NBER WORKING PAPER SERIES

ON FORECASTING INTEREST RATES: AN EFFICIENT MARKETS PERSPECTIVE

James E. Pesando

Working Paper No. 410

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

November 1979

The Research reported here is part of the NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research. The author is indebted to Michael Huberman for research assistance.

NBER Working Paper 410
November 1979

On Forecasting Interest Rates: An Efficient Markets Perspective

ABSTRACT

This paper reviews, from an applied forecasting perspective, the properties of short- and long-term interest rates in an efficient market. The paper emphasizes that efficient markets do not preclude economic agents from successfully forecasting movements in short-term interest rates. For brief forecast intervals, however, *ex ante* changes in long-term rates are sufficiently close to zero that economic agents are not likely to improve upon the no-change prediction of the martingale model. Economic agents, in effect, are not likely to succeed in forecasting *short-term* movements in long-term interest rates. An analysis of three sets of Canadian interest rate forecasts provides results which are consistent with the theoretical discussion. Further, these results parallel those obtained in recent studies of recorded forecasts in the United States, although the authors of these latter studies apparently failed to appreciate the nature of their findings.

James E. Pesando
Institute for Policy Analysis
150 St. George Street
Toronto, CANADA
M5S 1A1

416/978-8621

1. INTRODUCTION

In view of the substantial empirical evidence which supports the proposition that capital markets are efficient in their use of publicly available information, surprisingly little attention has been directed to the question of whether economic agents can successfully forecast interest rate movements. Impressed perhaps by the early evidence in favour of the random walk hypothesis,[1] most sophisticated agents tend to regard published forecasts of the future level of aggregate stock prices with a healthy degree of skepticism. As yet, however, such skepticism does not appear to extend to published forecasts of movements in either short- or long-term rates of interest, nor is it clear to many whether or not such skepticism is warranted.

At the theoretical level, the question of whether economic agents can successfully anticipate interest rate movements in an efficient market has received increased attention. Sargent (1976) and Pesando (1978) have noted that long-term interest rates in an efficient market will <u>approximately</u> follow a martingale sequence - and thus exhibit random walk characteristics - in the absence of time-varying term premiums. As Mishkin (1978) and Pesando (1979) caution, however, there is nothing in efficient markets theory which requires that short-term rates follow a random walk. Thus agents without access to inside information may well succeed in forecasting movements in short-term rates, but are not likely to succeed in forecasting <u>short-run</u> movements in long-term rates if the term premium accorded long-term bonds is indeed time-invariant.

The purpose of this paper is to extract from these theoretical results an appropriate perspective on the practical problem of forecasting interest rate movements. The first section of the paper briefly reviews the theoretical results, including the role of time-varying term premiums. The second explores the closeness of the martingale approximation for long-term rates in Canada, an exercise which bears directly on the usefulness of the no-change prediction implied by the martingale model. The third section examines the performance of a variety of recorded interest rate forecasts in Canada in light of the preceding theoretical arguments. A concluding section completes the paper.

2. THEORETICAL OVERVIEW

Let $R_{n,t}$ denote the interest rate on an n-period, non-coupon bond in period t , ϕ_t the information available to the market in period t, and ${}_{t+i}f_{1,t}$ the forward rate at time t for the one-period bond rate in period t+i. If pure expectations govern the term structure, and if the standard arithmetic approximation to the geometric average that links the long-term rate to current and future short-term rates is satisfactory, then - as noted by Sargent (1976) and Pesando (1978) - the *ex ante* change in the long-term rate can be written as follows:

$$E(\tilde{R}_{n,t}|\phi_{t-1}) - R_{n,t-1} = \frac{1}{n} * [E({}_{t+n-1}\tilde{f}_{1,t}|\phi_{t-1}) - R_{1,t-1}] \qquad (1)$$

The term on the right-hand side of equation (1), which represents the non-overlapping one-period rates, clearly approaches zero as n gets large. If $\psi_{n,t}$ represents the term premium accorded an n-period bond in period t, then (1) becomes:

$$E(\tilde{R}_{n,t}|\phi_{t-1}) - R_{n,t-1} = \frac{1}{n} * [E({}_{t+n-1}\tilde{f}_{1,t}|\phi_{t-1}) - R_{1,t-1}]$$

$$+ E(\tilde{\psi}_{n,t}|\phi_{t-1}) - \psi^{n}_{t-1} \qquad (2)$$

If this term premium is constant, then (2) reduces to (1) and hence the martingale approximation emerges:

