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NOTES ON OPTIMAL WAGE TAXATION AND UNCERTAINTY

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ABSTRACT

Most contributions to optimal tax theory have assumed that all prices, including that of leisure, are known with certainty. The purpose of this paper is to analyze optimal taxation when workers have imperfect information about their wages at the time they choose their labor supplies. Both efficiency and redistributive aspects of the problem are considered.

The paper begins with a discussion of the positive theory of wage taxation and labor supply under uncertainty. This is followed by a discussion of optimal taxation when individuals are identical, but their wages are stochastic. Finally, the case of simultaneous uncertainty and inequality is discussed. In this part of the paper it is assumed that the government's objective is to maximize a utilitarian social welfare function.

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I. Introduction

The literature of optimal taxation has provided useful principles for guiding thought about both efficient commodity taxation and optimal redistributive taxation.[1] However, most contributions to optimal tax theory have assumed that all prices, including that of leisure, are known with certainty at the time that resources are allocated. Although there have been a few discussions of the impact of uncertainty on optimal taxation [Diamond, et al., 1978; Mayshar, 1977], for the most part uncertainty appears to have had little influence upon optimal tax research. The purpose of this paper is to analyze optimal taxation when workers have imperfect information about their wages at the time they choose their labor supplies. The results are contrasted with those of the certainty case.

In Part II we set the stage for the normative analysis by outlining the positive theory of wage taxation and labor supply under uncertainty.[2] In Part III we discuss efficient taxation in the presence of uncertainty. Redistributive aspects of the problem are ignored by assuming that individuals are homogeneous. Attention is centered on the oldest and most fundamental result from optimal tax theory, that lump sum taxation is efficient. We show that when uncertainty is taken into account, this is no longer necessarily true. Because a wage tax reduces the riskiness of wage income, some combination of a lump sum tax and a wage tax generally will minimize excess burden.

Part IV analyzes the problem of redistributive income taxation when the government's objective is to maximize a utilitarian social welfare function. A series of numerical computations indicates how optimal tax rates can change in the presence of uncertainty. A concluding section provides a summary and some suggestions for future research.

We assume throughout this paper that the market fails to provide insurance against the vagaries of wage rates. We feel that this is typically the case. Problems of moral hazard associated with insurance in general are especially pervasive in the insurance of the returns to human capital.[3] The private insurer, unable to distinguish clearly between external events and the endogenous behavior of the insured party, would provide an incentive for an insured worker to work less hard, spend less time seeking a higher-paying job, or otherwise earn an income below potential. In such a situation the market is unlikely to provide insurance.

II. The Effects of Wage Taxation Under Uncertainty

Analyses of the effects of wage taxation on labor supply typically assume that workers know the real return on their labor effort when they allocate their time between labor and leisure.[4] As Block and Heineke [1973] point out, for many situations this assumption is unrealistic. Farmers who are deciding how much to work at planting time, for instance, do not know the effect that weather or vermin will have on their crop before harvest, or what market conditions will prevail when they sell their crop. More generally, workers who contract to work at fixed money wages do not know the effect that changes in consumer prices will have on their real wages during the contract period. In this section we show that wage uncertainty can affect the response of labor supply to taxes, in some cases potentially reversing the sign of results that obtain under certainty.

Utility maximization under certainty implies the following familiar relationships between taxes and labor supply: (i) Non-compensated increases in proportional wage taxation have an ambiguous effect on labor supply. This is due to the usual conflict between income and substitution effects. (ii) Increased taxation of exogenous non-labor income raises labor supply as long as leisure is non-inferior. (iii) Income-compensated increases in proportional wage taxes unambiguously reduce labor supply.

We show in this section that under wage uncertainty propositions (i) and (ii) continue to obtain under fairly general conditions. However, the important result (iii) on the sign of the compensated wage response is no longer necessarily true. An expected-income compensated increase in wage taxation may induce a rise in labor supply. Before proving these results, we first consider the effect of wage uncertainty *per se* on labor supply. The analysis provides a result necessary for understanding the impact of taxes on labor supply, as well as being interesting in and of itself.

