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TAXATION, PORTFOLIO CHOICE AND DEBT-EQUITY RATIOS: A GENERAL EQUILIBRIUM MODEL

Alan J. Auerbach

Mervyn A. King

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Cambridge MA 02138

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ABSTRACT

This paper explores the portfolio behavior of investors differing with respect to both tax rates and risk-aversion, emphasizing the role of constraints on individual and firm behavior in ensuring the existence of and characterizing portfolio equilibrium.

Under certain conditions on the securities available in the market, which also are required for shareholders to be unanimous in supporting firm value maximization, investors will be segmented by tax rate into two groups, one specialized in equity and the other in debt. Though the relative wealths of the two groups determines the aggregate debt-equity ratio, each firm will be indifferent to its financial policy.

Alan J. Auerbach
National Bureau of Economic Research
1050 Massachusetts Avenue
Cambridge, Massachusetts 01238

(617) 868-3927

Mervyn A. King
University of Birmingham
Department of Economics
P.O. Box 363
Birmingham, B15 2TT
England

021.472 1301 x3427

Taxation, Portfolio Choice and Debt-Equity Ratios:

A General Equilibrium Model[+]

A.J. Auerbach* and M.A. King**

1. Introduction

Our aim in this paper is to determine under what conditions there exists an equilibrium aggregate debt-equity ratio, and, if such exists, to examine its dependence on the risk preferences and tax rates of the investors in the market. Miller (1977) has conjectured that even if no optimal debt-equity ratio exists for an individual firm, there will be an equilibrium aggregate debt-equity ratio which will equal the relative wealth levels of those with tax preferences for debt as opposed to equity. This is true, however, only in special cases, and we examine a more general model below. In particular, we shall highlight the critical role which constraints (on, for example, personal borrowing or short sales) play in the determination of the market equilibrium.[1] The role of constraints when the tax system is non-neutral has been discussed by Black (1971, 1973), King (1974,1977) and Auerbach (1979).

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*Assistant Professor of Economics, Harvard University and Research Associate, National Bureau of Economic Research.

**Esmee Fairbairn Professor of Investment, University of Birmingham and Research Associate, National Bureau of Economic Research.

[1] In a footnote in his paper, Miller (op. cit.) alludes to the need for some constraints but is not explicit about which constraints are necessary, and his argument relies more on the capitalization of tax differentials than the role of constraints. In contrast, we shall argue that constraints are crucial.

The behaviour of the aggregate debt-equity ratio is important for the incidence of a corporation tax because the effects of such a tax depend on firms' financial policies, and on the change in the capital structure of the corporate sector resulting from a change in the tax. There appear to be very few econometric studies relating the aggregate debt-equity ratio to personal and corporate tax rates. In a study for the U.K., King (1977, chapter 7) found that an increase of one percentage point in the rate of corporation tax would lead to an increase of 1.5 points in the debt-equity ratio for the nonfinancial corporate sector. For the U.S., Gordon (1980) found that an increase in the rate of corporate income tax of one point would lead to a rise in the debt-equity ratio of 0.97 points. The results presented here provide a theoretical rationale for this relationship.

In Section 2 we present the basic model which adopts a simple mean-variance framework. This allows us to compare our results with those of the standard capital asset pricing model without taxes, and affords an explicit solution for market value which illuminates the role of taxes in the model. The importance of the assumption is that we are considering a model in which, in the absence of taxes, a firm is unable by altering its debt-equity ratio to affect the implicit prices of contingent commodities faced by its owners. Hence when there are no taxes the Modigliani-Miller theorem will hold.[2] We then investigate the introduction of taxes into such a world. If a firm may influence the implicit prices faced by its owners even when there are no taxes, then the capital structure of individual firms

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[2]See the discussion in King (1977, chapter 5).

will not be a matter of indifference and, in general, stockholders will disagree over the choice of debt-equity ratio. In other words, the aim here is to examine a model in which, if there were not taxes, both individual and the aggregate debt-equity ratios would be a matter of complete indifference, and we ask whether the existence of taxes leads to an equilibrium for either the individual or aggregate debt-equity ratio.

Taxes were introduced into the capital asset pricing model by Brennan (1970) and Gordon and Bradford (1979). Our contribution is to model the supply of corporate securities and to examine the interaction between demand and supply in a general equilibrium framework in which the interest rate is endogenous.

Section 3 discusses the conditions under which an equilibrium debt-equity ratio exists, and the optimal portfolios of different investors are examined in Section 4. In Section 5 we show that, provided there exist constraints of the appropriate type and there is a sufficiently large number of firms, value maximization is an objective which would command the unanimous support of the stockholders, and each firm would be indifferent as to its debt-equity ratio. The Modigliani-Miller Theorem would hold even in a world of distortionary taxes, but only if investors faced constraints. Finally, we examine the optimal degree of specialization in portfolios by pension funds and individual investors on their own account.