$$E(\tilde{R}_{n,t}|\phi_{t-1}) \doteq R_{n,t-1} \qquad (3)$$

Equation (3) indicates that the _ex ante_ change in the long-term rate is approximately zero if n is large, and hence that the optimal forecast of the long-term rate is simply its current value. For a ten-year bond whose (annual) yield is being forecast on a _quarterly_ basis, n equals 40 and the martingale approximation is likely to be close so long as the short-term rate is not "too nonstationary". If the yield on this same bond is being forecast on a monthly basis, n equals 120 and the _ex ante_ change in the long-term rate will be even closer to zero. In general, the shorter the forecast interval, the closer the martingale approximation. Equivalently, it is _short-run_ movements in long-term rates which will not be predictable under the joint hypothesis of market efficiency and a time-invariant term premium.

For _coupon_ bonds, as noted by Pesando (1979), the expression analogous to (1) is more complicated. Nonetheless, as illustrated later in this study, the martingale approximation remains quite close for _short-term_ movements in long-term rates in the absence of time-varying term premiums.

Intuitively, the martingale approximation stems from the fact that over the unit intervals (e.g. one quarter) in which agents typically forecast interest rate movements, the percentage change in bond prices necessary to equate the ex ante returns on short- and long-term securities (up to a time-invariant term premium) is very small. As a result, the implied ex ante changes in long-term rates are very close to zero. From this perspective, the martingale approximation for long-term rates is but a specific illustration of the more general proposition that short-run movements in speculative prices are not likely to be predictable.

Efficient market theory does not require, however, that the short-term rate follow a martingale, nor does empirical evidence suggest that this is the case. There is no arbitrage opportunity through which agents could eliminate any serial dependence in the short-term rate and - as emphasized by Modigliani and Shiller (1973) - an expectations "solution" to the term structure is consistent with any stochastic representation of the short-term rate. Using end-of-the quarter data on 90-day Treasury bills and 90-day finance company paper, so as to eliminate possible biases due to time aggregation (Working (1960)), alternative time series models can be fitted to the Canadian data. For the sample period 1957:1 - 1979:1, an ARMA (1, 1) or an AR (2) representation is clearly superior to the martingale model for both short-term rates.[2] The fact that the short-term rate need not follow a martingale sequence is "reconciled" with the martingale property of the long-term rate when it is remembered that the latter is only approximate. The long-term rate would exactly follow a martingale if and only if the short-term rate followed a martingale. In this case, the *ex ante* capital gain or loss on long-term bonds would always equal zero, and the long-term rate would simply equal the short-term rate plus the time-invariant term premium.

The martingale approximation for the long-term rate is derived on the assumption that the term premium accorded long-term bonds is time-invariant. Is the existence of time-varying term premiums likely to invalidate this approximation? The answer, of course, is moot, although a convincing case can be made that the "forecastability" of movements in long-term rates is not likely to be salvaged in this fashion. Those who work in the capital asset pricing framework of modern finance theory tend to treat the term premium - which is related to the covariance of bond returns and the return to the market portfolio - as constant over time.[3] Many - if not most - of those who have conducted empirical studies of the determinants of term premiums have concluded that they may well be time-invariant. McCulloch (1975) and Pesando (1975a) have presented evidence that the term premium accorded long-term government bonds in the United States and Canada, respectively, may be time-invariant. Mishkin (1978) and Pesando (1978) have found empirical support for the joint hypothesis that the bond market is efficient and term premiums are time-invariant. Attempts to enrich the traditional term structure equation of Modigliani-Sutch (1967) or Modigliani-Shiller (1973) with additional variables to capture the spirit of the "preferred habitat" model have, on the whole, been unsuccessful. Indeed, empirical support for the existence of time-varying term premiums is largely confined to structural models of the long-term bond market, such as those estimated for the United States by

Friedman (1977, 1979) and for Canada by Masson (1978). In these models, demand functions for long-term bonds are estimated for each major class of investor, such as banks, life insurance companies and so forth. When the aggregate of these demands is set equal to supply, the models yield a (restricted) reduced-form equation for the long-term rate. Although these reduced-form equations contain asset stocks, financial flows and other variables associated with time-varying term premiums, the repeated failure of researchers to find comparable effects in the traditional term structure equations is rather disconcerting. Clearly, the careful and successful inclusion of stock and flow variables into demand and supply functions is not sufficient to guarantee that the implicit equation for the long-term interest rate will better characterize its evolution than will the traditional term structure alternatives. Transactors exogenous to the model - such as investment dealers - may include Meiselman's risk-neutral, well-financed speculators whose arbitrage activities at the margin impose the expectations solution on the term structure. Indeed, the successful performance of these models in an *ex ante* forecasting experiment relative to the martingale alternative would offer perhaps the most impressive support for their successful incorporation of time-varying term premiums. (Such experiments could be conducted by extracting the implied equations for the long-term interest rate and then using optimal time series predictors of the variables exogenous to the model to create the *ex ante* forecasts.) As yet, no such evidence has been offered in support of these models. To conclude, no one has yet produced convincing evidence of the existence of well-delineated and predictable time-varying term premiums sufficient to salvage on an à priori basis the "forecastability" of short-run movements in long-term interest rates on these grounds.

