NBER Working Paper Series

ESTIMATING THE FAMILY LABOR SUPPLY FUNCTIONS DERIVED FROM THE STONE-GEARY UTILITY FUNCTION

Michael D. Hurd

Working Paper No. 228

CENTER FOR ECONOMIC ANALYSIS OF HUMAN BEHAVIOR AND SOCIAL INSTITUTIONS

National Bureau of Economic Research, Inc.
204 Junipero Serra Boulevard, Stanford, CA 94305

January 1978

This research was supported in part by NBER, and in part by a grant from the U.S. Department of Labor (Labor Grant J-9-M-6-0156). The author wishes to thank John Pencavel for many useful discussions on this work and Jeffrey Parker for excellent computational assistance.

NBER Working Paper 228
January 1978

ESTIMATING THE FAMILY LABOR SUPPLY FUNCTIONS DERIVED FROM THE STONE-GEARY UTILITY FUNCTION

Abstract

The Stone-Geary utility function defined over an index of goods, the leisure of the husband, and the leisure of the wife is used to derive the earnings functions of the husband and the wife. The parameters of the utility function are estimated from the parameters of the earnings functions in a way that accounts for a number of theoretical and statistical problems. The effect of family composition on utility is estimated by specifying and estimating adult equivalents in consumption and leisure of various categories of children. On the statistical side the following difficulties are all considered: nonlinear constraints across equations, endogenous marginal income tax rates, variations in tastes in the population, heteroscedasticity, and truncation of the left-hand variable. The data come from the 1967 Survey of Economic Opportunity. The results are generally good and support the view that the effects of family composition on utility can be estimated from behavioral relationships. Alternative results that ignore the complicated statistical problems are presented; they imply that the statistical problems are empirically important and should not be ignored.

Michael D. Hurd
NBER-West
204 Junipero Serra Boulevard
Stanford, CA 94305

1. Introduction

This paper gives the statistical theory and results of estimating the parameters of the family labor supply functions derived from the Stone-Geary utility function. The economic theory behind the labor supply functions is quite standard; but, the statistical problems associated with them are not, at least I know of no other work that treats all of them together. The statistical problems are: in one of two equations which have correlated error terms the right-hand variable is truncated; both equations have heteroscedastic errors; both equations have endogenous variables on the right-hand side; there are nonlinear cross-equation restrictions. A several-step procedure is developed that accounts for these problems to produce consistent estimates of the utility function parameters, and the procedure is applied to data.

In connection with some other work, a colleague and I want to estimate the parameters of the Stone-Geary utility function in a way that takes account of family composition so that the welfare effect of various income maintenance schemes can be studied.[1] We study only husband-wife families and we define utility over goods, the leisure of the husband and the leisure of the wife. The parameters of the utility function can, therefore, be estimated from the labor supply functions of the husband and the wife, or from the earnings functions of them. In fact, the earnings functions are slightly easier to work with; but, everything that is said in this paper can be applied directly to the estimation of labor supply

---

[1] My partner in the project is John Pencavel. He specified the form of the Stone-Geary utility function that is estimated here, and has provided valuable advice at several steps of this work. However, any errors in this work are mine.

functions. When the earnings functions of the husband and wife are written, one discovers that the error term in each function is heteroscedastic under the assumption that the error term arises partly because of variations in tastes for leisure and for goods.[2] Furthermore, earnings are zero for some family members when tastes for leisure are above a certain level or when tastes for goods are below a certain level. That is, some families are at corner solutions, and if, say, the offered wage rate were increased, the family member(s) would still desire not to work. Formally, this amounts to specifying that desired hours of work are functions of tastes and of the exogenous variables, and that for certain combinations of tastes and the variables, desired hours are negative.[3] In the data, however, we observe hours and earnings to be zero rather than negative. This is the truncation problem. Finally, the first-order conditions for utility maximization relate the net wage rate to desired hours of work. In that there is considerable variation in marginal income tax rates, we want to account for the income tax in the estimation. Marginal tax rates should be considered to be endogenous because they partly reflect tastes for work.[4]

One way to estimate regression functions with a limited dependent variable is the tobit estimator; however, tobit is not consistent when there is heteroscedasticity.[5] In our problem part of the heteroscedasticity arises from the specification of the utility function, and the theory indicates how it may be accounted for. The rest of the heteroscedasticity

---

[2] See Pollack and Wales.

[3] See Heckman. He states the equivalent condition that the reservation wage is greater than the offered wage.

[4] See my "Estimation of Nonlinear Labor Supply Functions."

[5] See my "Estimation in Truncated Samples."

in our model is due to the endogeneity of some of the right-hand variables. A considerable part of the work is concerned with accounting for this heteroscedasticity.

In this paper the discussion is limited almost completely to the estimation problems, the solutions, and the results. In other work we shall be concerned with the theory behind our formulation of how family composition affects utility, and with application of the results of the estimation.

2. Labor Supply and the Stone-Geary Utility Function

The form of the Stone-Geary utility function chosen to account for family composition is

$$U = U(l_1, l_2, x) = (1-B_1-B_2)\ \ln\left(\frac{x}{I_x} - a\right) + B_1\ln\left(b_1 - \frac{h_1}{I_1}\right) +$$

$$B_2\ln\left(b_2 - \frac{h_2}{I_2}\right)$$

where l_1 = husband's leisure, l_2 = wife's leisure, x = an index of goods, $h_1 = 1-l_1$ = husband's work, $h_2 = 1-l_2$ = wife's work. I_1, I_2 and I_x may be thought of as indexes that give the number of adult equivalents in the family; B_1 and B_2 are parameters to be estimated; and a, b_1 and b_2 are random variables that represent a distribution of tastes in the population.[6] In static utility maximization subject to the budget constraint

---

[6] I shall not attempt to justify here either the use of this utility function to derive labor supply functions or this way to account for family composition. We shall do that in other work as the emphasis here is on the estimation problem.

$$px = w_1h_1 + w_2h_2 + py - T(w_1h_1+w_2h_2+y),$$

the earnings functions of the wife and of the husband are given by

$$(1-t)e_1 = (1-B_1)\, b_1I_1(1-t)w_1 - B_1b_2I_2(1-t)w_2 - B_1(y+(e_1+e_2)t-T)$$

$$+ B_1aI_xp$$

$$(1-t)e_2 = -B_2b_1I_1(1-t)w_1 + (1-B_2)b_2I_2(1-t)w_2 - B_2(y+(e_1+e_2)t-T)$$

$$+ B_2aI_xp$$

where T( ) is the income tax function, t=T'=marginal tax rate, e_1 and e_2 are the husband's and wife's earnings respectively, y=nonlabor income, and p=price index of goods.

