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THE CONSUMER EXPENDITURE FUNCTION

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ABSTRACT

DARBY, M. R.--The Consumer Expenditure Function

A consumer expenditure function which integrates pure consumption and household investment in durable goods is formulated and estimated. Because of reduced reliance on the official classification of commodities as durable or nondurable, a considerable increase in ability to explain consumer expenditures results as compared to multiequation models. Further empirical investigation provides strong evidence that: (1) private sector income is significantly better than disposable personal income for explaining consumer expenditures, (2) the M_1 definition of money is similarly superior to both M_2 and M_3 definitions, and (3) the weight of current income in permanent income is about 10% per annum. Data appendix included.

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by

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I. Introduction and Summary

The functional relationship of aggregate consumer expenditures to income and other variables is one of the central elements of macroeconomic dynamics. Yet some time back in the 1950's, consumption theorists seem to have lost interest in this relation and began to concentrate on models of the pure consumption of service flows. This is not surprising perhaps, as explanations of pure consumption could be fairly directly derived from intertemporal choice theory and are sufficient to explain the broad movements of consumer expenditures.[1] But most cyclical variation in consumer expenditures would appear to arise in the adjustments of the stocks of consumer durable and semidurable goods and not in fluctuations in the growth of pure consumption. So macroeconomists should be concerned with a consumer expenditure function integrating the asset adjustment function and the pure consumption function.

The usual approach of those few economists who have concerned themselves with these distinctions has been to formulate a model in which investment in consumers' durable goods is estimated in a separate equation or equations from the determination of pure consumption. Empirically consumption is estimated as consumer expenditures less consumer expenditures on durables plus the estimated rental value of the stock of durable goods.

Such an approach depends critically upon the completeness of the empirical definition of consumer expenditures for durable goods. To the extent that goods which are behaviorally durable are in fact classified as nondurable, the model will be misspecified and omit a portion of the cyclical variation in consumer expenditures. In my restatement of the permanent income theory (1974), it was shown that on the order of half of the behaviorally defined durable goods are classified in the official data as nondurable goods and services.[2] So the standard approach indeed suffers from specification biases.

The most obvious approach is to correct the definition of durable goods so that the two or more equation approach can be directly applied. As a practical matter such a correction is impossible because of both a lack of finely disaggregated data and the generality of durability in a behavioral sense. To take a simple example related to the concept of human capital, surely a vacation is a durable good yielding benefits for many years in the form both of memories and of inflicting slide shows on relatives. A more promising approach followed in this paper is to formulate a model in which the role of specification bias is minimized. As it happens, an integrated consumer expenditure function not only serves this role but also refocuses attention on the basic macroeconomic concept.

The integrated consumer expenditure function is derived in Section II by inverting the standard theoretical definition of pure consumption so that consumer expenditures are defined in terms of pure consumption, household net investment in durable goods, and the yield on the stock of durable goods existing at the beginning of the period. This definition is converted into a consumer expenditure function by substitutions based upon the permanent income theory of pure consumption and a generalized stock adjustment model of household durables investment. Consumer expenditures are determined primarily by permanent income, transitory income, the real money stock, and the stock of consumers' durable goods with the long-term interest rate and relative price of durables playing minor roles because of their effect on stock demands. The model provides expected signs for most of these variables and explicates the relationships among their coefficients.

Section III applies the model to postwar U.S. data with remarkably favorable results. The estimated coefficients do not differ significantly from expectations and are consistent with the secular relation of consumption to saving. The most surprising finding is that the marginal propensity to spend (excess) real money balances is somewhat larger than the marginal propensity to spend current income for a one-year period. The theoretical model is shown to hold up well when disaggregated by use of estimated pure consumption and household durables investment. The explanatory power of the integrated model is considerably better than one based on separate consumption and household durables investment equations.

In Section IV, the consumer expenditure function is used to investigate three outstanding empirical questions: (1) Is disposable personal income or private sector income better at explaining consumer expenditures? (2) Which of the money definitions--M_1, M_2, or M_3--is best at explaining consumer expenditures? (3) What is the weight β of current income in the formation of permanent income? These questions were studied simultaneously by maximum likelihood estimation for each combination of income and money definitions for both quarterly and annual data. The data provided the following answers: The private sector income and M_1 (currency plus demand deposits) money definitions do significantly better than the alternative definitions. The likelihood function is rather flat for values of the β weight between 0 and 20% per annum but falls sharply for higher values of β, so the β weight of 10% per annum previously estimated for a pure consumption model is retained.

Concluding remarks and suggestions for future research are contained in Section V. The Data Appendix makes available to other researchers a considerable investment in constructing private sector income, permanent income, and the stock of household durable goods from the national income accounts as well as monthly M_3 data on the Federal Reserve definition for 1947 through 1958.

II. The Theoretical Model

This section presents an elaboration of the integrated model of consumer expenditures presented in Darby (1975). First a general framework is derived suitable for integrating all three-equation models of pure consumption c_t^f, household investment in durable goods Δd_t, and the (end-of-period) stock of consumers' durable goods d_t. A specific--but empirically quite general--model is then substituted into this framework to obtain the basic equation used in the empirical investigations.

The real stock d_t of consumers goods ("the durables stock") at the end of period t is computed by applying a depreciation rate of δ per period:

$$(1) \qquad d_t = (1 - 0.5\ \delta)\ c_t^d + (1 - \delta)\ d_{t-1},$$

where the coefficient of durable goods expenditures c_t^d adjusts for intraperiod depreciation on gross investment.[3] It follows directly that the net investment in durables Δd_t is

$$(2) \qquad \Delta d_t = (1 - 0.5\ \delta)\ c_t^d - \delta d_{t-1}\ .$$

The usual definition of pure consumption c_t^f is total consumer expenditures less the net investment in durables plus an imputed yield at the rate r per period on the average durables stock for the period:

$$(3) \qquad c_t^f = c_t^x - \Delta d_t + r\, 0.5\, (d_t + d_{t-1})$$

$$c_t^f = c_t^x - (1 - 0.5\, r)\ \Delta d_t + r d_{t-1}$$

Solving for c_t^x shows that consumer expenditures equal pure consumption plus net durables investment (adjusted for intraperiod yield[4]) less the yield on the beginning durables stock:

(4) $$c_t^x = c_t^f + (1 - 0.5r)\,\Delta d_t - rd_{t-1}.$$

Equation (4) is converted from an identity to a theory by substituting behavioral functions in the right hand side. Since the real value d_{t-1} of the durables stock at the beginning of period t is predetermined by past changes in the stock, functions must be specified only for pure consumption c_t^f and household investment in durable goods Δd_t.

For aggregate time-series data, the strict permanent income hypothesis has great theoretical and empirical appeal as an explanation of pure consumption:

(5) $$c_t^f = ky_{Pt}.$$

Pure consumption is assumed to be a constant fraction k of permanent income y_{Pt}. The use of permanent income is preferred to the life-cycle approach for two reasons: (1) Permanent income appears a more accurate method for estimating aggregate wealth (inclusive of human capital) in time series applications.[5] (2) This specification allows further empirical study of the reformulated permanent income theory presented in Darby (1974). Whether the life-cycle approach might in fact produce superior empirical results is an open issue for future research.

The change in the stock of durable goods is of the nature of a portfolio adjustment problem. Households will increase their holdings of durable goods in response to the increase in total assets from normal saving, in order to make up part of any remaining discrepancy between the desired and beginning stocks, in response to unexpected saving due to windfalls (transitory income y_{Tt}), and as a temporary response to disproportionately large money balances:

(6) $$\Delta d_t = (\Delta d_t)^e + \lambda_1(d_t^* - (\Delta d_t)^e - d_{t-1}) + \lambda_2 y_{Tt} + \lambda_3(m_t - m_t^*)$$

The model captures the main elements that are generally supposed to affect changes in the stock of durable goods.[6]

The model is completed by specifying the long-run durables stock demand d_t^*, the planned change in durable goods $(\Delta d_t)^e$, and real money demand m_t^*. Durables stock demand is assumed to be a linear function of permanent income, the relative price of durable goods $\frac{P_{Dt}}{P_{NDt}}$, and the long-term interest rate i_t:

(7) $$d_t^* = \alpha_0 + \alpha_1 y_{Pt} + \alpha_2 \frac{P_{Dt}}{P_{NDt}} + \alpha_3 i_t .$$

The planned change in durable goods through normal saving is approximately proportional to permanent income:

(8) $$(\Delta d_t)^e = \eta y_{Pt} .$$

The demand for real money balances is assumed to be a linear function of permanent income, transitory income,[7] and the long-term interest rate:

(9) $$m_t^* = \gamma_0 + \gamma_1 y_{Pt} + \gamma_2 y_{Tt} + \gamma_3 i_t .$$

Substitution in equation (6) yields the consumers' durable goods investment function

(10) $$\Delta d_t = (\lambda_1\alpha_0 - \lambda_3\gamma_0) + [(1 - \lambda_1)\eta + \lambda_1\alpha_1 - \lambda_3\gamma_1]y_{Pt} + (\lambda_2 - \lambda_3\gamma_2)y_{Tt} + \lambda_3 m_t - \lambda_1 d_{t-1} + \lambda_1\alpha_2 \frac{P_{Dt}}{P_{NDt}} + (\lambda_1\alpha_3 - \lambda_3\gamma_3)i_t .$$

The coefficient of real money balances is unambiguously positive and the coefficients of the lagged real durable goods stock and the relative price of durable goods is unambiguously negative. The other coefficients are of ambiguous sign.

