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ABSTRACT

This paper presents a simple model of market equilibrium to explain why firms that maximize the value of their shares pay dividends even though the funds could instead be retained and subsequently distributed to shareholders in a way that would allow them to be taxed more favorably as capital gains. The two principal ingredients of our explanation are: (1) the conflicting preferences of shareholders in different tax brackets and (2) the shareholders' desire for portfolio diversification, we show that companies will pay a positive fraction of earnings in dividends. We also provide some comparative static analysis of dividend behavior with respect to tax parameters and to the conditions determining the riskiness of the securities.

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The nearly universal policy of paying substantial dividends is the primary puzzle in the economics of corporate finance. Dividends are taxed at rates varying up to 70 percent and averaging nearly 40 percent for individual shareholders. In contrast, retained earnings imply no concurrent tax liability; the rise in the share value that results from retained earnings is taxed only when the stock is sold and then at least 60 percent of the gain is untaxed.¹ In spite of this significant tax penalty, U.S. corporations continue to distribute a major fraction of their earnings as

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¹Current law allows 60 percent of the gain to be excluded. This has the effect of taxing realized capital gains at only 40 percent of the regular income tax rate. When shares that are obtained as a bequest are sold, the resulting taxable income is limited to 40 percent of the rise in the value of the shares since the death of the previous owner.

dividends; during the past 15 years, dividends have averaged 45 percent of real after-tax profits. In effect, corporations voluntarily impose a tax liability on their shareholders that is currently more than \$10 billion a year.¹

Why do corporations not eliminate (or sharply reduce) their dividends and increase their retained earnings?² It is, of course, arguable that if all firms were to adopt such a policy it would raise the aggregate level of investment and therefore depress the rate of return on capital.³ But any individual firm could now increase its retained earnings without having to take less than

¹There would of course be no problem in explaining the existence of dividends if there were no taxes. The analysis of Modigliani and Miller (1958) shows that without taxes dividend policy is essentially irrelevant since shareholders can in principle offset any change in dividend policy by buying or selling shares. Even in the Modigliani-Miller world, the stability of dividend rates would require explanation.

²There is also in principle the possibility of repurchasing shares instead of paying dividends. The proceeds received by shareholders would be taxed at no more than the capital gains rate and therefore at no more than 40 percent of the rate that would be paid if the same funds were distributed as dividends. There are however significant legal impediments to a systematic repurchase policy. Regular periodic repurchases of shares would be construed as equivalent to dividends for tax purposes. Sporadic repurchases would presumably avoid this but would subject managers and directors to the risk of shareholder suits on the grounds that they benefited from insider knowledge in deciding when the company should repurchase shares and whether they as individuals should sell at that time. British law forbids the repurchase of shares. The present paper assumes that frequent repurchases would be regarded as income and therefore focuses on the choice between dividends and retained earnings. The possibility of postponed and infrequent share repurchases is expressly considered.

³The greater retained earnings could also partly or wholly replace debt finance.

the average market return on its capital if it used the additional funds to diversify into new activities or even to acquire new firms.

Several different possible resolutions of the dividend puzzle have been suggested. In reality there is probably some truth to all of these ideas but we believe that, even collectively, they have failed to provide a satisfactory explanation of the prevailing ratio of dividends to retained earnings. It is useful to distinguish five kinds of explanations.

First, there is the desire on the part of small investors, fiduciaries and nonprofit organizations for a steady stream of dividends with which to finance consumption. Although the same consumption stream might be financed on a more favorably taxed basis by periodically selling shares, it is argued that small investors might have substantial transaction costs and that some fiduciaries and nonprofit organizations are required to spend only "income" and not "principal." However, transaction costs could be reduced significantly if investors sold shares less frequently. Fiduciaries and nonprofit organizations can often eliminate any required distinction between income and principal.

Merton Miller and Myron Scholes (1978) have offered the ingenious explanation that the current limit on interest deductions implies that there is no marginal tax on dividends. Under current tax law, an individual's deduction for investment interest (i.e., interest other than mortgage and business interest) is limited to investment income plus \$25,000. An extra dollar of dividend income

raises the allowable interest deduction by one dollar. For a taxpayer for whom this constraint is binding, the extra dollar of dividends is just offset by the extra dollar of interest deduction, leaving taxable income unchanged. Although Miller and Scholes discuss how the use of tax-exempt annuities "should" make this constraint binding for all individual investors, in reality fewer than one-tenth of one percent of taxpayers with dividends actually had large enough interest deductions to make this constraint binding. Moreover, since the limit on interest deductions was only introduced in 1969, the Miller and Scholes thesis is irrelevant for earlier years.

A more plausible explanation is that dividends are required because of the separation of ownership and management. According to one form of this argument, dividends are a signal of the sustainable income of the corporation: management selects a dividend policy to communicate the level and growth of real income because conventional accounting reports are inadequate guides to current income and future prospects.² While this theory remains to be fully elaborated, it does suggest that the steadiness (or safety) of the dividend, as well as its average level, might be used in a dynamic setting. The dividend tax of more than \$10 billion does

¹The NBER TAXSIM model, using individual income tax returns for 1975, indicates that of 13 million returns with dividends totalling \$23.2 billion, the interest deduction constraint was binding for about 50 thousand.

²For a development of this view, see Bhattacharya (1979), Gordon and Malkiel (1979) and Ross (1977).

seem to be an inordinately high price to pay for communicating this information; a lower payment ratio would convey nearly the same information without such a tax penalty. Closely related to the signalling idea is the notion that shareholders distrust the management and fear that retained earnings will be wasted in poor investments, higher management compensation, etc. According to this argument, in the absence of taxation shareholders would clearly prefer "a bird in hand" and this preference is strong enough to pressure management to make dividend payments even when this involves a tax penalty. If investors would prefer dividends to retained earnings because of this distrust, it is hard to understand why there is not pressure for a 100 percent dividend payout.¹

Afan Auerbach (1979), David Bradford (1979) and Mervyn King (1977) have developed a theory in which positive dividend payments are consistent with shareholder equilibrium because the market value per dollar of retained earnings is less than one dollar. More specifically, if θ is the tax rate on dividends and c is the equivalent accrual tax rate on capital gains,² the net value of one dollar of dividends is $1 - \theta$ while the net value of one dollar

¹The argument that dividends reflect the separation of ownership and management appears to be supported by the fact that closely held companies pay little or no dividends. However, such companies can usually achieve a distribution of funds as management salary which is deductible.

²The equivalent accrual tax rate on capital gains is the present value of the tax liability that will eventually be paid, per dollar of dividend income.

of retained earnings is $(1-c)p$ where p is the rise in the market value of the firm's shares when an extra dollar of earnings is retained, i.e., p is the share price per dollar of equity capital. Auerbach, Bradford and King point out that shareholders will be indifferent between dividends and retained earnings if the share price per dollar of equity capital is $p = (1-\theta)/(1-c) < 1$. At any other value of p , shareholders would prefer either no dividends or no retained earnings but at $p = (1-\theta)/(1-c)$ any value of the dividend payout rate would be equally acceptable. Moreover, in the context of their model, the share price will satisfy this value of q when shares sell at the present value of after-tax dividends. In short, they argue that the existence of dividends is appropriate if the value of retained earnings capitalizes the tax penalty on any eventual distribution.