It is apparent that I make several assumptions that have been relaxed in the work of others. In particular, the wage rate is given to the individual, and there are no fixed costs associated with working.[7]

Let ξ_1, ξ_2, and ξ_a be the expected values of b_1, b_2 and a respectively. The husband's earnings function, for example, may be written as

$$(1-t)e_1 = (1-B_1)\xi_1I_1(1-t)w_1 - B_1\xi_2I_2(1-t)w_2 - B_1G + B_1\xi_aI_xp$$

$$+ (1-B_1)I_1(1-t)w_1z_1 - B_1I_2(1-t)w_2z_2 + B_1I_xpz_a$$

[7] Rosen within a restricted framework allows the wage rate to depend on hours of work. Hanoch and others have introduced fixed costs of work into the labor supply decision.

where $z_1 = b_1-\xi_1$, $z_2 = b_2-\xi_2$, $z_a = a-\xi_a$, and $G = y + (e_1+e_2)t-T$. The last three terms in each earnings function constitute the error term, the result of the distribution of tastes. The error terms are seen to be heteroscedastic, and to be correlated with each other. As things stand, the way to estimate the system is not apparent: 1-t and the z's are correlated; therefore, the expectation of the error term is not zero. (Of course, the expectation of the error term given the right-hand variables is not zero due to the endogeneity of 1-t; however, that problem may be attacked in the usual way.) I proceed by assuming that a is a parameter rather than a random variable. This is a compromise that imposes some restriction on the data; but in view of the other serious estimation problems it seemed sensible to impose the restriction rather than compromise on the solutions to the other problems.[8] If a is a constant the earnings functions become

$$e_1 = (1-B_1)\xi_1 I_1 w_1 - B_1\xi_2 I_2 w_2 - B_1 G/(1-t) + B_1\xi_a I_x \bar{p}/(1-t) + u_1'$$

$$e_2 = -B_2\xi_1 I_1 w_1 + (1-B_2)\xi_2 I_2 w_2 - B_2 G/(1-t) + B_2\xi_a I_x \bar{p}/(1-t) + u_2'$$

where u_1' and u_2' are the composite errors.

---

[8] It is perhaps surprising that assuming a to be a parameter is, in fact, a restriction. It is clear that if the variance of z_a is identified it is a restriction. One could imagine estimating the variances and the covariances of the z's from the regression of the square of the residual from the earnings functions on the squares and products of the variables that appear in the error terms after appropriately accounting for the endogeneity of 1-t. Even though it is a restriction, it is probably a weak restriction in these data because there is not much variation in $\bar{p}$. In the actual estimation an additional error is allowed to account for mistakes in maximization and observation. The variance of this error term appears as the coefficient on the constant term which is not too different from $\bar{p}$.

In the data e_1, the husband's earnings, is greater than zero in almost all households we study; however, e_2 is greater than zero only in about a third of the cases. There are a number of empirically equivalent ways to explain how this happens. First, the earnings functions may be interpreted as desired earnings functions, and one may recognize that if the tastes for leisure of the husband and the wife and the tastes for goods are of certain magnitudes desired hours of work of the wife will be negative. In the data this variable is truncated at zero. Second, one could consider inverting the earnings function of the wife to give the reservation wage as a function of earnings. Truncation occurs when the reservation wage at earnings of zero is greater than the offered or market wage.[9] Third, one might assume that u_2' has that distribution required to generate the observed values of earnings; that is, u_2' takes the value of minus the right-hand side for all those observations where earnings are zero.[10] The likelihood function is the same in all three ways of looking at the problem so, from a statistical point of view, the way is immaterial. I prefer the first so that will be the terminology used here.

The I's are index functions to be estimated as will be discussed below; there are many nonlinear restrictions across equations; and several of the right-hand variables are endogenous. Because u_1 and u_2 are correlated the expected values of both the u's given that e_2 is greater than zero are

---

[9] This is the way Heckman considers the problem.

[10] Let $y = x\beta+u$. Specify the distribution of u as follows: given $x\beta$ $p(u = -x\beta) = F(x\beta)$ and $p(u=t \mid t > -x\beta) = f(t)$ where $F(x\beta)$ is the probability $y = 0$ given x and $\int_{-x\beta}^{\infty} f(t)\, dt = 1 - F(x\beta)$. The likelihood of the sample is $\Pi(F(x_i\beta))^{1-d_i} (f(y_i-x_i\beta))^{d_i}$ where $d_i = 1$ if $y_i > 0$ and $d_i = 0$ otherwise. This likelihood function is the same as the function obtained by considering y to be truncated.

functions of the right-hand variables. This means that both the husband's and wife's earnings functions should be estimated by nonlinear methods. All of these problems mean that a first-round simultaneous estimation of both equations would be very complicated and expensive. Therefore, I adopted a several-stage method: first, the wife's earnings function is estimated consistently; second, the husband's earnings function is estimated consistently using some of the parameters from the first round to account for the truncation in the wife's earnings; third, both equations are simultaneously estimated using some of the previous estimates to account for the truncation but imposing the cross-equation restrictions on the coefficients of the right-hand variables. Nonlinear estimation is required at each step; however, a large very complicated nonlinear estimation problem is broken up into smaller less complicated nonlinear problems.

3. Estimation of the Wife's Earnings Function

A. Statistical specification

The index functions, I_1, I_2 and I_x are assumed to depend linearly on family characteristics:

$$I_1 = \mu_1' z \qquad I_2 = \mu_2' z \qquad I_x = \theta' z$$

where $\mu_1' = [1\ \mu_{11}\ \mu_{12}\ \mu_{13}]$, $z' = [1\ K_1\ K_2\ A]$ and similarly for μ_2 and θ.[12] z is a vector that describes some of the characteristics of the family: K_1 = number of children 0-5; K_2 = number of children 6-15; A = number of adults aged 16-64 exclusive of the husband and wife. The μ's and θ are vectors of parameters to be estimated. Define $W_h = W_1 Z$ and $W_w = W_2 Z$; these are 4-vectors of exogenous variables which give the interaction between the wage rates and family composition. Let $\alpha_1 = -B_2 \xi_1 \mu_1$, $\alpha_2 = (1-B_2)\xi_2\mu_2$, $\beta_1 = B_2 a\theta$ and $\beta_2 = -B_2$; all but the last are 4-vectors. Define $\beta' = [\beta_1'\ \beta_2]$ $Y' = [pz'/(1-t)\quad G/(1-t]$, a 5-vector. Then the wife's earnings function can be written as

$$e_2 = w_h'\alpha_1 + w_w'\alpha_2 + Y'\beta + u_2$$

where $$u_2 = w_h' \frac{\alpha_1}{\xi_1} z_1 + w_1' \frac{\alpha_2}{\xi_2} z_2 + \nu_2 .$$

It is assumed that z_1 and z_2 are normal random variables with mean zero, variance $\sigma_1{}^2$, $\sigma_2{}^2$ and covarance σ_{12}. ν_2 is normal with mean zero, variance $\sigma_{\nu_2}^2$ and it is independent of z_1 and z_2. The specification allows the

[12] See, for example, Brown and Deaton, and Muellbauer for discussions of incorporating family composition in the utility function.

random tastes for the leisure of the husband and the leisure of the wife to be correlated, but requires that maximization and observations errors be uncorrelated with the other random components. u is heteroscedastic, the variance depending on the first 8 right-hand variables.

e_2 is only observed when the right-hand side is positive; however, the conditional expectation of e, given that the right-hand side is positive, cannot be written immediately because of the endogeneity of Y. In particular, the expectation of u, given that the right-hand side is positive, is not given by the usual formula for the expectation of a truncated normal random variable; the same statement applies for the conditional density.

