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James J. Heckman
Robert J. Willis

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by

James J. Heckman
University of Chicago and NBER

Robert J. Willis
City University of New York-Graduate Center and NBER*

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## Introduction

In the past few years, there has been substantial progress in the application of the economic theory of household decisionmaking to human fertility behavior.[1] As yet, however, the theoretical and empirical scope of the economic theory of fertility has been quite limited. Observed fertility behavior is regarded as the outcome of utility maximizing choices by couples in which the costs and satisfactions associated with the number and "quality" of children are balanced against the costs and satisfactions of other activities unrelated to children. Theoretical emphasis has been given to the effects of the costs of parental time and money resources devoted to rearing children on the demand for the total number of children in a static framework under conditions of certainty. Empirical work has focused on explaining variation in the number of children ever born to women, who have completed their childbearing, as a function of measures of the household's total resources and the opportunity cost of time, especially the value of the wife's time. Empirical results have been of mixed quality. The value of the wife's time, as measured by her potential market wage or her education, is almost always found to have a significantly negative impact on completed fertility, but measures of husband's lifetime income are not always significant or consistent in sign.[2]

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[1] For a recent collection of papers on the economic analysis of fertility, and citations to earlier work, see T. W. Schultz, ed. (1973).

[2] A number of explanations for the puzzling inconsistency of the "income effect" on fertility has been advanced, but it is probably accurate to say that none has been universally accepted. See Becker (1960), Becker and Lewis (1973), Ben-Porath (1973), Sanderson and Willis (1971), Simon (1973) and Willis (1973).

One important objection to static theories of fertility is their failure to deal with the implications of the simple fact that reproduction is a stochastic biological process in which the number and timing of births and the traits of children (e.g. sex, intelligence, health, etc.) are uncertain and not subject to direct control. To control fertility, a couple can only attempt to influence the monthly probability of conception and, given conception, the probability that pregnancy will terminate in live birth by altering sexual behavior, contracepting or resorting to abortion. As recent work by Ben-Porath and Welch (1972) stresses, this implies that family fertility decisions are inherently sequential and that decisions about further children are made in light of experience with previous children. Moreover, a modest extension of this argument suggests that uncertainty may surround the valuation process itself: until a family has had one child it does not know what the costs and rewards of having a second one would be. Finally, it is evident that uncertainty concerning fertility decisions and realizations adds to and interacts with uncertainty surrounding other jointly determined household decisions about marriage and divorce, consumption and saving, labor supply and investment in human capital.

In this paper, we report some initial results of a study in progress whose goal is to develop an integrated theoretical and econometric model of fertility behavior within a sequential stochastic framework. The principal contribution of the paper is to the development of an appropriate econometric methodology for dealing with some new econometric problems that arise in such models. However, we also present in more tentative form the rudiments of a theoretical model of sequential fertility choice and some empirical estimates of the determinants of the monthly probability of conception in the first birth interval which utilizes our econometric methodology.

Recognizing the sequential and stochastic nature of family decisions, Ben-Porath suggests that "The proper framework for dealing with all the theoretical considerations [involved in the economic analysis of fertility] is a dynamic programming utility maximizing model with the various risks explicitly included." (Ben-Porath, 1973, p. 187). In Section I, we formulate a very simple model of this type to characterize the way in which a couple's contraception strategy evolves over its life cycle as a function of the cost of contraception, age, parity, the time paths of income and the cost of children. In each month of the child bearing period (excluding sterile periods following pregnancy), a couple's contraception decision is assumed to reflect (expected) utility maximizing choices in which the costs of contraception are balanced against the utility associated with each possible fertility outcome weighted by the probability of that outcome. Unfortunately, analytic results are difficult to achieve in such models, even with drastic simplification of the underlying structure of family decisionmaking. At its present stage of development, our theoretical model serves mainly to illustrate the stochastic structure in which fertility decisions are made and their consequences realized.

Even without a fully rigorous theory it is possible to utilize the conceptual framework of a stochastic theory of reproduction in order to determine empirically at what stages of the family building process and through which channels economic variables affect realized fertility outcomes. The full reproductive history of a woman (i.e. the timing of each birth and contraceptive choices in each birth interval) can be used together with the associated economic history of her family in order to investigate the impact of economic variables and accumulated experience on the sequence of contraception decisions beginning with marriage which determine the monthly probability of conception and, hence, the probability distribution of the timing, spacing and total number of births.

In Section II, we present methods to obtain consistent parameter estimates of the effect of economic variables in modifying the monthly probability of conception in the stochastic process. In order to obtain consistent parameter estimates, a number of new econometric problems arise. In particular, we demonstrate that it is important to account explicitly for sources of sample variation, including variation among individuals due to measured and unmeasured components. To avoid bias, it is especially important to take into account persistent variations in the monthly probability of conception among individuals caused by unmeasured differences in fecundity (i.e. the physiological capacity to reproduce), frequency of coition or efficiency of contraception which, in turn, are related to omitted economic variables and family characteristics which determine health, the cost of contraception and the demand for children.

Bias arises when persistent variation is ignored because of a selection mechanism which confounds changes in the behavior of an "average" couple in a sample caused by a change in an economic variable -- the relationship we seek -- with changes in the composition of the sample caused by differential probabilities of conception. For example, the group of women who begin a given birth interval may have an average monthly probability of conception of 0.2. If all women had identical probabilities, the conditional probability of conception in the second month of women who did not become pregnant in the first month would be 0.2. If they are not identical, however, women who survive the first month without conceiving are, on the average, those with the lowest probabilities. Hence, the conditional probability of conception would tend to decline over time because of a change in sample composition, not a change in behavior.

Further, we show that the mean probability of conception in the initial group of women is downward biased if persistent variation is ignored. Our econometric method enables us to estimate the fraction of persistent variance in total variance at the same time that we obtain unbiased estimates of the parameters of exogenous economic and demographic variables.

In Section III, we present parameter estimates of the model from data on the interval between marriage and first pregnancy from the 1965 Princeton National Fertility Study. Our empirical results suggest that the econometric problems discussed in Section III are of considerable practical importance.

## I. Contraception Strategies and Realized Fertility in Stochastic Models of Reproduction

Beginning with the seminal work of Perrin and Sheps (1964), mathematical demographers have developed stochastic models of reproduction in order to study the effects of variations in fecundity (i.e. the biological capacity to reproduce) and contraceptive practice on the number and timing of births over a woman's reproductive life cycle. In this section, we first describe the stochastic structure of these demographic models and then show how choice-theoretic economic models of fertility behavior can be embedded in it.

During any month a woman is in one of five possible states:

$S_0$ - nonpregnant and fecundable

$S_1$ - pregnant

$S_2$ - temporary sterile period due to anovulation following an abortion or miscarriage

$S_3$ - temporary sterile period following a still birth

$S_4$ - temporary sterile period following a live birth

The woman's family building history (i.e., the number and timing of pregnancies and births) is completely described by the sequence of visits she makes to these reproductive states and by the length of time spent in each state at each visit. For instance, the total number of pregnancies she has is equal to the number of transitions from $S_0$ to $S_1$ and the total number of births to the number of transitions from $S_1$ to $S_4$. Similarly, the timing of the first conception for a woman who begins marriage in a nonpregnant fecund state is equal to the length of her first stay in $S_0$ while the length of her first birth interval is equal to the time from marriage until the first transition from $S_1$ to $S_4$.

