Models of long-term interest rate determination*

Systematic analysis of the forces of demand and supply can provide a superior framework for forecasting interest rates.

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The determination of prices by the market-clearing nexus of demand and supply is perhaps the most fundamental concept in economics. Moreover, well developed markets for publicly traded securities in fact meet the idealized requirements underlying the market-clearing model more closely than do most product and labor markets. In the U.S. markets for corporate bonds and equities, for example, there are large numbers of securities investors (demanders) and securities issuers (suppliers), with even the largest still relatively small in comparison with the total market. In the U.S. Government securities market also, the number and size distribution of investors readily suggests almost a textbook market situation. Furthermore, these markets even have an equivalent of the mythical Walrasian auctioneer, in the form of underwriters and dealers, to make sure that the market actually clears. Hence the application of the demand-supply concept is especially appropriate in the context of financial asset prices and yields.

Nevertheless, economists' empirical models of the determination of long-term interest rates have traditionally side-stepped the explicit demand-supply apparatus and instead related long-term yields directly to short-term yields and other influences. Most recently, however, several researchers have turned to an explicit demand-supply framework to model long-term interest rates.[1]

This paper summarizes some recent work in which we have modeled long-term interest rate determination in an explicit demand-supply context, using multi-equation structural models and directly contrasts such models with unrestricted reduced-form models. Wholly apart from questions of disaggregation and institutional detail, the explicitly structural nature of demand-supply models necessitates additional theoretical constructs beyond those required by unrestricted reduced-form models. Some of these conceptual inputs are already available from established portfolio theory, and others represent objects of current or prospective research. Experience to date with structural models of long-term interest rate determination suggests, however, that the exploitation of the richer theoretical framework yields not only insights about portfolio behavior but, very likely, improved interest rate models as well.

We first discuss in further detail the distinction between structural and unrestricted reduced-form models of interest rate determination, emphasizing the importance of the portfolio theory underlying the explicit structural demand and supply relations. We then examine the portfolio theoretic constructs that we have either adapted or developed for this purpose. Next, we survey the results of our empirical implementation of structural models of interest rate determination in the U.S. corporate and Government bond markets. The final section explicitly compares the structural model for the corporate bond market to

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* We are grateful to David Garman for helpful suggestions, and to the National Science Foundation, the National Bureau of Economic Research, and the Alfred P. Sloan Foundation for research support. The opinions expressed here are solely our own; they do not necessarily represent the views of the Council of Economic Advisers or the Federal Reserve Bank of Kansas City.

[1] Footnotes appear at the end of the article.

corresponding unrestricted reduced-form models and then briefly summarizes the paper's principal conclusions.

THE CONCEPT OF THE STRUCTURAL MODEL

Since the concept of price determination by the market-clearing intersection of demand and supply is so central to the analysis of economic behavior, it seems at first only natural to approach the determination of financial asset prices and yields from an explicit demand-supply perspective. The total market demand for any given asset, A^D, is presumably some function of the asset's yield, r, and of other factors (such as yields on competing assets, variances, and covariances, etc.),

$$A^D = f^D(\ldots, r, \ldots), \qquad f^{D'} \geq 0 \qquad (1)$$

while the total market supply of the asset is an analogous function[2]

$$A^S = f^S(\ldots, r, \ldots), \qquad f^{S'} \leq 0. \qquad (2)$$

The requirement of market clearing,

$$A^D = A^S, \qquad (3)$$

closes the system and permits the model to determine not only the asset quantity $A(= A^D = A^S)$ but also the asset yield r. Any factor that influences the demand for or supply of an asset will also, *ceteris paribus*, influence the asset's yield (and price). Conversely, any factor that influences an asset's yield does so, *ceteris paribus*, only by influencing the relevant market demand or supply (or both).

In addition to its appeal from the general standpoint of economic theory, there are two further reasons why the explicit demand-supply perspective seems particularly appropriate for modeling asset prices and yields. First, the highly efficient markets for many actively traded financial assets should be cases for which, in comparison with many product and factor markets, the assumption of market clearing as in (3) requires relatively little sacrifice of realism. Second, a long tradition of economic analysis of portfolio behavior has provided a rich development of economic theory deriving the pertinent asset demand and supply relations, as in (1) and (2), from the constrained utility-maximizing behavior of market participants under a variety of assumptions about the specifications of the utility function and the nature of the associated constraints.

On the other hand, economists modeling the determination of yields on fixed-income assets of long duration have traditionally related long-term interest rates *directly* to short-term interest rates and/or various other factors assumed to influence the demand for and/or the supply of long-term bonds, using a single *unrestricted* reduced-form equation with the value of the long-term interest rate as the dependent variable. Familiar explanatory variables used in such unrestricted reduced-form long-term interest rate equations include the expected future path of short-term interest rates, usually represented by a distributed lag on past values of a short-term yield; a premium reflecting the differential liquidity of short- and long-term debt instruments; expected future price inflation in product markets, usually represented by a distributed lag on past values of a price inflation index; and an index of monetary policy other than interest rates themselves, such as the recent rate of growth of some monetary or reserve aggregate.

Since the explicit demand-supply model of (1)-(3) also implies an equation for the long-term interest rate, this structural model constitutes a valid alternative to the single-equation unrestricted reduced-form model. The structural model's implied expression for r is itself a reduced-form equation (except for any nonlinearities introduced by functional forms f^D and f^S) that is equivalent to the conventional equation except that it is *restricted* by the underlying structural demand and supply equations.

