{m}}_{t-1|t-1}) + e_1 \dot{m}_{t-1} + u_{1t}$

(26) $\dot{m}_t = a_2 X_t + b_2 (Y_t - \hat{Y}_{t|t} + c_2 Y_t + d_2 (Y_{t-1} - \hat{Y}_{t-1|t-1}) + e_2 Y_{t-1} + u_{2t}$

The reduced form of (25) and (26) is:

(27) $$\begin{pmatrix} Y_t \\ \dot{m}_t \end{pmatrix} = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \end{pmatrix} \begin{pmatrix} X_t \\ Y_{t-1} \\ \dot{m}_{t-1} \end{pmatrix} + \begin{pmatrix} \xi_{1t} \\ \xi_{2t} \end{pmatrix}$$

with

(28a) $\mu_{11} = (a_1 + a_2 c_1)k$

(29a) $\mu_{21} = (a_2 + a_1 c_2)k$

(29c) $\mu_{12} = c_1 e_2 k$

(29d) $\mu_{22} = e_2 k$

(28e) $\mu_{13} = e_1 k$

(28f) $\mu_{23} = c_2 e_1 k$

$k = [1-c_1 c_2]^{-1}$

$$(28g) \quad \xi_{1t} = \frac{u_{1,t}+(b_1+c_1)u_{2,t}+(1-c_1c_2)^{-1}[(c_1d_2+d_1(b_2+c_2))u_{1,t-1}+(d_1+c_1d_2(b_1+c_1))u_{2,t-1}]}{1-(b_1+c_1)(b_2+c_2)}$$

$$(28h) \quad \xi_{2t} = \frac{(b_2+c_2)u_{1,t}+u_{2,t}+(1-c_1c_2)^{-1}[(d_2+c_2d_1(b_2+c_2))u_{1,t-1}+(c_2d_1+d_2(b_1+c_1))u_{2,t-1}]}{1-(b_1+c_1)(b_2+c_2)}$$

The disturbance vector $\xi_t = (\xi_{1t}, \xi_{2t})'$ is serially correlated. With a single lag, both the contemporaneous variance—covariance matrix of the disturbances, $\Omega_5^0 = E(\xi_t \xi_t')$ and the covariance matrix at lag (or lead) 1, $\Omega_5^1 = E(\xi_t \xi_{t-1}')$ are non-zero. The covariance matrices at leads or lags greater than 1 are all zero. In general, with a T-period distributed lag, all covariance matrices at lags or leads less than or equal to the order of the lag are non-zero. For the equation system (27), we have

$$\Omega_5^o = \{\omega_{ij}^o\} = \left[1-(b_1+c_1)\ (b_2+c_2)\right]^{-2} x$$

$$\left[\begin{array}{c|c}
\left[1+\left(\frac{c_1d_2+d_1(b_2+c_2)}{1-c_1c_2}\right)^2\right]\sigma_{u_1}^2 + \left[(b_1+c_1)^2+\left(\frac{d_1+c_1d_2(b_1+c_1)}{1-c_1c_2}\right)^2\right]\sigma_{u_2}^2 & \omega_{21}^o \\
\hline
\left[b_1+c_1+\frac{\left(d_1+c_1d_2(b_1+c_1)\right)\left(c_2d_1+d_2(b_1+c_1)\right)}{(1-c_1c_2)^2}\right]\sigma_{u_2}^2 + \left[b_2+c_2+\frac{\left(c_1d_2+d_1(b_2+c_2)\right)\left(d_2+c_2d_1(b_2+c_2)\right)}{(1-c_1c_2)^2}\right]\sigma_{u_1}^2 & \left[(b_2+c_2)^2+\left(\frac{d_2+c_2d_1(b_2+c_2)}{1-c_1c_2}\right)^2\right]\sigma_{u_1}^2 + \left[1+\left(\frac{c_2d_1+d_2(b_1+c_1)}{1-c_1c_2}\right)^2\right]\sigma_{u_2}^2
\end{array}\right]$$

