NBER WORKING PAPER SERIES

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Working Paper No. 326

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

March 1979

This research is being supported by the National Science Foundation. I have benefited from comments by Moses Abramowitz, Bob Hall, Zvi Hercowitz, Robert King, and Leo Leiderman. The research reported here is part of the NBER's research program in economic fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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Summary

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Previous equilibrium "business cycle" models are extended by the incorporation of an economy-wide capital market. One aspect of this extension is that the relative price that appears in commodity supply and demand functions becomes an anticipated real rate of return on earning assets, rather than a ratio of actual to expected prices. From the standpoint of expectation formation, the key aspect of the extended model is that observation of the economy-wide nominal interest rate conveys current global information to individuals.

With respect to the effect of money supply shocks on output, the model yields results that are similar to those generated in simpler models. A new result concerns the behavior of the anticipated real rate of return on earning assets. Because this variable is the pertinent relative price for commodity supply and demand decisions, it turns out to be unambiguous that positive money surprises raise the anticipated real rate of return. In fact, this response provides the essential channel in this equilibrium model by which a money shock can raise the supply of commodities and thereby increase output. However, it is possible through a sort of "liquidity" effect that positive money surprises can depress the economy-wide nominal interest rate.

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March 1978
Revised February 1979

This paper extends previous equilibrium "business cycle" models of Lucas (1973, 1975) and myself (1976) by incorporating an economy-wide capital market. One aspect of this extension is that the relative price that appears in the supply and demand functions in local commodity markets becomes an anticipated real rate of return on earning assets, rather than a ratio of actual to expected prices. The analysis brings in as a central feature a portfolio balance schedule in the form of an aggregate money demand function. The distinction between the nominal and real rates of return is an important element in the model.

From the standpoint of expectation formation, the key aspect of the extended model is that observation of the economy-wide nominal rate of return conveys current global information to individuals. In this respect the present analysis is distinguished from Lucas's (1975) model, which considered only local (internal) finance. However, my analysis does not deal with the dynamics of capital accumulation, as considered by Lucas, and does not incorporate any other elements, such as inventory holdings, multi-period lags in the acquisition of information, or the adjustment costs for changing employment that were treated by Sargent (1977), that could produce persisting effects of monetary and other disturbances.

In order to retain the real effects of monetary surprises in the model, it is necessary that the observation of the current nominal rate of return, together with an observation of a current local commodity price, not convey full information about contemporaneous disturbances. Limitation of current information is achieved in the present framework by introducing a contemporaneously unobserved disturbance to the aggregate money demand function, along with an aggregate money supply shock and an array of disturbances to local excess commodity demands. Aggregate shocks to the commodity market (to the extent that they were not directly and immediately observable) could serve a similar purpose.

With respect to the effect of money supply shocks on output, the model yields results that are similar to those generated in earlier models. Notably, incomplete current information about the nature of underlying economic disturbances can produce a positive relation between money shocks and the level of output. Further, the coefficient that connects money disturbances to output responses tends to be inversely related to the variance of the money shocks.

A new result concerns the behavior of the anticipated real rate of return on earning assets. Because this variable is the pertinent relative price for commodity supply and demand decisions, it turns out to be unambiguous that positive money surprises raise the anticipated real rate of return. In fact, this response provides the essential channel in this equilibrium model by which a money shock can raise the supply of commodities and thereby increase output. However, it is possible through a sort of "liquidity" effect that positive money surprises can depress the economy-wide nominal interest rate. Given the increase in the real interest rate, this liquidity effect must involve a decrease in the anticipated rate of inflation. The downward movement of the nominal interest rate is less likely to obtain if money shocks exhibit positive serial correlation because the perceived part of money movements would then have a direct positive impact on anticipated inflation.

Since the relative price variable in the commodity supply and demand functions is a real rate of return on earning assets, rather than a ratio of actual to expected prices, there is a less clear connection than in previous models between money shocks and the current price level. Although the typical pattern would still be a positive, but less than one-to-one, short-run response of prices to money shocks, it is now possible that the required positive movement of the anticipated real rate of return would reflect an increase in the nominal interest rate rather than (or as a partial substitute for) a rise in the ratio of current to expected future prices. Abstracting from effects of serial correlation in the money supply process, the model suggests that a strong positive response of the current price level to money shocks would be associated with a strong negative response of the nominal interest rate and vice versa.

The first section sets up the model and presents the basic equilibrium conditions. The analysis proceeds, as in my (1976) paper, by postulating "plausible" forms for the supply and demand functions, rather than by presenting and solving an explicit underlying maximization problem. There is, however, an extended discussion of the specification of the relative price and wealth variables in the commodity supply and demand functions. The second section solves the model under conditions of full current information. Although these results involve an independence of the "real" variables from monetary disturbances (including the shock to the aggregate money demand function), they provide a useful frame of reference for the subsequent analysis. The third section solves the model under a specification of incomplete current information, where individuals are limited in their current knowledge of the economy to observations of the economy-wide nominal interest rate and a single local commodity price. A central aspect of the solution involves inferences from the observed current nominal interest rate and local price level to the expected value of the contemporaneously-unobserved money shock. The final part of this section provides some interpretations of the principal analytical results.

I. Setup of the Model

In the current period each individual transacts in two markets: a local commodity market indexed by z, and an economy-wide capital or loanable-funds market that deals in homogeneous, riskless, one-period loans. The number of commodity markets and the number of individuals are assumed constant, so that separate notation is not used below to distinguish aggregate from per capita quantities. The logarithm of the current price on the local commodity market is designated by $P_t(z)$ and the nominal, one-period rate of return on the capital market is denoted by R_t. Aside from holding capital market claims, individuals can hold fiat money, which is a liability of the "government." Changes in the aggregate quantity of money occur through positive or negative transfers from the government to individuals. The size of the transfer varies randomly across individuals and is independent of an individual's own holdings of cash. The analysis does not deal with any deadweight losses associated with governmental transfer/tax programs. The logarithm of the aggregate nominal quantity of money, denoted by M_t, is determined from

$$(1) \quad M_t \equiv M_{t-1} + \mu + m_t,$$

where μ is the constant long-run growth rate of money and m_t indicates the extent to which the current money growth rate departs from μ. I assume that m_t is generated in accordance with the first-order stochastic process,

$$(2) \quad m_t = \rho m_{t-1} + n_t,$$

where $\rho \gtrless 0$, $|\rho| < 1$, and n_t, the current innovation to money growth, is normally, independently distributed with zero mean and variance σ_n^2. In parts of the subsequent analysis the values of M_t and m_t are currently unobservable, but the lagged values, M_{t-1} and m_{t-1}, are assumed throughout to be contained in current individual information sets.

Each individual's portfolio allocation problem can be viewed as involving a tradeoff between the rate-of-return differential for holding capital market claims rather than money, R_t, and the marginal transactions benefits from holding cash.[1] Overall portfolio balance entails equality between the total amount of cash outstanding and the aggregate demand for money, which I assume can be described in the semi-log form,

$$(3) \quad M_t = M_t^d = P_t - \gamma R_t + \delta y_t + \phi_t,$$

where the constant, which would include the fixed number of individuals, has been normalized to zero. In equation (3) M_t is again in logarithmic terms, P_t is the (unweighted) average across markets of the local (log of) commodity prices, $\gamma > 0$ measures the interest rate sensitivity of real money demand, y_t is the (unweighted) average across markets of the local (log of) commodity outputs, and $\delta > 0$ is the elasticity of "per capita" real balances demanded with respect to "per capita output." The shock to aggregate money demand, ϕ_t, is assumed to be independently, normally distributed with zero mean and variance σ_ϕ^2. The realized value of ϕ_t is currently unobservable in some of the subsequent analysis.

The postulated commodity supply and demand functions for market z involve the specification of a relative price term and a wealth variable.

Speficiation of the Relative Price Variable

The relative price term compares current sale (purchase) opportunities in market z with those anticipated for next period in a randomly-selected market. For example, a sale this period is evaluated at the (log of) current price $P_t(z)$ plus the nominal rate of return from date t to t+1, R_t, to obtain a comparison with the expected (log of) price next period, $E_z P_{t+1}$. (The subscript z indicates that the expectation of P_{t+1} is conditioned on information available currently in market z.) The relative price term is then $r_t(z) \equiv P_t(z)-E_zP_{t+1}+R_t$--that is, the anticipated one-period real rate of return from the perspective of market z. It is this expression that would be considered in contemplating a shift of labor services, commodity purchases, etc. from date t to date t+1, assuming that any funds (plus or minus) held over time earn nominal interest at rate R_t. By comparison, the earlier analysis in Lucas (1973) and Barro (1976) amounts to treating $R_t = 0$, which is appropriate in a model where the only store of value is money that bears a zero nominal rate of return. The logarithm of local commodity supply, $y_t^s(z)$, is assumed to be positively related to $r_t(z)$, while the log of local demand, $y_t^d(z)$, is negatively related. The impact on demand amounts to the usual positive substitution effect on saving of the anticipated real rate of return. The nature of the income effect associated with a change in $r_t(z)$ is discussed in the section below that deals with the specification of a wealth variable.

