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BANK CAPITAL ADEQUACY, DEPOSIT INSURANCE AND SECURITY VALUES (Part I)

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Abstract

This paper provides a formal setting for the analysis of the capital adequacy of an institution with deposits insured by a third party. An insured depositor has a claim against the institution and a contingent claim against the insurer. This paper analyzes the effect of the riskiness of the asset mix and the relative amount of deposits and capital on the potential liability of the insurer. It shows that an increase in asset risk, holding value constant, increases the value of equity and raises the potential liability of the insurer.

BANK CAPITAL ADEQUACY, DEPOSIT INSURANCE AND SECURITY VALUES

(Part I)

W. F. Sharpe*

Introduction

Since the first owner of a gold depository discovered that profits could be made by lending some of the gold deposited for safekeeping, there has been a concern for the "capital adequacy" of depository institutions. The idea is simple enough. If the value of an institution's assets may decline in the future, its deposits will generally be safer, the larger the current value of assets in relation to the value of deposits. Defining capital as the difference between assets and deposits, the larger the ratio of capital to assets (or the ratio of capital to deposits) the safer the deposits. At some level capital will be "adequate" -- i.e. the deposits will be "safe enough".

In most countries depository institutions are regulated and examined frequently by regulatory authorities, and much of this effort is directed toward insuring capital adequacy, broadly construed.

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However, the concept of capital adequacy is generally left undefined, making it impossible to specify an explicit criterion by which one can judge whether capital is adequate or not.

This paper provides a formal setting for the analysis of the capital adequacy of an institution with deposits insured by a third party. We emphasize the case in which the insurer charges a fixed premium per dollar of deposits, since this is the policy of Federal insurance agencies in the United States. However, most of the analysis is applicable to cases in which insurance premiums vary with deposit risk. Much of the analysis can also be used to examine cases in which deposits are uninsured, although we will not consider such situations in detail.

To avoid circumlocution, we will refer to the depository institution as a bank, but the analysis applies as well to savings and loan companies and other depository institutions. Similarly, we will refer to the insurer as the FDIC (Federal Deposit Insurance Corporation), although the analysis applies as well to the Federal Savings and Loan Insurance Corporation and similar agencies.

This paper is designed to serve as an introductory analysis in a book reporting the results of a number of studies of capital adequacy undertaken by the National Bureau of Economic Research for the National Science Foundation.

Although much of the material is not new, the paper does bring together a number of aspects that have previously been discussed separately and attempts to provide a unified framework for dealing with the issues involved. In addition, new results are obtained and old ones given a more substantial theoretical basis.

The Value of the Insurer's Liability

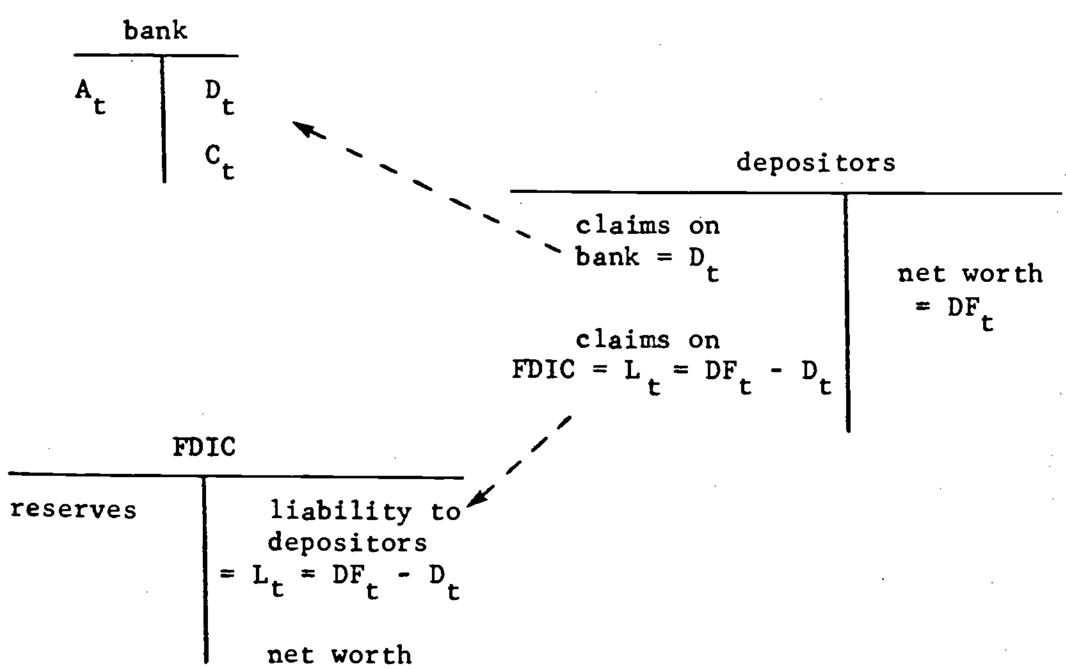
A highly simplified view of the balance sheet of a bank at time t is the following:

bank

Asset = A_t	Deposits = D_t
	Capital = C_t

All amounts are economic values -- the prices that the assets (A_t) or claims on assets (D_t, C_t) would sell for in a free market. Throughout this paper we will assume that values are calculated in this manner and that we are dealing with economic balance sheets, not traditional (accounting) balance sheets.

If there is any risk that the bank might not pay its depositors' claims in full and on time, the economic value of such claims will be less than it would be if there were no such risk. Define DF_t as the amount the deposit claims would be worth if they were default-free. An insured depositor has, in effect, two claims: one on the bank and another on the FDIC. One way to portray the situation is the following:



The depositors consider their claims default-free, with a corresponding value of DF_t. Since the bank may in fact default, its liability to the depositors is only worth D_t. The difference $L_t = DF_t - D_t$, is the present value of the FDIC's Liability.

Another way to portray the situation is the following:

bank		FDIC		depositors	
A_t	D_t ← - - - -	D_t	DF_t ← - - - - -	DF_t	net worth = DF_t
	C_t	reserves	net worth		

To avoid a negative net worth, *ex ante*, the FDIC should charge a premium that will bring in reserves equal to the present value of its liability. Conversely, if the premium is pre-determined, the FDIC should require that the value of the deposit claims (D_t) differs from the default-free value (DF_t) by no more than the premium.

