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# ON EXPECTATIONS, TERM PREMIUMS AND THE VOLATILITY OF LONG-TERM INTEREST RATES

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## ABSTRACT

The paper first identifies how large must be the range in which *ex ante* yields on long-relative to short-term bonds vary if term premiums are to account for a significant fraction of the variance of the holding-period yields on long-term bonds. The paper then extends Shiller's bound to the case of a time-varying term premium and readily identifies the variance in the term premium necessary to salvage the efficient markets model if the variance of these holding-period yields exceeds the bound implied by the rational expectations model. The role of transactions costs is noted and the possibility explored that evidence of excess volatility need not imply the existence of unexploited profit opportunities under the rational expectations model.

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# On Expectations, Term Premiums and the Volatility of Long-Term Interest Rates

by James E. Pesando*

## I. Introduction

In his classic article, Culbertson (1957) first drew attention to the volatility of holding-period yields on long-term bonds. In view of the relative stability of the interest rate (and hence yield) on short-term securities, he concluded that this volatility is *prima facie* evidence of the inappropriateness of the expectations model. In effect, he argued that the observed volatility is too great to be assigned to the receipt of new information, and that it must be assigned instead to shifts in demands and supplies in the segmented markets for fixed-income securities of different maturities. Only a term premium "story", Culbertson suggested, could account for the observed volatility of the holding-period yields on long-term bonds.

Recently, Shiller (1979) resurrected the question of whether the volatility of these yields is indeed inconsistent with the expectations model. Intuitively, the averaging inherent in the expectations model ought to impart a degree of smoothness to long- relative to short-term rates, suggesting a possible limit to the volatility of the holding-period yields on long-term bonds. By invoking the assumption of rational expectations formation, Shiller is able to show that the variance of the holding-period yield on long-term bonds is bounded by a scalar multiple of the variance of the short-term interest rate. His analysis indicates, contrary to Culbertson's supposition, that volatility *per se* does not discredit the expectations model. For an (annual) long-term interest rate of 8%, Shiller's inequality indicates that the variance of the 90-day holding-period yield

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on long-term bonds can exceed the variance of the 90-day rate by a factor of 25 without violating the expectations model. Nonetheless, Shiller provides evidence that these bounds may frequently be violated. More recently, Singleton (1980) conducts a series of tests to see if this and related bounds are violated by the data, and also finds some evidence of excess volatility.

The purpose of this paper is twofold. The first objective is to examine Culbertson's claim that term premiums can account for the volatility of holding-period yields on long-term bonds. This is accomplished by identifying the range in which *ex ante* yields on short- relative to long-term bonds must vary if term premiums are to account for a target fraction of the observed variance of holding-period yields. This exercise is made timely by the increased visibility of structural models of interest rate determination (Friedman (1977, 1980a, b, c), Masson (1978), Roley (1980)) which appear to assign much - if not most - of the explained variation in long-term interest rates to stock and/or flow variables which reflect the presence of term premiums. The second objective is to extend Shiller's bound to the case of a time-varying term premium. On the assumption that the short-term rate is stationary, the results obtained by Shiller suggest either that economic agents do not form rational expectations or that the expectations model is too restrictive, or both. The paper examines the question of whether the existence of time-varying term premiums can "salvage" market efficiency in the light of such empirical findings. On the assumption that the expectations model is valid, the paper also examines the question of whether this excess volatility implies the existence of unexploited profit opportunities. This issue focusses attention on the potential importance of transactions costs, which are - in the present context - analytically equivalent to the presence of a time-varying term premium.

The paper is organized as follows. In the first section, hypothetical bounds on the range in which *ex ante* yields on long- relative to short-term bonds can vary are motivated and then used to explore, in a perfect foresight model, the variance of holding-period yields on long-term bonds in the presence of time-varying term premiums. Both Canadian and United States data are examined in light of the discussion, and the implications for structural models of interest rate determination are noted. In the second section, Shiller's bound is extended to the case of a time-varying term premium and then applied to these as well as to Shiller's data. The role of transactions costs receives explicit attention at this time. A concluding section completes the paper.

## II. Term Premiums and the Volatility of Holding-Period Yields on Long-Term Bonds

### Time-Varying Term Premiums: Hypothetical Bounds on the Range in Which Ex Ante Yields May Vary

Let $H_{n,t}$ denote the holding-period yield on an n-period bond in period $t$ . Let $E_t$ denote the expectations operator conditional upon information available at time $t$ . Note that the *ex ante* yield at time $t$ on a one-period bond or $H_{1,t}$ is simply the one-period interest rate $R_{1,t}$ . Under the expectations hypothesis, market forces will ensure that the *ex ante* yields on the n-period and on the one-period bond are equal:

$$E_t(H_{n,t}) = R_{1,t} \tag{1}$$

In a world of uncertainty and risk-averse market participants, the expectations model (1) may prove too restrictive. In particular, suppose that the market accords a (known) term premium $(\psi_{n,t})$ to $H_{n,t}$ so that (1) becomes:

$$E_t(H_{n,t}) = R_{1,t} + \psi_{n,t} \tag{2}$$

In (2), the <u>ex ante</u> yields on the n-period and the one-period bonds are no longer forced to equality.