The endogenous part of Y is due to $1/(1-t)$ and $G=(y + (e_1+e_2)t-T)/(1-t)$. Because the marginal tax rate and actual taxes depend on labor supply and the wage rate, I assume that these two endogenous variables have reduced forms that depend on polynomials in all the exogenous variables in the problem. Suppose then that $Y = \pi x + v$. In that v incorporates u_2 it should be assumed that v is heteroscedastic. Therefore, when the reduced forms are used in the earnings functions another source of heteroscedasticity is introduced.

$$e_2 = w_h'\alpha_1 + w_w'\alpha_2 + \hat{Y}'\beta + u_2 + \hat{v}'\beta \quad \text{where}$$

$\hat{Y} = \hat{\pi}x$ and $\hat{v} = Y-\hat{Y}$. I assume that v is normal with mean zero and it has variance-covariance terms that arise from allowing a correlation between $1/(1-t)$ and G. The likelihood of e_2 can now be written as

$$p(e_2=s|e_2>0) = p(\text{RHS} = s|\text{RHS}>0)$$

$$= \frac{1}{\tau_2}\,\phi\left(\frac{s-w_h'\alpha_1 - w_w'\alpha_2 - x'\pi'\beta}{\tau_2}\right)\Big/\Phi\left(\frac{w_h'\alpha_1 + w_w'\alpha_2 + x'\pi'\beta}{\tau_2}\right)$$

where $\tau_2 = \text{var}(u_2 + v'\beta)$. τ_2 varies from observation to observation because both $v(u_2)$ and $v(v)$ vary. π is not known, but it can be estimated consistently. The likelihood of the observations with earnings of zero could also be written down; however, I only use the observations with positive earnings in estimating the earnings functions. The other observations are not used because the wage rate is generally not observed when earnings are zero. There are ways to use these observations by estimating a wage function. Considerable difficulties are introduced when this is done and the gain may be rather small: usually the R^2 in the wage equations are not large so that the amount of additional information obtained from using all the observations may not be large. Of course, as far as consistency of the estimators is concerned, either the conditional likelihood function or the unconditional likelihood function may be used.

If one assumes that π is known the only barrier to estimating the earnings function by maximum likelihood is that the variance of v is not known: the parameters that enter the variance of u are the α's multiplied by w_1 and w_2, and the functional forms are known from the theory. Unfortunately, the estimation of V(v) is not simple.

Some of the x's that appear on the right-hand side of the reduced forms for Y are functions of the wage rate and the square of the wage rate, but the wage is not always observed. One cannot estimate the reduced form over the observations with wage rates because the conditional likelihood is not known: the conditioning event is that earnings of the wife are positive, not that the right-hand side of the reduced form is positive. It is not at all obvious how that conditioning event relates to the error term in the reduced form because the tax schedules are nonlinear. My strategy is first to estimate a wage function;[13] second to estimate the reduced form for Y using all the observations and the fitted wage rate where necessary; and third to use all the observations to estimate the variance functions of v.

Suppose that

$w = s'\delta_1 + \varepsilon_1 \qquad w^2 = s'\delta_2 + \varepsilon_2$ where s is a vector of exogenous variables, and that δ_1, δ_2 $v(\varepsilon_1)$, $v(\varepsilon_2)$ and $\text{cov}(\varepsilon_1,\varepsilon_2)$ have been estimated consistently. If $\hat{w}$ and $\hat{w}^2$ are used where the observations are missing on w and w^2, $\hat{x}$ may be formed and π estimated over all the observations. This estimate of π is consistent. Finally, it is assumed the variance of each element of the random vector v depends linearly on the X; however, the straightforward regression of the square of the residuals on the X is not desired for two reasons. First, when the fitted values of w and w^2 are used as regressors, the variance of the v is changed according to a function of the variances of ε_1 and ε_2. Second, a functional form for the variance of v is desired that will reflect the fact that variances are positive. It is assumed that the functional relationship is loglinear; however, that specification requires nonlinear estimation because some of the estimates of the variance (the squared residuals less a correction for the use of the fitted wage) are negative making impossible the usual log transformation to linearity.

[13] In theory one ought to estimate the wage function conditional on labor force participation. See Gronau. In practice this has made little difference. Here I ignore any trucation in the wage functions.

All of this may be illustrated by considering the first equation, the equation for $y_1 = 1/(1-t)$. $y_1 = x\pi_1 + v_1$ where x, a vector of length 55, includes w, w^2 and interactions between w and other exogenous variables. If the fitted values of w and w^2 are used for certain observations

$$y_1 = \hat{x}\pi_1 + v_1 + (x-\hat{x})\pi_1 .$$

Despite the heteroscedasticity, consistent estiamtes of π_1 may be obtained by *ordinary least squares because there is no truncation: all of the* observations are used. The residuals for the i^{th} observation,

$$\hat{v}_1 = y_{1_i} - \hat{x}_i'\hat{\pi}_1, \quad \text{satisfies asymptotically}$$

$$\hat{v}_i^2 = v(v_{1_i}) + \pi_1' v(x_i - \hat{x}_i)\pi_1 + \text{residual where lim E(residual)=0.}$$[14]

$v(x_i - \hat{x}_i)=0$ for the observations with a known wage, and for the other observations many elements of $v(x-\hat{x})$ are 0 because most of the x vector is exogenous and known. Of course, $v(x-\hat{x})$ can be estimated for the observations with a fitted wage from the wage and wage squared regressions. Finally π_1 has been estimated *and asymptotically can be considered known, at least* for the purposes of consistent estimation.

---

[14] This form requires that the covariance between v_1 and the error terms in the wage functions are uncorrelated. This follows from the exogeneity of the wage rates.

I specify that $v(v_1) = e^{x\gamma_1}$. The variance function to be estimated becomes $\hat{v}_1^2 - d = e^{x\gamma_1}$ + residual where d is 0 when the wage is observed and $d = \pi_1' v(x-\hat{x})\pi_1$ when it is not observed. Asymptotically and for purposes of estimation $v(x-\hat{x})$ is composed of either zeros or elements of $v\binom{\varepsilon_1}{\varepsilon_2}$. Although the expectation of the left-hand side is positive, there is nothing to prevent it from being negative in the data; even asymptotically the $\hat{v}_1$ from some of the observations with a fitted wage will be close to zero. My solution is to estimate γ_1 by nonlinear methods which do not require that the log of the left-hand side be taken. That is, I find the γ_1 such that the sum of squares of $\hat{v}_1^2 - d - e^{x\gamma_1}$ is minimized.

Y has five elements; however, only two underlying random variables, 1/1-t and G, are combined with the exogenous variables on P and family composition to form the elements of Y. Therefore, only the determinants of 1/1-t and G need be estimated: the rest of Y and v(v) can be constructed from them. I specify that the variance function of G has the same form as that of 1/1-t, so it too must be estimated by nonlinear methods. Finally, I assume that the correlation coefficient between the residuals of 1/1-t and G is a constant. It may be estimated from the variance functions.