If it is assumed that the length of stay in each state and the outcome

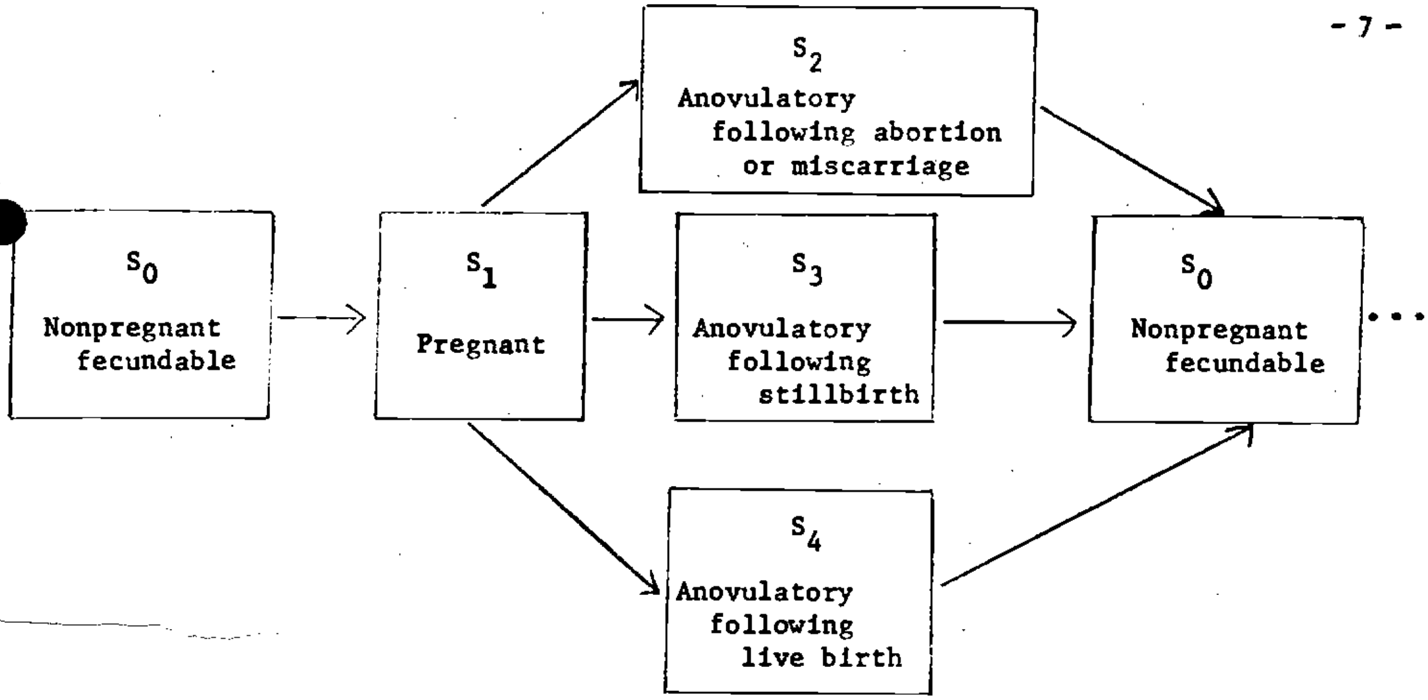


Figure 1: States of the stochastic model of reproduction

(note: adapted from Perrin and Sheps, 1964, p. 33)

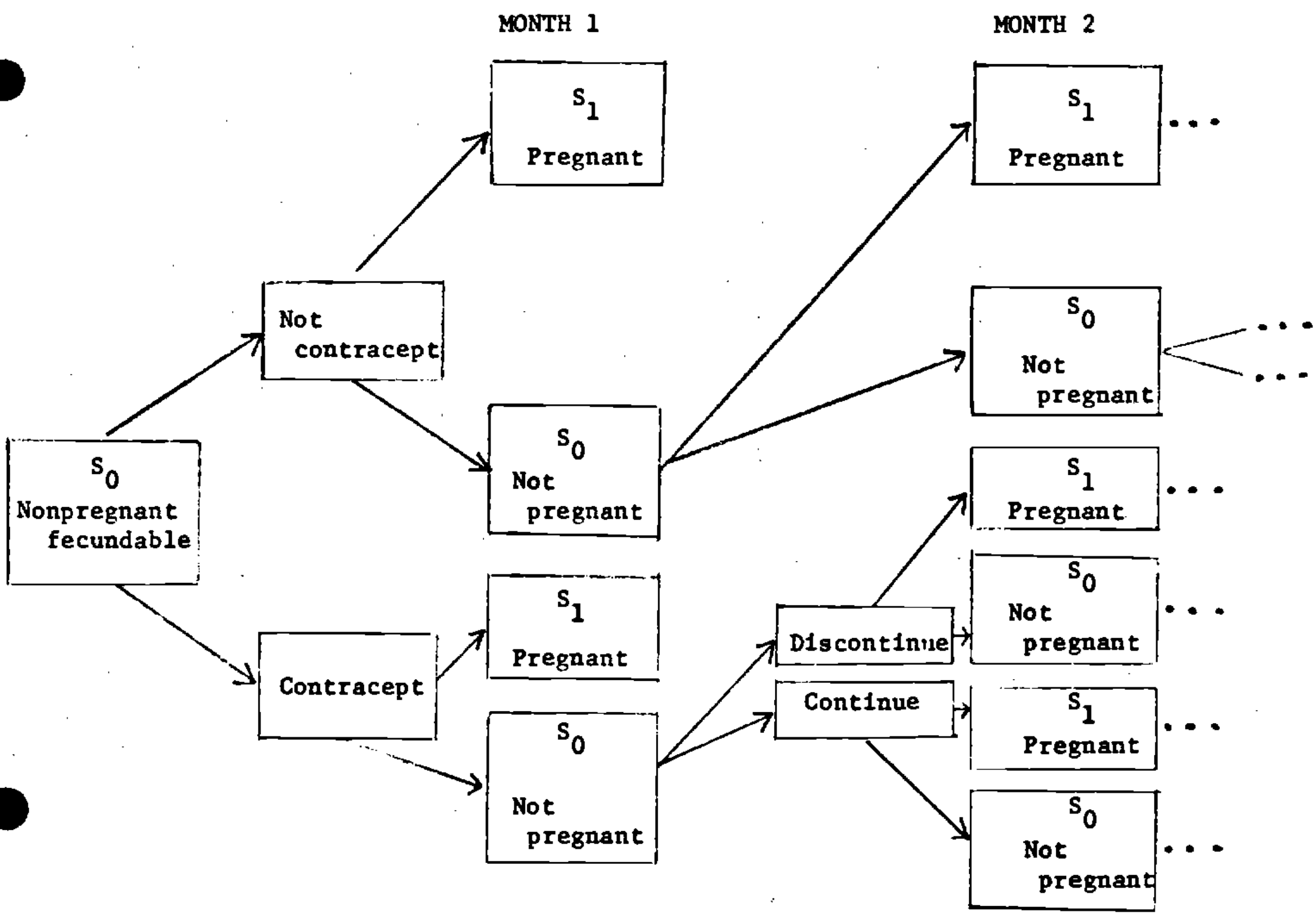


Figure 2: States of contraception decisions and pregnancy outcomes within one pregnancy interval

of each pregnancy are random variables, reproduction may be viewed as a stochastic process such as that represented in Figure 1. Assume that a woman begins marriage in a fecund nonpregnant state ($S_0$). Each month (the approximate length of the ovulatory cycle) she has some probability of conception. This probability is called fecundability by demographers. After a random length of time, she becomes pregnant, passing from $S_0$ to $S_1$. The length of time she stays in $S_1$ is a random variable whose mean and variance depend on the pregnancy outcome. For example, pregnancy lasts an average of perhaps less than three months when terminated by abortion or miscarriage and, of course, about nine months when terminated by a live birth. Finally, each type of pregnancy outcome has a given probability which governs the likelihood that she will pass from $S_1$ to $S_i$ ($i = 2,3,4$). After spending some random length of time in the post-partum sterile period, she reverts back to her initial nonpregnant fecund state $S_0$. Thus, the family building process may be viewed as a sequence of reproductive cycles such as the one represented in Figure 1, each of which is of random length and outcome.

It is clear in this model that a couple confronts considerable uncertainty about the number and timing of births. It is also clear that if fertility outcomes are subject to choice, this choice must be exercised (excluding abortion) through control of the monthly probability of conception, p, by means of contraception or by variations in the frequency and timing of coition over the menstrual cycle. The effect of contraception on the couple's chance of conception in any month may be expressed as

$$p^* = p\ (1 - e)$$

where p is the couple's "natural fecundability" (i.e., the monthly probability of conception in the absence of any deliberate attempt to control fertility) and (1-e) is the proportional reduction in fecundability achieved by contracepting with efficiency e.[3] The value of e depends on both the technical characteristics of the method chosen and the care with which it is used.

The nature of contraception decisions and pregnancy outcomes for a "typical individual" may be examined in more detail with the aid of the elementary branching process depicted in Figure 2. The process is assumed to begin in month 1 when the woman has first entered the nonpregnant fecund state ($S_0$) at marriage or after a previous pregnancy and ends with her passage into the pregnant state ($S_1$) in month t or at the end of the period of observation. Three types of contraception decisions are made within each pregnancy interval. (1) The couple is assumed to decide whether or not to contracept when the woman first enters $S_0$.[4] (2) If the decision is to contracept, the couple selects a given level of contraceptive efficiency, $e_t$, which determines the woman's monthly probability of conception, $p_t{}^* = p(1-e_t)$, $(t=0, 1...)$. (3) If, at the end of month t,

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[3]Natural fecundability is a somewhat misleading term because it depends not only on the physiological characteristics of a woman and her spouse but also on their "natural" pattern of sexual activity. Variations in sexual behavior may arise from differences in sexual preferences of a given couple at different times in their marriage, from variations in preferences among couples or from deliberate attempts to increase or decrease the chance of conception for couples with given preferences. In the latter case, of course, the frequency and pattern of coition should be considered as variation in contraceptive efficiency rather than natural fecundability. Apart from reported use of "rhythm" as a contraceptive method, however, it is difficult to distinguish these two sources of variation empirically.