A. Wage Uncertainty and Labor Supply[5]

We assume that consumer tastes may be represented by the smooth, twice-differentiable utility function $u(c,L)$ increasing in consumption, c, decreasing in hours worked, L, and exhibiting decreasing marginal utility of consumption. The worker's opportunity set is characterized by a random gross wage w which is taxed at rate t and by non-labor income in amount ω. Thus

$$(2.1) \qquad c = (1-t)wL + \omega .$$

The worker is assumed to choose L to maximize $E[u(c,L)]$ where expectations are based on a subjective probability density function of w, $f(w)$.

The first-order condition for a maximum is that

$$(2.2) \qquad E[u_1 \Theta w + u_2] = 0$$

where $\Theta \equiv 1-t$, $u_1 \equiv \partial u/\partial c$ and $u_2 = \partial u/\partial L$. Following the criterion of

Rothschild and Stiglitz [1971], increased wage uncertainty in the sense of an arithmetic mean preserving spread in w raises or lowers the maximizing value of L, denoted L^*, as $u_1\Theta w + u_2$ is convex or concave in w, or as

$$(2.3) \qquad (\Theta^2 L^*) \quad [2u_{11} + u_{111}\Theta w L^* + u_{211}L^*] \gtrless 0 \ .$$

Diminishing marginal utility of consumption implies that the first term is negative while diminishing or constant absolute risk aversion with respect to consumption implies that the second term is positive [see Leland, 1969]. Thus the overall effect of an increase in uncertainty is in general ambiguous.

If we assume constant relative risk aversion and additive separability of u, however, then

$$(2.4) \qquad u_{111} = - u_{11}(R+1)/c$$

where $R \equiv -u_{11}c/u_1$, the degree of relative risk aversion with respect to consumption. In this case (2.3) becomes

$$(2.5) \qquad - u_{11}\Theta^2 L^* [\frac{\Theta w L^*}{c}(1+R) - 2]$$

which is more likely to be positive: (1) the larger the degree of relative risk aversion R and (2) the larger the contribution of total labor income to consumption expenditure. These results are important in the analysis of the impact of wage taxation on labor supply, to which we now turn.

B. <u>Wage Taxation and Labor Supply</u>

To determine the effect of an increase in t, the proportional tax rate on wage income, on labor supply we differentiate (2.2) with respect to t to obtain

(2.6) $$\frac{dL^*}{dt} = -E[(u_1 + u_{11}\Theta wL^* + u_{21}L^*)w]\Delta^{-1}$$

where

$$\Delta \equiv -E[u_{11}(\Theta w)^2 + 2u_{21}\Theta w + u_{22}] > 0 .$$

The inequality is satisfied if the second order conditions obtain. Assuming, again, that the utility function is additively separable, (2.6) becomes

(2.7) $$- E[(u_1 w)(1-R\frac{\Theta wL^*}{c})]\Delta^{-1} .$$

As in the certainty case, no unambiguous statement can be made about the direction of the effect of increased wage taxation on labor supply. However, an increase in wage taxation is more likely to increase labor supply when risk aversion and the contribution of labor income to consumption expenditure are large.

The effect on labor supply of an increase in ω, non-labor income, is found by differentiation of (2.2):

(2.8) $$\frac{dL}{d\omega} = E(u_{11}\Theta w + u_{21})\Delta^{-1} .$$

If the utility function is additively separable $u_{21} = 0$ and the negativity of expression (2.8) is implied by diminishing marginal utility of consumption. Again, as in the certainty case an increase in non-labor income lowers labor supply.

We turn now to an analysis of compensated changes in the net wage. An increase in t which is accompanied by an increase in non-labor income of $\bar{w}L^*$, where $\bar{w}$ denotes the expected wage, does not affect expected consumption.[6] Such a change constitutes an <u>expected</u>-income compensated increase in wage taxation, denoted η, where

(2.9) $$\eta \equiv \frac{dL}{dt} + \bar{w}L^* \frac{dL}{d\omega} .$$

Incorporating (2.6) and (2.8) into (2.9) gives

$$(2.10) \qquad \eta = E[-u_1w-(u_{11}\Theta+u_{21})wL^*(w-\bar{w})]\Delta^{-1} .$$

The first term in expression (2.10), $-u_1w$, is unambiguously negative while, under certainty, $w = \bar{w}$, and the second term is zero. In the absence of wage uncertainty, an income-compensated increase in wage taxation reduces labor supply.