This line of reasoning is clearly important but raises several problems. First, it has been argued¹ that an equilibrium in which p is less than one is incompatible with new equity finance by the firm. While it is clearly inconsistent for firms to pay dividends and sell shares at the same time (except if dividends are paid for some of the other reasons noted above), the theory is not incompatible with firms having some periods when $p > 1$ and new equity is sold and other periods when $p < 1$ and dividends are paid but shares are not sold. In any case, new equity issues by established companies (outside the regulated industries where special

¹See, e.g., Gordon and Malkiel (1979).

considerations are applicable) are relatively rare,

A more important problem with the Auerbach-Bradford-King theory is that it is based on the premise that funds can never be distributed to shareholders in any form other than dividends. This implicitly precludes the possibility of allowing the company to be acquired by another firm or using accumulated retained earnings to repurchase shares. Either of these options permits the earnings to be taxed as capital gains after a delay.¹ The theory that we develop in the present paper explicitly recognizes this possibility.

A further difficulty with the theory is that any payout rate is consistent with equilibrium and therefore gives no reason for the observed stability of the payout rate over time for individual companies and for the aggregate. Although such stability could be explained by combining the Auerbach-King-Bradford model with some type of signalling explanation, our own analysis based purely on considerations of risk indicates that the payout rate is determinate and that it is likely to be relatively insensitive to fluctuations in annual earnings. (A more explicit dynamic analysis would be necessary to confirm this conclusion.)

The most serious problem with the Auerbach-Bradford-King hypothesis is the implicit assumption that all shareholders have

¹Such infrequent share repurchases are very different from a systematic program of substituting regular repurchases for dividends. They do not risk the adverse tax consequence referred to above and, unlike continuous repurchases in lieu of dividends, involve a different growth of equity.

the same tax rates (θ and c). In reality, there is very substantial variation in tax rates and therefore in the value of $p = (1-\theta)/(1-c)$ that is compatible with a partial dividend payout. For individuals in the highest tax bracket, $\theta = 0.7$ and the dividend-compatible q is approximately 0.33;¹ for tax exempt institutions, the corresponding value is one. The Auerbach-Bradford-King concept of shareholder equilibrium implies that, at any market value of p , almost all shareholders will prefer either no dividends or no retained earnings, depending on whether the market value of p was greater than or less than their own values of the ratio $(1-\theta)/(1-c)$. This condition would cause market segmentation and specialization; some firms would pay no dividend while others would have no retained earnings and each investor would own shares in only one type of firm. Such specialization and market segmentation is clearly counterfactual. Our own current analysis emphasizes the diversity of shareholder tax rates and shows that this is a key to understanding the observed policy of substantial and stable dividends.

In a previous paper with Eytan Sheshinski, we studied the long-run growth equilibrium of an economy with corporate and personal taxes (Feldstein, Green and Sheshinski, 1979). In this context, dividends appear as the difference between after-tax profits and the retained earnings that are consistent with steady-state growth

¹This assumes that postponement and the stepped up basis at death reduce the accrual equivalent capital gains tax to 10 percent.

and with the optimal debt-equity ratio. This limits aggregate retained earnings and implies positive aggregate dividends but does not explain why each firm will choose to pay positive dividends rather than to grow faster than the economy's natural rate. We suggested that each firm is constrained by the fact that more rapid growth would increase its relative size, thereby making it riskier and reducing the market price of its securities. An explicit model of this relation between size and the "risk-discount" was not presented in that paper but is one of the basic ideas of the general equilibrium analysis that we present here. Unlike the previous paper, the present analysis will not look at properties of the long-run steady state but will examine microeconomic choice in a one period model.

The idea of shareholder risk-aversion as a limit to a firm's growth and the existence of shareholders in diverse tax situations are the two central components of the analysis developed in the present paper. We consider an economy with two kinds of investors: taxable individuals and untaxed institutions (like pension funds and nonprofit organizations).¹ Firms can distribute profits currently as dividends or retain them, grow larger and ultimately distribute these funds to shareholders as capital gains.² In the

¹The same reasoning would apply if we consider "low tax rate" and "high tax rate" individuals. See Feldstein and Slemrod (1978) for the application of such a classification to analyzing the effect of the corporate tax system.

²This future capital gain distribution could be the result of the firm's shares being acquired by another firm or of a share repurchase by the firm itself.

absence of uncertainty, these assumptions would lead to segmentation and specialization. The taxable individuals would invest only in firms that pay no dividends even though, ceteris paribus, they prefer present dollars to future dollars while untaxed institutions would invest only in firms that retain no profits. In this equilibrium the share price per dollar of retained earnings would in general be less than one. This type of equilibrium with segmentation and specialization is not observed because of uncertainty. Because investors regard each firm's return as both unique and uncertain, they wish to diversify their investment. We show in this paper that each firm can in general maximize its share price by attracting both types of investors and that this requires a dividend policy of distributing some fraction of earnings as dividends. Only in the special case of little or no uncertainty or of a limited ability to diversify risks can the equilibrium be of the segmented-market form.

The first section of the paper presents the basic model of dividend behavior in a two-firm economy with two classes of investors. Some comparative statistics of the resulting equilibrium are developed in section 2. The third section examines the special case in which the two firms have equal expected yields and equal variances. Despite the diversity of taxpayers, both firms choose the same dividend rate. In section 4, the symmetry of this equilibrium is contrasted with the segmentation and specialization that can arise with riskless investments, or with risk-neutral individuals. The fifth section replaces the assumption that there

are only two firms with a specification of a very large number of firms of each of two types. The basic results of this large number case confirm the conclusions on diversification and divided policy that were based on the two-firm economy in the earlier sections of the paper. There is a final concluding section that suggests direction for further work.

1. Dividend Behavior in a Two-Company Economy

Our analysis of corporate dividend behavior uses a simple one-period model. At the beginning of the period, each firm has one dollar of net profits that must be divided between dividends and earnings. The firms announce their dividend policies and trading then takes place in the shares. The firms use the amounts that they have retained to make investments in plant and equipment. At the end of the period, the uncertain returns on these investments are realized and the companies are liquidated. All of the end-of-period payments are regarded as capital gains rather than dividends and will be assumed to be untaxed.

There are two kinds of investors in the economy. Households (denoted by a subscript H) are taxed at rate θ on dividend income but pay no tax on capital gains. Institutions (denoted by a subscript I) pay no taxes on either dividends or capital gains. At the beginning of the period, the two types of investors own the following numbers of shares in both companies: \bar{s}_{H1} , \bar{s}_{H2} , \bar{s}_{I1} and \bar{s}_{I2} , where the subscripts 1 and 2 indicate the companies. For notational simplicity, we normalize the number of shares in each company at 1. After the companies announce their dividend policies, the investors can sell their shares (at prices determined in the market that depend on the firms' dividend policies) and can buy other shares. Investors can also place some of the proceeds of their share sales in a riskless asset or can spend those funds on consumption; each dollar invested in this

riskless asset has an end of period value of R . We assume however that investors may not sell shares short. Both types of investors prefer present dollars to future dollars; one present dollar (obtained either as after-tax dividends or from the sale of shares) is worth R dollars. Although R might be expected to differ between households and institutions, we shall assume the same R for both groups.