2. The Model

We shall consider a two-period model with M investors and N firms. Each investor has an intial endowment of shares in firms, denoted by $\bar{n}_i^m$. We normalize such that

$$\sum_m \bar{n}_i^m = 1 \qquad \forall j \tag{1}$$

Firms are assumed to have made production and financial decisions before trading in financial assets takes place. The value (V) of each firm is the sum of the debt (D) and equity (E) which it issues

$$V_i = D_i + E_i \tag{2}$$

We shall ignore the possibility of bankruptcy.[3] The debt of all firms is, therefore, riskless, and investors are indifferent as to which firms' debt they hold. There are no bonds in the initial state, and the proceeds of bond issues are returned to the initial stockholders. We may write the budget constraint of investors as

$$D^m + \sum_{i=1}^{N} E_i^m = w^m = \sum_i \bar{n}_i^m V_i \qquad m=1\ldots M \tag{3}$$

where D^m is investor m's holdings of debt, E_i^m is investor m's holding of the equity of firm i = $n_i^m E_i$, and w^m is investor's wealth.

Investor preferences will be assumed to be characterized by a utility function defined over the mean and variance of the terminal value of his portfolio,

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[3]Bankruptcy is modelled in Auerbach and King (1979) and Strebel (1980); since the probability of bankruptcy is endogenous it is very difficult to analyze models of this type when there are many firms because the returns to each security are a truncated distribution.

$$U^m = U(\mu^m, (\sigma^m)^2) \tag{4}$$

If we denote by R the second-period return to corporate debt and by t_c, t_p, and t_e, the tax rates on corporate income, personal interest income and personal equity income respectively, the mean and variance of investor portfolios are given by

$$\mu^m = [w^m - \sum_i E_i^m]R(1-t_p^m) + \sum_i E_i^m[\frac{\mu_i - RD_i}{E_i}](1-t_c)(1-t_e^m) \tag{5}$$

$$(\sigma^m)^2 = \left[\sum_i\sum_j \frac{E_i^m E_j^m}{E_i E_j} C_{ij}\right] (1-t_c)^2(1-t_e^m)^2 \tag{6}$$

where μ_i is the mean pre-tax return to firm i and C_{ij} is the covariance of the returns to firms i and j.

We have assumed here that investors face given tax rates which are not a function of the level of income, and we shall discuss this further in section 3. The tax on equity income represents an effective tax rate based on the tax treatment of dividends and capital gains, and we shall assume that

$$t_e^m \leq t_p^m \qquad \forall m \tag{7}$$

In the absence of a satisfactory theory of dividend behavior it seems preferable to regard all equity income as being taxed at the rate t_e.

As we shall see below, constraints on investors play an important role in determining whether or not an equilibrium exists. There are two sets of constraints which must be considered, those on investors and those on firms.

In turn, there are two sources of arbitrage which give rise to the need for constraints. First, investors face different tax rates and have different marginal rates of substitution between assets. Secondly, firms and individuals have different tax rates producing an incentive to engage in financial transactions to exploit this difference. The constraints which we shall model will be the following. The first is a constraint on personal borrowing, and for simplicity, we assume that investors may not borrow at all.[4]

$$w^m - \sum_i E_i^m \geq 0 \qquad \forall m \tag{8}$$

Secondly, we shall suppose that investors are unable to sell equity short, and we will apply this constraint to an investor's total holdings of equity and not to holdings in individual firms.

$$\sum_i E_i^m \geq 0 \qquad \forall m \tag{9}$$

Finally, we shall impose a constraint on firms such that they may not sell either their own or other firms' equity short, nor may they hold negative amounts of debt

$$0 \leq D_i \leq V_i \qquad \forall i \tag{10}$$

If we attach multipliers λ_1^m and λ_2^m to the constraints (8) and (9) respectively, then we may form the Lagrangian

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[4]It is trivial to include a fixed positive borrowing limit for each investor.

$$\mathcal{L}^m = U(\mu^m, (\sigma^m)^2) + \lambda_1^m (w^m - \sum_i E_i^m) + \lambda_2^m \sum_i E_i^m \tag{11}$$

Substituting (5) and (6) into (11), and assuming investors to be price-takers, the first-order conditions for utility maximization obtained by differentiating with respect to each investor's holdings of equity in each firm may be written as[5]

$$(\mu_i - RD_i)A^m - RE_i B^m = \sum_j \left(\frac{E_j^m}{E_j} \right) C_{ij} \qquad \forall_{i,m} \tag{12}$$

where

$$A^m = \frac{1}{\gamma^m T^m (1-t_p^m)}$$

$$B^m = \left[\frac{1 + \frac{\lambda^m}{R(1-t_p^m)}}{\gamma^m (T^m)^2 (1-t_p^m)^2} \right]$$

$\gamma^m = -2 \frac{U_2}{U_1}$ where U_i is the derivative of U with respect to the i^{th} argument.

$T^m = \frac{(1-t_c)(1-t_e^m)}{(1-t_p^m)}$ which measures an investor's tax preference for equity rather than debt (indifference implies $T^m = 1$).

$\lambda^m = \lambda_1^m - \lambda_2^m$.

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[5]The second-order conditions are satisfied provided U is a concave function of the equity holdings and a suitable constraint qualification is satisfied.

If we sum the first order conditions over individual investors we obtain

$$(\mu_i - RD_i)A - RE_iB = C_i \qquad \forall_i \tag{13}$$

where $A = \sum_m A^m$

$B = \sum_m B^m$

$C_i = \sum_j C_{ij}$, the covariance of the returns to firm i with the total returns in the economy.