$$\Omega_5^1 = \left[1-(b_1+c_1)(b_2+c_2)\right]^{-2} x$$

$$\left[\begin{array}{c|c}
\frac{1}{1-c_1c_2}\left[c_1d_2+d_1(b_1+c_2)\right]\sigma_{u_1}^2 + \frac{1}{1-c_1c_2}\left[d_1+c_1d_2(b_1+c_1)\right](b_1+c_1)\sigma_{u_2}^2 & \frac{1}{1-c_1c_2}\left[c_1d_2+d_1(b_2+c_2)\right](b_2+c_2)\sigma_{u_1}^2 + \frac{1}{1-c_1c_2}\left[d_1+c_1d_2(b_1+c_1)\right]\sigma_{u_2}^2 \\
\hline
\frac{1}{1-c_1c_2}\left[d_2+c_2d_1(b_2+c_2)\right]\sigma_{u_1}^2 + \frac{1}{1-c_1c_2}\left[c_2d_1+d_2(b_1+c_1)\right](b_1+c_1)\sigma_{u_2}^2 & \frac{1}{1-c_1c_2}\left[d_2+c_2d_1(b_2+c_2)\right](b_2+c_2)\sigma_{u_1}^2 + \frac{1}{1-c_1c_2}\left[c_2d_1+d_2(b_1+c_1)\right]\sigma_{u_2}^2
\end{array}\right]$$

Consistent estimation of the reduced form (27) should allow for the fact that the disturbance term follows a generalised first-order moving average process.[9/] With the more general distributed lag structure of (23) and (24) the reduced form disturbance will follow a T^{th} order generalised moving average process. An asymptotically efficient estimation technique should in additon recognise that in general Ω_5^o and Ω_5^1 are unconstrained matrices. Assume that consistent and asymptotically efficient estimates of the reduced form coefficients $\hat{\mu}_{ij}$, i=1,2; j=1,2,3; and $\hat{\Omega}_5^i$, i=1,2 have been obtained. If $e_1 \neq o$ *a priori*, c_2 is identified and can be calculated as $\hat{\mu}_{23}/\hat{\mu}_{13}$. If $e_2 \neq o$ *a priori*, c_1 is identified and can be calculated as $\hat{\mu}_{22}/\hat{\mu}_{12}$. Even if $e_2=e_1=0$ *a priori*, c_1 and c_2 can be identified, as in the model of (1) and (11).

If c_1 and c_2 are identified, e_1 and e_2 are also identified. Therefore, provided the effect in the structural equation (25) of lagged anticipated monetary growth on output is not zero *a priori*, the effect of current anticipated monetary growth on output is identified. The effects of unanaticipated monetary growth - current (b_1+c_1) and lagged (d_1+e_1) - and the effects of unanticipated output - current (b_2+c_2) and lagged (d_2+e_2) - can be identified, if at all, only via the information contained in Ω_5^o and Ω_5^1. Without further *a priori* restrictions on Ω_5^o and Ω_5^1 neither b_1+c_1 nor d_1+e_1 are identified. Even the restriction $b_2+c_2=0$ is not sufficient to identify the effects of current or lagged unanticipated monetary growth on output. Probably the weakest set of over-identifying

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9/
$$\xi_t = \begin{bmatrix} \ell & \ell(b_1+c_1) \\ \ell(b_2+c_2) & \ell \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} + \begin{bmatrix} \ell k\left(c_1d_2+d_1(b_2+c_2)\right) & \ell k\left(d_1+c_1d_2(b_1+c_1)\right) \\ \ell k\left(d_2+c_2d_1(b_2+c_2)\right) & \ell k\left(c_2d_1+d_2(b_1+c_1)\right) \end{bmatrix} \begin{bmatrix} u_{1\ t-1} \\ u_{2\ t-1} \end{bmatrix}$$

$$\ell = \left(1-(b_1+c_1)(b_2+c_2)\right)^{-1}$$

$$k = (1-c_1c_2)^{-1}$$

constraints that permit the identification of b_1+c_1 is $b_2+c_2=0$ and $d_2=0$.10/ In that case Ω_5^1 reduces to a singular matrix, i.e.