In general, however, $(w-\bar{w})$ is not zero and it is therefore non-trivial to sign the second term of (2.10). To do so,[7] we begin by defining

$$(2.11) \qquad \Gamma(w) \equiv -(u_{11}\Theta+u_{21})wL^*$$

so that (2.10) can be re-written

$$(2.12) \qquad \eta = -E[u_1w]\Delta^{-1} - E[\Gamma(w)(w-\bar{w})]\Delta^{-1}$$

Now observe that

$$(2.13a) \qquad \int_{-\infty}^{\bar{w}} \Gamma(w)(w-\bar{w})f(w)dw \gtrless \Gamma(\bar{w}) \int_{-\infty}^{\bar{w}} (w-\bar{w})f(w)dw$$

and

$$(2.13b) \qquad \int_{\bar{w}}^{\infty} \Gamma(w)(w-\bar{w})f(w)dw \gtrless \Gamma(\bar{w}) \int_{\bar{w}}^{\infty} (w-\bar{w})f(w)dw$$

as $\Gamma'(w) \gtrless 0$. Adding the inequalities (2.13a) and (2.13b) we observe that the right-hand side is zero. The term $E[\Gamma(w)(w-\bar{w})]$ is thus positive or negative as $\Gamma'(w) \gtrless 0$. But differentiation of (2.11) yields

$$(2.14) \qquad \Gamma'(w) = -[u_{11}\Theta + u_{21} + (u_{111}\Theta+u_{211})\Theta wL^*]L^*$$

Under conditions of additive separability of utility and constant relative risk aversion, (2.14) becomes

$$(2.15) \qquad \Gamma'(w) = -\Theta u_{11}[1-(1+R)\Theta wL^*/c]$$

which can be positive. The possibility of a positive response of labor supply to an income-compensated increase in wage taxation is therefore a real one. To illustrate this possibility consider a situation in which the wage equals two or zero with equal probability. If the utility function is additively separable and if relative risk aversion is constant then expression (2.10) may be written

$$(2.16) \qquad \eta = E[-u_1(1-R\frac{\Theta L^*}{2\Theta L^*+\omega})]$$

which if $\omega = 0$ and $R > 2$, for instance, is positive.

Intuitively, a positive response of labor supply to an income-compensated increase in wage taxation can arise because wage taxation reduces not only the expected wage but its variability as well. If the second effect tends to raise the supply of labor, then the direction of the overall response can be opposite to the one usually expected. Note that expression (2.15) can be positive only when (2.5) is negative: an income-compensated increase in wage taxation can raise labor supply only when increased wage uncertainty reduces labor supply.

III. Efficient Taxation and Uncertainty

By definition, a lump sum tax is independent of an individual's behavior. Such a tax does not break the equality between the marginal rate of substitution in consumption and the marginal rate of transformation in production, and hence generates no excess burden. In this section we show that in the presence of uncertainty, lump sum taxation is not efficient.

To develop an intuition for this result, consider an individual whose wage is uncertain. In the face of uncertainty, the individual's welfare will increase if he can obtain insurance. But a proportional earnings tax in effect acts as insurance -- it lowers risk because the government shares in both losses and gains. Indeed, if labor supply were completely exogenous, the individual would desire a 100% earnings tax, with the expected value of earnings returned as a lump sum.[8]

This extreme result is a consequence of the unrealistic assumption that the individual's gross earned income stream is exogenous. We now discuss the efficient taxation of an individual with an uncertain wage stream who faces a labor-leisure tradeoff. Throughout the analysis we assume that the government chooses a tax schedule which satisfies an expected revenue requirement and which maximizes expected taxpayer utility. We also assume that the parameters of the tax system must be set *ex ante*, in ignorance of the true wage outcome for each individual. For this reason expected taxpayer utility constitutes the appropriate social objective function.[9]