[4]In Figure 2, we assume that a woman who initially decides not to contracept will never decide to contracept later in this pregnancy interval. The main reason for this assumption is that our data record whether or not a woman contracepted in a given interval and when (and if) she discontinued contraception but do not record when she began contracepting. Since the purpose of contraception is to delay or prevent pregnancy, it seems most plausible, given our data, to assume that she begins contraception as soon as she is at risk (i.e., enters $S_0$).

the woman remains nonpregnant, the couple decides whether or not to discontinue contraception.

Observed fertility outcomes follow as a probabilistic consequence of the contraception strategy adopted by a couple. The length of each pregnancy interval is a random variable whose mean and variance are determined by contraception decisions made within the interval. The sequence of these decisions across intervals determines the probability distribution of the total number of pregnancies and births over a woman's reproductive span. The contraception strategy chosen by a couple is assumed to reflect the interaction of the couple's demand for children (including both number and timing dimensions and embodying their attitude toward risk), the costs of contraception and their past childbearing experience.

The effect of costly contraception on strategy choices and realized fertility can be made clearer with the aid of a simple economic model. Let us assume that a couple receives a flow of c units of child services per child per year for as long as the child remains in the household and that it also receives a flow of s units of other satisfactions unrelated to children. The couple's lifetime utility function is assumed to be: (1) intertemporally additive, (2) of identical form in each year, and (3) characterized by a constant rate of time preference. It is written as

$$U = \sum_{t=o}^{T} d^t \{u(c\,N_t,\ s_t) - f_t\} \tag{1}$$

where T is the family's time horizon in months from the date of marriage at $t = 0$, d is the rate of time preference, u(.) is the flow of utility per month from its consumption of c and s, $N_t$ is the number of children in the household in month t and $f_t$ is the cost of contraception in month t measured in utils.[5] We assume that the monthly contraception cost function takes the form

$$f = f(e) \qquad (2)$$

where e is the efficiency of contraception, noncontraception is costless [i.e. $f(0) = 0$] and increases in efficiency are achieved at increasing cost (i.e. $\frac{df}{de} = f' > 0$).

The couple is assumed to maximize expected lifetime utility subject to its lifetime resource constraint. For simplicity, we make the following additional assumptions: (4) that the household's full income is an exogenous flow of $I_t$ per month; (5) that $\pi_{ct}$ and $\pi_{st}$, the full resource (i.e. time and money) costs per unit of c and s in period t, are exogenous and (6) that no borrowing or lending is possible so that full monthly income is equal to monthly expenditure on c and s. Thus, the flow budget constraint is

$$I_t = \pi_{ct} c N_t + \pi_{st} s_t . \qquad (3)$$

Let us first examine the implications of the model under deterministic conditions by assuming that contraception is costless (i.e. $f_t \equiv 0$) and that there is no biological constraint on fertility (i.e. the couple may choose with certainty to have a birth any time it wishes). At the beginning

[5]In principle, the costs of contraception may include both resource costs (i.e. time and money) and psychic (i.e. util) costs. For simplicity, we have assumed that all costs are psychic. One implication of this is that variations in contraception costs shift the utility function, not the budget constraint. Consequently, variations in these costs cause no income effects.

of marriage, the couple's constrained lifetime utility maximization problem is

$$\max L = \sum_{t=1}^{T} d^t \{u(cN_t, s_t) + \lambda_t (-I_t + \pi_{ct} c\, N_t + \pi_s\, s_t)\} \quad (4)$$

where the $\lambda_t$'s are lagrangian multipliers. It is convenient to rewrite this as an unconstrained maximization problem by substituting the flow budget constraint for $s_t$ in the flow utility functions to obtain the problem

$$\max L = \sum_{t=1}^{T} d^t v_t(N_t) \quad (5)$$

where

$$v_t(N_t) = v(cN_t, I_t, \pi_{ct}, \pi_{st}) = u[cN_t, 1/\pi_{st}(I_t - \pi_{ct}\, cN_t)]$$
$$= u\,(cN_t, s_t)$$

is the couple's indirect flow utility function in period t and where the number of units of child services per month, c, received from each child is set equal to one. Once born, a child is assumed to remain in the household permanently so that the stock of children can never be decreased (i.e. $N_0 \leq N_1 \leq \ldots \leq N_t$).

The couple's utility flow in any month t = 1, ..., T is determined by the number of children, $N_t$, present in the household during that month according to the indirect flow utility function, $v_t(N_t)$, which is a concave function of the form illustrated in Figure 3.

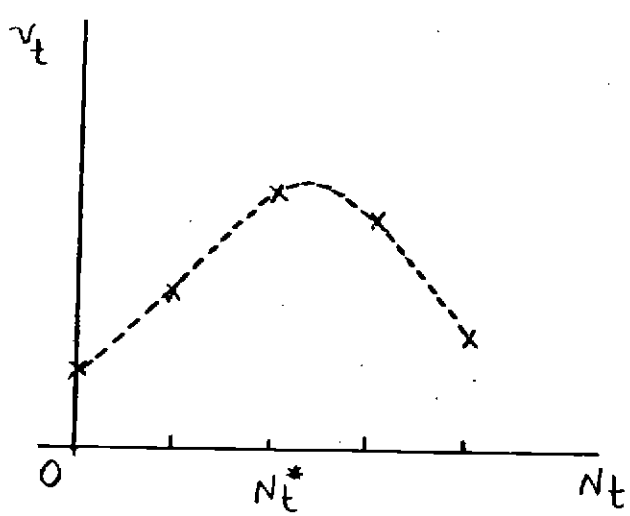


Figure 3

Let $N_t^*$ be the integer value of $N_t$ that maximizes $v_t$. Given assumptions (1) - (6) above, the time path of $N_t^*$ depends on the time paths of full income, $I_t$, and the relative resource costs of child services, $\pi_{ct}/\pi_{st}$. In the simplest case, for example, $N_t^*$ would be a constant over the life cycle if $I_t$ and $\pi_{ct}/\pi_{st}$ were constant because the $v_t$ functions would be identical over time and, therefore, each would be maximized by the same number of children. If $I_t$ grew during the life cycle, and child services have a positive income elasticity, the time path of $N_t^*$ would tend to be an increasing step function.[6] Similarly, holding $I_t$ constant, an increasing time path of $\pi_{ct}/\pi_{st}$ would generate a time path of $N_t^*$ which is a decreasing step function.

In the absence of any biological constraint on obtaining children, a couple's optimal stock of children at any time $t_0$ is equal to $N_{t_0}^*$ provided that the future time path of $N_t^*$ is constant or increasing; if it is decreasing, the optimal value of $N_{t_0}$ is less than (or equal to) $N_{t_0}^*$ because the family cannot decrease its stock of children when that stock becomes "too large." In the case of constant or decreasing $N_t^*$, the couple would optimally have all of its children simultaneously at the beginning of marriage and, in the case of rising $N_t^*$, births would be spaced. These implications suggest that births are more likely to be widely spaced, the more rapidly rising is the life cycle profile of full

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[6]Since $N_t$ can take only integer values, income must grow by a finite amount in order to increase the flow utility maximizing number of children by one. It should be noted that the time path of $N_t^*$ would be unrelated to the time path of $I_t$ in a perfect capital market because monthly resource expenditures would be constrained by wealth rather than current income. This argument also abstracts from any functional relationship between $I_t$ and $\pi_{ct}/\pi_{st}$ operating through the value of time (see Willis, 1973).

income, and are more likely to be closely spaced the more rapidly rising is the time path of relative resource cost of child services.[7]

We now relax the assumption that a couple may costlessly choose any number and timing pattern of births it wishes with certainty. Instead, we assume that the couple chooses in any month of the woman's childbearing period (excluding sterile periods due to pregnancy or postpartum anovulation) a monthly probability of conception, $p_t^* = p\,(1-e_t)$ by using contraception with efficiency $e_t$ at a cost in utils of $f_t = f(e_t)$ so as to maximize expected lifetime utility in the remaining T-t months of life.