Once π and the variance functions have been estimated, the estimation of the earnings function of the wife can proceed. The estimated likelihood of an observation is

$$\hat{p}(e_2 = s \mid e_2 > 0) = \frac{1}{\hat{\tau}_2}\,\phi\left(\frac{s - w_h'\alpha_1 - w_w'\alpha_2 - x'\hat{\pi}'\beta}{\hat{\tau}_2}\right) \Bigg/ \Phi\left(\frac{w_h'\alpha_1 + w'\alpha_2 + x'\hat{\pi}'\beta}{\hat{\tau}_2}\right)$$

where $\hat{\tau}_2^2$ may be written as $v(u_2) + 2\text{cov}(u_2, \hat{v}'\beta) + \beta'\hat{v}(v)\beta$; $\hat{v}(v)$ is a 5x5 matrix constructed from the variance function of 1/1-t and G; it is fixed in this part of the estimation.

$$v(u_2) = (w_h' \frac{\alpha_1}{\xi_1})^2\sigma_1^2 + (w_1' \frac{\alpha_2}{\xi_2})^2\sigma_2^2 + 2w_h' \frac{\alpha_2}{\xi_1} w_w' \frac{\alpha_2}{\xi_2} \sigma_{12} + \sigma^2_{\nu_2} .$$

I have assumed that the correlation between u and $v'\beta$ is a constant, ρ_2: this is not entirely satisfactory, but the alternative, allowing separate correlations between each of the components of u and each of the v's, requires the introduction of six more variance parameters and is not practical.

The parameters of the earnings function are estimated by maximizing the estimated likelihood function. Asymptotically this is equivalent to maximizing the likelihood function because the estimate likelihood function converges in probability to the actual function.[15] The nonlinear problem is, therefore

$$\max_{P} - \sum_i \log \hat{\tau}_{2_i} - \frac{1}{2} \sum_i \left(\frac{s_i - w_h'\alpha_1 - w_w'\alpha_2 - x'\hat{\pi}'\beta}{\hat{\tau}_{2_i}}\right)^2 - \sum_i \log\Phi\left(\frac{w_w'\alpha_1 + w_w'\alpha_2 + x'\hat{\pi}'\beta}{\hat{\tau}_{2_i}}\right)$$

where $P = \{ \alpha_1, \alpha_2, \beta, \sigma_1^2, \sigma_2^2, \sigma_{12}, \sigma^2_{\nu_2}, \rho_2 \}$.

---

[15] This statement requires that the variance functions of 1/1-t and G be exact. This was implicitly assumed in the estimation of these functions. There are no statistical measures, such as a R^2, to verify this.

3.B. Data and Results

The data are from the 1967 Survey of Economic Opportunity. These data have been used extensively by many researchers and do not need to be described here.[11] The variables are defined in the Appendix and the criteria for inclusion in the sample are discussed. Briefly, I use observations on husband-wife families for which the husband is not self-employed, has an observed wage, has earnings, is white, and is between the ages of 16 and 64. In the estimation of the functions determining 1/1-t and G (the tax and nonlabor income functions), the observations on all the wives are used; in the estimation of the earnings function only the wives with positive earnings are used.

The results from the estimation of the tax and transfer equations and the wage equation are not of particular interest here: the first two equations do not have a behavioral interpretation as they combine parameters of the labor supply function with the parameters of the functions that determine tax rates given income. The wage function is quite standard in the literature and the results are similar to what others have found. More details are given in the Appendix. Of more interest are the variance functions because they are not often estimated and used as they are here. The main issue is whether there is sufficient heteroscedasticity to warrant the calculations involved in the estimation of the variance functions. Table 1 gives the estimated distributions of the variances of the tax and nonlabor income equations.

---

[11] See Hall for example.

Table 1

Variance Distributions

Rate		Nonlabor Income (thousands)	
Interval * 10^3	Frequency %	Interval	Frequency %
0 - .74	.5	0 - .67	11.8
.74 - 1.48	1.5	.67 - 1.34	13.0
1.48 - 2.22	15.0	1.34 - 2.01	12.5
2.22 - 2.96	37.9	2.01 - 2.68	8.0
2.96 - 3.70	24.2	2.68 - 3.35	26.7
3.70 - 4.44	10.6	3.35 - 4.02	13.6
4.44 - 5.18	5.1	4.02 - 4.69	5.3
5.18 - 5.92	2.4	4.69 - 5.36	3.1
5.92 - 6.66	1.0	5.36 - 6.03	1.8
6.66+	1.6	6.03+	4.2

It may be seen that the tax function seems to have less heteroscedasticity than the nonlabor income function. This is varified in a scale independent measure of heteroscedasticity $\sqrt{v(\sigma)}/E(\sigma)$, where $v(\sigma)$ is the estimate of the variance of the standard error and $E(\sigma)$ is an estimate of its mean. The value of this summary measure is .18 for the rate function and .37 for the nonlabor income function. It is not easy to judge what a large value of this measure is. However, in some other work on the effects of heteroscedasticity on the kind of estimator employed here (but ignoring the heteroscedasticity) I found that in a simple problem with only one right-hand variable, values of this measure of heteroscedasticity were associated with fairly large misestimates of the slope parameter. The comparison is only suggestive, of course; however, it does suggest that the amount of heteroscedasticity found here is not an inconsequential amount.

The correlation coefficient between the error term in the tax function and the error term in the nonlabor income function was estimated to be .315. The results for the wife's earnings equation are given in Table 2. In that they are only intermediate, the discussion of them will be brief.

As a description of the earnings function, the results are about what one has come to expect from the many studies of the labor supply of wives: an increase in the husband's wage causes a decrease in hours, although if there are several children this is no longer the case; an increase in the own wage increases hours; an increase in the number of children decreases hours. One would, however, expect the coefficient on Y_2 to be negative as that is the response to nonlabor income. More will be said about this later. As implicit estimates of the parameters of the utility function the results are not so good. For example, the consistent estimates of e_1 and e_2 are both negative. They are supposed to be the adult equivalents in consumption of children aged zero to five and of children aged six to fifteen. One explanation of this is that most of the utility function parameters have large asymptotic standard errors. The asymptotic variances are given by

$$v[f(z)] = \frac{\partial f'}{\partial z} v(z) \frac{\partial f}{\partial z}$$

where z is the vector of estimated earnings function coefficients and f is the function that gives the utility function estimates. For example

$$\hat{\theta}_1 = \frac{-.503}{.452} = 1.11 \text{ with estimated standard error of } 2.42.$$

Table 2

Wife's Earnings Equation

Variable	Estimated Coefficient	Estimated Standard Error
W_h	-.133	.025
W_hK_1	.089	.048
W_hK_2	.050	.022
W_hA	.063	.023
W_w	1.497	.172
W_wK_1	.002	.345
W_wK_2	.213	.148
W_wA	-.139	.091
Y_1	.452	.309
Y_1K_1	-.503	.787
Y_1K_2	-.827	.341
Y_1A	-.066	.131
Y_2	.083	.018
σ_1^2	1.635	.174
σ_2^2	.263	.025
σ_{12}	-.595	.056
$\sigma_{\nu_2}^2$	1.076	.118