$$\Omega_5^1 = \begin{bmatrix} \omega_{11}^1 & \omega_{12}^1 \\ \omega_{21}^1 & \omega_{22}^1 \end{bmatrix} = \left[1-(b_1+c_1)\right]^{-1} \begin{bmatrix} \frac{1}{1-c_1c_2} d_1(b_1+c_1)\sigma_{u_2}^2 & \frac{d_1}{1-c_1c_2}\sigma_{u_2}^2 \\ \frac{1}{1-c_1c_2} c_2d_1(b_1+c_1)\sigma_{u_2}^2 & \frac{c_2d_1}{1-c_1c_2}\sigma_{u_2}^2 \end{bmatrix}$$

$b_1+c_1 = \omega_{11}^1/\omega_{12}^1 = \omega_{21}^1/\omega_{22}^1$. Of course, if $d_1=d_2=0$ (and if $b_1+c_2=0$)

b_1+c_1 can be identified from $\Omega_5^o = \begin{bmatrix} \omega_{11}^o & \omega_{12}^o \\ \omega_{12}^o & \omega_{22}^o \end{bmatrix}$ with $b_1+c_1 = \omega_{12}^o/\omega_{22}^o$. 11/

Thus when output is a function of lagged as well as current anticipated and unanticipated monetary growth, the identification of the current (and lagged) effects of anticipated monetary growth is easier than when only current anticipated and unanticipated monetary growth are included as arguments in the output equation. The identification of the effect of current monetary innovations still requires the *a priori* restriction that current unanticipated output does not affect monetary growth. This is not now sufficient for identification of b_1+c_1, however. The simplest sufficient condition appears to be that $b_2+c_2=0$ and $d_2=0$.

10/ $d_2=0$ means that anticipated and unanticipated lagged output have the same effect on monetary growth.

11/ If c_1 and c_2 are identified, d_1 can also be obtained from Ω_5^o.

Throughout it was assumed that the maximal orders of the distributed lags are known *a priori*. If anticipated and unanticipated money and output enter their respective equations with distributed lags of unknown order, or if the u_{it} are not white noise but instead follow a general arima process, identifying and estimating the response of output to money growth and tests of the relevant hypotheses become much more difficult. The maximal orders of the distributed lags must be inferred from the data and the lagged endogenous variables are less informative from the point of view of identification (see Sims [1980] and Wallis [1980]).

3) Anticipated and unanticipated money in the output equation and the role of monetary policy.

In this section of the paper it is assumed that the problems of reliably identifying and estimating anticipated and unanticipated money have somehow been overcome. It is argued, however, that the output equation used by Barro (23) is a highly restrictive special case and that plausible structural macromodels will yield "semi-reduced form" output equations that include anticipated and unanticipated money in a number of ways not included in equation (23). The most general output equation is given in equation (29).

$$(29)\quad Y_t = a_i X_t + \sum_{i=0}^{T_1} \sum_{j=0}^{S_1} b_{ij} [\dot{m}_{t-i} - E(\dot{m}_{t-i} | I_{t-i-j})] + \sum_{i=0}^{T_1} c_i \dot{m}_{t-i}$$

$$+ \sum_{k=1}^{\Omega_1} \sum_{i=0}^{T_2} \sum_{j=0}^{S_2} d_{ijk} [E(\dot{m}_{t+i} | I_{t-j}) - E(\dot{m}_{t+i} | I_{t-j-k})]$$

$$+ \sum_{k=1}^{\Omega_2} \sum_{i=1}^{T_3} \sum_{j=0}^{S_3} e_{ijk} [E(\dot{m}_{t-i} | I_{t-i-j}) - E(\dot{m}_{t-i} | I_{t-i-j-k})]$$

$$+ \sum_{i=0}^{T_4} \sum_{j=0}^{S_4} f_{ij}\, E(\dot{m}_{t+i} | I_{t-j}) + u_{1t}.$$

Three kinds of channels through which monetary policy can affect real output that are excluded from Barro's semi-reduced form (23) are included in (29).

a) A distributed lag on forecast horizons

In all empirical work unanticipated monetary growth is represented by a distributed lag on unperceived contemporaneous monetary growth.12/ This is generalised in (29) to prediction errors from forecasts of $\dot{m}_{t-i}$, $i = 0,1, \ldots, T_1$, made at the beginning of period $t-i$ and in earlier periods $t-i-j$, $j=1, \ldots, S_1$. Such a reduced form will emerge e.g. if the money wage in period t has to be set in open-loop fashion in a period before t (see Fischer [1977], Phelps and Taylor [1977], Taylor [1979]).

