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THE RATE OF OBSOLESCENCE OF KNOWLEDGE, RESEARCH GESTATION LAGS, AND THE PRIVATE RATE OF RETURN TO RESEARCH RESOURCES

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The Rate of Obsolescence of Knowledge, Research Gestation Lags and the Private Rate of Return to Research Resources

SUMMARY

This paper points out the conceptual distinction between the rates of decay in the physical productivity of traditional capital goods and that of the appropriate revenues accruing to knowledge-producing activities, and notes that it is the latter parameter which is required in any study which constructs a stock of privately marketable knowledge. The rate of obsolescence of knowledge is estimated from a simple patent renewal and the estimates are found to be comparable to evidence provided by firms on the lifespan of the output of their R&D activities. These estimates, together with mean R&D gestation lags, are then used to correct previous estimates of the private excess rate of return to investment in research. We find that after the correction, the private excess rate of return to investment in research, at least in the early 1960's, was close to zero, which may explain why firms reduced the fraction of their resources allocated to research over the subsequent decade.

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THE RATE OF OBSOLESCENCE OF KNOWLEDGE, RESEARCH GESTATION LAGS, AND THE PRIVATE RATE OF RETURN TO RESEARCH RESOURCES

The recent interest of economists in knowledge-producing activities has had two main strands. The first is an attempt to explain the growth in the measured productivity of traditional factors of production by incorporating research resources in production function and social accounting frameworks; (for a review see Griliches, 1973, and *idem,* forthcoming). The second derives from two fundamental characteristics of knowledge as an economic commodity, its low or zero cost of reproduction and the difficulty of excluding others from its use. These features give knowledge the character of a public good and suggest that the structure of market incentives may not elicit the socially desirable level (or pattern) of research and development expenditures. In particular, it has been argued that market incentives may create an underinvestment in knowledge-producing activities (see Arrow, 1962, for a discussion). In order to investigate this possibility, economists have applied the techniques of productivity analysis and estimated the private (and social) rate of return to research from production functions incorporating research resources as a factor of production. The estimates of the *private* return for the late 1950's and early 1960's fall in the range of 30-45 percent.[1] Despite these high estimated private rates of return, the share of industrial resources allocated to research expenditures declined steadily over the succeeding decade.[2] This suggests

[1]For example, see Griliches (forthcoming) and Mansfield (1965). Note that both these studies are based on firm or micro data. More aggregative data bases are not directly relevant to estimates of the private rate of return to research.

[2]The share of net sales of manufacturing firms devoted to R&D has declined from 0.046 in 1963 to 0.029 in 1974, or by about 40 percent. See NSF (1976, Table B-36).

a paradox: why has research effort been receiving less attention from industrial firms if the private rate of return to research is so attractive?

Two important parameters in these calculations of the private return to research are the rate of decay of the private revenues accruing to industrially-produced knowledge and the mean lag between the deployment of research resources and the beginning of that stream of revenues. These parameters, of course, are necessary ingredients in *any* study which involves a measurement of the stock of privately marketable knowledge. The rate of decay in the returns from research has not previously been estimated. In this paper we present a method of explicitly estimating that parameter. We also use information provided by others to calculate the approximate mean R & D gestation lags. Since previous research has not included the latter and seems to have seriously understated the rate of decay of appropriable revenues in calculations of the private rate of return to research expenditures, we then use our estimates to improve on previous results on this rate of return.

Of course, all previous work in this area has been forced to make some assumption, either implicit or explicit, about the value of the decay rate. The problem arises because it has been assumed to be similar to the rate of decay in the *physical* productivity of traditional capital goods. The fact that the rate of deterioration of traditional capital and the rate of decay in appropriable revenues from knowledge arise from two different sets of circumstances seems to have been ignored.[3]

The employment of research resources by a private firm produces new knowledge, with some gestation lag. The new knowledge or innovation may be

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[3] It is common to find an assumed rate of decay of the knowledge produced by firms of between 0.04 and 0.07 (Mansfield, 1965). Griliches (forthcoming 1978) noting some of the conceptual distinctions between the rates of decay in traditional capital and in research, assumes an upper bound of 0.10 for the latter.