Assume, for the moment, that (1) is valid. Assume also that the distribution of $R_{1,t}$ is stationary, with constant mean $(\mu(R_{1,t}))$ and constant variance $(\sigma^2(R_{1,t}))$. If economic agents had perfect foresight, then $\sigma^2(H_{n,t})$ would equal $\sigma^2(R_{1,t})$. As first emphasized by Culbertson, however, the stylized fact is that $\sigma^2(H_{n,t})$ greatly exceeds $\sigma^2(R_{1,t})$. Thus the assumption of perfect foresight or the assumption that the expectations model is valid, or both, must be relaxed. If the assumption of perfect foresight is to be relaxed, the obvious replacement is the assumption of rational expectations formation. Indeed, Shiller employs a linearized version of (1) in conjunction with the rational expectations hypothesis to explore the possibility that there is excessive volatility in the holding-period yields on long-term bonds. In order to focus on Culbertson's claim that the volatility of the holding-period yields on long-term bonds must be assigned to the presence of term premiums, it is useful to retain the assumption of perfect foresight and to relax the assumption that (1) is valid. In a world of perfect foresight, of course, the theoretical rationale for the existence of term premiums would disappear. This counterfactual assumption, corresponding to the limiting case of no new information, is nonetheless useful for analytical purposes.

In order to assess the credibility of the term premium story, an analytically tractable formulation of the role of term premiums is required. To draw attention to an often neglected issue, and to facilitate the later discussion of transactions costs, it is useful to focus on the range in which the *ex ante* yield on long- relative to short-term bonds is permitted to vary. For an arbitrary positive constant $k$, let the range in which $\psi_{n,t}$ can vary be bounded as follows:

$$-kR_{1,t} \leq \psi_{n,t} \leq kR_{1,t} \qquad (3)$$

Assuming that $R_{1,t}$ is bounded below at zero, which is not restrictive, (3) enables the role of arbitrage forces to be collapsed into the single parameter $k$. The larger is the value of $k$, the weaker is the implicit strength of arbitrage forces across the markets for one-period and n-period bonds. Equation (3) is useful because, when combined with (2), it implies a bound on the range in which $E_t(H_{n,t})$ can vary:[1]

$$(1-k)R_{1,t} \leq E_t(H_{n,t}) \leq (1+k)R_{1,t} \qquad (4)$$

When perfect foresight is invoked, the expectations operator in (4) becomes superfluous, and an expression involving the observed values of $H_{n,t}$ emerges:

$$(1-k)R_{1,t} \leq H_{n,t} \leq (1+k)R_{1,t} \qquad (5)$$

Equation (5) provides the basis for the "mental experiment" designed to reflect on the credibility of the claim that term premiums can account for the observed volatility of holding-period yields on long-term bonds.

In the traditional term structure literature, (4) is motivated as follows. If risk-neutral speculators at the margin impose the expectations solution on the term structure, as argued by Meiselman (1962), then $k$ is equal to zero and (4) reduces to (1). If the markets for one-period and n-period bonds are truly segmented, as argued by Culbertson, then their ex ante yields may diverge sharply and a "large" value of $k$ is suggested. In the one-period capital asset pricing model of Sharpe and Lintner, $E_t(\psi_{n,t})$ equals $B(E_t(R_{m,t}) - R_{1,t})$ where $R_{m,t}$ is the return on the market portfolio and $B$ is the covariance between $H_{n,t}$ and $R_{m,t}$ divided by the variance of $R_{m,t}$. In applying the Sharpe-Lintner model to the bond market, Roll (1971), McCallum (1975) and others have treated $E_t(\psi_{n,t})$ as a constant. This need not, of course, be the case. Assuming constant relative risk aversion and joint normally distributed security returns, Friedman (1980b) demonstrates that the divergence between the expected return on the risky asset (or a market portfolio of risky assets under an appropriate separation theorem) and the risk-free rate depends on the degree of risk aversion of investors, the distribution of wealth among investors heterogeneous in their degrees of risk aversion, and asset supplies.[2] So long as the degree of risk aversion is not small, shifts in the relative supplies of debt instruments and/or shifts in wealth ownership among investors could cause substantial movement in the term premium. Further, as argued in the traditional literature, the term premium goes to zero as the degree of risk aversion of investors becomes small, or as wealth is effectively concentrated in the hands of investors with a low degree of risk aversion. In short, a low degree of risk aversion on the part of investors suggests a narrow range for the term premium and thus a k-value close to zero. A high degree of risk aversion is a necessary (but not sufficient) condition for the term premium to vary over a large range and might thus be reflected in a "large" value for the arbitrage parameter.

The "Mental Experiment"

If the range within which ex ante yields vary must be large in order for term premiums to account for a significant fraction of the observed variance of holding-period yields, then cet. par. the credibility of Culbertson's term premium story is reduced. This observation motivates the "mental experiment" described below. The exercise also draws attention to the fact that structural models of interest rate determination may require these ex ante yields to vary over a considerably wider range than the casual reader might at first suspect.

