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MEASURING THE VARIANCE-AGE PROFILE OF LIFETIME INCOME

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ABSTRACT

Measuring the Variance-Age Profile of Lifetime Income

This paper presents an operational meaning to the concept of the variance in lifetime income in terms of the discounted variance of T mutually uncorrelated, sequentially realized, random variables. It is then shown how the logical implications of the lifecycle consumption model can be used to estimate this series of variances, called the variance-age profile of lifetime income, and we refer to an earlier paper by Eden (1977) to show how this variance-age profile can be used to compare the riskiness of alternative labor income paths. Finally the estimation technique is applied to Israeli data in order to compare the riskiness of the earnings path of those who attended college with that of those who terminated their education at the high school level in that economy, and to consider data requirements and estimation problems in greater depth.

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A. Introduction

This paper is concerned with providing measures of the total and the time profile of the realizations in the variance in lifetime income. Some preliminary definitions will clarify these terms. Let $\underset{\sim}{W} = [w_1, w_2, \ldots, w_T]$ where T is the length of the planning horizon, designate the individual's random labor income path and note that the accumulation of information will cause the distribution of W to change over time. Now if r is the safe interest rate, and A_t is the known value of the individual's assets at t, then we define the distribution of $Y^t = \sum_{j=t}^{T} (\frac{1}{1+r})^{j-t} w_j + A_t$ conditional on all information available at t, to be the distribution of lifetime income in period t. The variance in $Y^0 (Var(Y^0))$ is termed the total variance in lifetime income, while $\sigma^2_{t+1} = Var(Y^t) - Var(Y^{t+1})$ for t=0, 1, ... T-1, is that portion of $Var(Y^0)$ which is realized in period t. Since $Var(Y^0) = \sum_{j=0}^{T} (1+r)^{-2j} \sigma^2_j$ by construction (see appendix) we only consider estimates of the vector $\underset{\sim}{\sigma}^2 = [\sigma^2_1, \sigma^2_2, \ldots, \sigma^2_T]$. $\underset{\sim}{\sigma}^2$ will be called the variance-age profile of lifetime income. If its elements are large there is a high degree of uncertainty in the associated income stream, while if $\underset{\sim}{\sigma}^2 = [\sigma^2_1, 0, 0, \ldots, 0]$ all of this uncertainty is resolved by the end of the first period.

Our interest in $\underset{\sim}{\sigma}^2$ stems from the life-cycle permanent-income theory of individual decision-making. An important empirical implication of that model is that individual decisions which affect their income paths can be analyzed solely in terms of the effect of those decisions on $\underset{\sim}{Y} = [Y^0, Y^1 \ldots Y^T]$.[1]

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1 That is one need not consider alternate joint distributions of the entire sequence of random wage vectors generated by alternative information sets.

Our purpose is to present a method for estimating a set of second order moments of this distribution which are particularly relevant to the decision maker. That is, since information is useful in the sense that it allows one to plan more accurately, it can be shown that, caeterus parabus, income paths with lower $Var(Y^o)$ and more of the variance in lifetime income realized in earlier periods, will be preferred over alternatives.[2] Estimates of $\underset{\sim}{\sigma}^2$ ought, therefore, to be helpful in analyzing an assortment of decisions - including those on schooling, on-the-job training, migration, changing sectors of employment, and joining a union.

In addition, the procedure for estimating $\underset{\sim}{\sigma}^2$ presented here leads naturally to a definition of, and an estimator for, the best predictor of future consumption given present information. In this context, this paper extends the work of Hall (1979), on predicting aggregate consumption, to micro data and provides an explicit means of estimating the error variance in this prediction.

The theoretical rational behind the estimation procedure is discussed in the next section. Section C adds the appropriate disturbances and describes the identification scheme. The model is then applied to estimating and comparing the variance-age profiles of individuals who went to college and those who terminated their education after high school in the Israel economy. In the empirical section attention is given to an assortment of estimation problems. Finally, a short summary is provided.