In the present model it turns out that expected future values of r are constant--essentially, departures of r(z) from the "normal" real rate of return represent a temporary situation that cannot be predicted to arise in one direction or the other for future periods. Therefore, it is only the current value of the perceived real rate of return that will appear in the commodity demand and supply functions.

The present treatment of the anticipated real rate of return, $r_t(z)$, considers only the supply and demand effects associated with the conditional first moment of P_{t+1}. Generally, higher moments of the conditional distribution of the future price level would also be relevant. For example, suppose that the pertinent relative price variable for determining commodity demand and supply were $E_z[\hat{P}_t(z)e^{R_t}/\hat{P}_{t+1}(z')]$, where a caret denotes the level of a variable rather than its logarithm, and z' specifies the randomly-selected market visited next period. Assuming $\hat{P}_{t+1}(z')$ to be log-normally distributed, the log of this expected relative price variable is given by[2]

$$P_t(z) + R_t - E_z P_{t+1} + \frac{1}{2}\sigma^2,$$

where P_{t+1} is the average across the markets of $P_{t+1}(z')$ and σ^2 is the conditional variance of $P_{t+1}(z')$. This expression differs from the $r_t(z)$ variable specified above by the inclusion of the variance term, $\frac{1}{2}\sigma^2$. As long as σ^2 is constant, the use of this expression rather than $r_t(z)$ would modify the subsequent analysis only by adding effects of once-and-for-all shifts in σ^2 on the mean (natural) values of output and the real rate of return. It would also be necessary to relate σ^2 to the underlying parameters of the model, including the variances of the exogenous disturbance terms. Additional variance effects on the supply and demand functions would, of course, arise in a serious analysis of individual choice under uncertainty.

Specification of the Money/Wealth Variable

Since the model does not encompass changes in physical or human capital, the specified wealth variable considers only movements in the money stock.[3] The net money/wealth variable that is pertinent to commodity demand and supply involves four elements: current money stocks, expected future monetary transfers from the government, current money demand, and expected future money demand.

Let $\hat{M}_t(z)$ denote the quantity of nominal money held at the start of the period (before any market trading occurs, but after the transfers from the government) by the aggregate of individuals located currently in market z. Individuals anticipate for future periods an infusion of cash that will accrue as transfer payments from the government. The size of the transfer varies randomly across individuals--in particular, the amount is independent of individual money holdings or of current or future market location. The expected individual nominal transfer for period t+i is therefore equal, aside from a constant of proportionality, to the expected change in total money outstanding for the period, $E_z(\hat{M}_{t+i} - \hat{M}_{t+i-1})$, where $i \geq 1$.

Denote the sequence of anticipated future one-period nominal interest rates as $R_{t+1}, R_{t+2}, \ldots$. I treat these future yields as though they were known with certainty, although this assumption is not crucial for the results obtained below.[4] The discount factor applicable to date t+i is then given by

$$\nu_i = \frac{1}{(1+R_t)(1+R_{t+1})\ldots(1+R_{t+i-1})} .$$

The nominal present value of current money holdings plus expected future transfers is equal to

$$\hat{M}_t(z) + \sum_{i=1}^{\infty} \nu_i E_z(\hat{M}_{t+i} - \hat{M}_{t+i-1}),$$

where constants of proportionality have been omitted.

Commodity demand and supply will be influenced by this wealth variable net of the expected portion of wealth "expended" on current and future demands for cash. If the nominal demand for money in period t+i by individuals who are located at date t in market z is $\widehat{M^d_{t+i}}(z)$, then the nominal interest earnings in period t+i+1 for these individuals are reduced by $R_{t+i}\widehat{M^d_{t+i}}(z)$ relative to a situation where zero cash is held. The expected nominal present value of these interest payments foregone is given by

$$\frac{R_t \hat{M}^d_t(z)}{(1+R_t)} + \sum_{i=1}^{\infty} \nu_{i+1} R_{t+i} E_z \hat{M}^d_{t+i}(z) .$$

Therefore, the net monetary wealth variable for current participants of market z can be written as

$$\hat{M}_t(z) + \sum_{i=1}^{\infty} \nu_i E_z(\hat{M}_{t+i} - \hat{M}_{t+i-1}) - \frac{R_t \hat{M}_t^d(z)}{(1+R_t)} - \sum_{i=1}^{\infty} \nu_{i+1} R_{t+i} E_z \hat{M}_{t+i}^d(z).$$

Using the condition, $\nu_i = (1+R_{t+i})\nu_{i+1}$, the second term can be modified to yield the equivalent net wealth expression,

$$\hat{M}_t(z) - \frac{1}{(1+R_t)} E_z \hat{M}_t + \sum_{i=1}^{\infty} \nu_{i+1} R_{t+i} E_z \hat{M}_{t+i} - \frac{R_t \hat{M}_t^d(z)}{(1+R_t)} - \sum_{i=1}^{\infty} \nu_{i+1} R_{t+i} E_z \hat{M}_{t+i}^d(z).$$

Suppose now that individuals are identical in the sense that their expected nominal demands for cash in period t+i, $E_z \widehat{M_{t+i}^d}(z)$, where $i \geq 1$, is equal, aside from a constant of proportionality, to the expected total nominal demand for the period, $E_z \hat{M}_{t+i}^d$. The two summations terms above can then be written as

$$\sum_{i=1}^{\infty} \nu_{i+1} R_{t+i} E_z (\hat{M}_{t+i} - \hat{M}_{t+i}^d),$$

which equals zero since individuals appropriately anticipate portfolio balance to obtain in every (future) period. The simplified net wealth expression can then be written in two equivalent forms:

$$(4) \quad [\hat{M}_t(z) - E_z \hat{M}_t] - \frac{R_t}{(1+R_t)}[\hat{M}_t^d(z) - E_z \hat{M}_t] = \frac{1}{(1+R_t)}(\hat{M}_t - E_z \hat{M}_t) + \frac{1}{(1+R_t)}[\hat{M}_t(z) - \hat{M}_t] + \frac{R_t}{(1+R_t)}[\hat{M}_t(z) - \hat{M}_t^d(z)].$$

The first form of the net wealth expression in equation (4) indicates that a net monetary wealth effect arises in market z only when the money held in this market at the start of the period, $\hat{M}_t(z)$, or the local demand for money,

$\widehat{M^d_t}(z)$, differs from the perceived value of the aggregate money stock, $E_z\hat{M}_t$. Equal movements in $\hat{M}_t(z)$, $\widehat{M^d_t}(z)$ and $E_z\hat{M}_t$ yield no net effect because the interest foregone associated with current and expected future money demand exactly offsets the present value of current cash plus expected future transfers. For a given value of $E_z\hat{M}_t$, the net wealth position is raised by an increase in $\hat{M}_t(z)$ and lowered by an increase in $\widehat{M^d_t}(z)$ (because of the interest foregone, $R_t\widehat{M^d_t}(z)$, on this period's relatively high cash holdings).

The second form of the net wealth expression in equation (4) is convenient because it separates out the last two terms, which add to zero in summations across the markets. The middle term depends on $\hat{M}_t(z) - \hat{M}_t$, which expresses the relative cash position of participants of market z at the start of the period.[5] Since this term adds to zero in summations across the markets, it may represent a shift to relative commodity demand and supply, but it would not (in a linear model) represent an aggregate shift. Since relative commodity demand and supply disturbances to market z are included separately below, it is satisfactory to omit further consideration of this term in the construction of the money/wealth variable.

The last part of the net wealth expression in equation (4) depends on $\hat{M}_t(z) - \hat{M}^d_t(z)$.[6] Since $\hat{M}_t(z)$ represents cash held at the start of the period by participants of market z, portfolio balance does not require this term to equal zero. That is, market z could turn out to be a net importer or exporter of cash during period t. However, overall portfolio balance for the current period does require this term to add to zero (in equilibrium) in summations across the markets. As in the case above, this term can be viewed as a component of the relative commodity demand and supply terms that are introduced separately. Therefore, this term may also be neglected in the construction of the money/wealth variable.

The key element in the money/wealth variable is the first term on the right side of equation (4), which involves the expression, $(\hat{M}_t - E_z\hat{M}_t)$. Note that, since $E_z\hat{M}_t \neq \hat{M}_t$ in each market or on average over z, this term can represent an aggregate net wealth effect that does not vanish in summations across the markets.