Finally equations (5) and (10) are substituted into equation (4) to obtain the consumer expenditure function:

$$(11) \qquad c_t^x = \beta_0 + \beta_1 y_{Pt} + \beta_2 y_{Tt} + \beta_3 m_t + \beta_4 d_{t-1} + \beta_5 \frac{P_{Dt}}{P_{NDt}} + \beta_6 i_t \ .$$

Where:

$$\beta_0 = (1 - 0.5r)(\lambda_1\alpha_0 - \lambda_3\gamma_0)$$

$$\beta_1 = k + (1 - 0.5r) \ [(1 - \lambda_1)\eta + \lambda_1\alpha_1 - \lambda_3\gamma_1]$$

$$\beta_2 = (1 - 0.5r)(\lambda_2 - \lambda_3\gamma_2)$$

$$\beta_3 = (1 - 0.5r) \ \lambda_3 > 0$$

$$\beta_4 = -r - (1 - 0.5r) \ \lambda_1 < 0$$

$$\beta_5 + (1 - 0.5r) \ \lambda_1\alpha_2 < 0$$

$$\beta_6 = (1 - 0.5r)(\lambda_1\alpha_3 - \lambda_3\gamma_3)$$

Although unambiguous signs are assigned only to β_3, β_4, and β_5, it would be surprising if the direct positive effects of permanent income and transitory income were completely offset by their indirect effects operating through the demand for money. Variations in the magnitudes of $\lambda_3\gamma_0$, $\lambda_3\gamma_1$, $\lambda_3\gamma_2$, and $\lambda_3\gamma_3$ will cause some variation below in the estimates of β_0, β_1, β_2, and β_6 for alternative money definitions.

In sum, equation (11) serves as a reasonably straightforward method of incorporating standard notions about factors influencing pure consumption and household investment in consumers' durable goods into a consumer expenditure function. Alternative routes could be used to derive the same equation with somewhat different interpretations placed on the coefficients, but the current approach seems the most atractive to this author.

III. Estimation of the Model

Basic estimates of the model and a comparison with the multiequation approach are presented in this section. Discussion of some important empirical issues with respect to the computation of permanent income and the definitions of income and money is postponed for fuller consideration in Section IV.

Data Definitions[8]

A major empirical finding of this paper is that data definitions of durable goods, income, and money make a real difference in the ability to explain consumer behavior. So it is necessary to devote particular attention to the precise definitions of data sources used. Some important data series have been constructed and are made available in the Data Appendix for use by others.

Four basic series are available directly:

c_t^x Personal consumption expenditures in constant (1958) dollars (quarterly data at seasonally adjusted quarterly rates (SAQR)).

c_t^d Personal consumption expenditures for durable goods in constant (1958) dollars (quarterly data at SAQR).

m_t Money supply M_1 (average of monthly data) deflated by the implicit price deflator for personal consumption expenditures.

i_t Yield on long-term U.S. government bonds (average of monthly data).

The stock of durable goods at the end of the quarter t is computed according to equation (1) for $\delta = 0.05$, so[9]

$$d_t = 0.975\, c_t^d + 0.95\, d_{t-1}. \qquad (12)$$

Annual regressions use the end of the year (fourth quarter) data extracted from the quarterly estimates.

Two alternative current income measures are compared in Section IV, one corresponding to the accrual of purchasing power and the other to cash receipts. Each is adjusted for an imputed 10% per annum real yield r on the beginning durables stock.[10] The basic accrual concept of income is private sector income y_t^{PS}--see Darby (1976, Chapter 2)--which is the amount (implicit in the national income accounts) available to the private sector (ultimately consumers) for consumption or addition to wealth.[11] The cash-receipts concept is based on disposable personal income y_t^{DP}. Both series are deflated by the implicit price deflator for personal consumption expenditures and quarterly observations are at SAQR. Thus, on the accrual definition current income is

$$(13) \qquad y_t = y_t^{PS} + r\, d_{t-1},$$

where r = 0.10 for annual data and 0.025 for quarterly data. Where the cash-receipts definition is used, y_t^{DP} replaces y_t^{PS} in equation (13).

Permanent income is computed in the usual way as

$$(14) \qquad y_{Pt} = \beta y_t + (1 - \beta)(1 + g)\, y_{P,t-1} .$$

The implied geometrically declining weights were shown in Darby (1974) to be implied by a perpetual inventory model of total (human and nonhuman) wealth where β is the real yield on wealth and g is the trend growth rate of income.[12] The value of β is estimated by search over the interval $0 \leq \beta \leq 1$ for the value which minimizes the sum of squared residuals in the consumer expenditure regression.

Transitory income is computed as the difference between the estimates of current and permanent income:

(15) $$y_{Tt} = y_t - y_{Pt}.$$

The relative price of durable to nondurable goods and services is computed by dividing the implicit price deflator for personal consumption expenditures on durable goods by the corresponding deflator for nondurable goods and services. The latter unpublished deflator is derived as the ratio of expenditures on nondurable goods and services in current dollars to the expenditures in constant (1958) dollars.[13]

For purposes of comparison with the multiequation approach to explaining consumer expenditures, estimates of household investment in durable goods Δd_t and pure consumption c_t^f are based on the Commerce Department definitions of durable goods:

(16) $$\Delta d_t = d_t - d_{t-1};$$

(17) $$c_t^f = c_t^x - (1 - 0.5r)\ \Delta d_t + rd_{t-1},$$

where the imputed yield on durable goods r is the same as used in estimating current income.

Estimates of the Consumer Expenditure Function

The consumer expenditure function (11) was estimated in both quarterly and annual versions for the entire period 1947-1973 for which complete data was available. The basic estimates--for reasons to be discussed in Section IV--are based on the accrual (private sector) income definition and the narrow (M_1) money definition.

The annual estimate is[14]

(18)
$$c_t^x = \underset{(-2.57)}{-148.9} + \underset{(16.69)}{1.08} y_{Pt} + \underset{(6.87)}{0.406}\, y_{Tt} + \underset{(4.96)}{0.681}\, m_t$$

$$- \underset{(-5.29)}{0.376}\, d_{t-1} + \underset{(0.80)}{29.0} \frac{P_{Dt}}{P_{NDt}} + \underset{(1.11)}{1.49}\, i_t$$

$\hat{\beta} = 0.15\ [0\ ,\ 0.23]$, S.E.E. = 1.98, R^2(adj.) = .9996,

D-W = 2.39 .

The corresponding quarterly regression is

(19)
$$c_t^x = \underset{(-3.21)}{-28.52} + \underset{(27.16)}{0.90}\, y_{Pt} + \underset{(12.67)}{0.455}\, y_{Tt} + \underset{(7.59)}{0.189}\, m_t$$

$$- \underset{(-4.39)}{0.042}\, d_{t-1} + \underset{(0.53)}{2.95} \frac{P_{Dt}}{P_{NDt}} + \underset{(1.66)}{0.37}\, i_t$$

$\hat{\beta} = 0.01\ [0,\ 0.06]$, S.E.E. = .744, R^2(adj.) = .9992,

D-W = 1.08

The two estimates correspond very closely when it is recalled that, in view of the stock-flow relationships, c_t^x, y_{Pt}, and y_{Tt} are measured at quarterly rates in the quarterly regression.[15] The low quarterly Durbin-Watson statistic suggests autocorrelation of the residuals not present in the annual regression, however. This autocorrelation may be due either to correlated data errors such as from the seasonal adjustment or else to an omitted variable such as lagged transitory income which is not important at the annual level. Since autocorrelation suggest overly optimistic standard errors, the discussion below will emphasize the more reliable annual regression.

Because of the important trend element, the adjusted R^2 is a meaningless measure of explanatory power.[16] More useful is the ratio of the standard error of estimate to the mean value of dependent variable. This value is 0.58 percent for the annual regression and 0.86 percent for the quarterly regression. If the consumer expenditure functions were converted to private saving functions by use of the identities (see Darby (1975), equation (12)), the standard errors would be 5.0 percent of mean private saving for annual data and 7.5 percent for quarterly data. Further the annual standard error of estimates is only 34.0 percent of the standard error for the naive model of footnote 16 and 50.5 percent of the standard error for a Keynesian consumption function.

In the annual regression, the coefficient of y_{Pt} exceeds unity because the effect operating through the stock demand for durables is large relative to the offset due to the demand for money. The long-run effect of permanent income would include induced effects on the durables and money stocks. Of special interest are the implied long-run values for the ratio k of pure consumption to total accrued income and the ratio σ of private saving to private sector income (exclusive of the imputed yield on the durables stock). These values are estimated at 0.90 and 0.08 respectively on the basis of regression (18).[17] In view of the nonlinear transformations and auxiliary information used in their computations, these values are better regarded as rough checks on the consistency of the regression than as good estimates of k and σ.

The short-run marginal propensity to consume is given by

$$(20)\qquad \frac{dc_t^x}{dy_t} = \frac{dc_t^x}{dy_{Pt}}\frac{dy_{Pt}}{dy_t} + \frac{dc_t^x}{dy_{Tt}}\frac{dy_{Tt}}{dy_t}$$

$$\frac{dc_t^x}{dy_t} = (1.08)(0.15) + (0.406)(0.85) \simeq 0.51 .$$

For the quarterly regression, the corresponding value is 0.46. The lower quarterly value reflects the smaller impact of current income on permanent income within a quarter as compared to over a year. The estimates of this β weight bracket the 0.1 per annum (0.025 per quarter) value which was estimated in Darby (1974) on the basis of pure consumption. They will be analyzed further in Section IV.