a cost-reducing process, a product, or some combination of the two. The knowledge-producing firm earns a return either through net revenues from the sale of its own output embodying the new knowledge, or by license and non-monetary returns collected from other firms which lease the innovation. Since the private rate of return to research depends on the present value of the revenues which accrue to the sale of the knowledge produced, the conceptually appropriate rate of depreciation is the rate at which the appropriable revenues decline for the innovating firm. However, as Boulding (1966) noted, knowledge, unlike traditional capital, does not obey the laws of (physical) conservation. The rate of decay in the revenues accruing to the producer of the innovation derives not from any decay in the productivity of knowledge but rather from two related points regarding its market *valuation,* namely, that it is difficult to maintain the ability to appropriate the benefits from knowledge and that new innovations are developed which partly or entirely displace the original innovation. Indeed, the very use of the new knowledge in any productive way will tend to spread and reveal it to other economic agents, as will the mobility of scientific personnel. One might expect then that the rate of decay of appropriable revenues would be quite high, and certainly considerably greater than the rate of deterioration in the physical productivity of traditional capital.[3a]

In Section I we examine two independent pieces of evidence bearing on the rate of decay of appropriable revenues. Next, the information from various sources on the mean lag between R&D expenditures and the beginning of the

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[3a]It should be noted that the models used in this paper do not assume that the rate of decay in appropriable revenues is exogenous to the firm's decision-making process. In a dynamic context, a firm possessing an innovation has to choose between increasing present revenues and inducing entry, and charging smaller royalties in order to forestall entry. This choice is the basis of Gaskin's (1971) dynamic limit pricing analysis of

associated revenue stream is summarized. In Section III we attempt to get a rough idea of how seriously the existing estimates of the private rate of return to research overstate the true private rate of return. Brief concluding remarks follow.

3.1. *The Rate of Decay of Appropriable Revenues*

The first piece of evidence on the rate of decay in appropriable revenues (hereafter, the rate of decay) is based on data presented in Federico (1958). Federico provides observations on the percentage of patents of various ages which were renewed by payment of mandatory annual renewal fees during the period 1930-39 in the United Kingdom, Germany, France, the Netherlands, and Switzerland. A theoretical model of patent renewal will lead directly to a procedure for estimating the rate of decay from these data.

Consider a patented innovation whose annual renewal requires payment of a stipulated fee. Letting r(t) and c(t) denote the appropriable revenues and the renewal fee in year t, the discounted value of net revenues accruing to the innovation over its lifespan, V(T), is

(1) $$V(T) = \int_0^T [r(t) - c(t)]\, e^{-it} dt,$$

where i is the discount rate and T is the expiration date of the patent. Differentiating (1) with respect to T, the optimum expiration date, T*, is written implicitly as

(2) $$r(T^*) = c(T^*)$$

provided that $r'(t) < c'(t)$ for all t. Equivalently, the condition for renewal of the patent in year t is that the annual revenue at least covers the cost of the renewal fee

(3) $$r(t) \geq c(t).$$

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situations involving temporary monopoly power. Gaskin's model can be used to show that the optimal revenue stream will decline over time and that the rate of decline will depend on certain appropriability parameters.

Let the annual renewal fee grow at the rate g, and let the appropriable revenues decline at rate δ. Then condition (3) can be written as

(4) $$r(0) \geq c(0)e^{(g+\delta)t}.$$

Partition δ into an economy average rate of decay (δ^*) and a patent specific deviation from that average (δ_i), and let f(r) denote the relative density function of initial appropriable revenues adjusted for differences in the decay rate among patents $[r_i = r_i(0)e^{-\delta_i}]$. Then the percentage of patents renewed in each year, P(t), is

(5) $$P(t) = \int_{C(t)}^{\infty} f(r)dr,$$

where $C(t) = c_0 e^{(g+\delta^*)t}$.

It follows that

(6a) $$P'(t) = -C'(t)f(C)$$

and

(6b) $$P''(t) = -f(C)C''(t)[1 + C\frac{f'(C)}{f(C)}],$$

where the primes denote derivatives. That is, as long as $(g + \delta) > 0$, the percentage of patents renewed will decline with their age. The curvature of P(t), however, will depend on the distribution of the values of the innovations patented.[4] In particular, if f(r) is lognormal, P(t) will have one point of inflection, being concave before it and convex thereafter (see curve 1, Figure 1a).