We again assume that the pretax wage w is a random variable with probability density function $f(w)$. The distribution is identical and independent for each worker and each worker knows the function f . As in section II the utility of the typical worker may be represented by a smooth, concave function $u(c,L)$. The worker pays a proportional wage tax at rate t and receives from the government a (positive or negative) lump-sum transfer, T . Thus,

$$(3.6) \qquad c = \Theta wL + \omega + T ,$$

where, again, ω denotes non-labor income and $\Theta \equiv 1 - t$.

The government seeks to extract an average amount of revenue G from each worker at minimum social cost, where

(3.7) $G = tE(wL) - T$.

The optimal tax problem is to choose t and T which satisfy (3.7) and maximize the worker's expected utility.

The worker chooses L to maximize

(3.8) $$\int u(\Theta wL + \omega + T, L)\, f(w)dw$$

given Θ , T and $f(w)$. Let L^* denote the value of L which maximizes (3.8). The government, then, must find t and T which satisfy the budget constraint

(3.9) $T = t\bar{w}L^* - G$.

The first term on the right-hand side of (3.9) is expected wage tax revenue. We assume that the number of workers is sufficiently large to insure that average labor income per worker is $\bar{w}L^*$.

Substituting (3.9) and L^* into (3.8) and differentiating with respect to t gives, as a first-order condition for a maximum,

(3.10) $E\{u_1 \cdot [\Theta w\eta - L^*w + t\bar{w}\eta + L^*\bar{w}] + u_2\eta\} = 0$

where η is the effect on L^* of an expected-income compensated increase in wage taxation, as defined in (2.9). Recall, however, the first-order condition for individual utility maximization,

(3.11) $E[u_1\Theta w + u_2] = 0$.

Substituting (3.11) into (3.10) yields the optimum condition

(3.12) $E\{u_1[L^*(\bar{w}-w) + t\bar{w}\eta]\} = 0$.

Under certainty, $w = \bar{w}$ and (3.12) is satisfied at $t = 0$. This is the well-known result that, under certainty, lump-sum taxation is optimal.

Under uncertainty, however, when (3.12) is evaluated at $t = 0$, we have

$$(3.13) \qquad L^*E[u_1(\bar{w}-w)] = L^*[E(u_1)E(\bar{w}-w) + \text{cov}\ (u_1,\bar{w}-w)] \ .$$

The first term in square brackets on the right-hand side of (3.13) is zero. The second term, since $du_1/dw = u_{11}L^* < 0$, is positive. Expected utility at the point $t = 0$ is <u>strictly increasing</u> in t. Whenever the wage rate is uncertain and the marginal utility of income is diminishing, a welfare improvement is obtained by raising the wage tax rate above zero.

In contrast with the case in which labor supply is exogenous, however, the optimal wage tax rate is not 100%. To show this, consider first values of t equal to or greater than unity. For such values net wage income is non-positive and optimal labor supply is zero. At $L^* = 0$ expression (3.12) reduces to

$$(3.14) \qquad t\bar{w}\ E[u_1\eta \ \big|_{L^* = 0}] \ .$$

Let $\tilde{t}$ denote the lowest marginal tax rate at which $L^* = 0$. For $t > \tilde{t}$ expression (3.14) is zero while, to the immediate left of $\tilde{t}$ it is negative. (The negativity of η at $L^* = 0$ follows immediately from equation (2.10).) Thus welfare is nonincreasing in t over the range $(\tilde{t}, \infty)$ and must strictly decrease for increases in t which occur to the immediate left of $\tilde{t}$. There exists, therefore, a tax rate $t < \tilde{t}$ which yields higher welfare than $t = 1$ -- 100% wage taxation is inefficient. Since welfare is increasing in t at $t = 0$ and decreasing in t at or below $t = \tilde{t}$ there exists a locally efficient wage tax rate t^* between zero and 100%.