The two key advantages of the structural model are (a) its ability to use the theory of portfolio behavior to restrict the implied equation for the long-term interest rate, and (b) the facility that it provides for directly investigating hypotheses about portfolio behavior. In return, the structural approach imposes upon the researcher the discipline of explicitly acknowledging that, since financial asset yields (that is, asset prices) are proximately determined in a market in which assets are bought and sold,[3] any factor hypothesized to influence the long-term interest rate can do so only by influencing some issuer's supply of bonds or some investor's demand for bonds, or both. To the extent that (a) expectations of future short-term yields are relevant via substitution effects that enforce the usual term-structure relationship, (b) less-than-infinite elasticities of substitution create "preferred habitats" that render quantity variables relevant, and (c) less-than-infinite adjustment speeds render quantity flow variables relevant as well as quantity stock variables, all these factors affect the determination of long-term interest rates in the structural model only by influencing the portfolio behavior of borrowers and lenders.

Several methodological aspects of the structural approach to modeling long-term interest rate determination that we apply in Section III also merit explicit comment.

First, since the long-term interest rate is clearly a jointly determined variable in the structural model of (1)-(3), it is necessary to use an estimation technique that avoids the inconsistency of ordinary least-squares procedures. Because of the level of disaggregation at which we have modeled the corporate and government bond markets, however, the complete models include too many predetermined variables to permit straightforward implementation of the two-stage least-squares method. To derive consistent estimators, therefore, we have followed the procedure of Brundy and Jorgenson [4], which uses as instrumental variables not only the leading principal components of the full-system set of predetermined variables but also, on an equation-by-equation basis, the single-equation sets of predetermined variables themselves.

Second, the structural approach largely avoids the problem of spurious correlations inherent in unrestricted estimation of flexible distributed lags on past interest rates, which are typically the heart of interest rate models based on the expectations theory of the term structure. In a structural model, any such distributed lags simply appear as arguments of the individual demand and supply equations, where spurious correlation is both less likely and less harmful.

Third, the single-equation unrestricted reduced-form model of long-term interest rate determination will always "fit" historical interest rate data at least as well as the restricted expression. Hence, it is possible that the structural model may buy its key associated advantages — its ability to use and test explicit behavioral hypotheses — at great cost in terms of performance as measured by within-sample fit. The key methodological finding described and shown explicitly below, however, is that the sacrifice of empirical performance required by the structural approach is relatively minor. The portfolio-theoretic restrictions placed on the structural model's demand and supply equations apparently "pay their freight" in terms of enriching the model's ability to draw general behavioral implications without substantially eroding even its within-sample "predictive" performance.

MODELS OF PORTFOLIO BEHAVIOR

Individual investors in financial assets presumably decide simultaneously, within a multi-period horizon, not only how much to save but also in what form to hold their savings. Since in general the individual's lifetime consumption stream will depend on the interaction of the amount saved and the form of saving, it would be sub-optimal to separate these two decisions except under special restrictions.

As is well known, however, the explicit analysis of a model with such complications as a multi-period horizon and simultaneous saving and portfolio decisions, not to mention transactions costs and other relevant factors, leads in general to specifications of asset demand equations that are not empirically tractable and that may even obscure the principal determinants of portfolio behavior. Our investigation of investors' demands for financial assets therefore focuses on the more limited context of the allocation, within a one-period horizon, of existing portfolio wealth. Mossin [22] and Fama [5], for example, have shown that there are plausible circumstances — in particular, constant relative risk aversion, as we assume immediately below — under which individuals' observed multi-period behavior is analytically equivalent to single-period behavior. In addition, this framework of analysis is especially appropriate for institutional investors, such as pension funds and insurance companies, since the assumption of an approximately given saving flow is valid for such institutions in at least the short and medium runs.

In the context of this framework, we represent investors' portfolio behavior by a model that determines separately the desired long-run equilibrium portfolio allocation and the short-run adjustment toward that equilibrium allocation.[4]

DESIRED PORTFOLIO ALLOCATION. An investor's desired portfolio allocation is best determined using the theory of expected utility maximization. This conceptual apparatus is especially advantageous in the portfolio context in that it explicitly represents the risk and return trade-off that is fundamental to portfolio selection behavior. As has become commonplace since the pioneering work of Markowitz [19] and Tobin [28], it is useful to restrict the class of representations of utility to those that reduce to preference orderings in terms of the mean and variance of some outcome resulting from the portfolio allocation (usually end-of-period wealth). As is clear in the original work of Markowitz and Tobin, subsequently bolstered by the work of Samuelson [26] and Tsiang [29], the justification for the mean-variance framework is not any presumption of its precise universal validity but rather its appeal as a tractable approximation useful for a variety of analytical purposes — especially when the amount of risk involved is small relative to initial wealth, as will be the case in either a continuous-time model or a discrete-time model with a small time unit.

We derive mean-variance portfolio behavior from the assumptions that: (a) each investor's utility may be represented by any power (or logarithmic) function of wealth exhibiting constant relative risk aversion; and (b) that investors perceive asset returns to be joint normally (or lognormally) distributed. While any of a number of familiar alternative sets of assumptions is sufficient to derive mean-variance behavior, we use this particular pair of assumptions because, as we have shown in [18],only they yield asset demand functions with the two desirable properties of wealth homogeneity and linearity in expected asset returns — properties often simply assumed *a priori* in work in portfolio behavior in the monetary economics literature.