If we now sum over firms we have

$$(\mu - RD)A - REB = C \tag{14}$$

where

$\mu = \sum_i \mu_i$

$D = \sum_i D_i$

$E = \sum_i E_i$

$C = \sum_{ij}\sum C_{ij}$

This gives the market interest rate as

$$R = \frac{A}{B}\left(\frac{\mu - \frac{C}{A}}{1 - D(1 - \frac{A}{B})}\right) \tag{15}$$

The equilibrium interest rate is a function of the aggregate debt-equity ratio unless A = B, a condition to which we return below.

From (13) we may solve for the equilibrium market value of each firm

$$V_i = D_i + E_i$$

$$= \frac{A}{B}\left[\frac{\mu_i - \frac{C_i}{A}}{R}\right] + D_i\ (1 - \frac{A}{B}) \qquad (16)$$

Equation (16) is the capital asset pricing model adjusted for taxes and heterogeneous investors. If A = B we obtain the usual valuation result for a mean-variance model in that the value of the firm is simply the risk-adjusted discounted flow of profits and is independent of its capital structure. The necessary and sufficient condition for the value of a firm to be independent of its debt-equity ratio is that A = B.[6] This is also the necessary and sufficient condition for the riskless interest rate to be independent of the amount of debt in the economy. An examination of the conditions under which A = B leads us into a discussion of the existence of a market equilibrium.

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[6]The derivative of market value with respect to debt is $1 - \frac{A}{B}$, which reduces to the results of Modigliani and Miller (1963) and Brennan (1970) when there are no constraints and given their respective assumptions about tax rates.

3. Market Equilibrium

Before trading takes place we suppose that firms announce both their production plans and the amount of debt which they will issue. Under what conditions will an equilibrium exist, and when will there be unique market values for firms which are independent of their debt-equity ratios?

Consider, first, the case of perfect certainty (when $C_{ij} = 0 \; \forall_{i,j}$). It is clear from (12) that in the absence of constraints individual excess demands for securities are unbounded, and hence no equilibrium exists, except in the special case where all investors have values of unity for the tax preference variable T^m. This will occur with either a comprehensive income tax or a comprehensive expenditure tax, but will not be true of any tax system that discriminates between different types of income from capital. Even if investors have identical values for T^m but these differ from unity, no equilibrium is possible unless constraints are placed on firms to prevent tax arbitrage between the personal and corporate sectors. In this case the equilibrium would be for the corporate sector to be all debt or all equity depending on whether T^m was less than or greater than unity, respectively. One route by which values of T^m might be equalized would be for individuals with above-average values of T^m to borrow from investors with below-average values and, provided interest payments were tax-deductible, to continue this process until marginal tax rates were equalized.[7]

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[7]When equity income is viewed more realistically as a combination of dividends and capital gains then in a multi-period model the individual who borrows from other individuals is not necessarily the investor with the highest personal tax rate (this is analyzed in King, 1977, Chapter 6).

There are, however, several objectives to this way of modelling the equilibrium. First, it is rare for the marginal personal tax rate to be a continuous function of taxable income, a property which is required for the previous argument to hold. Secondly, some investors are "endowed" with tax rates that are independent of the level of income; the obvious example here is the existence of tax-exempt institutions. Thirdly, the tax authorities will probably impose constraints to prevent the tax avoidance which results from the creation of these personal loans.[8] But even if personal tax rates were to be equalized, only by chance would this be at a value for the common T^m of unity. In general, the resulting equilibrium would be one in which the corporate sector was all debt or all equity.

An interior equilibrium aggregate debt-equity ratio for the corporate sector can be obtained only by imposing constraints on individual investors. If short sale constraints are imposed on both debt and equity (i.e. lending between individuals is prohibited) then those investors who wish to hold debt will have to hold corporate debt. In equilibrium the aggregate corporate sector debt-equity ratio will equal the ratio of the wealth of those investors with a tax preference for debt ($T^m < 1$) to the wealth of those with a tax preference for equity ($T^m > 1$). Note, however, that this requires constraints to rule out personal borrowing. Where personal borrowing is feasible it will in general be more profitable for highly taxed investors to issue debt than for this to be done by corporations.

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[8] See King (1974, 1977).

Exactly the same considerations carry over to a world of uncertainty. We shall assume that each firm is "small" relative to the market so that a firm is unable to affect the implicit prices (i.e. the individual valuations) of consumption in each state of the world. With this assumption investors will wish firms to maximize market value. From (16) it is clear that the optimal debt-equity ratio for a "small" firm is given by the solution to the equation

$$\frac{dV_i}{dD_i} = (1 - \frac{A}{B}) \tag{17}$$

When there are no constraints on investors, each firm will be either all debt or all equity according to whether $B \gtrless A$, a condition which we may interpret in terms of a "market" preference for debt or equity.[9] As in the case of certainty, the absence of constraints on investors leads to an all debt or all equity equilibrium for the corporate sector.