Each firm has an initial amount of one dollar available for distribution and retention. Company i pays dividend d_i at the beginning of the period and therefore invests amount $1-d_i$. The end-of-period of company i ($i=1,2$) is r_i per dollar of funds that are retained and invested; the rate of return on the firm's capital is thus r_i-1 .¹ The expected value of this uncertain return is r_i^e and its variance is σ_{ii} . The covariance of the returns of the two firms is σ_{12} . In the analysis that follows, we consider the general case in which the yields and variances are unequal. We then examine in detail the character of the equilibrium in the case in which the mean yields and variances of the two firms are identical. We show that in this situation the degree of uncertainty (as measured by the common variance) and the opportunity for effective diversification (as measured by the correlation between the returns) determine whether both companies pay

¹We assume that firms do not borrow and that the stochastic return per dollar of investment does not depend on the amount that is invested.

dividends and are owned by both types of investors or there is market segmentation in which one company pays no dividends and is owned by the household investors.

Our strategy of analysis is as follows. We first derive the share demand equations for the two types of investors. These demands depend on the prices of the shares and on their stated dividend policies. We then use the fact that the available number of shares of each type of stock is fixed to calculate the price functions. The price of each type of share depends in general on the dividend policy of that firm and of the other type of firm. We assume that firms select the dividend policy that maximizes the firm's value, i.e., that maximizes the price per share.¹ This maximization yields the optimal dividend for each firm. When these dividend values have been obtained, we shall examine the characteristics of the equilibrium and the comparative static response to changes in the tax rate.

Investors' Demands for Shares

We derive each investor's demand functions for shares by maximizing the investor's expected utility subject to the wealth constraint implied by the investor's initial shareholdings and

¹Maximizing the share price is pareto efficient but not uniquely optimal. There are other plausible criteria by which management might in general decide its dividend policy even in a one period model such as the current one, e.g., majority voting of the shareholders.

the equilibrium share prices. We assume that the investors' utility functions are quadratic and focus our attention on the role of taxes by assuming that all investors have exactly the same utility function. The nature of the utility function implies that the demand for each type of share is independent of the individual's wealth; we can therefore derive aggregate demand functions for each type of shareholder by treating all of the investors of each type as if they were a single investor.

Consider first the investment problem of the households. If the market equilibrium share prices for the two companies are p_1 and p_2 , the value of their initial portfolio is $p_1 \bar{s}_{H1} + p_2 \bar{s}_{H2}$. The initial wealth is exchanged for s_{H1} shares of company 1, s_{H2} shares of company 2, and z dollars of the monetary asset. The new portfolio must satisfy the wealth constraint:

$$(1.1) \quad p_1 \bar{s}_{H1} + p_2 \bar{s}_{H2} = p_1 s_{H1} + p_2 s_{H2} + z_H.$$

With dividend payouts of d_1 and d_2 , the households' total after-tax funds at the beginning of the period are $(1-\theta)d_1 s_{H1} + (1-\theta)d_2 s_{H2} + z_H$. The additional funds received at the end of the period are the uncertain amount $(1-d_1)s_{H1}r_1 + (1-d_2)s_{H2}r_2$. Combining these two with each dollar of beginning-of-period funds equivalent to R dollars of the end-of-period funds yields the argument of the household's utility function:

$$(1.2) \quad W_H = R(1-\theta) [s_{H1}d_1 + s_{H2}d_2] + Rz_H + s_{H1}(1-d_1)r_1 + s_{H2}(1-d_2)r_2 .$$

The quadratic character of the utility function implies that expected utility can be written as a linear combination of the mean and variance of W_H :

$$(1.3) \quad E[U(W_H)] = E(W_H) - 0.5\gamma \cdot \text{var}(W_H)$$

where $\gamma > 0$ is a measure of risk aversion (and the 0.5 is introduced to simplify subsequent calculations.) Equation 1.2 implies that

$$(1.4) \quad E(W_H) = R(1-\theta)(s_{H1}d_1 + s_{H2}d_2) + Rz_H + s_{H1}(1-d_1)r_1^e + s_{H2}(1-d_2)r_2^2$$

and

$$(1.5) \quad \text{var}(W_H) = s_{H1}^2(1-d_1)^2\sigma_{11} + s_{H2}^2(1-d_2)^2\sigma_{22} + 2s_{H1}s_{H2}(1-d_1)(1-d_2)\sigma_{12}$$

The households' optimum portfolio is found by maximizing equation 1.3 subject to the constraint of equation 1.1.¹ The first-order conditions for maximizing expected utility are:

$$(1.6) \quad 0 = R(1-\theta)d_1 + (1-d_1)r_1^e - Rp_1 - \gamma[s_{H1}(1-d_1)^2\sigma_{11} + s_{H2}(1-d_1)(1-d_2)\sigma_{12}]$$

and

¹We indicate below the important circumstances under which the demands implied by this maximization would violate the "no short sale" constraints. This "limited risk avoidance" case will be considered explicitly in section 4.

$$(1.7) \quad 0 = R(1-\theta)d_2 + (1-d_2)r_2^e - Rp_2 - \gamma [s_{H2}(1-d_2)^2\sigma_{22} \\ + s_{H1}(1-d_1)(1-d_2)\sigma_{12}]$$

Collecting terms, we may write the households' pair of demand equations as:

$$(1.8) \quad \gamma \begin{bmatrix} (1-d_1)^2\sigma_{11} & (1-d_1)(1-d_2)\sigma_{12} \\ (1-d_1)(1-d_2)\sigma_{12} & (1-d_2)^2\sigma_{22} \end{bmatrix} \begin{bmatrix} s_{H1} \\ s_{H2} \end{bmatrix} = \\ \begin{bmatrix} R(1-\theta)d_1 + (1-d_1)r_1^e - Rp_1 \\ R(1-\theta)d_2 + (1-d_2)r_2^e - Rp_2 \end{bmatrix}$$

or, in matrix notation,

$$(1.9) \quad \gamma A \underline{s}_H = \underline{a}_H - \underline{Rp} \quad .$$

where the elements of A and \underline{a}_H are clear from 1.8. If the matrix A is not singular, 1.8 can be solved for the share demands \underline{s}_H . It is important to note that A is singular when either stock is riskless or when the correlation between the two yields is one; in either case, holding a mixed portfolio does not achieve any reduction in risk. The optimal portfolio in this case is an investment in only one type of stock. More generally, when the variances are small or the correlation high, the solution of equation 1.9 may imply demands for shares that violate the constraint on short-selling. The feasible optimum again requires a specialized portfolio and induces extreme dividend behavior in which one company pays no dividend and the other keeps no retained earnings.

We return below to examine the characteristics of this "low risk avoidance" equilibrium. Now however we shall focus on the case in which A is nonsingular and the solution of equation 1.9 does not violate the other constraints on portfolio behavior.¹

Solving equation 1.9 yields the households' share demand equation under the assumption that $s_H \geq 0$ (i.e., that short-selling would not be optimal):

$$(1.10) \quad \underline{s}_H = \gamma^{-1} A^{-1} [\underline{a}_H - R\underline{p}] .$$

Analogous share demand equations hold for the institutional investors:

$$(1.11) \quad \underline{s}_I = \gamma^{-1} A^{-1} [\underline{a}_I - R\underline{p}] .$$

The share demands differ only because \underline{a}_H contains the tax variable ($\theta > 0$) while in \underline{a}_I the tax variable is implicitly zero.²

Price Functions and Optimal Dividends

By equating the share demands of 1.10 and 1.11 to the fixed share supplies, we can solve for the market clearing share prices that would correspond to any combination of dividend policies. Since the number of shares of each company was normalized to one,

¹We later show that such equilibria can exist for plausible parameter values.