$Y_1 = p/(1-t)$

$Y_2 = (\text{nonlabor income} + \text{tax grant})/(1-t)$

Of more interest are the variance estimates because they are not often made in this way. σ_1^2 and σ_2^2 are the estimated variances in the maximum hours of the husband and the wife respectively. According to these results there is much greater variation in tastes for work among husbands than among wives. It should be emphasized that this is not the result of only using the part of the sample with working wives because the conditional likelihood function accounted for the sample selection given that the distributions are normal. The covariance between the tastes for work of the husband and the wife is negative and large: it implies a correlation coefficient of -.91. This is strong evidence in support of the kind of marriage sorting suggested by Becker: he gives the condition for negative sorting that $\frac{\partial^2 v}{\partial b_1 \partial b_2}$ be negative, where v is the indirect utility function and b_1 and b_2 are characteristics of the husband and wife. This is the case in the indirect utility function based on the Stone-Geary utility function.

4. Estimation of the Earnings Function of the Husband

By using some of the estimates of the coefficients in the wife's earnings function the earnings function of the husband can be estimated by fairly simple nonlinear methods. This is possible because estimates of the probability that the wife works will be available, so that the effects of sample truncation can be considered to be known. As will be seen, the nonlinearity of the problem arises because it is necessary to make nonlinear constraints on the coefficients in the equation.

The husband's earnings function may be written as

$$e_1 = w_h'\tilde{\alpha}_1 + w_w'\tilde{\alpha}_2 + \hat{y}'\tilde{\beta} + u_1 + (y-\hat{y})'\tilde{\beta}$$

where $\tilde{\alpha}' = (\tilde{\alpha}_1' \tilde{\alpha}_2')$ and $\tilde{\beta}$ are functions of the parameters of the utility function, and $u_1 =$

$$\frac{w_h'\tilde{\alpha}_1}{\xi_1} z_1 + \frac{w_w'\tilde{\alpha}_2}{\xi_2} z_2 + \nu_1 \quad .$$

$$E(e_1|e_2>0) = w'\tilde{\alpha} + (\pi x)'\tilde{\beta} + \rho\tau_1 m \left(\frac{w'\tilde{\alpha} + (\pi x)'\beta}{\tau^2}\right)$$

where $\tau_1 = v(u_1 + v'\tilde{\beta})$ and ρ is the correlation coefficient between the residual in the husband's earnings equation and the residual in the wife's earnings function. That is,

$$\rho = {}^{\tau_{12}}/\tau_1\tau_2 \quad \text{where} \quad \tau_{12} = \text{cov}(u_1 + v'\tilde{\beta}, \quad u_2 + v'\beta).$$

The last part of the expectation accounts for the truncation. This form shows that in the estimation of labor supply functions based on family utility maximization, the truncation of the wife's earnings function (or labor supply function) should be taken into account when estimating the supply function of the husband. Only if ρ is zero can consistent estimates of the supply function be obtained when the truncation is ignored. In this system ρ is generally not zero. Except for the special case in which there is no variation in tastes for leisure, ρ is zero only if the utility of leisure of the husband and wife are both zero.

From the estimation of the wife's earnings function many of the parameters that enter τ_1 and ρ, and the entire argument of m may be considered to be known. As shown in the Appendix, the estimation reduces to a nonlinear problem in three parameters which takes the following form:

$$\min_{B_1,\ B_2,\ \sigma^2_{\nu_1}} \quad SSR(B_1,\ B_2,\ \sigma^2_{\nu_1})$$

where B_1 and B_2 are parameters of the utility function and $\sigma^2_{\nu_1} = v(\nu_1)$; $SSR = \Sigma(e-\hat{e})^2$ and $\hat{e}$ is found from the regression of $e - f_1(B_1,\ B_2)$ on $w, \hat{y}$ and $f_2(B_1,\ B_1,\ \sigma^2_{\nu_1})$; f_1 and f_2 are known functions of B_1, B_2 and $\sigma^2_{\nu_1}$. The iterations consist, therefore, of a search over B_1, B_2 and $\sigma^2_{\nu_1}$, and at each step in the search least-squares residuals are found conditional on the values of B_1, B_2 and $\sigma^2_{\nu_1}$. In this step the heteroscedasticity of $\varepsilon_1 = u_1 + v'\tilde{\beta} - E(u_1 + v'\tilde{\beta} \mid e_2>0)$ is ignored. The estimators are still consistent.

The only parameters estimated in this step that will be used in later calculations are B_1 and $\sigma^2_{\nu_1}$. B_1 was estimated to be $-.140$, which implies that an increase in nonlabor income will lead to an increase in the husband's hours. As was the case with B_2 in the wife's equation, a possible cause of this anomalous result will be discussed later. $\sigma^2_{\nu_1}$, the variation attributed to maximization errors and errors of observation in the husband's earnings equation, was estimated to be 1.107, again a substantial value.

5. Simultaneous Estimation of the Wife's and Husband's Earnings Functions

From the separate estimates of the earnings functions of the husband and of the wife consistent estimates of all the parameters have been obtained. No cross-equation constraints were imposed, however, so that there are multiple estimates of many of the parameters of the utility function. In this step the cross-equation constraints are imposed to estimate the parameters that influence the means of the earnings function in the complete sample while using the previous estimates of those parameters and the variance parameters to account for the truncation and heteroscedasticity. Being able to take those effects to be known is an important empirical simplification. Of course, the estimators will not be as efficient as the full-scale maximum likelihood estimators because the estimation does not take account that the parameter estimates that correct for truncation and heteroscedasticity should be the same as the estimates produced by this step. The full-scale maximum likelihood problem is not at all easy to solve numerically: the solution of the likelihood function describing the wife's earnings function required substantial programmer and computation time, and it is much simpler than the complete maximum likelihood problem.