Assume that for the relevant period the insurance premium is ρDF_t. Then the required condition is:

$$\rho DF_t + D_t \geq DF_t$$

or:

$$\frac{DF_t - D_t}{DF_t} \leq \rho$$

If this condition is met, capital is adequate; if not, capital is inadequate. As we will see, the ratio on the left is a function of capital coverage and asset and liability risk. The determination of a bank's capital adequacy thus requires both an assessment of the economic values of all assets and liabilities (including intangible assets such as the value of a charter, monopoly power and/or superior management and options such as acceptances, lines of credit, etc.) and the estimation of all relevant risks.

The second depiction of the relationships among the three parties is particularly useful in one respect: it highlights the fact that the FDIC has the major interest in monitoring and policing the behavior of the bank, since it must bear the consequences of any default.

In the United States there is both explicit and implicit deposit insurance. The FDIC insures only some deposit claims; excluded are foreign deposits, claims owned by other banks (most of the "Federal funds") and portions of deposits above $40,000 per private account and above $100,000 per government account. However, the Federal Reserve System often provides a kind of de facto insurance for its member banks by furnishing liquidity to a troubled bank so that uninsured depositors can be paid off before the bank is actually closed. Moreover, the FDIC tries, wherever possible, to avoid actually closing a bank; arranging instead for another bank to assume all the deposit claims. One way or another, almost all deposits are insured.

The cost of such insurance is the explicit FDIC premium -- a percentage of (virtually) all deposits, including those nominally uninsured -- plus at least part of the interest forgone on reserves required to be held at a Federal Reserve bank by members of the Federal Reserve system.

We will ignore these complexities, assuming that all deposits are insured and, where relevant, that the premium is a fixed amount (ρ) per dollar of deposits (measured at their default-free values). In fact, this is quite an accurate characterization of the actual situation in the United States.

A One-period Case

We deal here with an extremely simple case in which there is only one relevant period. This makes it possible to focus attention on asset default risk. More complex cases in which there is also interest rate risk will be analyzed in a subsequent paper.

A bank issues a certificate of deposit (CD) which promises a payment of P_1 dollars one year hence. The CD is "sold" for D_o dollars (thus D_o is the current value of the bank's deposits). In addition, the bank issues common stock for which it receives C_o dollars (C_o is thus the current value of the bank's capital). The total amount is invested in an asset mix with a current value of $A_o (= D_o + C_o)$, and the balance sheet is:

assets = A_o	deposits = D_o
	capital = C_o

At the end of one year, the assets will have a value of $\tilde{A}_1 = (1+\tilde{r}_{01})A_o$, where $\tilde{r}_{01}$ is the rate of return on the asset mix between time zero and time 1 and tildes indicate variables whose actual values are uncertain ex ante. If $\tilde{A}_1$ exceeds P_1, the CD holders will be paid in full and the stockholders will retain the difference $(\tilde{A}_1 - P_1)$. Otherwise, the CD holders will receive all that is available $(\tilde{A}_1)$ and the stockholders will receive nothing.

Two decision variables are of interest: (1) the relative amounts of deposits and capital and (2) the riskiness of the asset mix. Both affect the value of the deposits.

We will use two different approaches to analyze this case. The first relies on a probabilistic analysis and stochastic dominance arguments but is somewhat incomplete due to the absence of a general equilibrium model of capital asset prices. The second approach does not suffer from this drawback, since it does utilize a general equilibrium model (the time-state preference paradigm). It also suggests a simple yet potentially valuable empirical relationship between deposit risk and changes in capital value.

A Probabilistic Approach

Given a probability distribution for $\tilde{r}_{01}$ and an initial asset value A_o, the cumulative distribution of $\tilde{A}_1$ is determined. It is shown by curve OXY in Figure 1. Given the nature of the certificates of deposit, the cumulative distribution of $\tilde{D}_1$, the amount paid to the depositors, is shown by curve OXQZ in Figure 1. The expected payment, $E(D_1)$ is a weighted average of the likely values, using the probabilities as weights. The expected value can never exceed P_1, and will generally be less.

What will be the effect of a ceteris paribus increase in assets relative to deposits? A_o will be larger relative to P_1 and the cumulative distribution of $\tilde{A}_1$ will have moved to the right, increasing the probability that P_1 will be paid and decreasing the probabilities associated with smaller payments. As shown in Figure 2, the cumulative distribution of D_1 will shift from OXQZ to OX'QZ, increasing the expected payment from $E(D_1)$ to $E(D_1')$.

If assets are very large relative to deposits and risk is small enough, it may be virtually certain that deposits will be paid in full and on time. In such a situation the expected value of the payment will equal the promised amount (i.e. $E(\tilde{D}_1) = P_1$) and the deposits will be default-free. Let π_{ot} represent the market rate of interest at time zero on default-free securities