Since $L^* = 0$ for $t \geq 1$, an increase in t cannot increase welfare in this range. However, casual examination of the first-order condition (3.12) does not rule out the possibility that $t < 0$, i.e., that a wage subsidy may be optimal. As shown in Section II, $\eta > 0$ is possible. For $t < 0$, then, expression (3.12) may become negative. Nevertheless, a wage subsidy is never optimal. To prove this consider a worker's expected utility when the tax rate is $\hat{t} < 0$. Let $L^*(\hat{t})$ be the associated labor supply. Expected utility is then

$$(3.15) \qquad E\{u[w(1-\hat{t})L^*(\hat{t}) + \omega + \hat{t}\bar{w}L^*(\hat{t}), L^*(\hat{t})]\}$$

Now, if he were to continue to work $L^*(\hat{t})$ hours when the tax rate was zero his expected utility would be

$$(3.16) \qquad E\{u[wL^*(\hat{t}) + \omega, L^*(\hat{t})] \ .$$

In each case consumption consists of a deterministic component, $t\bar{w}L^*(\hat{t}) + \omega$, and a random component, $w(1-t)L^*(\hat{t})$. For any given value of t, expected consumption is the same in both (3.15) and (3.16), but the weight of the random component rises as t falls. Thus, a fall in t constitutes a mean preserving spread in the distribution of consumption. As Rothschild and Stiglitz [1970] show, for all individuals with $u_{11} < 0$, a mean preserving spread in consumption reduces expected utility. Thus (3.66) exceeds (3.15). Facing a zero wage tax and non-labor income of ω, however, a worker will not choose to work $L^*(\hat{t})$. Denoting the optimal labor supply when $t = 0$ as $L^*(0)$, expected utility given $t = 0$ is

$$(3.17) \qquad E\{u[wL^*(0) + \omega, L^*(0)]\}$$

which, since $L^*(0)$ is an optimum for $t = 0$, must exceed expression (3.16). By transitivity (3.17) thus exceeds (3.15); a zero wage tax must always dominate a wage subsidy. We conclude that under wage uncertainty, when labor supply is endogenous, the optimal wage tax lies strictly between zero and 100%.

IV. Optimal Redistributive Wage Taxation

So far we have analyzed an economy in which individuals are identical. When people differ according to their abilities and in their endowments of non-human wealth, both utilitarian and Rawlsian principles of social justice suggest that society impose taxes to redistribute income. To the extent that taxes impose distortions, optimal redistribution does not require equal post-tax incomes for all. Redistribution continues only as long as the incremental gains from more equality exceed the incremental excess burdens. Analyses of optimal income taxation under certainty indicate that, when labor supply is endogenous, the optimal degree of progressivity of the income tax may indeed be quite low.[10] In view of our finding that, under uncertainty, a positive wage tax is optimal even when all workers are identical *ex ante*, one might suppose that, when individuals differ in terms of their expected productivities, uncertainty in wages raises the optimal progressivity of the income tax.

Unfortunately, general conditions under which this conjecture is true are difficult to obtain. Just as in the literature on optimal income taxation in certainty models, it appears necessary to make fairly specific assumptions in order to obtain many interesting results. In this section we develop a model in which two classes of individuals with random wages must allocate their time between labor and leisure and the decisions are made *ex ante*.[11] Some numerical results suggest that the presence of uncertainty may have a substantial impact on optimal marginal tax rates. Overall, the analysis implies that because previous studies ignored uncertainty, they may have generated incorrect estimates of optimal tax rates.

In our model, workers must determine their labor supplies on the basis of the subjective probability distributions of their wages. Inequality represents differences in wage distributions which workers do know at the time they make their labor supply decisions. In an economy with equality and uncertainty all workers choose their labor supplies with the same expectations about their realized earnings. Conversely, in an economy with inequality and no uncertainty, workers know exactly what their wages are at the time they choose their labor effort. This is the case typically analyzed in the optimal income tax literature. The same distribution of wages may be observed in both situations, but the allocation of labor effort and optimal tax policy may differ substantially in the two situations. In the presence of pure wage uncertainty, for instance, if workers' tastes are homogeneous then we should observe the same effort on the part of all workers. This is not the case in the presence of inequality.

