

NBER WORKING PAPER SERIES

ACCELERATED DEPRECIATION AND THE EFFICACY
OF TEMPORARY FISCAL POLICY: IMPLICATIONS
FOR AN INFLATIONARY ECONOMY

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Working Paper No. 596

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

December 1980

This paper was presented at the NBER's 1980 Summer Institute in Economic Fluctuations. The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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Implications for an Inflationary Economy

ABSTRACT

The effect on investment of temporary tax rate changes depends on the age profile of depreciation deductions. If the depreciation allowance schedule is accelerated, then temporary cuts in the corporate tax rate could reduce investment. Inflation causes the age profile of real depreciation deductions to become accelerated and thus could make temporary tax cuts have a contractionary effect on investment. Two currently proposed reforms are shown to exacerbate this effect. Under these proposals, temporary tax cuts are likely to have opposite effects on investment in short-lived and long-lived capital, thereby complicating the conduct of countercyclical fiscal policy.

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I. Introduction

Since allowable depreciation deductions are fixed in nominal rather than real terms, an increase in the rate of inflation reduces the present value of real depreciation deductions. Recent work by Auerbach (1979), Kopcke (1979), and Abel (1980a) has analyzed the implications of this reduction in real depreciation deductions for the choice of capital durability. Feldstein (1979) has compared the effects of two reforms--indexing depreciation deductions and shortening the depreciable life of capital--focusing on the extent to which they offset the reduction in real depreciation deductions. In all of these studies, the corporate tax rate is assumed to remain constant over time.

In this paper, we examine the role of depreciation allowances in determining the effects on investment of permanent and temporary changes in the corporate tax rate. In particular, if the age profile of real depreciation deductions is accelerated, then temporary cuts in the corporate tax rate could reduce investment even if permanent tax cuts would stimulate investment. Since increases in the rate of inflation tend to make the age profile of depreciation deductions more accelerated, higher inflation could create the situation in which temporary tax rate cuts reduce investment.¹ It is shown that two proposed depreciation allowance reforms designed to mitigate the effects of inflation--the Conable-Jones 10-5-3 proposal and the Auerbach-Jorgenson proposal--actually exacerbate the acceleration of the age profile of real deductions and increase the tendency toward perverse effects of temporary fiscal policy. Specifically, if the 10-5-3 proposal were in effect permanently, temporary

¹However, higher inflation increases the tendency for permanent tax rate cuts to stimulate rather than restrain investment.

cuts in the corporate tax rate would stimulate investment in short-lived capital and would actually restrain investment in long-lived capital. Since the Auerbach-Jorgenson proposal has the most accelerated profile possible, temporary tax cuts, especially brief ones, would reduce investment if this proposal were permanently in place. The effects of countercyclical fiscal policy would possibly be perverse under either of these proposals.

In Section II we develop an adjustment cost model of the firm which gives rise to a model of investment similar to Tobin's (1969) q theory of investment. Since the model incorporates perfect foresight, it is well suited to the analysis of the dynamic effects of temporary as well as permanent tax rate changes. In Section III, we analyze the effect of permanent and temporary tax rate changes on q_t , which is the determinant of investment. After defining a non-distortionary age profile of depreciation deductions, we show that under such a depreciation scheme the effect on q of temporary changes in the corporate tax rate is always in the same direction as, and smaller in magnitude than, a permanent change of the same size and direction. However, under accelerated depreciation deduction schemes, a temporary cut in the corporate tax rate can be contractionary even if a permanent tax rate decrease is expansionary.

After demonstrating the importance of the shape of the age profile of depreciation deductions in Section III, we derive non-distortionary depreciation deduction schedules corresponding to certain patterns of physical decay. Among the perhaps counterintuitive results is that if the physical depreciation of capital is straight line depreciation (a constant absolute amount over the life of capital), then the sum-of-years-digits (SYD) depreciation deduction schedule has a non-distortionary profile. Thus SYD depreciation allowances are not accelerated in the sense used in this paper.

In Section V we demonstrate that an increase in the rate of infla-

tion will cause a non-distortionary age profile of depreciation deductions to become accelerated. The implications of this effect for the efficacy of temporary tax rate changes are analyzed in the context of the proposed 10-5-3 depreciation allowance reform. It is shown that for some reasonable values of interest rates and the rate of inflation, temporary tax cuts, which might be considered as part of countercyclical fiscal policy, will reduce rather than increase the rate of investment, in both long-lived and short-lived capital. More often, however, temporary cuts in the corporate tax rate will stimulate investment in short-lived capital but will restrain investment in long-lived capital. These allegedly perverse effects of countercyclical fiscal policy would be less likely to occur if, instead of shortening the depreciation lives of capital, depreciation deductions are indexed.

II. A Model of Investment

Consider a price-taking equity-financed² firm which faces exogenously determined paths of output prices, p_t , and nominal wage rates, w_t . The path of the after-tax nominal discount rate, r_t , is also exogenous to the firm. The output of the firm is determined by a linearly homogeneous production function $f(K_t, L_t)$ where K_t is the capital stock and L_t is the labor input at time t . The investment technology is given by a convex adjustment cost function $c(I_t)$, where I_t is gross investment at time t , with $c' > 0$ and $c'' > 0$ for $I_t > 0$.

The capital stock at any point in time is the sum of the undepreciated units of all past investment. Since this paper focuses on the role of physical depreciation and depreciation allowances in the investment

²The Appendix extends the analysis of this paper to the case of a firm financed entirely by debt.

decision, we introduce a physical depreciation scheme which is more general than the commonly assumed proportional depreciation. Let $m(x)$ be the fraction of capital which survives to age x so that the capital stock at time t is given by

$$K_t = \int_0^{\infty} I_{t-x} m(x) dx \quad (\text{II.1})$$

Specific forms of the function $m(x)$ will be discussed in Section IV.³

The tax environment in this paper is based on the extension of the Hall-Jorgenson (1967, 1971) analysis to adjustment cost models by Abel (1980c). Corporate taxable income at time t is taxed at rate τ_t . In calculating taxable income, firms are permitted depreciation deductions equal to a fraction $DD(x)$ of original cost for capital of age x . Thus, the total depreciation deduction at time t , on all units of capital, is

$$\int_0^{\infty} c(I_{t-s}) DD(s) ds. \quad (\text{II.2})$$

Finally we treat the investment tax credit as an immediate rebate of a fraction k of the cost of gross investment.

We assume that the firm chooses the time paths of I_t and L_t to maximize the present-value of cash flow. At time 0, this present value is

$$V = \int_0^{\infty} \{ (1-\tau_t) [p_t f(K_t, L_t) - w_t L_t] + \tau_t \int_0^{\infty} c(I_{t-s}) DD(s) ds - (1-k)c(I_t) \} \rho_t dt$$

(II.3)

where $\rho_t = e^{-\int_0^t r_v dv}$.

³At this point, it appears that any function $m(x)$ which satisfies $m(x) \geq 0$ for all $x \geq 0$, $m(0) = 1$, $\lim_{x \rightarrow \infty} m(x) = 0$, and $m(x_1) \geq m(x_2)$ for

$x_1 \leq x_2$ would be a reasonable specification.

The expression in curly brackets, which is the after-tax cash flow at time t , contains three terms. The first term is the after-tax income prior to depreciation deductions. The second term is the tax bill savings due to the depreciation allowance and the third term is the cost of gross investment net of the investment tax credit.