Finally, the preceding discussion has implicitly assumed that agents do not have access to inside information. Following Lucas (1976), there also exists considerable doubt as to whether agents who might possess inside information - such as central banks - can successfully forecast movements in long-term interest rates. For central banks, the forecasting exercise typically consists of the attempt to simulate the impact on long-term rates of contemplated policy initiatives. There exists the fundamental concern that standard simulation exercises may <u>not</u> provide the central bank with reliable "forecasts" of movements in long-term rates conditional upon hypothetical changes in policy instruments. If such policy initiatives diverge from those which characterize the period over which the policy-maker's macroeconometric models have been estimated, or alter the time series properties of key variables (such as the short-term rate) which drive expectations, then little faith can be attached to the implicit assumption that the underlying structure of the economy will remain invariant to the contemplated activity. In short, even agents <u>with</u> access to inside information may have difficulty in forecasting short-term movements in long-term interest rates, a result which provides a re-inforcing perspective on the negative conclusion regarding the "forecastability" of such movements contained in the prior discussion.

3. THE MARTINGALE APPROXIMATION: SOME EMPIRICAL EVIDENCE

Let $H_{n,t}$ denote the holding-period return on long-term bonds in period t, c_{t-1} the (known) coupon to be paid during the period t, P_t the price of the bond at the end of period t and ϕ_{t-1} the information available to the market at the end of period t-1. Then:

$$E(\tilde{H}_{n,t}|\phi_{t-1}) = \frac{E(\tilde{P}_t|\phi_{t-1}) - P_{t-1} + c_{t-1}}{P_{t-1}} \tag{4}$$

If the bond is trading at par, so that $\frac{c_{t-1}}{P_{t-1}}$ is simply the long-term interest rate $R_{n,t-1}$, then (4) can be written

$$E(\tilde{H}_{n,t}|\phi_{t-1}) = \frac{E(\tilde{P}_t|\phi_{t-1}) - P_{t-1}}{P_{t-1}} + R_{n,t-1} \tag{5}$$

Let ψ_n be the constant term premium accorded the n-period bond. Assuming that investors have a one-period horizon, and noting that $R_{1,t-1}$ is the <u>ex ante</u> return on holding one-period bonds, the relationship between the <u>ex ante</u> returns on short- and long-term securities can be written as follows:

$$R_{1,t-1} + \psi_n = E(\tilde{H}_{n,t}|\phi_{t-1}) = \frac{E(\tilde{P}_t|\phi_{t-1}) - P_{t-1}}{P_{t-1}} + R_{n,t-1} \tag{6}$$

Implicit in (6) is the <u>ex ante</u> change in the long-term interest rate necessary to equilibrate the two returns, up to the presumed time-invariant term premium. Let x_t represent the expected capital gain or loss implied by (6), or $(R_{1,t-1} + \psi_n - R_{n,t-1})$. Since the price of a consol is just the coupon divided by the interest rate, the *ex ante* change in the long-term rate *if the bond were a consol* ($\Delta R^A_{\infty,t}$) is simply:

$$\Delta R^A_{\infty,t} = \frac{-x_t}{1 + x_t} * R_{\infty,t-1} \tag{7}$$

If the long-term bond is trading at par, and if its term to maturity of n-periods is sufficiently large so that its change during the period can be ignored, then the *larger* change in the long-term rate necessary to produce the same capital gain or loss is approximately:[4]

$$\Delta R^A_{n,t} = \Delta R^A_{\infty,t} * \frac{1}{1 - \frac{1}{(1+R_{n,t-1})^n}} \tag{8}$$

Note, in general, that the unit of observation is crucial to the determination of $\Delta R^A_{\infty,t}$ and hence $\Delta R^A_{n,t}$. For quarterly data, for example, the interest rates in (6) are expressed as quarterly rates, and the implicit capital gain or loss is only one-fourth of that required in the case of annual data. The smaller the unit of observation, *cet. par.* the smaller is the required capital gain or loss and hence the smaller the *ex ante* change (expressed at an annual rate) in the long-term interest rate. Since economists traditionally forecast macroeconomic variables on a quarterly basis, the unit of observation will be standardized at one quarter in the subsequent analysis.