Solving out the constrained maximization problem that results from this formulation leads, after linearization with respect to first and second moment variables, to the investor's optimal portfolio allocation among N assets in the form

$$\alpha^*_{it} = \frac{A^*_{it}}{W_t} = \sum_k^N \beta_{ik} r^e_{kt} + \sum_k^N \gamma_{ik} v_{kt} + \sum_k^N \sum_{j \neq k}^N \delta_{ikj} c_{kjt} + \pi_i, \qquad i = 1, \ldots, N \qquad (4)$$

where the α^* are desired portfolio shares (in fractions), the A^* are desired asset holdings (in dollars), W is total portfolio wealth, the r^e are expected asset returns, and the v and c are respectively, the variances and covariances associated with those returns. The β, γ, δ, and π are fixed coefficients that satisfy the usual "adding-up" constraints

$$\sum_i \beta_{ik} = \sum_i \gamma_{ik} = 0$$

for all k,

$$\sum_i \delta_{ikj} = 0$$

for all k and j, and

$$\sum_i \pi = 1.$$

Further possible properties of these coefficients, like symmetry, constitute testable hypotheses concerning risk-averse portfolio behavior. In addition, in most cases expected own-yields have unambiguously non-negative coefficients, and coefficients on competing asset yields have signs, dependent on combinations of variances and covariances, that are unknown *a priori*. A further area for related investigation is the formation (and empirical representation) of the expected holding-period yields themselves. Our work described below has focused on all of these issues.

SHORT-RUN PORTFOLIO ADJUSTMENT. In order to translate the implications of desired portfolio allocation of the general form (4) into an operational model of an investor's behavior, some model of portfolio adjustment is necessary. Since transactions costs constitute, in the first instance, the underlying motivation for using a model that admits discrepancies between actual and desired asset holdings, it is worth while to model the implications of transactions costs with some care. Four desirable features of such a model that the standard stock-adjustment model does *not* exhibit are: (a) effects of differential transactions costs between the investor's allocation of a new investable financial flow and the corresponding reallocation of existing asset holdings; (b) dependence of the allocation of new investable financial flows on desired equilibrium asset holdings; (c) effects of new investable financial flows on the reallocation of existing asset holdings; and (d) asymmetric effects from positive and negative new investable financial flows. The portfolio models used by Brainard and Tobin [3], Modigliani [20], and Bosworth and Duesenberry [2] exhibit property (a) but not (b)-(d).

The "optimal marginal adjustment" model, developed by Friedman [9] and implemented in much of our modeling of both the corporate and Government bond markets, exhibits properties (a)-(c). Given an investor's initial (beginning-of-period) wealth and current-period new investable financial flow, the optimal marginal adjustment model expresses the actual change in portfolio holdings during the period in the form

$$\Delta A_{it} = \sum_k^N \theta_{ik} (\alpha^*_{kt} W_{t-1} - A_{k,t-1}) + \alpha^*_{it} \Delta W_t, \qquad i = 1, \ldots, N \qquad (5)$$

where the θ are fixed coefficients satisfying the adding-up constraint

$$\sum_i \theta_{ik} = \bar{\theta}$$

for all k (with $\bar{\theta}$ arbitrary). An intuitive interpretation of this model is that the first term on the right-hand side of (5) represents the reallocation, according to a standard stock-adjustment model, of the investor's previous asset holdings, while the second term represents the allocation of the new investable flow according to the desired proportions α^* determined by (4). In the long run this model converges to the same equilibrium given by the standard stock-adjustment model, but the dynamics of the adjustment in the short run are in general different.

A still more general model of portfolio adjustment, developed by Roley [23] and implemented in his modeling of the U.S. Government bond market, distinguishes between positive and negative reallocations of existing assets, and between positive and negative financial flows, so as to exhibit all four properties (a)-(d) noted above. In addition, for modeling the investment behavior of commercial banks, Roley further generalized this model by disaggregating the new financial flow in terms of individual predetermined balance sheet items.

A FINAL METHODOLOGICAL NOTE. The models that we have used to represent desired equilibrium portfolio allocation (4) and short-run portfolio adjustment (5 or its generalization) describe an investor's demands for all assets (or liabilities), so that each of the derived structural demand equations is implicitly an element of a set of demand equations that satisfy the specified "adding-up" constraints. While there is no inconsistency involved in estimating, for any investor or investor group, the demand equation for only one asset rather than the entire set, in principle a complete model including all investors and all markets (that is, all assets and liabilities) would be preferable, in that it would permit the researcher to examine the implications for other asset demand equations of the presence of a given variable in any one asset demand equation. The construction of such a complete model, however, lies beyond the scope of our research reviewed in this paper.

STRUCTURAL MODELS OF THE CORPORATE AND GOVERNMENT BOND MARKETS

In a series of papers, we have applied the demand-supply approach described above to modeling interest rate determination in the U.S. corporate and Government bond markets.[5]

THE CORPORATE BOND MARKET. The demand side of the corporate bond market model consists of six equations representing the net purchases on corporate bonds by six distinct categories of investors that together hold approximately 94% of all corporate bonds issued in the United States, and the model's supply side consists of two equations representing the net new issues of corporate bonds by two distinct groups of bond issuers that together account for approximately 87% of all U.S. corporate bonds (see Table 1).[6] The specification of each of these bond demand and supply equations combines the linear homogeneous model (4) of the selection of optimal portfolio allocation for given wealth (or, for issuers, given external deficit) and the optimal marginal adjustment model (5) of portfolio adjustment out of equilibrium.