It will be convenient to collect terms in $c(I_t)$ so we rewrite (II.3) as

$$V = \int_0^{\infty} \{ (1-\tau_t)[p_t f(K_t, L_t) - w_t L_t] - (1-k) \int_0^{\infty} \tau_{t+s} DD(s) \rho_{t+s} \rho_t^{-1} ds \} c(I_t) \rho_t dt \\ + \int_0^{\infty} \tau_t \int_t^{\infty} c(I_{t-s}) DD(s) ds \rho_t dt. \quad (II.4)$$

The firm is assumed to choose I_t and L_t to maximize V subject to (II.1) and the history of past investment. Note that only the first line of (II.4) involves I_t and L_t so that the second line of (II.4) may be ignored in the optimization problem. The first order conditions, which hold at every time t , are

$$f_L(K_t, L_t) = w_t / p_t \quad (II.5a)$$

$$(1-k) \int_0^{\infty} \tau_{t+s} DD(s) \rho_{t+s} \rho_t^{-1} ds c'(I_t) \\ = \int_t^{\infty} (1-\tau_s) f_K(K_s, L_s) m(s-t) \rho_{t+s} \rho_t^{-1} ds. \quad (II.5b)$$

The interpretation of (II.5a) is that at every point in time, the firm chooses its level of labor input to equate the marginal product of labor with the exogenously given real wage rate. According to (II.5b), the firm chooses the rate of investment which equates the marginal cost of investment net of the investment tax credit and the depreciation allowance with the present value of after-tax rentals accruing to capital.

Note that since $f(K, L)$ is linearly homogenous, equation (II.5a)

implies that the optimal capital/labor ratio is an increasing function of the real wage rate, and hence the marginal product of capital is a decreasing function of w_t/p_t . Defining ϕ_t to be the marginal physical product of capital at time t , $f_K(K_t, L_t)$, we obtain

$$(II.6) \quad \phi_t = \phi(w_t/p_t), \quad \phi > 0, \quad \phi' < 0.$$

The important implication of (II.6) for our purposes is that since the real wage rate is exogenously given, ϕ_t is also exogenously determined.

Now consider the following specification of the corporate tax rate policy

$$(II.7) \quad \tau_t = \begin{cases} \tau_1, & 0 \leq t < t^* \\ \tau_0, & t \geq t^* \end{cases}.$$

This specification allows us to analyze the effects of a temporary change in the tax rate which remains in effect until t^* .

Given the specification of tax policy in (II.7), it will be useful to introduce the following notation for any $t < t^*$

$$(II.8) \quad \begin{aligned} \text{a.} \quad & \hat{D}_t = \rho_t^{-1} \int_t^{t^*} DD(s-t) \rho_s ds \\ \text{b.} \quad & \hat{D}_t = \rho_t^{-1} \int_{t^*}^{\infty} DD(s-t) \rho_s ds \\ \text{c.} \quad & D_t^* = \hat{D}_t + \hat{\hat{D}}_t \\ \text{d.} \quad & \hat{N}_t = \rho_t^{-1} \int_t^{t^*} p_s \phi_s m(s-t) \rho_s ds \\ \text{e.} \quad & \hat{N}_t = \rho_t^{-1} \int_{t^*}^{\infty} p_s \phi_s m(s-t) \rho_s ds \\ \text{f.} \quad & N_t^* = \hat{N}_t + \hat{\hat{N}}_t. \end{aligned}$$

Observe that \hat{D}_t is the present value of the stream of depreciation deductions between t and t^* per dollar of cost of gross investment at time t . Similarly, $\hat{\hat{D}}_t$ is the present value of the stream of depreciation deductions from t^* onward per dollar of cost of gross investment at time t . D_t^* , which is the sum of \hat{D}_t and $\hat{\hat{D}}_t$, corresponds to Hall and Jorgenson's (1967, 1971) z , the present value of depreciation deductions over the life of capital. Observe that N_t^* is the present value of the stream of pre-tax rentals accruing to capital installed at time t . N_t^* is partitioned into the rentals accruing before t^* , \hat{N}_t , and the rentals accruing from t^* onward, $\hat{\hat{N}}_t$.

Using the various definitions in (II.8), the first-order condition in (II.5b) can be rewritten as

$$(II.9) \quad a. \quad c'(I_t) = q_t$$

where

$$b. \quad q_t = \frac{(1-\tau_1)\hat{N}_t + (1-\tau_0)\hat{\hat{N}}_t}{(1-k-\tau_1)\hat{D}_t - \tau_0\hat{\hat{D}}_t}$$

Since $c(I_t)$ is convex, (II.8a) could be inverted to yield investment as an increasing of q_t . Note that q_t is the ratio of the shadow price of installed capital to the net purchase price of uninstalled capital, in the spirit of Tobin's (1969) q , as explained in Abel (1980b,c). Since the effects of tax policy on investment operate entirely through q_t , we can examine the comparative effects of various tax policies on investment by examining the comparative effects on q_t .

The optimal value of investment can be determined graphically as shown in Figure 1. The upward sloping solid line represents the net marginal cost of investment, $(1-k-\tau_1\hat{D}_t - \tau_0\hat{\hat{D}}_t)c'(I_t)$, which is an increasing function of the rate of investment. The horizontal solid line represents the present value of after-tax rentals accruing to a unit of capital

installed at time t , which is invariant to changes in the instantaneous rate of investment. At the intersection of the two schedules E , (II.5b) is satisfied. Therefore, I^* represents the optimal rate of investment.

Now consider a decrease in τ_1 . Since the decrease in τ_1 increases the after-tax rentals accruing until time t^* , the horizontal schedule in Figure 1 shifts upward as shown by the dashed horizontal line. This increase in after-tax rentals tends to increase investment. However, the reduction in τ_1 also reduces the tax bill saving associated with the depreciation allowance \hat{D}_t and hence shifts the net investment cost schedule upward, thereby tending to decrease investment. The new optimal rate of