Matching data on 90-day Treasury bills with long-term Government of Canada bonds, and 90-day finance company paper with long-term corporate bonds,[5] the *ex ante* changes in the long-term rates can be calculated in two steps using (7) and (8).[6] A representative term to maturity of 17 years was deemed to be appropriate for both of the long-term bond indexes. The time-invariant term premiums in (6) were set equal to the average spreads between the corresponding long- and short-term rates during the sample period 1957:1 - 1979:1. These premiums equalled 136 basis points for Canada bonds, and 138 basis points for corporate bonds.

The results of these calculations, summarized in Table 1, indicate that the *ex ante* changes in the long-term rates are indeed quite small. The mean absolute change is only 2.07 *basis points* for Government of Canada bonds, and 2.60 *basis points* for corporate bonds.[7] The maximum changes were 7.21 and 9.67 basis points, respectively. A full eighty-eight percent of the *ex ante* changes for Canada bonds were smaller than 4 basis points in absolute value. The corresponding figure for corporate bonds was 80%. By contrast, the actual changes in these long-term rates were typically quite large. Of particular interest is the striking difference in the size of the variances of the *ex ante* changes and the variances of the actual changes. The ratio of the former to the latter equalled 0.62% for Canada bonds and 1.26% for corporate bonds. Since anticipated and unanticipated changes in long-term rates must be uncorrelated if the market is efficient, these figures imply that 99.38% and 98.74% of the variances of the observed changes in Canada and corporate bonds, respectively, must be due - under the joint hypothesis of market efficiency and time-invariant term premiums - to the receipt of new information. Equivalently, under this joint hypothesis, regressions of the current change in the long rate on any set of variables known at the beginning of the period are not likely to explain more than one percent or so of the variance of the dependent variable. A higher R^2 or explained variance would suggest the presence of spurious correlation, and suggest that the observed relationship is not likely to prove useful in an out-of-sample or forecasting context.

To sum up, the ex ante changes in the long-term rates are so close to zero that the no-change prediction of the martingale model is a very close approximation to the optimal forecast under the joint hypothesis. This benchmark is employed in the subsequent analysis of the recorded forecasts of long-term interest rates in Canada.[8] Note also that the small size of these ex ante changes suggests that the relatively large changes in long-term interest rates implicit in many recorded forecasts imply substantial variation in the ex ante return on long- relative to short-term securities. This point receives attention in the subsequent discussion of the recorded forecasts.

4. AN ANALYSIS OF RECORDED INTEREST RATE FORECASTS IN CANADA

The Conference Board in Canada has made quarterly macroeconomic forecasts since late 1974, and histories of these forecasts for long-term corporate bonds (McLeod, Young, Weir (MYW) 10 industrials) and 90-day finance paper are available. Data Resources Incorporated (DRI) of Canada has published macroeconomic forecasts since 1974, and historical data are available on five separate interest rate series: 90-day Treasury bills, 90-day finance company paper, long-term Government of Canada bonds, long-term corporate bonds (MYW 10 industrials) and long-term provincial bonds (MYW 10 provincials). For a large part of the sample period, DRI forecasts are available monthly. To provide comparability with the other recorded forecasts, only four forecasts per year - corresponding to the calendar quarters - were employed in the forecasting experiments. Since December of 1974, McLeod, Young, Weir and Company Limited has conducted quarterly surveys of interest rate forecasts, receiving responses from 35 to 40 individuals from the financial community. The survey respondents are asked to provide one and two quarter ahead forecasts of nine Canadian and three U.S. interest rates. Since (1) the concern in this paper is with Canadian data and (2) several of the series (such as the prime lending rate of the chartered banks) are not determined in auction markets, only three series proved appropriate for this study: 90-day finance company paper, long-term corporate bonds (MYW 10 industrials) and long-term provincial bonds (MYW 10 provincials).

In comparing the recorded forecasts to the predictions of the martingale model, care must be made to ensure that the martingale forecasts (i.e. the current value of the relevant interest rate) do not have access to information not available at the time the recorded forecasts were made. Since the published data on the several interest rates series are available only on a monthly basis, the martingale forecasts were set equal to the interest rate at the end of the latest month which unambiguously preceded the month in which the recorded forecasts were made. In so doing, this procedure confers an informational advantage to the recorded forecasts. This advantage typically consisted of about two weeks, but in some cases approached a full month.