TABLE 1

CORPORATE BONDS OUTSTANDING AS OF YEAR-END 1978

	Amount	Percentage
Total Bonds Outstanding	$422.5 billion	100.0%
Bonds Issued By:		
* Nonfinancial Corporate Businesses	318.3	75.3
* Finance Companies	51.3	12.1
Foreign Issuers	43.2	10.2
Commercial Banks	5.9	1.3
Savings and Loan Associations	2.2	0.5
Real Estate Investment Trusts	1.6	0.4
Bonds Held By:		
* Life Insurance Companies	158.5	37.5
* State and Local Government Retirement Funds	80.2	19.0
* Households	63.2	15.0
* Private Pension Funds	53.8	12.7
* Mutual Savings Banks	21.8	5.2
* Other Insurance Companies	18.2	4.3
Foreign Investors	10.6	2.5
Commercial Banks	7.6	1.8
Mutual Funds	6.2	1.5
Brokers and Dealers	2.3	0.5

Notes:

Source: Board of Governors of the Federal Reserve System.

Groups marked by asterisk are endogenous in the corporate bond market model.

Detail may not add to total because of rounding.

All data are at par value, except for foreign issues and foreign holdings.

The model's ninth equation is a market-clearing equilibrium condition, as in (3), that enables the structural model to determine the interest rate on bonds — which is itself an argument of each structural demand or supply equation. The particular bond interest rate used in this model is the observed new-issue yield on long-term bonds issued by utility companies rated Aa by Moody's Investors Service, Inc. Aa-rated utility bonds provide the greatest continuity, in terms of the frequency of new issues; they are also most representative of new-issue activity in the U.S. corporate bond market. Previous studies of long-term interest rate determination using the unrestricted reduced-form approach have relied on indices of yields on either new issues or seasoned issues, but the new-issue yield is likely to be superior for several reasons including greater trading volume, fewer measurement problems, and absence of any term-coupon bias.

The principal methodological finding of our empirical research to date with the corporate bond market model is that, in addition to the explicit demand-supply model's advantages for purposes of investigating portfolio behavior per se, the model also performs surprisingly well as a within-sample "predictor." Table 2 shows the "fit" statistics for the estimation of the individual demand and supply equations using quarterly data over the 1960:I-1973:IV sample period, as well as for a within-sample dynamic simulation of the full nine-equation model.[7] This simulation is fully dynamic in that, after the first quarter of the simulation period, the solution uses internally generated values for the lagged bond stock variables in all eight equations, as well as internally generated values for the lagged bond interest rate that appears in several equations. (In other words, the stock of bonds held by any investor or owed by any issuer at the beginning of, say, 1970:I equals the exogenously given stock at the beginning of 1960:I plus the sum of 40 quarters of solved values for net purchases or net issues.)

The results summarized in Table 2 for the eight flow demands for and supplies of bonds are broadly consistent with the portfolio model summarized above. The estimated equations explain a large percentage of the variation of the changes of bond holdings and bond outstandings for most of the eight

TABLE 2

SUMMARY STATISTICS FOR THE CORPORATE BOND MARKET MODEL

Variable	Single Equation Results SE	Single Equation Results $\overline{R}^2$	Dynamic Simulation Results ME	Dynamic Simulation Results RMSE
Flow Demands for Bonds by:				
Life Insurance Companies	213	0.80	1.0	181
Other Insurance Companies	66	0.92	0.4	64
Private Pension Funds	198	0.67	1.0	213
State and Local Government Retirement Funds	156	0.83	1.9	156
Mutual Savings Banks	134	0.89	0.9	127
Households	496	0.79	1.8	423
Flow Supplies of Bonds by:				
Nonfinancial Corporate Businesses	311	0.95	13.2	405
Finance Companies	210	0.68	−6.2	197
Own-yield on Corporate Bonds	—	—	−0.01	0.21

Notes:

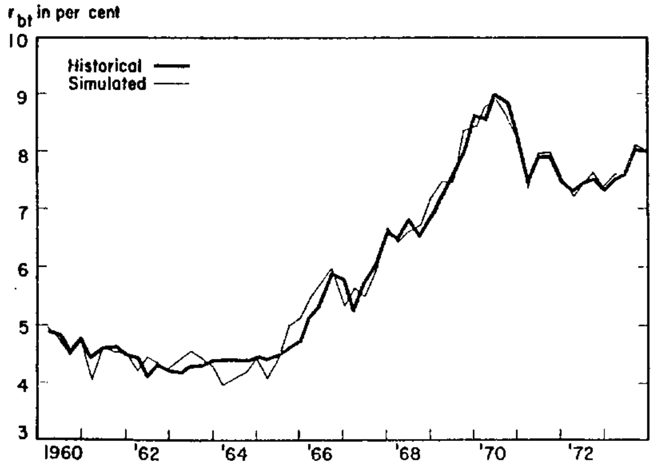
SE = estimated standard error.
$\overline{R}^2$ = coefficient of multiple correlation, adjusted for degrees of freedom.
ME = mean error.
RMSE = root-mean-square error.
All flow variables in millions of dollars; yield variable in percent.

groups.[8] The dynamic simulation results show no significant bias for any of the eight flow variables, and the root-mean-square simulation errors are about in line with the standard errors of the corresponding estimated equations, thereby indicating that the errors made by individual equations have no observable tendency to compound one another.