To derive the optimal linear wage tax in the presence of both inequality and wage uncertainty we first consider the individual's optimization problem. An individual in class i has a random wage w^i with p.d.f. $f^i(w^i)$. He chooses L^i to maximize

$$(4.1) \qquad E[u(\Theta w^i L^i + T + \omega \, , L^i)]$$

Let L^{i*} denote the value of L^i that maximizes (4.1). Expected social welfare is then given by

$$(4.2) \qquad \int Eu[(\Theta w^i L^{i*} + T + \omega \, , L^{i*})] g(i) di$$

where $g(i)$ gives the relative number of members of class i.

Revenue from the wage tax T_Y is

$$T_Y \equiv \int (1-\Theta) E(w^i) L^{i*} g(i) di$$

which, under our assumption that risks are independent, is non-random. The optimal tax problem is then to choose Θ to maximize (4.2) and satisfy

$$T_y - T - G \geq 0$$

where G denotes government revenue requirements.

A number of interesting questions arise in this context: Does introducing uncertainty in wages increase the optimal progressivity of the income tax? To what extent do revenue requirements affect this result? Given the ex post wage distribution, is the optimal progressivity of the tax necessarily greater when uncertainty, rather than inequality, is generating the observed wage distribution? As is the case with optimal taxation problems under certainty, a fruitful approach toward answering these questions is to consider specific numerical examples.

We consider an economy consisting of two classes, one and two, each with different wage distributions. Members of each class have constant elasticity of substitution utility functions

$$u^i = \frac{1}{\gamma}[\alpha(c^i)^{-\mu}+(1-\alpha)(1-L^i)^{-\mu}]^{-\gamma/\mu} \quad ,$$

where γ determines the degree of risk aversion $(R = 1 - \gamma)$, μ determines the elasticity of substitution between consumption and leisure $(\sigma = 1/(1 + \mu))$, and α determines in part the distribution of full income between consumption and leisure.

We compute the linear income tax which maximizes $E(u^1) + E(u^2)$ under several alternative assumptions about wage distributions and parameter values.[12] For all the calculations, we set $\mu = 1.45$, a value that Stern [1976] has shown is implied by a number of econometric studies of labor supply. The share parameter α was set at 0.33.

Four sets of simulations were done, which we now discuss in turn:

1. Effects of Changing Risk Aversion

The purpose of these simulations is to determine how uncertainty and risk aversion change optimal tax rates. We assume at the outset that members of class one receive a wage of 0.5 or 1.5 each with probability 1/2 while members of class two receive a wage of 1.0 or 3.0, again each with probability 1/2. In order to focus on the effects of uncertainty *per se*, it is assumed that both revenue requirements and non-wage income are zero. We then compute optimal tax rates conditional on various values of γ. The results are shown in the first row of Table 1. The rates increase as γ decreases. Thus, as individuals become more risk averse, they value more highly the insurance aspects of wage taxation, and higher tax rates become appropriate.

It is of some interest to determine how optimal tax rates change if the wage is known with certainty. We therefore compute optimal tax rates when each individual receives his mean wage with probability of one. These results are exhibited in the second row of Table 1. In every case, uncertainty leads to higher tax rates than those associated with certainty.[13] The differences are most pronounced when the degree of risk aversion is low.

2. Effects of Changing Revenue Requirements

We now investigate the impact of different revenue requirements upon optimal tax rates. We set $\gamma = -1.5$, a figure consistent with recent empirical evidence. [See Friend and Blume, 1975, p. 919.] Negative values of G, of course, are algebraically equivalent to positive unearned income.

The results for both the certainty and uncertainty cases are shown in Table II. For both cases, positive revenue requirements raise optimal tax rates. This result parallels one found by Feldstein [1973] in the context of a certainty model.