There are many ways in which (5) can be used to explore this issue. To place the term premium story in a favourable light, it is useful to assume that $\psi_{n,t}$ is distributed uniformly across the interval $-kR_{1,t}$ to $kR_{1,t}$. One can then establish, in the perfect foresight model, how large a value of $k$ is required if the variance of $H_{n,t}$ defined in (5) is to represent a significant fraction of the observed variance of the holding-period yield on long-term bonds.[3] The assumption of a uniform distribution for the term premium is not, of course, designed to be realistic. (In the later discussion, when ex ante yields can vary only due to the presence of transactions costs, it may prove to be more realistic.) Under this assumption, term premiums neither exhibit serial correlation nor are constrained to "change slowly" over time, as is frequently presumed in empirical research.[4]

If term premiums do change slowly and smoothly, then the assumption that they are uniformly distributed across their hypothetical range will lead to an overstatement of the fraction of the variance of holding-period yields that can be assigned to them. The exercise thus seeks to identify the minimum range in which ex ante yields must vary if term premiums are to account for a target fraction of the observed variance of holding-period yields on long-term bonds.

Assume then that $\psi_{n,t}$ is uniformly distributed and let $f(\psi_{n,t}|R_{1,t})$ represent its conditional probability distribution. Then:

$$f(\psi_{n,t}|R_{1,t}) = \frac{1}{2kR_{1,t}} , \quad -kR_{1,t} < \psi_{n,t} < kR_{1,t} \tag{6}$$

$$= 0 , \quad \psi_{n,t} < -kR_{1,t} \text{ and } \psi_{n,t} > kR_{1,t}$$

From (6), it follows that $E(\psi_{n,t}|R_{1,t}) = 0$ and $\text{var}(\psi_{n,t}|R_{1,t}) = \frac{k^2}{3}R^2_{1,t}$. The unconditional mean and variance of $\psi_{n,t}$ follow immediately. Clearly, $E(\psi_{n,t}) = 0$, and it is straightforward[5] to show that $\text{var}(\psi_{n,t}) = \frac{k^2}{3}(\sigma^2(R_{1,t}) + \mu^2(R_{1,t}))$. Clearly, $\text{var}(H_{n,t}) = \text{var}(R_{1,t}) + 2\,\text{cov}(R_{1,t}, \psi_{n,t}) + \text{var}(\psi_{n,t})$. Noting that $\text{cov}(R_{1,t}, \psi_{n,t}) = 0$, this result implies:

$$\sigma^2(H_{n,t}) = \sigma^2(R_{1,t}) + \frac{k^2}{3}(\sigma^2(R_{1,t}) + \mu^2(R_{1,t})) \tag{7}$$

Equation (7) indicates how the variance of $H_{n,t}$ varies with $k$ on the assumptions of (i) perfect foresight in (5) and (ii) the uniform distribution of $\psi_{n,t}$ in (6). Let $\text{var}_k(H_{n,t})$ indicate the variance so calculated. By identifying how large must be the value of $k$ if $\text{var}_k(H_{n,t})$ is to be a significant fraction of the observed $\text{var}(H_{n,t})$, this exercise provides an important perspective on the validity of Culbertson's claim.

Empirical Evidence

Since previous investigations of the volatility of interest rates have used United States and/or United Kingdom data, it was decided to use Canadian data in the present study. For comparative purposes, a set of United States data were employed as well. Equation (7) imposes no special requirements on the data to be selected. The expression used in this paper to translate observed data on long-term interest rates into holding-period yields, however, requires that the long-term bonds be trading at par. The McLeod, Young, Weir 10 Industrials, an index of the yield on long-term corporate bonds in Canada, is a new issue series which should satisfy the above requirement. Two different holding-periods were considered: 30-days and 90-days. Data on 30-day and 90-day finance company paper were the "one-period rates" employed for each of these respective holding-periods. The sample period 1957:1 to 1979:1 was used for the quarterly data, while the absence of data on 30-day finance company paper prior to 1960 required a corresponding reduction in the length of the sample period for the monthly data. No attempt was made to incorporate the most recent data into the analysis since the sharp rise in long-term rates in the second half of 1979 and early 1980 resulted in the virtual absence of new issues, while rendering untenable the assumption that issues in the existing index were trading near par. Finally, to provide an international perspective, data on new long-term AA utility bonds in the United States together with the 90-day commercial paper rate were also analyzed for the shorter period, 1960:1 - 1979:1.[6]

For an n-period bond trading at par, the holding-period return during period t - consisting of the interest payment and the capital gain or loss - is approximately equal to:[7]

$$H_{n,t} = R_{n,t} - \left(\frac{R_{n-1,t+1} - R_{n,t}}{R_{n-1,t+1}}\right) * \left(1 - \frac{1}{(1+R_{n,t})^n}\right) \quad (8)$$

The capital gain/loss term on the r.h.s. of (8) reflects the fact that an n-period bond at time $t$ becomes an n-1 period bond at time $t+1$. This distinction is ignored in the implementation of (8), which is applied to the average yield in each of the respective indexes. For a consol $(n = \infty)$, this capital gain or loss is just the negative of the percentage change in the long-term rate. Finally, (8) requires an estimate of the term to maturity of the respective indexes of long-term interest rates. Such estimates, in general, are clouded by the existence of call options and sinking funds. For the McLeod, Young, Weir series of long-term corporate rates, a representative maturity of 17 years appeared to be appropriate.[8] For the series of AA utility rates, which consists of bonds with deferred call protection, a representative term to maturity of 10 years was assumed.