It is represented by the term $\sum_{i=0}^{T_1} \sum_{j=0}^{S_1} b_{ij} (\dot{m}_{t-i} - \hat{\dot{m}}_{t-i|t-i-j})$ in (29). It can be shown that even if the hypothesis $c_0 = \ldots = c_{T1} = 0$ is not rejected by the data or held to be correct *a priori*, the true model can have $b_{ij} > 0$ for $j > 0$. Consider the following example, which makes no claim to descriptive realism. $T_1 = 0$, $S_1 = 1$ and all other channels through which anticipated and unanticipated money could affect output are absent. Let $X_t = [G_t, \dot{m}_{t-1}]'$ where G_t denotes real public spending which is assumed exogenous and known with certainty in the current and past periods.

$$(30) \quad Y_t = a_{11} G_t + b_{00}(\dot{m}_t - \hat{\dot{m}}_{t|t}) + b_{01}(\dot{m}_t - \hat{\dot{m}}_{t|t-1}) + c_0 \dot{m}_t + u_{1t} \qquad 13/$$

$$(31) \quad \dot{m}_t = a_{21} G_t + a_{22} \dot{m}_{t-1} + u_{2t}$$

12/ In practice this is approximated by prediction errors from forecasts, made at the beginning of the unit period of analysis, of monetary growth during that period.

13/ For simplicity it is assumed that lagged monetary growth does not affect output.

Therefore

$$\dot{m}_t - \hat{\dot{m}}_{t|t} = u_{2t}$$

$$\dot{m}_t - \hat{\dot{m}}_{t|t-1} = u_{2t} + a_{22}(\dot{m}_{t-1} - \hat{\dot{m}}_{t-1|t-1}) = u_{2t} + a_{22}\, u_{2t-1}$$

The reduced form of (30) and (31) is given by (31) and (32).

$$(32)\quad Y_t = (a_{11}+c_0\, a_{21})G_t + c_0\, a_{22}\, \dot{m}_{t-1} + u_{1t} + (c_0+b_{00}+b_{01})u_{2t} + b_{01}\, a_{22}\, u_{2t-1}$$

Contrast this model with the model of equations (33) and (31). Equation (33) is the special case of (29) when $T_1 = 1$, $S_1 = 0$ and all other potential monetary transmission mechanisms are absent.

$$(33)\quad Y_t = a_{11}\, G_t + b_{00}(\dot{m}_t-\hat{\dot{m}}_{t|t}) + b_{10}(\dot{m}_{t-1}-\hat{\dot{m}}_{t-1|t-1}) + c_0\, \dot{m}_t + c_1\, \dot{m}_{t-1} + u_{1t}$$

The reduced form of (33) and (31) is given by (31) and (34).

$$(34)\quad Y_t = (a_{11}+c_0\, a_{21})G_t + (c_0\, a_{22}+c_1)\dot{m}_{t-1} + u_{1t} + (c_0+b_{00})u_{2t} + b_{10}\, u_{2t-1}$$