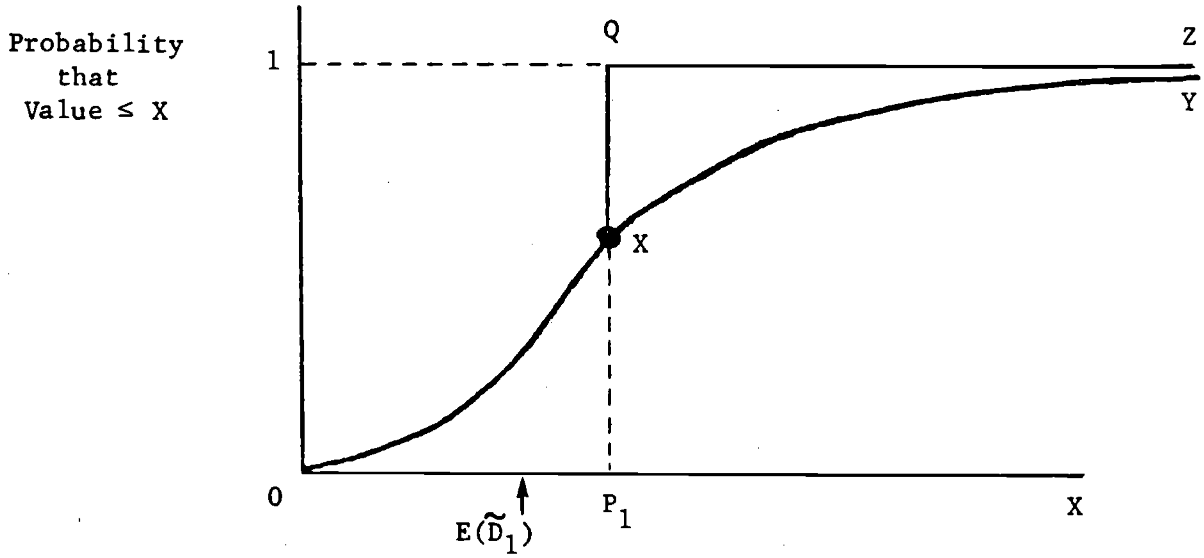


Figure 1

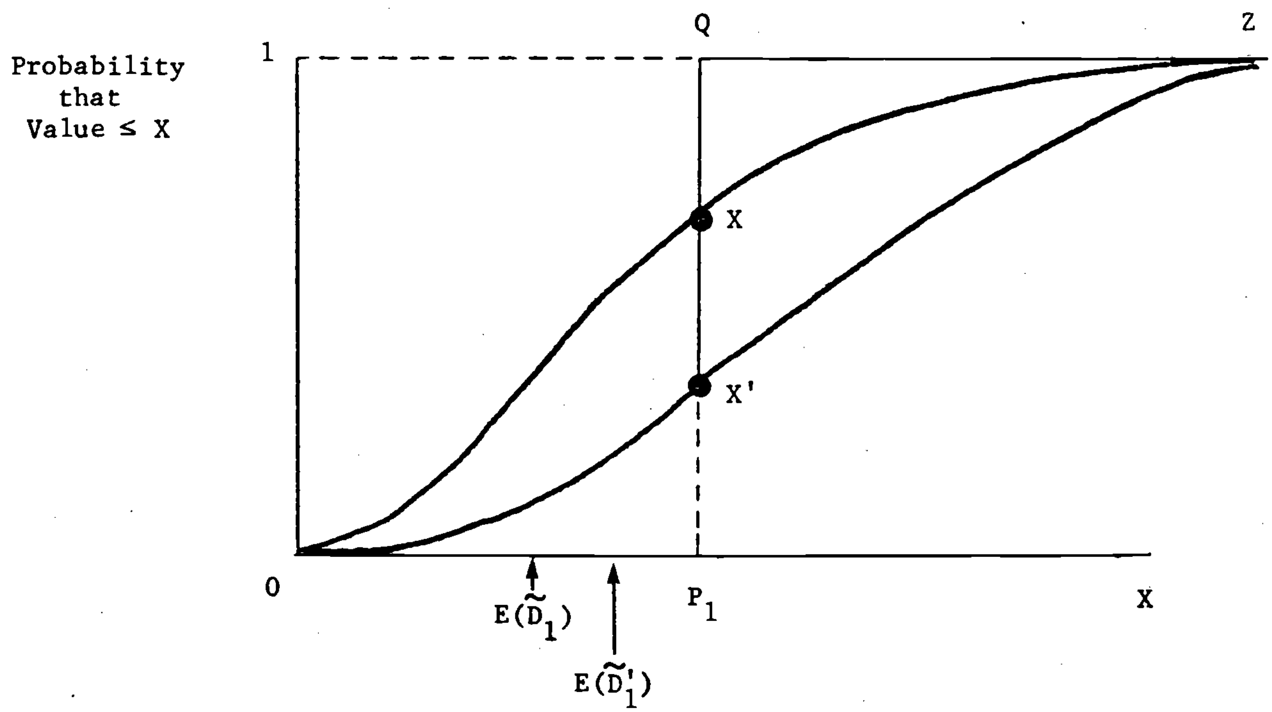


Figure 2

maturing at time t. Then the present value of a certificate of deposit promising to pay P_t t periods hence will be:

$$DF_o = \frac{P_t}{(1 + \pi_{ot})^t}$$

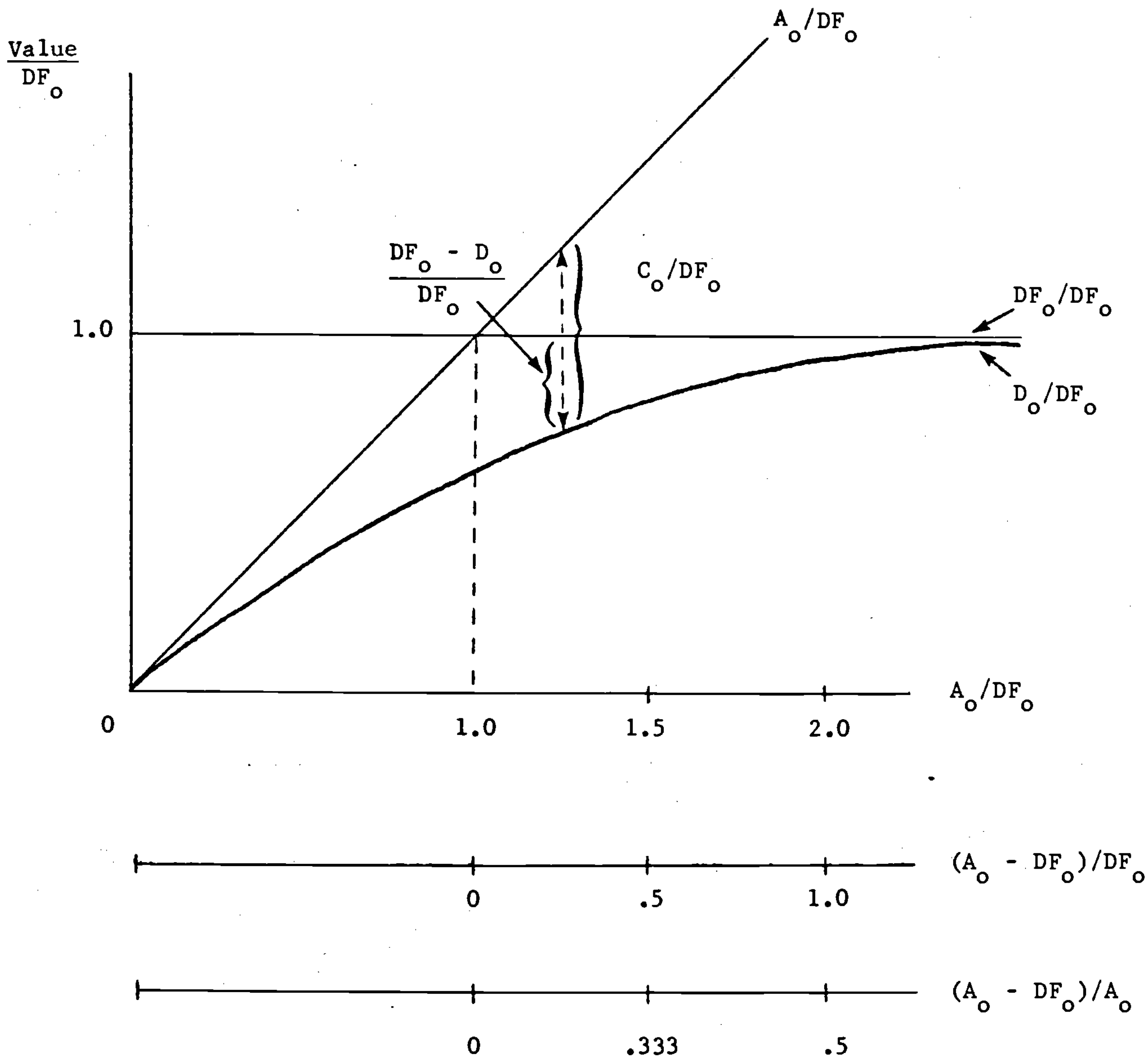
Deposits that are risky will be worth less than their default-free value for two reasons. First, their expected payments will be less than promised. Second, the expected return required by investors on such securities may be greater than that required for default-free investments. In general, the greater the difference between $E(\tilde{D}_1)$ and P_1, the greater will be the difference between D_o (the true value of the deposits) and DF_o (their default-free value).