Commodity demand and supply will depend on the "real value" of the term, $(\hat{M}_t - E_z\hat{M}_t)/(1+R_t)$. If the real value were defined as the ratio of this nominal magnitude to the current local price level $\hat{P}_t(z)$, then a change in $r_t(z)$ would--with this "real wealth" concept held fixed--involve an important income effect. Since the real rate of return expected at date t for date t+1 onwards turns out to be constant in the present model, the natural definition of real wealth is in terms of date t+1 commodity values. With this wealth concept held fixed, a change in $r_t(z)$ has an income effect that involves only the one-period opportunity for a high (or low) anticipated real rate of return. Since this income effect can reasonably be neglected in a context where decisions are based on "permanent income" over a long horizon, it is then satisfactory to assume that the substitution effects of $r_t(z)$, as discussed above, are, in fact, the dominant responses to a change in the current anticipated real rate of return. The expected real value for date t+1 of the money/wealth variable, abstracting from the two terms on the right side of equation (4) that cancel in summations across the markets, is

$$(5)\quad \frac{1}{(1+R_t)}(\hat{M}_t - E_z\hat{M}_t)\cdot(1+R_t)E_z(1/\hat{P}_{t+1}) = (\hat{M}_t - E_z\hat{M}_t)E_z(1/\hat{P}_{t+1}).$$

In order to remain within the setting of a linear model, it is necessary to make some approximations to the form of the money/wealth variable. Basically, these approximations amount to neglecting some effects of higher moments of the distribution of money growth--that is, errors that are of the same order of magnitude as those committed above when the future price variance was neglected as a component of $r_t(z)$. From equation (1), assuming $|\mu| << 1$, it is assumed that

$$\hat{M}_t \simeq \hat{M}_{t-1}(1+\mu)(1+m_t),$$

where m_t is the non-systematic part of the money growth rate. With $\hat{M}_{t-1}$ observable at date t, $E_z\hat{M}_t$ is similarly approximated by $\hat{M}_{t-1}(1+\mu)(1+E_z m_t)$, which neglects an effect of a term that depends on the conditional variance of m_t. The money/wealth variable can then be approximated by

$$(6) \quad (m_t-E_z m_t)\hat{M}_{t-1}(1+\mu)E_z(1/\hat{P}_{t+1}).$$

The principal implication of this analysis is that the net money/wealth variable in expression (6) depends on discrepancies between actual and currently perceived money growth, $m_t - E_z m_t$. The net money/wealth variable equals zero when $m_t = E_z m_t$, independently of the anticipated rate of growth of money or prices. It should be noted from the forms of expressions (4) and (5) that this general type of result does not hinge on the approximations made above. Further, this conclusion does not depend on the form of the money demand function--which did not enter the analysis--or on the specification of individual information sets, other than their inclusion of the last period's money stock.

Two aspects of the derivation of the net money/wealth variable in the form of equation (6) should be stressed. First, the analysis involves the capitalization of transfers and interest-foregone over an infinite horizon.[7]

In this context it is not surprising that an increase in current real balances that is accompanied by an equal, permanent increase in the demand for real balances would not generate a net wealth effect that would influence commodity demand and supply. On the other hand, in a finite horizon setting the liquidation value of terminal real balances would produce a positive net wealth effect when actual and desired real cash rose by the same amounts. This point is analogous to the issue of whether interest-bearing government bonds constitute net wealth. The finiteness of life can generate a net wealth effect for shifts between public debt and taxes because the tax liabilities on future generations are not fully counted. Similarly, a net wealth effect from the level of real money balances would result if the interest-foregone associated with the demand for money (net of government transfers) by future generations were not considered. As in the interest-bearing public debt case discussed in Barro (1974), the presence of operative intergenerational transfers can make finite-lived individuals act as though they were infinitely-lived with respect to calculations of effective wealth. With a tie to subsequent generations and the knowledge that descendants will also have a demand for money (as well as a claim to future government transfers), an increase in actual and permanently desired real cash balances would not exert a direct wealth effect on commodity demand and supply. In this sense the derivation of the net money/wealth term in the form of equation (6) can apply even when the finiteness of life is brought into the model.

Finally, the major limitation of the present analysis is its failure to incorporate the real role of money as an economizer of transaction costs, etc.--that is, to bring in the real factors that underlie the demand for money. Although these considerations would not seem to invalidate the specification of the net wealth term in expression (6), some other effects might be missed.

For example, an increase in the average inflation rate that reduces the average holdings of real cash and correspondingly raises average transaction costs incurred could also influence the work-leisure decision, the demands for productive factors, etc. These effects would depend on cross-substitutions between the demand and supply of commodities and the demand for money. Sidrauski's (1967) deterministic model, in which real balances provide utility, where labor supply is exogenous, and where utility is additive over time with a constant utility rate of discount, is an example of a setting where these effects do not arise in the steady state.

Specification of Commodity Supply and Demand Functions

Formally, the local commodity supply and demand functions are written as the semi-log expressions,

$$(7)\quad y_t^s(z) = k^s(z) + \alpha_s r_t(z) - \beta_s(m_t - E_z m_t) + \varepsilon_t^s(z),$$

$$(8)\quad y_t^d(z) = k^d(z) - \alpha_d r_t(z) + \beta_d(m_t - E_z m_t) + \varepsilon_t^d(z),$$

where y denotes the log of the quantity of commodities (and services), the k-terms--assumed to be invariant over time--represent any systematic supply and demand forces that are not captured by the other terms, $r_t(z) \equiv P_t(z) - E_z P_{t+1} + R_t$ is the relative price term discussed above, $(\alpha_s, \alpha_d) \geq 0$ are relative price elasticites, $(\beta_s, \beta_d) \geq 0$ are wealth elasticities, and the $\varepsilon_t(z)$'s represent local shocks to commodity supply and demand. The realized values of these shocks are not currently observable in some of the subsequent analysis. Aggregate real shocks could be added to equations (7) and (8), as in Barro (1976, pp. 4,5), without altering the nature of the main analysis. The present model does not deal with capital accumulation, inventory changes, population growth, technological change, etc., which could be described by exogenous or endogenous movements over time in the k-terms of equations (7) and (8). Note that the α_d-term in equation (8) corresponds to the usual inverse effect on commodity demand (investment and/or consumption) of the anticipated real rate of return. The α_s-term in equation (8) corresponds to the type of relative price effect on supply (of labor services, etc.) that was stressed in Lucas and Rapping (1969). As seems appropriate, this relative price is measured by an anticipated real rate of return--that is, in a manner that is symmetric to the specification of commodity demand.[8]

In order to preserve the linearity of the model (so as to be able to calculate expectations), I have entered the money/wealth variable from expression (6) as a linear term in $(m_t - E_z m_t)$. Essentially, the dependence of the β-coefficients attached to this variable in equations (7) and (8) on the level of normal real balances has been lost in this restricted specification. The β_d-term in equation (8) expresses the usual positive wealth effect on demand. The β_s-term in equation (7) can be viewed as a negative wealth effect on the supply of services--that is, a positive wealth effect on leisure. The general analysis would not be altered--although some ambiguities would be resolved--if the wealth effect on the supply side were omitted.

It is convenient to use the definitions,

$$\alpha \equiv \alpha_s + \alpha_d,$$

$$\beta \equiv \beta_s + \beta_d,$$

$$k = k(z) \equiv k^d(z) - k^s(z),$$ [9]

$$\varepsilon_t(z) \equiv \varepsilon_t^d(z) - \varepsilon_t^s(z),$$

where $\varepsilon_t(z)$ is assumed to be normally, independently distributed with zero mean and variance σ_ε^2. The values of the $\varepsilon_t(z)$'s are assumed to net to zero in summations across the markets.[10]

Market-Clearing Conditions

The local commodity price, $\hat{p}_t(z)$--or, equivalently, the anticipated real rate of return from the perspective of market z, $r_t(z)$--must be such as to satisfy the local market-clearing condition, $y^s_t(z) = y^d_t(z)$. This equilibrium condition follows from the constraint that commodities not travel from one local market to another during the current period. However, the existence of a global capital market means that a particular market z can assume a net export (import) position in cash that corresponds to the opposite net position in interest-bearing assets. If relative shocks to money supply and demand in market z are neglected (or viewed as part of the $\varepsilon(z)$ terms, as in the present analysis), the net cash and interest-bearing asset positions of market z will depend, from equation (3), on the relative values of local commodity price and output, which will turn out to depend on the realized values of the local commodity market shocks, $\varepsilon^d_t(z)$ and $\varepsilon^s_t(z)$.

Using equations (7) and (8), the local commodity market-clearing condition requires

$$(9)\quad r_t(z) = (1/\alpha)[k + \beta(m_t - E_z m_t) + \varepsilon_t(z)],$$

which implies the expression for local (log of) output,

$$(10)\quad y_t(z) = (\alpha_s/\alpha)[k^d(z)+\varepsilon^d_t(z)] + (\alpha_d/\alpha)[k^s(z)+\varepsilon^s_t(z)] + \frac{(\alpha_s\beta_d-\alpha_d\beta_s)}{\alpha}(m_t-E_z m_t).$$

It follows that economy-wide average values, for which the $\varepsilon_t(z)$ terms vanish, are given by

(11) $r_t = (1/\alpha)[k + \beta(m_t - \overline{E_z m_t})]$,

and

(12) $y_t = (\alpha_s/\alpha)k^d + (\alpha_d/\alpha)k^s + \frac{(\alpha_s\beta_d - \alpha_d\beta_s)}{\alpha}(m_t - \overline{E_z m_t})$,

where $\overline{E_z m_t}$ is the economy-wide average value of $E_z m_t$. It is convenient to define

$$y^* \equiv (\alpha_s/\alpha)k^d + (\alpha_d/\alpha)k^s,$$

which is the level of y_t that corresponds to $m_t = \overline{E_z m_t}$ in equation (12).