Selected summary statistics for the interest rate data are presented in Table 1. The fact that $var(H_{n,t})$ greatly exceeds $var(R_{1,t})$, thus requiring that (1) and/or the perfect foresight assumption must be relaxed, is readily apparent. For the 90-day holding-period, $var(H_{n,t})$ exceeds $var(R_{1,t})$ by factors of 26.1 and 29.4 for the Canadian and United States data, respectively. For the 30-day holding-period, the corresponding factor is just under 50. The first question to be addressed, in view of the claim put forward by Culbertson, is whether this volatility can reasonably be assigned to the presence of term premiums.

The ratios of $var_k(H_{n,t})$ to $var(H_{n,t})$ for k-values from 0.0 to 3.0 are presented in Table 2. If the term premium is distributed uniformly across its permitted range, then a k-value in excess of two is required if term premiums are to account for even 50% of the observed variance of $H_{n,t}$ for 90-day holding-periods. A k-value of two implies that arbitrage forces are sufficiently weak that the <u>ex ante</u> yield on long-term bonds can vary from between minus one to three times the <u>ex ante</u> yield on short-term bonds. For the reasons cited earlier, this is clearly the <u>minimum</u> range within which <u>ex ante</u> yields must vary if term premiums are to account for 50% of $\mathrm{var}(H_{n,t})$. For the monthly data, a k-value of three is required to achieve this 50% target. This result is a direct reflection of the higher value (expressed at an annual rate) of $\mathrm{var}(H_{n,t})$ for the monthly data. To many, the results reported in Table 2 are likely to strain the credibility of Culbertson's term premiums story.

As noted, structural models of interest rate determination, which consist of equations to explain investors' demands and borrowers' supplies together with appropriate market-clearing conditions, have recently attained increased visibility. These models typically isolate asset stocks, flows of funds and other variables identified with the presence of term premiums as important determinants of the long-term rate of interest. This result stands in sharp contrast to the evidence obtained by many researchers (McCulloch (1975), Modigliani and Shiller (1973), Modigliani and Sutch (1967), and Pesando (1975)) that expectations models - with a time-invariant term premium - appear to fit the data well. The preceding discussion suggests that the differences between these two approaches may be sharpened further by focusing on the range within which <u>ex ante</u> yields on long- versus short-term bonds are presumed to vary. In the expectations models, the <u>ex ante</u> yield spread cannot change. In the structural models, the range in which *ex ante* yields vary may typically be large.

In a representative exercise, Friedman (1980b) employs his model of the long-term bond market to assess the impact of the redistribution of savings flows from households to pension funds and finds that "major" effects on the term structure do occur for wealth shifts of plausible magnitude. In his model, and typical of structural models in general, the impact of such shifts is the same regardless of whether or not they are anticipated. The implication is that the *ex ante* yield on long-term bonds, which may contain a large and anticipated capital gain or loss, can diverge sharply from the *ex ante* yield on short-term bonds.[9] Indeed, these experiments suggest that much of the movement in long-term rates - and thus in the holding-period yields on long-term bonds - explained by the model may be due to term premiums. If so, the results reported in Table 2 *imply* that substantial *ex ante* yield variation must occur within the structural model. Thus the simulation exercises not only draw attention to a neglected feature of structural models, which may well sharpen their contrast to the expectations-oriented alternatives, but also suggest that these models permit - and perhaps require - *ex ante* yield spreads to vary over a considerable range. Whether or not the builders of such models intend this to be the result is unclear, since the issue never receives explicit attention.

## III. Extension of Shiller's Bound to the Case of a Time-Varying Term Premium

### The Extended Bound

As noted, Shiller chooses to explore the ceiling imposed on $\text{var}(H_{n,t})$ when (1) is retained, but the assumption of rational expectations formation replaces that of perfect foresight. Using a linearized expression for the holding-period return on long-term bonds $(\tilde{H}_{n,t})$ and assuming (only) that $R_{1,t}$ is stationary, Shiller uses (1) to show that $\text{var}(\tilde{H}_{n,t})$ cannot exceed a scalar multiple of $\text{var}(R_{1,t})$. For an n-period bond, the bound is:[10]

$$\sigma^2(\tilde{H}_{n,t}) \leqq \frac{1}{1-\gamma_n^2}\sigma^2(R_{1,t}) \tag{9}$$

where $\gamma_n = \gamma(1-\gamma^{n-1})/(1-\gamma^n)$, $\gamma = \frac{1}{1+\bar{R}}$, and $\bar{R}$ is the average or normal long-term rate of interest around which the expression for the holding-period yield is linearized. Since both Shiller and Singleton provide evidence that (9) is violated for at least some data sets, the question naturally arises as to whether such violation would prove robust to the relaxation of the assumption that time-varying term premiums do not exist.