This is illustrated in Figure 3. For generality all values are expressed as proportions of the initial default-free value of the deposits. The primary scale for the horizontal axis is the ratio of assets to the default-free value of deposits but monotonic transformations can be used to obtain scales for capital-deposit and capital-asset ratios, if the "book value of capital" -- the amount obtained by subtracting the <u>default-free</u> value of deposits from the economic value of assets -- is utilized:

$$\frac{A_o - DF_o}{DF_o} = \frac{A_o}{DF_o} - 1$$

$$\frac{A_o - DF_o}{A_o} = 1 - \frac{1}{A_o/D_o}$$

As shown in Figure 3, <u>ceteris paribus</u>, the greater the amount of assets covering deposits, the smaller will be the difference between the actual value



Value / DF_o
A_o/DF_o
DF_o - D_o / DF_o
C_o/DF_o
DF_o/DF_o
D_o/DF_o
1.0
0
1.0
1.5
2.0
A_o/DF_o
(A_o - DF_o)/DF_o
0
.5
1.0
(A_o - DF_o)/A_o
0
.333
.5

Figure 3

of the deposits and the default-free value. Of course the balance sheet must balance, since the sum of the claims on a set of assets is worth neither more nor less than the assets. Thus C_o must equal $A_o - D_o$, and C_o/DF_o must equal $(A_o - D_o)/DF_o$, as shown. The distance between the curve and the horizontal line is also of interest -- it is the value of the FDIC liability per unit of deposits $\left(\frac{DF_o - D_o}{DF_o}\right)$; if this is less than the premium (ρ), capital is adequate; if not, it is not.

What would be the effect of a <u>ceteris paribus</u> increase in the riskiness of the asset mix? The answer depends on the specification of the <u>ceteris paribus</u> conditions. Most relevant for present purposes is a change in which the value of the asset mix does not change while the "spread" of end-of-period asset values increases. We will term this a <u>value-preserving spread</u>.

Denoting the new situation by primes, we have (by assumption):

$$A'_o = A_o$$

and thus:

$$C'_o + D'_o = C_o + D_o$$

Figure 4 shows the situation before and after the change. The new cumulative distribution of $\tilde{A}_1$ is assumed to cross the old distribution at only one point (A_1^*). Thus the increase in "spread" can be considered unambiguous.*

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* The point at which the distributions cross need not be the mean of the initial distribution. For example, increasing the spread, holding the mean constant may well decrease value; if so, the new distribution must have a higher mean in order to keep value constant. A plausible situation would thus have the new distribution cross the old one below the p = .50 level. This contrasts with the more familiar construct of a <u>mean-preserving spread</u>. It corresponds more closely to the notion of a <u>utility-preserving spread</u>, but to apply the latter concept one would have to invoke a social utility function or assume a limited degree of variation in individuals' utility curves. The concept of a value-preserving spread is more general and leads to economically (and empirically) more interesting implications.

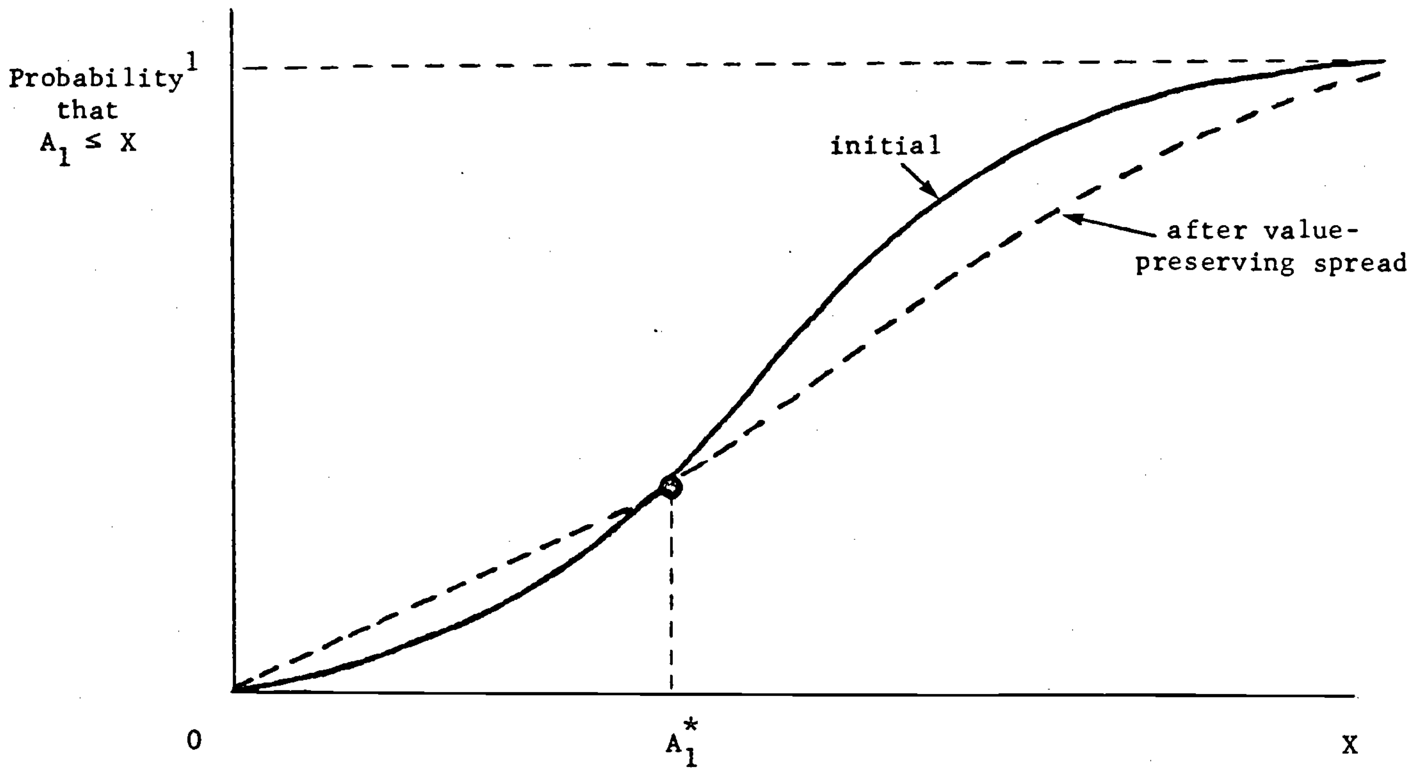
Probability that $A_1 \leq X$
1
initial
after value-preserving spread
0
A_1^*
X