It is useful to note that equations (9) - (12) have been derived without regard to the form of the demand function for money, the form of the process for m_t, or any specification of current information sets other than their inclusion of M_{t-1}. Equations (9) - (12) are not final solutions for anticipated real rates of return and outputs because they contain the endogenous expectation, $E_z m_t$. However, several results are already apparent:

1) $r_t(z)$ and $y_t(z)$ will depend only on "real" factors--that is, the k- and ε-terms in the present setup--unless money growth differs from its perceived value, $m_t \neq E_z m_t$.[11] Of course, this property depends on the form of the net money/wealth term, as given in expression (6).

2) The anticipated real rate of return is positively related to unperceived money shocks. Typical Keynesian analysis under fixed wages and/or prices argues that (unperceived?) monetary expansion has a depressing, "liquidity" effect on the (nominal and real) rate of return,[12] which leads to an expansion of aggregate demand. Since this scenario leaves unexplained the motivation for increased

supply, it is necessary to view output determination in this context as involving an initial excess supply/quantity rationing situation in which production and sales and/or employment are willingly raised without additional price incentives in response to increases in aggregate demand. In the present equilibrium context the initial monetary expansion produces an excess demand for commodities that must be closed by an increase in the anticipated real rate of return.

3) Output can be positively related to unperceived monetary expansion. Demand is directly stimulated (in accordance with the coefficient β_d) by the monetary movement, and supply is raised (in accordance with the coefficient α_s) by the increase in the anticipated real rate of return. However, because of the offsetting wealth and relative price effects, as represented by the coefficients β_s and α_d, the sign of the output response is generally ambiguous. The net effect depends on the same combination of elasticities, $\alpha_s\beta_d-\alpha_d\beta_s$, that appeared in my earlier model that omitted a capital market (1976, p. 11). If the dominant influences are the wealth effect on demand (β_d) and the relative price effect on supply (α_s)--in particular, if the wealth effect on supply β_s is minor--then unperceived monetary expansion will have a positive output effect.

The full solution of the model involves also the determination of R_t and $P_t(z)$. Defining the combination of supply and demand parameters,

$$H \equiv \alpha_s\beta_d - \alpha_d\beta_s,$$

the nominal rate of return can be written from equation (3), with y_t substituted from equation (12) and M_t from equation (1), as

$$(13) \quad R_t = -(1/\gamma)\{M_{t-1}+\mu+m_t-P_t-\phi_t-\delta[y^*+(H/\alpha)(m_t-\overline{E_z m_t})]\},$$

where y^* is defined below equation (12). The solution for $P_t(z)$ can be written by using the condition $P_t(z) \equiv r_t(z) + E_zP_{t+1} - R_t$, where $r_t(z)$ is determined from equation (9) and R_t from equation (13), as

$$(14)\quad P_t(z) = \frac{k}{\alpha} + \frac{\beta}{\alpha}(m_t - E_zm_t) + \frac{1}{\alpha}\varepsilon_t(z) + E_zP_{t+1} + \frac{1}{\gamma}\{M_{t-1}+\mu+m_t-P_t-\phi_t-\delta[y^*+\frac{H}{\alpha}(m_t-\overline{E_zm_t})]\}.$$

Note that E_zP_{t+1} and E_zm_t are expectational variables contained on the right side of equation (14).

The solution of the model hinges on the structure of current local information. I assume throughout that information on all lagged variables, including M_{t-1} and m_{t-1}, is available during period t. I work out first the case of full current information--which includes direct observations or sufficient indirect information to infer the values of the three current shocks, m_t, ϕ_t and $\varepsilon_t(z)$--and second the case where current information is limited to that contained in the observation of the economy-wide nominal interest rate R_t and a single local commodity price $P_t(z)$.[13] The background of the full current information case is useful in discerning the monetary effects on output, the anticipated real rate of return, etc., that emerge under conditions of incomplete current information.

II. Solution of the Model Under Full Current Information

Since $E_zm_t = m_t$ obtains under complete current information, the solutions for $r_t(z)$ and $y_t(z)$ follow immediately from equations (9) and (10). Using asterisks to denote the full current information case, the results are

$$(15)\quad r_t^*(z) = (1/\alpha)[k + \varepsilon_t(z)],$$
$$y_t^*(z) = (\alpha_s/\alpha)[k^d(z) + \varepsilon_t^d(z)] + (\alpha_d/\alpha)[k^s(z) + \varepsilon_t^s(z)].$$

Therefore, in terms of economy-wide average values, the results are

$$(16)\quad r_t^* = k/\alpha,$$
$$y_t^* = (\alpha_s/\alpha)k^d + (\alpha_d/\alpha)k^s \equiv y^*.$$

Given the availability of full current information, the average across markets of the anticipated real rate of return corresponds to its "natural" value, k/α, and is independent of the quantity of money, M_t, the current money shock, m_t (or n_t), the aggregate money demand shock, ϕ_t, or the long-run money growth rate, μ. The absence of anticipated inflation-type effects on real rates of return and output depends on the form of the net money/wealth term in expression (6). The average anticipated real rate of return would be affected positively by any aggregate real disturbances that affected excess commodity demand. The local anticipated real rate of return, $r_t(z)$, is positively related to the local excess commodity demand shock, $\varepsilon_t(z)$.

As with the anticipated real rate of return, the level of output under conditions of full current information is independent of M_t, m_t, ϕ_t or μ.[14] The (geometric) average of outputs across the markets is fixed at its "natural" value, $y^* \equiv (\alpha_s/\alpha)k^d + (\alpha_d/\alpha)k^s$.

As a prelude to the incomplete current information case, it is useful to apply a solution procedure for $P_t(z)$ and R_t under full current information that is more formal than would be necessary for this case alone. The method is the one of undetermined coefficients that has been applied before in models that omitted a global capital market in Lucas (1973, 1975) and Barro (1976). Specifically, given the form of the price solution in equation (14)--which involves the expectations, E_zP_{t+1} and E_zm_t--and given that m_t is generated from the first-order process that is shown in equation (2), it is apparent for the full current information case that the present "state of the economy" for a local commodity market would be fully described by a specification of values for the variables, $[M_{t-1}, m_t, \phi_t, \varepsilon_t(z)]$. I limit attention in the present analysis to solutions that are determined as a stationary function of this state vector--that is, non-stationary price solutions are not considered.

In the present linear model, the local price solution will end up as a linear function of the state variables--that is,

$$(17) \quad P_t(z) = \pi_0 + \pi_1 M_{t-1} + \pi_2 m_t + \pi_3 \phi_t + \pi_4 \varepsilon_t(z),$$

where the π's are a set of yet-to-be-determined coefficients. The (geometric) average price across the markets is determined by averaging the $\varepsilon_t(z)$'s to zero in equation (17) to be

$$(18) \quad P_t = \pi_0 + \pi_1 M_{t-1} + \pi_2 m_t + \pi_3 \phi_t.$$

The expected price for next period in a randomly-selected market is given by taking expectations of an updated form of equation (18) to be

$$(19) \quad E_z P_{t+1} = \pi_0 + \pi_1 (M_{t-1} + \mu + E_z m_t) + \rho \pi_2 E_z m_t,$$

where equations (1) and (2) and the conditions, $E_z \phi_{t+1} = E_z \varepsilon_{t+1}(z) = 0$, have been used. Under full current information, $E_z m_t = m_t$ can be substituted in equation (19).

The forms for $P_t(z)$, P_t and $E_z P_{t+1}$ (and the condition $E_z m_t = m_t$) can be substituted into the price level relation that is shown in equation (14). The five π-coefficients are then determined by requiring this price condition to hold identically in $[M_{t-1}, m_t, \phi_t, \varepsilon_t(z)]$. The solution from this straightforward exercise turns out to be

$$\pi_0^* = \gamma k/\alpha - \delta y^* + \mu(1+\gamma),$$

$$\pi_1^* = 1,$$

$$\pi_2^* = 1 + \gamma\rho/(1+\gamma-\gamma\rho),$$

$$\pi_3^* = -1/(1+\gamma),$$

$$\pi_4^* = 1/\alpha.$$

The implied price level solutions are then

$$(20)\quad P_t^*(z) = \gamma k/\alpha - \delta y^* + \mu\gamma + (M_{t-1}+\mu+m_t) + m_t\left(\frac{\gamma\rho}{1+\gamma-\gamma\rho}\right) - \frac{\phi_t}{1+\gamma} + \frac{\varepsilon_t(z)}{\alpha},$$

and

$$(21)\quad (E_z P_{t+1})^* = \gamma k/\alpha - \delta y^* + \mu\gamma + (M_{t-1}+\mu+m_t) + \mu + \rho m_t\left(\frac{1+\gamma}{1+\gamma-\gamma\rho}\right),$$

where $(M_{t-1}+\mu+m_t) = M_t$ from equation (1) could be substituted in the above expressions. Note that, via inverse effects on money demand, there is a negative effect on the price level of normal output y^* and a positive effect of the long-run money growth rate μ. As would be expected, the level of the nominal money stock has a one-to-one, positive price level effect.