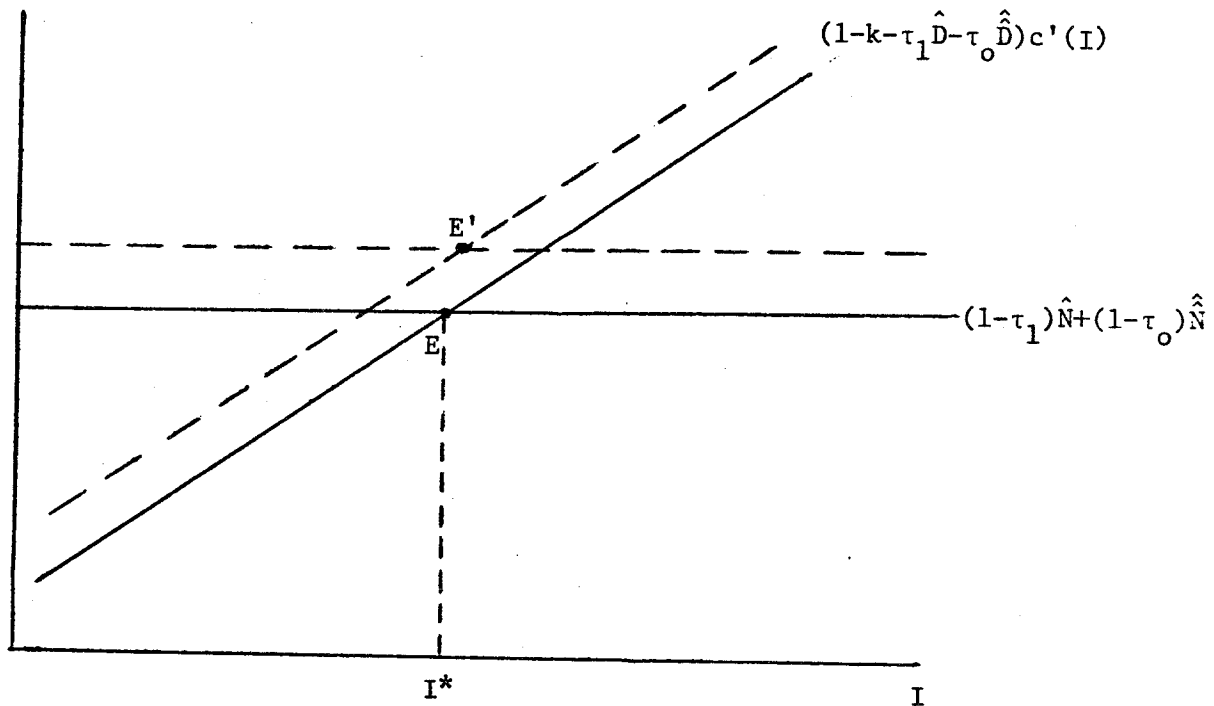


Figure 1

investment is determined at E' . If the present value of after-tax rentals shifts upward by more than the net marginal investment cost shifts upward at I^* , the optimal rate of investment increases. However, if the net marginal cost schedule shifts upward at I^* by more than the present-value of rentals shifts upward, there is a decrease in the rate of investment.

Intuitively, the larger is \hat{D} relative to \hat{N} , the more likely is this "perverse" effect. In the next section, we formalize this intuitive explanation.

III. The Comparative Effects of Permanent and Temporary Tax Rate Changes

In this section we compare the effects of permanent and temporary tax rate changes under the assumption that the after-tax discount rate, r_t , is invariant to the tax rate. This assumption corresponds to the case of a firm financed entirely by equity; the case of a debt-financed firm is treated in the Appendix.

Since the effects of tax policy on investment operate through q_t , we focus on the effect of tax rate changes on q_t . Recall from (II.9b) that

$$(III.1) \quad q_t(\tau_1, \tau_0) = \frac{(1-\tau_1)\hat{N}_t + (1-\tau_0)\hat{N}_t}{1-k-\tau_1\hat{D}_t - \tau_0\hat{D}_t}$$

To determine the effect of a temporary change in the tax rate, we calculate the partial derivative of $q_t(\tau_1, \tau_0)$ with respect to τ_1 . To facilitate the analysis, we evaluate this partial derivative at $\tau_1 = \tau_0 = \tau$ to obtain

$$(III.2) \quad \left. \frac{\partial q_t(\tau_1, \tau_0)}{\partial \tau_1} \right|_{\tau_1 = \tau_0 = \tau} = \frac{1}{(1-k-\tau D_t^*)^2} \{-(1-k-\tau D_t^*)\hat{N}_t + (1-\tau)N_t^*\hat{D}_t\}.$$

By evaluating this partial derivative at $\tau_1 = \tau_0 = \tau$, we are examining the effect of changes in the tax rate between times t and t^* , starting from an initial situation in which the tax rate is expected to have the constant value τ forever. Rearranging the term in brackets (III.2) we obtain

$$(III.3) \quad \left. \frac{\partial q_t(\tau_1, \tau_0)}{\partial \tau_1} \right|_{\tau_1 = \tau_0 = \tau} = \frac{-1}{(1-k-\tau D_t^*)^2} \{(1-k-D_t^*)\hat{N}_t + (1-\tau)(D_t^*\hat{N}_t - N_t^*\hat{D}_t)\}.$$

Equation (III.3) can be used to analyze the effects of permanent as well as temporary tax rate changes. First consider a permanent change so that $\hat{N}_t = N_t^*$, $\hat{D}_t = D_t^*$, $\hat{N}_t = 0$ and $\hat{D}_t = 0$. Noting that $D_t^*\hat{N}_t = N_t^*\hat{D}_t$ in this case, we easily obtain

$$(III.4) \quad \left(\frac{\partial q_t}{\partial \tau} \right)^P = \frac{-1}{(1-k-\tau D_t^*)^2} (1-k-D_t^*) N_t^*$$

where $(\partial q_t / \partial \tau)^P$ will be used to represent the effect on q_t of a permanent change in the tax rate. Note that a permanent reduction in the corporate tax rate will lead to an increase in q_t and hence an increase in investment if and only if the present value of depreciation deductions, $D_t^* c(I_t)$, is less than the cost of investment, net of the investment tax credit, $(1-k)c(I_t)$. In the case in which $D_t^* = 1-k$, permanent changes in the tax rate have no effect on q_t or on investment.⁴

Combining equations (III.3) and (III.4), we obtain the following expression for the effect on q of a temporary tax rate change

$$(III.5) \quad \left. \frac{\partial q_t(\tau_1, \tau_0)}{\partial \tau_1} \right|_{\tau_1 = \tau_0 = \tau} = \frac{\hat{N}_t}{N_t^*} \left(\frac{\partial q_t}{\partial \tau} \right)^P + \frac{(1-\tau) N_t^* D_t^*}{(1-k-\tau D_t^*)^2} \left(\frac{\hat{D}_t}{D_t^*} - \frac{\hat{N}_t}{N_t^*} \right).$$

First consider the case in which

$$(III.6) \quad \frac{\hat{D}_t}{D_t^*} = \frac{\hat{N}_t}{N_t^*} \quad 0 \leq t < t^*$$

If (III.6) holds, the depreciation allowance schedule will be said to have a non-distortionary age profile. In this case, (III.5) simplifies to

$$(III.7) \quad \frac{\partial q_t}{\partial \tau_1} = \frac{\hat{N}_t}{N_t^*} \left(\frac{\partial q_t}{\partial \tau} \right)^P.$$

Since \hat{N}_t / N_t^* is a positive number less than (or equal to) one, we find that a temporary tax rate change has an effect on investment which is smaller than, but in the same direction as, the effect of a permanent tax rate change of the same magnitude. Furthermore, for a given date t , the longer

⁴Note that in general this neutrality condition ($D_t^* = 1-k$) differs from the Hall-Jorgenson neutrality condition which is $k + \tau D_t^* = \tau$. The condition in the text is the condition for ceteris paribus changes in τ to have no effect on investment. The Hall-Jorgenson neutrality condition is the condition for the existence of taxes to have no effect on investment. See Abel (1980c).

the policy is expected to last the greater will be \hat{N}_t/N_t^* . Thus a longer-lived policy will lead to a larger effect on the rate of investment at time t . Note that these results depend on the assumption that the depreciation allowance schedule has a nondistortionary age profile.