In fact, the extension of (9) is quite straightforward since the presence of a term premium is analytically equivalent to an increase in the variance of the short-term rate. The "term-premium-equivalent" short-term rate $(r_{1,t})$, for the purpose of deriving Shiller's inequality restriction, is simply:

$$r_{1,t} = R_{1,t} + \psi_{n,t} \tag{10}$$

Shiller derives his bound by first noting that (1) implies that $H_{n,t} - R_{1,t}$ must have the usual properties of a forecast error under rational expectations formation; that is, $H_{n,t} - R_{1,t}$ must be uncorrelated with information known at time $t$. In particular, since $R_{n,t}$ is known at time $t$, $\text{cov}(H_{n,t} - R_{1,t}, R_{n,t}) = 0$. This covariance restriction is then used to derive (9). Under (2), $H_{n,t} - R_{1,t} - \psi_{n,t} = H_{n,t} - r_{1,t}$ must have the properties of a forecast error, implying that $\text{cov}(H_{n,t} - r_{1,t}, R_{n,t}) = 0$, and Shiller's derivation of (9) generalizes in a straightforward manner to yield:

$$\sigma^2(H_{n,t}) \leq \frac{1}{1-\gamma_n^2}\sigma^2(r_{1,t}) = \frac{1}{1-\gamma_n^2}(\sigma^2(R_{1,t}) + 2\,\text{cov}(R_{1,t}, \psi_{n,t}) + \sigma^2(\psi_{n,t})) \quad (11)$$

Equation (11) could be used, for example, to identify how large $\sigma^2(\psi_{n,t})$ must be - given an assumption about $\text{cov}(R_{1,t}, \psi_{n,t})$ - if the violation of Shiller's original bound for a particular data set is to be reversed through the relaxation of his assumption of a time-invariant term premium.[11]

If an assumption is made about the distribution of the term premium, (11) can be used to identify the range in which the term premium must vary if any evidence of excessive volatility is to be dissipated. Assume again that the term premium is distributed uniformly across its hypothetical range. Since $R_{1,t}$ and $\psi_{n,t}$ are uncorrelated, as noted in the derivation of (7), inequality (11) yields:

$$\sigma^2(\tilde{H}_{n,t}) \leq \frac{1}{1-\gamma_n^2}\sigma^2(r_{1,t}) = \frac{1}{1-\gamma_n^2}[\sigma^2(R_{1,t}) + \frac{k^2}{3}(\sigma^2(R_{1,t}) + \mu^2(R_{1,t}))] \quad (12)$$

Equation (12), in effect, translates the required variance of the term premium identified by (11) into the corresponding k-value implied by the assumption of a uniform distribution of the term premium in (6).

## Empirical Findings

Implementation of Shiller's bound, in either its original or extended form, requires the linearization of the expression for the holding-period yield. This linearized yield $(\tilde{H}_{n,t})$, for a bond trading at or near par, is

$$\tilde{H}_{n,t} = \frac{R_{n,t} - \gamma_n R_{n-1,t+1}}{1 - \gamma_n} \qquad (13)$$

Holding-period yields calculated by (13) are also presented in Table 1. As expected, the returns calculated by (8) and (13) are highly correlated, with correlation coefficients ranging from .9915 to .9965 for the three data sets.

For the 30-day holding-period, Shiller's original bound (9) is not violated (254.7 < 290.7 = 56.4 * 5.15) . For the 90-day holding-period, the bound is violated for both the Canadian (136.9 > 82.3 = 16.9 * 4.88) and the U.S. (156.2 > 71.2 = 14.9 * 4.79) data. Assuming that the sample variances accurately reflect the population parameters, the latter results provide support for Shiller's contention that there appears to be too much volatility in the holding-period yields on long-term bonds to accord with the rational expectations model.

If, for simplicity, it is assumed that $cov(\psi_{n,t}, R_{1,t}) = 0$ ,[12] then (11) readily identifies the variance in the term premium necessary to eliminate evidence of excess volatility. The variance of the term premium necessary to ensure that the equality in (11) obtains is usefully expressed as a fraction of the variance of the short-term rate. For the quarterly Canadian and U.S. data, $\sigma^2(\psi_{n,t})$ must equal 0.66 and 1.20, respectively, of $\sigma^2(R_{1,t})$ if the evidence of excess volatility is to be eliminated. For the six data sets originally examined by Shiller, $\sigma^2(\psi_{n,t})$ must vary between 0.01 to 6.21 of $\sigma^2(R_{1,t})$ to achieve this result (Table 3).

If is is assumed that term premiums are uniformly distributed across their range, then (12) indicates that k-values of 0.48 and 0.68 are necessary to eliminate the evidence of excess volatility for the Canadian and U.S. data examined in this study. For Shiller's data, k-values from 0.05 to 1.59 are necessary to achieve this result, although in only one case must the k-value exceed one. Although the (perhaps) unrealistic assumption of a uniform distribution may - in the present context - understate the k-value necessary to reverse the finding of excess volatility, these results combined with those in Table 2 serve to formalize an important point. One cannot simultaneously maintain that term premiums can account for a significant portion of $\sigma^2(H_{n,t})$ *and* that there is indeed excess volatility in these holding-period yields.

## Transactions Costs

Under the assumption that the expectations model is valid, the question naturally arises as to whether the excess volatility of holding-period yields on long-term bonds implies the existence of unexploited profit opportunities. This question immediately draws attention to the possible importance of transactions costs. If arbitrage agents at the margin face non-trivial transactions costs, then the equality of *ex ante* yields required by (1) need not hold even if the expectations model is valid. There is, in effect, a neutral band within which no additional arbitrage is profitable since transactions costs exceed aribtrage profits. This effect may be captured, for example, by adding a disturbance term $\varepsilon_t$ to the r.h.s. of (1):

$$E_t(H_{n,t}) = R_{1,t} + \varepsilon_t \tag{14}$$

Comparing (14) to (2) highlights the fact that the presence of transactions costs is analytically equivalent to the existence of a time-varying term premium. Now, however, the assumption that the term premium or $\varepsilon_t$ has a uniform distribution becomes realistic, as the ex ante yeild spread may reasonably be presumed to fall with equal likelihood anywhere within the neutral zone created by the existence of transactions costs. This fact suggests that transactions costs be stated in terms of the k-value corresponding to the range within which arbitrage is no longer profitable, so that (12) can be used to determine whether evidence of excess volatility can be explained in terms of the existence of transactions costs.