The anticipated rate of inflation from the perspective of market z is given by

$$(22)\quad (E_z P_{t+1})^* - P_t^*(z) = \mu + \frac{\rho m_t}{(1+\gamma-\gamma\rho)} + \frac{\phi_t}{1+\gamma} - \frac{\varepsilon_t(z)}{\alpha}.$$

The average across markets of the expected rate of inflation is influenced positively by the long-run money growth rate μ, by the short-run part of anticipated money growth as represented here by ρm_t, and by the (temporary) aggregate money demand shift, ϕ_t.

Finally, the nominal rate of return, which corresponds to the sum of the average anticipated real rate r_t^* from equation (16) and the average across markets of the anticipated inflation rates shown in equation (22), is equal to

$$(23) \quad R_t^* = k/\alpha + \mu + \frac{\rho m_t}{(1+\gamma-\gamma\rho)} + \frac{\phi_t}{1+\gamma} .$$

The nominal interest rate is independent of the level of the money stock, but varies one-to-one with the long-run money growth rate μ. The excess of current money expansion over μ, m_t, has a temporary positive effect on anticipated inflation if $\rho > 0$, which implies a positive effect of m_t on R_t^*. The effect of monetary disturbances on the nomonal rate of return becomes substantially more complicated and interesting under conditions of incomplete current information, as discussed in the next section.

III. Solution of the Model Under Incomplete Current Information

The full current information setup is now replaced by a specification in which current information for a participant of market z is limited to that contained in the observations of the local commodity price, $P_t(z)$, and the global nominal interest rate, R_t. Because this analysis becomes algebraically very complicated, I have simplified the model by eliminating serial correlation in the money growth process--that is, by assuming $\rho = 0$ in equation (2). In this restricted specification m_t is independently, normally distributed with zero mean and variance σ_m^2. This simplification of the model is probably not serious because the main effect of allowing $\rho \neq 0$ seems to be the corresponding influence of the perceived part of m_t on the anticipated inflation rate and the nominal rate of return, as shown in equations (22) and (23). These effects of serial correlation in money growth did not have any impact on the anticipated real rates of return or levels of output, as given in equation (15), because these variables depend, as shown in equations (9) and (10), only on the unperceived part of money growth, $m_t - E_z m_t$.

The basic equilibrium conditions that are expressed in terms of $r_t(z)$, $y_t(z)$, R_t, and $P_t(z)$ in equations (9), (10), (13) and (14) continue to apply. The forms of the solutions for $P_t(z)$, P_t and $E_z P_{t+1}$ that are shown in equations (17) - (19) also remain valid.[15] The key difference in the incomplete information setup is that the $E_z m_t$ terms that appear in equations (14) and (19) cannot simply be replaced by m_t.

In the present context $E_z m_t$ is conditioned on the observations of $P_t(z)$ and R_t (and M_{t-1}). It is apparent from equation (17) that the current information contained in the observation of $P_t(z)$ amounts to knowledge of a certain linear combination of the three current shocks--$\pi_2 m_t + \pi_3 \phi_t + \pi_4 \varepsilon_t(z)$. Equation (13), which reflects averaging-out of market-specific effects, indicates that the observation of R_t conveys information about a particular linear combination of the two aggregate disturbances, which can be denoted as $c_2 m_t + c_3 \phi_t$, where the c's are yet-to-be-determined coefficients. If the model had contained only a single aggregate shock (or no market-specific shock)--for example, if $\sigma_\phi^2 = 0$ had been assumed[16]--then the observations of $P_t(z)$ and R_t would amount to full current information. In other words, the setup with one type of relative disturbance and two types of aggregate disturbances is the simplest stochastic structure within the present general framework that would reveal the consequences of incomplete current information. It would, of course, be possible to introduce additional shocks--for example, aggregate real disturbances to the commodity market can readily be incorporated.

Under the assumption of normally distributed shocks, the conditional expectation of m_t will turn out, as discussed in the appendix, to be a linear combination of the two pieces of current information--that is,

$$(24) \quad E_z m_t = b_1[\pi_2 m_t + \pi_3\phi_t + \pi_4\varepsilon_t(z)] + b_2(c_2 m_t + c_3\phi_t).$$

The formulae for the b_1 and b_2 coefficients are derived in the appendix.

The average value $\overline{E_z m_t}$, which appears in equations (13) and (14), can be calculated by subtracting $b_1\pi_4\varepsilon_t(z)$ from the $E_z m_t$ expression that is shown in equation (24). Note that it is now not possible to determine the anticipated real rate of return and the level of output before obtaining the solutions for $P_t(z)$ and R_t. Because $r_t(z)$ and $y_t(z)$ depend on $E_z m_t$, which depends in turn on the realized values of $P_t(z)$ and R_t, it is now necessary to start with the full solution of the model.

The two c-coefficients, which express the dependence of R_t on m_t and ϕ_t, can be readily related to the π-coefficients by using the expression for R_t that is given in equation (13). Substituting on the right side for P_t from equation (18) and for $\overline{E_z m_t}$ from the use of equation (24), and writing $R_t = \ldots + c_2 m_t + c_3\phi_t$ (where the dots denote dependence on M_{t-1} and a constant, which are not of interest here) leads to the conditions,

$$c_2(1 + b_2\delta H/\gamma\alpha) = -(1/\gamma)[1 - \delta H/\alpha - \pi_2(1 - b_1\delta H/\alpha)],$$

$$c_3(1 + b_2\delta H/\gamma\alpha) = (1/\gamma)[1 + \pi_3(1 - b_1\delta H/\alpha)],$$

where it may be recalled that $H \equiv \alpha_s\beta_d - \alpha_d\beta_s$.

The solution for the π-coefficients now involves the use of the price level condition from equation (14). The procedure is first to substitute for $P_t(z)$ from equation (17), $E_z m_t$ from equation (24), $E_z P_{t+1}$ from equation (19), and P_t from equation (18), using the above two conditions to substitute out for the c_2 and c_3 coefficients in the expression for $E_z m_t$. The five π-coefficients are then determined, as in the full current information case, by requiring the resulting equation to hold identically in $[M_{t-1}, m_t, \phi_t, \varepsilon_t(z)]$. Not surprisingly, the constant, π_0, and the M_{t-1} coefficient, π_1, correspond to the full current information values. However, the other three coefficients generally differ from those associated with full current information. After a large amount of algebra, the solution for these coefficients turns out to be

$$\pi_2 = \frac{(1-b_2)(\alpha-\beta-\delta H) + \beta(1+\gamma)}{(b_1-b_2)(\alpha-\beta-\delta H) + [\alpha(1-b_1)+b_1\beta](1+\gamma)},$$

$$\text{(25)} \quad \pi_3 = -\frac{(1-b_2)\alpha + b_2\beta}{(b_1-b_2)(\alpha-\beta-\delta H) + [\alpha(1-b_1)+b_1\beta](1+\gamma)},$$

$$\pi_4 = 1/[\alpha(1-b_1) + b_1\beta].$$

Defining the denominator of the π_2 and π_3 expressions as

$$A \equiv (b_1-b_2)(\alpha-\beta-\delta H) + [\alpha(1-b_1) + b_1\beta](1+\gamma)$$

and neglecting the constant and M_{t-1} parts of the answer (which correspond to those from the full current information case), the price level solutions are given from equations (17) and (19) by

$$(26) \quad P_t(z) = \ldots + (1/A)[(1-b_2)(\alpha-\beta-\delta H) + \beta(1+\gamma)]m_t - (1/A)[(1-b_2)\alpha + b_2\beta]\phi_t + \varepsilon_t(z)/[\alpha(1-b_1) + b_1\beta]$$

and

$$(27) \quad E_zP_{t+1} = \ldots + (1/A)[(b_1-b_2)(\alpha-\beta-\delta H) + b_1\beta(1+\gamma)]m_t - (\alpha/A)(b_1-b_2)\phi + b_1\varepsilon_t(z)/[\alpha(1-b_1) + b_1\beta].$$

These results imply the anticipated inflation rate from the perspective of market z,

$$(28) \quad E_zP_{t+1}-P_t(z) = \ldots - (1/A)(1-b_1)(\alpha+\beta\gamma-\delta H)m_t + (1/A)[\alpha(1-b_1)+b_2\beta]\phi_t - (1-b_1)\varepsilon_t(z)/[\alpha(1-b_1)+b_1\beta].$$