Now consider the case in which the depreciation allowance schedule is such that

$$(III.8) \quad \frac{\hat{D}_t}{D_t^*} > \frac{\hat{N}_t}{N_t^*} \quad 0 \leq t < t^*.$$

If (III.8) holds, the depreciation allowance schedule will be said to have an accelerated age profile. In the subsequent discussion, we restrict our attention to cases in which $D_t^* \leq 1-k$ so that $(\partial q_t / \partial \tau)^P \leq 0$, i.e., the situation in which permanent tax rate cuts stimulate investment. Combining (III.5) and (III.8), we find that with an accelerated age profile of depreciation allowances

$$(III.9) \quad \frac{\partial q_t}{\partial \tau_1} > \frac{\hat{N}_t}{N_t^*} \left(\frac{\partial q_t}{\partial \tau} \right)^P.$$

If the profile of depreciation allowances is sufficiently accelerated, it is possible for $\partial q_t / \partial \tau_1$ to be positive even though $(\partial q / \partial \tau)^P$ is negative. That is, with a sufficiently accelerated depreciation allowance, a temporary tax decrease can actually decrease investment, even though a permanent tax decrease would raise investment. The reason is that the tax saving due to the depreciation allowance is lower with a lower tax rate. If depreciation allowances are disproportionately heavy at the beginning of the life of capital, the reduced tax saving from the depreciation allowance may induce a fall in investment. In terms of Figure 1, a tax rate decrease in the presence of a disproportionately large value of \hat{D}_t will lead to a larger upward shift in the marginal cost schedule than in the present value

of after tax rentals; hence investment will fall.⁵

Recall from (III.4) that if $D_t^* = 1-k$, then $(\partial q_t / \partial \tau)^P = 0$ and permanent tax rate changes are neutral with respect to investment. However, if permanent tax changes are neutral, then from (III.9) it is clear that a temporary tax rate increase will increase the rate of investment if the age profile of depreciation allowances is accelerated. This result contrasts with the fact that with a non-distortionary age profile, temporary tax rate changes are neutral if and only if permanent changes are neutral.

Auerbach and Jorgenson (1980) have recently proposed to allow firms an immediate write-off of some fraction of capital expenditure with no subsequent depreciation deductions.⁶ Under this proposal, \hat{D}/D^* is equal to one, which corresponds to the most accelerated age profile of depreciation deductions. Note from III.5 that for a given stream of rentals and a given present value of real depreciation deductions, $\partial q / \partial t$ is maximized for \hat{D}/D^* equal to one. Thus, the Auerbach-Jorgenson proposal maximizes the tendency for temporary tax cuts to reduce rather than stimulate investment.

A decelerated age profile of depreciation allowances is such that

$$(III.10) \quad \frac{\hat{D}_t}{D_t^*} < \frac{\hat{N}_t}{N_t^*} \quad 0 \leq t \leq t^*.$$

With a decelerated profile of depreciation allowances we obtain

$$(III.11) \quad \frac{\partial q_t}{\partial \tau_1} < \frac{\hat{N}_t}{N_t^*} \left(\frac{\partial q_t}{\partial \tau} \right)^P$$

⁵Eisner (1969) points out that temporary tax increases can raise investment if the depreciation allowance is sufficiently accelerated.

⁶See Abel (1980c) Section V for a dynamic analysis of investment behavior under the Auerbach-Jorgenson depreciation allowance scheme.

In this case, if $(\partial q/\partial \tau)^P$ is negative, $\partial q_t/\partial \tau_1$ is also negative. If the profile is sufficiently decelerated, then $\partial q_t/\partial \tau_1$ could be algebraically less than $(\partial q_t/\partial \tau)^P$. That is, if the effect of a permanent tax cut is to increase investment, the effect of a temporary tax cut will also be to increase investment but by even more than the permanent tax cut if the depreciation allowance profile is sufficiently decelerated. Recall from Figure 1 that a tax cut reduces the tax savings associated with the depreciation allowance and shifts the net marginal cost curve upward. This upward shift tends to dampen the increase in investment. However, if \hat{D} is small, the upward shift in the marginal cost curve is small and hence the dampening effect is small. Thus a temporary tax cut can cause a larger increase in investment than a permanent tax cut.

IV. Depreciation Allowance Schedules with Non-distortionary Profiles

In Section III we demonstrated that with a non-distortionary age profile of depreciation allowances, the effect of a temporary tax rate change on investment is in the same direction as, but smaller in magnitude than, the effect of a permanent tax rate change. Without a non-distortionary profile of depreciation allowances, temporary tax rate changes can have larger effects than, or effects in the opposite direction from, the effects of permanent tax rate changes. In this section we specify non-distortionary depreciation allowance schedules which correspond to various patterns of physical depreciation.

Recall from equation (III.6) that a non-distortionary profile of depreciation allowances satisfies

$$(IV.1) \quad \frac{\hat{D}_t}{D_t^*} = \frac{\hat{N}_t}{N_t^*} \quad 0 \leq t < t^*$$

This condition requires that the time profile of deductions be proportional

to the time profile of marginal value products of capital. Thus, if the schedule of depreciation deductions for capital installed at time t is

$$(IV.2) \quad DD_t(s-t) = \lambda p_s \phi_s m(s-t), \quad s \geq t,$$

for some positive constant λ , then the age profile will be non-distortionary.

This may be verified by noting that in this case $\hat{D}_t = \lambda \hat{N}_t$ and $D_t^* = \lambda N_t^*$ so that $\hat{D}_t/D_t^* = \hat{N}_t/N_t^*$.

Given that (IV.2) specifies a non-distortionary profile of depreciation deductions, we can now specify non-distortionary depreciation allowance schedules for specific patterns of physical depreciation. To simplify the analysis we assume that the output price P_t and the marginal product of capital ϕ_t are constant over time. First consider capital which depreciates at a constant proportional rate so that the fraction of capital which survives to age x is

$$(IV.3) \quad m(x) = e^{-\delta x}.$$

In this case, the non-distortionary depreciation allowance schedule is proportional to $e^{-\delta x}$ which is the same as the depreciation allowance schedule often used in the literature.⁷

Now consider capital which is subject to straight line physical depreciation. That is, capital depreciates by a constant absolute amount over time. In this case, the output produced by a given unit of capital declines linearly over its life. Letting L be the life of a unit of capital, we obtain

$$(IV.4) \quad m(x) = \begin{cases} 1 - x/L & 0 \leq x \leq L \\ 0 & x > L \end{cases}.$$

⁷This form of depreciation allowance is used by Hall and Jorgenson (1967, 1971), Auerbach (1979), and Abel (1980a). See Abel (1980c) Section V for a dynamic analysis of investment behavior under this particular depreciation schedule.

Since a non-distortionary depreciation allowance schedule is proportional to $m(x)$, we find that sum-of-years-digits (SYD) depreciation, not straight-line (SL) depreciation, yields a non-distortionary age profile of depreciation deductions for capital with straight line physical depreciation. Although SYD allowances are more accelerated than SL allowances, SYD allowances are not accelerated in the sense defined in Section III, if physical depreciation is SL.

To demonstrate further that SYD depreciation allowances are non-distortionary if physical depreciation is SL, consider the case in which the discount rate is zero. In this case the value of a unit of capital is equal to the (undiscounted) sum of its marginal products over its remaining lifetime. With SL physical depreciation we can view capital as a coupon bond with coupons which decline linearly over time. At any point of time value of such a coupon bond is equal to the sum of its remaining coupons. The loss in value of this bond per unit time is equal to the value of the coupon which was redeemed during that time. Since the coupons decline linearly over time, the loss in value per unit time, that is, true economic depreciation per unit time, declines linearly over time. Thus with a zero discount rate, SYD depreciation allowances are equivalent to true economic depreciation for capital with SL physical depreciation. The use of the term "accelerated depreciation" to describe SYD depreciation seems quite inappropriate in this case. Even if the interest rate is positive, SYD depreciation allowances are non-distortionary in the sense of this paper, although SYD allowances differ from true economic depreciation.⁸ Further discussion of true economic depreciation appears in the Appendix, where it is shown that true economic depreciation has a non-distortionary age profile for a debt-financed firm.