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[4] Since the value of patents [equation (1)] is a monotonic transformation of the adjusted initial revenues, we shall use the two terms interchangeably.

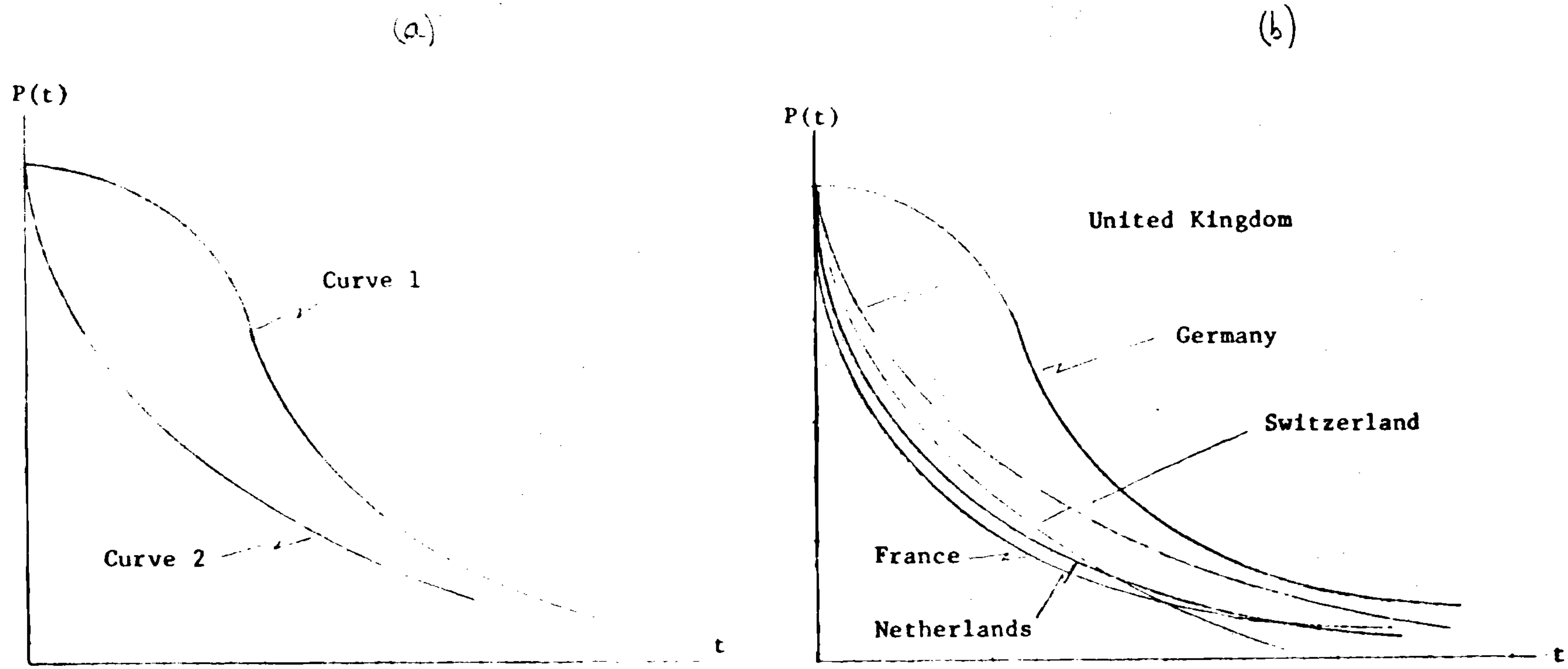
(a)
(b)
P(t)
Curve 1
Curve 2
t
P(t)
United Kingdom
Germany
Switzerland
France
Netherlands
t

Figure 1

Alternatively, Scherer (1965) cites evidence presented in Sanders and others (1958) which indicates that the value of patents tends to distribute Pareto-Levy. If f(r) is strictly declining (e.g. Pareto-Levy), P(t) will be a strictly decreasing, convex function of patent age, as shown by curve 2. Figure 1b presents the actual time paths of P(t) from Federico (1958). Four of the five curves tend to support Sander's data and are consistent with an underlying Pareto-Levy distribution of patent values. The time path for Germany, however, indicates that the underlying distribution for that country has at least one mode. Since it is futile to estimate both the parameters of the underlying lognormal (for example) and the decay rate in appropriable revenues from only 18 observations available on Germany, we shall disregard the German data in the remainder of the empirical work.