The bid-asked spread for actively traded corporate bonds in Canada is one-half of a point ($5.00) or ½% of each bond's par value of $1,000.00. Assume that all agents have a holding-period of one quarter, and that the transactions costs associated with buying and selling finance company paper - which are small - are in fact equal to zero. Then the neutral zone in which the ex ante yield on corporate bonds can vary relative to the 90-day paper rate without inducing profitable arbitrage activity is readily determined to be 200 basis points (expressed at an annual rate).[13] This translates into a k-value of 0.30 based on the mean value of the paper rate during the sample period. Using (12), it has already been established that k-values of 0.48 and 0.68 are necessary to eliminate evidence of excess volatility in the Canadian and U.S. data examined in this study. For the quarterly data examined by Shiller, k-values of 0.77 and 1.59 are required. If the transactions costs noted above are amortized over a 30-day holding-period, the equivalent k-value is 0.90. This is greater than the 0.58 required to eliminate the evidence of excess volatility in Shiller's monthly data set. On balance, transactions costs - at least as defined above - would not appear to be of sufficient magnitude to reverse the previous findings. Nonetheless, they clearly weaken existing evidence regarding the excess volatility of holding-period yields under the rational expectations model.

## IV. Conclusion

The simulation experiments indicate that *ex ante* yield spreads must vary over a wide range if term premiums *per se* are to account for a significant fraction of the observed variance of holding-period yields on long-term bonds. They thus identify in a more precise fashion how *weak* arbitrage forces must be if Culbertson's term premium "story" is to be credible. The exercises also highlight the fact that structural models in which much of the explained variance of the long-term rate is due to variables reflecting the presence of term premiums must imply that *ex ante* yield spreads can - *and do* - vary over a wide range.

The data examined in this study provide further support for Shiller's claim that the variance of holding-period yields on long-term bonds appears to be too great to accord with the rational expectations model. The existence of a time-varying term premium or non-trivial transactions costs could in principle reverse this finding. On balance, transactions costs do not appear to be sufficiently large to reverse the evidence of excess volatility, although they clearly weaken it. For the quarterly Canadian and U.S. data examined in this study, term premiums - if uncorrelated with the short-term rate - must have variances equal to 0.66 and 1.20 of the variances of the respective short-term rates if evidence of excess volatility is to be eliminated. For certain of Shiller's data sets, the ratios are somewhat higher. Finally, by focusing again on the range within which *ex ante* yield spreads can vary, the analysis highlights the fact that one cannot simultaneously maintain that term premiums can account for a significant fraction of the variance of holding-period yields on long-term bonds *and* that there is evidence of excess volatility in these holding-period yields.

## Footnotes

[1] One could, of course, add a positive constant (c) to (3) to ensure that the term premium is an average positive:

$$-kR_{1,t} + c \leqq \psi_{n,t} \leqq kR_{1,t} + c \qquad \text{(F1)}$$

Note, however, that the range in which $H_{n,t}$ can vary in (5) would not be altered.

[2] Mossin (1969), Lintner (1970), Friend and Blume (1975) and others have analyzed the determinants of the market price of risk, defined to equal $(E(R_{m,t}) - R_{1,t})/\sigma^2(R_{m,t})$. Their analyses recognize that the market price of risk, and hence the term premium implied by the capital asset pricing model, need not be constant over time.

[3] One could also, for example, derive (simulate) the theoretical (empirical) ceiling on $var(H_{n,t})$ in (5) in order to provide a further perspective on Culbertson's claim. Since the assumption of a uniform distribution already places this story in a favourable - although perhaps unrealistic - light, these further exercises are perhaps unnecessary. They are, however, available from the author upon request.

[4] Modigliani and Shiller (1973), for example, in their well-known tests of the "preferred habitat" model, employ an eight quarter moving standard deviation of the short rate as a proxy for uncertainty and hence for the role of the term premium. There is, however, no consensus that the term premium need necessarily be slow moving. In the context of Friedman

(1980b), discussed previously in the text, the term premium could - at least in principle - fluctuate rapidly in the presence of frequent, transitory shifts in (exogenous) asset supplies and/or in wealth ownership among investors heterogeneous in their degree of risk aversion. See also the discussion of transactions costs in a later section of the text.

5 As is well known (Keeping (1962), p. 398), the unconditional variance of a random variable can be regarded as the sum of two parts, the expectation of the conditional variance and the variance of the conditional expectation. In the present context, the variance of the conditional expectation is zero since $E(\psi_{n,t}|R_{1,t}) = 0$ for all $R_{1,t}$. Thus the unconditional variance of $\psi_{n,t}$ is simply equal to the expectation of the conditional variance, or $\text{var}(\psi_{n,t}) = \int_{R_{1,t}} \frac{k^2}{3} R_{1,t}^2 f(R_{1,t}) dR_{1,t} = \frac{k^2}{3} (\sigma^2(R_{1,t}) + \mu^2(R_{1,t}))$.