The wife's and husband's earnings functions are:

$$e_1 = w'\tilde{\alpha} + \hat{y}'\beta + p_1 + \varepsilon_1$$

$$e_2 = w'\alpha + \hat{y}'\beta + p_2 + \varepsilon_2$$

where $p_1 = \tau_{12}\ m(\frac{w'\alpha + \hat{y}'\beta}{\tau_2}) / \tau_2$

and $p_2 = \tau_2 m(\frac{w'\alpha + \hat{y}'\beta}{\tau_2})$. Therefore $E(\varepsilon_1|e_2>0) = 0$ and $E(\varepsilon_2|e_2>0) = 0$.

$$V\begin{bmatrix}\varepsilon_1 & | & e_2>0\\ \varepsilon_2 & & \end{bmatrix} = \begin{bmatrix}\tau_1^2(\rho^2\tau + 1-\rho^2) & \tau_1\tau_2\rho\tau\\ \tau_1\tau_2\rho\tau & \tau\tau_2^2\end{bmatrix} = \Omega$$

where $\tau = [1 - tm(t) - m(t)^2]$ and $t = \frac{w'\alpha + \hat{y}'\beta}{\tau_2}$. Ω will, of course,

vary from observation to observation. The generalized least squares estimators are

$$\min_{\alpha,\ \tilde{\alpha},\ \beta,\ \tilde{\beta}} \quad \sum_i \begin{bmatrix}e_1 - \hat{e}_1\\ e_2 - \hat{e}_2\end{bmatrix}' \Omega_i^{-1} \begin{bmatrix}e_1 - \hat{e}_1\\ e_2 - \hat{e}_2\end{bmatrix} \quad \text{subject to the}$$

nonlinear constraints $g(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}) = 0$. $\hat{e}_1 = w'\hat{\tilde{\alpha}} + \hat{y}'\hat{\tilde{\beta}} + p_1$ and similarly for $\hat{e}_2$. Ω, p_1, and p_2 have been consistently estimated and are considered to be known. Taken together α, $\tilde{\alpha}$, β and $\tilde{\beta}$ have a total of 26 components; but, there are only 14 underlying parameters from the utility functions so that 12 restrictions are imposed.

The results of the joint estimation are given in Table 3. What I have called the standard errors in that table are at best only suggestive because they do not take into account two facts: part of the systematic part of the earnings functions, p_1 and p_2, are not known but are estimated; the parameters that appear in the systematic part, the α, α^2, β and $\tilde{\beta}$, also appear in the variances. One can develop the formulas for calculating the true asymptotic standard errors; but, the calculations are very complicated, and I have not done it. See Amemiya for a discussion of maximum likelihood when part of the likelihood function has been estimated.

Table 3

Joint Estimation of the Husband's and Wife's Earnings Equations

Parameter	Interpretation	Estimate	"Standard Error"
ξ_1	Mean of husband's max. hrs.	1.233	.031
ξ_2	Mean of wife's max. hrs.	1.687	.025
ξ_a	Minimum goods	-7.817	.841
θ_1	Goods Index: K_1	.612	.124
θ_2	K_2	.150	.057
θ_3	A	.759	.107
μ_{11}	Husband's Hours Index: K_1	-.122	.019
μ_{12}	K_2	.036	.012
μ_{13}	A	-.179	.011
μ_{21}	Wife's Hours Index: K_1	-.145	.012
μ_{22}	K_2	-.142	.008
μ_{23}	A	-.012	.012
B_1	Husband's Marginal Propensity to Consume Leisure	-.135	.008
B_2	Wife's Marginal Propensity to Consume Leisure	.003	.003

Note: K_1 indicates number of children 0-5 years old.
K_2 indicates number of children 6-15 years old.
A indicates number of other adult family members.

Most of the parameter estimates seem reasonable and have a natural interpretation. ξ_1 and ξ_2 are the means of the distributions of maximum hours of work. Perhaps it is surprising that the husband's mean is less than the wife's mean; however, these parameters are supposed to represent tastes for work and, from that point of view, there is no reason to suppose that ξ_1 should be greater than ξ_2. One purpose of the estimation is to discover whether observed differences are due to systematic differences in the exogenous variables or not. Apparently, in these data differences in hours are due to differences in the exogenous variables and in the other parameters. In addition, the variance of the husband's maximum hours is much greater than the variance of the wife's maximum hours (1.635 and .263) so that some husbands have very large maximum hours. For example, about 2.5% of the husbands would have maximum hours greater than 3800, whereas about 2.5% of the wives would have maximum hours greater than 2700.

The θ have a natural interpretation of adult equivalents in goods consumption. Because of the normalization of the θ vector, the unit of measure, the adult equivalent, is a "husband-wife." Notice that without considering data on single-headed households, one cannot estimate per capita equivalents. According to these estimates, the first child between the ages of zero and five has a weight in consumption of .612 of a husband-wife. This seems too large: if one assumed that there were no returns-to-scale in husband-wife households without children, the per capita consumption of each adult would be .5, yet the consumption weight of the first child is .61. This may be partly caused by the strong returns-to-scale imposed by the functional form of the goods index. For example, a second child aged 0-5 has a weight of .23 in consumption, and a third child a weight of .12. Similarly, the consumption weight of the first additional adult is surely too large. All of the composition variables interact in the sense that the additions of children to a family which already has a complicated structure will reduce the effective consumption less than the addition in a family with a simple structure. Not too much weight should be placed on the returns-to-scale as an empirical finding, however, because the functional form imposes this as long as the θ are positive.

The interpretation of the μ's as adult equivalents in hours of work poses conceptual difficulties. As far as their place in the utility function is concerned, they act to change the utility associated with hours of work because they enter the utility function through the variable $-\frac{h}{I}$ where h is hours of work and I is the index. Negative values of μ mean that increasing the number of children increases the disutility associated with a given number of hours of work, and, if leisure is a normal good, this will cause a decrease in hours of work. Of course, because no distinction is made between time spent in home production and time spent at leisure, this change could be caused by an increase in efficiency of time spent in home production. It may be noted that the results on the θ and the μ do not explain why people have children: if all the θ were positive and the μ were negative, having more children would always decrease family utility. This is almost the case with these estimates. Unlike the case with the θ, however, it is not at all clear what reasonable magnitudes of the μ are. For example, adding a young child increases the weight given to an hour of the husband's work by about 12%, and to an hour of the wife's work by about 15%. While these values seem plausible, I have no prior notions about what they might be, nor do there seem to be other estimates in the literature with which to compare them.

The estimate of B_1 is not reasonable. Its being negative implies that the marginal utility of leisure of the husband is negative, and that increases in nonlabor income will cause the husband to work more. I find this completely implausible. In the estimating equation, the identification of B_1 comes from nonlabor income which is probably badly measured both in these data and in other data sets. Other investigators have estimated the marginal utility of leisure of the husband to be negative, so that these results are not completely anomalous in the literature.[16] One can think of at least one reason why the estimate of B_1 is negative: because most of nonlabor income flows from accumulated assets, families with high incomes in past years would tend to have large nonlabor income. The heads of those families would have worked more than average both in past years and in the present year because tastes for work probably change rather slowly. This argument that nonlabor income is endogenous at the individual level would lead to a positive relationship between measured nonlabor income and hours worked if the taste component of work is large compared with the other components. In this model the taste component is quite large; as previously mentioned the variance in maximum hours of husbands is estimated to be about 1600 hours per year. Due to these considerations, I decided to re-estimate the parameters under the assumption that nonlabor income is endogenous. The variable in which nonlabor income appears is

---

[16]See, for example, J. DaVanzo, D. DeTray and D. Greenburg; H. Rosen and R. Quandt; J. Ham; and my "The Estimation of Nonlinear Labor Supply Functions."