The coefficient of real money balances is quite significant in both the economic and statistical senses. Its high value would appear to support the substitution hypothesis of the real balance effect. This may be interpreted in two equivalent ways: (1) Bonds and durables are substitutes in the household portfolio and the demands for both are affected by an excess supply of money. (2) Money supply, given its demand, is a good proxy for the unavailable real yields on substitutes for durable goods. Another possibly complementary liquidity hypothesis would stress the critical role of cash balances in providing down payments for the purchase of consumers' durable goods because of the illiquidity of other forms of assets. A substantive implication of the effect of real money balances on consumer expenditures is that--in terms of the Hicksian cross--an increase in the money supply shifts the IS (as well as the LM) curve to the right.

The negative coefficient on the real durables stock is significantly larger than the yield on the stock. This indicates that both the direct substitution of durables yield for nondurable goods and services and the indirect adjustment of the durables stock affects consumer expenditures. Abnormally high durables sales during a boom imply a period of abnormally low durables sales later while low sales during a recession imply high sales later.

The coefficient of the relative price of durable goods is insignificant and of the wrong sign. Although not surprising--given that about half of behaviorally defined durable goods are represented in the denominator--this result is disappointing. An unsuccessful attempt was made to estimate durables stock and price series inclusive of clothing and shoes. Although the relative price coefficient became negative, insurmountable difficulties in estimating the initial stock and depreciation resulted in a slight deterioration in the standard error. Even were a definitionally "pure" estimate available, there would be two other factors making for an insignificant--or even perversely signed--coefficient for the relative price of durables: (1) The behavior of the relative price of durables is dominated by a downward trend over the postwar period. Given the costs of maintaining current price information on infrequently purchased items, the price as perceived by consumers not actively in the market would be better represented by a trend than by the actual price. Since permanent income is trend-dominated, the high implicit coefficient on permanent income in the durables stock demand will reflect not only the true income effect but also the effect of the negative trend in prices. (2) If--contrary to the usual macroeconomic assumption--the supply curve of durable goods is not infinitely elastic at a given price, the relative price coefficient would reflect the interaction of demand and supply effects and be of indeterminante sign.

The nominal interest rate coefficient is slightly positive. This indicates that the positive effect (from decreasing the demand for money) slightly outweighs the negative effect (from decreasing the durable stock demand). Since no attempt was made to adjust for expected inflation, the nominal interest rate would not be expected to have much effect in the durables stock demand. It is perhaps surprising then--if money demand is significantly interest elastic--that the interest rate coefficient does not more substantially exceed zero.

The early part of the period, say from 1947 through 1953, appeared suspect for three possible reasons: (1) the constraint on durables goods purchases during World War II, (2) possible inaccuracies in the starting benchmarks for permanent income and the durables stock, and (3) the effect on the demand for money of the abandonment during 1951-1953 of pegged interest rates on government bonds. The equations were restimated for 1954-1973, but there was no hint of a structural change or even a significant change in any of the coefficients.[18] So the entire period is retained for the statistical analysis.

It is customary to include in consumer expenditure functions--or rather consumption functions fitted to consumer expenditure data--the lagged dependent variable. The usual justification is the Koyck transformation in which lagged consumption serves as a proxy for lagged permanent income. This justification does not hold for lagged consumer expenditures however, since it has been seen above that short-run fluctuations in consumer expenditures are due primarily to fluctuations in transitory income, real money, and durables stock. Although estimation of regressions such as (18) by a maximum likelihood search routine is somewhat costly, the resulting standard errors and Durbin-Watson statistics do not suffer from the biases anticipated in regressions inclusive of the lagged dependent variable. Where the precise estimate of the weight β of current income in permanent income is not of concern, regressions can be run conditional upon a particular β weight.

Disaggregation into Consumption and Durables Investment Equations

It serves as a useful check on the derivation and interpretation of the integrated consumer expenditure function (11) to estimate the underlying pure consumption function (5) and household durables investment function (10). The difficulty in doing this is that the main reason for using the integrated approach is a lack of good data on household durables investment and pure consumption. Nevertheless for illustrative purposes, the official definition of durable goods was used to construct estimates (as explained under "Data Definitions") of pure consumption c_t^f and household durables investment Δd_t.

Table 1 presents the regression results. Equation (5) is estimated by regressions number 1 and 4 for annual and quarterly data respectively. The previous indirect calculation of k as 0.90 corresponds well to the direct estimate of 0.88. Since it was argued that a pure consumption estimate based on the official durables definition would in fact include considerable household investment in misclassified durables, regression 2 and 5 apply the consumer expenditure function to the estimated "pure consumption." Regressions 3 and 6 apply the household durables investment function (10) to the estimated net investment in (officially classified) durables good.

In comparing regressions 2 and 3, it is clear that the estimated net investment contains on the order of half of total net investment in a behavioral sense.[19] The only significant problem--not present in the quarterly regressions--is the larger coefficient on the lagged durables stock in regression 2 than in regression 3. This apparently offsets a slightly high estimated β weight of current income in permanent income while the quarterly regressions 5 and 6 display the opposite bias due to a low β weight.[20] The signs of the coefficients of the relative price of durables are just the reverse of what would be expected, but not much can be made of the statistically insignificant results for that variable.

In sum, the disaggregated version of the model is very much as would be guessed from the derivation of the model and the estimates of the integrated consumer expenditure function. The only significant divergence between the annual and quarterly results--autocorrelation aside--are apparently due to the use of a slightly too high value of β in the annual regressions and slightly too low a value of β in the quarterly regressions.

The disaggregation done in Table 1 takes advantage of the estimated β weight of current income in permanent income from the integrated consumer expenditure function. A standard multiequation model would make separate estimates of equations (5) and (10) and combine them by use of the identity (4) if a prediction of total consumer expenditures were required. The β estimate of the durables investment function will be unbiased but imprecise because of the low coefficient of permanent income. Since the estimates of pure consumption include elements of durables investment, the demonstration of upward bias of β from Darby (1974) applies directly. Nevertheless the biased permanent income estimates will provide more accurate predictions of c_t^f than regressions 1 and 4. In practice an even more favorable estimate of c_t^f based on the Koyck transformation would likely be used instead of equation (5):

$$(21) \qquad c_t^f = a_1 + a_2 y_t + a_3 c_t^f.$$

Table 2 compares the root mean squared errors of these two disaggregate estimation techniques with the root mean squared error of the consumer expenditure function. The integrated consumer expenditure function does much better than either disaggregated approach for the annual data. But for quarterly data, the method utilizing the Koyck transformation does nearly as well. The quarterly national income accounts data appear to spread receipts and expenditures over adjacent quarters however, so that the Koyck transformation in this case displays a spurious accuracy.

The consumer expenditure function has been successfully estimated in this section with no significant departures from expected signs or magnitudes of coefficients. The estimated coefficients are internally consistent. The disaggregated estimates are consistent with the original hypothesis that all coefficients other than permanent income enter because of household investment in durable goods but that nearly half of durable goods in a behavioral sense are included in the official data on nondurable goods and services. As a result, disaggregate estimates of consumer expenditures derived from separate models of pure consumption and household durables investment compare poorly with the estimates of the integrated consumer expenditure function.

IV. Analysis of Three Empirical Issues

The consumer expenditure function is used in this section to investigate further three empirical issues: (1) the definition of current income which best explains consumer expenditures; (2) the definition of money which best explains consumer expenditures; and (3) the value of the weight β of current income in the determination of permanent income.

The two income definitions which are compared are the accrual concept and the cash-receipts concept.[21] These two definitions reflect the two basic alternative conceptions of consumer behavior. The accrual concept is harmonious with a view of the consumer as a rational decision-maker constrained by total wealth. The cash-receipts concept is sensible if consumer behavior is more a matter of spending nearly all that is received. Until recently, the use of a cash-receipts concept (disposable personal income) was the standard, traditional practice. A number of studies in the last decade have moved toward the accrual concept by adding undistributed corporate profits (as an estimate of accrued capital gains).

There are many other competing income definitions which could be considered. For example, Barro (1974) and Kochin (1974) have recently argued--following David Ricardo--that government bonds are not viewed by the private sector as net wealth. In that case an accrual definition of income would be essentially net national product less government expenditures for goods and services plus the increase in high-powered (base) money.[22] Feldstein (1974) on the other hand argues for inclusion of an estimate of increases in "social security wealth." Another issue concerns the transfer of purchasing power to the government through inflation. This would suggest subtraction of the rate of inflation times high-powered money and government bonds (if government bonds are net wealth). In view of the high estimation costs of dealing with many alternative income definitions simultaneously with the other two main empirical issues, it was decided only to compare the basic accrual and cash-receipts definitions, leaving for further research comparison of finer differences conditional on a particular money definition and β weight.

Section III only discussed the M_1 (currency plus demand deposits) definition of money. Two other money definitions have received considerable attention by monetary economists: M_2 (M_1 plus time deposits at commercial banks[23]) and M_3 (M_2 plus savings and loan and mutual savings bank deposits). These alternative money definitions are compared with M_1 in this section.

The M_2 data used are an average of the monthly data deflated by the implicit price deflator for personal consumption expenditures. Unfortunately Federal Reserve data for M_3 is available only from January 1959 onward while the Friedman and Schwartz (1970) data contain no series using the official M_3 definition. Monthly estimates of M_3 for 1947 through 1958 were made on the basis of the Friedman and Schwartz data on savings and loan and mutual savings bank deposits.[24] The M_3 data used in this series are averages of this monthly data deflated by the implicit price deflator for personal consumption expenditures.

In Darby (1974), removal of the specification bias resulted in an estimated β weight of 0.1 per annum in terms of an essentially pure consumption model. This section examines whether that estimate stands up for the consumer expenditure function under alternative definitions of income and money. Were β not estimated for each combination it could bias the choice of the best combination of income and money.