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2 It should be noted that these preferences do not depend on any cardinal properties of the intertemporal choice function, such as its concavity. In fact, Eden (77) has shown that provided there exists a market which offers fair bets whose outcome is known in the near future, such as portfolios or stocks with an average rate of return greater than or equal to r, estimates of $\underset{\sim}{\sigma}^2$ will suffice to provide a partial ordering of income streams with the same initial expected value. Strictly speaking our use of Eden's criteria requires either that $\underset{\sim}{Y}$ be distributed joint normally, or that the choice function be additive in a quadratic instantaneous utility function.

B. Using the Life-Cycle Model to Measure $\underset{\sim}{\sigma}^2$.

In the life-cycle consumption model each individual at every t plans a random consumption vector for the remainder of the planning horizon by maximizing an intertemporal objective function subject to the random lifetime budget constraint, $Y^t = \sum_{j=t}^{T} (\frac{1}{1+r})^{j-t} C_j$, where C_j is consumption in period j. This section will show how the difference between the consumption planned for period t+1 in period t, and actual consumption in period t+1, can be used as an indicator of σ^2_{t+1}. To do so we associate with every random income path a new vector $\underset{\sim}{\eta} = [\eta_o, \eta_1 \ldots \eta_T]$. η_o is defined to be the expected value of lifetime income at the beginning of the planning horizon, while η_{t+1} is the difference in the expected value of lifetime income that occurs because of information which is available in period t+1 but is not available in period t. That is, if $\underset{t}{E}$ denotes the mathematical expectation operator conditional on all information available at t, then:

$$\eta_o = \underset{o}{E} Y^o \qquad \text{(a)}$$

and (1)

$$\eta_{t+1} = \underset{t+1}{E} \sum_{j=t+1}^{T} w_j^{t+1} (1+r)^{j-(t+1)} - \underset{t}{E} \sum_{j=t+1}^{T} w_j^t (1+r)^{j-(t+1)}$$

$$\text{for } t=0, 1, \ldots, T-1. \qquad \text{(b)}$$

The appendix proves that, for every income path, there exists an $\underset{\sim}{\eta}$ whose elements are mutually uncorrelated, and that the expected value of η_i given the information in any previous period is zero, for i = 1, 2, ... T. It follows that the variance-covariance matrix of $\underset{\sim}{\eta}$ is diagonal with the principle diagonal elements being $\underset{\sim}{\sigma}^2$, the variance-age profile of lifetime income.

We assume that the distribution of the consumption paths chosen under η will be the same as those chosen under the random income path which defines it. Since, by construction, the first two moments of the lifetime budget constraint are the same under both vectors for all t, a sufficient condition for this assumption is that the distribution of $\underset{\sim}{Y}$ be entirely determined by these moments.[3]

Let C^*_{t+1} be the consumption planned for period t+1 in period t if the realization of η_{t+1} is 0, and C_{t+1} be actual consumption. Since η_{t+1} is the addition to expected lifetime income over the preceding period, it is reasonable to assume that:

$$C_{t+1} - C^*_{t+1} = \beta_{t+1}\,\eta_{t+1} \tag{2}$$

where $0 < \beta_{t+1} < 1$.[4] Temporarily ignore differences in preferences as well as measurement errors, and consider a sample of N individuals which is randomly drawn from a population of the same age and who have chosen similar income paths. If we let i index individuals, then

$$\text{plim } 1/N \sum_{i=1}^{N} (C_{t+1,\,i} - C^*_{t+1,\,i})^2 = \beta^2_{t+1}\,\sigma^2_{t+1} \tag{3}$$

Equation (3) can be used to identify σ^2_{t+1} provided that β_{t+1} and C^*_{t+1} can be identified.

To identify C^*_{t+1} we assume that the consumer's maximization problem at every t is:

$$\max_{C_t \ldots C_T} \underset{t}{E} \sum_{j=0}^{T-t} \left(\frac{1}{1+\delta}\right)^j U(C_{t+j}) \tag{4}$$

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3 This is not a necessary condition since the intertemporal objective function may be such that all consumption programs are determined by the first two moments of $\underset{\sim}{Y}$ regardless of the latter's distribution. See Levhari and Srinvanson (1969) for a discussion of this point.