If we now impose constraints on investors then the value of B becomes a function of the endogenous multipliers corresponding to the constraints. When the constraints are binding an interior equilibrium will exist in which A = B (provided the market contains some investors who have a tax preference for equity and some with a tax preference for debt). At this point there is an equilibrium aggregate debt-equity ratio for the corporate sector, but each firm is indifferent as to its own debt-equity ratio and market values are independent of debt-equity ratios. The important point to note here is that two conditions are required for the result. First, each firm must be small relative to the market, and, secondly, constraints on individual investors are necessary to produce an interior equilibrium for the corporate sector.

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[9] We assume here constraints on firms to ensure existence of an equilibrium.

4. Optimal Investor Portfolios

Given the existence of a market equilibrium, we may use the individual first-order conditions for utility maximization (12) to solve for the optimal portfolio of each investor. The results may be seen as a generalization of the special case in which there are no taxes and a separation theorem dictates that each investor holds the same "market" portfolio of equities. When individuals possess different "tax preferences" for debt and equity, just as when their expectations differ, optimal portfolios will vary, but in a way which may be explained intuitively.

Substituting (13) into (12), and using the fact that $E_i^m = n_i^m E_i$, we obtain

$$\sum_j n_j^m C_{ij} = B^m[\frac{C_i}{B} + (\mu_i - RD_i)(\frac{A^m}{B^m} - \frac{A}{B})] \qquad \forall_{i,m} \tag{18}$$

Stacking these conditions for each individual m yields:

$$\Gamma \cdot \underset{\sim}{n}^m = B^m[\Gamma \cdot \frac{\underset{\sim}{1}}{B} + (\underset{\sim}{\mu} - R\underset{\sim}{D})(\frac{A^m}{B^m} - \frac{A}{B})] \qquad \forall_m \tag{19}$$

where

$$\Gamma = \begin{bmatrix} C_{11} \cdots C_{1N} \\ \vdots \qquad \vdots \\ C_{N1} \cdots C_{NN} \end{bmatrix} ; \quad \underset{\sim}{n}^m = \begin{bmatrix} n_1^m \\ \vdots \\ n_N^m \end{bmatrix} ; \quad \underset{\sim}{\mu} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_N \end{bmatrix} ; \quad D = \begin{bmatrix} D_1 \\ \vdots \\ D_N \end{bmatrix} ; \quad \underset{\sim}{1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Assuming the variance-covariance matrix Γ is of full rank, we can multiply both sides of (19) by Γ^{-1} to obtain the vector of individual m's demands for equity:

$$\underset{\sim}{n}^m = B^m[\frac{\underset{\sim}{1}}{B} + \Gamma^{-1}(\underset{\sim}{\mu} - R\underset{\sim}{D})(\frac{A^m}{B^m} - \frac{A}{B})] \tag{20}$$

In the absence of taxes, $\frac{A^m}{B^m} \equiv 1 \equiv \frac{A}{B}$, so that individual m holds the same fraction of each firm's equity, $\frac{B^m}{B}$; this is the separation theorem alluded to above.

With taxes present, patterns of equity holdings will normally vary across investors, although there are some special situations when the separation theorem will still obtain. For example, if the ratio of mean return received by equity, $\mu_i - RD_i$, relative to covariance with the market, C_i, is some constant α for all firms, (20) reduces to

$$\underset{\sim}{n}^m = B^m[\frac{1}{B} + \alpha(\frac{A^m}{B^m} - \frac{A}{B})]\cdot\underset{\sim}{1} \tag{21}$$

It is interesting how (21) combines the results for investor equity demands which derive from the simpler models in which either taxation or uncertainty is ignored. The relative influence of the two terms in brackets on the right-hand side of (21) depends on α, which measures how risky the market is. If the amount of undiversifiable risk is large, and α small, then demands approximately resemble those derived in the absence of taxes. On the other hand, when risk is slight and α large, investors approach the certainty outcome of being specialized in either equity or debt, according to whether T^m is greater than or less than $\frac{A}{B}$.[10]

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[10] To see this, recall that $T^m = \frac{A^m}{B^m}$ except where investor m is specialized in one type of security.

The reason why individual portfolios will differ in the presence of taxes may be understood by considering first the special case where firm returns are independent (Γ is diagonal), and (20) becomes:

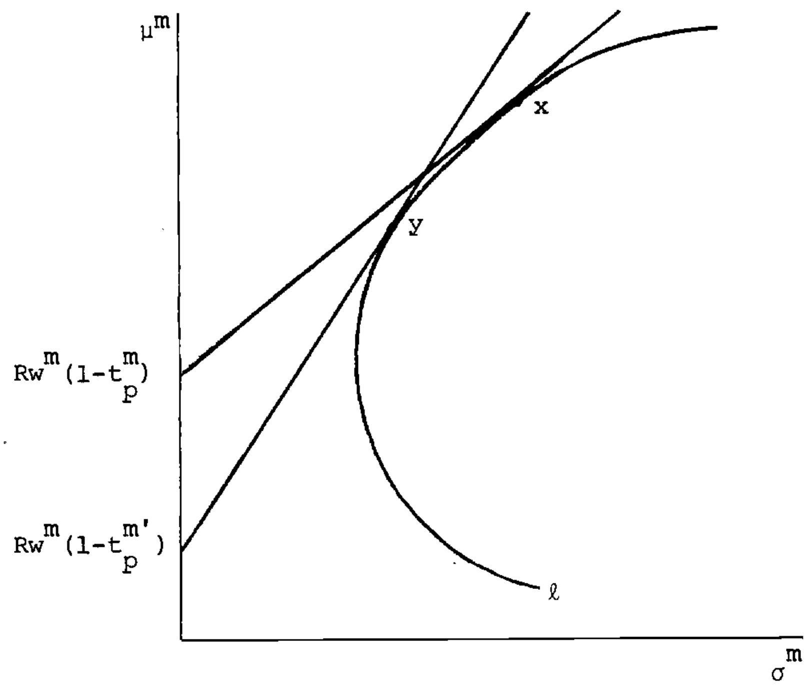
$$n_i^m = B^m[\frac{1}{B} + (\frac{\mu_i - RD_i}{C_i})(\frac{A^m}{B^m} - \frac{A}{B})] \qquad \forall_{i,m} \tag{22}$$