Under the null-hypothesis that anticipated money has no real effects ($c_0 = 0$ in (32), $c_0 = c_1 = 0$ in (34)) the reduced form output equations are

$$(32')\quad Y_t = a_{11}\, G_t + u_{1t} + (b_{00}+b_{01})u_{2t} + b_{01}\, a_{22}\, u_{2t-1}$$

$$(34')\quad Y_t = a_{11}\, G_t + u_{1t} + b_{00}\, u_{2t} + b_{10}\, u_{2t-1}$$

(32') and (34') are observationally equivalent. Yet in (32') the probability distribution function of Y_t does depend on the known parameter, a_{22}, of the monetary feedback rule while in (34') the deterministic part of the monetary rule is irrelevant for the behaviour of real output. The only way to discriminate between (32') and (34') is along the lines suggested by Sargent [1976]. If a change in the monetary regime (i.e. a different value of a_{22}) can be established, (34') would be invariant under such a change while (32') would be affected. The presence or absence of such a change in the coefficient of u_{2t-1} can be established by testing for a change in the contemporaneous variance-covariance matrix of the reduced form disturbances of (32') and (31) and in the covariance matrix at lag 1, at the moment that the policy change is known to have occurred.

The distinction between (32') and (34') bringsout a by now familiar policy issue. Even if only unanticipated money affects output, deterministic monetary feedback rules affect real output if $b_{ij} \neq 0$ for some $j > 0$, say j', and if monetary growth in period t can respond to information more recent than $I_{t-j'}$. Provided at least some private actions affecting Y_t depend on forecast errors from private forecasts of $\dot{m}_{t-i}$ that are based on earlier, and therefore less complete, information than the information used by the policy authorities to determine the value of $\dot{m}_{t-i}$, known contingent policy rules will affect y_t. This policy effectiveness result holds even if $c_i = 0$ for all i. (See e.g. Buiter [1980 a, b, c, d]). In the example of equations (30) and (31), the authorities can influence the monetary forecast error $\dot{m}_t - \hat{\dot{m}}_{t|t-1}$ by making $\dot{m}_t$ a function of $\dot{m}_{t-1}$. If $c_0 = 0$, the value of a_{22} that minimizes the variance of output is zero.14/ In a more realistic model in which Y_t (and $\dot{m}_t$) can be functions of e.g. Y_{t-1}, the conclusion that zero feedback is optimal will not in general follow.

14/ This follows from (32') and the orthogonality of u_{2t} and u_{2t-1}.

b) Anticipated future monetary growth

Anticipated future inflation and therefore anticipated future monetary growth will affect real output in equilibrium models if output is a function of the real stock of capital and money is not superneutral. (See Tobin [1965] and Fischer [1979]). If e.g. the demand for money depends on the nominal interest rate while the rate of investment (or the equilibrium capital stock) depends on the real interest rate, the density function of real output will not be invariant under alternative deterministic money supply rules, even if these rules are open-loop such as constant growth rate rules for the nominal money stock. A general formulation of this "inflation tax" effect of anticipated future monetary growth is represented in (29) by the term $\sum_{i=o}^{T_4} \sum_{j=o}^{S_4} f_{ij} \hat{\dot{m}}_{t+i|t-j}$. This transmission mechanism of monetary policy was not considered in Barro's work on anticipated and unanticipated money. The nature and magnitude of the bias imparted to estimates of b_{ij} and c_i if anticipated future money growth is omitted from a regression model when $f_{ij} \neq 0$ for some i and j can only be established on a case-by-case basis for alternative money supply processes.

c) Revisions in money supply forecasts

Consider the Sargent-Wallace supply function

$$(35) \quad y_t = \beta\left(p_t - E(p_t | I_{t-1})\right) + u_t^y \qquad \beta > 0$$

The information set I_t is now assumed to contain all current and past endogenous variables, exogenous variables and random disturbances. y_t is the log of output, p_t the log of the general price level and u_t^y a white noise disturbance term. From the discussion of multi-period forecast horizons, it will be clear that if m_t could be made a known function of I_t , the forecast error $p_t - \left(E(p_t | I_{t-1})\right)$, and therefore output, will be influenced by such a contingent monetary rule. It is less obvious that the probability density function of y_t can also be affected by a deterministic monetary feedback rule if m_t is a function of I_{t-1} , of I_{t-2} of of any I_{t-i} , $i > 1$. Recent papers by Turnovsky [1980] and Weiss [1980] have shown how the current price forecast error (and thereby current output) can be influenced by revisions in forecasts of future money supplies and how feedback rules of the type $m_t = \psi(I_{t-i})$ $i \geq 1$, can influence these money supply forecasts. (See also Buiter [1980 b, c, d], Buiter and Eaton [1980]).