These three empirical issues are examined simultaneously by the regressions reported in Tables 3 (annual data) and 4 (quarterly data). The message of these tables is very clear: The accrual income concept and the M_1 money concept do much better in explanatory power (as judged the sum of squared residuals or standard error of estimate criterion) than the alternative definitions. Further the β weight of 0.1 per annum previously estimated on the basis of pure consumption continues to hold up for the consumer expenditure function.

Consider first the definition of income: For each money definition and for both annual and quarterly data, the accrual definition of income does better than the cash-receipts definition.[25] The sum of squared residuals of the best cash-receipts definition regression exceeds that of the corresponding accrual definition by 41.8 percent for the annual data and 10.6 percent for the quarterly data.[26] It would have been surprising, given the success of the model which is based on rational consumers faced with a wealth constraint, had the accrual definition not done considerably better than the cash-receipts definition of income.

As to the empirical definition of money, the results are similar. For either definition of income, the M_1 definition does better than either M_2 or M_3. Comparing the best M_1 estimate with the best alternative (M_3), the alternative definition's sum of squared residuals is 32.9 percent higher for annual data and 11.1 percent higher for quarterly data. The coefficients of real money balances would be expected to decline in moving from M_1 to M_2 to M_3 (because of the increasing absolute magnitudes), but the fact that the standard errors decline less rapidly (so that t-values fall) is suggestive that M_2 and M_3 are properly interpreted as proxies for M_1. The fact that M_3 does a bit better than M_2 is suggestive that consumers find bank and nonbank time deposits much better substitutes for each other than they find all kinds of time deposits for M_1.

As already discussed in Section III, the estimates of the β weight bracket--but in no case significantly differ from--the previous estimate of 0.1 per annum or .025 per quarter. As discussed in footnote 20 above, the high correlation of the durables stock and permanent income for low β weights make precise estimation impossible. However, it is clear from the behavior of the likelihood function that the actual β weight must lie in the neighborhood of 0.1 per annum (0.025 per quarter). This is illustrated in Figures 1 and 2 which are graphs of the sum of squared residuals as a function of the β weight for annual and quarterly data, respectively. The critical value of the sum of squared residuals for a two-tailed likelihood ratio test at the 90% significance level is indicated in each figure by SSR^{crit}. Between 0 and 0.2 for annual data and, equivalently, between 0 and 0.05 for quarterly data, the sum of squared residuals is rather flat so that the minimizing β weights of 0.15 and 0.01, respectively, are little better than any other value within that range. From 0.2 to 0.6 per annum (0.05 to 0.2 per quarter), the sum of squared residuals rises very rapidly to a much higher plateau. So the estimation of β is imprecise within the range from 0 to 0.2 per annum, but any value much above that range--including Friedman's original (1957) biased[27] estimate of 0.35 per annum--can be easily rejected.

Regressions number 7 and 13--presented earlier as equations (18) and (19)--are not acceptable as final estimates of the consumer expenditure function since they have inconsistent β weights which bias upwards (in absolute value) the durables stock coefficient in the annual version and bias it downwards in the quarterly version. Since the average of the two β weights is 9.5% per annum and there is no reason to reject the previous 10% per annum based on a pure consumption model, the final estimates of the consumer expenditure function are based on a β weight of 0.10 per annum and 0.025 per quarter. The annual estimate is[28]

(22)
$$c_t^x = \underset{(-2.52)}{-147.5} + \underset{(17.42)}{1.005}\, y_{Pt} + \underset{(7.98)}{0.446}\, y_{Tt} + \underset{(5.07)}{0.729}\, m_t - \underset{(-4.60)}{0.289}\, d_{t-1} + \underset{(0.83)}{30.3}\, \frac{P_{Dt}}{P_{NDt}} + \underset{(1.47)}{1.96}\, i_t$$

$$\beta = 0.1,\ \text{S.E.E.} = 2.00,\ R^2(\text{adj.}) = .9996,\ \text{D-W} = 2.39.$$

The corresponding quarterly estimate is

(23)
$$c_t^x = \underset{(-3.37)}{-30.47} + \underset{(25.88)}{0.971}\, y_{Pt} + \underset{(12.85)}{0.460}\, y_{Tt} + \underset{(7.51)}{0.187}\, m_t - \underset{(-6.01)}{0.065}\, d_{t-1} + \underset{(0.49)}{2.76}\, \frac{P_{Dt}}{P_{NDt}} + \underset{(1.43)}{0.33}\, i_t$$

$$\beta = 0.025,\ \text{S.E.E.} = .746,\ R^2(\text{adj.}) = .9992,\ \text{D-W} = 1.07.$$

Strong evidence has been presented in this section for the following empirical propositions: (1) The accrual (private sector income) definition of income explains consumer expenditures better than the cash-receipts (disposable personal income) definition. (2) The narrow M_1 definition of money is an important determinant of consumer expenditures and significantly better in explanatory power than either broad definition M_2 or M_3. (3) The weight β of current income in permanent income lies in the range from 0 to 20% per annum.

V. Concluding Remarks

The central theme of this paper is the empirical value of an integrated consumer expenditure function in explaining consumer expenditures. The theoretical value of the consumer expenditure function is that it concentrates directly on the variable of prime interest to macroeconomists. But the alternative treatment of household investment in durable goods as a component of an enlarged definition of total investment does not lack theoretical appeal either. The basic attraction is therefore the empirical one: The integrated approach is much less subject to biases introduced by the essentially arbitrary classification of commodities between durable and nondurable goods and services.

An empirical question can be answered only by examination of the data. An unusually clear answer was provided by the research reported here: The consumer expenditure function explains the data well and significantly better than the multiequation pure consumption-household investment approach. The reason for this superior performance is found in the fact that the official data on durable goods expenditures include only about half of total durables expenditures as defined behaviorally.

The data also provided strong evidence that (1) an accrual (private sector) definition of income better explains consumer expenditures than a cash-receipts (disposable) personal income definition: (2) the narrow M_1 definition similarly does better than either M_2 or M_3: and (3) the β weight of current income in the formation of permanent income lies somewhere in the range from 0 to about 20% per annum. While there is no a priori presumption about the best money definition, the results on the income definition and the β weight reinforce the basic conception underlying the model--that consumers are rational decision-makers constrained by total (human and nonhuman) wealth as estimated by permanent income. The rationality of consumers would certainly be questionable if they responded to cash receipts rather than accrued income. A β weight of about 10% per annum--which is the estimated real yield on total wealth--is certainly more acceptable than the higher weights estimated in many previous studies.[29]

The empirical advantages of an integrated consumer expenditure function seem clear. Future research might be directed at substituting a life cycle model for the permanent income explanation of pure consumption to compare their explanatory powers. Other areas for possible improvement would be either the generalized stock adjustment hypothesis (6) or the underlying stock demand functions (7) and (9). A somewhat different line of research would utilize the consumer expenditure function to examine finer definitions of accrued income adjusted for increases in government debt, in social security wealth, or the inflationary tax on base money and possibly government debt.

DATA APPENDIX

Several data series of general applicability were estimated in the course of this project. In order to make them available for future research by others in this and other areas, the most important are reproduced here with instructions for updating as revised data become available.

Table 5 presents annual data for nominal and real private sector income, the current and permanent (real) income on the accrued definition, the real durables stock, and the nominal M_3 money supply. Table 6 contains quarterly data for the same series. Table 7 presents the monthly nominal M_3 data through 1959 when they tie in with the Federal Reserve System's published data.

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Table 1

Disaggregation of the Consumer Expenditure Function Using Estimated Pure Consumption and Household Durables Investment

Reg. No.	Dep. Var.	Observation Frequency	Regression Coefficients for: Constant	y_{Pt}	y_{Tt}	m_t	d_{t-1}	P_{Dt}/P_{NDt}	i_t	β	S.E.E.	R^2(adj.)	D-W
1	c_t^f	Annual	-	0.880 (418.2)	-	-	-	-	-	0.15	4.53	.9982	0.28
2	c_t^f	Annual	-44.4 (-1.08)	0.976 (21.4)	0.219 (5.22)	0.208 (2.14)	-0.190 (-3.78)	-2.95 (-0.12)	3.51 (3.69)	0.15	1.40	.9998	2.06
3	Δd_t	Annual	-110.0 (-2.08)	0.106 (1.80)	0.198 (3.66)	0.498 (3.98)	-0.090 (-1.39)	33.7 (1.02)	-2.12 (-1.73)	0.15	1.81	.9358	2.01
4	c_t^f	Quarterly	-	0.876 (619.3)	-	-	-	-	-	0.01	1.53	.9967	0.08
5	c_t^f	Quarterly	-7.65 (-1.36)	0.794 (38.0)	0.265 (11.7)	0.078 (4.96)	0.008 (1.32)	-1.24 (-0.35)	0.857 (6.01)	0.01	0.47	.9997	1.05
6	Δd_t	Quarterly	-21.1 (-3.01)	0.104 (4.00)	0.193 (6.78)	0.112 (5.70)	-0.025 (-3.32)	4.24 (0.96)	-0.488 (-2.74)	0.01	0.59	.8951	0.95

Notes: t-values are given in parentheses below the coefficient estimates.
β estimates are from equations (18) and (19).

Table 2

Comparison of Prediction Errors

Consumer Expenditure Function vs. Disaggregated Estimates

Estimation Approach	Mean Prediction Errors[a] Annual	Quarterly
Consumer Expenditure Function[b]	1.704	0.720
Equations (5) and (10)[c]	3.458	1.150
Koyck and equation (10)[d]	2.635	0.731

[a]Square root of the mean squared error, 1947-1973

[b]Regression equations (18) and (19)

[c]Maximum likelihood estimates of equations (5) and (10) combined by equation (4)

[d]Maximum likelihood estimates of equations (21) and (10) combined by equation (4).