The results of the forecast comparisons,[9] summarized in Table 2, indicate that the recorded forecasts of long-term rates in general failed to outperform the no-change prediction of the martingale model, while this was not the case for the recorded forecasts of short-term rates. In view of the theoretical discussion, this result is not surprising. Only the two-quarter ahead forecasts by DRI of the long-term Canada rate proved superior to the martingale forecasts. Even this modest success merits qualification, however, in view of (1) the informational advantage accorded the recorded forecasts and (2) the inferior performance of the one-quarter ahead forecasts relative to those of the martingale model. This latter result suggests that the improvement of the two-quarter ahead forecasts may be somewhat of a statistical artifact and not likely to obtain over a longer sample period.

Both the DRI and MYW survey forecasts of the short-term rates did prove significantly superior to the martingale model. Further, the forecasts of the short-term rate by the Conference Board were about as accurate as the martingale model, while their forecasts of the long-term rate were distinctly inferior. These results, in general, enhance the credibility of the null results with respect to the forecasts of the long-term rates. In general, there always remains - especially with survey data - the possibility that the reported figures do not accurately reflect the expectations that actually drive market behavior. In this regard, it is interesting to note that Friedman (forthcoming) provides evidence that the Goldsmith-Nagan interest rate forecasts fail to pass a now standard battery of tests designed to measure their "rationality". As noted by Pesando (1975b), such failures raise the interpretive question of whether the recorded forecasts do indeed reflect those of active economic agents, in which case the latter fail to form "rational" expectations, or whether such recorded forecasts must <ins>not</ins> be those of the market on the maintained hypothesis that the market efficiently processes information. Lynch (1979) performs this standard battery of tests - the unbiasedness of the forecasts, the orthogonality of forecast errors to costlessly available information, and efficiency and consistency in the use of autoregressive information - on the MYW survey data and, on the whole, the results are favourable to the hypothesis that the expectations are rationally formed.

Finally, the magnitude of the changes in the long-term interest rates implicit in many of the recorded forecasts merits comment. The maximum (absolute) change forecast for the corporate bond rate, based on the two-period compared to the one-period prediction (so as to eliminate the uncertainty regarding the date - and hence the value of this rate - when the forecasts were made), equalled 22 basis points in the MYW survey data, 30 basis points for the DRI forecasts and 70 basis points (!) for the Conference Board forecasts. The mean absolute changes were 11.2, 12.7 and 13.8 basis points, respectively. In view of the magnitude of the *ex ante* changes summarized in Table 1, these recorded forecasts clearly imply substantial variation in the *ex ante* return on long- relative to short-term securities. For the extreme forecast of the Conference Board, for example, the implied *ex ante* return on long-term bonds is 32.60%. During this same quarter, the Conference Board was predicting a 6.86% return on 90-day finance company paper. On a *prima facie* basis, this result appears to be unreasonable and serves to highlight the conceptual importance of translating predicted changes in long-term rates into comparable statements about their ex ante holding-period return. Although corresponding calculations for the DRI and MYW survey forecasts are less dramatic, they also imply a very sharp divergence between the ex ante returns on short- and long-term securities.[10] Both forecasters and those who monitor forecasts would be well advised to focus on ex ante returns in assessing the likely path of long-term interest rates during their forecast horizons.

5. SUMMARY AND CONCLUSIONS

The failure of the recorded forecasts of long-term interest rates to outperform the no-change prediction of the martingale, and the reversal of this result for short-term rates, is not surprising on theoretical grounds. In fact, however, this point does not appear to be well understood by many financial market participants. These results, for example, mirror those found in two studies of the accuracy of recorded forecasts in the United States. The authors of these studies, however, apparently failed to fully understand the nature of their findings. Prell (1973), analyzing Goldsmith-Nagan survey data for the period 1969:3 - 1972:4, found that the recorded forecasts of short-term rates (Federal funds, 90-day Eurodollars and 90-day Treasury bills) outperformed the no-change prediction, while those of the long-term rates (Aaa utility bonds) did not. A similar, although less precise, analysis by Fraser (1977) of pooled forecasts of the National Association of Business Economists, Wharton Econometric Forecasting Associates and Chase Econometrics, Inc. for the period 1974-1976 suggests comparable results. In both studies, the authors refer with some surprise to the inability of forecasters to outperform the "naive" no-change extrapolation for long-term rates, and apparently fail to anticipate that their results are not surprising in light of the evidence in support of market efficiency.