For the bond yield itself, there is again no bias in the simulation (see also Figure 1). As we explain below in some detail, the 0.21% root-mean-square simulation error compares very favorably with the within-sample fit achieved by other researchers using single-equation unrestricted reduced-form models. This performance seems quite creditable, especially since, as we have already emphasized, the structural model does not include an unrestricted equation estimated directly for the bond yield but instead implies an equation for the bond yield that is restricted by the portfolio-theoretic behavior hypotheses specifying the underlying demand and supply equations.[9]

FIGURE 1

DYNAMIC SIMULATION RESULTS FOR Aa UTILITY NEW ISSUE YIELD



Research thus far with the nine-equation model of interest rate determination in the corporate bond market has also produced a number of interesting substantive findings.

First, our results overall provide support for the optimal marginal adjustment model (5) according to which financial flow variables do influence portfolio behavior. In contrast to previous empirical work, which has largely followed the unrestricted reduced-form approach and has typically found only a minor role at best for such effects, our results from the structural approach based on explicit behavior equations specified according to the optimal marginal adjustment model confirm the importance of "preferred habitat" influences operating through financial flow variables.

Second, while previous researchers have modeled the corporation's bond issuing decision in such a way as to provide no role for substitution effects based on the relative yields of long- and short-term financing, Friedman's bond supply equations in [10,11] indicate that the relative equilibrium levels of long- versus short-term borrowing costs do influence the net amount of bonds issued in precisely the way that familiar substitution effects would suggest.

Third, Friedman's bond supply equation for nonfinancial corporations in particular also supports two additional hypotheses about the influence of risk aversion in determining borrowers' choice of liabilities. Corporations attempt to "match the maturities" of assets and liabilities, in that the desired long-term financing share of a given cumulated external deficit is positively related to their (illiquid) fixed investment in plant and equipment, and they are more willing to bear the exposure of short-term indebtedness if internally generated cash flow is large relative to total indebtedness.

Fourth, our results in [10,12,17] support a direct "Fisher effect" by which bond issuers sell more bonds, and bond investors purchase fewer bonds, as their expectations of price inflation are greater. These results imply a Fisherian "inflation premium" of about ⅔% for each 1% of expected price inflation.

Fifth, work incorporating a structural model of long-term interest rate determination within the MIT-Penn-SSRC econometric model, reported in [16], suggests that fiscal policy may have somewhat stronger effects, and monetary policy somewhat weaker effects, than previous research using that model had indicated.

Finally, the substantive findings of this research bear a number of potential further implications for debt management and regulatory policies that as yet remain largely unexploited. As one example of the possibilities for such research, Friedman [6,11] took advantage of the disaggregation of the corporate bond market model to investigate the implications, for the structure of interest rates, of several proposals for reform of the funding of pension funds. Such reform would, under most proposals, shift investable cash flows away from households, which typically prefer to invest in short-term assets, toward pension institutions, which typically prefer to invest in long-term assets. Since the relevant demand equations of the corporate bond market model represent the differing portfolio preferences of these classes of investors, as well as the dependence of their portfolio behavior on their respective cash flows, partial-equilibrium simulations of the model indicate how this shift of cash flows alters the economy's aggregate asset preference structure and hence the structure of interest rates. The simulation results suggest that familiar pension proposals would, if implemented, cause a sizeable "tilt" in the prevailing yield structure.

In sum, research with the corporate bond market model has been able to exploit the advantages of the structural modeling approach to find empirical evidence on a broad selection of questions about bond investors' and bond issuers' portfolio behavior, in addition to providing a model of long-term interest rate determination.

THE U.S. GOVERNMENT SECURITIES MARKET. Roley [23,24] has modeled interest rate determination in two separate maturity sub-markets of the U.S. Government securities market: short-intermediate-term and long-term. (These two maturity classes serve to illustrate the determination of market yields on two unambiguously disjoint groups of securities; future research will consider the two remaining maturity classes of U.S. Government securities.) The available data for disaggregated U.S. Government securities holdings consist of weighted maturity classes defined in terms of four "definite" maturity areas — within 1 year (short-term), 2-4 years (short-intermediate-term), 6-8 years (long-intermediate-term), and over 12 years (long-term) — with securities in the three borderline maturity areas allocated to the definite classifications according to a weighting scheme so as to avoid the perverse effects that would otherwise occur when large debt issues cross fixed-maturity boundaries.

The demand side of each of these two U.S. Government securities submarket models consists of either ten or nine equations representing the net purchases of securities by distinct categories of investors that together hold approximately 97% of the outstanding U.S. Treasury securities net of Federal Reserve and foreign holdings (see Table 3). The specification of each of these 19 securities demand equations combines the linear homogeneous model (4) of equilibrium portfolio allocation and either the optimal marginal adjustment model (5) or one of its generalizations.

TABLE 3

U.S. GOVERNMENT SECURITIES AS OF YEAR-END 1978

Securities Held By:	Amount	Percentage
Foreign Investors	$137.1 billion	25.5%
Federal Reserve System	110.6	20.5
* Commercial Banks	97.2	18.1
* State and Local Government General Funds	70.4	13.1
* Households	58.8	10.9
* Private Pension Funds	16.3	3.0
* Other Insurance Companies	11.0	2.0
* State and Local Government Retirement Funds	10.1	1.9
* Savings and Loan Associations	5.3	1.0
* Mutual Savings Banks	5.0	0.9
* Life Insurance Companies	4.8	0.9
Security Brokers and Dealers	3.1	0.6
* Nonfinancial Corporate Businesses	2.6	0.5
Investment Companies	1.9	0.4
Sponsored Credit Agencies	1.5	0.3
Money Market Funds	1.5	0.3
Credit Unions	1.3	0.2
Total	538.5	100.0

Notes:

Source: Board of Governors of the Federal Reserve System.
Groups marked by asterisk are endogenous in the U.S. Government securities market model.
Detail may not add to total because of rounding.
Agency issues and non-negotiable savings bonds are excluded.