Table 3

Consumer Expenditure Functions Estimated for Alternative Income and Money Definitions
Annual Data, 1947-1973

Reg. No.	Income Concept	Money Concept	Regression Coefficients for Constant	y_{Pt}	y_{Tt}	m_t	d_{t-1}	P_{Dt}/P_{NDt}	i_t	$\hat{\beta}$	S.E.E.	R^2(adj.)	D-W	SSR
7	Accrual	M_1	-148.9 (-2.57)	1.08 (16.69)	0.406 (6.87)	0.681 (4.96)	-0.376 (-5.29)	29.0 (0.80)	1.49 (1.11)	0.15	1.98	.9996	2.39	78.4
8	Cash	M_1	-90.0 (-1.35)	1.09 (15.71)	0.630 (7.46)	0.461 (2.62)	-0.402 (-5.38)	15.3 (0.36)	-0.159 (-0.10)	0.10	2.36	.9995	2.25	111.2
9	Accrual	M_2	-14.7 (-0.28)	0.92 (14.04)	0.410 (5.57)	0.183 (3.51)	-0.247 (-3.49)	-14.1 (-0.36)	2.08 (1.32)	0.15	2.33	.9995	2.29	108.4
10	Cash	M_2	-8.20 (-0.15)	1.03 (13.46)	0.628 (6.46)	0.116 (1.96)	-0.363 (-4.82)	-10.9 (-0.26)	-0.08 (-.05)	0.125	2.48	.9994	2.36	123.4
11	Accrual	M_3	6.75 (0.13)	0.83 (9.14)	0.337 (4.18)	0.146 (3.35)	-0.203 (-2.58)	-16.6 (-0.43)	2.65 (1.59)	0.225	2.28	.9995	2.26	104.2
12	Cash	M_3	2.41 (0.05)	0.95 (8.86)	0.581 (5.89)	0.106 (2.15)	-0.331 (-3.77)	-10.4 (-0.25)	0.91 (0.52)	0.175	2.40	.9995	2.32	115.5

Notes: t-values are given in parentheses below the coefficient estimates.

SSR indicates the sum of squared residuals.

Regression number 7 was discussed as equation (18) in Section III.

Table 4

Consumer Expenditure Functions Estimated for Alternative Income and Money Definitions
Quarterly Data, 1947-1973

Reg. No.	Income Concept	Money Concept	Regression Coefficients for: Constant	y_{Pt}	y_{Tt}	m_t	d_{t-1}	P_{Dt}/P_{NDt}	i_t	$\hat{\beta}$	S.E.E.	R^2(adj.)	D-W	SSR
13	Accrual	M_1	-28.5 (-3.21)	0.90 (27.16)	0.455 (12.67)	0.189 (7.59)	-0.042 (-4.39)	2.95 (0.53)	0.37 (1.66)	0.01	0.744	.9992	1.08	55.9
14	Cash	M_1	-18.3 (-2.02)	0.96 (25.76)	0.597 (11.38)	0.138 (5.08)	-0.061 (-5.84)	1.30 (0.22)	-0.14 (-0.61)	0.00	0.782	.9991	0.94	61.8
15	Accrual	M_2	-0.29 (-0.04)	0.82 (23.06)	0.444 (10.73)	0.054 (6.10)	-0.035 (-3.52)	-4.37 (-0.76)	0.50 (2.06)	0.02	0.797	.9991	0.95	64.2
16	Cash	M_2	1.50 (0.20)	0.92 (21.51)	0.615 (10.92)	0.039 (4.20)	-0.060 (-5.57)	-3.86 (-0.66)	-0.03 (-0.11)	0.01	0.810	.9990	0.90	66.3
17	Accrual	M_3	3.12 (0.42)	0.79 (14.45)	0.381 (8.42)	0.042 (6.00)	-0.044 (-3.63)	-4.61 (-0.82)	0.60 (2.31)	0.06	0.784	.9991	0.94	62.1
18	Cash	M_3	6.01 (0.83)	0.79 (14.25)	0.571 (9.98)	0.036 (5.04)	-0.040 (-3.54)	-3.84 (-0.68)	0.30 (1.17)	0.01	0.785	.9991	0.93	62.2

Notes: t-values are given in parentheses below the coefficient estimates
SSR indicates the sum of squared residuals.
Regression number 13 was discussed as equation (19) in Section III.

Table 5

Annual Data Series

year	Y_t^{PS} (a)	y_t^{PS} (b)	y_t^{PSDY} (c)	y_{Pt} (d)	d_t (e)	M_{3t} (f)
1946	163.22	——	——	223.77	72.43	——
1947	176.05	225.96	233.20	232.43	81.33	172.71
1948	200.45	243.36	251.49	242.35	90.01	176.74
1949	199.35	244.00	253.00	251.78	99.09	178.10
1950	215.17	259.67	269.58	262.24	112.21	183.88
1951	235.42	265.77	276.99	272.76	119.75	191.82
1952	246.85	272.81	284.79	283.37	125.42	204.35
1953	258.70	282.12	294.66	294.27	134.05	215.89
1954	263.97	285.46	298.87	304.88	141.22	229.28
1955	285.02	307.21	321.33	317.04	154.10	240.53
1956	300.37	317.00	332.41	329.51	162.61	250.40
1957	314.90	322.48	338.74	341.79	169.93	261.83
1958	322.82	322.81	339.80	353.38	172.69	278.57
1959	345.65	341.29	358.56	366.08	180.20	295.62
1960	355.80	345.78	363.80	378.48	187.32	305.57
1961	369.60	355.63	374.36	391.12	192.38	326.65
1962	392.97	374.70	393.94	404.89	201.22	350.86
1963	411.05	387.31	407.43	419.11	212.54	380.07
1964	447.52	416.86	438.12	435.46	226.49	410.07
1965	486.17	446.71	469.36	453.87	244.78	444.83
1966	526.20	471.67	496.15	473.75	264.24	476.19
1967	556.55	486.46	512.88	494.00	281.17	511.70
1968	596.87	503.97	532.09	514.84	302.65	554.49
1969	633.10	512.47	542.74	535.39	323.88	588.93
1970	683.57	528.42	560.81	556.39	339.49	613.94
1971	744.15	553.84	587.78	578.72	360.34	692.55
1972	804.55	581.95	617.99	602.60	388.58	778.92
1973	905.25	620.52	659.37	629.06	419.04	862.13

Columns (a) through (e) based on final data through 1971. Federal Reserve estimates of M_3 are subject to change back to 1959.

Notes: a. Private sector income in current dollars (billions of dollars). To update or extend: Y_t^{PS} = net national product - governement purchases of goods and services - government surplus (NIA basis) - statistical discrepancy (NIA) - Federal government transfer payments to foreigners (net) - personal transfer payments to foreigners.

b. Private sector income in constant dollars (billions of 1958 dollars). To update or extend: $y_t^{PS} = Y_t^{PS}$ deflated by the implicit price deflator for personal consumption expenditures.

Notes to Table 5 (continued):

c. Private sector income in constant dollars adjusted for the imputed yield on the stock of consumers' durable goods (billions of 1958 dollars). The accrual concept of income. To update or extend: $y_t^{PSDY} = y_t^{PS} + 0.1\ d_{t-1}$.

d. Permanent income based on y_t^{PSDY} and a β weight of 0.1 (billions of 1958 dollars). To update or extend: $y_{Pt} = 0.1\ y_t^{PSDY} + 0.9344862\ y_{Pt-1}$.

e. Stock of consumers' durable goods at the end of the year (billions of 1958 dollars). To update or extend: Fourth quarter data from Table 6.

f. Money stock M_3 in current dollars (billions of dollars). To update or extend: Average of monthly Federal Reserve data beginning 1959.