⁸ By "true economic depreciation" we mean the loss in value as discussed by Samuelson (1964).

As a third pattern of physical depreciation, consider a one hoss shay which produces at a constant rate for a length of time L and then disappears. For a one hoss shay we have

$$(IV.5) \quad m(x) = \begin{cases} 1, & 0 \leq x \leq L \\ 0, & x > L \end{cases}$$

In this case SL depreciation deductions are non-distortionary. Even though there is no physical depreciation until age L , a one hoss shay unit of capital does lose value as it ages. As explained in the case of SL physical depreciation, a non-distortionary profile of depreciation allowances reflects the true loss in economic value which would occur if the interest rate were zero. Hence, even in the absence of physical depreciation, there must be positive depreciation deductions to achieve a non-distortionary profile.

In this section we have shown that three methods of calculating accounting depreciation --proportional (declining balance), sum-of-years-digits, and straight line--represent non-distortionary age profiles for three commonly used patterns of physical depreciation--proportional, straight line, and one hoss shay, respectively. We must note, however, that the definition of non-distortionary depreciation allowances was based on the assumption that the after-tax discount rate is invariant to changes in the tax rate, as is the case for an equity-financed firm. In the Appendix, we examine the case in which the pre-tax discount rate is invariant to the tax rate so that changes in the tax rate cause the after-tax discount rate to move in the opposite direction, as is the case for a debt-financed firm.

V. The Effect of Inflation on the Efficacy of Tax Rate Changes

Since depreciation deductions are based on historical cost, an increase in the rate of inflation will reduce the real value of future depreciation deductions and hence will reduce the real present value of depreciation deductions D^* . By increasing the net cost of investment, this reduction in D^* tends to reduce the optimal rate of investment.⁹ This contractionary effect of inflation on investment could be offset by indexing depreciation deductions or by allowing faster write-offs of capital, as discussed by Feldstein (1979). Feldstein finds that for moderate values of the real discount rate and the rate of inflation, and for reasonable lives of capital, the 10-5-3 proposal for faster write-off of capital is similar to indexing depreciation deductions. More precisely, the two proposed reforms have similar effects on D^* but, as will be shown below, they have quite different effects on the shape of the age profile of real depreciation deductions. As explained in Section III, the shape of the age profile of depreciation deductions is important for assessing the relative impact on investment of permanent and temporary tax rate changes. In this section, we examine the impact of inflation on the age profile of depreciation deductions and the implications for the efficacy of short-run fiscal policy.

In order to simplify the analysis, we assume that the equity-financed firm faces a constant nominal discount rate r and a constant rate of inflation π over the entire future. Therefore, the real discount

⁹ The reduction in D^* also distorts the choice of durability of capital, as discussed in Auerbach (1979), Kopcke (1979) and Abel (1980a).

rate $i = r - \pi$ is constant over time. Normalizing the price level so that $p_s = e^{\pi(s-t)}$, $s \geq t$ (and assuming the marginal product, ϕ_s , is unity for all s), equation (II.8) can be rewritten as

$$(V.1) \quad \begin{aligned} \text{a.} \quad \hat{D}_t &= \int_t^{t^*} DD(s-t)e^{-r(s-t)} ds = \int_t^{t^*} (e^{-\pi(s-t)} DD(s-t))e^{-i(s-t)} ds \\ \text{b.} \quad D_t^* &= \int_t^{\infty} DD(s-t)e^{-r(s-t)} ds = \int_t^{\infty} (e^{-\pi(s-t)} DD(s-t))e^{-i(s-t)} ds \\ \text{c.} \quad \hat{N}_t &= \int_t^{t^*} m(s-t)e^{-i(s-t)} ds \\ \text{d.} \quad N_t^* &= \int_t^{\infty} m(s-t)e^{-i(s-t)} ds. \end{aligned}$$

Note that in the expressions for \hat{D}_t and D_t^* , the nominal depreciation deductions $DD(s-t)$ are discounted by the nominal discount rate or, equivalently, the real depreciation deductions $e^{-\pi(s-t)} DD(s-t)$ are discounted by the real discount rate. In the expressions for \hat{N}_t and N_t^* , the real marginal products of capital are discounted by the real discount rate.

To see the effect of the rate of inflation, consider the following depreciation deduction scheme which would be non-distortionary in the presence of a constant price level

$$(V.2) \quad DD(x) = \lambda m(x).$$

Under this depreciation deduction scheme, we would obtain

$$(V.3) \quad \begin{aligned} \text{a.} \quad \hat{D}_t &= \lambda \int_t^{t^*} e^{-\pi(s-t)} m(s-t)e^{-i(s-t)} ds \\ \text{b.} \quad D_t^* &= \lambda \int_t^{\infty} e^{-\pi(s-t)} m(s-t)e^{-i(s-t)} ds. \end{aligned}$$

If the rate of inflation were zero, then $\hat{D}_t = \lambda \hat{N}_t$ and $D_t^* = \lambda N_t^*$ so that the

depreciation allowance scheme would have a non-distortionary age profile as claimed above. However, with a positive rate of inflation, the age profile of real depreciation deductions $\lambda e^{-\pi x} m(x)$ would slope downward (as a function of age x) more steeply than the age profile of real marginal products so that $\hat{D}_t/D_t^* > \hat{N}_t/N_t^*$. That is, with a positive rate of inflation, the age profile of depreciation deductions becomes accelerated. With an accelerated age profile of depreciation allowances, it is possible for a temporary tax cut to reduce investment, even in situations in which a permanent tax cut would increase investment.

We have shown that inflation has two effects on the schedule of depreciation deductions: (1) an increase in the rate of inflation reduces the real present value of depreciation deductions D^* ; and (2) an increase in the rate of inflation causes the age profile of real depreciation deductions to become more accelerated. Clearly, if nominal depreciation deductions are indexed to the price level, these two effects will disappear. An alternative reform of current depreciation deduction schedules would be to allow a faster write-off of capital. Although allowing a faster write-off of capital would tend to offset the reduction in D^* (see Feldstein 1979), it would tend to exacerbate the accelerated nature of the age profile of depreciation deductions and could reverse the response of investment to temporary tax rate changes. The empirical relevance of this possibility is assessed below, in the context of 10-5-3 depreciation.

Suppose that in line with recently proposed legislation firms receive a 10 percent investment tax credit and are allowed to depreciate equipment based on a five-year tax-life using the sum-of-years method.^{10,11} Also suppose that the equipment has a useful life of L years and depreciates linearly over time.¹² Note that if the life L is equal to five years and if the price level is constant then, as discussed in Section IV, the age pro-

file of depreciation deductions will be non-distortionary. However, if the useful life L exceeds five years and/or if the rate of inflation is positive, then the age profile of depreciation deductions will be accelerated. The effects on q of permanent and temporary tax rate increases are shown in Table I, based on a corporate tax rate of 46%.

Each cell in Table 1 contains three numbers which indicate the effect on q_t of an increase in the tax rate, $\partial q/\partial \tau$, calculated under three alternative sets of assumptions.¹³ The first number in each cell gives the effect on q_t of a tax increase lasting for one year under the assumption that the useful life of capital, L , is five years. The second number in each cell also gives the effect of a one-year tax increase except that L is assumed to equal ten years rather than five years.¹⁴ The third number in each cell gives the effect on q_t of a permanent tax rate increase, based on (III.4).