²If any of the non-negativity constraints on \underline{s}_H or \underline{s}_I are binding, the optimum is no longer given by equations 1.10 and 1.11

we have

$$(1.12) \quad \underline{s}_H + \underline{s}_I = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

or

$$(1.13) \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \gamma^{-1} A^{-1} [\underline{a}_H + \underline{a}_I - 2R\underline{p}] .$$

Solving equation 1.13 for this price vector yields:

$$(1.14) \quad \underline{p} = \frac{1}{2R} \begin{bmatrix} R(2-\theta)d_1 + 2(1-d_1)r_1^e \\ R(2-\theta)d_2 + 2(1-d_2)r_2^e \end{bmatrix} \\ - \frac{\gamma}{2R} \begin{bmatrix} (1-d_1)^2\sigma_{11} + (1-d_1)(1-d_2)\sigma_{12} \\ (1-d_2)^2\sigma_{22} + (1-d_1)(1-d_2)\sigma_{12} \end{bmatrix}$$

The price of each type of share is positively related to its own expected yield and negatively related to the variance of that yield and its covariance with the yield of the other type of stock.

We assume that each firm selects its dividend payout rate to maximize its share price and takes the dividend of the other firm as given.¹ The first order condition for firm 1 is:

¹We show in the appendix that the current process is stable under this assumption. Section 5 develops a model with a large number of firms in which it is more natural to assume that each firm treats the dividends of other firms as parameters.

$$(1.15) \quad \frac{\partial p_1}{\partial d_1} = 0$$

$$= \frac{1}{2R} [R(2-\theta) - 2r_1^e + \gamma[2(1-d_1)\sigma_{11} + (1-d_2)\sigma_{12}]]$$

and implies that the firm's optimal dividend rate (d_1^*) satisfies:

$$(1.16) \quad 1-d_1^* = \frac{2(r_1^e - R) + \theta R}{2\gamma\sigma_{11}} - \frac{\sigma_{12}(1-d_2)}{2\sigma_{11}}$$

Equation 1.16 describes the first firm's optimal reaction to the dividend policy of the second firm. Symmetrically we obtain the dividend policy reaction function of the second firm:

$$(1.17) \quad 1-d_2^* = \frac{2(r_2^e - R) + \theta R}{2\gamma\sigma_{22}} - \frac{\sigma_{12}(1-d_1)}{2\sigma_{22}}$$

If the returns to the investments by the two firms are not independent ($\sigma_{12} \neq 0$), the optimal dividend policy of each firm depends on the dividend policy of the other firm. The two dividend policy functions can be solved simultaneously to obtain the equilibrium dividend policy of each firm:

$$(1.18) \quad \begin{bmatrix} 1-d_1^* \\ 1-d_2^* \end{bmatrix} = \frac{1}{1 - \frac{1}{4} \cdot \frac{\sigma_{12}^2}{\sigma_{11}\sigma_{22}}} \begin{bmatrix} \frac{2(r_1^e - R) + \theta R}{2\gamma\sigma_{11}} - \frac{1}{2} \frac{\sigma_{12}}{\sigma_{11}} \left[\frac{2(r_2^e - R) + \theta R}{2\gamma\sigma_{22}} \right] \\ \frac{2(r_2^e - R) + \theta R}{2\gamma\sigma_{22}} - \frac{1}{2} \frac{\sigma_{12}}{\sigma_{22}} \frac{2(r_1^e - R) + \theta R}{2\gamma\sigma_{11}} \end{bmatrix}$$

The stability of this solution is verified in an appendix.

2. Some Comparative Statics

It is immediately clear from equation 1.18 that each firm's optimal retained earnings depends positively on its own expected return and negatively on its own variance.¹ A higher expected yield makes it optimal to retain and invest more in the company while an increase in the uncertainty of that return makes the immediate payment of dividends more appealing.

If the returns of the two firms are positively correlated ($\sigma_{12} > 0$), each firm's optimal retained earnings varies inversely with the attractiveness of investment in the other firm (i.e., with the other firm's expected yield and the inverse of its variance). Intuitively, when retained earnings in one firm are more attractive and therefore increase, the riskiness of retaining earnings in the other firm increases if the yields of the two firms are positively correlated.

The effect of an increase in the rate of tax on dividends is particularly interesting. For firm 1,

$$(2.1) \quad \frac{\partial d_1^*}{\partial \theta} = - \frac{1}{1 - \frac{1}{4} \frac{\sigma_{12}^2}{\sigma_{11}\sigma_{22}}} \cdot \frac{R}{2\gamma\sigma_{11}} \cdot \left[1 - \frac{1}{2} \frac{\sigma_{12}}{\sigma_{22}} \right]$$

The first two terms on the right hand side are unambiguously

¹Since $\sigma_{12}^2/\sigma_{11}\sigma_{22}$ is the square of the correlation coefficient between the two yields and therefore necessarily less than unity, the common multiplier of both terms is positive.

positive. If the yields of the two firms are uncorrelated ($\sigma_{12}=0$), an increase in the tax rate on dividends necessarily reduces the firm's payout. However, when the yields are correlated the effect of the tax rate is ambiguous, i.e., the sign of the final term in equation 2.1 can be either positive or negative. Since σ_{12}/σ_{22} is the regression coefficient of the return for the first firm's investment on the return for the second firm's investment,¹ it could exceed 2 and make the final expression negative.

It is easy to understand why a strong covariance between the yields could produce the apparently counterintuitive result that an increase in the tax rate on dividends can actually raise a firm's optimal payout. Note first that an equation similar to 2.1 holds for firm 2:

$$(2.2) \quad \frac{\partial d_2^*}{\partial \theta} = - \frac{1}{1 - \frac{1}{4} \frac{\sigma_{12}}{\sigma_{11}\sigma_{22}}} \cdot \frac{R}{2\gamma\sigma_{22}} \cdot \left[1 - \frac{1}{2} \frac{\sigma_{12}}{\sigma_{11}} \right]$$

Adding these two expressions gives the effect of an increase in θ on the total dividends of the two firms combined:

$$(2.3) \quad \frac{\partial (d_1^* + d_2^*)}{\partial \theta} = - \frac{1}{1 - \frac{1}{4} \frac{\sigma_{12}}{\sigma_{11}\sigma_{22}}} \cdot \frac{R}{2\gamma\sigma_{11}\sigma_{22}} [\sigma_{22} + \sigma_{11} - \sigma_{12}]$$

¹This regression coefficient is closely related to the beta of capital market theory but refers here to the yields expressed as a return on physical capital rather than share value.

It is easy to show that this is unambiguously negative. This is clearly so if $\sigma_{12} < 0$. To see that this is also true when $\sigma_{12} > 0$, note that the variance of the difference $r_1 - r_2$ is $\sigma_{22} + \sigma_{11} - 2\sigma_{12}$; since this is a variance it is necessarily positive, implying $\sigma_{22} + \sigma_{11} > 2\sigma_{12}$ and therefore that $\sigma_{22} + \sigma_{11} - \sigma_{12} > \sigma_{12} > 0$. Thus an increase in the tax rate on dividends unambiguously reduces total dividends. The dividends of one of the firms may increase but not the dividends of both of them. The dividends of one firm will increase when the decrease in the dividends of the other is so large that, given the positive covariance between the returns, the greater risk associated with retained earnings in the first firm outweighs the direct effect of the tax.