4 See Levhari and Srinvanson (1969) for the intertemporal objective function which justifies (1).

subject to

$$Y^t = \sum_{j=0}^{T-t} (1+r)^{t-j} C_{t+j} \tag{5}$$

where U( ) is strictly-concave one-period utility function, δ is the rate of subjective time preference, and it is understood that the expectation is taken conditional on all information available at t. It can be shown (see Hall, 1979) that the optimum consumption program in this case will satisfy the first order condition:

$$\underset{t}{E}U'(C_{t+1}) = \frac{1 + \delta}{1 + r} U'(C_t) \tag{6}$$

As noted by Hall (1979), equation (5) implies that consumption in period t + 1 should be predicted by C_t alone.[5] We approximate (5) by

$$C^*_{t+1} = \underset{t}{E}C_{t+1} = \alpha + \lambda C_t \tag{7}$$

This approximation is good if U' is close to linear. In this case $\lambda = \frac{1 + \delta}{1 + r}$ and $\alpha = \alpha_0(\lambda-1)$. A reasonable prior seems to be $\alpha = 0$ and $\lambda = 1$. We shall use this prior later.

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[5] Note that the additive welfare function is required for this result.

C. Disturbance Terms and The Identification Scheme[6]

Following Friedman (1957) and others, it is assumed that measured consumption, c_t, is related to the latent consumption variable which appears in the model, C_t, by

$$c_t = C_t + \varepsilon_t \tag{8}$$

where ε_t is a zero mean disturbance term which is assumed to be uncorrelated with C_t and to have a constant variance over two-year intervals. It arises as a result of mismeasurement and differences in preferences. Substituting (7) and (2) into (8) we obtain,

$$c_{t+1} = \alpha + \lambda c_t + \beta_{t+1} \eta_{t+1} + \varepsilon_{t+1} - \lambda\varepsilon_t \tag{9}$$

Comparing (9) to (2) one finds that allowing for disturbance terms has the implication that the difference between observed consumption in period t+1, and the consumption expected in period t+1 given the information in period t, $\alpha + \lambda c_t$, is not a perfect indicator of $\beta_{t+1} \eta_{t+1}$, since it may also be a result of a non-zero realization of $\varepsilon_{t+1} - \lambda\varepsilon_t$. In addition, the disturbance term is, by construction, correlated with c_t. Therefore, an ordinary least squares regression on (8) will provide inconsistent estimates of α and λ.

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6 Econometrically the model to be presented here belongs to a class of latent variable models discussed in detail by Jöreskog (1973). Our presentation of the maximum likelihood estimates for the model, is similar to that of Chamberlin (1976).

The latter problem can be solved by assuming:

$$\text{Cov}(\varepsilon_t w_t) = \text{Cov}(\varepsilon_{t+1}, w_t) = 0 \qquad \text{(a)}$$

$$\text{Cov}(\eta_{t+1} w_t) = 0 \qquad \text{(b)} \qquad (10)$$

$$\text{Cov}(c_t w_t) \neq 0 \qquad \text{(c)}$$

Since assumption (10a) has been discussed extensively by other researchers, it will not be discussed here.[7] (10b) and (10c) are reasonable since η_{t+1} is uncorrelated with any information available at t, including w_t; and c_t is a function of w_t. Given (10), consistent estimates of the parameters in (9) can be derived by using w_t as an instrument on c_t.