¹⁰ Under the proposed Capital Cost Recovery Act of 1979 (HR4646 of the 96th Congress), structures could be depreciated in ten years, equipment (except autos and light trucks) could be depreciated in five years, and autos and light trucks could be depreciated in three years. This legislation also proposes a 10 percent investment tax credit on equipment (except autos and light trucks) and a 6 percent investment tax credit for autos and light trucks.

¹¹ Following Feldstein (1979), we use the "half-year" convention in which 50 percent of a full year's depreciation is taken during the first year. Therefore, the annual depreciation deductions are 20, 32, 24, 16, and 8 percent of initial cost in years one through five, respectively.

¹² We also adopt the "half-year" convention for examining the marginal product of capital. Therefore, the specification of the age profile of marginal products $m(x)$ is $m(1) = \omega/L$ and $m(x) = (2\omega/L^2)(L + 1 - x)$ for $x=2, \dots, L$, where ω is the undiscounted sum of marginal products over the life of the unit of equipment.

¹³ The numbers in Table 1 for $\partial q/\partial \tau$ give the change in the level of q per unit change in τ . To obtain the effect of a five point increase in the tax rate from .46 to .51, the numbers in Table 1 must be multiplied by .05. Observe that we have normalized ω (defined in footnote 12.) to be 1.0; the implied value of q_t ranges from .56 to 1.16. Note, however, that ω is simply a scale factor. A slightly higher value of ω would yield values of q that appear more empirically reasonable. If ω is increased, then all entries in Table I are increased proportionately.

TABLE I

EFFECTS of TAX RATE CHANGES on q

| | | $\pi = 0$ | $\pi = 2\%$ | $\pi = 4\%$ | $\pi = 10\%$ | $\pi = 20\%$ |
|-------|------|-----------|-------------|-------------|--------------|--------------|
| i=1% | L=5 | .0723 | .0373 | .0098 | -.0449 | -.0915 |
| | L=10 | .2831 | .2387 | .2034 | .1310 | .0663 |
| | | .3560 | .1155 | -.0704 | -.4297 | -.7146 |
| i=3% | L=5 | .0230 | -.0026 | -.0231 | -.0650 | -.1021 |
| | L=10 | .2075 | .1754 | .1494 | .0946 | .0437 |
| | | .1098 | -.0669 | -.2063 | -.4832 | -.7120 |
| i=4% | L=5 | .0033 | -.0187 | -.0365 | -.0734 | -.1066 |
| | L=10 | .1773 | .1498 | .1273 | .0794 | .0341 |
| | | .0156 | -.1372 | -.2586 | -.5031 | -.7089 |
| i=5% | L=5 | -.0138 | -.0328 | -.0483 | -.0808 | -.1107 |
| | L=10 | .1511 | .1274 | .1079 | .0658 | .0254 |
| | | -.0637 | -.1964 | -.3027 | -.5193 | -.7049 |
| i=7% | L=5 | -.0416 | -.0561 | -.0679 | -.0934 | -.1176 |
| | L=10 | .1082 | .0904 | .0755 | .0428 | .0104 |
| | | -.1873 | -.2887 | -.3710 | -.5425 | -.6945 |
| i=9% | L=5 | -.0629 | -.0740 | -.0833 | -.1034 | -.1232 |
| | L=10 | .0751 | .0615 | .0500 | .0243 | -.0020 |
| | | -.2756 | -.3543 | -.4190 | -.5564 | -.6819 |
| i=10% | L=5 | -.0717 | -.0815 | -.0896 | -.1076 | -.1255 |
| | L=10 | .0614 | .0495 | .0393 | .0164 | -.0073 |
| | | -.3099 | -.3795 | -.4371 | -.5607 | -.6750 |

Note that the changes examined in Table 1 are permanent and temporary changes in the corporate tax rate. It is assumed that the schedule of depreciation deductions is, and always will be, given by the 10-5-3 proposal, with a five year life; we are not examining temporary depreciation allowance policies.

Examination of Table 1 reveals several striking features of the effects on q_t of a change in τ . Recall from (III.4) that since $N_t^* \geq 0$, the sign of $(\partial q / \partial \tau)^P$ depends only on the sign of $1-k-D^*$, which depends on the nominal interest rate but not on the real interest rate. This fact is evident in Table 1 where $(\partial q / \partial \tau)^P$, the third entry in each cell, is positive if the nominal interest rate is 4 percent or less and is negative if the nominal interest rate is 5 percent or greater. That is, a permanent cut in the corporate tax rate will increase investment if the nominal discount rate is greater than or equal to 5 percent. However, if the nominal discount rate is 4 percent or less, then the present value of depreciation deductions, D^* , is sufficiently large and offers a big enough shield from taxes that a permanent tax rate decrease will actually reduce investment.

Another feature of the effect of permanent tax rate changes can be seen by looking across the rows of Table 1. It is clear that $(\partial q_t / \partial \tau)^P$ is a decreasing function of the rate of inflation, holding constant the real

¹⁴The first two numbers in each cell are calculated based on (III.5). Although we use the half-year convention for marginal products and depreciation deductions, we discount all flows to the beginning rather than the middle of the first year. Thus, for example,

$$D^* = \sum_{i=1}^5 DD(i)(1+r)^{-i} \quad \text{where } DD(i) \text{ are as given in footnote (11).}$$

discount rate.^{15,16} That is, for a given real interest rate, a permanent tax cut tends to have a more expansionary (or equivalently, less contractionary) effect on q and investment as the rate of inflation is increased. More generally, since an increase in the rate of inflation reduces D^* , it increases the tendency for permanent tax cuts to stimulate investment, as is evident from (III.4).

Now consider the case of zero inflation as shown in the first column of Table 1. The first element in each cell in this column is based on the assumption that the useful life of capital, L , is five years which equals the taxable life. Thus, the depreciation deduction scheme has a non-distortionary age profile. Therefore, the sign of $\partial q/\partial \tau$ for a temporary tax change must be the same as the sign of $(\partial q/\partial \tau)^P$ for a permanent tax rate change, as indicated by (III.7). Now consider the second element in each cell of column 1, which is based on the assumption that the useful life of capital, L , is ten years and the taxable life is 5 years. In this case, the depreciation deduction schedule has an accelerated age profile, and as explained in Section III, it could be the case that a temporary tax cut is contractionary even though a permanent tax cut would be expansionary. This situation arises in the first column of Table 1 when the real discount rate is greater than or equal to 5 percent.

¹⁵ It can be shown that $\frac{\partial}{\partial \pi} (\partial q/\partial \tau)^P = N^* D^* / (1-k-\tau D^*)^3$. $\{(1-2\tau)(1-k) + \tau D^*\}$ where D^* is $\partial D^*/\partial \pi$. Since $D^* < 0$, $\frac{\partial}{\partial \pi} (\partial q/\partial \tau)^P$ has the opposite sign as the term in curly brackets. (We assume that $(1-k-\tau D^*)$ is positive so that the cost of investment^P is positive.) For the parameter values $\tau = 0.46$ and $k = 0.1$, $\frac{\partial}{\partial \pi} (\partial q/\partial \tau)^P$ is negative.