We complete a simple macro model by adding a Cagan-style demand function for real money balances

$$(36)\quad m_t - p_t = \alpha_1 y_t - \alpha_2 [E(p_{t+1} | I_t) - p_t] + u_t^m \qquad \alpha_1 , \alpha_2 > 0$$

m_t is the log of the nominal money stock and u_t^m a while noise disturbance term. Assuming stability, the solution for the price forecast error is given by:

$$(37)\quad p_t - E(p_t | I_{t-1}) = \frac{1}{1 + \alpha_1 \beta + \alpha_2} [m_t - E(m_t | I_{t-1})]$$

$$+ \frac{\alpha_2}{(1 + \alpha_1 \beta + \alpha_2)(1 + \alpha_2)} \sum_{i=0}^{\infty} \left(\frac{\alpha_2}{1 + \alpha_2}\right)^i \left[E\left(m_{t+1+i} | I_t\right) - E\left(m_{t+1+i} | I_{t-1}\right)\right]$$

$$- \frac{1}{1 + \alpha_1 \beta + \alpha_2} \left(\alpha_1 u_t^y + u_t^m\right)$$

Thus, the current price forecast error and current output are functions of the revision, between period $t - 1$ and period t , in the forecasts for all current and future money supplies. Consider the non-stochastic monetary feedback rule given in (38).

$$(38)\quad m_t = \sum_{i=0}^{\infty} \left[\mu_{i,y}\, u^y_{t-i} + \mu_{i,m}\, u^m_{t-i} \right]$$

The $\mu_{i,y}$ and $\mu_{i,m}$ are the known policy response coefficients.

Substituting this into (37) yields

$$(39)\quad p_t - E\left(p_t | I_{t-1}\right) = \frac{1}{1 + \alpha_1 \beta + \alpha_2} \left(\mu_{0,y}\, u^y_t + \mu_{0,m}\, u^m_t\right)$$

$$+ \frac{\alpha_2}{(1 + \alpha_1 \beta + \alpha_2)(1 + \alpha_2)} \sum_{i=0}^{\infty} \left(\frac{\alpha_2}{1 + \alpha_2}\right)^i \left(\mu_{1+i,y}\, u^y_t + \mu_{1+i,m}\, u^m_t\right)$$

$$- \frac{1}{1 + \alpha_1 \beta + \alpha_2} \left(\alpha_1\, u^y_t + u^m_t\right)$$

The feedback rule cannot merely <u>influence</u> $p_t - E(p_t | I_{t-1})$, in the present example it can completely eliminate the forecast error. Any choice of $\mu_{i,y}$ and $\mu_{i,m}$ such that

$$(40a)\quad \mu_{0,y} + \frac{\alpha_2}{1 + \alpha_2} \sum_{i=0}^{\infty} \left(\frac{\alpha_2}{1 + \alpha_2}\right)^i \mu_{i+1,y} - \alpha_1 = 0$$

and

$$(40b)\quad \mu_{0,m} + \frac{\alpha_2}{1 + \alpha_2} \sum_{i=0}^{\infty} \left(\frac{\alpha_2}{1 + \alpha_2}\right)^i \mu_{i+1,m} - 1 = 0$$

will achieve $p_t = E(p_t | I_{t-1})$ 15/

15/ Instead of equating output to its *ex post* natural level u^y_t , we could also use the monetary rule to equate output to its *ex ante* natural level, zero.

Note that this perfect stabilization can be achieved even if $\mu_{0,y} = \mu_{0,m} = 0$, i.e. if no response of the current money supply to the current information set is permitted. In this model it doesn't matter <u>when</u> the money supply responds to new information. The identical effect on current output can be achieved by current response or by delayed response. For future money supply forecast revisions to enter (37) it is necessary that expectations of future endogenous variables conditioned at different dates be present in the model. Thus if (36) were replaced by e.g.

$$(36') \quad m_t - p_t = \alpha_1 y_t - \alpha_2 \left[E(p_{t+1}|I_{t-1}) - E(p_t|I_{t-1})\right] + u_t^y$$

monetary feedback rules making m_t a function of I_{t-i}, $i > 0$, would have no effect on $p_t - E(p_t|I_{t-1})$ or on y_t. (See Buiter [1980c] for a further discussion).