The solution for the nominal rate of return, which is found by means of the above conditions for the c-coefficients, is

$$(29) \quad R_t = \ldots - (1/A)(1-b_1)(\alpha-\beta-\delta H)m_t + (1/A)[\alpha(1-b_1)+b_1\beta]\phi_t.$$

The locally-anticipated real rate of return is determined from the use of equations (9) and (24) to be

$$(30) \quad r_t(z) = r_t^*(z) + (1/A)(1-b_1)\beta(1+\gamma)m_t + (1/A)(b_1-b_2)\beta\phi_t - (\beta/\alpha)b_1\varepsilon_t(z)/[\alpha(10b_1)+b_1\beta],$$

where $r_t^*(z)$ is the full current information solution (which includes a dependence on $\varepsilon_t(z)$) that is shown in equation (15). Finally, the result for local output is

$$(31) \quad y_t(z) = y_t^*(z) + (\alpha_s\beta_d-\alpha_d\beta_s)\{(1/A)(1-b_1)(1+\gamma)m_t + (1/A)(b_1-b_2)\phi_t - (b_1/\alpha)\varepsilon_t(z)/[\alpha(1-b_1) + b_1\beta]\},$$

where $y_t^*(z)$ is the full current information solution (which includes a dependence on $\varepsilon_t^d(z)$ and $\varepsilon_t^s(z)$), as shown in equation (15).

The above solutions involve the b_1 and b_2 coefficients. The analysis in the appendix relates these coefficients to the underlying parameters of the model, including the variances, σ_m^2, σ_ϕ^2 and σ_ε^2. The appendix calculations do raise the possibility of multiple solutions for b_1 and b_2, although a unique solution is guaranteed for a plausible range of parameter values. Since I do not presently understand the economics of the multiple solution case, I have limited attention in the text to situations where (b_1, b_2) are uniquely determined. For present purposes, the most important properites of the solutions for b_1 and b_2--in the case of a unique solution--are, assuming that σ_m^2, σ_ϕ^2 and σ_ε^2 are all non-zero,

$$0 < b_1 < 1$$

$$-\infty < b_2 < 1 \text{ if } (\alpha-\beta-\delta H) > 0,$$

$$b_1 > b_2 \text{ if and only if } (\alpha-\beta-\delta H) > 0.$$

The analysis implies also that the A-parameter, as defined above equation (26), is unambiguously positive.

I focus the analysis on the case where $H \equiv \alpha_s\beta_d-\alpha_d\beta_s > 0$ and $\alpha-\beta-\delta H \equiv \alpha-\beta-\delta(\alpha_s\beta_d-\alpha_d\beta_s) > 0$. If the income elasticity of money demand, δ, were equal to unity and if α_d and β_s were negligible, this last condition would, assuming $\beta < 1$, require $\alpha > \beta/(1-\beta)$. Larger values of $\alpha_d\beta_s$ imply a less stringent condition--for example, $\alpha > \beta$ would be required when $\alpha_d\beta_s = \alpha_s\beta_d$. In general the assumed inequality requires a high relative price elasticity of excess commodity demand, α, in comparison to the wealth elasticity, β.

Effects of Money Shocks

Consider now the effects of a money shock, m_t. The central conclusion, which was suggested much earlier from the form of equation (9), is that the anticipated real rate of return, $r_t(z)$, rises with m_t, as shown in equation (30). (Note that $1-b_1 > 0$ and $A > 0$ apply.) Unanticipated monetary expansion causes excess commodity demand, which requires an increase in the anticipated real rate of return in order to restore market clearing. These movements imply a positive response of output to m_t (assuming that $\alpha_s\beta_d-\alpha_d\beta_s > 0$), as shown in equation (31). However, as $\sigma_m^2 \rightarrow \infty$, the results from the appendix imply that $b_1 \rightarrow 1$ (all price movements are viewed in this situation as reflecting monetary stimuli on a one-to-one basis), while A remains finite, so that the coefficients on m_t in the $r_t(z)$ and $y_t(z)$ expressions approach zero.[17] When monetary disturbances become the primary source of price fluctuations, the confusion between monetary and other disturbances vanishes, which implies a disappearance of the "real" effects of m_t (on $r_t(z)$ and $y_t(z)$). This phenomenon is an example of Lucas's (1973) effect of monetary variance on the slope of the "Phillips curve."

A basic implication of the model is that the responses of $P_t(z)$, E_zP_{t+1} and R_t to m_t must be consistent with the positive response of $r_t(z) \equiv P_t(z) - E_zP_{t+1} + R_t$. However, the response of the three individual components of the anticipated real rate of return turns out to be sensitive to changes in the specification of the model. In the present setup, assuming $\alpha > \beta+\delta H$, it follows from equation (26) (using the definition of A and the condition $b_2 < 1$) that $P_t(z)$ responds positively and less than one-to-one with m_t. It also follows from equation (27) (recalling that $b_1 > b_2$) that E_zP_{t+1} responds positively and less than one-to-one with m_t. Further, the response of E_zP_{t+1} is smaller than that of P_t, so that the anticipated inflation rate, as shown in equation (28), declines with m_t. Finally, equation (29) indicates a negative response of the nominal interest rate to m_t. This behavior corresponds to the usual "liquidity" effect of monetary expansion--with the quantity of money rising more than prices in the "short run," as implied by equation (26), aggregate portfolio balance requires a decline in R_t (assuming, as guaranteed by the condition $\alpha > \beta+\delta H$, that the rise in y_t does not, by itself, raise money demand sufficiently to balance the increase in supply). Note that the downward response of R_t to m_t is consistent with the upward movement of $r_t(z)$. The intervening variable between the nominal and anticipated real interest rates, which is the expected rate of inflation, moves downward sufficiently--that is, $P_t(z)$ rises sufficiently relative to E_zP_{t+1}--to allow R_t and $r_t(z)$ to respond in opposite directions to monetary disturbances. The "short-run flexibility" of the anticipated infaltion rate is obviously a crucial element in this analysis. If inflationary expectations responded only sluggishly to current disturbances, it would not be possible for the nominal interest rate to move substantially in the short run in a direction opposite to that of the anticipated real rate of return.

The pattern of response to monetary shocks may be altered if the relative price sensitivity of excess demand, α, is sufficiently weak that $\alpha < \beta+\delta H$ applies. Equation (29) indicates that R_t would now be positively related to m_t--because the response of money demand from the output channel is stronger than before. The response of $P_t(z)$ to m_t becomes of ambiguous sign and would be negative if $\alpha-\beta-\delta H$ were negative and of sufficient magnitude. However, the response of E_zP_{t+1} to m_t that is shown in equation (27) is still positive (as can be seen by substituting for b_1-b_2 from the formula given in the appendix), so that the anticipated inflation rate now rises with m_t. A conclusion here is that a weak contemporaneous response of prices to money shocks is possible if it is accompanied by a positive relation between money shocks and the nominal rate of return. The positive response of the anticipated real rate of return now involves an increase in the nominal interest rate that exceeds the rise in the anticipated inflation rate. In the previous (perhaps more likely) case, the rise in $r_t(z)$ involved a decline in the anticipated inflation rate that dominated over a decline in the nominal rate of return.

Another consideration that would affect the relation of money shocks to prices and the nominal interest rate involves serial correlation in the money growth process. This possibility was considered in the initial analysis in the form, $m_t = \rho m_{t-1} + n_t$, but was dropped subsequently on computational grounds. This extension would not seem substantially to alter the determination of $r_t(z)$ and $y_t(z)$, which depend from equations (9) and (10) only on the unperceived parts of monetary expansion. However, anticipated future growth rates of money would involve the term, $\rho E_z m_t = \rho(\rho m_{t-1} + E_z n_t)$. For the case where $\rho > 0$, the current monetary innovation n_t would raise $E_z n_t$ and thereby raise the short-term anticipated money growth rate. This expectation would produce an increase in the short-term anticipated inflation rate and "thereby" raise the nominal interest rate. (A positive effect on price levels would also arise here because of the inverse dependence of money demand on R_t.) The negative "liquidity" effect of monetary expansion on R_t that is shown in equation (29) (when $\alpha > \beta+\delta H$ applies) would therefore be offset by this direct money growth anticipation effect.

Because of the various possibilities for the effects of monetary surprises on price levels and the nominal interest rate, it seems that the most interesting conclusion from the present analysis is that the average anticipated real rate of return, $r_t \equiv P_t - E_z P_{t+1} + R_t$, would rise with m_t. This result is of special interest because it distinguishes qualitatively an implication of the "equilibrium business cycle" approach from the hypothesis of an inverse effect of m_t on r_t that would arise under usual Keynesian analysis.