6 The McLeod, Young, Weir 10 Industrials are compiled by McLeod, Young, Weir and Company Limited, while the other Canadian data are compiled by the Bank of Canada. All are published in the monthly issues of the Bank of Canada _Review_. The United States data are drawn from Salomon Brothers, _An Analytical Record of Yields and Yield Spreads_. All interest rate data refer to the end of the quarter, or to the end of the month, as appropriate. The implication is that the arbitrage or adjustment period equals the unit of observation.

7 Let c be the coupon paid each period and let F be the face value of an n-period bond. After the summation, the standard valuation formula reduces to:

$$P = \frac{c}{R_n} + \frac{(F - \frac{c}{R_n})}{(1 + R_n)^n} \qquad (F2)$$

Differentiating (F2) and dividing the resulting expression by price yields:

$$\frac{dP}{P} = \frac{\left( - \frac{c}{R_n^2} - \frac{nF}{(1+R_n)^{n+1}} + \frac{n(\frac{c}{R_n})}{(1+R_n)^{n+1}} + \frac{\frac{c}{R_n^2}}{(1+R_n)^n} \right) dR_n}{\frac{c}{R_n} + \frac{(F - \frac{c}{R_n})}{(1+R_n)^n}} \qquad (F3)$$

If the bond is selling at par, so that $\frac{C}{F}$ equals $R_n$ , then this expression reduces to:

$$\frac{dP}{P} = \frac{-dR_n}{R_n} * (1 - \frac{1}{(1 + R_n)^n}) \qquad (F4)$$

When this capital gain or loss is added to the interest component $(\frac{c}{F} = R_n)$ of the holding-period return, the discrete analogue is (8) in the text.

[8] The convention used in selecting the average term to maturity of the index is to set the maturity equal to the maturity date if the bond is selling below par, and to the earliest call date if the bond is selling above par. Note also that a 17-year bond implies $n = 4 * 17 = 68$ for a 90-day holding-period in the implementation of (8).

[9] In Friedman (1980b), a $25 billion shift in savings flows from households to pension funds exerts about 2½ times the impact on the term structure of a $10 billion shift. Since this impact is the same regard-

less of whether or not the shifts are anticipated, the implication is that changes in wealth or savings flows can call forth approximately linear changes in *ex ante* yields. Although, as emphasized by Lucas (1976), caution must be exercised in assessing the impact of shocks which are dissimilar to those which occurred in the sample period in which the model was estimated, the size in principle of the *ex ante* yield changes implied by very large savings shifts is potentially troublesome. During the first quarter of the simulation period, for example, the $25 billion savings shift causes the spread between the long- and short-term rates to fall (his Figure 1) by about 90 basis points relative to the control solution. If this were to occur solely by a reduction in the long-term rate, the capital gain - and thus the holding-period yield - when expressed at an annual rate would be very large. (If the long-term rate were a consol - which it is not - and if its level were 9 percent, then the capital gain would be 10 percent or 40 percent at an annual rate.) An attractive, although technically difficult, means of addressing this issue would be to impose a rational expectations solution on the structural model, so that *ex ante* yields changes implied by such shifts would feed simultaneously back to investors' demands.

10 In fact, Shiller derives a tigher bound in which $\rho^2_{rR}(\sigma^2(R_{1,t})$ replaces $\sigma^2(R_{1,t})$ on the r.h.s. of (9), where $\rho^2_{rR}$ is the square of the correlation coefficient between the short- and the long-term rate and is thus less than or equal to one. When (9) is generalized to (11), $\rho^2_{\tilde{r}R}$ then appears as the multiplicative factor, where $\rho^2_{\tilde{r}R}$ is the square of the correlation coefficient between the "term-premium equivalent" short-term rate in (10) and the long-term rate. Since this correlation coefficient -

unlike its predecessor - cannot be estimated from the data, only the bounds reported in the text are employed in the empirical analysis.

11 Following Shiller (1979), one could place a lower bound on $\sigma(\tilde{H}_{n,t})$ based on the $\chi^2$ distribution by assuming that $\tilde{H}_{n,t}$ is normally distributed and uncorrelated over time. This lower bound, rather than $\sigma(\tilde{H}_{n,t})$, could then be used to isolate the variance of the term premium necessary to satisfy (11), or the range identified in (12). Since $(\tilde{H}_{n,t} - R_{1,t})$ alone should be uncorrelated under the rational expectations model and since holding-period yields tend to be leptokurtic, the assumption of a $\chi^2$ distribution for $\sigma(\tilde{H}_{n,t})$ might be questioned. Further, as discussed by Shiller, it is not obvious how to place an upper bound on the r.h.s. of (9) since small sample estimates of $\sigma^2(R_{1,t})$ could understate the true population variance in the presence (say) of long cycles in the short-term rate.

12 Kessel (1965) argues that term premiums should vary positively with the level of short-term interest rates and thus that the covariance term should be positive. Nelson (1970) argues the reverse.