In each of these two sub-market models, the supply of U.S. Government securities within the maturity class is exogenously determined by fiscal and debt-management policies, Federal Reserve holdings are exogenously determined by monetary policy, and foreign holdings are also exogenously determined. The market-clearing condition, as in (3), requires that domestic private investors absorb the total supply, less Federal Reserve and foreign holdings. In each case this condition enables the model to determine the respective own-rate of interest: the Treasury's published yield series on "three-five year" and on "long-term" U.S. Government securities.

Tables 4 and 5 show, for both of these submarket models, the "fit" statistics for the estimation of the individual demand equations over the 1960:I-

TABLE 4

SUMMARY STATISTICS FOR THE SHORT-INTERMEDIATE-TERM U.S. GOVERNMENT SECURITIES MARKET

	Single Equation Results		Synamic Simulation Results	
Variable	SE	$\bar{R}^2$	ME	RMSE
Flow Demands for Securities by:				
Commercial Banks	712	0.69	−0.4	337
Households	406	0.79	3.0	348
Life Insurance Companies	21	0.82	−0.4	31
Mutual Savings Banks	50	0.69	−0.4	50
Nonfinancial Corporate Businesses	160	0.80	−0.6	206
Other Insurance Companies	56	0.54	−0.2	55
Private Pension Funds	47	0.75	0.2	43
Savings and Loan Associations	161	0.57	−4.3	164
State-Local General Funds	190	0.41	3.4	164
State-Local Retirement Funds	25	0.40	−0.3	26
Own-yield on 3-5 Year Securities	—	—	−0.01	0.34

Notes:

SE = estimated standard error.
$\bar{R}^2$ = coefficient of multiple correlation, adjusted for degrees of freedom.
ME = mean error.
RMSE = root-mean-square error.
All flow variables in millions of dollars; yield variable in percent.

1975:IV sample period, and for within-sample simulations of the two complete models that are fully dynamic in the sense described above in connection with the corporate bond market model. Most of the multiple correlations for the 19 demand equations are comparable to those reported in Table 2 for the corporate bond demands, with only several exceptions across the two maturity classes. On the whole, these equations explain much of the variation of the relatively more volatile demands in the U.S. Government securities market. The individual securities demand equation estimates also provide support for allocation model (4) and several variants of portfolio adjustment model (5).

TABLE 5

SUMMARY STATISTICS FOR THE LONG-TERM U.S. GOVERNMENT SECURITIES MARKET

	Single Equation Results		Dynamic Simulation Results	
Variable	SE	$\bar{R}^2$	ME	RMSE
Flow Demands for Securities by:				
Commercial Banks	79	0.64	0.8	123
Households	167	0.66	0.5	196
Life Insurance Companies	40	0.92	−0.1	39
Mutual Savings Banks	52	0.71	0.4	48
Other Insurance Companies	24	0.65	0.0	25
Private Pension Funds	38	0.80	0.2	37
Savings and Loan Associations	52	0.67	−1.1	63
State-Local General Funds	110	0.50	0.4	96
State-Local Retirement Funds	108	0.63	−0.9	119
Own-yield on Long-term Securities	—	—	−0.02	0.20

Notes:

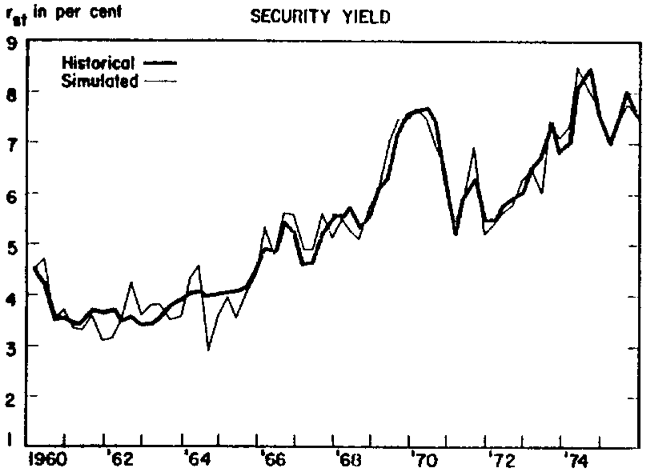
SE = estimated standard error.
$\bar{R}^2$ = coefficient of multiple correlation, adjusted for degrees of freedom.
ME = mean error.
RMSE = root-mean-square error.
All flow variables in millions of dollars; yield variable in percent.

Several further aspects of the estimation results are of particular interest. First, as in the corporate bond demand equations, in each case the own-yield response is significantly greater than zero at a high level of confidence. The magnitudes of these estimated responses appear to be reasonable (with the possible exception of the single largest value for each maturity class), and the short-run response is always positive and always smaller than the equilibrium response. Second — again as in the corporate bond demands, but in contrast to most other estimated asset demand functions — these equations explicitly include variances of asset holding-period yields (here modeled as four- or eight-quarter moving-average variances of observed returns). Based on their statistical significance, variances appear in 15 of the 19 estimated securities demand equations. Third, the results indicate asymmetric effects from positive and negative cash flows in the manner indicated in the fully general portfolio adjustment model. This asymmetry is apparent for commercial banks, mutual savings banks, nonfinancial corporate businesses, and savings and loan associations. Fourth, as discussed in detail in [25], non-yield variables such as demand deposits, time deposits, and loans significantly affect commercial banks' short-run portfolio allocation. In turn, changes in these variables alter the Treasury yield curve due to banks' preferred habitat in short-term securities reflecting liquidity considerations.