It is interesting to consider the magnitude of this sensitivity of the payout policy with respect to the tax parameter. Equation 1.18 can be used to calculate the elasticity of the aggregate retained earnings with respect to θ . Although it is easy to obtain a general expression, the interpretation of the elasticity is clearer if we assume that the "excess yield" ($r^e - R$) is the same for both assets.¹ With this assumption, equation 1.18 implies the elasticity

¹When the excess returns differ for the two firms, $r^e - R$ is replaced by a weighted average including the variances and covariances of the yields.

$$(2.4) \quad \frac{\theta}{2-d_1^*-d_2^*} \frac{\partial(2-d_1^*-d_2^*)}{\partial\theta} = \frac{\theta R}{2(r^e-R)+\theta R}$$

In the special case in which the expected yield is equal to the yield on the riskless asset (i.e., $r^e=R$), there are retained earnings only because of the tax effect and the elasticity of the retained earnings with respect to the dividend tax rate is unity. When there is a positive expected excess return on retained earnings, the tax effect is less important and the elasticity is less than one.¹

¹In an early empirical study of the effect of taxes on the dividend policy of British firms, Feldstein (1970) estimated that the elasticity of the dividend rate with respect to the inverse of θ was 0.9. Since dividends were about two-thirds of retained earnings in that sample period, the estimated elasticity of 0.9 corresponds to an elasticity of retained earnings with respect to θ of approximately 0.6, and is therefore quite compatible with equation 2.4.

3. Characteristics of the Symmetric Equilibrium

The special case in which the two firms have equal expected yields and equal variances is particularly interesting to analyze. Together with the assumptions that we have made about the similarity of the two types of investors, this assumption about the firm implies that the only essential source of difference in the model is in the different tax treatments of households and institutions. We commented in the introduction, and show formally in section 4 below, that when the advantage of diversification is small (i.e., low risk or high correlation) this difference in taxation leads to specialization of ownership and corner solutions for the firms' dividend policies, i.e., the firm that remains in business pays no dividend. We now examine the characteristics of the equilibrium in the case in which there is sufficient risk and opportunity for diversification and show that in this case both firms do pay dividends. The opportunity for advantageous diversification by investors induces positive dividends by firms.

With $r_1^e = r_2^e$ and $\sigma_{11} = \sigma_{22}$ equation 1.18 shows immediately that $d_1^* = d_2^*$, i.e., both firms have the same optimal dividend. In contrast to the "no diversification" case in which the dividend policies are at opposite extremes, advantageous diversification produces identical dividend policies. This common dividend policy satisfies:

$$(3.1) \quad 1 - d^* = \frac{2(r^e - R) + \theta R}{\gamma\sigma(2+\rho)}$$

where r^e is the common expected yield, σ is the common variance, and ρ is the correlation between the yields.¹

Note first that $\theta = 0$ and $r^e = R$ together imply $d^* = 1$; when there is no tax on dividends and no "excess return" on funds retained in the firm, all profits will be paid out. The economic reason for this is clear: with no tax or yield incentive for retention, full payout avoids the risk of retained earnings without any loss in after-tax yield.

A small tax on dividends clearly makes $1-d^* > 0$ and therefore $d^* < 1$, i.e., both firms pay out some but not all of their profits as dividends. A positive but partial dividend payout is clearly optimal despite a tax that discriminates against dividends. Of course, a large enough value of θ can make $1-d^* \geq 1$ and therefore imply $d^* = 0$; when the tax discrimination against dividends is strong enough, no dividends will be paid. Note that the excess return on retained earnings affects the optimal dividends in the same way as the dividend tax. Starting at $\theta = 0$ and $r^e = R$, a small increase in r^e will cause positive but partial dividend payout while a large enough excess return on retained

¹With $\sigma_{11} = \sigma_{22}$, $\rho = \sigma_{12}/\sigma_{22} = \sigma_{12}/\sigma_{11}$. Equation 3.1 follows directly from 1.18 when it is noted that the common multiplier in 1.18 is the inverse of $1 - (\rho/2)^2$ and that $1 - \frac{1}{2}(\sigma_{12}/\sigma_{11}) = 1 - (\rho/2)$; the ratio of these two is the inverse of $1 + (\rho/2)$.

earnings will cause all dividends to stop.¹

Consider next the price per share that prevails in this case when both firms adopt the optimal dividend policy. This share price is the value that investors place on the initial dollar of available profits inside the firm.² Since dollars

¹It is tempting to ask what happens as ρ tends to unity. When $\rho = 1$, there is no opportunity for diversification. The economics implies that in this case there will be specialization of ownership and therefore of dividend policy. This cannot be seen by setting $\rho = 1$ in equation 3.1 because equation 3.1 does not hold when $\rho = 1$. When $\rho = 1$, the matrix A of equation 1.9 is singular and the share demand equations (1.10 and 1.11) from which 3.1 is derived do not hold.

²There is an extensive literature on this value, which is sometimes referred to as "Tobin's q." It has been common to assume that the equilibrium value of q is one, an assumption that we accepted in Feldstein, Green and Sheshinski (1979). Auerbach (1979), Bradford (1979) and King (1977) analyze a model without uncertainty and with only taxable shareholders; they conclude that if firms are paying positive but partial dividends, the share price must equal $1 - \theta$, i.e., a dollar of profits inside the firm must be valued at the amount that can be paid net of tax to the shareholder. (Their analysis also allows for a tax on capital gains; which also influences the share price; in the absence of this tax their share price formula reduces to $1 - \theta$.) Studies using the capital asset pricing model to measure the value of a marginal dollar inside the firm produce estimates that vary substantially over time with an average that is in the range of unity or somewhat less; see Gordon and Bradford (1979) and the studies that they cite. Green (1979) shows that changes in share prices on their ex dividend days cannot be used to estimate the value of a marginal dollar of funds inside the firm.

retained in the firms have equal expected yields and equal variances, their share prices must also be equal. Equation 1.14 confirms this and shows that the common price is:

$$(3.2) \quad p = R^{-1} [(1-\theta/2)dR + (1-d)r^e - \frac{1}{2}\gamma(1-d)^2\sigma(1+\rho)] .$$

The three terms on the right hand side of 3.2 show that the price depends on the net-of-tax value of the current dividend $[(1-\theta/2)d]$, the expected present value of the retained earnings $[(1-d)r^e R^{-1}]$, and the offset for the risk associated with the retained earnings $[\gamma(1-d)^2\sigma(1+\rho)]R^{-1}$. Substituting the optimal value of the dividend payout rate from equation 3.1 and rearranging terms yields:

$$(3.3) \quad p = 1 - \frac{\theta}{2} + \frac{\gamma\sigma}{2R} (1-d^*)^2$$

or

$$(3.4) \quad p = 1 - \frac{\theta}{2} + \frac{[r^e - R + R\theta/2]^2}{2R\gamma\sigma(1+\rho/2)^2} .$$