$Y_2 = (y+g)/(1-t)$ where y is nonlabor income, g is the tax grant necessary to linearize the curving budget constraint, and t is the marginal income tax rate. The variable Y_2 was originally taken to be endogenous due to the endogeneity of g and t; but, y was taken to be exogenous by allowing it to appear on the right-hand side of the reduced form for Y_2. I made y endogenous by re-estimating the reduced form for Y_2; y was excluded, and a set of explanatory variables for y such as age and geographic information was included. To my surprise, the estimate of B_1 was even more negative, and, therefore, I conclude that the endogeneity of nonlabor income is not the cause of the negative estimates.

6. Conclusion

A number of statistical problems in the estimation of labor supply functions were taken into account and they complicated considerably the estimation. The final results seem generally good; but, one naturally wonders whether the difficult procedures made a difference in the results. Here, therefore, I shall concentrate on the methodological issues. In particular, I shall try to indicate the importance of accounting for the truncation and the heteroscedasticity.

As indicated near the beginning of Section 5, the conditional earnings functions are $e_1 = w'\tilde{\alpha} + \hat{y}'\tilde{\beta} + p_1 + \varepsilon_1$ and $e_2 = w'\alpha + \hat{y}'\beta + p_2 + \varepsilon_2$, and when written in this way the conditional earnings functions are true regression functions in that the expected values of the error terms are zero given the right-hand variables including p_1 and p_2. If p_1 and p_2 are small, ignoring the p's will cause little error in the estimation of the earnings functions. In these data, p_1 has a mean of -.06 with a standard deviation of .33. These statistics compare with a mean and standard deviation of husband's earnings of 6.97 and 3.04. In that there is no constant term in the earnings function, the mean of the p's will have considerable influence on the estimation of the slope parameters of the earnings functions. The mean of p_1 is small; but the standard deviation is not small compared to the standard deviation of the husband's earnings.

p_2 has mean and standard deviation of .20 and .30, which are substantial fractions of the wives' earnings mean and standard deviation of 3.20 and 1.97. Another way to make the comparison is the ratio of p_2 to earnings; this variable has mean and standard deviation of .27 and 1.39.

Apparently the observations with low actual earnings were given high p_2. This is, of course, what the statistical theory of truncated random variables would suggest.

The estimated heteroscedasticity is substantial: in the husband's equation the standard error of the conditional error term has a mean of 2.07 with standard deviation of 1.77, a ratio of .86. This is, for example, much more heteroscedasticity than what was found in the tax and transfer functions in Section 3. In the wife's equation the conditional standard error has mean of 1.48 with standard deviation of .751. Again, this is considerable heteroscedasticity.

The heteroscedasticity and p_2 are substantial enough that one would think accounting for them would change the estimates of the parameters. To provide a comparison I estimated the earnings functions only imposing the cross-equation constraint. That is, both the heteroscedasticity and the p's were ignored. The results are given in Table 4. They are quite different from the results of Table 3: for example, the signs on five of the coefficients changed. I would judge that the estimates produced by accounting for truncation and heteroscedasticity are superior: two of the θ, which are supposed to represent adult equivalents in consumption, are negative in Table 4. This makes their interpretation difficult. The marginal utility of the wife's leisure became negative. The means of the maximum hours of work were reduced considerably from what I consider to be small values of Table 3.

I conclude that accounting for truncation and heteroscedasticity makes a difference in the estimates, and that the estimates produced by the theoretically appropriate estimation method are superior to the simpler estimates. Whether this will hold in other bodies of data remains to be seen; however, there is nothing about this problem or these data to suggest that this conclusion will not hold.

Table 4

Joint Estimation of the Husband's and Wife's Earnings Equations; Truncation and Heteroscedasticity Ignored

Parameter	Estimate
ξ_1	.702
ξ_2	.714
ξ_a	-22.845
θ_1	.090
θ_2	-.094
θ_3	-.084
μ_{11}	-.108
μ_{12}	.254
μ_{13}	.172
μ_{21}	-.326
μ_{22}	-.141
μ_{23}	.013
B_1	-.130
B_2	-.060

7. Appendix

A.1 Data and Sample Selective

The data are from the 1967 Survey of Economic Opportunity. See Hall for a description of the data. The variables were generated in the following way:

Annual earnings in 1966: reported directly. For husbands or wives who were unemployed in 1966 earnings were adjusted to account for the unemployment. The theoretical left-hand variable is desired earnings, and the reported weeks unemployed were interpreted as weeks during which desired earnings were the same as actual earnings during the weeks employed. Desired annual earnings are actual earnings plus desired earnings while unemployed.

Wage rate: gross labor earnings in the Survey week of March 1967 divided by hours worked in that week.

Marginal tax rate: calculated from the tax tables assuming standard deductions.

Nonlabor income: actual reported income from assets plus inputed returns to assets and liabilities. See Hall.

Price: assigned one of 20 values according to geographical information in the SEO. The geographical information includes which of the 12 largest SMSA's, which of four CPS areas, and information on whether the residence was in an urban or rural area.

The sample was selected to include husband-wife families headed by a white. Neither husband nor wife was self-employed and neither had health difficulties that influenced the amount or kind of work done. The husband was between the ages of 16 and 64, and there were no other family members over age 64. Families that received welfare income were excluded. Families in which the husband did not have an observed wage rate or in which the husband had no earnings were excluded. One additional observation was excluded; the wife was recorded to have had a wage rate of $120.

A.2 Wage and Wage Squared Functions

Each of these functions had 37 categorical right-hand variables. There were variables indicating age, variables indicating education, variables indicating geographical location, variables indicating union status, and variables indicating whether the observation was from the self-weighting part of the SEO or not. The error terms were estimated to have standard errors of 1.16 and 20. 87 and covariance of 20.17. The only use to which the wage functions were put is in the estimation of the tax functions and the variance functions for the tax functions; they did not enter directly the estimation of the earnings functions because the earnings functions were only estimated over the part of the sample with observed wage rates.

A.3 Husband's Earnings Function

The husband's earnings function can be written as

$$e_1 = w_1'\tilde{\alpha}_1 + w_2'\tilde{\alpha}_2 + \hat{Y}'\tilde{\beta} + u_1 + v'\tilde{\beta}$$

where $$u_1 = \frac{w_1'\tilde{\alpha}_1}{\xi_1} z_1 + \frac{w_2'\tilde{\alpha}_2}{\xi_2} z_2 + \nu_1,$$

and z_1 and z_2 are bivariate normal with zero mean and variance-covariance matrix $\begin{bmatrix} \sigma_1{}^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2{}^2 \end{bmatrix}$. ν_1 is normal $(0, \sigma^2_{\nu_1})$ independent of the z's.