Consider an individual who holds both debt and equity in equilibrium and is therefore not constrained. For this investor, $T^m = \frac{A^m}{B^m}$, and (22) says that taxes will lead him to concentrate his holdings more in those equities which are safe ($\frac{\mu_i - RD_i}{C_i}$ is large) if $T^m > \frac{A}{B}$, or those which are risky ($\frac{\mu_i - RD_i}{C_i}$ is small) if $T^m < \frac{A}{B}$. Those with a tax preference for equity wish to hold as much equity as possible for <u>tax</u> purposes. To do so without incurring excessive risk, they will tend to hold more of their portfolio in low risk stocks. The opposite is true of individuals for whom debt is superior for tax reasons. These investors will want to hold as little equity as possible to obtain the "desired degree" of risk, and will do so by holding small amounts of risky firms.

This result carries over to the general case where firm returns are not independent. Figure 1 depicts the opportunity set of a typical investor in terms of available combinations of mean μ^m and standard deviation, σ^m. The position of the efficient frontier of equity combinations, labelled ℓ, depends on the equity tax rate $t_e{}^m$. The available riskless return is $Rw^m(1-t_p{}^m)$, and the corresponding optimal equity portfolio is at point x. The investor will hold some combination of this portfolio and riskless debt. An increase in the tax rate on interest

Figure 1

The Effect of Taxes on the Individual Portfolio



income to $t_p^{m'}$ changes the composition of the optimal portfolio to that at point y, where both mean and standard deviation are lower, but $\frac{\mu^m}{\sigma^m}$, and hence $\frac{\mu^m}{(\sigma^2)^m}$, is higher than at point x. The shift of the investor's tax preference toward equity caused by the increase in t_p^m leads to a safer equity portfolio. It is important to note that this alone does not imply that the investor will bear less risk, because it says nothing about the effect of the change in t_p^m on the allocation of wealth between debt and equity.

One final observation to be made concerns the question of which investors will be constrained in their portfolio decisions by the short-sale and borrowing restrictions we have assumed to exist. As is evident from (20), whether an individual is constrained in equity or debt depends not only on his tax preference, but also on his degree of risk aversion, as measured by γ. Investors might be observed specialized in equity, for example, who prefer debt for tax purposes but are nearly risk-neutral. Thus, the knowledge of tax preferences alone will be insufficient to identify the groups of investors who will specialize in either form of security, and hence to determine the equilibrium aggregate debt-equity ratio.

5. Objectives of the Firm

Thus far, we have concentrated on the behavior of individual investors and the determination of market equilibrium, taking as given the real and financial decisions of firms. This section considers the question of how firms should behave in their choice of financial policy.

In the absence of taxes, the Modigliani-Miller Theorem dictates that firm financial policy is irrelevant, having no effect on investor utilities at all. This will not generally be true with taxes present, but there may still exist conditions under which a firm's stockholders are unanimous in their agreement that the firm should choose its debt-equity ratio to maximize its total market value.

To explore this issue, we consider the effect on the utility of a typical investor of an increase in the debt issued by firm k. First, note that the expression for the mean return μ^m, in (5), may be rewritten using the definitions of T^m and n_i^m as

$$\mu^m = (1-t_p{}^m)[Rw^m + \sum_i n_i^m((\mu_i - RD_i)T^m - RE_i)] \tag{23}$$

Using (3), the definition of w^m, and (13), we obtain

$$\mu^m = (1-t_p{}^m)[R \sum_i \bar{n}_i^m V_i + \frac{T^m}{A} \sum_i n_i^m C_i + (\frac{B}{A}T^m - 1)R \sum_i E_i^m] \tag{24}$$