We now use this simplified model to obtain rough estimates of the decay rate of appropriable revenues δ^*. Consistent with the evidence in Figure 1b, the relative density function of initial revenue is taken to be of the Pareto-Levy type:

(7) $$f(r) = \beta r_m^{\beta} r^{-(\beta+1)}, \qquad r_m > 0, \quad \beta > 0$$

where r_m is the minimum value of r in the population. Then the percentage of patents which are renewed in year t can be expressed as

(8) $$P(t) = [r_m/c(0)]e^{-\beta(g+\delta^*)t}.$$

There are two error terms differentiating the observed value of the logarithm of P_t, $\log P^m_t$, from the value predicted by equation (8).[5] The first, v_1, is a sampling or measurement error while the second, v_2, is a structural error in the model. Assuming that P^m_t is derived from a binomial sampling process around the actual value, that is, $P^m_t \sim b(P_t, N)$,

it follows that

$$\sigma^2_{v_1} = V(\log P^m_t) \simeq \frac{1 - P}{PN},$$

where N is the (unobserved) number of patents sampled. The structural error, v_2, will be assumed to be an independent, indentically distributed normal deviate with variance σ^2.

Letting j index a country, for sufficiently large N_j the logarithmic transform of P^m_{tj} can be written as

(9) $$\log P^m_{tj} = \alpha_{0j} + \alpha_{1j} t_j + \mu_{tj}$$

where $\mu_{tj} \sim N[0, \sigma^2 + (1 - P_{tj})/P_{tj}N_j]$, $\alpha_{1j} = -\beta_j(g_j + \delta^*_j)$, and $\alpha_{0j} = \log(\frac{r_m}{c_0})_j$.

Consistent estimates of α_{0j}, α_{ij} and their standard errors can be derived from the following two-stage procedure. First estimate (9) by ordinary least squares. Next define e^2 as the squared residuals from (9) and regress.

(10) $$e^2_{tj} = \sigma^2 + \frac{1}{N_j} \frac{1 - P^m_{tj}}{P^m_{tj}} .$$

Letting F be the fitted value from (10), use $F^{-\frac{1}{2}}$ to weight, and perform weighted least squares on (9).

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5 P_t is bounded by zero and unity. As a result, the composite error cannot be independently and identically distributed. The analysis which follows corresponds closely to the treatment of similar problems in logit regressions. See Berkson (1953) and Amemiya and Nold (1975).

If our model is correct, and if β and δ^* do not vary between countries, then

(11) $$\alpha_{ij} = -\beta\delta^* - \beta g_j .$$

Since g_j, the rate of growth of renewal fees, is available from Federico's data,[6] (11) can be tested by using $F^{-\frac{1}{2}}$ to weight and performing weighted least squares on the equation

(12) $$\log P^m_{tj} = \alpha_{0j} - \beta\delta^* t_j - \beta(g_j t_j) + \mu_{tj},$$

where all symbols are as defined above.

If (11) is the true specification, then -2 times the logarithm of the likelihood ratio from (12) and the weighted least-squares version of (9) will distribute asymptotically as a χ^2_2 deviate. Moreover, equation (12) will provide estimates of both the rate of decay of appropriable revenues (δ^*), and the underlying distribution of patent values (β).

6 These growth rates were calculated from a semi-log linear regression of costs against time for each country. The growth rates (and their standard errors) for the Netherlands, the United Kingdom, France, and Switzerland were 0.085 (0.002), 0.129 (0.015), 0.089 (0.006) and 0.143 (0.008).

Table 1. Estimates from the Patent Renewal Model[a/] [b/]

A. Common parameters (57 observations)	
$\beta\delta^*$	0.14 0.01
β	0.57 0.07
δ^*	
Point estimate[c/]	0.25
Confidence Interval[d/]	0.18-0.36
σ^2	0.0002 0.0007
R^2	0.996

B. Country-specific parameters	α_{0j}	$1/N_j$
France	0.04 0.02	0.00014 0.00016
United Kingdom	0.55 0.02	0.00016 0.00014
Netherlands	0.09 0.02	0.00040 0.00008
Switzerland	0.32 0.02	0.00032 0.00012

a/ The data on patent renewal are taken from Federico (1958). These data cover the percentage of patents of different ages in force during 1930-39 which were renewed by payment of a mandatory annual renewal fee. For example, if a patent was granted in 1925, it would appear in the data as 5 years old in 1930, 6 years old in 1931, and so on. Therefore, the percentage of patents renewed after 5 years is based on the total number of patents issued 5 years earlier in a particular country.

b/ Small numerals are standard errors.

c/ $\hat{\delta}^* = \beta\hat{\delta}^*/\beta$.

d/ The confidence interval corresponds to the 95 per cent Fieller bounds on δ^*.