The wife's earnings function is

$$e_2 = w_1{}'\alpha_1 + w_2{}'\alpha_2 + \hat{Y}'\beta + u_2 + v'\beta$$

where $u_2 = \frac{w_1{}'\alpha_1}{\xi_1} z_1 + \frac{w_2{}'\alpha_2}{\xi_2} z_2 + \nu_2$. Then

$$E(e_1|e_2>0) = w_1{}'\tilde{\alpha}_1 + w_2{}'\tilde{\alpha}_2 + \hat{Y}'\tilde{\beta} + \tau_{12}\, m(\frac{w'\alpha + (\pi x)'\beta}{\tau_2})/\tau_2 \ ,$$

where $\tau_{12} = \text{cov}(u_1 + v'\tilde{\beta},\ u_2 + v'\beta)$. α, β, π, and τ_2 have been estimated; however, τ_{12} is not a parameter but a function of other parameters and of the data for each observation.

$$\tau_{12} = \text{cov}(u_1, u_2) + \text{cov}(u_1, v'\beta) + \text{cov}(v'\tilde{\beta}, u_2) + \text{cov}(\tilde{\beta}'v, v'\beta).$$

From the assumption on (z_1, z_2), cov (u_1, u_2) can be calculated as

$$k_1x_1 + k_2x_2 \quad \text{where} \quad k_1 = (B_1 - 1)/B_2,\ k_2 = B1/(B_2 - 1)$$

and x_1 and x_2 are known functions of the data and previously estimated parameters. As in the wife's equation u_1 and $v'\beta$ are assumed to have a constant correlation coefficient, ρ_1; $v(u_1)$ can be written as

$k_1{}^2x_3 + k_1k_2x_4 + k_2{}^2x_5 + \sigma_{\nu_1}^2$ where x_3, x_4 and x_5 depend on previously estimated parameters and the data; $\sigma_{\nu_1}^2$ is an unknown parameter to be estimated; $v(v'\beta)$ has been consistently estimated from the wife's earnings function. Therefore, $\mathrm{cov}(u_1, v'\beta)$ can be written as

$$\rho_1 \; ((k_1^2 x_3 + k_1 k_2 x_4 + k_2^2 x_5 + \sigma_{\nu_1}^2) \; x_6)^{1/2}$$

where $x_6 = v(v'\beta)$. Because $\tilde{\beta} = k\beta$ where $k = B_1/B_2$;

$$\mathrm{cov}(v'\tilde{\beta}, u_2) = k \; \mathrm{cov}(v'\beta_1, u_2) \text{ and } \mathrm{cov}(\tilde{\beta}'v, v'\beta) = k \; v(v'\beta).$$

$\mathrm{cov}(v'\beta, u_2)$ and $v(v'\beta)$ have been consistently estimated from the wife's earnings function. All of these facts may be used to write τ_{12} as

$$k_1 x_1 + k_2 x_2 + \rho_1((k_1^2 x_3 + k_1 k_2 x_4 + k_2^2 x_5 + \sigma_{\nu_1}^2) \; x_6)^{1/2} + kx_7$$

where

$$x_7 = \mathrm{cov}(v'\beta, u_2) + v(v'\beta).$$

In τ_{12} there are three unknown parameters, B_1, ρ_1 and $\sigma_{\nu_1}^2$; however, in the estimation I did not impose the value of B_2 obtained from the wife's earnings equation because the estimated variance of $1/B_2$ is very large.

Because $m(\frac{w'\alpha + (x\pi)'\beta}{\tau_2})/\ \tau_2$ in the husband's earnings equation can be calculated, one can write

$$e_1 = w'\tilde{\alpha} + \hat{Y}_1'\tilde{\beta}_1 + c_1(B_1, B_2) + \rho_1 c_2(B_1, B_2, \sigma_{\nu_1}^2) + \varepsilon_1 \quad \text{where}$$

$\varepsilon_1 = u_2 + v'\tilde{\beta} - E(u_1 + v'\tilde{\beta}|e_2>0)$; $\hat{Y}_1$is a vector of the first

four entries of the five-vector $\hat{Y}$, and $\tilde{\beta}_1$ is the corresponding part of the parameter vector $\tilde{\beta}$; $c_1(B_1, B_2) = (k_1x_1 + k_2x_2 + kx_7)m/\tau_2 - B_1\hat{Y}_2$ and $\hat{Y}_2$ in the last entry in $\hat{Y}$; $c_2(B_1, B_2, \sigma_{v_1}^2) = ((k_1{}^2x_3 + k_1k_2x_4 + k_2{}^2x_5 + \sigma_{v_1}^2)x_6)^{1/2}\ m/\tau_2$.

ε_1 is heteroscedastic, but that is ignored in this step. The parameters are estimated by least squares. Given estimates of B_1, B_2 and $\sigma_{v_1}^2$, $\tilde{\alpha}$, $\tilde{\beta}$, and ρ_1 which comprise 13 parameters can be estimated from the linear regression of $e_2 - c_1$ on w, $\hat{Y}'$, and c_2. That is, the problem of minimizing the sum of squared residuals over all the parameters can be decomposed into two parts:

$$\min_{B_1,\ B_2,\ \sigma_{v_1}^2} [\min_{\tilde{\alpha},\tilde{\beta}_1,\rho_1} \Sigma(e_1 - c_1(B_1, B_2) - w'\tilde{\alpha} - \hat{Y}_1'\tilde{\beta}_1 - \rho_1 c_2(B_1,B_2,\sigma_{v_1}^2))^2].$$

The part of the problem inside the braces has a closed-form solution so that the iterative search need only be made over three parameters.

Amemiya, T., "The Specification and Estimation of a Multivariate Logit Model," Technical Report No. 211, IMSSS, Stanford University.

Becker, G., "A Theory of Marriage, Part I," Journal of Political Economy, July/August, 1973.

Brown, A. and A. Deaton, "Models of Consumer Behavior: A Survey," Economic Journal, December 1972.

DaVanzo, J., D.N. DeTray and D.H. Greenburg, "The Sensitivity of Male Labor Supply Estimates to Choice of Assumptions," Review of Economics and Statistics, 58(1976), pp. 313-25.

Gronau, R., "Wage Comparisons--A Selectivity Bias," Journal of Political Economy, November/December 1974.

Hall, R.E., "Wages, Income, and Hours of Work in the U.S. Labor Force," in Income Maintenance and Labor Supply, G. Cain and H. Watts, eds., Chicago: Rand McNally, 1973.

Ham, J.C., "Rationing and the Supply of Labor: An Econometric Approach," Princeton Industrial Relations Section Working Paper No. 103, revised November 1977.

Hanoch, G., "Hours and Weeks in the Theory of Labor Supply," RAND Publication R-1787, August 1976.

Heckman, J., "Shadow Prices, Market Wages, and Labor Supply," Econometrica, July 1974.

Hurd, M., "The Estimation of Nonlinear Labor Supply Functions with Taxes from a Truncated Sample," Technical Report No. 217, IMSSS, Stanford University.

Hurd, M., "Estimation in Truncated Samples when there is Heteroscedasticity," Working Paper 85, IMSSS, Stanford University.

Muellbauer, J., "Testing the Barten Model of Household Composition Effects and the Cost of Children," Economic Journal, September 1977.

Pollak, R.A. and T.J. Wales, "Maximum Likelihood Estimates of the Linear Expenditure System," Econometrica, October 1969.

Rosen, H., "Taxes in a Labor Supply Model with Joint Wage-Hour Determination," Econometrica, May 1976.

Rosen, H. and R.E. Quandt, "Estimation of a Disequilibrium Aggregate Labor Market," Princeton Industrial Relations Section Working Paper No. 96, June 1977.