Differentiation of $U^m(\mu^m,(\sigma^m)^2)$ with respect to D_k under the assumption that firm k is too small to influence the market parameters R, A and B, yields (from the expressions for $(\sigma^m)^2$, V_i and μ^m; (6), (16) and (24), respectively)

$$\frac{dU^m}{dD_k} = U_1^m(1-t_p^m)[R\bar{n}_k^m(1-\frac{A}{B}) + \frac{T^m}{A}\sum_i \frac{dn_i^m}{dD_k}C_i + (\frac{B}{A}T^m-1)R\frac{d(\sum_i E_i^m)}{dD_k}]$$

$$+ U_2^m[(T^m)^2(1-t_p^m)^2 \cdot 2\sum_i\sum_j \frac{dn_i^m}{dD_k}n_j^m C_{ij}] \qquad (25)$$

$$= U_1^m(1-t_p^m)\{R[\bar{n}_k^m(1-\frac{A}{B}) + (\frac{B}{A}T^m-1)\frac{d(\sum_i E_i^m)}{dD_k}] + T^m[\frac{1}{A}\sum_i\frac{dn_i^m}{dD_k}C_i - \frac{1}{A^m}\sum_i\sum_j\frac{dn_i^m}{dD_k}n_j^m C_{ij}]\}$$

By combining conditions (12) and (13), we are able to rewrite (25) as

$$\frac{dU^m}{dD_k} = U_1^m R(1-t_p^m)[\bar{n}_k^m(1-\frac{A}{B}) + (\frac{B}{A}T^m-1)\frac{d(\sum_i E_i^m)}{dD_k} - T^m(\frac{B}{A} - \frac{B^m}{A^m})\sum_i E_i\frac{dn_i^m}{dD_k}] \qquad (26)$$

But, since

$$\sum_i E_i \frac{dn_i^m}{dD_k} = \frac{d(\sum_i E_i^m)}{dD_k} - \sum_i \frac{dE_i}{dD_k}n_i^m = \frac{d(\sum_i E_i^m)}{dD_k} + (\frac{A}{B})n_k^m \qquad (27)$$

equation (26) simplifies to

$$\frac{dU^m}{dD_k} = U_1^m R(1-t_p^m)[\bar{n}_k^m(1 - \frac{A}{B}) + (T^m\frac{B^m}{A^m} - 1)\frac{d(\sum_i E_i^m)}{dD_k} + T^m\frac{B^m}{A^m}(\frac{A}{B} - \frac{A^m}{B^m})n_k^m] \qquad (28)$$

From the definition of T^m, A^m and B^m, it is clear that $(T^m\frac{B^m}{A^m} - 1) = 0$ unless individual m is constrained. Moreover, if the constraint on short sales of equity is in force, $\sum_i E_i^m = 0$. Thus, the second term in brackets in (28) is non-zero only if $\sum_i E_i^m = w^m$, and we may thus combine the first two terms in (28) to obtain

$$\frac{dU^m}{dD_k} = U_1^m R(1-t_p^m)T^m \frac{B^m}{A^m}[(1 - \frac{A}{B})\bar{n}_k^m + (\frac{A}{B} - \frac{A^m}{B^m})n_k^m] \tag{29}$$

Note that with no taxes or binding constraints, this expression is uniformly zero, in accordance with the Modigliani-Miller Theorem.

Expression (29) says that a firm's financial policy affects individual utility in two ways. The first represents the wealth effect of a change in firm value (since $\frac{dV_k}{dD_k} = 1 - \frac{A}{B}$). Were this the only term, all shareholders would agree on the objective of value maximization. As the following argument demonstrates, the second term represents the effect of the firm's debt policy on the opportunity set available to the investor. Suppose that individual m plans to purchase a certain amount of equity in firm k, and the firm increases its issuance of debt. To undo the effect of this decision, the investor can purchase some of this debt, in the same proportion to his equity in the firm as the total issue of debt has to the firm's equity. This has two effects on the individual's welfare (aside from the wealth effect discussed above). First, it raises the total cost of his holdings in the firm at a rate proportional to $\frac{dV_k}{dD_K} n_k^m = (1 - \frac{A}{B})n_k^m$. Second, it changes the form in which the firm's returns flow to the investor, switching a fraction from debt to equity. In the absence of constraints, this would increase taxes at a rate proportional to $(T^m-1)n_k^m$. With constraints, the "preference" for equity is $\frac{A^m}{B^m}$ (see (12)), and this expression becomes $(\frac{A^m}{B^m} - 1)n_k^m$. Thus, the loss to the investor involved in undoing the firm's leverage change in the way described is proportional to $(1 - \frac{A}{B})n_k^m + (\frac{A^m}{B^m} - 1)n_k^m = (\frac{A^m}{B^m} - \frac{A}{B})n_k^m$, which is the second term in brackets in expression (29). This term will uniformly disappear only if the constraints λ^m on each investor m take on values which make $\frac{A^m}{B^m} = \frac{A}{B}$.

Suppose that this condition were satisfied, and that value maximization would therefore be optimal. As we showed in section 3, value maximization would lead to an interior equilibrium in which A=B. Hence $A^m=B^m$. In turn, this implies that investors who have a tax preference for equity $(T^m>1)$ must have $\lambda^m<0$ and hence be specialized in equity, and investors with a tax preference for debt $(T^m<1)$ must have $\lambda^m>0$ and be specialized in debt. This outcome would lead to the equilibrium characterized by Miller (1977), in which all investors are specialized in either debt or equity according to their tax rates, each firm would be indifferent to its own debt-equity ratio, and the corporate sector debt-equity ratio would equal the ratio of the wealth of those investors with a tax preference for debt to the wealth of those investors with a tax preference for equity.