Table 6

Quarterly Data Series

year & quarter	Y_t^{PS} (a)	y_t^{PS} (b)	y_t^{PSDY} (c)	y_{Pt} (d)	d_t (e)	M_{3t} (f)
1946 4	169.9	——	——	227.17	72.43	——
1947 1	170.4	223.62	230.87	229.39	74.56	169.19
1947 2	172.0	223.96	231.41	231.58	76.76	171.88
1947 3	179.3	228.99	236.67	233.88	78.89	174.06
1947 4	182.5	227.27	235.16	236.09	81.33	175.72
1948 1	190.5	234.90	243.03	238.47	83.63	176.74
1948 2	199.5	243.29	251.66	241.03	85.83	176.27
1948 3	205.1	246.22	254.80	243.63	88.02	176.96
1948 4	206.7	249.04	257.84	246.26	90.01	176.99
1949 1	201.1	244.35	253.35	248.74	91.77	177.05
1949 2	198.9	243.15	252.33	251.15	94.01	178.04
1949 3	199.8	245.76	255.16	253.60	96.48	178.34
1949 4	197.6	242.75	252.40	255.94	99.09	178.97
1950 1	209.5	257.69	267.60	258.63	101.86	180.75
1950 2	209.7	256.67	266.86	261.25	104.59	183.50
1950 3	217.3	260.24	270.70	263.93	109.11	184.87
1950 4	224.2	264.08	274.99	266.67	112.21	186.42
1951 1	226.0	257.11	268.33	269.20	115.28	187.97
1951 2	234.6	265.99	277.51	271.93	117.00	190.00
1951 3	240.0	271.19	282.89	274.74	118.43	192.80
1951 4	241.1	268.78	280.63	277.46	119.75	196.50
1952 1	241.9	268.78	280.75	280.13	121.08	199.80
1952 2	242.6	269.26	281.36	282.78	122.51	202.66
1952 3	248.0	273.73	285.98	285.51	123.40	205.85
1952 4	254.9	279.50	291.84	288.33	125.42	209.08
1953 1	258.0	282.58	295.13	291.20	127.75	211.72
1953 2	260.1	284.26	297.04	294.06	129.97	214.98
1953 3	259.6	282.17	295.17	296.84	132.05	217.26
1953 4	257.1	279.46	292.66	299.51	134.05	219.60
1954 1	260.6	281.42	294.83	302.19	135.61	229.13
1954 2	261.3	282.18	295.74	304.86	137.34	225.62
1954 3	263.6	285.28	299.01	307.56	139.08	229.51
1954 4	270.4	292.96	306.86	310.42	141.22	232.87
1955 1	276.6	298.70	312.83	313.38	144.08	236.60
1955 2	283.7	306.37	320.78	316.49	147.45	239.40
1955 3	287.4	309.36	324.11	319.64	151.00	241.87
1955 4	292.4	314.41	329.51	322.88	154.10	244.27
1956 1	293.9	313.99	329.40	326.06	156.46	246.53
1956 2	297.8	315.80	331.45	329.24	158.63	249.10
1956 3	302.3	317.21	333.07	332.41	160.50	251.50
1956 4	307.5	320.98	337.03	335.63	162.61	254.47
1957 1	311.1	321.72	337.98	338.83	164.84	257.63
1957 2	314.2	322.92	339.40	342.01	166.72	260.60
1957 3	318.0	324.16	340.83	345.18	168.37	263.43
1957 4	316.3	321.12	337.95	348.22	169.93	265.67
1958 1	314.4	315.66	332.65	351.09	170.72	269.60
1958 2	317.6	317.60	334.67	353.96	171.20	276.47

Table 6 (continued)

year & quarter	Y_t^{PS}	y_t^{PS}	y_t^{PSDY}	y_{Pt}	d_t	M_{3t}
1958 3	325.1	324.77	341.89	356.97	171.83	281.80
1958 4	334.2	333.20	350.38	360.14	172.69	286.43
1959 1	339.8	337.77	355.04	363.38	174.35	290.90
1959 2	348.1	344.99	362.43	366.75	176.43	294.73
1959 3	345.3	339.86	357.50	369.95	178.60	297.87
1959 4	349.4	342.55	360.41	373.17	180.20	299.00
1960 1	354.1	346.14	364.16	376.43	182.25	299.80
1960 2	356.9	347.52	365.74	379.68	184.26	302.07
1960 3	357.2	346.80	365.22	382.87	186.01	307.50
1960 4	355.0	342.66	361.27	385.91	187.32	312.93
1961 1	357.7	344.60	363.34	388.95	188.11	318.27
1961 2	366.0	352.94	371.75	392.15	189.24	323.97
1961 3	372.6	358.27	377.19	395.44	190.62	329.43
1961 4	382.1	366.70	385.76	398.90	192.38	334.93
1962 1	386.9	370.24	389.48	402.39	194.48	341.47
1962 2	391.6	374.02	393.47	405.93	196.48	348.33
1962 3	394.4	375.62	395.27	409.46	198.77	353.27
1962 4	399.0	378.92	398.79	413.02	201.22	360.37
1963 1	402.6	381.25	401.37	416.59	203.88	368.63
1963 2	406.7	383.68	404.07	420.17	206.60	376.50
1963 3	414.1	389.92	410.58	423.85	209.53	383.70
1963 4	420.8	394.38	415.33	427.60	212.54	391.43
1964 1	433.5	405.52	426.77	431.58	215.95	397.73
1964 2	445.7	415.38	436.97	435.75	219.61	404.83
1964 3	453.2	421.97	443.93	440.02	223.35	414.27
1964 4	457.7	424.58	446.92	444.31	226.49	423.47
1965 1	469.0	433.46	456.11	448.76	231.03	432.20
1965 2	477.0	438.42	461.52	453.27	235.15	439.50
1965 3	493.6	452.84	476.36	458.08	239.75	448.37
1965 4	505.1	462.12	486.10	463.07	244.78	459.27
1966 1	513.5	465.97	490.45	468.08	250.26	468.13
1966 2	520.0	467.21	492.23	473.06	254.71	474.77
1966 3	529.6	473.28	498.75	478.12	259.60	478.63
1966 4	541.7	480.23	506.19	483.29	264.24	483.23
1967 1	544.5	480.58	507.01	488.40	268.12	492.40
1967 2	550.8	484.01	510.82	493.53	272.75	505.17
1967 3	560.7	488.41	515.69	498.69	276.98	519.33
1967 4	570.2	492.83	520.52	503.90	281.17	529.90
1968 1	579.6	496.23	524.35	509.12	286.27	538.60
1968 2	595.7	504.83	533.46	514.49	291.45	548.13
1968 3	602.3	506.99	536.13	519.84	297.21	558.90
1968 4	609.9	507.83	537.55	525.14	302.65	572.33
1969 1	613.8	506.44	536.70	530.34	308.46	582.90
1969 2	626.5	510.59	541.44	535.57	314.00	589.30
1969 3	642.7	517.06	548.45	540.90	319.02	590.73
1969 4	649.4	515.81	547.71	546.12	323.88	592.80
1970 1	660.6	518.12	550.51	551.34	328.28	594.87
1970 2	681.3	529.78	562.61	556.77	332.78	604.30

Table 6 (continued)

year & quarter	Y_t^{PS}	y_t^{PS}	y_t^{PSDY}	y_{Pt}	d_t	M_{3t}
1970 3	695.2	536.01	569.28	562.29	337.01	619.60
1970 4	697.2	529.79	563.49	567.58	339.49	637.00
1971 1	722.5	544.46	578.41	573.15	344.38	659.47
1971 2	741.4	552.87	587.31	578.86	349.19	685.43
1971 3	748.9	554.74	589.66	584.54	354.60	703.80
1971 4	763.8	563.27	598.73	590.36	360.34	721.50
1972 1	775.2	566.67	602.70	596.19	366.65	744.03
1972 2	791.4	574.73	611.39	602.14	373.42	765.97
1972 3	809.1	583.35	620.69	608.23	380.78	790.67
1972 4	842.5	603.08	641.16	614.74	388.58	815.00
1973 1	873.3	617.61	656.47	621.53	397.72	835.87
1973 2	893.3	619.06	658.83	628.27	406.03	854.50
1973 3	915.1	622.52	663.12	635.01	413.59	870.47
1973 4	939.3	622.88	664.24	641.68	419.04	887.70

Columns (a) through (d) are at seasonally adjusted annual rates; divide by 4 to obtain the seasonally adjusted quarterly rates used in the text. Columns (a) through (e) based on final data through 1971 4. Federal Reserve estimates of M_3 are subject to change back to 1959.

Notes: a. Private sector income in current dollars (billions of dollars). To update or extend: Y_t^{PS} = net national product - government purchases of goods and services - government surplus (NIA basis) - statistical discrepancy (NIA) - Federal government transfer payments to foreigners (net) - personal transfer payments to foreigners.

b. Private sector income in constant dollars (billions of 1958 dollars). To update or extend: $y_t^{PS} = Y_t^{PS}$ deflated by the implicit price deflator for personal consumption expenditures.

c. Private sector income in constant dollars adjusted for the imputed yield on the stock of consumers' durable goods (billions of 1958 dollars). The accrual concept of income. To update or extend: $y_t^{PSDY} = y_t^{PS} + 0.1\ d_{t-1}$.

d. Permanent income based on y_t^{PSDY} and a β weight of 0.025 per quarter (billions of 1958 dollars). To update or extend: $y_{Pt} = 0.025\ y_t^{PSDY} + 0.9843473\ y_{Pt-1}$.

e. Stock of consumers' durable goods at the end of the quarter (billions of 1958 dollars). To update or extend: $d_t = 0.24375\ c_t^d + 0.95\ d_{t-1}$, where c_t^d is consumption expenditures for durable goods in constant (1958) dollars as seasonally adjusted annual rates.

f. Money stock M_3 in current dollars (billions of dollars). To update or extend: Average of monthly Federal Reserve data beginning 1959.

Table 7

Monthly M_3 Money Stock Estimates, 1947 - 1959

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1947	168.4	169.0	170.2	171.1	171.8	172.8	173.2	174.0	175.0	175.2	176.0	176.0
1948	176.7	176.9	176.6	176.3	176.1	176.4	176.9	177.1	177.0	177.1	176.9	176.9
1949	176.7	177.2	177.2	177.7	178.3	178.2	178.3	178.4	178.3	178.6	178.9	179.4
1950	179.9	180.7	181.6	182.7	183.7	184.1	184.6	184.9	185.1	185.8	186.5	186.9
1951	187.5	187.9	188.5	189.5	190.0	190.5	191.8	192.5	194.1	194.8	196.9	197.8
1952	198.9	199.7	200.7	201.7	202.6	203.6	204.6	205.8	207.1	207.9	209.2	210.2
1953	210.7	211.4	213.0	214.1	215.3	215.5	216.5	217.4	217.9	218.7	219.7	220.4
1954	231.5	232.4	223.5	224.1	225.9	226.8	228.4	229.6	230.6	231.7	233.0	233.9
1955	235.3	237.1	237.4	238.4	239.6	240.2	241.1	241.7	242.8	243.7	244.1	245.0
1956	245.8	246.5	247.3	248.4	248.9	250.0	250.6	251.3	252.6	253.4	254.5	255.5
1957	256.6	257.6	258.7	259.6	260.7	261.5	262.6	263.5	264.2	264.9	265.7	266.4
1958	266.9	269.8	272.1	274.4	276.3	278.7	280.0	282.0	283.4	284.8	286.6	287.9
1959	290.0	290.7	292.0	293.4	294.8	296.0	297.5	297.8	298.3	298.5	299.1	299.4

Notes: Money stock M_2 plus deposits at mutual savings banks and savings and loan associations (Federal Reserve System definition). Seasonally adjusted monthly averages (billions of dollars).