¹⁶ It is also true that for the real rates of interest and the rates of inflation displayed in Table I, $\frac{\partial}{\partial \pi} (\partial q/\partial \tau) < 0$.

$$\text{Let } g = \frac{\partial q}{\partial \tau} = \frac{-N^*}{1-k-\tau D^*} \left[\frac{\hat{N}}{N^*} - \frac{(1-\tau)\hat{D}}{1-k-\tau D^*} \right]. \text{ It can be shown that}$$

$$\frac{\partial g}{\partial \pi} = \frac{\tau D^*}{1-k-\tau D^*} \left\{ g + \frac{(1-\tau)N^*}{\tau D^*} \left[\frac{\hat{D}}{\pi(1-k-\tau D^*)} + \tau \hat{D} \frac{D^*}{\pi} \right] \right\} \text{ where } D^* = \frac{\partial D^*}{\partial \pi} < 0 \text{ and}$$

It was suggested above that the presence of inflation will change a non-distortionary age profile of nominal deductions into an accelerated profile and could lead to a situation in which a permanent tax rate cut stimulates investment but a temporary tax rate cut actually reduces investment. In terms of Table 1, this effect would appear as a cell in which the first number is positive and the third number is negative. This effect is evident in only one cell ($i = 1\%$, $\pi = 4\%$). Thus, for the depreciation allowance schedule underlying Table 1, and for temporary tax rate cuts lasting one year, this particular effect of inflation is possible but not widespread.

Directing attention to capital with a ten-year useful life, note that, except for two cases in which the rate of inflation is 20 percent, a temporary tax cut will reduce investment (the second entry in all but two cells is positive). However, if instead of shortening the taxable life of ten-year capital as in Table 1, depreciation deductions were based on the sum-of-years digits method with a ten-year life and with indexing, the effects of temporary tax rate changes would be quite different. The indexing scheme in this example would yield a non-distortionary age profile of depreciation deductions, even in the presence of inflation. Thus the effect on q_t of temporary tax rate changes would be in the same direction as the effect of permanent tax rate changes. Hence, for nominal interest rates greater than or equal to 5 percent, a temporary tax rate

$\hat{D}_\pi = \frac{\partial \hat{D}}{\partial \pi} < 0$. Note that $g > 0$ is a sufficient condition for $\frac{\partial g}{\partial \pi} < 0$. A necessary and sufficient condition for $\frac{\partial g}{\partial \pi} < 0$ is $\frac{\hat{N}}{N^*} - \frac{1-\tau}{\tau} \frac{\hat{D}_\pi}{D_\pi^*} < \frac{2(1-\tau)\hat{D}}{1-k-\tau D^*}$. For the real rates of interest and rates of inflation displayed in Table 1, the maximum value of \hat{N}/N^* is .2032 for $i = 1$ percent and $L =$ five years; the minimum value of $\frac{2(1-\tau)\hat{D}}{1-k-\tau D^*}$ is .2532 for $r = 30$ percent. Since $\hat{D}_\pi/D_\pi^* > 0$, it is clear that $\frac{\partial g}{\partial \pi} < 0$ for the range of real interest rates and inflation rates displayed in Table 1.

reduction would stimulate investment in capital with a ten-year life, in contrast to the effect displayed in Table 1.¹⁷

The most disconcerting feature of Table 1 from the point of view of countercyclical fiscal policy is that for most reasonable values of the real interest rate and the rate of inflation, the first and second entries in each cell are of opposite sign. That is, temporary tax cuts have opposite effects on investment with 5-year and 10-year useful lives. Temporary tax cuts tend to stimulate investment in less durable capital but reduce investment in more durable capital. With these opposing effects it becomes extremely difficult to assess the effect of countercyclical tax policy on total investment in short-lived and long-lived capital.

VI. Concluding Remarks

If temporary changes in the corporate tax rate are to be considered in the formulation of countercyclical fiscal policy, then the age profile of depreciation deductions and the age profile of the marginal product of capital must be carefully analyzed. If the age profile of depreciation deductions is non-distortionary, then temporary tax cuts will stimulate investment if and only if a permanent tax cut would stimulate investment. However, if the age profile of depreciation deductions is accelerated, then temporary tax cuts could be contractionary even though a permanent cut would be expansionary.

¹⁷Under the sum-of-years digits method based on a 10-year life, $D^* = .9040$ if the nominal discount rate r is 4%, and $D^* = .8824$ if r is 5%. With a 10% investment tax credit, $(\partial q / \partial \tau)^P$ is positive if $r \leq 4\%$, and $(\partial q / \partial \tau)^P$ is negative if $r \geq 5\%$.

It is well known that increases in the rate of inflation reduce the present value of real depreciation deductions and thereby inhibit investment. Three reforms of the current nominal depreciation allowance laws have been proposed: (1) the Conable-Jones 10-5-3, (2) the Auerbach-Jorgenson immediate write-off scheme and (3) indexing. The first two of these three proposals would exacerbate the accelerated nature of real depreciation deductions induced by inflation and could lead to the situation in which a temporary tax cut would reduce investment in very durable capital, even though a permanent tax cut would stimulate investment in this durable capital. One cannot simply recommend a temporary tax rise to stimulate investment in very durable capital since this policy would reduce investment in short-lived capital. Temporary tax rate changes are plagued by the fact that they have opposite effects on investment in long-lived and short-lived capital under two of the currently proposed reforms to depreciation allowances.

Appendix

In the text we analyzed the investment behavior of an equity-financed firm with an after-tax discount rate invariant to changes in the corporate tax rate. This appendix extends the analysis to the case of a debt-financed firm with after-tax discount rate $(1-\tau)r$. We assume that the pre-tax discount rate r is invariant to changes in the corporate tax rate so that changes in τ induce opposite changes in the after-tax discount rate. It will be shown that if depreciation allowances are equal to economic depreciation, then the age profile of depreciation deductions is non-distortionary. In general, the non-distortionary profile of depreciation deductions will differ according to whether the firm is equity-financed or debt-financed.

The analysis of this Appendix extends Samuelson's (1964) tax invariance theorem to the case in which the corporate tax rate varies over time. Although the extension is straightforward, it merits attention since Samuelson suggested that his analysis does not apply to the case of time-varying tax rates.¹⁸ In order to conduct the analysis in a framework similar to Samuelson's, we rewrite (II.9b) as

$$(A.1) \quad (1-k)q_t = R_t$$

$$\text{where } R_t = \int_t^{\infty} (1-\tau_s) p_s \phi_s m(s-t) \rho_s^* (\rho_t^*)^{-1} ds + q_t \int_t^{\infty} \tau_s DD_t(s-t) \rho_s^* (\rho_t^*)^{-1} ds$$

$$\text{and } \rho_s^* = e^{-\int_0^s (1-\tau_v) r_v dv} \quad \text{is the after-tax discount factor.}$$

¹⁸Samuelson (1964, p. 605) states "All of this analysis presupposes a tax rate that is uniform over time for each person. Obviously, if a man is to be subject to different rates, with T being a function of time, his optimal decision will be distorted by this fact."

To interpret (A.1), recall from (II.9a) that at the optimal rate of investment q_t is equal to the marginal cost of investment, so that $(1-k)q_t$ is the marginal cost of investment net of the investment tax credit. Note that R_t is the value of installed capital, including the value of tax savings due to the depreciation allowance. We will show that if the depreciation deduction, $q_t DD(s-t)$, equals the decrease in the value of the unit of capital at time s , then R_t is invariant to the tax rate. If the right-hand side of (A.1) is invariant to changes in the tax rate, then q_t must also be independent of the time path of the tax rate.

We begin by defining W_s to be the value at time s of capital installed at time t , $t \leq s$, calculated under the assumption that the tax rate τ is identically zero.

$$(A.2) \quad W_s = \int_s^{\infty} p_v \phi_v m(v-t) \rho_v (\rho_s)^{-1} dv$$

where $\rho_v = e^{-\int_0^v r_t dt}$ is the pre-tax discount factor.