To separate the two sources of the difference between c_{t+1} and $\alpha + \lambda c_t$, namely $(\varepsilon_{t+1} - \lambda\varepsilon_t)$ and $\beta_{t+1}\,\eta_{t+1}$, we use the property that c_t is correlated with $\varepsilon_{t+1} - \lambda\varepsilon_t$, but not with η_{t+1}. Letting a hat over a parameter indicate its estimated value and defining $e_{t+1} = c_{t+1} - \hat{\alpha} - \hat{\lambda} c_t$ - that is e_{t+1} is the difference between observed consumption and the estimate of planned consumption - we have

$$e_{t+1} = \beta_{t+1}\,\eta_{t+1} - \lambda\varepsilon_t + \varepsilon_{t+1} + 0(N^{-1}) \qquad (11)$$

where $0(N^{-1}) = [(\alpha - \hat{\alpha}) + (\lambda - \hat{\lambda})c_t]$. Since $0(N^{-1})$ converges to zero with sample size, N, it does not affect the maximum likelihood estimates, and can be ignored if N is large.

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7 See, for example, Liviatan (1961) and (1963). Liviatan's (1961) analysis of the demand for individual products uses c_{t+1} as an indicator of permanent income in period t + 1, an idea similar to the one used here.

If it is assumed that $\lambda = 1$ (see the next section for a test of and the reason for imposing this constraint), it follows that;

$$\mathrm{Var}(e_{t+1}) = \beta^2_{t+1}\,\sigma^2_{t+1} + 2\sigma^2_{\varepsilon}(1 - \rho) \qquad \text{(a)}$$

and (12)

$$\mathrm{Cov}(e_{t+1}c_t) = -\sigma^2_{\varepsilon}(1 - \rho) \qquad \text{(b)}$$

where $\mathrm{Var}(\varepsilon) = \sigma^2_{\varepsilon}$ and $\mathrm{Cov}(\varepsilon_t, \varepsilon_{t+1}) = \rho\sigma^2_{\varepsilon}$. Let $S(x\ y)$ denote the sample covariance of x and y; then (12) implies that

$$\mathrm{plim}\ [S(e_{t+1}e_{t+1}) + 2S(c_t e_{t+1})] = \beta^2_{t+1}\sigma^2_{t+1}\ . \qquad (13)$$

Finally, to identify σ^2_{t+1}, we need to estimate β_{t+1}. This can be done by using the relationship between β_{t+1} and λ. This relationship is derived by Hall (1979) who shows that

$$\beta_{t+1} = \frac{1}{1 + \frac{\lambda}{1 + r} + \ldots + (\frac{\lambda}{1 + r})^{T-t-1}} \qquad (14)$$

Thus, a complete identification of the model requires information about the planning horizon, T, and the interest rate, r.

D. An Example: Estimates of $\underset{\sim}{\sigma}^2$ For Different Schooling Groups

In order to illustrate the use of our technique and to consider relevant estimation problems, we estimated and compared the $\underset{\sim}{\sigma}^2$ vectors associated with the earnings paths of high school and university graduates in the Israel economy. The data was gathered by Israel's Central Bureau of Statistics and covers families (both spouses present) who were interviewed as to their consumption decisions in 1963/64 and then again in 1964/65. In each case the husband was between the ages of 21 and 65 and had at least five years of schooling.[8] w_t and c_t were defined to equal the wage income of the male head of the household from his primary labor activity and the consumption expenditures of the household, respectively.

In any particular year the ex-ante expectations of the distribution of η_{t+1} may not equal the ex-post distribution of its realizations, since as a result of a macro (or a year) effect we may sample only a portion of the distribution of η_{t+1}. When the GNP is below its expected average, consumers will tend to experience negative realizations of η_{t+1} and will adjust by choosing lower (than average) levels of C_{t+1}. If we limit the sample to one time period, we will, in this case, obtain a downward bias in both the estimate of C^*_{t+1} and of σ^2_{t+1}. Similarly, when the labor market for the group we are interested in is unexpectedly buoyant, we will overestimate C^*_{t+1} and again underestimate σ^2_{t+1}. An econometric solution to this problem requires data over many years. Our data, however, consists of observations over two years only. In the absence of appropriate data, one may either use the prior $\alpha = 0$ and $\lambda = 1$, or examine whether it is reasonable to assume that the particular year chosen was typical and ignore the year effect. Both possibilities will be explored here.