Figure 4

Two cases need to be considered. First, assume that $P_1 < A_1^*$. As shown in Figure 5, when asset risk increases, the cumulative distribution D_1 shifts from curve OXQZ to curve OX'QZ. The change clearly makes depositors worse off, for the probability that $\tilde{D}_1 \leq X$ is greater or equal to the former value for all X. Thus D_o will be smaller and C_o larger as a result of the increase in asset risk.

Now consider the other possible case -- i.e. one in which $P_1 \geq A_1^*$. Figure 6 shows the cumulative distribution of $\tilde{C}_1$ -- the amount paid to shareholders -- before and after the change. Clearly the shift from curve OP_1XZ to curve $OP_1X'Z$ makes equity holders better off, for the probability that $\tilde{C}_1 \leq X$ is less than or equal to its former value for all X. Thus C_o will be larger and D_o smaller as a result of the increase in asset risk.

Both cases lead to the same conclusion. An increase in asset risk, holding value constant, generally increases the value of equity and decreases the value of deposit claims. The riskier the asset mix, the lower will be the D_o/DF_o curve in Figure 3. For any given ratio of assets to the default-free value of deposits, a larger portion of value will be represented by capital (C_o) and a smaller portion by deposits (D_o) as the risk of the assets increases.

Recall that the FDIC liability per dollar (L_t) equals $\frac{DF_o - D_o}{DF_o}$. Other things equal, the greater the asset coverage (A_o/DF_o) and the smaller the asset risk, the safer the deposits and the smaller this ratio.

<u>A Complete Market Approach</u>

The very existence of financial intermediaries provides evidence that transactions and information cost money and that some individuals do not find it desirable to make financial arrangements directly with one another. However,