Table 1 summarizes the empirical results. The observed value of the χ^2 test statistic is 5.4, while the five percent critical value is 5.99. Though a little high, the test statistic does indicate acceptance of the hypothesis in (11). The estimates of $1/N_j$ and σ^2 are all positive thereby lending support to the weighting procedure described above.

Turning to the parameters of interest, the point estimate of β was 0.57 with a standard error of 0.07. One can check this estimate against an independent source of information. As mentioned earlier, Sanders *et. al*. (1958) provide evidence on the distribution of the value of patents in the United States. Fitting a Pareto-Levy distribution to these data, we obtain a point estimate for β of 0.63 with a standard error of 0.06. That is, the estimate of β from Sanders' data is very close to that obtained using our model and Federico's data.[7]

Our primary interest is in δ^*, the (average) decay rate in appropriable revenues. The point estimate of δ^* is 0.25, while a 95 percent confidence interval places the true value of δ^* between 0.18 and 0.36.[8] An estimated

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[7] These data correspond to appropriable revenues minus costs associated with the patent, but since cost data were not available we were forced to use net-value data. A cumulative Pareto-Levy distribution was fitted to the five positive net-value observations on expired patents which therefore have observable net values. The R^2 from this regression was 0.97. Note that Pareto-Levy distributions with $\beta < 1$ do not have either a finite mean or variance and hence do not behave according to the law of large numbers. Therefore, if the distribution of patent values approximates the distribution of project values, diversification into many independent projects will not reduce risk. This point was originally made by Nordhaus (1969). Of course, if the returns to different projects are negatively correlated, diversification may still reduce risk.

[8] Note that since the estimate of δ^* is obtained as the ratio of two coefficients, its confidence interval (obtained by using Fieller bounds) is not symmetric around its point estimate. Three remarks on the robustness of these results are also in order. First, the assumption that the revenue stream can be described by an exponential rate of decay can be viewed as a first-order (logarithmic) approximation to a more general stream of

δ^* of 0.25, though consistent with theoretical arguments concerning the unique characteristics of knowledge as an economic commodity, implies that earlier researchers have assumed values of δ^* which are far too small. In particular, the *lower* bound of the 95 percent confidence interval for δ^* is nearly twice the *maximum* value of the rate of decay of private returns used in previous research.

There is, of course, the possibility of sample selection bias in our estimate of δ^*. The rate of decay of patented innovations may differ from that of all innovations. The direction of the bias is indeterminate since it depends on the correlation between the patent selection process and the rates of obsolescence in the universe of all innovations. However, there are two reasons to believe that the estimates of δ^* may be biased downwards. First, the fact that patents create property rights in the embodied knowledge may result in a lower rate of obsolescence for those innovations which *can* be patented. Second, given an innovation which can be patented, it is easy to show that the innovator will actually take out a patent only if patenting lowers the rate of decay. As we show presently, however, evidence

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revenue. We also experimented with a second-order approximation, namely, $r(t) = r(0)e^{At+Bt^2}$. The estimates of B and its standard error were both zero to two places of decimals, and the rest of the results were almost identical to those reported here. Apparently, market-induced obsolescence is well approximated by an exponential pattern. Griliches (1963) reaches the same conclusion with respect to the obsolescence component of the deterioration in the value of traditional capital goods. Second, the results from the unweighted regressions were similar to the ones reported above. The unweighted version of (12) yielded a point estimate of $\delta^* = 0.22$, with Fieller bounds of 0.16 to 0.33, and an estimate of $\beta = 0.62$.