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8 These data are described more fully by the Central Bureau of Statistics (1967). We are grateful to Reuben Gronau for allowing us to use his key for this data set.

Since the variance-age profile of lifetime wealth is a new concept we start by presenting measures of it for an average member of the sample. Table 1 presents the instrumental variable estimates of equation (8) for two subsamples; those below and those at or above, age 45. The X_2^2 deviate tests the joint hypothesis that $\alpha = 0$ and $\lambda = 1$. Neither of the X_2^2 statistics are surprising. Their sum equals 3.10 which is less than the expected value of a X_4^2 deviate under the null hypothesis. Thus, it seems that the year effect is not significant in our sample. We shall proceed by imposing $\alpha = 0$, $\lambda = 1$. As one might expect, the results for the case of free parameters were similar.

Table 2 presents the estimated variance components and some relevant moments. Note that the variance in wages, $\sigma_w^2 = 1/2\ \mathrm{Var}(w_t) + 1/2\ \mathrm{Var}(w_{t+1})$, is always greater than the variance in consumption, $\sigma_c^2 = 1/2\ \mathrm{Var}(c_t) + 1/2\ \mathrm{Var}(c_{t+1})$. This is consistent with the life-cycle hypothesis. Further, note that σ_w^2 is about fifty percent larger in the older age group. This does not imply, however, that more uncertainty is resolved at older ages, since, over time, information is accumulated with respect to positions in the cross-section distribution of earnings. The variance of $c_{t+1} - c_t$ also increases with age but this increase is much less than the increase in σ_w^2. To obtain the variance of $\beta\eta_{t+1}$, $\mathrm{Var}(c_{t+1} - c_t)$ must be purged of observed consumption changes caused by mismeasurement. Column (4) in Table 2 presents the estimates of $\sigma_\varepsilon^2(1 - \rho)$, while column (5) uses these estimates to calculate $\beta^2\sigma_\eta^2$. Since $\sigma_\varepsilon^2(1 - \rho)$ is much larger for the older age group, $\beta^2\sigma_\eta^2$ is larger for younger group. Table 3 uses (13) and (14) to calculate β_t and σ_η^2 assuming $T = 70$ and $t = 33$ and 55 in the younger and older age groups respectively, under alternative assumptions on r.

Table 1. Instrumental Variable Estimates of the Recursive Consumption Equation[a/]

	$\hat{\alpha}$	$\hat{\lambda}$	Sample size (N)	χ_2^2 [b/]
Younger	3.78	1.05	434	2.28
Ages 21-44	*(67.0)*	*(0.09)*		
Older	-28.66	1.06	353	0.82
Ages 45-64	*(97.20)*	*(0.14)*		

a/ Numbers appearing in parentheses below the coefficients are standard errors.

b/ Test statistic for $\lambda = 1$, $\alpha = 0$.

Table 2. Estimated and Sample Moments for the Young and Old Age Groups[a/]

	σ_W^2 (1)	σ_c^2 (2)	$Var(c_{t+1} - c_t)$ (3)	$\sigma_\varepsilon^2(1-\rho)$ (4)	$\beta^2\sigma_\eta^2$ (5)	N (6)
Younger	197,787	86,704	74,404	34,930	6,017	434
Ages 21-44				*(4,158)*	*(6,837)*	
Older	297,476	123,538	111,917	54,283	3,554	353
Ages 45-64				*(6,855)*	*(12,124)*	

a/ Numbers appearing in parentheses below the coefficients are standard errors. All moments presented in this and other tables are in hundreds of 1963 Israeli lirot per year.