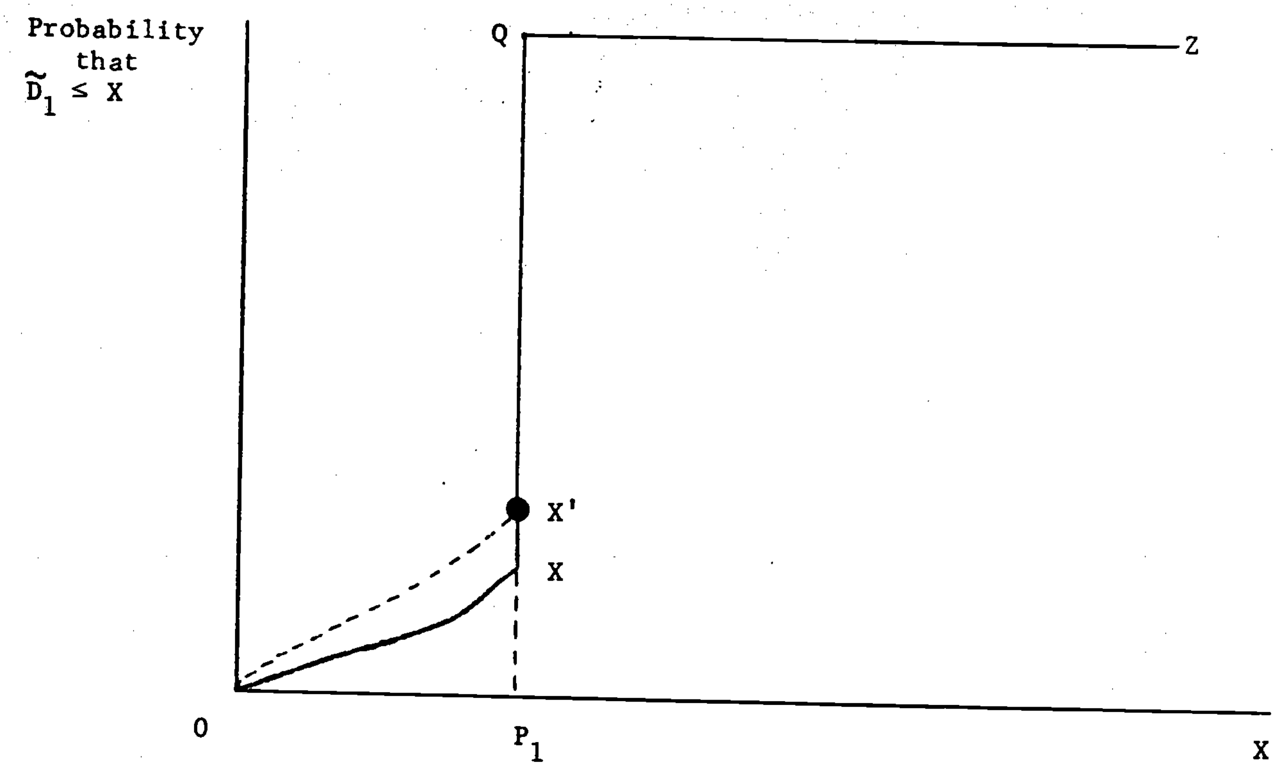

Figure 5

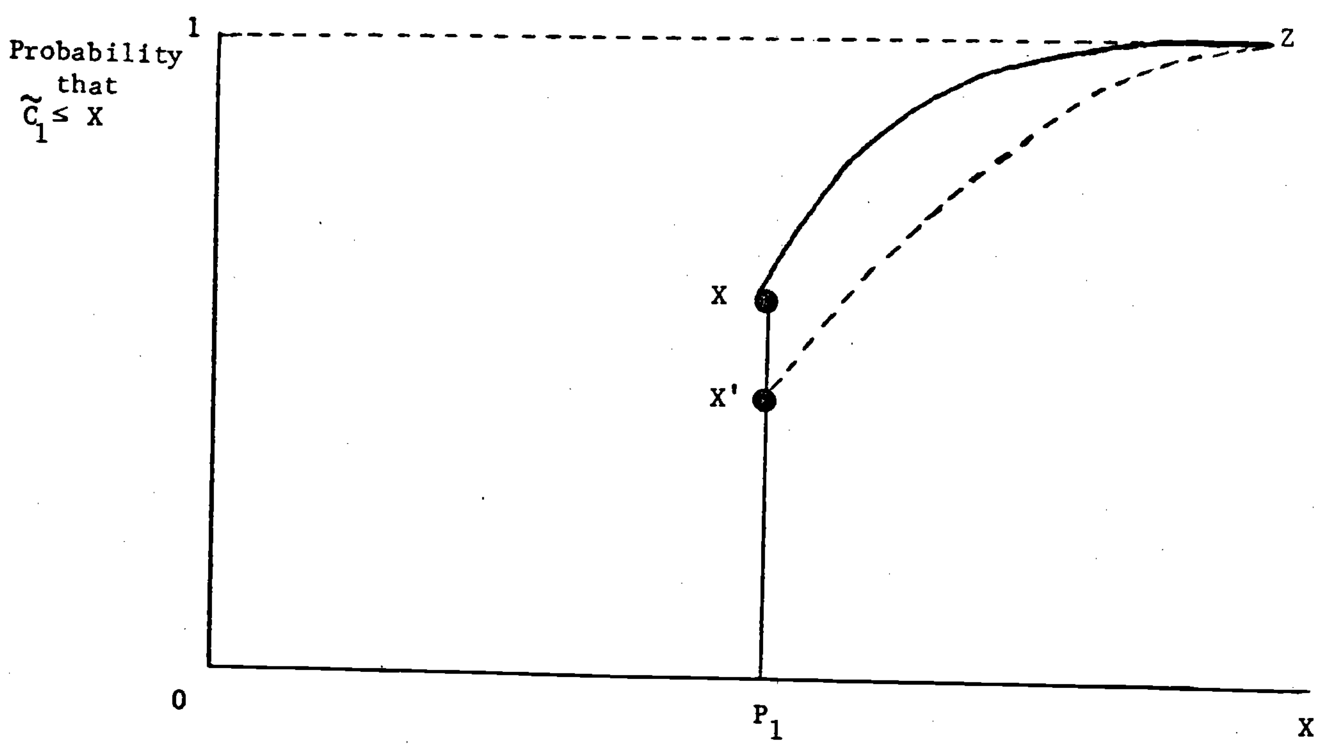

Figure 6

many of the qualitative conclusions obtained by analyzing a transactions- and information-cost-free market may apply as well to real (competitive) financial markets.

Assume that there are S possible states of the world one year hence, and that the return on a bank's assets from time zero to time 1 in state s is r_{o1s}. Then the payment to depositors in state s will be:

$$D_{1s} = \min \left[A_o(1+r_{o1s}),\ P_1 \right]$$

and the payment to stockholders will be:

$$C_{1s} = A_o(1+r_{o1s}) - \min \left[A_o(1+r_{o1s}),\ P_1 \right]$$

If, by buying and selling existing securities, an investor can obtain any desired proportions of payments in different states, the financial market is said to be <u>complete</u>. Equilibrium in such a market is characterized by a series of implicit or explicit prices for state-contingent claims -- prices which are the same whether one wishes to purchase or sell such claims (since transactions costs are assumed to be zero).

Now, let:

P_{o1s} = the price in time zero certain dollars of a default-free promise to receive $1 if and only if state s occurs one year hence

Then the pure interest rate π_{o1} is defined by:

$$\sum_{s=1}^{S} P_{o1s} = \frac{1}{1 + \pi_{o1}} \qquad (1)$$

and:

$$A_o = \sum_{s=1}^{S} P_{ols} A_{1s} = \sum_{s=1}^{S} P_{ols}(1+r_{ols})A_o$$

or: $$\sum_{s=1}^{S} P_{ols}(1+r_{ols}) = 1 \qquad (2)$$

In such a market, the value of the bank's CD would be:

$$D_o = \sum_{s=1}^{S} P_{ols} \left\{\min\left[A_o(1+r_{ols}), P_1\right]\right\} \qquad (3)$$

and the value of its common stock would be:

$$C_o = \sum_{s=1}^{S} P_{ols} \left\{A_o(1+r_{ols}) - \min\left[A_o(1+r_{ols}), P_1\right]\right\} \qquad (4)$$

Clearly:

$$C_o + D_o = \sum_{s=1}^{S} P_{ols} \left\{A_o(1+r_{ols})\right\} = A_o \qquad (5)$$

Thus an uninsured bank could raise just enough capital to pay the market value for its assets, no matter which mix of deposits and stock it elected to employ. Moreover, each source of capital would be priced appropriately. While this is almost tautological, it does show that in a complete financial market, there is no "optimal" financing mix.*

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* This assumes that no resources are lost in the event of bankruptcy. If this assumption is dropped, with all others maintained, any situation that could lead to bankruptcy in any state would be suboptimal, as shown in John H. Kareken and Neil Wallace "Deposit Insurance and Bank Regulation: A Partial Equilibrium Exposition", Staff Report #15, Research Dept., The Federal Reserve Bank of Minneapolis, January 1977.

Given this formulation, we can show how the information required to draw Figure 4 could be obtained and how the curves would actually appear. Note that:

$$DF_o = \sum_{s=1}^{S} p_{o1s} P_1 \tag{6}$$

and thus:

$$D_o = \sum_{s=1}^{S} p_{o1s} \left\{ \min \left[A_o(1+r_{o1s}),\ P_1 \right] \right\} \leq DF_o \tag{7}$$

Without loss of generality, we will assume that the S states are numbered so that $r_{o1s} \leq r_{o1,s+1}$.

Given A_o and P_1, there will be a set of states 1, ..., K in which the CD's will receive less than promised, and a set of states K+1, ..., S in which they will receive the full amount promised. Moreover, as A_o/P_1 grows, K will decrease (but only at discrete points).