The nature of the decision-making problem may be illustrated by considering the couple's decision of whether or not to contracept in the first month after marriage on the assumption that the woman is initially childless, nonpregnant and fecund. At the beginning of the month, the couple selects a value of contraceptive efficiency, $e_1$, ( $0 \leq e_1 \leq 1$) at a cost of $f(e_1)$ where, of course, the choice of $e_1 = 0$ corresponds to a decision not to contracept and noncontraception is costless (i.e. $f(e_1) = f(0) = 0$). The woman's chance of conception during the month is $p_1^* = p(1-e_1)$ and her chance of remaining nonpregnant is $1- p_1^*$, where p is her natural fecundability (i.e. chance of conception in the absence of contraception). For simplicity, assume that all conceptions result in live births and that all children survive to the end of the couple's time horizon, T.

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[7] It should be stressed again that the present model is a very simple one which should be elaborated before hypotheses derived from it are taken too seriously. As obvious examples, allowance might be made for (1) variations in child "quality" (e.g. by letting the number of units of child services per child be a choice variable), (2) variation in the scale and time intensity of resources devoted to children as a function of their age (e.g. plausibly, children become less time intensive as they age) or (3) investment in human capital by the husband and wife and its interactions with the cost of children.

The couple's expected lifetime utility at the beginning of the second month of marriage is conditional on which event, conception or nonconception, occurs in the first month. If the woman conceives in the month 1, let $V_2(b_1)$ be the couple's expected lifetime utility at the beginning of month 2 on the assumption that the couple follows an optimal, expected utility maximizing contraception strategy in all subsequent time periods conditional on beginning month 2 in a pregnant state. Similarly, let $V_2(\sim b_1)$ be expected lifetime utility at the beginning of month 2 conditional on entering that month in a nonpregnant state.[8] The couple's expected lifetime utility at the beginning of marriage may then be written as

$$V_{01} = p_1^* \{V_2(b_1) - f(e_1)\} + (1-p_1^*) \{V_2(\sim b_1) - f(e_1)\}^9 \quad (6)$$

where, recall, $p_1^* = p\,(1-e_1)$ .

We may now examine the conditions under which a couple will contracept in month 1 and, if so, how efficiently. If the couple chooses not to contracept (i.e. it selects $e_1 = 0$ and, since $f(0) = 0$, it incurs no costs of contraception), its expected lifetime utility is

$$V_{01} = p\, V_2(b_1) + (1-p)\, V_2(\sim b_1) = V_2(\sim b_1) - p\Delta V_2(\sim b_1) \quad .$$

The term $\Delta V_2(\sim b_1) = V_2(\sim b_1) - V_2(b_1)$ is the expected lifetime utility

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[8] More generally, we may use the notation $V_t(b_n)$ to denote the expected utility in the remaining portion of life of a couple that conceives in month t and whose parity (i.e. number of previous births) is n-1 at the beginning of month t-1 and $V_t(\sim b_n)$ for the corresponding case of nonconception. Later, we shall illustrate the meaning of these terms more concretely.

[9] The general notation for expected utility over the remaining portion of life for a couple with a stock of n children at the beginning of month t is $V_{nt}$. For expositional simplicity, the flow utility from zero children during month 1, $v_1(0)$, is omitted from equation (6) since it does not depend on whether or not the woman conceives and, therefore, does not affect the couple's decisions. Similarly, the term $v_t(n)$ is omitted in the more general expression for $V_{nt}$ in equation (9) below.

of preventing a conception in month 1. If $\Delta V_2(\sim b_1)$ is positive, the couple will choose to contracept (assuming that the marginal cost of contraception, $f' = \frac{df}{de}$, is zero in the neighborhood of $e = 0$) and, if it is negative, the couple will choose not to contracept.

Assuming that $\Delta V_2(\sim b_1)$ is positive, the couple selects the value of contraceptive efficiency that maximizes $V_{01}$ in equation (6). The first order condition for a maximum is

$$\frac{dV_{01}}{de_1} = p\,\{V_2(\sim b_1) - V_2(b_1)\} - f' = p\Delta V_2(\sim b_1) - f' = 0 \qquad (7)$$

and the second order condition is

$$\frac{d^2V_{01}}{de_1^2} = -f'' < 0. \qquad (8)$$

In words, the first order condition states that the optimal value of $e_1$ is such that the marginal cost of efficiency, $f'$, is equal to the expected marginal benefit of efficiency, $p\Delta V_2(\sim b_1)$, where $p = -\frac{dp_1^*}{de_1}$ is the rate of decrease in the chance of conception with respect to contraceptive efficiency and, as before, $\Delta V_2(\sim b_1)$ is the expected utility of preventing a conception. The second order condition implies that the marginal cost of efficiency must be rising if values of $e_1$ strictly greater than zero or less than one can be optimal.

This analysis is illustrated diagramatically in Figure 4 where the horizontal curves $MB_a$, $MB_b$ and $MB_c$ correspond to three possible values of the expected marginal benefit of preventing a conception (i.e. $MB = p\Delta V_2(\sim b_1)$) and the curve 0d is the marginal cost curve of contraceptive efficiency which reaches its upper limit of one at point d. If the value of preventing a birth is sufficiently high (e.g. $MB_a$), the couple will contracept perfectly ($e_a = 1$), perhaps by practicing abstinence. Given a lower marginal benefit such as $MB_b$, the couple will practice contraception imperfectly and confront the risk $p_1^* = p(1-e_b)$ of having an "accidental"

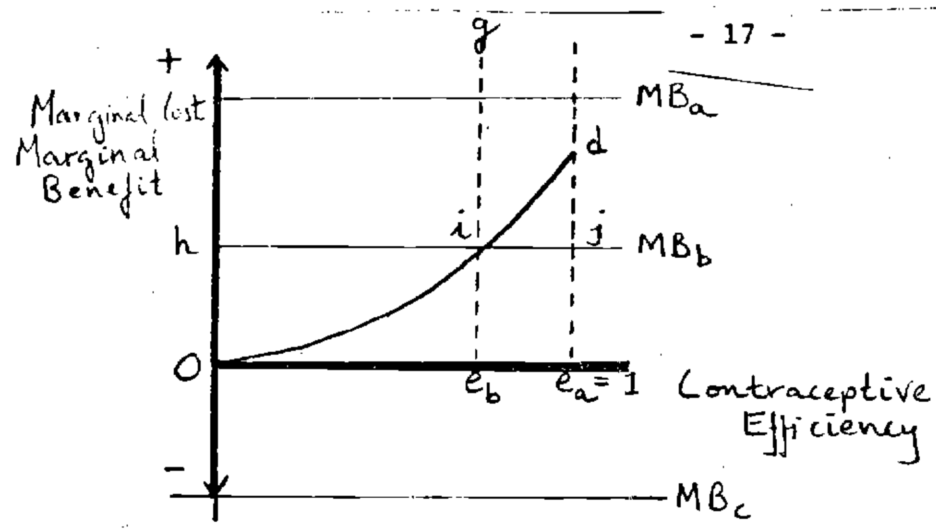


Figure 4

conception in the first month of marriage. Finally, if the expected utility of preventing a conception is negative (e.g. $MB_c$), the couple will not contracept and will have a probability p of having a "desired" conception in month 1.

The preceding analysis is easily extended to the contraception decisions of a couple with n children at the beginning of month t.[10] Generalizing equation (6), the couple's expected utility over the remaining portion of life is

$$V_{nt} = p_t^* \{V_{t+1}\ (b_{n+1}) - f(e_t)\} + (1-p_t^*)\ \{V_{t+1}\ (\sim b_{n+1})\ - f(e_t)\}$$

$$= V_{t+1}\ (\sim b_{n+1}) - p_t^*\ \Delta V_{t+1}(\ b_{n+1}) - f(e_t) \qquad (9)$$

where $\Delta V_{t+1}\ (\sim b_{n+1}) = V_{t+1}\ (\sim b_{n+1}) = V_{t+1}(b_{n+1})$ is the expected utility of preventing the conception of the n+1 child in month t. As before, the couple's optimal decision is not to contracept if $\Delta V_{t+1}(\sim b_{n+1})$ is negative and, if it is positive, to select $e_t$ such that $f' = p\Delta V_{t+1}(\sim b_{n+1})$. The sequence of contraception decisions made by a couple depends on how the sign and magnitude of $\Delta V_{t+1}(\sim b_{n+1})$ varies with time and parity and with the probabilistic outcome of these decisions in terms of the actual

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[10] It is assumed, of course, that the woman is in a nonpregnant fecund state at this time.

timing and number of pregnancies and births the couple experiences.