Table 3. β_t *and* σ^2_η

r	β_t		σ^2_η	
	Younger t = 33	Older t = 55	Younger t = 33	Older t = 55
r = .10	0.09	0.11	742,876	438,815
r = .20	0.17	0.17	208,211	122,990
r = .30	0.23	0.23	113,748	67,191

The main conclusion from these tables is that more information on lifetime income is accumulated per unit of time early on in the life-cycle. The standard errors of the estimated variance components indicate, however, that this conclusion cannot be held too firmly.[9] The major cause of the lack of precision in the estimates of $\beta^2\sigma_\eta^2$ is the magnitude of $\sigma_\varepsilon^2(1 - \rho)$.[10] A data base with a more precise measure of consumption services would do better on this count, but, failing that, one could add indicators of mis-measurement (e.g. large one-time expenditures for durable goods) or of transitory changes in consumption (e.g. changes in the number of persons in the household) to equation (8) and reestimate the model.[11] For example, had we been able to decrease the variance of the disturbance in equation (8) to equal that of $\beta\eta$, and if all other parameters remained unchanged, the standard error of $\beta^2\sigma_\eta^2$ would have fallen from IL 12,124 to IL 2,374 in the older age group, and from IL 6,832 to IL 2,431 in the younger. Note also that the variance of $\beta^2\sigma_\eta^2$ is of the order 1/N so that if, under these same assumptions, the size of the sample were increased to 5,000, the standard errors would have decreased further to IL 716 and IL 613 respectively.

Next we compare the variance-age profile of lifetime income experienced by those who went to college with that for individuals who only attended high school. Each educational group was split into an older and a younger

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9 In the limit the estimated variance components are normally distributed about their true values with the standard errors reported in Table 2.

10 The variance of $\beta^2\sigma_\eta^2$ is an increasing, convex function of $\sigma_\varepsilon^2(1 - \rho)$.

11 The addition of variables to equation (13) would necessarily decrease the variance of the estimate of $\beta^2\sigma_\eta^2$ but would have a cost in terms of an increase in the variance of λ. Since, in our case, the regression coefficients are not of primary importance, the tradeoff seems worthwhile.

age group[12] and equation (8) was estimated using w_t as an instrument on c_t for each of the age-schooling groups separately. None of the X_2^2 values were signficant, nor was their sum, so that we can accept the hypothesis that $\alpha = 0$, $\lambda = 1$ for each group and for the sample as a whole. Table 4 presents the relevant moments together with sample size for each group. All of the general comments made above apply to these numbers as well; in particular, σ_c^2 is always less than σ_W^2, $\sigma_\varepsilon^2(1 - \rho)$ increases with age in both educational groups and is always large, causing imprecise estimates of $\beta^2\sigma_\eta^2$. Comparing the point estimates of $\beta^2\sigma_\eta^2$ across groups, we find that among those who did not go to college more of the variance of lifetime income is realized per unit of time in the earlier part of the life-cycle, while among those who did go to college the opposite is true. That is, the college-educated have to wait relatively longer to acquire information on their lifetime income. Therefore, even if the total variance in lifetime income were the same for both groups, the variance-age profile of those who did not attend college would be preferred to the profile for those who did. In fact, however, the variance-age profile of the college-educated lies entirely above that of those who did not go to college, indicating that college education leads to a more risky earnings path in the sense made explicit in Eden (77). Again, the standard errors of our estimates are large and as a result any conclusions from them should be considered as preliminary.[13]

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12 In fact there were only about 5 units in the 20-25 and 60-65 age groups in each educational class and they contributed a great deal of $\sigma_\varepsilon^2(1 - \rho)$, so the younger group in Table 4 was redefined to equal ages 25-44 and the older group to be ages 45-59.

13 Three caveats are worthy of note. First, in this early version of our analysis, we have ignored problems induced by self-selection. That is individuals who go to (do not go to) college may expect to experience more desirable variance-age profiles as a result of going to (not going to) college than a random member of the population would. In addition, there is the question of the stability of these profiles. To make inferences about individual decision-making from the information contained in a single corss section one must assume that an individual who attends college at the age of (cont.)