The definition of K insures that:

$$(1+r_{o1k})A_o < P_1 \text{ for } k = 1, \ldots K$$

$$(1+r_{o1k})A_o \geq P_1 \text{ for } k = K+1, \ldots S$$

Thus:

$$D_o = \sum_{s=1}^{K} \left[p_{o1s}(1+r_{o1s})A_o \right] + \sum_{s=K+1}^{S} (p_{o1s}P_1) \tag{8}$$

Now, define:

$$\frac{dD_o}{dA_o} = s_K$$

as the slope of the curve relating D_o to A_o when default occurs in states 1, ..., K. Then:

$$s_K = \sum_{s=1}^{K} \left[p_{ols}(1+r_{ols})\right] \tag{9}$$

And:

(1) s_K is constant for ranges of asset values over which K is unchanged

(2) $s_K \leq 1$ (by (9) and (2))

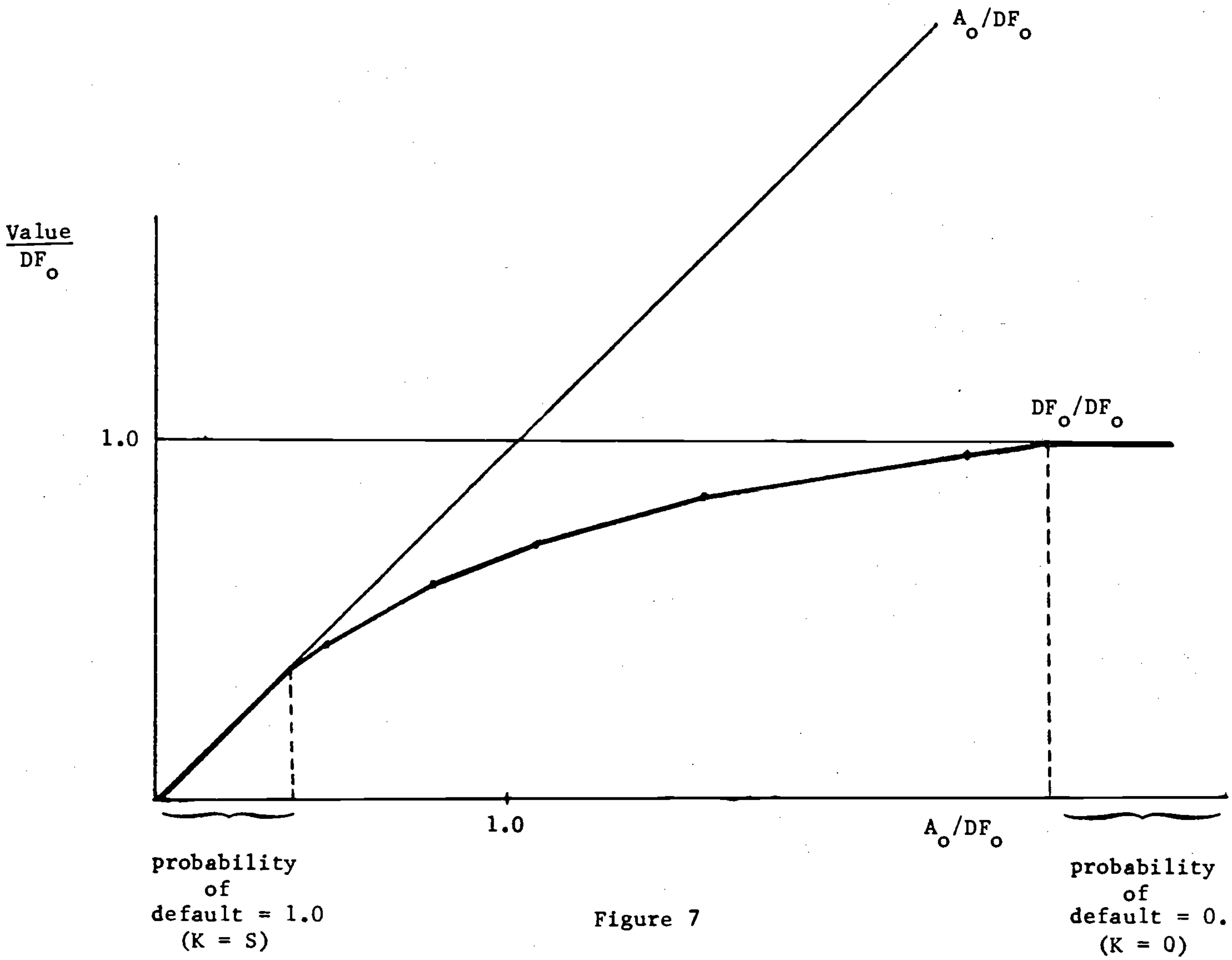
(3) s_K is smaller, the smaller is K

(4) $s_s = 1$

(5) $s_0 = 0$

These relationships imply that the curve relating D_o to A_o (and thus the curve relating D_o/DF_o to A_o/DF_o) is piecewise linear, concave, and bounded by both the 45^o line from the origin and the horizontal line for which $D_o = DF_o$, as illustrated in Figure 7.

The larger the number of states, the larger the number of linear segments, and the closer the piecewise linear curve in Figure 7 will approach the smooth curve in Figure 3.



A_o/DF_o
Value / DF_o
1.0
DF_o/DF_o
1.0
A_o/DF_o
probability of default = 1.0 (K = S)
probability of default = 0. (K = 0)

Figure 7

Now consider a change in the risk of the asset mix unaccompanied by any change in the default-free value of the deposits. As in the previous section, we wish to keep the value of the assets unchanged while increasing the "spread" of the outcomes in an unambiguous manner. The most straightforward approach is the following. Let primes denote the new values. Then:

$$r'_{o1s} = r_{o1s} + \Delta_s \tag{10}$$

where:

$$\Delta_s \leq 0 \text{ for } s = 1, \ldots, s^*$$

$$\Delta_s \geq 0 \text{ for } s = s^* + 1, \ldots, S$$

and:

$$\sum_{s=1}^{S} p_{o1s} \Delta_s = 0 \tag{11}$$

Note that these relationships imply that:

$$\sum_{s=1}^{k} (p_{o1s} \Delta_s) \leq 0 \quad \text{for all } K \tag{12}$$

Now, consider a spread small enough so that default still occurs in the same states (1, ..., K). Then:

$$D'_o = \sum_{s=1}^{K} \left[p_{o1s}(1+r'_{o1s})A_o\right] + \sum_{s=K+1}^{S} \left[p_{o1s}P_1\right]$$

$$= \sum_{s=1}^{K} \left[p_{o1s}(1+r_{o1s} + \Delta_s)A_o\right] + \sum_{s=K+1}^{S} \left[p_{o1s}P_1\right]$$

$$= \sum_{s=1}^{K} \left[p_{o1s}(1+r_{o1s})A_o\right] + \sum_{s=K+1}^{S} \left[p_{o1s}P_1\right] + \sum_{s=1}^{K} \left[p_{o1s}\, \Delta_s\, A_o\right]$$

$$= D_o + \left(\sum_{s=1}^{K} p_{o1s} \Delta_s\right) A_o \tag{13}$$

But, from (12), $D_o' \leq D_o$ and such an increase in asset risk will generally decrease the value of the deposit claims.