Finally, we can use the unconstrained version of the estimating equation to provide six different estimates of the combination (β, δ^*). The constrained version, in effect weights the various estimates in an optimal manner. Five of these unconstrained estimates were (0.63, 0.22), (0.49, 0.32), (0.55, 0.26), (0.65, 0.21), and (0.47, 0.34). The sixth is obtained from comparing the estimates of α_{1j} and g_j from France and the Netherlands. However, since $g_N = g_F$ to two decimal places this comparison contains almost no new information and therefore receives almost no weight in the constrained regression.

of a completely different nature tends to show that whatever bias exists is negligible.

The second source of evidence on the magnitude of the decay rate of appropriable revenues is derived from data presented in Wagner (1968) on the lifespan of applied research and development expenditures. Survey data on applied research and development were collected from about 35 firms with long R & D experience in 33 product fields, using the product field description employed by the NSF in its annual industry reports. Included in the survey was a question on the lifespan of R & D, defined as the period after which the product of the R & D was "virtually obsolete."

This definition does not correspond directly to the decline in the appropriable revenues accruing to research and development. However, a rough correspondence can be established by assuming that R & D is virtually obsolete when the appropriable revenues reach some small fraction of the initial value, and then experiment with different fractions to examine the sensitivity of the implied decay rate to the assumption. Table 2 presents the average lifespan of R & D for durable and non-durable product field categories, product and process-oriented R & D, and the implied decay rates based on various reasonable definitions of virtual obsolescence. While the implied decay rates do vary with the definition of virtual obsolescence, the range of values is nearly identical to the Fieller bounds on δ^* in Table 1.[9]

The responses of firms to Wagner's question can also be used to check the reasonableness of the rates of obsolescence commonly assumed in the

[9]The only other estimate of the decay rate in produced knowledge of which we are aware is reported in a footnote in Griliches (forthcoming). A regression of productivity growth against R & D flow and stock intensity variables in his micro data set yielded an estimated δ^* of 0.31. Griliches points out the discrepancy between this result and the rest of his analysis but offers no reconciliation.

Table 2. Estimates of δ^* from Average Lifespan of R & D[a/]

Ratio of revenue in year T to initial revenue (x) (i.e., 'virtual obsolescence')	0.15	0.10	0.05
Durable-goods R & D	Revenue decay rate[b/]		(δ^*)
Product (T = 9)	0.21	0.26	0.33
Process (T = 11)	0.17	0.21	0.27
Nondurable-goods R & D			
Product (T = 9)	0.21	0.26	0.33
Process (T = 8)	0.24	0.29	0.38

a/ Taken from Wagner (1968, p. 196, Table 5), which refers to 'applied research and development' (AR&D). These lifespan figures (in parentheses) are averages of survey responses, weighted by 1965 product-field expenditures and by frequencies of the response distribution.

b/ Calculated as $\delta^* = -(\log x)/T$.

literature. If in fact $\delta^* = .05$ (.10), that would imply (using T = 9 from Table 2) that firms consider the product of their R & D virtually obsolete even though the annual revenue flow is still 64 (41) percent of its initial value.

3.2. Mean R & D Lags

Two independent sources of information are used to estimate the mean R & D lag, defined as the average time between the outlay of an R & D dollar and the beginning of the associated revenue stream. This lag consists of a mean lag between project inception and completion (the gestation lag), and the time from project completion to commercial application (the application lag).

Rapoport (1971) presents detailed data on the distribution of costs and time for 49 commercialized innovations and the total innovation time for a subset of 16 of them in three product groups - chemicals, machinery, and electronics. The innovation process is decomposed into five stages: applied research, specification, prototype or pilot plant, tooling and manufacturing facilities, and manufacturing and marketing start-up. Since the expenditures on manufacturing and marketing start-up are not included in the NSF definition of R & D expenditures, the time involved in that stage is treated here as the application lag. The remaining data are used to calculate the gestation lag. The first part of Table 3 summarizes the R & D gestation and application lags for the three product groups.[10]

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[10] The details of all calculations are omitted here for the sake of brevity but are available upon request. However, the limitations of these estimates should be noted. First, all the projects analyzed by Rapoport resulted in significant innovations, and as Scherer (1965) and Mansfield (1968) have noted, mean lags tend to be longer for more significant technical advances. Second, we have not taken into account the time

Table 3. Estimates of the Mean R & D Lag (θ)

	R & D gestation lag	Application lag	Total lag
<u>Rapoport</u>			
Chemicals	1.48	0.24	1.72
Machinery	2.09	0.31	2.40
Electronics	0.82	0.35	1.17
<u>Wagner</u>			
Durables	1.15	1.47	2.62
Nondurables	1.14	1.03	2.17

Source: Calculated from data contained in Rapoport (1971) and Wagner (1968).