Table 4. Estimated and Sample Moments for Age-Education Groups

	σ_W^2	σ_C^2	$Var(c_{t+1} - c_t)$	$\sigma_\varepsilon^2(1 - \rho)$	$\beta^2\sigma_\eta^2$	N
High School						
Younger	164,693	69,899	59,098	28,733	5,566	176
				(5,281)	*(7,811)*	
Older	247,191	105,896	98,992	48,104	3,122	112
				(10,538)	*(16,791)*	
College						
Younger	261,852	100,332	85,317	26,604	38,195	96
				(9,070)	*(17,165)*	
Older	451,981	149,215	159,153	59,786	45,848	80
				(17,492)	*(30,028)*	

Conclusion and Summary

This paper has presented an operational meaning to the concept of the variance in lifetime income in terms of the discounted sum of the variance in T mutually uncorrelated, sequentially realized, random variables. The logical implicatons of the lifecycle consumption model can be used to estimate this variance age-profile and Eden (77) has shown how these estimates can be used to compare the riskiness of alternative income streams.

The estimation technique was applied to Israeli data in order to compare the riskiness of the earnings path of those who attended college with that of those who terminated their education at the high school level. The estimates of $\underset{\sim}{\sigma}^2$ derived for these two groups are preliminary, both because of their large standard errors and because, at this early stage of analysis, we ignored the influence of such phenomenon as sample selection. The results indicate that the total variance in lifetime income is smaller for high-school graduates and that, in contrast to the college educated, most of the uncertainty in the income streams of high school graduates is resolved in younger ages. Thus, the earnings path experienced by college graduates seems to be more risky. Moreover, the example has shown how richer data could overcome both the selection and the precision problems associated with estimating the vector $\underset{\sim}{\sigma}^2$ in a variety of circumstances.

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twenty expects to experience at the age of thirty the same variance which is experienced now by a person who matriculated ten years ago. Third the $\underset{\sim}{\sigma}^2$ vectors estimated here are averages over individual $\underset{\sim}{\sigma}^2$ vectors in a particular group and the actual $\underset{\sim}{\sigma}^2$ vectors may have some within-group variance. We add, however, that none of these problems are new and they also appear in the more traditional estimates of the first order moments of labor income streams.

Appendix: The Derivation of $\underset{\sim}{\eta}$

To provide an intuitive derivation of $\underset{\sim}{\eta}$ we introduce slightly different notation. Let $L = \sum_{t=1}^{T} w_t(1+r)^{-t}$, I_t be the information set in period t, and define:

$$\eta^*_{t+1} = E[L|I_{t+1}] - E[L|I_t] \qquad (1)$$

where it should be noted that η^*_{t+1} is just η_{t+1} discounted to the beginning of the planning horizon, i.e.; $\eta^*_{t+1} = (1+r)^{-(t+1)}\ \eta_{t+1}$ for t = 0, 1, 2 ..., T-1. Conditioning both sides of (1) on I_{t-j}, for $0<j<t+1$, and passing through another expectations operator, one obtains $E[\eta^*_{t+1}|I_{t-j}] = 0$. Similarly, further use of the double expectations operator proves that $Cov(\eta_i,\eta_q|I_{t-j}) = 0$ for all $i\neq q$ i, j, q = 1, 2, ..., T. Now solve (1) recursively and note that $E[L|I_T] = L$ so that :

$$L = \eta^*_T + \eta^*_{T-1} + \dots + \eta^*_1 + \eta_o \qquad (2)$$

where $\eta_o = E[L|I_o]$ which equals $E[Y^o]$ as defined in the text. That is, L is just the sum of T mutually uncorrelated, sequentially realized, random variables and a constant term. Combining (1) and (2) with the definitions given in the text, if follows that:

$$Var(Y^o) = Var(L|I_o) = \sum_{t=1}^{T} Var(\eta^*_t) = \sum_{t=1}^{T} (1+r)^{-2t}\ \sigma^2_t$$

and

$$Var(Y^t) - Var(Y^{t+1}) = [Var(L|I_t) - Var(L|I_{t+1})]\ (1+r)^{2(t+1)} = \sigma^2_{t+1}\ .$$

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