Consider the output equation corresponding to (37),

$$(41) \quad y_t = \frac{\beta}{1 + \alpha_1 \beta + \alpha_2} \left[m_t - E(m_t|I_{t-1})\right]$$

$$+ \frac{\beta\alpha_2}{(1 + \alpha_1 \beta + \alpha_2)(1 + \alpha_2)} \sum_{i=0}^{\infty} \left(\frac{\alpha_2}{1 + \alpha_2}\right)^i \left[E(m_{t+1+i}|I_t) - E(m_{t+1+i}|I_{t-1})\right] + \frac{(1 + \alpha_2)\, u_t^y - \beta\, u_t^m}{1 + \alpha_1 \beta + \alpha_2}$$

Equation (41) is observationally equivalent to an output equation omitting the influence of future money supply revisions such as (42),

$$(42) \quad y_t = k\left(m_t - E(m_t|I_{t-1})\right) + \eta_t$$

where η_t is a white noise disturbance term.

The reason is that $m_t - E(m_t|I_{t-1})$ and all terms such as $E(m_{t+1+i}|I_t) - E(m_{t+1+i}|I_{t-1})$ are linear combinations of the same random disturbances, u_t^y and u_t^m. These represent the new information that has accrued between periods t and t-1 .[16/] Thus models in which monetary feedback from lagged information is irrelevant for real output are observationally equivalent to models in which such lagged feedback rules can affect real output. In (29) the "forecast revision effect" is represented by

$$\sum_{k=1}^{Q_1} \sum_{i=0}^{T_2} \sum_{j=0}^{S_2} d_{ijk} [\hat{\dot{m}}_{t+i|t-j} - \hat{\dot{m}}_{t+i|t-j-k}]$$

$$+ \sum_{k=1}^{Q_2} \sum_{i=1}^{T_3} \sum_{j=0}^{S_3} e_{ijk} [\hat{\dot{m}}_{t-i|t-i-j} - \hat{\dot{m}}_{t-i|t-i-j-k}] \quad .$$

The empirical work of Barro and others working in this area has not considered the possibility that even if anticipated money is irrelevant for real output, deterministic monetary feedback rules may affect real output by influencing revisions in forecasts of future money supplies.

16/ If the money supply rule is a stochastic function of the disturbances u_t^y and u_t^m, the "news" in period t would include the random component of the monetary rule.

Conclusion

The main conclusions are stated in the introduction. In future empirical work on the real effects of anticipated and unanticipated money, the specification and estimation of the money supply process will require even greater attention. The crucial issue of whether unanticipated output affects monetary growth will have to be resolved. Since most empirical work uses monetary aggregates that are wider than the monetary base, the interpretation of the money supply process as a policy reaction function seems overly simple. It may be necessary to model the behaviour of the private banking sector whose liabilities constitute most of M_1, M_2 or M_3. Another surprising (or even worrying) feature of past empirical work on the money supply process is that no structural break has been reported in that process at the time of the demise of the Bretton Woods adjustable peg exchange rate regime.

As regards the output equation, a firm distinction needs to be made between the proposition that anticipated (current and past) money has no real effects ($c_1 = 0$ for all i) and the proposition that deterministic monetary feedback rules have no real effects. Known contingent monetary rules can influence monetary forecast errors (Fischer [1977]) and revisions in money supply forecasts (Turnovsky [1980], Weiss [1980]). This may give monetary policy a handle on the real economy. Serious identification problems make the empirical resolution of these issues doubtful.

Finally, changes in anticipated future monetary growth rates will, by altering the anticipated real rate of return on money *vis a vis* real assets, alter the composition and (in the long run) the magnitude of "full-information" real output. The importance of this monetary transmission channel can in principle be empirically evaluated in a Barro-type framework.