Data for 1947 through 1958 are estimated by adding to M_2 the Friedman & Schwartz (1970) data on mutual savings bank and savings and loan association deposits. Monthly savings and loan data were interpolated between annual (1947-1949) and quarterly (1950-1954) benchmarks by the use of mutual savings bank deposits. Data for 1959 is Federal Reserve data.

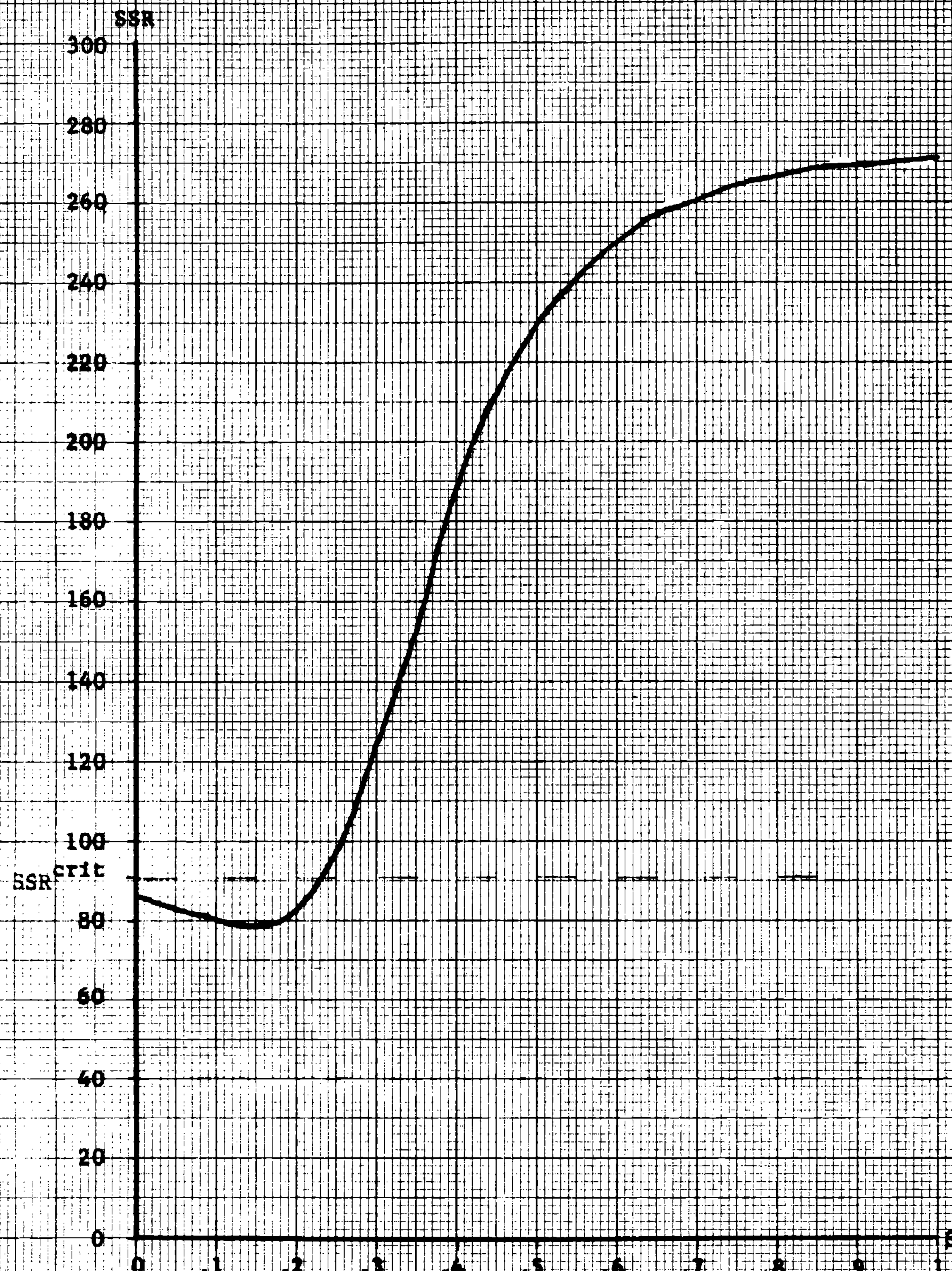

Figure 1

Sum of Squared Residuals as a Function of the ß Weight
Annual Consumer Expenditure Functions

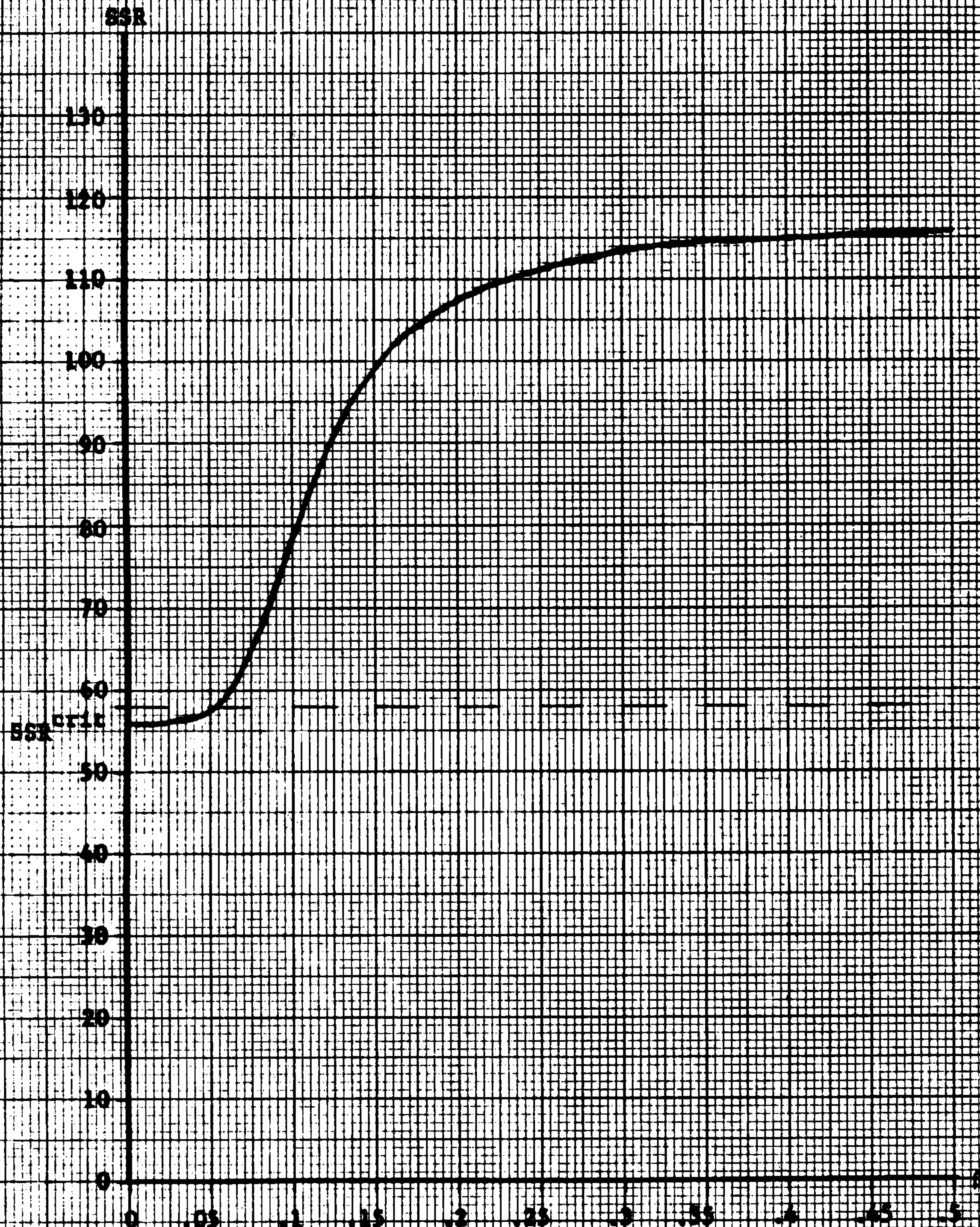

Figure 2

Sum of Squared Residuals as a Function of the δ Weight
Quarterly Consumer Expenditure Functions

FOOTNOTES

*This research was written during the author's Harry Scherman Research Fellowship at the National Bureau of Economic Research but is not an official report of the National Bureau. Nurhan Helvacian provided valued research assistance.

[1]The old vaudeville joke comes to mind: First drunk--"Why are you looking for your wallet here if you lost it over there?" Second drunk--"The light's much better here by the lamp post."

[2]A rough definition of behavioral durability is responsiveness to transitory income. Consumer expenditures for durable goods and for clothing and shoes are about equally responsive to changes in transitory and permanent income, but all other expenditures are only about one quarter as responsive to changes in transitory income as to permanent income. Thus the official Commerce Department definition does appear to capture goods significantly more durable than those classified as nondurable goods and services (with the exception of clothing and shoes). Given the relative magnitudes, however, the remaining durable elements in "nondurable goods and services" are nevertheless quite significant.