The dynamic simulation results for the two sub-market models largely exhibit the favorable properties discussed above in connection with the corporate bond market model. For the own-yields on short-intermediate-term and long-term U.S. Government securities, the root-mean-square simulation errors are 0.34% and 0.20%, respectively (see also Figures 2 and 3). As is to be expected, this measure is larger for the yield on short-intermediate-term securities because of the greater volatility of shorter term yields. Nevertheless, the magnitudes of both of these root-mean-square errors indicate again that the structural modeling methodology is capable of a high degree of within-sample "predictive" accuracy. The results for the dynamic simulations shown in Tables 4 and 5 also show no evidence of bias in the individual demand equations, nor any irregularities in the individual root-mean-square errors. In addition, these

FIGURE 2

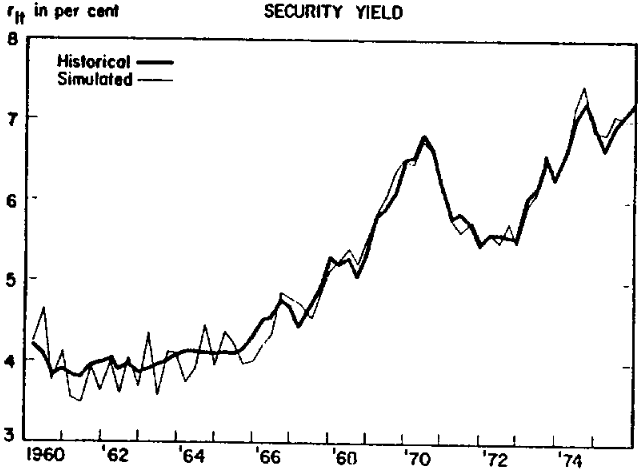
DYNAMIC SIMULATION RESULTS FOR 3-5 YEAR U.S. GOVERNMENT SECURITY YIELD



dynamic simulation results indicate (see again Figures 2 and 3) that both models performed somewhat better in more recent years despite the greater recent volatility of actual market yields. In particular, for the subsample period beginning with 1965:I, the root-mean-square errors for the short-intermediate-term and long-term yields on U.S. Government securities are only 0.30% and 0.14%, respectively. This superior performance during more recent years holds also for the corporate bond market model, and the ability to track well during periods of greater yield volatility is an attractive feature of both of the structural models.

FIGURE 3

DYNAMIC SIMULATION RESULTS FOR LONG-TERM U.S. GOVERNMENT SECURITY YIELD



As is the case for research with the corporate bond market model, therefore, research with structural models of two U.S. Government securities sub-markets has not only provided empirically successful models of interest rate determination but also found evidence on several substantive questions about investors' portfolio behavior, including issues with a wide range of potential policy implications.

AN EXPLICIT COMPARISON OF STRUCTURAL AND REDUCED-FORM MODELS

In order to highlight our finding that the portfolio-theoretic restrictions embodied in the structural models of interest rate determination do not involve a major sacrifice of within-sample predictive ability, we have estimated a number of unrestricted reduced-form equations directly comparable to the structural corporate bond yield model described above.

Table 6 summarizes these comparisons. The table first reports the results of a *non*dynamic simulation of the corporate bond yield using the model in Friedman [9,11]. The root-mean-square error is again 0.21%, as in the dynamic simulation described above, and the simulation errors (processed into a Durbin-Watson statistic, as if they were single-equation residuals) show no significant serial correlation.

TABLE 6

COMPARISON OF STRUCTURAL VERSUS REDUCED-FORM MODELS OF THE CORPORATE BOND YIELD

Structural Model	RMSE	DW
Friedman nine-equation model (nondynamic simulation)	0.21%	2.36
Reduced-Form Models	SE	DW
Feldstein-Eckstein equation (re-estimated)	0.31%	1.40
Modigliani-Shiller equation (re-estimated)	0.20	1.55
Feldstein-Chamberlain equation (re-estimated)	0.42	2.75
Friedman model information set (unweighted denominator)	0.15	2.55
Friedman model information set (constrained denominator)	0.13	2.71

Notes:

RMSE = root-mean-square error.
SE = estimated standard error.
DW = Durbin-Watson statistic.
Yield variable is Aa utility new-issue yield.
Sample period is 1960:I-1973:IV.

Next, the table reports the results of reestimating three familiar unrestricted reduced-form equations for the long-term interest rate, using the Aa utility new-issue yield and the 1960:I-1973:IV sample period as in the corporate bond market model. Feldstein and Eckstein's [7] preferred equation originally had a standard error of only 0.09% for the less volatile Aaa yield over the sample period 1954:I-1969:II, and Modigliani and Shiller's [21] preferred equation originally had a standard error of 0.13% for the Aaa yield over 1955:III-1971:II; but the apparent within-sample superiority of both equations disappeared on reestimation, and there is some evidence of serial correlation. Feldstein and Chamberlain's [6] equation (3.4) originally had a standard error of 0.24% for the Aaa yield over 1954:I-1971:I, but it deteriorated sharply on reestimation.