To conclude, economic agents ought to regard published forecasts of short-term movements in long-term interest rates with a healthy degree of skepticism, although the same does not necessarily apply to future movements in short-term rates. (The very small *ex ante* changes in long-term rates isolated for the quarterly data examined in this paper do not, of course, necessarily imply trivial future movements in the short-term rate. In the absence of yield data on noncoupon bonds with sequential maturity dates, however, such implied movements are not easily extracted.) Those seeking to forecast movements in long-term rates over longer horizons ought to explore further the use of the *ex ante* changes in the long-term rate implicit in the term structure itself as a means of refining the no-change prediction of the martingale model. In this regard, the large forecast changes in long-term rates in the recorded data analyzed in this paper - which imply substantial *ex ante* variation in the returns to long-term relative to short-term bonds - merit note. These results highlight, in the absence of convincing evidence regarding the existence of time-varying term premiums, the fact that economic agents ought to observe two rules: (1) predicted changes in long-term rates must be "small", at least over short forecast intervals, if they are to be creditable; and (2) predicted changes in long-term rates ought to be recast into statements regarding *ex ante* returns on long-term bonds, in part to facilitate comparison with the *ex ante* returns (i.e. the interest rate) on short-term securities.

Footnotes

[1] If stock prices follow a random walk (i.e. $P_t - P_{t-1} = \varepsilon_t$ where ε_t is an independent and identically distributed disturbance term with mean zero), then equilibrium returns are zero. If stock prices follow a random walk with a particular form of drift (i.e. $P_t - P_{t-1} = \varepsilon_t + kP_{t-1}$), then equilibrium returns are constant (and equal to k). The statement in the text is meant to draw attention to the fact that forecasts of the future level of common stocks which translate into large (in absolute value) *ex ante* returns on a diversified portfolio are appropriately discounted by the market. The same skepticism does not seem to apply to forecasts of changes in long-term interest rates, perhaps because the latter are not so easily translated into statements about *ex ante* returns and/or the case for market efficiency is viewed as less persuasive for bonds than for common stocks.

[2] The standard errors of the regressions, in basis points, for the Treasury bill and finance company paper rates were: ARMA (1,1), 74.92 and 92.90; AR (2), 75.98 and 93.65; AR (1), 78.32 and 94.08. In the AR (1) regression, the null hypothesis that the coefficient of the lagged rate is equal to one could not be rejected. The standard errors for the martingale or ARIMA (0,1,0) models, which constrain this coefficient to equal unity, are clearly larger than those noted for the AR (1) specification.

[3] See, for example, Roll (1971) and McCallum (1975).

[4] Equation (8) is derived as follows. For a consol, $\frac{dP}{P} = \frac{-dR_\infty}{R_\infty}$. Since the ex ante changes in the long-term rate implied by (6) are so small, this expression produces ex ante changes ($dR_\infty = \frac{-dP}{P} * R_{\infty,t-1} = -x_t * R_{\infty,t-1}$) which are almost identical to the exact changes implied by (7). Consider the standard valuation formula for an n-period coupon bond. Let c be the coupon payment and F be the face value of the bond. After summation, the standard valuation formula reduces to:

$$P = \frac{c}{R_n} + \frac{(F - \frac{c}{R_n})}{(1 + R_n)^n} \qquad \text{(F1)}$$

Differentiating (F1) and dividing the resulting expression by price yields:

$$\frac{dP}{P} = \frac{\left(-\frac{c}{R_n^2} - \frac{nF}{(1+R_n)^{n+1}} + \frac{n(\frac{c}{R_n})}{(1+R_n)^{n+1}} + \frac{\frac{c}{R_n^2}}{(1+R_n)^n} \right) dR_n}{\frac{c}{R_n} + \frac{(F - \frac{c}{R_n})}{(1+R_n)^n}} \qquad \text{(F2)}$$

If the bond is selling at par, so that $\frac{c}{F}$ equals R_n , then this expression reduces to:

$$\frac{dP}{P} = \frac{-dR_n}{R_n} * (1 - \frac{1}{(1 + R_n)^n}) \qquad \text{(F3)}$$

Hence, for a given $\frac{dP}{P}$ ($= x_t$) , the larger change in the interest rate on an n-period bond is related to the change in the interest rate on a consol by (8). Note, of course, that $dR_n \to dR_\infty$ as $n \to \infty$.

[5] The corporate bond series is the McLeod, Young, Weir 10 Industrials, compiled by McLeod, Young, Weir and Company Limited. The other series are compiled by the Bank of Canada. The corporate bond rates are those prevailing on the last business day in the quarter. All of the other rates are those prevailing on the last Wednesday in the quarter.

[6] Calculating the ex ante change in the long-term rate as if the long-term rate were a consol provides a useful check on the results reported in the text. If these changes are not trivial, then the larger (in absolute value) changes for the finite maturity bonds clearly cannot be trivial either. The mean absolute values of the ex ante changes so calculated are 1.42 and 1.92 basis points for the government and corporate bonds, respectively.