In the first birth interval (i.e. the interval between marriage and first pregnancy), parity remains constant at zero, but time varies. As is indicated schematically in Figure 2, a number of alternative sequences of decisions within this interval are possible. If $\Delta V_t(\sim b_1)$ is initially negative and remains so over time, the couple will not contracept during the interval and, therefore, faces a constant monthly probability of conception, p. The length of time it takes the woman to conceive is a random variable, distributed geometrically with mean $1/p$ and variance $(1-p)/p^2$.[11] If $\Delta V_t(\sim b_1)$ is positive and remains roughly constant over time, the couple will contracept at some given level of efficiency such as $e_b$ in Figure 4 until an "accidental" pregnancy occurs. In this case, the monthly probability of conception is the constant $p^* = p(1-e_b)$ and the mean and variance of waiting time to conception are increased to $1/p^*$ and $(1-p^*)/p^{*2}$, respectively.

Another possibility is that $\Delta V_t(\sim b_1)$ is initially positive, but decreases over time until it becomes negative as would be indicated in Figure 3 by a progressive decrease in the marginal benefit of contraceptive efficiency from $MB_b$ to $MB_c$. In this case, according to Figure 4, the couple initially contracepts with efficiency $e_b$ and, assuming the woman remains nonpregnant, continues to contracept, but with decreasing efficiency, until MB becomes negative at which time the couple discontinues contraception. It follows that $p_t^*$ continuously increases until it equals p at the time of discontinuation.[12]

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[11] See, for example, Sheps (1964).

[12] Another less plausible possibility is that $\Delta V_t(\sim b_1)$ is initially negative and increases over time until it becomes positive. In this case, of course, the couple would begin contracepting once $\Delta V_t(\sim b_1)$ became positive, provided that the woman did not become pregnant in the initial period of noncontraception.

To the extent that decreasing contraceptive efficiency involves a switching of contraceptive techniques rather than using a given technique with less care, the decline in efficiency may be substantially less than would be indicated by following the marginal cost curve 0d in Figure 3 as $MB_t$ decreases. As an extreme example, suppose that a couple initially chooses a technique such as the IUD which has a "technologically" fixed level of efficiency equal to $e_b$. Further suppose that the monthly cost of wearing an IUD is zero once it has been inserted.[13] The "supply" of contraceptive efficiency, given the choice of an IUD, is then the dotted vertical line $e_b g$ in Figure 4. As long as $\Delta V_t(-b_1)$ is positive, the couple contracepts with efficiency $e_b$ and faces the constant probability of conception $p^* = p\,(1-e_b)$; when $\Delta V_t(-b_1)$ becomes negative, the woman has the IUD removed.

The theory can easily be extended to deal with the choice of contraceptive techniques such as the IUD which involve fixed costs as well as variable monthly "user" costs. To take the simplest example, suppose the couple must choose either the IUD or not contracept at all during the first birth interval. The maximum price (in utils) that the couple would be willing to pay to have an IUD inserted at the beginning of marriage is equal to the discounted sum of expected utility gains from wearing an IUD. This "demand price" is $\sum_{t=0}^{t_0} d^t(1-p^*)^t MB_t e_b$ where $MB_t e_b$ is the total expected utility gained from contracepting with efficiency $e_b$ in month t, $t_0$ is the duration of contraception before voluntary discontinuation, d is the rate of

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[13] This assumption abstracts from that possibility that the IUD produces unpleasant and "costly" side effects such as cramping. We also abstract from the possibility that the device may be expelled involuntarily.

time discount, $p^* = p(1-e_b)$ and $(1-p^*)^t$ is the probability that the woman goes t months without conceiving. If the demand price exceeds the cost (in utils) of inserting an IUD, the couple will contracept. It is easy to see that the probability of choosing the IUD is greater the more effective is the IUD, the higher the marginal benefit of preventing a conception in each month, the lower is the rate of discount, and the longer is the desired duration of use.[14]

So far, we have discussed how the marginal (and fixed) costs of contraception interact with various possible time paths of the marginal benefit of contraception within the first birth interval to generate the sequence of decisions to contracept or not contracept, to select optimal levels of contraceptive efficiency and to discontinue contraception which are depicted schematically in Figure 2. Clearly, a similar analysis of contraception decisions in subsequent birth intervals is possible. For example, a decision to contracept in the first month - say $t = t_2$ - that a woman enters the nonpregnant fecund state $\mathcal{S}_0$ after the birth of her first child would be optimal if the marginal benefit of contraception, $p\Delta V_{t_2}(\bar{b}_2)$, is positive. The sign and magnitude of $\Delta V_{t2}(\bar{b}_2)$ depends both on the woman's parity - she now has one child - and on the timing of her first birth which is the probabilistic outcome of contraception decisions in the first birth interval.

In general, our model suggests that the reproductive history of a woman (i.e. the number and timing of pregnancies and births) may be

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[14]The extension of this analysis to choice among many alternative forms of contraception which have different fixed and variable costs is straightforward, but beyond the scope of this paper. For an analysis of the choice of contraceptive technique in a static framework, see Michael and Willis (1973).

regarded as the realization of a stochastic process whose parameters are determined by the biological capacity of a couple to reproduce and by the sequence of contraception decisions the couple makes. These decisions, in turn, depend on the costs of contraception and the sign and magnitude of the marginal benefits of contraception ($p\Delta V_t(-b_n)$) as it varies with time and parity.

It is apparent that any hypotheses that may emerge from our model about the effect of economic variables on contraception decisions and realized fertility depend crucially on our capacity to derive the relationship between these variables and the $\Delta V_t(-b_n)$. Formally, the model of optimal decisionmaking that we have specified requires a couple to solve a stochastic dynamic programming problem at the beginning of each month from marriage to menopause whose answer is summarized by the sign and magnitude of $\Delta V_t(-b_n)$. Unfortunately, the rigorous analysis of these dynamics programming problems remains on our agenda of future research.[15]

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[15]Two issues that may occur to the reader at this point deserve brief comment. First, it is known that stochastic dynamic programming problems are difficult to solve and often do not yield many predictions. We are encouraged on this issue by the recent work of McCabe and Sibley (1973) who have obtained comparative static results using dynamic programming techniques in a model of sequential fertility behavior which assumes perfect fertility control but allows for uncertainty about future income and wage rates. Second, it may strain the credibility of the reader to suppose that behavior is in fact governed by the complex calculations implied by our model. Without attempting to add to or resolve the ancient controversy concerning the realism or relevance of deriving hypotheses by assuming optimizing behavior, we shall simply assert that it is plausible to imagine that "rules of thumb" or "behavioral norms" which emerge to guide decisionmaking in complex situations tend to be perpetuated to the extent they approximate optimal decisions. If this is the case, optimizing models can be a fruitful source of empirical hypotheses about behavior.

It is possible, however, to use a simple two period dynamic programming model to illustrate the meaning of $\Delta V_t$ $(-b_n)$ more concretely than we have done so far and to show how current contraception decisions are influenced by a positive probability of "accidental" pregnancies in the future under conditions of costly and imperfect contraception. Paradoxically, for example, we can show that a couple might find it optimal to contracept when contraception is costly in situations in which it would not contracept if contraception were perfect and costless. This implies that, under certain conditions, a decrease in the marginal cost of contraception may decrease the probability that a couple contracepts. A motivation for such behavior is suggested by the demographer Nathan Keyfitz (1971) who argues that the increase in the efficiency of modern birth control techniques has allowed couples to concentrate their childbearing in the early years of marriage instead of spacing them widely to avoid the chance of ending up with "excess" fertility.

To examine the plausibility of Keyfitz's argument for "precautionary contraception", consider the following two period model. Let us suppose that a couple has $N^* - 1$ children at the beginning of period 1 and that the per period flow of utility from children is such that $v(N^*-1) < v(N^*) > v(N^*+1)$ so that we may say that $N^*$ is the desired stock of children.[16] The couple's decision problem is to decide whether or not to contracept during period 1.

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[16] For expositional simplicity, we assume that children are conceived at the beginning of a period and born at the end of the period after which they provide utility to their parents; that the rate of time preference is zero and that there is no sterile period following birth. Under conditions of certainty, the couple's maximum lifetime utility at the beginning of period 1 would be $v(N^*) + v(N^*)$.