Additional information on the average R & D and application lags is provided in Wagner (1968). Survey data on process and product oriented R & D were gathered from about 36 firms with long R & D experience in a variety of durable and nondurable goods industries. Included was information on the duration of applied research and development, project duration for projects successfully completed in 1966, the distribution of R & D expenditures for successfully completed projects classified by project duration, the percentage of total funds accounted for by projects abandoned before completion, together with the time of abandonment, and the interval between the completion of R & D and commercial application of the innovations. These data are used to calculate both an application lag and a mean gestation lag which, unlike those based on Rapoport's data, take into account expenditures on both technically successful and unsuccessful projects. The results are given in the second part of Table 3.

The gestation lags based on Wagner's data are broadly similar to those derived from Rapoport but the application lags are considerably longer,[11] causing some discrepancy between the two sets of results. Mansfield (1968), using data gathered from extensive personal interviews with R & D project

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overlap between stages which, according to Rapoport, is considerable. Both of these factors would tend to cause upward biases in our estimates of θ. On the other hand, the R & D costs of technically unsuccessful projects should be taken into account, which would tend to raise the estimates of θ.

[11] Since Wagner does not precisely define the "end of AR&D" or the "application of innovations," some caution should be exercised in interpreting the application lags. Wagner does indicate that the longer application interval in durables reflects in large part the defense-space-atomic-energy oriented fields, so that the application lag for other industries is probably closer to the nondurables estimate. On the other hand, Rapoport's 'manufacturing and marketing start-up' stage may understate the actual application lag.

evaluation staff, concluded that the mean application lag was about 0.53 years. Substitution of this number for Wagner's would bring the two sets of results closer together and put the total lag at about 1.75 years. For present purposes, however, a range of values somewhere between 1.2 and 2.5 years is good enough.[12]

3.3. *Implications for Measuring the Private Rate of Return to Investment in Research*

The preceding sections of this paper provided estimates of the decay rate and the mean R & D lag whose values are substantially higher than those assumed in previous research. These estimates are now used to get a rough indication of the implications for production function estimates of the private rate of return to research expenditures.

Let Q_R denote the increment in value added (or sales) generated by a unit increase in research resources θ years earlier. Then the equation for the private (internal) rate of return to investment in research is

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[12] It is interesting to note that the maximum of this range is considerably shorter than the midpoint of the interval between project inception and marketing, reflecting the fact that the distribution of research expenditures on projects is considerably skewed to the left.

(13) $$1 = \int_{\theta}^{\infty} Q_R e^{-(r+\delta^*)\tau} d\tau ,$$

where θ and δ^* were defined earlier and r is the private rate of return, or the implicit discount rate that would make investment in research marginally profitable. In the special case where $\theta = 0$, (13) reduces to $r = Q_R - \delta^*$, which corresponds to the equation used by previous researchers. In general, however, the private rate of return is found by solving the following equation for r

(14) $$r - Q_R e^{-(r+\delta^*)\theta} + \delta^* = 0.$$

Two points should be noted here. First, since research expenditures are already included in the measures of traditional capital and labor expenditures in the production functions used to estimate Q_R, the calculated private rate of return to research represents an excess return above and beyond the normal remuneration to traditional factors (see Griliches, 1973). That is, r may be interpreted as the risk premium attached to research activity. Second, the estimates of Q_R obtained by other researchers are calculated by multiplying the estimated sales (value-added) elasticity of the stock of knowledge times the ratio of sales to the stock of knowledge. The stock of knowledge is taken as the undepreciated sum of research expenditures over the period of observation. For the calculations in (14) to be consistent, however, the stock of knowledge must be calculated according to declining-balance depreciation. We therefore calculate the depreciated sum of research expenditures with a decay rate of δ^* and then use this stock of knowledge to convert the estimated sales elasticity into a value of Q_R.[13] This has been done for the three values of δ^*, corresponding to the point estimate and Fieller bounds obtained earlier (0.18, 0.25, and 0.36), and for three different values of Q_R (0.25, 0.30, and 0.35). The results are presented in Table 4 for values of θ equal to 1.75 and 2.0.