[7] For corporate bonds, the term to maturity figure of 17 years is clouded by the existence of call options and sinking funds. Fortunately, the results of the simulation exercises are not sensitive to moderate changes in the assumed term to maturity of the bond indexes. For an assumed term to maturity of 10 years, the mean absolute value of the ex ante changes is 3.57 basis points rather than the 2.60 basis points reported in the text. For an assumed term to maturity of 20 years, the corresponding figure is 2.44 basis points.

Similarly, variations in the assumed value of the time-invariant term premium do not produce major changes in the mean absolute values of these ex ante changes. If, contrary to most empirical studies, the term premium is set equal to zero (i.e. the pure expectations hypothesis is assumed), then these values rise to 3.50 and 4.02 basis points for Canada and corporate bonds, respectively. These higher values reflect the tendency of long-term to exceed short-term rates, which thus requires - on average - *ex ante* increases in long-term rates to produce the capital losses necessary to equilibrate *ex ante* returns.

[8] An obvious question is whether the use of the *ex ante* change to refine the martingale prediction (i.e. $\hat{R}_{n,t} = R_{n,t-1} + \Delta R^A_{n,t}$ rather than $\hat{R}_{n,t} = R_{n,t-1}$) will improve its forecasting accuracy and thus provide a more useful benchmark. In fact, root-mean-squared errors of the refined martingale as defined above were slightly smaller than that of the martingale model for the period 1957:2 - 1979:1. These errors equalled 34.87 and 29.75 basis points for government and corporate bonds, respectively. compared to those of 35.15 and 30.81 basis points for the unadjusted martingale model. These results, in general, invited the estimation of the following regression:

$$\Delta R_{n,t} = \alpha + B(\Delta R^A_{n,t}) + \varepsilon_t \qquad \text{(F4)}$$

Under the joint hypothesis of market efficiency and a time-invariant term premium, $\alpha = 0$ and $B = 1$ while the residuals are devoid of serial correlation. The appropriate test statistic for the null hypothesis ($\alpha = 0$, $B = 1$), distributed $F(2,86)$, equalled 0.83 and 2.97 for government and corporate bonds, respectively. The 5 per cent significance level for this test statistic is 3.12 and thus the null hypothesis could not be rejected for either of the two data sets. The Durbin-Watson statistics, as required by the joint hypothesis, provided no evidence of serial correlation. Finally, as anticipated in the text, the explanatory power of these equations is quite low. R^2 equalled .022 in the corporate bond regression, and only .005 in the government regression.

[9] The forecasts differ in their release dates, and pertain to both quarterly average (Data Resources and the Conference Board) and end-of-the-quarter (MYW surveys) interest rates. As a result, the accuracy of the forecasts cannot be compared across forecasting agents.

[10] The maximum change predicted by DRI implied an *ex ante* return on long-term bonds of 17.71 per cent at a time when DRI forecast a 9.49 per cent return on finance company paper. For the MYW survey, the corresponding figures are 17.23 per cent and 9.29 per cent.

REFERENCES

Fraser, D.R. "On the Accuracy and Usefulness of Interest Rate Forecasts". Business Economics XII: No. 4 (September 1977): 38-44.

Friedman, Benjamin M. "Financial Flow Variables and the Short-Run Determination of Long-Term Interest Rates". Journal of Political Economy 85, No. 4 (August 1977): 661-89.

__________. "Substitution and Expectations Effects on Long-term Borrowing Behavior and Long-term Rates". Journal of Money, Credit and Banking 11: No. 2 (May 1979): 131-150.

__________. "Survey Evidence on the Rationality of Interest Rate Expectations". Journal of Monetary Economics, forthcoming.

Lucas, Robert E. "Econometric Policy Evaluation: A Critique", in Karl Brunner and Allan H. Meltzer, eds., The Phillips Curve and Labour Markets, Carnegie-Rochester Conference Series on Public Policy, vol. 1 (Amsterdam, North-Holland, 1976): 19-46.

Lynch, Kevin. "A Test of the Rationality of Interest Rate Expectations Using Survey Data". Mimeograph, April 1979.

Masson, Paul R. "Structural Models of the Demand for Bonds and the Term Structure of Interest Rates". Economica 45 (November 1978): 363-377.

McCallum, John S. "The Expected Holding Period Return, Uncertainty and the Term Structure of Interest Rates". Journal of Finance 30 (May 1975): 307-23.