The next step is to discover the conditions under which $\frac{A^m}{B^m} = \frac{A}{B}$. From equation (19) we know that this can occur only if

either (1) Γ is of full rank; hence, Γ^{-1} exists and investors hold the market portfolio, with individual m holding a fraction $\frac{B^m}{B}$ of each firm's equity,

or (2) Γ is not of full rank; there exists one firm which is perfectly correlated in its underlying returns with some linear combination of other firms.

It is easy to see that outcome (1) is impossible for the market as a whole. It implies that $\frac{B^m}{B} = 0$ for individuals specialized in debt, and since

$\frac{B^m}{B} = \frac{A^m}{A}$ by assumption, it follows that $A^m = 0$ for all such individuals. This will be true only if $\gamma^m = \infty$ coincidentally.

Thus, the segmented equilibrium in which value-maximization is necessarily optimal can occur only if firms are small and there is a redundant firm. Moreover, the existence of such a firm is also a sufficient condition for such an equilibrium to obtain. This can be shown by examining condition (18). It is easiest to imagine that there are two firms with identical returns, although this is in no way restrictive. In this case, the left-hand side of (18) is the same for each firm. For the right-hand sides to be equal also requires that $\frac{A^m}{B^m} = \frac{A}{B}$, assuming the debt levels of the firms to be different. This has a straightforward explanation. Without the existence of constraints, these firms would present the opportunity for riskless tax arbitrage. By buying equity in the firm which is less highly levered, and selling short the other firm's equity, each investor can create a safe composite asset which is taxed at equity rates and costs $(\frac{A}{B})$ times the cost of debt yielding the same gross return. Thus, by borrowing one unit of debt and purchasing $\frac{B}{A}$ units of this safe equity, investor m can generate a safe return after-tax of $\frac{B}{A}R(1-t_c)(1-t^e_m) - R(1-t_p^m) = R(1-t_p^m)(\frac{B}{A}T^m-1)$. Thus, if $T^m > \frac{A}{B}$, investor m would engage in positive amounts of this transaction; conversely, for $T^m < \frac{A}{B}$, the arbitrage could be accomplished by selling the safe equity short to purchase debt. The presence of constraints prevents these transactions from occurring without bound, but all investors will engage in them until a constraint is reached, unless $T^m = \frac{A}{B}$ and they are indifferent. This is because any interior position can be improved upon by simply trading one safe asset for another without in any way affecting the portfolio's risk characteristics. Further, because this arbitrage incurs no other costs, the shadow price of the constraint which prevents it must fully offset its arbitrage value; that is, $\frac{A^m}{B^m}$ must equal $\frac{A}{B}$.

The "Miller Equilibrium" thus requires that there exist a perfect equity substitute for debt. In a more general model, with debt being risky due to bankruptcy or other causes, the corresponding requirement would be that there exist an equity substitute for each form of debt. This makes clearer why the segmented equilibrium so resembles the outcome when there is no risk at all or individuals are risk-neutral: each investor's decision is now decomposable into two parts--first, decide on the optimal portfolio, then decide how it shall be taxed.

One final point to be made here concerns the exact form of the constraints chosen. It is essential to recognize the importance of the symmetry between the restrictions on borrowing and those on short sales. Had we instead imposed restrictions on short sales of individual firms, as might seem more plausible, it is clear that even a singular variance-covariance matrix for the underlying returns is not sufficient to ensure that value maximization (and hence a segmented equilibrium) is optimal. This is because the argument we used above to show sufficiency required the investors to engage in short sales of the equity of some firms.[11] When the constraints are imposed on short sales of individual firms' equity, the conditions for value maximization become more stringent. The basic condition is that investors must be able to obtain their

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[11] Problems would arise also if the tax treatment of short sales were not the mirror image of the treatment of positive holdings.

desired pattern of returns across states of the world in whichever form is preferred for tax purposes, debt or equity. If holdings of each security must be nonnegative, then in general we require not just that the set of underlying returns to firms span all states of the world (and that each firm is small), but that the returns to equity and the returns to debt both span all possible states. Essentially, this means that for each "real" firm there are two firms identical in all respects, except that one is all equity and the other all debt.

Since this is a stringent requirement, it is clear that the plausibility both of the equilibrium described by Miller and of the assumption of value maximization depends a great deal on which constraints are thought to be relevant.

6. Portfolio Behavior of Pension Funds

In the model explored above the factor which determined whether or not an equilibrium existed for an interior value of the corporate sector debt-equity ratio was the absolute tax advantage of different securities for different investors. In this model if all investors have an absolute tax preference for debt, say, the aggregate debt-equity ratio is infinite. We may now extend the model to the case where an investor's overall portfolio consists of two separate funds taxed at different rates. In this case the equilibrium depends upon comparative tax advantages. The obvious example here is an investor who holds a certain fraction of his wealth directly on his own account, and the remainder is invested on his behalf in a pension fund which is tax-exempt. Even if the investor has an absolute tax preference for debt, if his personal tax rate is positive he will have a comparative advantage in equity.