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[13] We thank Zvi Griliches for pointing out this problem and suggesting a solution. The data for the calculations are taken from NSF (1976, Table B-1). Three additional points should be noted. First, Griliches' aggregate gross excess rate of return estimate was about 0.30, while his estimates for research intensive and non-research intensive industries were 0.40 and 0.20 respectively. Mansfield's estimates (averaged over the ten firms he used) range from about 0.20 to 0.30 depending on the specific assumptions made. Since Griliches' data are far more comprehensive, we based the calculation of the conversion factor on them. Second, if private returns to knowledge do in fact decay, there is an error in the measured stock of knowledge used as an independent variable in Griliches' regression. However, it can be shown that the ratio of the variance in measurement error to the variance in the true stock is less than 0.0026, which can safely be ignored. That is, the sales elasticity of the stock of knowledge can be taken directly from Griliches' regression. Finally, the private rates of return are much less sensitive to changes in θ than to different values of δ^*.

Table 4. Estimates of the Net Excess Private Rate of Return to Research

δ^*	Q_R		
	0.25	0.30	0.35
θ = 1.75			
0.18	0.053	0.085	0.114
0.25	0.015	0.047	0.081
0.36	-0.051	-0.015	0.021
θ = 2.0			
0.18	0.043	0.073	0.100
0.25	0.004	0.033	0.065
0.36	-0.067	-0.032	0.000

Turning to the results, it is apparent that the measured private (net excess) rates of return to investment in research are greatly reduced by our adjustments. For example, Griliches (forthcoming) reports a gross excess return of 0.30 which, after correction for an assumed decay rate of 0.10, translates into a net excess rate of return of 0.20. By contrast, Table 4 indicates that the rate of return associated with $Q_R = 0.30$, evaluated at our point estimate of δ^*, is between 0.033 and 0.047. In view of the abnormal riskiness associated with research expenditures, these risk premia would appear to be modest.

In short, Table 4 suggests that the private rates of return to investment in research and traditional capital are roughly equated at the margin. Another way of checking this possibility is to ask: what is the decay rate of appropriable revenues implied by the assumption that firms equate, at the margin, the private rates of return to investment in research and traditional capital? With a mean R & D lag of θ, the return to a dollar of research is $(r+\delta^*+1)(1+r)^{-\theta}$, while for traditional capital, with depreciation rate δ^*_c, it is $(r + \delta^*_c + 1)$. Using $\delta^*_c = 0.06$ and $r = 0.08$ from Griliches (forthcoming), the value of δ^* which equates these two terms is $\delta^* = 0.22$ if $\theta = 1.75$ and $\delta^* = 0.25$ if $\theta = 2.0$. These values are nearly identical to the point estimate of δ^* in Table 1.

3.4. Concluding Remarks

In this paper we stress the conceptual distinction between the rates of decay in the physical productivity of traditional capital goods and that of the appropriable revenues which accrue to knowledge-producing activities. An estimate of the private rate of obsolescence of knowledge is necessary in any study which requires constructing a stock of privately marketable knowledge. We estimate this parameter from a simple patent renewal model and find the estimate comparable to evidence provided by firms on the lifespan of the output of their R & D activities. The empirical results indicate that the rate of obsolescence is considerably greater than the rates typically assumed in the literature. The estimated decay rates, together with mean R & D gestation lags, are used to calculate the private excess rate of return to investment in research. Our results suggest that the *private* rate of return to research expenditures, at least in the early 1960's, was *not* unreasonably high. It is important to emphasize, however, that in order to come to conclusions regarding the divergence between the private and social rates of return to knowledge-producing activities, information on the social rate of return must be added to the information contained in this paper.[14] Nonetheless, if our calculations of the private excess rate of return are even approximately correct, they do suggest a partial resolution to the paradox presented in the introduction to this chapter: why did private firms steadily reduce the share of their resources devoted to R & D if their previous research effort was so highly profitable? Part of the answer may be that research was not as privately profitable as has been thought.

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[14] In this connection, it should be noted that the social rate of decay may well be smaller than the rate of decay of appropriable revenues. See Hirshleifer's (1971) distinction between real and distributive effects in the production of knowledge.

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