Finally, since the structural model takes as predetermined a large amount of information not required by these three unrestricted reduced-form equations, Table 6 also reports the results of directly estimating the structural model's implied reduced-form equation for the corporate bond yield. From the optimal marginal adjustment model (5), each structural demand or supply equation for corporate bonds includes the own-yield in the form

$$\Delta A_t^j = \ldots + \beta_1^j \Delta W_t^j r_t + \beta_2^j W_{t-1}^j r_t + \ldots, \qquad j = 1, \ldots, 8 \qquad (6)$$

where β_1^j and β_2^j are coefficients to be estimated and the j superscript indicates the j-th category of market participant. (The estimated $\hat{\beta}_2^j$ are non-zero only for life insurance companies, households, and nonfinancial corporations.) Substituting from (6) into the market-clearing equation

$$\sum_i^8 \Delta A_t^j = 0$$

and solving for the own-rate gives

$$r_t = \frac{1}{\sum_i^8 \beta_1^j \Delta W_t^j + \sum_i^8 \beta_2^j W_{t-1}^j} (\ldots\ldots\ldots\ldots\ldots) \qquad (8)$$

where the right-hand parentheses include the sum of all terms *not* involving the own-yield, from the respective right-hand sides of all eight structural equations.

The first reduced-form equation shown in Table 6 for the full information set used in the structural model deals with the nonlinearity of (8) by dividing each right-hand-side term by the simple unweighted sum

$$\sum_i^8 \Delta W_t^j,$$

thereby imposing the restrictions $\beta_1^j = \bar{\beta}_1$, $\beta_2^j = 0$, all j. The result is a reduced-form equation that slightly outperforms the (fully restricted) structural model (standard error only 0.15%) and that exhibits slightly more serial correlation. The second reduced-form equation shown for the full information set partially exploits the restricted structural model estimates by dividing each right-hand side term by the fraction shown in (8), calculated using the structurally estimated $\hat{\beta}_1^j$ and $\hat{\beta}_2^j$ values. Incorporating this further information — which is unavailable in genuine unrestricted reduced-form estimation — leads to a slight further improvement to a 0.13% standard error and to yet slightly more serial correlation.

It is true, of course, that the unrestricted reduced-form model exhibits a closer within-sample fit than the structural model incorporating all of its portfolio-theoretic restrictions. Nevertheless, the difference for the two models based on the full information set (0.15% standard error versus 0.21% root-mean-square error) hardly seems damaging, especially since obtaining the best possible within-sample tracking performance is not the prime objective of structural interest rate modeling. Furthermore, as Table 6 shows, the structural model in fact shows no within-sample inferiority at all in comparison to reduced-form equations using compact information sets, as are familiar in the interest rate literature.

SUMMARY OF CONCLUSIONS

Structural demand-supply models constitute a valid alternative to the more familiar unrestricted reduced-form models of interest rate determination. The essential difference between the two kinds of models is equivalent to the distinction between restricted and unrestricted estimation. The two major advantages of the structural model that follow from its explicit demand-supply framework are, first, its ability to use the theory of portfolio behavior to constrain the implied equation for interest rate determination and, second, the facility it provides for directly investigating hypotheses about portfolio behavior.

We have implemented structural models of this nature based on explicit hypotheses about portfolio selection, derived from the theory of expected utility maximization under uncertainty and risk aversion, and on a generalization of the theory of portfolio adjustment that specifically allows for effects of differential transactions costs between stock and flow allocations. These models, for the U.S. corporate bond market and for two maturity class sub-markets of the U.S. Government securities market, provide broad support both for the structural modeling approach to interest rate determination and for several of the specific underlying hypotheses about portfolio selection and portfolio adjustment.

In comparison with previous researchers' single-equation unrestricted reduced-form models, these structural models perform surprisingly well as historical "predictors" of interest rates in dynamic simulation tests. In addition, the estimation results for the corporate bond supply equations and the corporate and government bond demand equations provide useful evidence on a wide range of familiar questions about borrowers' and lenders' behavior.

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[1] In addition to our own work cited below, see, for example, Silber [27], Bosworth and Duesenberry [2], Hendershott [18], and Backus et al. [1].

[2] In some contexts, such as the Government securities market, the supply of the asset depends on some different process.

[3] The "proximate determination" of long-term interest rates in the bond market is not inconsistent with the principle of general equilibrium in the asset markets or for the economy as a whole.

[4] The discussion in this section focuses entirely on the behavior of an investor deciding on asset purchases; the models used by Friedman to represent the behavior of borrowers deciding on liability sales are analogous.

[5] References [8-17, 23-25] below represent most of our work to date on this line of research.

[6] The primary data source for the stock and flow quantities used in the model is the Federal Reserve System's flow-of-funds accounts.

[7] The statistics shown in Table 2 are for the six demand equations in [9], the two supply equations [11], and the dynamic simulation in [9].

[8] Moreover, the six demand equations from [9] are preliminary in that they include only limited efforts to capture expectational effects. The corresponding fit statistics ($SE/\bar{R}^2$) from the more fully developed model in [12] are uniformly better: 154/0.89 for life insurance companies, 42/0.96 for other insurance companies, 198/0.63 for private pension funds, 95/0.94 for state-local retirement funds, 97/0.94 for mutual savings banks, and 382/0.86 for households.

[9] The results in [14], which show a root-mean-square error of only 0.26% for an analogous two-equation demand-supply model, make clear that the favorable performance of the structural model does not depend heavily on the degree of disaggregation chosen.

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