Deposit Risk and Change in Capital Value

Formula (13) can also be used to show an empirically useful relationship between the effect of a shift in asset risk and the initial riskiness of the deposits.

The change in deposit value is:

$$\Delta D_o = D_o' - D_o = \sum_{s=1}^{K} (p_{o1s}\, \Delta_s) A_o \tag{14}$$

Given a vector Δ_s, the parenthesized expression is related to K as shown in Figure 8. To the left of some s*, the function is monotonic downward; to the right it is monotonic upward. Recall that K is the number of states in which default occurs. Unless this is large ($K > s^*$), the following relationship holds:

> The larger the initial risk of the deposits (number of states in which default will occur), the larger the decline in deposit value due to an increase in asset risk.

Recall, however, that $D_o + C_o = A_o$. Since a value-preserving spread leaves A_o unchanged, $\Delta C_o = -\Delta D_o$. The relationship between ΔC_o and K (shown in Figure 9) is thus the mirror image of that shown in Figure 8. Unless the number of states in which default occurs is large ($K > s^*$):

> The larger the initial risk of the deposits, the greater the increase in capital value due to an increase in asset risk unaccompanied by a change in asset value.

Within limits, it may be possible to measure the effect on the economic (market) value of capital of a change in asset risk, holding asset value constant. The greater the magnitude of the "risk shift" in value from the FDIC to a bank's stockholders, the greater the initial value of the FDIC liability. The magnitude of this observed responsiveness may thus be an observable surrogate for the value of the FDIC liability. Subsequent studies will attempt to exploit this relationship using both time-series and cross-sectional data.

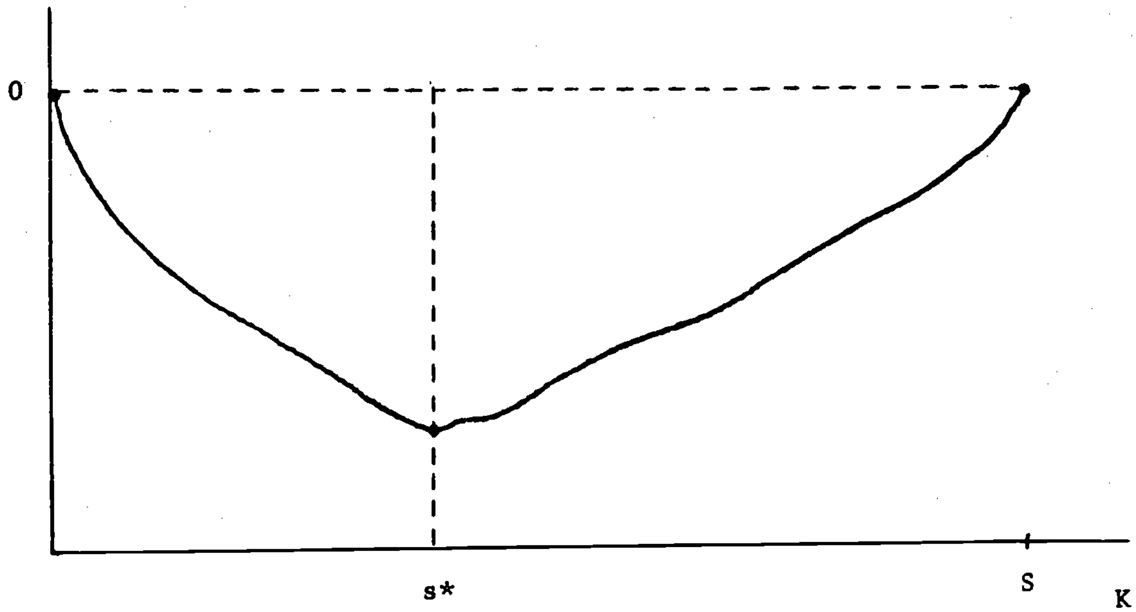


Figure 8

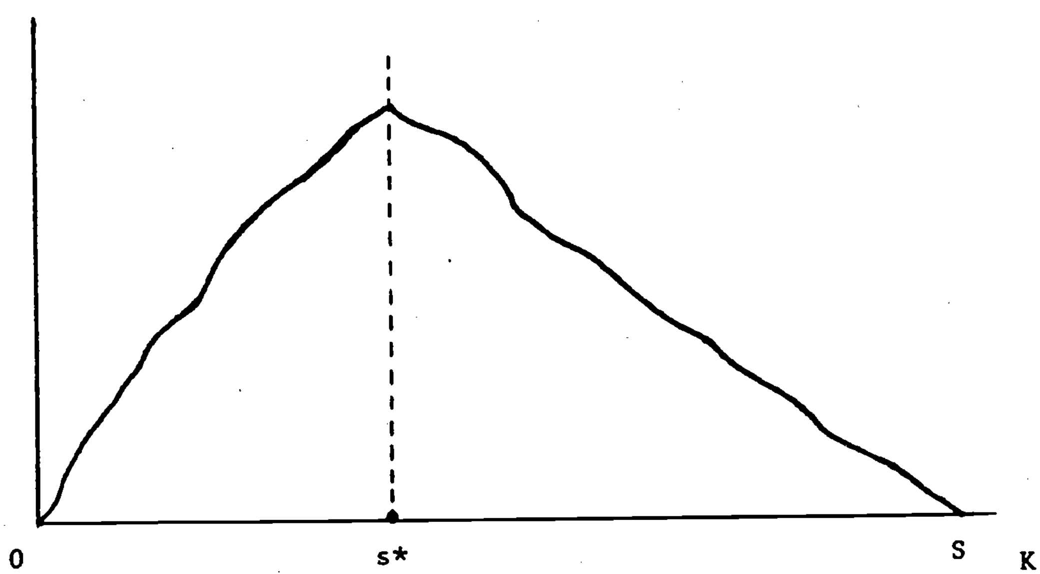


Figure 9