13 Let $a$ be the basis point equivalent of the bid-asked spread on corporate bonds if amortized over an annual holding-period. Assume that the transactions costs involved in buying and selling finance company paper are zero. An investor holding cash at the beginning of the period, who has funds to invest for exactly <u>one quarter</u>, will buy bonds rather than paper if $E_t(H_{n,t}) - R_{1,t} > \underline{4}a$, thus establishing $R_{1,t} + 4a$ as the upper limit on $E_t(H_{n,t})$. An investor holding bonds at the beginning of the period, and again having a holding-period of one quarter, will sell bonds now (rather than at the end

of the period) if $E_t(H_{n,t}) < R_{1,t}$ , thus establishing $R_{1,t}$ as the lower limit on $E_t(H_{n,t})$ . The neutral zone in this particular illustration - which is not centered on the equality of expected returns - is thus 4a, or 200 basis points. If the transactions costs associated with buying and selling finance company paper were equal to ½a , then the upper bound on $E_t(H_{n,t})$ would be $R_{1,t} + 2a$ , while the lower limit would be $R_{1,t} - 2a$ . Thus the neutral zone would remain 4a or 200 basis points.

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Table 1

ALTERNATIVE DATA SETS: SUMMARY STATISTICS

| | CANADA<br>90-day Holding-Period<br>1957:1 - 1979:1 | CANADA<br>30-day Holding-Period<br>1960:1 - 1979:3 | UNITED STATES<br>90-day Holding-Period<br>1960:1 - 1979:1 |
|---|---|---|---|
| $\mu(R_{1,t})$ | 6.16 | 6.07 | 5.72 |
| $\sigma(R_{1,t})$ | 2.21 | 2.27 | 2.19 |
| $\mu(R_{n,t})$ | 7.56 | 7.87 | 6.86 |
| $\sigma(R_{n,t})$ | 1.99 | 1.91 | 1.89 |
| $\mu(H_{n,t})$ | 5.02 | 5.65 | 4.61 |
| $\sigma(H_{n,t})$ | 11.30 | 15.94 | 11.88 |
| $\mu(\tilde{H}_{n,t})$ | 5.28 | 5.92 | 5.14 |
| $\sigma(\tilde{H}_{n,t})$ | 11.70 | 15.96 | 12.50 |
| max $H_{n,t}$ | 32.28 | 73.44 | 31.59 |
| min $H_{n,t}$ | -32.04 | -58.56 | -19.80 |

Notes: $H_{n,t}$ refers to holding-period return calculated by (8); $\tilde{H}_{n,t}$ refers to linearized holding-period return calculated by (13). All rates are expressed in percent per annum.

Table 2

RATIOS OF $var_k(H_{n,t})$ to $var(H_{n,t})$

| k-value | CANADA 90-day Holding-Period | CANADA 30-day Holding-Period | UNITED STATES 90-day Holding-Period |
|---|---|---|---|
| 0.0 | 0.038 | 0.020 | 0.034 |
| 0.1 | 0.039 | 0.021 | 0.035 |
| 0.2 | 0.043 | 0.022 | 0.037 |
| 0.3 | 0.048 | 0.025 | 0.042 |
| 0.4 | 0.056 | 0.028 | 0.048 |
| 0.5 | 0.066 | 0.032 | 0.056 |
| 0.75 | 0.101 | 0.047 | 0.084 |
| 1.0 | 0.149 | 0.068 | 0.123 |
| 1.25 | 0.213 | 0.096 | 0.171 |
| 1.50 | 0.290 | 0.129 | 0.234 |
| 1.75 | 0.380 | 0.168 | 0.306 |
| 2.0 | 0.485 | 0.214 | 0.389 |
| 2.5 | 0.737 | 0.364 | 0.588 |
| 3.0 | 1.044 | 0.516 | 0.832 |

Notes: See text for explanation of notation.

Table 3

VARIANCE OF TERM PREMIUM NECESSARY TO REVERSE SHILLER'S FINDINGS OF EXCESS VOLATILITY

| Data Set/Description | Excess Volatility | Term Premium to Reverse Excess Volatility | | |
|---|---|---|---|---|
| | $(\sigma^2(\tilde{H}_{n,t}) \div \frac{1}{1-\gamma_n^2}(\sigma^2(R_{1,t}))$ | $\sigma^2(\psi_{n,t})$ from (11) | $\sigma^2(\psi_{n,t}) \div \sigma^2(R_{1,t})$ | k-value from (12) |
| 1. U.S., qtly, 1966:1 - 1977:2 | 5.20 | 13.37 | 4.22 | 0.77 |
| 2. U.S., monthly, 1969:1 - 1974:1 | 3.80 | 8.83 | 2.82 | 0.58 |
| 3. U.S., annual, 1960 - 1976 | 7.18 | 11.99 | 6.21 | 0.83 |
| 4. U.S., annual, 1919 - 1958 | 1.02 | 0.04 | 0.01 | 0.05 |
| 5. U.K., qtly, 1956:1 - 1977:2 | 5.76 | 38.80 | 4.81 | 1.59 |
| 6. U.K., annual (avg.), 1824 - 1929 | 1.12 | 0.18 | 0.13 | 0.29 |

Notes: See Shiller (1979), Table 1 for detailed results. Calculation of $\sigma^2(\psi_{n,t})$ from (11) is under the assumption that $cov(\psi_{n,t}, R_{1,t}) = 0$. Since $\mu(R_{1,t})$ is not reported by Shiller, calculation of k-value in (12) is under the assumption that mean of the short-term rate is 100 basis points less than the mean of the corresponding long-term rate. All rates are expressed at percent per annum.