Since half of the shareholders pay tax at rate θ while half pay no tax, the average tax rate is $\theta/2$ and $1 - \theta/2$ is the net-of-tax income per dollar of dividends. Equation 3.3 shows that when it is optimal to pay out all profits as dividends ($d^* = 1$), the share price equals the net-of-tax value of the dividend.¹ More

¹This special case thus corresponds to the Auerbach-Bradford-King share price equation extended to the case of heterogeneous taxpayers. It holds however only when all profits are paid as dividends by both firms.

generally, when the firms retain some of their earnings, the price per share exceeds the net-of-tax amount that could be distributed. This is shown clearly in equation 3.3. The equivalent expression in equation 3.4 indicates why this is so. Since it is optimal for a firm to retain some of its earnings when the returns inside the firm exceed their opportunity cost or when there is a tax penalty on dividends, either of these reasons to limit dividends causes an increase in the share price vis-a-vis the price that would prevail if $d^* = 1$. This is seen explicitly in equation 3.4. To the extent that there is an excess return on retained earnings ($r^e > R$) or that the average tax rate on dividends is positive ($\theta/2 > 0$), the price exceeds the net amount that could be distributed. An increase in risk aversion (γ) or in the riskiness of retained earnings ($\sigma(1+\rho/2)$) decreases the magnitude of this premium.

It is certainly interesting to note that the price per dollar of earnings inside the firm may be less than, equal to or greater than unity. When $d^* = 1$, the price is clearly less than 1. A high value of excess return can of course produce a share value greater than one. But even if $r^e = R$, the price lies between $1 - \theta/2$ and 1.¹

¹Clearly when $r^e = R$ and $d^* = 0$, the value of the firm is the discounted expected value of the subsequent payout ($r^e/R = 1$) minus any adjustment for risk. When $r^e = R$ but $d^* > 0$, p lies between this upper bound and $1 - \theta/2$.

Our discussion in this section has implicitly assumed that the investors hold both assets in an optimal portfolio but this has not been formally demonstrated. Consider therefore the share demand equations 1.10 and 1.11. In the symmetric case of $r_1^e = r_2^e$ and $d_1^* = d_2^*$, the two elements of \underline{a}_H are equal. Moreover, $\sigma_{11} = \sigma_{22}$ implies that A is symmetric and therefore that A^{-1} is symmetric. This implies that $s_{H1} = s_{H2}$. The value of s_{H1} is easily shown to be

$$(3.5) \quad s_{H1} = \frac{R(1-\theta)d + (1-d)r^e}{\gamma\sigma_{11}(1+\rho)}$$

which is clearly greater than zero. Thus $s_{H1} = s_{H2} > 0$ and there is diversification by households. A similar derivation shows $s_{I1} = s_{I2} > 0$, which implies both that institutional investors diversify and that neither group demands 100 percent of the shares of any firm.

A Numerical Example

To conclude this analysis of the case in which the opportunity for advantageous diversification causes nonspecialization and positive but partial dividend payout, it is useful to present a numerical example in which these properties hold. Consider the case in which the expected return on investment in both firms is $r^e = 1.3$ and the correlation between the return is $\rho = 0.5$. Let the tax rate be $\theta = 0.5$ and the riskless yield on the alternative asset be $R = 1.1$. The common variance of the returns does not

matter as such, only the product of the variance and the risk aversion coefficient ($\gamma\sigma$). The dividend payout rate (d) and the combined risk parameter ($\gamma\sigma$) must satisfy the dividend payout condition (equation 3.1) and the condition that the demand for shares by households and institutions (given by equations 1.10 and 1.11) together equal unity for each firm and separately do not violate the condition that investors may not sell short. The symmetry of the current problem implies that each type of investor will hold equal amounts of both types of shares. These conditions are satisfied if the dividend payout rate is $d = 0.8$ and the risk parameter is $\gamma\sigma = 1.87$. Equation 3.3 implies that the corresponding price per share is $p = 0.79$.

4. The Segmented Market Equilibrium

We have been analyzing the case in which firms are identical but in which there is enough opportunity for advantageous diversification to cause investors to hold mixed portfolios. Firms pay out positive dividends in a value-maximizing equilibrium. Qualitatively, these results are not surprising. It is however somewhat odd that the equilibrium of our model in the symmetric case is itself symmetric: both firms choose the same dividend payout rate and each investor holds an equal share in the two firms. The conflict between diversification and tax-avoidance is completely resolved in favor of the former. One might have thought that the firms would "locate" at different points in the dividend spectrum attracting a different clientele, one more heavily taxed on average than the other, and that investors would accept this incomplete diversification in equilibrium in order to reap the tax advantages.

At present we do not know whether this striking symmetry property is the result of the mean-variance utility, the "two class" model of investors, or whether it is a phenomenon of more fundamental generality. (The next section will show, however, that it persists with competitive as well as duopolistic behavior by firms).

In this section we will show that this symmetric equilibrium, which is unique whenever investors are holding shares of both firms, coexists with asymmetric "locational" equilibria when

the non-negativity conditions for portfolios are binding. Such a situation arises when there is little variance in yields or a high correlation between the two firms so that diversification is of only limited benefit.

The phenomenon of asymmetric, segmented market equilibrium is seen most clearly in the extreme case of certainty: $\sigma_{11} = \sigma_{22} = 0$. This lack of risk implies that each investor values shares at the present value of their payouts, net of taxes. For either firm, one dollar paid as dividends is worth $(1-\theta)R$ to households and R to institutions; while one dollar of retained earnings is worth r to both types of investors.

Consider the case in which $R > r^e > R(1-\theta)$, i.e., in which funds inside the firm have a lower yield than outside the firm ($R > r^e$) but are worth more than funds outside the firm if a dividend tax has to be paid ($r^e > (1-\theta)R$).¹ In this case, the untaxed institutional investor prefers immediate payout ($d=1$) because the value of the dividend (R) exceeds the expected value of the funds left in the company (r^e). In contrast, the taxed household investor prefers no dividend payout ($d=0$) because the value of the net-of-tax dividend ($(1-\theta)R$) is less than the expected value of the funds left in the company (r^e). The market will accommodate this conflict of preferences by specialization of ownership and dividend policies.

¹ The alternative case of $r^e > R$, both investors will prefer to have no dividends and both firms will therefore choose $d=0$. The firms behave identically and there is no market segmentation.

Let us examine the equilibrium prices that would lead to $d_1 = 0$, $d_2 = 1$, with portfolios $s_{H1} = 1$, $s_{H2} = 0$, $s_{I1} = 0$, $s_{I2} = 1$. First, it is clear, that unless the initial ownership of shares gives the two classes equal portfolio wealth, the equilibrium prices of the two firms may not be equal. This is not incompatible with the value maximizing assumption because the firm cannot achieve the other's value by mimicking its dividend policy. Both values will change in this process.

We will show that the equilibrium prices are given by

$$(4.1) \quad \begin{aligned} p_1 &= R^{-1} r^e \\ p_2 &\in [(1-\theta), 1] \end{aligned}$$

where the precise value of p_2 in this interval is determined in such a way that the portfolios described above are compatible with the budget equation 1.1. For households to hold shares of firm 1 in positive quantity we need $p_1 \leq R^{-1} r^e$ and if they don't hold firm 2, then $p_2 \geq 1 - \theta$. Similarly, the implications that can be derived from institutions' portfolios are $p_2 \leq 1$, $p_1 \geq R^{-1} r^e$. Combining these we see that 4.1 is required.