It is worth emphasizing that the model's results discussed above apply to the anticipated real rate of return--which is the variable that affects commodity supply and demand--and not directly to the realized value of this return. In fact, under the present partial information setup that involves the setting of an economy-wide nominal interest rate, it turns out that the anticipated and realized real rates of return respond in opposite directions to money shocks. Aside from effects that involve the dependence of P_{t+1} on realized values of date t+1 disturbances, the economy-wide average anticipated real rate of return differs from the average realized return because of a difference between the economy-wide average value of E_zP_{t+1} and the value of this expectation that would have been formed under full date t information. The latter expectation, labeled EP_{t+1}, follows immediately from updated forms of equations (26) and (20) as $EP_{t+1} = \ldots + m_t$--that is, under full current information, the effect on EP_{t+1} of m_t is one-to-one (assuming $\rho = 0$) and the effect of ϕ_t is nil. The economy-wide average realized real rate of return depends on the realized values of date t+1 disturbances and on the expression, $R_t - (EP_{t+1} - P_t)$. The last term is determined, using the above expression for EP_{t+1} and the formulae for $P_t(z)$ and R_t from equations (26) and (29), as

$$(32)\quad R_t - (EP_{t+1}-P_t) = \ldots - (1/A)(1-b_1)(\alpha-\beta)(1+\gamma)m_t - (1/A)(b_1-b_2)(\alpha-\beta)\phi_t.$$

In contrast with the anticipated real rates of return, as shown in equation (30), the average realized rate moves inversely to m_t.

It seems clear that this result could not obtain if the capital market specified a real interest rate rather than a nominal rate. In the context of an economy-wide real interest rate, the movements in anticipated and realized real rates of return would be coincident and would involve a positive response to money shocks. The "indexation" of nominal returns on financial assets to the average realized value of inflation would therefore alter the model's conclusions concerning the behavior of realized real rates of return.[18]

Effects of Money Demand Shocks

Consider now the impact of an aggregate money demand shock, ϕ_t. The model yields the surprising conclusion that this disturbance has a positive effect on output. However, it should be stressed that the present analysis does not admit the possibility of negative correlation between the money demand shock and a disturbance to aggregate excess commodity demand (which was not included in the model). The existence of this sort of correlation would likely reverse the association between money demand shocks and output movements.

It is clear from equations (9) and (10) that the effect of ϕ_t on $r_t(z)$ and $y_t(z)$ would operate in the present framework solely through an effect on $E_z m_t$. An increase in ϕ_t implies, as would be expected, a decrease in $P_t(z)$ from equation (26) (assuming $\alpha > \beta+\delta H$ so that $b_2 < 1$) and an increase in R_t from equation (29). The former effect leads, in accordance with the coefficient b_1, to a decrease in $E_z m_t$. The latter effect involves, through the coefficient b_2, an ambiguous effect on $E_z m_t$. However, if the latter effect is positive, it must, assuming $\alpha > \beta+\delta H$, be of smaller magnitude than the former effect. Therefore, $E_z m_t$ declines with ϕ_t, which implies--for a given value of m_t--that $r_t(z)$ and $y_t(z)$ would increase. Although a money demand shock is contractionary in the sense of reducing price levels, it is expansionary in terms of leading, via an increase in $(m_t - E_z m_t)$ and a corresponding increase in anticipated real rates of return, to an increase in outputs. It is worth noting that the present example is one in which an output expansion is accompanied by a decline in current prices relative to expected future prices. The result in equation (32) indicates that the effect on realized real rates of return is again opposite to that on anticipated returns.

Effects of Relative Shocks

Finally, a relative excess demand shock $\varepsilon_t(z)$ raises $P_t(z)$ (and does not affect R_t) and thereby implies an increase in $E_z m_t$. Consequently, $r_t(z)$ and $y_t(z)$ are determined below their full current information values, as shown in equations (30) and (31). The output solution in equation (31) implies that, in comparison with the full current information case (and assuming $\alpha > \beta + \delta H$ and $\alpha_s \beta_d - \alpha_d \beta_s > 0$), the incomplete current information solution involves "excessive" response to the global disturbances, m_t and ϕ_t, and "insufficient" response to the local disturbances, $\varepsilon_t(z)$. Similar behavior emerged in the model without a global capital market that was constructed in my earlier paper (1976, p. 17), although the aggregate money demand/portfolio shock, ϕ_t, did not enter into that analysis.

IV. Concluding Remarks

This theoretical study has focused on the anticipated real rate of return on earning assets as the relative price variable that links monetary shocks to output responses. A monetary disturbance that creates excess demand for commodities raises this anticipated return and thereby eliminates the excess demand. If this relative price variable does have a key role in the transmission of monetary effects, it is likely that the same variable would be important for analyzing the output effects of other variables, such as government purchases of goods and services. (See Hall, 1978 and Evans, 1978, in this context.) For example, the rate of return mechanism might explain the tendency of total output to rise strongly during wartime. A possible analysis would involve the following elements. 1) Aggregate demand rises initially because the government spending is, first, not a close substitute for private consumption or investment, and second, is not perceived as permanent. These two considerations imply a small initial offsetting decrease in private commodity demand. 2) The consequent increase in the anticipated real rate of return would reduce private demand and also stimulate an increase in the overall supply of goods and services. A strong response on the supply side might account for the observed responsiveness of total output to wartime spending.

There are numerous related issues that could be pursued theoretically--notably, a mechanism for explaining the persisting output effects of monetary and other disturbances could be added to the model. However, I suspect that empirical research would potentially constitute the most fruitful complement to the present theoretical analysis. Key empirical questions are whether monetary disturbances exert the hypothesized positive contemporaneous effect on the anticipated real rate of return and whether the response of this relative price variable can be documented as a central channel for the transmission of real effects of monetary disturbances. The treatment of expectations will be a crucial part of this empirical analysis.

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FOOTNOTES

[1]If these net benefits are always positive then portfolio balance would require R_t to be positive. The form of the money demand function, equation (3) below, does not exhibit this property, although a modification along these lines would not seem to have important implications for the main analysis.

[2]See Aitchison and Brown (1969, p. 8).

[3]A change in $r_t^*(z)$ would generally have an income effect. See the discussion below.

[4]The important condition is $\nu_i = (1+R_{t+i})\nu_{i+1}$, where ν_i is the discount factor for date t+i, as defined below.

[5]This term corresponds to Lucas's (1972) mechanism for generating relative disturbances across markets.

[6]This term is analogous to Mishan's (1958, p. 107) "cash balance effect [that] comes into operation when the cash available to the community for transactions purposes . . . diverges from the amount of cash that the community desires to hold for this purpose." This concept appears also in Archibald and Lipsey (1958).

[7]In this respect the analysis parallels the infinite horizon, optimizing model of Sidrauski (1967). The superneutrality of money in his model--that is, the independence of the steady-state real interest rate and capital-labor ratio from the growth rate of money--depends on this infinite horizon setup. For additional discussion see Barro and Fischer (1976, section 3).

[8]Sargent (1973, p. 434) and Sargent and Wallace (1975, pp. 242-43) specify a model in which commodity demand depends on the anticipated real rate of return, but where commodity supply depends on the ratio of the current price to the price that was anticipated for today as of last period.

[9]The equality $k = k(z)$ for all z follows from an arbitrage condition (in the absence of mobility costs over one period of time) that requires all markets to look equally desirable, ex ante, from the standpoint of suppliers and demanders.

[10]This netting to zero is essentially a matter of defining a relative disturbance to local commodity markets. Aggregate real commodity shocks could be considered separately in equations (7) and (8). Serial independence of the $\varepsilon(z)$'s can be viewed as a consequence of the arbitrage condition described in n. 9 above.

[11]An interaction between monetary "neutrality" results for output and interest rates has been stressed by Sargent (1973, pp. 442-44).

[12]The downward effect of money on the rate of return in this type of model follows unambiguously only if a direct wealth effect of money on consumer demand is omitted.

[13]It is assumed throughout that the observation of one's own transfer from the government conveys negligible additional information over the observations of R_t and $P_t(z)$. Similarly, the analysis neglects the information provided by observations of one's own money demand shift.

[14]The locally anticipated real rate of return and level of output also respond in the same manner to permanent demand or supply forces, $k^d(z)$ and $k^s(z)$, as to temporary stimuli, $\varepsilon_t^d(z)$ and $\varepsilon_t^s(z)$. The distinction between permanent and temporary shock effects that arose in my earlier model (1976, appendix 1) does not appear here because of the adjustment of the nominal interest rate (to the value of k in equation (23) below). The permanent/temporary distinction would reemerge if the commodity excess demand response to (perceived) "permanent" movements in $r_t(z)$ were differentiated from the

response to (perceived) "temporary" changes. It would be anticipated that excess commodity demand would be substantially more responsive--because of the larger set of intertemporal substitution possibilities--to movements in $r_t(z)$ that were viewed as transitory opportunities for above or below normal real rates of return.

[15]The term $\pi_5 m_{t-1}$ would have to be added to equations (17) and (18) if $\rho \neq 0$ were permitted. The term $\rho\pi_5 E_z m_t$ would then appear in equation (19).