The couple begins period 2 with either $N^*$ or $N^*-1$ children depending on whether or not it conceived in period 1. If it begins period 2 with $N^* - 1$ children, it maximizes expected utility in the final period by not contracepting. In this case, using the notation we defined for the general T period model, its expected utility at the beginning of period 2 is

$$V_2(\sim b_n^*) = v(N^* - 1) + p\, v(N^*) + (1-p)v(N^* - 1).$$

If the couple begins period 2 with $N^*$ children, it is optimal to contracept in order to reduce the chance of "excess fertility". Its expected utility is

$$V_2(b_n^*) = v(N^*) + p_2^* \, v(N^* + 1) + (1-p_2^*)\, v\, (N^*) - f(e_2)$$

where $p_2^* = p\,(1-e_2)$ is the probability of having the $N^* + 1$ child, $f(e_2)$ is the cost of contraception in period 2, and $e_2$, the optimal level of contraceptive efficiency, is chosen such that the marginal benefit [(i.e. $p[v(N^*) - v(N^* + 1)]$)] and marginal cost (i.e. f') of contraception are equated.

The couple's decision about whether or not to contracept at the beginning of period 1 depends on the sign of $\Delta V_2(\sim b_n^*) = V_2(\sim b_n^*) - V_2(b_n^*)$ which, as before, is interpreted as the expected utility of preventing a conception in period 1 on the assumption that the couple pursues optimal (expected utility maximizing) decisions in future period(s). Using the expressions derived above, we see that

$$\Delta V_2(\sim b_n^*) = (2-p)\ \{v(N^*-1) - v(N^*)\} + p_2^*\ \{v(N^*) - v(N^* + 1)\} + f(e_2).$$

If contraception is perfect (i.e. $p_2^* = 0$) and costless (i.e. $f(e_2) = 0$), $\Delta V_2(\sim b_n^*)$ is negative since $v(N^* - 1) < v(N^*)$. In this case, the couple will not contracept in period 1 in order to maximize its chance of having the $N^*$ child. If, however, contraception is costly and imperfect, the positive terms, $p_2^*\ \{v(N^*) - v(N^* + 1)\} + f(e_2)$, may be of sufficient magnitude to make $\Delta V_2(\sim b_n^*)$ positive and, as Keyfitz conjectured, lead the couple to contracept before reaching its "desired" number of children.

These positive terms have a simple economic interpretation as the total opportunity cost of imperfect contraception. This may be illustrated in Figure 4 (page 17 above) on the assumption that $e_2 = e_b$ and $p\{v(N^*) - v(N^*+1)\} = f' = MB_b$. The total opportunity cost of imperfect contraception is equal to the area $0ije_a$ which, in turn, is equal to the sum of the direct cost of contraception, $f(e_b)$, given by the area $0ie_b$ under the marginal cost curve and the expected loss of potential utility from "excess" fertility, $p_2^* \{v(N^*) - v(N^* +1)\} = MB_b(1-e_b)$, which is equal to the area of rectangle $e_b ije_a$. The upper limit of the opportunity cost of imperfect contraception is equal to the direct cost of perfect contraception (i.e. $f(e_2) = f(1)$) given by area $ode_a$ in Figure 4.

In our two period example, it is evident that a necessary condition for a couple to engage in precautionary contraception is that the loss of potential utility from one child too many, $v(N^*) - v(N^*+1)$, is substantially greater than the loss from one child too few, $v(N^*) - v(N^*-1)$. While this might be true, it need not be. Indeed, on grounds of symmetry it might be argued that, on the average, the losses from one too few children and one too many children are about equal so that precautionary contraception would occur in only a minority of cases. Possibly, the incentive to engage in precautionary contraception is greater in the general multi-period case because of the chance of higher levels of excess fertility (i.e. the chance of having births $N^* + 2$, $N^* + 3$ and so on). Unfortunately, examination of this possibility must await rigorous analysis of the more general model.

We shall conclude this section by considering the effects of variations in economic variables on the optimal path of contraception decisions a couple would follow under the simplifying assumption that it may contracept perfectly at zero cost. In this way, we eliminate consideration of the effect on current decisions of the risk of future contraception costs and risks of "accidental" pregnancies while contracepting since $f(e_t) = 0$ and $p_t^* = 0$ in every month in which $\Delta V_t(-b_n)$ is positive. The analysis is nearly identical to our earlier discussion of fertility behavior in the absence of a biological constraint on fertility, except that now the couple cannot obtain children as rapidly as it wishes.

Recall, for example, that we showed that if the flow of full income, $I_t$, and the relative cost of child services, $\pi_{ct}/\pi_{st}$, are constant over the life cycle, the optimal stock of children, $N_t^*$, is also a constant - say $N^*$ - in every month. In this case, the couple will not contracept until a parity of $N^*$ is reached and will contracept perfectly thereafter. Although sufficient changes in the levels of income or cost of child services may change the optimal stock of children, they will have no effect on behavior (e.g. the monthly probability of conception) until $N^*$ is reached. For instance, if $N^*$ is always greater than one child, variations in income and the cost of children will not influence contraception decisions in the first birth interval.

If the cost of child services follows a rising time path (e.g. because of an increasing wage profile of the wife) and $I_t$ is constant, our earlier discussion implies that the optimal stock of children will tend to decrease at discrete time intervals during the life cycle. Provided that the optimal stock at the beginning of marriage exceeds one child, the couple will not contracept during the first birth interval. Since the timing of the first birth is a random variable, the optimal stock of children at the beginning of the second birth interval will vary across individual households which initially had identical "fertility goals". Those couples who had their first child quickly would have larger optimal stocks of children at the beginning of the second interval than those who took longer to conceive the first child. Consequently, the probability that a couple will go on to have a second child is negatively related to the length of the first birth interval. Extending the argument to subsequent birth intervals, the probability that a couple terminates childbearing with the nth child is positively related to the length of time it has taken the couple to achieve parity n. Thus, in the case of an exogenously rising time path of the cost of child services, the completed fertility of a group of initially identical households is dependent on the realized timing of births.[17]

A different pattern of behavior is implied by the assumption of a rising time path of full income assuming constant $\pi_{ct}/\pi_{st}$ since, as we showed earlier, the optimal stock of children, $N_t^*$, will tend to increase at discrete times during the life cycle. If $N_t^* = 0$ for a period of time, the couple will contracept at the beginning of marriage, then discontinue contraception when income has risen sufficiently to make $N_t^* = 1$.[18] If the first child is born before $N_t^*$ increases to two, the couple will again practice contraception in the second interval, discontinue when $N_t^* = 2$ and so on until the highest value of $N_t^*$ is reached at the peak of the income profile (assuming that $I_t$ remains constant thereafter). Once actual parity reaches this level (there is, of course, some probability that it will not),

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[17] An interesting extension of this analysis would be to consider the interaction between contraception strategy and the wife's accumulation of human capital via labor force experience. See Mincer and Polachek (1974) for evidence that female wage rates are quite responsive to labor force experience which, in turn, is strongly related to the wife's reproductive history.

[18] Assuming that a major purpose of marriage is to have children, a (potential) couple may delay marriage until $N^* = 1$. Another possibility, of course, is that marriage may be delayed until an actual parity of one is imminent. Despite these considerations, we treat the date of marriage as an exogenous event in this paper.

the couple will contracept permanently. This analysis suggests that the more steeply rising is the income profile, the more likely it is that couples will contracept in order to space their births.[19] It also implies that the probability that a couple will contracept for spacing purposes in the second or higher intervals is greater, the faster its earlier births occurred. Finally, an upward shift in the level of the income profile (or decrease in the cost of child services) will tend to increase $N_t^*$ for all $t=1,\ldots, T$, thus reducing the probability that a couple will contracept at any given time and increasing the maximum value of $N_t^*$.

In this section, we have shown how a choice-theoretic economic model of fertility behavior can be embedded in the stochastic structure of demographic models of reproduction depicted in Figures 1 and 2. Our model implies that the sequence of decisions to contracept, the choice of contraceptive efficiency and decisions to discontinue contraception that are made as a couple proceeds through its reproductive life cycle may be interpreted as a contraception strategy in which decisions at each time and parity level are based on current and future values of income, costs of child services and costs of contraception. It also implies that a woman's actual reproductive history can be interpreted as the probabilistic consequence of this strategy.

It is clear that much remains to be done before a complete economic model of fertility behavior within a sequential stochastic framework is achieved. The rather simple model specified in this paper has not yet been fully analyzed in the general T period case under conditions of imperfect

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[19] As we noted earlier, in a perfect capital market the value of $N_t^*$ depends only on the present value of the income profile and is independent of its shape. In this case, the slope of the time path of $N_t^*$ is rising, constant, or falling according to whether the rate of interest is greater than, equal to, or less than the rate of time preference.

contraception. Consequently, we are not yet certain what implications the model has for effects of variation in the levels and time paths of income and the cost of children on optimal contraception decisions when there are risks of future "accidental" pregnancies.