Consider, first, the case of a defined contribution pension plan so that the prospective pension depends entirely on the performance of the fund. Then the investor will be concerned solely with the performance of the total portfolio consisting of the securities owned on his own account and those owned by the pension fund. Suppose, further, that the relative sizes of the two accounts are fixed exogenously (by law, say, which limits the size of contributions to a tax-exempt fund). There exists the possibility of infinite arbitrage between the two accounts to exploit the comparative tax advantages of the two accounts. No equilibrium exists unless constraints are imposed. Assume that constraints are imposed on short sales (of both equity and debt) as in our previous model. Then it is trivial to show that the optimal portfolio is to let the pension fund own the debt which the investor wishes to hold and to purchase equity on his own account.[12] If the relative amounts of debt and equity which he wishes to own are exactly equal to the fixed relative wealths of the two accounts, then both investors on their own account and pension funds will be completely specialized. In general, however, one of the accounts will be specialized and the other will contain both debt and equity.[13] If there are no constraints on borrowing, but only on short sales of equity then pension funds will always be all-debt. This is because if a pension fund contained some equity, the individual investor would prefer to switch the equity from the fund to his own account, borrowing to finance the purchase. His liability on the loan would be offset by the extra holding of debt which the fund would purchase out of the proceeds of selling the equity. The capital structure of the total portfolio would be unaffected by the switch but the tax burden on the returns would be less. Again we see the critical role played by constraints. Some constraints are necessary for existence; constraints only on short sales of equity lead to pension funds being completely specialized in debt; constraints on short sales of both debt and equity result in one of the accounts being specialized but, in general, this may be either the pension fund (in debt) or the investor's own account (in equity). It is clear that if individuals do not face borrowing constraints we would expect to find all investments in defined contribution pension plans

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[12]The reader may easily see this from the appropriately revised formulations of the budget set and first-order conditions in (5), (6), and (12).

[13]If the conditions required for firms to seek value maximization are satisfied, then it is clear that pension and personal accounts will each specialize in debt or equity according to their own absolute tax preferences. Thus, the pension fund will always specialize in debt, and the individual will hold only equity or only debt according to whether T^m is greater than or less than one. This result is thus a special case of the general findings just discussed.

(such as TIAA-CREF, Keogh Plans or IRA's) in debt.[14] Since discussion with our colleagues suggests this is not the case, there is an interesting puzzle to resolve about the investment strategy of pension funds.

So far we have assumed that the proportions of total wealth held directly and in pension funds were given exogenously. If individuals are free to allocate their wealth between the two accounts then, assuming that sufficient constraints exist to ensure the existence of equilibrium, they will exploit the absolute tax advantage of tax-exempt funds and put all their wealth in pension funds. In practice, of course, our two-period model does not capture the fact that pension wealth is tied up until retirement and there will be an upper limit on the proportion of wealth investors choose to put into such funds.

The above argument refers to tax-exempt funds from which the investor will benefit directly. But many pension plans are defined benefit plans, and the performance of the fund affects not prospective pension recipients but the owners of the liability to pay future pensions, usually stockholders of the company offering the pension plan. In this case, the analogue with our previous model is that the company decides how much to put into its pension fund (the degree of funding) and the portfolio strategy of the fund and on its own account (i.e., its own debt-equity ratio). Exactly the same arguments as we used before apply here, the only difference being that the investor is now the company and the portfolio consists of the assets of the company and its unfunded pension liability (which is equal to gross pension liabilities less the assets of the pension fund). This case has been discussed by Black (1980) and Tepper (1980).

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[14] Assuming that interest payments on personal borrowing are tax-deductible.

The comparative tax advantage is that the company should hold equity while the pension fund should hold debt. If the pension fund holds equity the company would prefer the fund to sell the equity and purchase debt. This would reduce the implicit riskiness of the unfunded liability thus allowing the company to issue more debt on its own account leaving the capital structure of the company (including unfunded liabilities) unchanged. The tax saving from issuing more debt is a pure arbitrage profit. Clearly, constraints on short sales of equity by the pension fund are necessary for existence of an equilibrium and since companies are allowed to borrow the resulting equilibrium is one in which pension funds are completely specialized in debt. This result for defined benefit plans is exactly analogous to the case of defined contribution plans discussed above. The absolute tax advantage would suggest that, ceteris paribus, companies would choose to fully fund their pension schemes.

7. Conclusions

We have shown that in a world in which investors face different tax rates, no equilibrium exists unless constraints are imposed. The exact nature of those constraints will have a critical bearing on the nature of the equilibrium and should therefore be modelled explicitly. In section 5 we showed that, given certain conditions on the securities available in the market, investors will be unanimous in supporting value maximization and firms will be indifferent as to their choice of debt-equity ratio. When this is the case the equilibrium will be segmented, investors will be completely specialized, and the aggregate debt-equity ratio will equal the ratio of the wealth of those who, for tax reasons, prefer debt to the wealth of those who prefer equity. But when the conditions do not hold, value maximization is no longer the unambiguous objective of the firm, and investors will in general hold both debt and equity. It is difficult to reconcile value maximization with the existence of portfolios which contain both debt and equity.

Finally, we may note that for the Modigliani-Miller Theorem to hold in a world without taxes, short sales of debt and equity must be allowed, whereas, once we allow for taxes, constraints are essential for an equilibrium to exist and indeed it is the constraints which enable an equilibrium to be reached in which firms are indifferent as to their choice of debt-equity ratio.

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