Differentiating W_s with respect to time, and denoting this time derivative as \dot{W}_s , we obtain

$$(A.3) \quad \dot{W}_s = -p_s \phi_s m(s-t) + r_s W_s.$$

Observe that $-\dot{W}_s$ is the loss in value at time s of a unit of capital installed at time t . Following Samuelson (1964), we set the depreciation deduction $q_t DD(s-t)$ equal to $-\dot{W}_s$. Using this specification of depreciation allowances, and using (A.3) to substitute for $m(s-t)$ in (A.1), we can calculate the value of capital, in the presence of taxes, at time $t' \geq t$ as

$$(A.4) \quad R_{t'} = \int_{t'}^{\infty} (1-\tau_s) r_s W_s \rho_s^* (\rho_{t'}^*)^{-1} ds - \int_{t'}^{\infty} \dot{W}_s \rho_s^* (\rho_{t'}^*)^{-1} ds.$$

Using integration by parts to simplify the first integral in (A.4), we obtain

$$(A.5) \quad R_{t'} = (\rho_{t'}^*)^{-1} [-\rho_S^* W_S \Big|_{s=t'}^{s=\infty}].$$

Since $\rho_S^* W_S$ approaches zero as s approaches infinity, (A.5) becomes

$$(A.6) \quad R_{t'} = W_{t'}, \quad t' \geq t.$$

Since $R_{t'} = W_{t'}$, for all $t' \geq t$, $\dot{R}_{t'} = \dot{W}_{t'}$, and economic depreciation in the presence of taxes, $-\dot{R}_{t'}$, is indeed equal to $-\dot{W}_{t'}$, which was used in the above derivation.

To summarize the analysis above, we have shown that if the loss in value is taken as a depreciation deduction, then R_t is independent of the value of the tax rate, regardless of whether or not the tax rate is constant. Since R_t is independent of the tax rate, (A.1) implies that q_t is independent of the tax rate. Therefore, if depreciation allowances are equal to economic depreciation, investment is unaffected by any changes in the tax rate, whether they are permanent or temporary.

Now consider a depreciation allowance schedule which is proportional to the path of economic depreciation given in (A.3),¹⁹

$$(A.7) \quad DD_t(s-t) = -\lambda \dot{W}_s, \quad \lambda > 0.$$

Using this depreciation allowance schedule, the value at time t' of a unit of capital installed at time t is

$$(A.8) \quad R_{t'} = \int_{t'}^{\infty} (1-\tau_s) r_s W_s \rho_S^* (\rho_{t'}^*)^{-1} ds - \int_{t'}^{\infty} (1-\tau_s (1-\lambda)) \dot{W}_s \rho_S^* (\rho_{t'}^*)^{-1} ds,$$

which is the analogue of (A.4) when we allow λ to differ from unity. Using integration by parts to simplify the first integral, we obtain

$$(A.9) \quad R_{t'} = W_{t'} + (\lambda-1)A_{t'}$$

where $A_{t'} = -\int_{t'}^{\infty} \tau_s \dot{W}_s \rho_S^* (\rho_{t'}^*)^{-1} ds = -\int_{t'}^{\infty} \tau_s \dot{W}_s e^{-\int_{t'}^s (1-\tau_v) r_v dv} ds.$

¹⁹In general, when $\lambda \neq 1$, it is not the case that $\dot{R}_s = \dot{W}_s$ so that the depreciation allowance in (A.7) is not proportional to economic depreciation.

Note that A_t , would be the present value at time t' of the future tax savings accruing to the depreciation allowance, if firms were allowed to deduct \dot{W}_s from taxable income.

In the case in which $\lambda = 1$, (A.9) indicates that the value of R_t , is invariant to the tax rate as explained above. In order to analyze the cases in which $\lambda \neq 1$, we must determine certain characteristics of A_t . We limit our attention to cases in which $\dot{W}_s \leq 0$ for all $s \geq t$ so that depreciation deductions in (A.7) are non-negative. Since τ_s is nonnegative, it follows that $A_t \geq 0$. Note also that a decrease in the tax rate over the time interval from t' to t^* tends to decrease the value of A_t , for two reasons. First a decrease in the tax rate at any point of time decreases the value of tax savings due to the depreciation allowance at that point of time. Furthermore, a decrease in the tax rate increases the after-tax discount rate and hence reduces the present value of future tax savings. Hence, A_t , falls in response to a temporary decrease in the tax rate extending to time t^* . Furthermore, the longer the tax cut is expected to be in effect, the greater will be the decrease in A_t .

Now consider the case in which $\lambda < 1$ so that depreciation deductions are less than $-\dot{W}_s$ at every point in time. In this case, a decrease in the tax rate, whether permanent or temporary, would decrease A_t , and hence would increase R_t and increase investment. Note that the longer the tax cut remains in effect, the larger will be the increase in R_t and the larger will be the increase in investment at time t . In the case in which $\lambda > 1$ so that depreciation allowances are, in a sense, overly generous, we find that a tax cut reduces R_t and actually reduces investment at time t . The reduction in investment at time t will be greater the longer the tax cut is expected to remain in effect.

To summarize the effects of tax rate changes when depreciation allowances are determined according to (A.7), we have found that investment is invariant to both temporary and permanent changes in the tax rate when $\lambda=1$. When $\lambda \neq 1$, the response of investment to a temporary tax rate change is in the same direction as, but smaller in magnitude than, the response of investment to a permanent tax rate change of the same amount. When $\lambda < 1$, tax cuts increase investment, and when $\lambda > 1$, tax cuts reduce investment.

Now that we have determined the general form of a non-distortionary age profile of depreciation allowances, we briefly discuss non-distortionary age profiles corresponding to certain patterns of physical depreciation. As in Section IV, we assume that the price p_t and the marginal physical product of capital, ϕ_t , are constant for all t . If physical depreciation is proportional to the existing stock so that the fraction of capital which survives to age x is given by (IV.3), then economic depreciation is equal to

$$(A.10) \quad -\dot{W}_x = \frac{p\phi\delta}{r+\delta} e^{-\delta x}.$$

In this particular case, the non-distortionary age profiles of depreciation deductions for both debt-financed and equity-financed firms are proportional to the age profiles of physical depreciation and economic depreciation. The fact that these different concepts of depreciation coincide for the case of proportional depreciation should not obscure the fact that these concepts are distinct.²⁰

Consider the case of one loss shay physical depreciation. It can be shown that at age x

²⁰ See Feldstein and Rothschild (1974) for a discussion of different concepts of depreciation.

$$(A.11) \quad -\dot{W}_x = p\phi e^{-r(1-x)}$$

where L is the life of a unit of capital and r is the pre-tax discount rate. According to (A.11), economic depreciation rises over time and reaches its maximum value when capital reaches age L . Clearly, this schedule is not proportional to the flat profile which is non-distortionary in the case of equity financing. Furthermore, neither of these schedules is proportional to the pattern of physical depreciation which is zero everywhere except for a spike at age L .²¹

²¹In the case of straight line physical depreciation, economic depreciation at age x is $\frac{p\phi}{rL} (1 - e^{-r(L-x)})$ where r is the discount rate and L is the length of life of a unit of capital. In general there is no proportionality among any pair of: 1) economic depreciation, 2) physical depreciation, and 3) non-distortionary depreciation deductions in either the ATI or PTI case.

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