It is also evident that the specification of the model abstracts from a number of aspects of family decisionmaking and the environment in which these decisions are made which probably have a substantial impact on contraception strategy. For example, we have assumed that the flow of child services from a given child and the costs of producing these services are independent of the child's age, sex or other traits and the presence and characteristics of other children. We have also assumed that the flow of services from a child cannot be increased by the expenditure of resources on child "quality". Obviously, specification of a household production function for child services which incorporated these factors might considerably alter the implications of the model for desired spacing patterns under perfect contraception and attitudes toward the risk of unwanted pregnancies under imperfect contraception. Other factors that deserve consideration include the effect on fertility decisions of uncertainty about future income and wage rates; decisions concerning investments in human capital and life cycle labor supply by husbands and wives; and decisions about the timing of marriage and choice of spouse's characteristics.

While further theoretical progress is highly desirable, it is of equal importance to design and implement empirical methods by which we may determine the effect of economic variables on realized fertility as these effects are channeled through the sequence of decisions we have called a couple's contraception strategy. Our ultimate empirical objective is to use data on the full reproductive histories of women to estimate the effect of economic variables and prior experience with the fertility process on contraception decisions in successive birth intervals. By directly estimating the constituent probabilities of the fertility process (i.e. the probability of contracepting, the monthly probability of contraception conditioned on contraception and the probability of discontinuing contraception) as it evolves over the reproductive life cycle, we can explain completed fertility as well as the timing, spacing and contraception decisions which lead to completed fertility. We can then use the estimated probabilities to simulate the effects of economic variables on the aggregate birth rate, and can determine at what stages and in what decisions economic variables contribute to the explanation of observed fertility outcomes.

It is obvious, however, that many additional, usually unmeasured and frequently persistent factors influence contraception decisions and fertility outcomes. Among these are variations among couples in natural fecundability due to differences in health or taste for sexual activity and variations in contraceptive efficiency caused by differences in the taste for children or distaste for using contraceptives. As we demonstrate theoretically in the next section and empirically in the final section, these unmeasured components of persistent variation in p and e raise a serious statistical problem in obtaining unbiased estimates of the effect of economic variables on the monthly probability of conception of the representative or average couple in a sample. We now turn to an examination of this problem and present a method for resolving it as one step toward our longer run objective of estimating the stochastic structure of an economic model of reproduction.

## II. Serial Correlation Problems

In the previous section, we presented an economic model of fertility behavior within a sequential stochastic framework. It is important to note that this structure, as represented by the schema in Figures 1 and 2 of the previous section, has been presented only for a typical individual. Unless very strong statistical assumptions are made, the simple semi-Markov structure does not lead to a sample likelihood function in which estimated parameterized probabilities can be said to predict accurately the probabilities of observed events for individuals. To see that this is so, it is important to distinguish three sources of variation in observed birth intervals among individuals: (1) purely random factors that arise independently in each time period, and are independent of random factors in other time periods, (2) random factors, including unobservable variables, that are correlated across time periods, (3) deterministic variables such as income and education that can be measured, and which are assumed to affect the probabilities.

To fix ideas, suppose we are concerned solely with estimating the probability process determining whether a woman has a first pregnancy. Inherent in the model is the notion of a time series of events. A woman has a first pregnancy in month $j$ only if she has not had a first pregnancy in months $1,\ldots,j-1$. The most general way to model this probability is to imagine a set of continuous random variables $S_1$, $S_2$, ..., which may be thought of as index functions. The $S_i$, $i=1, \ldots, \infty$, are assumed to be intercorrelated. The event of a woman becoming pregnant in the first interval depends on what value the "wheel of chance" throws up for $S_1$. Suppose that her education E is the only economic variable of interest. We may then define $\alpha_o + \alpha_1 E$ so that if $S_1 < \alpha_o + \alpha_1 E$ a woman becomes pregnant in the first interval and leaves the sample while if the inequality is reversed, the woman is not pregnant and stays in the sample. The probability of a woman becoming pregnant in the jth interval is thus

$$\Pr\ (S_1 > \alpha_o + \alpha_1 E, \ldots, S_{j-1} > \alpha_o + \alpha_1 E_1, S_j < \alpha_o + \alpha_1 E) \quad . \qquad (10)$$

If we assume that the $S_i$ are independently and identically distributed, this probability may be written as

$$\prod_{i=1}^{j-1} \Pr\ (S_i > \alpha_o + \alpha_1 E) \quad \Pr(S_j < \alpha_o + \alpha_1 E) \quad . \qquad (11)$$

If each $S_i$ is assumed to be distributed normally with mean zero, and variance $\sigma_s^2$, the probability statement may be written using the probit function,

$$\left[ \int_{\frac{\alpha_o + \alpha_1 E}{\sigma_s}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \right]^{j-1} \int_{-\infty}^{\frac{\alpha_o + \alpha_1 E}{\sigma_s}} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \qquad (12)$$

If the $S_i$ were assumed to be logistically distributed, a similar probability statement using cumulative logistics could easily be written.

If the $S_i$ for all women are generated by the same random process, we may use the principle of maximum likelihood to estimate $\frac{\alpha_o}{\sigma_s}$ and $\frac{\alpha_1}{\sigma_s}$ by taking a sample of women with different birth intervals, and choosing parameter values which maximize the probability of observing the sample distribution of birth intervals.

Note, however, a crucial step in the argument. We assumed that over time, the $S_i$ were independently distributed. This assumption rules out serial correlation in the S sequence. Such serial correlation may naturally arise if there are unmeasured random variables which remain at, or near the same level, over time for a given individual, but which are randomly distributed among individuals. For example, unmeasured components of fecundability (e.g. semen counts of husbands, tastes for coital activity, and variations in contraceptive efficiency) plausibly have a persistent component for the same individual across time periods although these components may vary widely among individuals.[20] Similarly, important economic variables may be missing in a given body of data.[21]

Following a convention in the analysis of covariance, we may decompose $S_i$ into two components

$$S_i = U_i + \varepsilon \tag{13}$$

where $U_i$ is a random variable with mean zero and variance $\sigma_u^2$, and $\varepsilon$ is a random variable with mean zero, and variance $\sigma_\varepsilon^2$. We further assume that

$$E(U_i U_j) = 0 \quad , i \neq j \tag{14}$$

$$E(U_i \varepsilon) = 0 \quad , i = 1, \ldots, \infty .$$

Then $S_i$ is a random variable with mean

$$E(S_i) = 0 \tag{15}$$

and

$$E(S_i S_j) = \sigma_\varepsilon^2, \; i \neq j \tag{16}$$

$$= \sigma_\varepsilon^2 + \sigma_j^2, \; i = j .$$

Thus, the correlation coefficient between $S_i$ in any two periods, $\rho$, may be defined as

$$\rho = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2} \tag{17}$$

Clearly, it is possible to imagine more general intercorrelation relationships

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[20] The problem of heterogeneity is considered in a demographic context by Sheps (1964), Potter and Parker (1964), Sheps and Menken (1972) and Sheps and Menken (1973).

[21] In this paper, we abstract from the further problem that the unobserved components may be correlated with the included variables.

such as a first-order Markov process. These generalizations are straightforward and, since they are not of direct interest in this paper, are not pursued here.

If intercorrelation applies because there are persistent omitted variables, the probability of a woman becoming pregnant in interval j can no longer be written in the simple form of equation (10) (or if S is assumed normal, as in equation (12)). To see what the appropriate probability statement becomes, note that in general we may write the probability of the event conditional on a given value of $\varepsilon$ as

$$\Pr(S_1 > \alpha_0+\alpha_1 E, \ldots, S_{j-1} > \alpha_0+\alpha_1 E,\ S_j < \alpha_0+\alpha_1 E \mid \varepsilon) \ . \tag{18}$$

But note that if $\varepsilon$ is held fixed, the distribution of $S_1$ conditional on $\varepsilon = \tilde{\varepsilon}$ must satisfy the following properties:

$$E(S_i \mid \tilde{\varepsilon}) = \tilde{\varepsilon} \ ,$$

$$E(S_i S_j \mid \tilde{\varepsilon}) = \begin{cases} \tilde{\varepsilon}^2, & i \neq j \\ \sigma_u^2 + \tilde{\varepsilon}^2, & i = j \end{cases} \tag{19}$$

and, since the $U_i$ are independent, the conditional values of $S_i$ are also independent. Then we see that

$$\Pr(S_1 > \alpha_0+\alpha_1 E, \ldots, S_{j-1} > \alpha_0+\alpha_1 E,\ S_j < \alpha_0+\alpha_1 E \mid \tilde{\varepsilon}) \tag{20}$$

$$= \Pr(S_1 > \alpha_0+\alpha_1 E \mid \tilde{\varepsilon}) \Pr(S_2 > \alpha_0+\alpha_1 E / \tilde{\varepsilon}) \ldots \Pr(S_j < \alpha_0+\alpha_1 E \mid \tilde{\varepsilon})$$

so that conditional on $\varepsilon = \tilde{\varepsilon}$, we reach precisely the same functional form as in equation (11) where persistent omitted variables are ignored. However, to solve back to the probability statement of interest, where $\varepsilon$ is permitted to vary between plus and minus infinity, we note that the unconditional probability may be written as

$$\int_{-\infty}^{\infty} \Pr(S_1 > \alpha_0+\alpha_1 E \mid \varepsilon) \Pr(S_2 > \alpha_0+\alpha_1 E \mid \varepsilon) \ldots \Pr(S_j < \alpha_0+\alpha_1 E \mid \varepsilon) h(\varepsilon) d\varepsilon \tag{21}$$

where $h(\varepsilon)$ is the marginal density function of $\varepsilon$ , and $\varepsilon$ is permitted to vary over all possible values, as before.