[3]In my earlier work (1972, 1975), this adjustment was neglected because of the small difference from unity for quarterly data (with $\delta = .05$, $1 - 0.5\,\delta = 0.975$).

[4]This adjustment too is small for quarterly data. Using $r = 2.5\%$ per quarter or 10% per annum, $1 - 0.5r = 0.9875$ for quarterly data or 0.95 for annual data.

[5]Darby (1974) demonstrates the interpretation of permanent income as a perpetual inventory of wealth.

[6]Other possible influences have been omitted either here or below in completing the specification because of the difficulty in obtaining good data and the paucity of true degrees of freedom. The empirical results that follow do not seem to have suffered much.

[7]The coefficient of permanent income will capture the effect of wealth and of secular trends in institutions, payments technology, and so forth. The coefficient of transitory income reflects both effects of windfalls on portfolio adjustment and of cyclical variations in transactions (see Darby (1972)).

[8]Basic data series were all drawn from the N.B.E.R.'s data bank.

[9]This amounts to the usual 10-year-life, double-declining-balance method. The initial value for December 31, 1946, was computed from Raymond Goldsmith's (1962) data as 72.43 billion 1958 dollars. See Darby (1972, pp. 931-32) for details. This calculation requires that c_t^d be measured at quarterly rates in order to integrate flows into stocks.

[10]A theoretically more attractive definition would base the imputation on the average durables stock for the period $d_{t-1} + 0.5\ \Delta d_t$. As this might impart spurious correlation--particularly in the disaggregated estimation of the Δd_t equation--this was not done.

[11]That is, private sector income equals disposable personal income + undistributed corporate profits + wage accruals less disbursements + corporation inventory valuation adjustment - other personal outlays. For computational purposes, an equivalent definition is net national product -

taxes net of transfers (i.e., government expenditures ± NIA surplus) - government and private transfers to foreigners - statistical discrepancy.

[12]This growth rate is implicit in saving plans. The required growth rates and initial values y_{P0} (0 = 1946 for annual data and 1946-IV for quarterly data) were estimated by a loglinear trend (see Darby (1974) for details) as:

Income Concept	$\hat{g}$	$\hat{y}_{P0}$
Accrual (annual data)	.03832	223.773
Accrual (quarterly data)	.00959	56.7934
Cash receipts (annual data)	.04003	213.973
Cash receipts (quarterly data)	.01001	54.3573

[13]This series is the most problematical. It was pointed out in the introduction that roughly half of behaviorally durable goods are included among nondurable goods and services. Thus their prices will be included in the denominator instead of the numerator.

[14]The t-values are given in parentheses. The square brackets indicate a greater than 90 percent probability confidence interval computed on the basis of the asymptotic distribution of the logarithm of the likelihood function. For the annual regression, the values $\hat{\beta}$ = 0, 0.025, 0.050, ..., 0.975, 1.000 were searched for the weight of current income in permanent income that maximized the likelihood function. For the quarterly regression, the values $\hat{\beta}$ = 0, .01, .02, ..., .99, 1.00 were used.

[15]See the analysis of the coefficients below equation (11). The annual coefficients should be approximately four times the quarterly coefficients except for those of y_{Pt} and y_{Tt}. Only the amount by which β_1 exceeds k is multiplied by 4 for y_{Pt}. The coefficient β_2 of y_{Tt} should be essentially

unchanged as the lower quarterly value of λ_3 is offset by a higher value of γ_2 so that the only expected change is due to the slightly higher quarterly value of the $(1 - 0.5r)$ adjustment factor.

[16]For example, a naive trend regression shows a very high R^2 for the annual data:

$$c^x_t = \underset{(-1.14)}{-4.72} + \underset{(88.47)}{1.054}\, c^x_{t-1}$$

where S.E.E. = 5.82, R^2(adj.) = .9968, D-W = 2.13.

[17]The value of k is estimated on the basis of $k = \beta_1 - (1 - \theta/2r)$ $[(1 - \lambda_1)\eta + \lambda_1\alpha_1 - \lambda_3\gamma_1]$. The imputed yield r = 0.1 From regression (18), $\hat{\beta}_1 = 1.08$; $\hat{\lambda}_1 = -(\hat{\beta}_4 + r) / (1 - 0.5r) \approx 0.29$; $\hat{\lambda}_3 = \hat{\beta}_3 / (1 - 0.5r) \approx 0.72$. The values of $\hat{\alpha}_1$ and $\hat{\gamma}_1$ are estimated by dividing the total sample period change in the durables stock and real money stock respectively by the total change in permanent income. So $\hat{\alpha}_1 \approx 0.79$ and $\hat{\alpha}_2 \approx 0.087$. The estimated value of η is computed as $\hat{\eta} = (\Delta d_t)^e / y_{Pt} = \alpha_1(\Delta y_{Pt}/y_{Pt})^e \approx \hat{\alpha}_1[\hat{g}/(1 + \hat{g})]$ $\approx .029$ where $\hat{g} = .03832$ from footnote 12 above. Substitution yields $\hat{k} \approx$ $1.08 - 0.95\ (0.021 + 0.229 - 0.063) \approx 0.90$. The estimate of σ is computed by noting that in the long-run $y^{PS}_t = y_{Pt} - rd_{t-1}$ and $(1 - \sigma) = c^x_t / y^{PS}_t =$ $\dfrac{y_{Pt}}{y^P_t - rd_{t-1}} \dfrac{c^x_t}{y_{Pt}}$. Substituting equation (4) for c^x_t yields

$$1 - \sigma = \frac{1}{1 - rd_{t-1}/y_{Pt}} \left[k + (1 - \theta/2r)\frac{\Delta d_t}{Y_{Pt}} - r\frac{d_{t-1}}{y_{Pt}}\right].$$

Taking $d_{t-1}/y_{Pt} \approx \hat{\alpha}_1 - \hat{\eta} \approx 0.76$, $1 - \hat{\sigma} = (1/0.924)$

$[0.90 + (0.95)(0.029) - (0.10)(0.76)] = 0.92$. So $\hat{\sigma} = 0.08$.

[18]The estimates of the β weight of current income in permanent income were a bit closer to the value of 0.1 per annum estimated in Darby (1974). The only other noticeable--though statistically insignificant--changes were generally higher (in absolute value) estimates for the coefficients of money and the durables stock. All these changes are consistent with the hypothesized shift, but the standard errors of estimate actually deteriorated slightly in the truncated period.

[19]The smaller coefficient of transitory income y_{Tt} in regression 3 than in 2 reflects the larger offset due to the higher money coefficient. Note that, rounding error aside, regressions 2 plus 3 less 0.1 d_{t-1} equal regression equation (18). Similarly, regressions 5 plus 6 less 0.025 d_{t-1} equal regression equation (19).

[20]I am indebted to Tom Mayer for the observation that for low β weights permanent income and the durables stock are closely related because of the high correlation between transitory income and fluctuations in household durables investment. A high estimate of β applies too low a weight to past transitory income and can be offset by a more negative coefficient on the durables stock. This correlation is the probable explanation for the relatively flat likelihood function at the low end of the β range as discussed in Section IV below. In regressions based on the β weight of 0.1 per annum (but not reproduced here), the coefficient of d_{t-1} is -0.092 for the c_t^f dependent variable and -0.102 for the Δd_t dependent variable. Consumer expenditure functions for $\beta = 0.1$ per annum and 0.025 per quarter are presented in Section IV.

[21]As explained in the first part of Section III, the accrual concept is private sector income adjusted for the yield on the durables stock while the cash-receipts concept is disposable personal income with the same adjustment. The conclusions as to the relative merits of the two concepts are not affected by omission of the durables yield adjustment.

[22]To be precise, transfers to foreigners and the statistical discrepancy should also be subtracted.

[23]Exclusive of large negotiable certificates of deposits.

[24]The monthly M_3 data for January 1947 through December 1959 are reported in the Data Appendix. Monthly savings and loan deposits were interpolated between annual (1947-1949) and quarterly (1950-1954) benchmarks by the use of mutual savings bank deposits.

[25]The quarterly sums of squared residuals are biased downwards by the autocorrelation indicated by the low Durbin-Watson statistics. Note that this statistic is 1.08 for the accrual definition and 0.94 for the cash receipts definition (regression 13 and 14 respectively). Only regressions 17 and 18 for the quarterly M_3 comparison are close to a dead heat. That presumably reflects some peculiarity in the data which also accounts for the unusually high β estimate of 0.06 per quarter in regression 17.

[26]In comparing definitions such as these, the hypotheses are not strictly nested and no generally acceptable significance test exists. Consider the following, however. If the difference between the accrual and the cash-receipts definitions were allowed to enter with a weight μ (to be estimated) between 0 and 1, the cash-receipts definition would be nested (with the restriction $\mu = 1$) in the more general hypothesis that

income is the sum of the accrual concept plus μ times the difference. For this model, the sum of squared residuals could not be greater than the sum of squared residuals for $\mu = 0$ (the accrual income definition). If we suppose that this upper limit on the unconstrained sum of squares is the actual value--which is favorable to accepting the cash-receipts definition--the likelihood ratio test could be used. The critical value at the 5% significance level for the excess sum of squares would then be 15.3% for annual data and 3.6% for quarterly data. So even on this apparently generous test, the cash-receipts definition is significantly worse than the accrual definition. The same argument and critical values would apply to the money definitions discussed below.

[27] See Darby (1974, esp. pp. 233-34).

[28] The t-values are given in parentheses. The greater than 90% confidence interval for β is [0, 0.23] for annual data and [0,0.06] for quarterly data.

[29] The analysis of this specification bias is contained in Darby (1974). A 10% per annum β weight was estimated there on the basis of a pure consumption model.