3. Effects of Inequality versus Uncertainty

If there were neither inequality nor uncertainty, optimal tax rates would be zero. Either inequality or uncertainty is sufficient to generate positive tax rates, but how important is each effect? To answer this question, we first assume that our two individuals are identical, but that they both face an uncertain wage: 0.5 with a probability of 1/2 and 1.5 with a probability of 1/2 . The associated optimal tax rates for $G = 0$ and various values of γ are shown in the first row of Table III.

These rates are then contrasted with those which emerge when individuals one and two have wages of 0.5 and 1.5, respectively, with certainty. These (See the second row of Table III.) For most cases, the certainty-inequality case yields higher marginal tax rates, although the difference diminishes as γ increases, and the relationship is actually turned around when $\gamma = 0.5$.

An alternative interpretation of the results of Table III is useful. The first row shows the optimal tax rates when labor supply decisions are made ex ante, and the second when they are made ex post. The table suggests that even if individuals must make their decisions before the wage is known, it does not necessarily increase the progressivity of the optimal linear earnings tax. The outcome depends upon the degree of risk aversion, and for a wide range of values uncertainty leads to less progressivity.

4. Effects of Alternative Loci of Uncertainty

Casual observation suggests that not all individuals face equal amounts of uncertainty. We therefore study how optimal tax rates depend upon who is bearing the risk. To begin, we first compute the optimal tax rate when there is equality (in the sense that both individuals have the same expected wage), but only one person faces uncertainty. More specifically, assume that individual one faces a wage of 0.5 or 1.5, each with probability 1/2; and individual two faces a wage of 1.0 with certainty. (For this set of simulations, $\gamma = -1.5$ and $G = 0$.) The optimal tax rate is 0.44. This is a striking result when it is recalled that if both individuals face certainty, the optimal tax rate is zero, while if both individuals face uncertainty it is 0.52 (from the second column of Table III). When one-half the population faces uncertainty, the optimal rate is much closer to the one which prevails if all face uncertainty than if none faces uncertainty.

We now examine a situation in which only one individual faces uncertainty and there is also inequality. Consider first a case in which individual one faces a wage of 0.5 or 1.5, each with probability of 1/2, and individual two faces a wage of 2.0 with certainty. Then imagine giving individual one a wage of 1.0 with certainty, but individual two facing a wage of 1.0 or 3.0, each with probability of 1/2. The results are recorded in Table IV. When the "poor" individual's income is uncertain, a more progressive tax structure emerges than when the "rich" man is the risk-bearer. This is true even though the ratio of the variance of the wage to its mean is the same for both uncertainty cases.

Comparison of these results with those of the third column of Table I is also quite useful. When either individual faces uncertainty, the optimal tax rate is considerably above the certainty rate, and when the poor person's income is the source of the uncertainty, the result is strikingly close to the value of 0.55, which occurs when both individuals face uncertainty. We are lead to conjecture that even if a relatively small fraction of the population experiences wage uncertainty, there may be a significant impact on optimal tax rates.

As stressed above, since these results are based on a simple and unrealistic model, we do not claim that they reflect actual optimal marginal tax rates for any real economy. The analysis does show, however, the complex ways in which uncertainty and inequality interact in the optimal taxation problem.

V. Conclusion

Our goal has been to re-examine the theory of optimal taxation in a framework which explicitly recognizes uncertainty. An important result is that lump sum taxation is not necessarily efficient. This is a consequence of the fact that a tax on wages serves partially to insure the individual against random wage movements. Although related results have been shown in the literature on insurance and uncertainty, up to now such considerations have generally been ignored in the optimal tax literature. Consider, for example, Bradford's [1978] summary of Atkinson and Stiglitz's discussion of the subject:

> Admitting [lump sum] taxes into the usual second best optimality problem leads to the expected conclusion: Because of the efficiency advantages lump sum taxes will be used to the fullest extent possible . . . As Atkinson and Stiglitz point out, it is the distributional function, inherently deriving from differences in individuals, that leads us to consider taxes on transactions as additional instruments [p. 42].