The empirical work on anticipated and unanticipated money has not so far brought us much closer to an assessment of the stabilization and structural (or allocative) roles of monetary policy.

References

Attfield, C.L.F., D. Demery and N.W. Duck, 1979, "Unanticipated monetary growth, output and the price level: U.K. 1946-77", University of Bristol, mimeo, July.

Attfield, C.L.F., D. Demery and N.W. Duck, 1979, "A quarterly model of unanticipated monetary growth and output in the U.K. 1963-78", University of Bristol, mimeo, September.

Barro, Robert J., 1977, "Unanticipated money growth and unemployment in the United States", American Economic Review, 67, March, 101-15.

Barro, Robert J., 1978, "Unanticipated money, output and the price level in the United States", Journal of Political Economy, 86, August, 549-81.

Barro, Robert J., 1979, "The equilibrium approach to business cycles", unpublished, November.

Barro, Robert J. and M. Rush, 1979, "Unanticipated money and economic activity", National Bureau of Economic Research working paper no. 339 (Cambridge, MA: NBER, April).

Buiter, Willem H., 1980a, "The macroeconomics of Dr. Pangloss: a critical survey of the new classical macroeconomics", Economic Journal, 90, March, 34-50.

Buiter, Willem H., 1980b, "Monetary, financial and fiscal policies under rational expectations", forthcoming in IMF Staff Papers, December.

Buiter, Willem H., 1980c, "The role of economic policy after the new classical macroeconomics", forthcoming in D. Currie and D. Peel eds, Contemporary economic analysis, vol. IV, Croom Helm.

Buiter, Willem H. (1980d), "The superiority of contingent rules over fixed rules in models with rational expectations", forthcoming in Economic Journal, September 1981.

Buiter, Willem H. and Jonathan Eaton, 1980, "Policy decentralization and exchange rate management in interdependent economies", National Bureau of Economic Research working paper no. 531, August.

Deissenberg, Christopher, 1979, "Optimal stabilisation policy with delayed controls and imperfect state measurements", Fakultät Für Wirtschaftswissenschaften und Statistik, Universität Konstanz, working paper, series A, no. 133, November.

Fischer, Stanley, 1977, "Long-term contracts, rational expectations and the optimal money supply rule", Journal of Political Economy, 85, February, 191-206.

Fischer, Stanley, 1979, "Capital accumulation on the transition path in a monetary optimizing model", Econometrica, 47, November, 1433-1440.

Gordon, Robert J., 1979, "New evidence that fully anticipated monetary changes influence real output after all", National Bureau of Economic Research working paper no. 361, (Cambridge MA: NBER, June).

Lucas, R.E., 1975, "An equilibrium model of the business cycle", Journal of Political Economy, 83, December, 1113-44.

McCallum, B.T., 1979, "On the observational inequivalence of classical and Keynesian models", Journal of Political Economy, 87, April, 393-402.

Phelps, Edmund S. and John B. Taylor, 1977, "Stabilising powers of monetary policy under rational expectations", Journal of Political Economy, 85, February, 163-90.

Sargent, T.J., 1976, "The observational equivalent of natural and unnatural rate theories of macroeconomics", Journal of Political Economy, 84, June, 631-40.

Sims, Christopher A., 1980, "Macoreconomics and reality", Econometrica, 48, January, 1-48.

Taylor, John B., 1979, "Staggered wage setting in a macro model", American Economic Review, 69, Papers and Proceedings, May, 108-113.

Tobin, James, 1965, "Money and economic growth", Econometrica, 33, October, 671-84.

Turnovsky, Stephen J., 1980, "The choice of monetary instruments under alternative forms of price expectations", Manchester School, March, 39-62.

Wallis, Kenneth F., "Econometric implications of the rational expectations hypothesis", Econometrica, 48, January, 49-73.

Weiss, Laurence, 1980, "The role for active monetary policy in a rational expectations model", Journal of Political Economy, 88, April, 221-233.