To verify that these prices are indeed equilibria, it is necessary to see what changes would be induced by different dividend policies. This problem is a little curious in that even if dividends were to vary, the same prices and the same portfolios could still persist. Thus the equilibrium sustained by extreme dividend policies is compatible with value-maximization only in the sense that firms are indifferent to these choices.

It is of interest to note that the symmetric equilibrium $d_1 = d_2 = 0$ and $p_1 = p_2 = R^{-1}r^e$ is also an equilibrium here.¹ The paradox of symmetric vs. segmented equilibria is resolved by noting that the latter are produced when the non-negativity constraints for portfolios are binding.

Moreover one can observe that since no taxes are actually collected in either of these cases, the consumption patterns, and hence welfare considerations are identical.

The results of the riskless case can be extended to the case of small variance or high correlation without changing the essential conclusion. In such cases, the share demands implied by equations 1.10 and 1.11 would violate the no short-selling constraint. The constrained optimum would involve a corner solution in which ownership is specialized. The dividend policy of each company would then be adjusted to the tax situation of its homogeneous group of shareholders. The lack of such homogeneity and the presence of dividends for the majority of major publicly owned companies suggest that the opportunities for advantageous diversification are sufficient to prevent shareholder specialization.²

¹ Any share ownership will sustain this, and individuals will be indifferent.

² Other possibilities include a non-homogeneity of beliefs which are not perfectly correlated with tax status, locked-in investors due to the taxation of capital gains on realization, or inter-temporal considerations which are of practical importance but are difficult to model.

5. The Many-Firm Case

The analysis above has been cast entirely in duopolistic terms. Each firm, contemplating a change in its dividend policy, is altering the entire pattern of returns available to investors in a significant way. A more "competitive" view of the asset market would entail many firms, each of which is such a small part of the portfolio of the average investor that its dividend policy has only a marginal effect on the set of attainable mean-variance pairs. We shall show that the main results of the two-firm model, including all of the qualitative characteristics of the symmetric equilibria on which we have concentrated, remain valid in the many-firm case. Some modifications of the comparative statics will arise.

Our approach to the many firm case will be asymptotic. That is, we will consider the equilibrium of an economy with a large finite number of firms and will analyze the behavior of the model as this number increases. The reason for this methodology is conceptual as well as mathematical. If we were to deal immediately with the idealized case of a continuum of firms whose returns were all infinitesimal, the effect of any one firm's dividend policy on the attainable set would be literally zero,¹ and its value would therefore be a constant.

¹ There is the additional mathematical problem that it is impossible to have a continuum of random variables that are mutually independent; thus such a model would be ill-specified from the beginning.

Without a genuine solution to the value-maximization problem, our model has no motive force. On the other hand, if we were to consider the characteristics of each firm as fixed, and increase the number of firms and the number of investors at the same rate, the central limit theorem tells us that each investor will be able to achieve a riskless portfolio in the limit. Therefore, in order to preserve both a non-degenerate attainable set of portfolios for the investor and a non-trivial value-maximization problem for the firms, it is necessary to think of a sequence of economies in which the firm-specific variances are growing at the same rate as the number of firms and the number of investors. (The fraction of each firm initially owned by each investor will thus be decreasing at the square of this rate.)

Let there be N firms of each of two types, denoted 1 and 2 as above. The return to firm i per dollar of retained earnings is given by the sum of two random variables

$$r_i = x_i + x$$

where x_i is a firm specific return which is independent over all firms both within a type and across types, and x is a common return affecting all firms of both types. Moreover all the x_i 's are independent of x .

We assume that the variance of x_i is equal to $N\tau_1$ for all firms of type 1, and $N\tau_2$ for all firms of type 2. The means of x_i are denoted \bar{x}^1 and \bar{x}^2 respectively. The common component has mean \bar{x} and variance τ_0 .

We will study the equilibria of this model which have the characteristic that all the firms within each type are choosing the identical dividend policy. To compute the behavior of a single firm of type 1, we hypothesize a given dividend policy, d_1 , common to the $N-1$ other firms of type 1, and a policy, d_2 , common to the N firms of type 2. Then, by varying the dividend policy of the firm in question, we maximize its value in a manner completely analogous to that in Section 1.

An equilibrium is a pair (d_1, d_2) such that d_j is the optimal policy for a firm of type j given that the $2N-1$ other firms act in the indicated way.

It follows from the symmetry of the actions hypothesized that any optimal portfolio consists of equal ownership shares in all of the firms of a given type that are choosing a common dividend policy. Thus the three relevant random variables for portfolio choice are r , the return to the type 1 firm in question; $r^1 = \sum r_i$, where the summation runs over all firms of type 1 except this one; and $r^2 = \sum r_i$, where the summation runs over all firms of type 2. Under our assumptions, the variances and covariances are given by

$$\begin{aligned} \text{var } r &= N\tau_1 + \tau_0 \\ \text{var } r^1 &= N(N-1)\tau_1 + (N-1)\tau_0 \\ \text{var } r^2 &= N^2\tau_2 + N\tau_0 \\ \text{cov } r, r^1 &= (N-1)\tau_0 \\ \text{cov } r, r^2 &= N\tau_0 \\ \text{cov } r^1, r^2 &= N(N-1)\tau_0 \end{aligned}$$

Consider a taxable investor, H. Let s_H, s_{H1}, s_{H2} be the ownership shares he holds in each of the three categories of firms. Let p, p_1, p_2 be the prices of the shares of these three groups of firms. Writing the investor's objective function as in 1.3, the first-order conditions for his problem are expressed as

$$\gamma B \begin{pmatrix} s_H \\ s_{H1} \\ s_{H2} \end{pmatrix} = \begin{pmatrix} b_H \\ b_{H1} \\ b_{H2} \end{pmatrix} - R \begin{pmatrix} p \\ p_1 \\ p_2 \end{pmatrix}$$

where

$$B = \begin{bmatrix} (1-d)^2 (N\tau_1 + \tau_0) & (1-d)(1-d_1)(N-1)\tau_0 & (1-d)(1-d_2)N\tau_0 \\ (1-d)(1-d_1)(N-1)\tau_0 & (1-d_1)^2 (N(N-1)\tau_1 + (N-1)\tau_0) & (1-d_1)(1-d_2)N(N-1)\tau_0 \\ (1-d)(1-d_2)N\tau_0 & (1-d_1)(1-d_2)N(N-1)\tau_0 & (1-d_2)^2 (N^2\tau_2 + N\tau_0) \end{bmatrix}$$

$$b_H = R(1-\theta)d + (1-d)\bar{x}_1$$

$$b_{H1} = R(1-\theta)(N-1)d_1 + (1-d_1)(N-1)\bar{x}_1$$

$$b_{H2} = R(1-\theta)Nd_2 + (1-d_2)N\bar{x}_2$$

As above, we find the induced equilibrium prices for the assets by solving the equation

$$(5.1) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} Ns_H + Ns_I \\ Ns_{H1} + Ns_{I1} \\ Ns_{H2} + Ns_{I2} \end{pmatrix} = \frac{N}{\gamma} B^{-1} \begin{bmatrix} b_H + b_I \\ b_{H1} + b_{I1} \\ b_{H2} + b_{I2} \end{bmatrix} - 2R \begin{pmatrix} p \\ p_1 \\ p_2 \end{pmatrix}$$

for the prices, where the subscript 1 is the analogous quantity for institutions (i.e. setting $\theta = 0$).