McCulloch, J. Huston. "An Estimate of the Liquidity Premium". Journal of Political Economy 83: No. 1 (February 1975): 95-119.

Mishkin, Frederic, S. "Efficient-Markets Theory: Implications for Monetary Policy". Brookings Papers on Economic Activity 3, 1978: 707-52.

Modigliani, Franco and Shiller, Robert. "Inflation, Rational Expectations and the Term Structure of Interest Rates". Economica 40, No. 157 (February 1973): 12-43.

Modigliani, Franco and Sutch, Richard. "Debt Management and the Term Structure of Interest Rates: An Empirical Analysis of Recent Experience". Journal of Political Economy 75, No. 4, pt. 2 (August 1967): 569-89.

Pesando, James E. "On the Efficiency of the Bond Market: Some Canadian Evidence". Journal of Political Economy 86, No. 6 (December 1978): 1057-1076.

________________. "The Impact of the Conversion Loan on the Term Structure of Interest Rates in Canada: Some Additional Evidence". Canadian Journal of Economics 8: No. 2 (May 1975): 281-88. (a)

________________. "A Note on the Rationality of the Livingston Price Expectations". Journal of Political Economy 83: No. 4 (August 1975): 849-58. (b)

________________. "On the Random Walk Characteristics of Short- and Long-term Interest Rates in an Efficient Market". Journal of Money, Credit and Banking, forthcoming, 1979.

Prell, M.J. "How Well Do the Experts Forecast Interest Rates?" Federal Reserve Bank of Kansas City Monthly Review, September-October 1973, pp. 3-13.

Roll, Richard. "Investment Diversification dn Bond Maturity". Journal of Finance 26: No. 1 (March 1971): 51-66.

Sargent, Thomas J. "A Classical Macroeconometric Model for the United States". Journal of Political Economy 84, No. 2 (April 1976): 207-237.

Working, Holbrook. "Note on the Correlation of First Differences of Averages in a Random Chain". Econometrica 28: No. 4 (October 1960): 916-18.

TABLE 1A

ACTUAL AND ANTICIPATED QUARTERLY CHANGES IN LONG-TERM INTEREST RATES IN CANADA, 1957:1 - 1979:1

	Government Bonds		Corporate Bonds	
	Actual	Anticipated	Actual	Anticipated
Mean absolute change	26.38	2.07	23.08	2.60
Maximum absolute change	127.00	7.21	119.00	9.67
Mean change	6.73	-0.03	5.90	0.05
Standard deviation	34.50	2.71	30.23	3.39

TABLE 1B

CUMULATIVE FREQUENCY DISTRIBUTION OF ABSOLUTE VALUES OF ANTICIPATED CHANGES IN LONG TERM INTEREST RATES

	Government Bonds		Corporate Bonds	
Absolute Change	No.	Percent	No.	Percent
Less than 2	51	57	42	47
" " 4	79	88	71	80
" " 6	84	94	82	92
" " 8	89	100	86	97
" " 10	89	100	89	100
	89	100	89	100

In both tables all numbers are in basis points. See text for discussion of the calculation of the anticipated changes.

TABLE 2

Recorded Forecasts of Short- and Long-Term Interest Rates Versus the Martingale Alternative

Source	Sample	Series	ROOT MEAN SQUARE FORECASTING ERRORS: One Period, Recorded	One Period, Martingale	Two Period, Recorded	Two Period, Martingale
DATA RESOURCES OF CANADA	1975:2 - 1978:4	90-day Treasury bills	48.47	74.50	79.89	127.80
		90-day Finance paper	61.66	63.86	108.40	119.42
		Long-term Canada bonds	25.79	24.36	36.57	44.87
		McLeod, Young, Weir 10 industrial bonds	33.80	19.62	42.56	36.54
		McLeod, Young, Weir 10 provincial bonds	27.77	20.48	36.40	37.22
THE CONFERENCE BOARD IN CANADA	1975:1 - 1978:4	90-day Finance paper	71.16	69.68	120.72	129.40
	1974:3 - 1978:4	McLeod, Young, Weir 10 industrial bonds	40.12	28.44	76.22	45.56
MCLEOD, YOUNG, WEIR SURVEYS	1974:4 - 1978:4	90-day Finance paper	94.03	108.40	137.51	154.50
		McLeod, Young, Weir 10 industrial bonds	36.37	33.83	54.55	51.83
		McLeod, Young, Weir 10 provincial bonds	38.96	37.19	55.89	41.79

Notes: Martingale forecasts equal the interest rate at the end of the month prior to the date at which the recorded forecasts were made. Forecast dates for the recorded figures vary within the quarter and are thus not strictly comparable across forecast sources.