The results of this paper suggest that even in a world of identical individuals, efficient taxation does not require sole reliance on lump sum taxes.

Is the widespread reliance upon taxes on transactions due in part to efficiency considerations? At this time, not enough information is available even to venture a guess. Indeed, the analysis of section IV indicates that severe difficulties may be involved in attempts to disentangle uncertainty and distributional considerations.

In this paper we have focused on the distinction between wage differences which are known at the time labor supply is determined and those which do not arise until after the work decision is made. Our perspective has been a utilitarian one in that we have viewed all wage differences in terms of their impact on individuals' expected utility. It is not obvious, however, that society does in fact view income differences arising from different sources equivalently. Even egalitarians may view those income differentials systematically related to race and sex as worse than those arising from purely random occurrences. Defining an appropriate social objective function in a world with uncertainty is worth further investigation.

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Footnotes

1: Sandmo [1976] discusses many of the important optimal taxation results; a less technical presentation is provided by Bradford and Rosen [1976].

2: The effects of proportional income taxation on portfolio allocation have been discussed by Stiglitz [1969] and on savings and investment behavior under uncertainty by Eaton [1978]. Block and Heineke [1973] discuss labor supply under wage uncertainty, but there is no taxation in their model.

3: Arrow [1970] discusses problems of moral hazard as they arise in insurance markets.

4: See, for example, Kosters [1969].

5: Results related to those of this sub-section are derived by Block and Heineke [1973, pp. 381-383]. However, we provide an expression for the impact of wage uncertainty on labor supply which is explicitly in terms of the risk aversion parameter. Block and Heineke also do not consider the expected-income compensated response in labor supply to increased wage taxation, as we do below.

6: That the appropriate compensation is $\overline{w}L^*$ is easily shown. Since expected income $\overline{Y}$ is given by $(1-t)\overline{w}L^* + \omega$, then $\frac{d\overline{Y}}{dt} = -\overline{w}L^* + \frac{d\omega}{dt}$. The condition for expected income to be constant, i.e., $d\overline{Y}/dt = 0$, is satisfied by $d\omega/dt = \overline{w}L^*$.

7: The technique used here is similar to that developed by Levhari and Weiss [1974].

8: This result parallels that of Mayshar [1977], who considers the effects of uncertainty on optimal subsidization.

9: As Mirrlees [1974] points out, in the Arrow-Debreu framework, the individual's beliefs as well as his tastes are taken as data. We adopt this point of view, although we recognize that it is a controversial one.

10: See Stern [1976] for a review of these results.

11: For the case in which the labor supply decision occurs ex post, wage uncertainty and wage inequality are formally identical: increased wage uncertainty is equivalent to increased wage inequality.

12: The computations were performed using GQOPT, a numerical optimization package. See Goldfeld and Quandt [1971] for details. Optimal marginal tax rates were found on the basis of a grid search between zero and one in intervals of 0.01.

13: The tax rates vary with γ even with certainty because γ affects the marginal social benefit of redistribution. Indeed, it is impossible to disentangle the role of γ as a risk aversion parameter from its role as a distributional weight.

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Table I

Risk Aversion and Optimal Tax Rates

	γ = -2.5	γ = -1.5 (G=0)	γ = -0.5	γ = 0.5
uncertainty	0.57	0.55	0.52	0.47
certainty	0.41	0.37	0.31	0.23

Table II

Revenue Requirements and Optimal Tax Rates

	G = -0.2	G = -0.1 (γ= -1.5)	G = 0.1	G = 0.2
uncertainty	0.47	0.51	0.59	0.63
certainty	0.30	0.33	0.41	0.46

Table III

Uncertainty, Inequality, and Optimal Tax Rates

	γ = -2.5	γ = -1.5 (G=0)	γ = -0.5	γ = 0.5
uncertainty-equality	0.54	0.52	0.49	0.44
certainty-inequality	0.60	0.57	0.52	0.42

Table IV

Locus of Inequality and Optimal Tax Rates

(γ = -1.5, G = 0)

Uncertainty for "Poor"	Uncertainty for "Rich"
0.52	0.46