We are interested in the behavior of the optimum d , given d_1 and d_2 , for large values of N . Taking the derivative of the solution for p from 5.1 with respect to d and letting N go to infinity, we find that the limiting value of d satisfies

$$(5.2) \quad 1-d^* = \frac{-1}{2\tau_1} [(1-d_1)\tau_0 + (1-d_2)\tau_0] - \frac{R(2-\theta)}{2\gamma\tau_1} + \frac{\bar{r}_1}{\gamma\tau_1}$$

Interchanging 1 and 2 a similar formula applies to type 2 firms optimal dividend policy.

To find an equilibrium, set $d^* = d_1$ in 5.2 and, in the analogous expression for type 2, set $d^* = d_2$. Solving for d_1 and d_2 we find that

$$(5.3) \quad (1-d_1) \left(1 + \frac{\tau_0/2\tau_1}{1+\tau_0/2\tau_2}\right) = \frac{2\bar{x}^1 - R(2-\theta)}{2\gamma\tau_1} - \frac{\tau_0/2\tau_1}{1+\tau_0/2\tau_2} \left[\frac{2\bar{x}^2 - R(2-\theta)}{2\gamma\tau_2} \right]$$

and similarly for type 2.

If $\tau_0 = 0$, so that all firms' returns are independent, we get

$$1-d_1 = \frac{2\bar{x}^1 - R(2-\theta)}{2\gamma\tau_1}$$

The conditions for an interior optimum for the firm $1 > d_1 > 0$ are

$$\gamma\tau_1 > \bar{x}^1 - R(1-\theta/2) > 0$$

The right hand inequality is satisfied as long as \bar{x}^{-1} exceeds R ; the left hand inequality requires that the uncertainty in returns not be too small, or else the firm will want to retain everything. This is analogous to 1.18.

6. Comparative Statics of the Many Firm Case

As in the case of duopoly, the retention rate of firms of a given type is increasing in the mean return, decreasing in the variance of return and decreasing in R , the relative attractiveness of the alternative asset. Because we have modeled the returns across firms as having a common component, they are necessarily positively correlated. The negative influence of the other type's mean payout, and the positive influence of its firm-specific variance which was a characteristic of the two-firm model whenever $\sigma_{12} > 0$, is also preserved.

One difference between the two models concerns the potential for a positive effect of the tax rate on the retentions of any firm. Recall that in 2.1 it was shown that if the regression coefficient of firm 1's returns on firm 2's returns exceeds 2, then this apparently perverse phenomenon would hold. From equation 5.3 we see that

$$\frac{\partial(1-d_1)}{\partial\theta} = \frac{R}{2\gamma\tau_1} \left(1 - \frac{\tau_0}{2(\tau_2 + \tau_0)}\right) > 0$$

It follows, a fortiori, that the aggregate retention rate rises in response to increased dividend taxation.

One might believe that this result is due to our implicit assumption that the regression coefficient of one firm's returns on that of any other is necessarily less than one ($\tau_0/\tau_0 + \tau_j$, where j is the type of firm in question). This is

not true. Suppose that the model were generalized slightly so that the returns to firms of type one were

$$r_i = x_i + Kx.$$

The regression of type 1 on type 2 returns is now $\frac{K\tau_0}{\tau_0 + \tau_1}$. For K sufficiently large this exceeds 2, the critical value beyond which increased taxation leads to firm 1's decreased retentions in the duopolistic model. In the many firm case, however, the equation analogous to 1.18 is

$$\begin{aligned} (1-d_1) \left[1 + \frac{\tau_0}{2\tau_1 K} - \frac{\tau_0^2}{4\tau_1 \tau_2 (1+\tau_0/2\tau_2)} \right] \\ = \frac{2\bar{r}_2 - R(2-\theta)}{2\gamma\tau_2} \cdot \left(\frac{1}{1+\tau_0/2\tau_2} \right) \\ + \frac{2\bar{r}_1 - R(2-\theta)}{2\gamma\tau_1 K^2} \cdot \end{aligned}$$

Since this is a linear equation for the retention rate, its derivative with respect to θ is just the derivative of the right hand side, divided by the bracketed expression on the left. The latter can be negative when K is large and τ_1 is small, suggesting the potential for a negative, perverse dependence here as well. This is not possible, however, because retentions would themselves have to be negative in such a case. We would really be in the corner solution where everything is paid out. In summary, the competitive model has the property that wherever there are some retentions in equilibrium they will increase in magnitude when

dividends are taxed more heavily. This is in contrast to 2.1 where a high variance of firm 1's return and a high covariance could produce a reduction in retentions downward from a positive equilibrium level.

In the symmetric case, where the means and variances are the same for the two types of firms, the same formula as 2.4 applies for the elasticity of aggregate retentions with respect to the tax rate. This is a further rough confirmation of the applicability of this model to the empirical results mentioned above in footnote 1, p. 24.

7. Conclusion

This paper has provided a simple model of market equilibrium to explain why firms that maximize the value of their shares pay dividends even though the funds could instead be retained and subsequently distributed to shareholders in a way that would allow them to be taxed more favorably as capital gains. Our explanation does not rely on any asymmetry of information or divergence of interests between management and shareholders. The heterogeneity of tax rates and the existence of uncertainty and of risk aversion are explicitly recognized. Indeed, it is the combination of the conflicting preferences of shareholders in different tax brackets and their desire for portfolio diversification in the face of uncertainty that together cause all firms to pay dividends in our model.

The very simple framework that we have used here should be extended in several directions. The possibility of borrowing by corporations and investors should be specifically recognized. An explicit multiperiod analysis with growing capital stocks should be developed. The relationship between each corporation's rate of investment and its equilibrium rate of return can be examined in this extended framework.

The present study indicates that the existing tax treatment of dividends distorts corporate financial dividends and may cause a misallocation of total investment. It will be important to see whether these adverse effects remain in the more general analytic framework.

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Appendix

We verify the stability of the equilibrium defined in 1.1B by examining the reaction functions in 1.16 and 1.17. Note that

$$\frac{dd_1^*}{dd_2} = \frac{-\alpha_{12}}{2\alpha_{11}}$$

$$\frac{dd_2^*}{dd_1} = \frac{-\alpha_{12}}{2\alpha_{22}}$$

If we linearize the adjustment process

$$\frac{d}{dt} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} \begin{pmatrix} d_1^* - d_1 \\ d_2^* - d_2 \end{pmatrix}$$

(where α_1 and α_2 are arbitrary positive speed of adjustment coefficients) around the equilibrium, we are lead to examine the

$$J = \begin{pmatrix} -\alpha_1 & -\alpha_1 \frac{\alpha_{12}}{2\alpha_{11}} \\ -\alpha_2 \frac{\alpha_{12}}{2\alpha_{22}} & -\alpha_2 \end{pmatrix}$$

in which the trace $(-(\alpha_1 + \alpha_2))$ is clearly negative and the determinant $(\alpha_1 \alpha_2 (1 - \frac{\alpha_{12}^2}{4\alpha_{11}\alpha_{22}}))$ is clearly positive. Therefore

the system is locally stable.