[16]However, the equilibrium solution can break down in the present model when $\sigma_\phi^2 = 0$. The problem is that the observed nominal interest rate cannot impart information about the underlying money shock when the interest rate is invariant with money--as one would expect when $\sigma_\phi^2 = 0$. The introduction of a non-zero value for ρ would eliminate this problem. A more general discussion of this type of equilibrium problem is contained in King (1978).

[17]I have as yet been unable to ascertain whether the relation between σ_m^2 and these coefficients is monotonic.

[18]The simultaneous presence of economy-wide nominal and real interest rates would imply a qualitative shift in the information structure of the model. The two pieces of current global information implied by this setup would seem to constitute full current information in the present model that includes only two types of aggregate shocks. A satisfactory analysis of this model would seem to require the introduction of additional disturbance terms.

Appendix

Derivation of the Conditional Expectation of Money Growth

The conditional expectation, $E_z m_t$, is expressed in equation (24) in terms of the two pieces of current information and the two coefficients, b_1 and b_2. The b_1-coefficient multiplies the current information implicit in the observation of $P_t(z)$, $\pi_2 m_t + \pi_3 \phi_t + \pi_4 \varepsilon_t(z)$, while the b_2-coefficient applies to the current information contained in the observation of R_t, $c_2 m_t + c_3 \phi_t$. The three π-coefficients are shown in equation (25) while the two c-coefficients are those attached to m_t and ϕ_t in the formula for R_t in equation (29).

Since the three current disturbances, m_t, ϕ_t and $\varepsilon_t(z)$, are independently, normally distributed with zero mean and known variances, the determination of the b_1 and b_2 coefficients emerges from a straightforward, but tedious, calculation of a conditional expectation. A formula for the present multivariate normal case appears in Graybill (1961, p. 63). An intuitive feel for this formula can be obtained by viewing b_1 and b_2 as least-squares estimates--using the known population variances and covariances--that would emerge from a regression of m_t on the two variables, $\pi_2 m_t + \pi_3 \phi_t + \pi_4 \varepsilon_t(z)$ and $c_2 m_t + c_3 \phi_t$. The results can be written from the usual least-squares regression formulae as

$$b_1 = -(c_3/\Delta)(c_2\pi_3 - c_3\pi_2)\sigma_m^2\sigma_\phi^2,$$

$$b_2 = (1/\Delta)[\pi_3(c_2\pi_3 - c_3\pi_2)\sigma_m^2\sigma_\phi^2 + (\pi_4)^2 c_2 \sigma_m^2\sigma_\varepsilon^2],$$

where the determinant Δ is given by

$$\Delta = (c_2\pi_3 - c_3\pi_2)^2\sigma_m^2\sigma_\phi^2 + (c_2\pi_4)^2\sigma_m^2\sigma_\varepsilon^2 + (c_3\pi_4)^2\sigma_\phi^2\sigma_\varepsilon^2.$$

After substitution for the π- and c-coefficients and a substantial amount of manipulation, the two b-coefficients can be expressed as

$$\text{(A1)} \qquad b_1 = \frac{\alpha\beta\sigma_m^2\sigma_\phi^2}{\alpha\beta\sigma_m^2\sigma_\phi^2 + \left[\frac{(1-b_1)(\alpha-\beta-\delta H)}{\alpha(1-b_1)+b_1\beta}\right]^2 \sigma_m^2\sigma_\varepsilon^2 + \sigma_\phi^2\sigma_\varepsilon^2} ,$$

$$\text{(A2)} \quad b_2 = \frac{\alpha\beta\sigma_m^2\sigma_\phi^2 - \frac{(1-b_1)(\alpha-\beta-\delta H)}{[\alpha(1-b_1)+b_1\beta]^2}\left[(1+\gamma)[\alpha(1-b_1)+b_1\beta] + b_1(\alpha-\beta-\delta H)\right]\sigma_m^2\sigma_\varepsilon^2}{\alpha\beta\sigma_m^2\sigma_\phi^2 - b_1(1-b_1)\left[\frac{\alpha-\beta-\delta H}{\alpha(1-b_1)+b_1\beta}\right]^2 \sigma_m^2\sigma_\varepsilon^2 + \sigma_\phi^2\sigma_\varepsilon^2} .$$

Unfortunately, the solutions for b_1 and b_2 cannot be written as closed-form expressions.

The solution in equation (A1) can be expressed as a cubic in the coefficient b_1. It is possible to obtain necessary and sufficient conditions in terms of the parameters, $(\sigma_m^2, \sigma_\phi^2, \sigma_\varepsilon^2, \alpha, \beta, \delta H)$, for the existence of a single real root. The general conditions are very complicated, but a sufficient condition for one real root turns out to be

$$\text{(A3)} \qquad (\beta^2\sigma_\phi^2)/\sigma_\varepsilon^2 > (8/27)[(\alpha-\beta-\delta H)/\alpha]^2 .$$

Therefore, if σ_ϕ^2 is not too small relative to σ_ε^2 and/or if $|\alpha-\beta-\delta H|$ is small, a single real root is guaranteed. However, there seems to be a range of parameter values that yield three real roots for b_1.

The solution in equation (A2) relates b_2 one-to-one to the value of b_1. Therefore, a unique solution for b_1 implies a unique solution for b_2 and vice versa.

There seems to be a range of parameter values that yield three separate real solutions for (b_1, b_2). I do not presently see the economic meaning of these multiple equilibria. For some discussion of this type of problem, see King (1978). For present purposes I carry out the rest of the analysis under the assumption that the parameter values are such as to imply a single real solution for (b_1, b_2).

It is convenient to have an expression for the difference between b_1 and b_2, which can be written as

$$\text{(A4)}\quad (b_1-b_2) = \frac{(1-b_1)(1+\gamma)(\alpha-\beta-\delta H)\sigma_m^2\sigma_\varepsilon^2}{[\alpha(1-b_1)+b_1\beta]\left\{\alpha\beta\sigma_m^2\sigma_\phi^2 - b_1(1-b_1)\left[\frac{\alpha-\beta-\delta H}{[\alpha(1-b_1)+b_1\beta]}\right]^2\sigma_m^2\sigma_\varepsilon^2 + \sigma_\phi^2\sigma_\varepsilon^2\right\}}.$$

The subsequent discussion assumes that the parameters, $(\sigma_m^2, \sigma_\phi^2, \sigma_\varepsilon^2, \alpha, \beta)$, are all positive. The condition $0 < b_1 < 1$ then follows from inspection of equation (A1).

The condition $b_2 < 1$ if $\alpha-\beta-\delta H > 0$ is implied by equation (A2). The result follows if the denominator on the right side of the equation can be shown to be positive because the second term of the numerator (which is negative if $\alpha-\beta-\delta H > 0$) can, since $0 < b_1 < 1$, be readily shown to be of larger magnitude than the middle term of the denominator. The denominator is positive for some values of $(\sigma_m^2, \sigma_\phi^2, \sigma_\varepsilon^2)$--for example, as $\sigma_m^2 \to 0$ and $b_1 \to 0$, the denominator would become positive since $\sigma_\phi^2\sigma_\varepsilon^2 \neq 0$. Further, the denominator cannot pass through zero because this expression equalling zero can be shown to be inconsistent with the expression for b_1 that is given in equation (A1). With b_2 a continuous function of the σ^2's (in the case of unique solutions for b_1 and b_2), it follows that the denominator must be positive throughout.

The condition $b_1 > b_2$ if and only if $\alpha-\beta-\delta H > 0$ follows from equation (A4), because $0 < b_1 < 1$ and the expression in large brackets in the denominator of the right side of the equation is positive from the argument in the above paragraph. It also follows from the form of the expression for (b_1-b_2) in equation (A4) that the A parameter, as defined above equation (26) in the text, is unambiguously positive.

The following limiting conditions for the b-coefficients are implied by equations (A1) and (A2):

$$\sigma_m^2 \to 0 \;\; \Rightarrow \;\; (b_1,\; b_2) \to 0,$$

$$\sigma_m^2 \to \infty \;\; \Rightarrow \;\; (b_1,\; b_2) \to 1,$$

$$\sigma_\phi^2 \to 0 \;\; \Rightarrow \;\; b_1 \to 0, \quad b_2 \to -\infty \cdot \mathrm{sign}(\alpha-\beta-\delta H),$$

$$\sigma_\phi^2 \to \infty \;\; \Rightarrow \;\; (b_1,\; b_2) \to \alpha\beta\sigma_m^2/(\alpha\beta\sigma_m^2+\sigma_\varepsilon^2) \in [0,1],$$

$$\sigma_\varepsilon^2 \to 0 \;\; \Rightarrow \;\; (b_1,\; b_2) \to 1,$$

$$\sigma_\varepsilon^2 \to \infty \;\; \Rightarrow \;\; b_1 \to 0, \quad b_2 \to - \frac{(\alpha-\beta-\delta H)(1+\gamma)\sigma_m^2}{\alpha\sigma_\phi^2} = -\;(+)\cdot \mathrm{sign}(\alpha-\beta-\delta H).$$