In the special case with S normally distributed with zero mean and variance $\sigma_\varepsilon^2 + \sigma_u^2$, equation (21) becomes

$$\int_{-\infty}^{\infty} \left[ \int_{(\alpha_o+\alpha_1 E)}^{\infty} \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{1}{2}\frac{(U-\varepsilon)^2}{2\sigma_u^2}} du \right]^{j-1} \left[ \int_{-\infty}^{(\alpha_o+\alpha_1 E)} \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{1}{2}\frac{(U-\varepsilon)^2}{2\sigma_u^2}} du \right] \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} e^{-\varepsilon^2/2\sigma_\varepsilon^2} d\varepsilon .$$

Letting $t = \frac{U}{\sigma_u}$ , and $q = \frac{\varepsilon}{\sigma_\varepsilon}$ , and using the definition of $\rho$ in equation (17), this integral may be written as

$$\int_{-\infty}^{\infty} \left[ \int_{\frac{\alpha_o^*+\alpha_1^* E+\rho^{1/2} q}{(1-\rho)^{1/2}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \right]^{j-1} \left[ \int_{-\infty}^{\frac{\alpha_o^*+\alpha_1^* E+\rho^{1/2} q}{(1-\rho)^{1/2}}} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \right] \frac{1}{\sqrt{2\pi}} e^{-q^2/2} dq \qquad (22)$$

where $\alpha_o^* = \frac{\alpha_o}{(\sigma_u^2+\sigma_\varepsilon^2)^{1/2}}$ and $\alpha_1^* = \frac{\alpha_1}{(\sigma_u^2+\sigma_\varepsilon^2)^{1/2}}$ .

If no serial correlation is present ($\rho=0$), this expression collapses to equation (12). In the more general case, $\rho$ allows us to measure the proportion of total variance in the index explained by systematic correlated components.

Notice that there is an alternative "incidental parameters" argument that leads directly to equation (22). Suppose it is argued that in an ordinary probit model a disturbance "$\varepsilon$" appears. This may be viewed as an incidental parameter with density function $h(\varepsilon)$. Following a suggestion of Kiefer and Wolfowitz (1956), the problem of incidental parameters has precisely the solution written in equation (21) and for the normal case this solution becomes equation (22). In a simple one period probit model, such as one designed to explain the purchase of refrigerators in a cross section, the "incidental parameters" problem becomes irrelevant as long

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as the incidental parameter is normally distributed. Thus, if $j=1$, equation (22) may be written as

$$\int_{-\infty}^{\alpha_o^*+\alpha_1^* E} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

so that correlated and temporally random components cannot have separate effects, as is intuitively obvious.

Yet another interpretation of these results is possible. An individual may be imagined as having a geometric probability process characterizing the probabilities of pregnancy at each interval for a given value of $\varepsilon$. "$\varepsilon$" is, in fact, a random variable governed by a density function $h(\varepsilon)$. Then the true probability of pregnancy at month $j$ is a continuous mixture of geometric processes and is given by equation (21).[22]

The Implications of Serial Correlation

In this section we demonstrate that estimates of the coefficients $\alpha_o^*$ and $\alpha_1^*$, defined in the previous section that are based on techniques which ignore serial correlation will, in general, be biased, although it is not possible to sign the bias. To see this, we first consider the case of no serial correlation.

In this case, the conditional probability of a woman of education level E becoming pregnant in interval j, given that she was not pregnant in the j-1 previous intervals is

$$m_j = \frac{[\Pr(S > \alpha_o+\alpha_1 E)]^{j-1}\Pr(S < \alpha_o+\alpha_1 E)}{[\Pr(S > \alpha_o+\alpha_1 E)]^{j-1}} = \Pr(S < \alpha_o+\alpha_1 E) \qquad (23)$$

and is clearly the same for all intervals $j=1,2,\ldots$. However, in the case

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[22] For a discussion of mixtures, see Kendall and Stuart, Vol. I, (1969) Pearson (1894), Quandt (1972) and Zellner (1973).

of serial correlation, this conditional probability becomes

$$\tilde{m}_j = \frac{\int_{-\infty}^{\infty}[\Pr(S > \alpha_0+\alpha_1 E|\varepsilon)]^{j-1}\Pr(S < \alpha_0+\alpha_1 E|\varepsilon)h(\varepsilon)\,d\varepsilon}{\int_{-\infty}^{\infty}[\Pr(S > \alpha_0+\alpha_1 E|\varepsilon)]^{j-1}h(\varepsilon)\,d\varepsilon} \quad . \tag{24}$$

Using the fact that $\Pr(S < \alpha_0+\alpha_1 E|\varepsilon) = 1-\Pr(S > \alpha_0+\alpha_1 E|\varepsilon)$, the conditional probability $\tilde{m}_j$ becomes

$$\tilde{m}_j = 1 - \frac{\int_{-\infty}^{\infty} \Pr(S > \alpha_0+\alpha_1 E|\varepsilon)^{j}h(\varepsilon)\,d\varepsilon}{\int_{-\infty}^{\infty} \Pr(S > \alpha_0+\alpha_1 E|\varepsilon)^{j-1}h(\varepsilon)\,d\varepsilon} \quad . \tag{25}$$

**It can be proved that the conditional monthly probability of conception** declines for successive months. Using the fact that

$$\ln\int_{-\infty}^{\infty} [\Pr(S > \alpha_0+\alpha_1 E|\varepsilon)]^{j}h(\varepsilon)\,d\varepsilon$$

is a convex function of $j$ (Hardy, Littlewood, and Polya, 1934 ) the difference between two successive conditional probabilities of becoming pregnant is

$$\tilde{m}_{j+1}-\tilde{m}_j = \frac{\int_{-\infty}^{\infty}[\Pr(S>\alpha_0+\alpha_1 E|\varepsilon)]^{j+1}h(\varepsilon)\,d\varepsilon}{\int_{-\infty}^{\infty}[\Pr(S>\alpha_0+a_1 E\ \varepsilon)]^{j}\,h(\varepsilon)\,d\varepsilon} - \frac{\int_{-\infty}^{\infty}[\Pr(S>\alpha_0+\alpha_1 E|\varepsilon)]^{j}\,h(\varepsilon)\,d\varepsilon}{\int_{-\infty}^{\infty}[\Pr(S>\alpha_0+\alpha_1 E\ \varepsilon)]^{j-1}h(\varepsilon)\,d\varepsilon} \tag{26}$$

$$= \frac{-\left[\int_{-\infty}^{\infty}[\Pr(S>\alpha_0+\alpha_1 E|\varepsilon)]^{j}h(\varepsilon)\,d\varepsilon\right]^2 + \int_{-\infty}^{\infty}[\Pr(S>\alpha_0+\alpha_1 E|\varepsilon)]^{j+1}h(\varepsilon)\,d\varepsilon\int_{-\infty}^{\infty}[\Pr(S>\alpha_0+\alpha_1 E|\varepsilon)]^{j-1}h(\varepsilon)\,d\varepsilon}{\int_{-\infty}^{\infty}[\Pr(S>\alpha_0+\alpha_1 E|\varepsilon)]^{j}h(\varepsilon)\,d\varepsilon \int_{-\infty}^{\infty}\Pr[S>\alpha_0+\alpha_1 E|\varepsilon]^{j-1}h(\varepsilon)\,d\varepsilon}$$

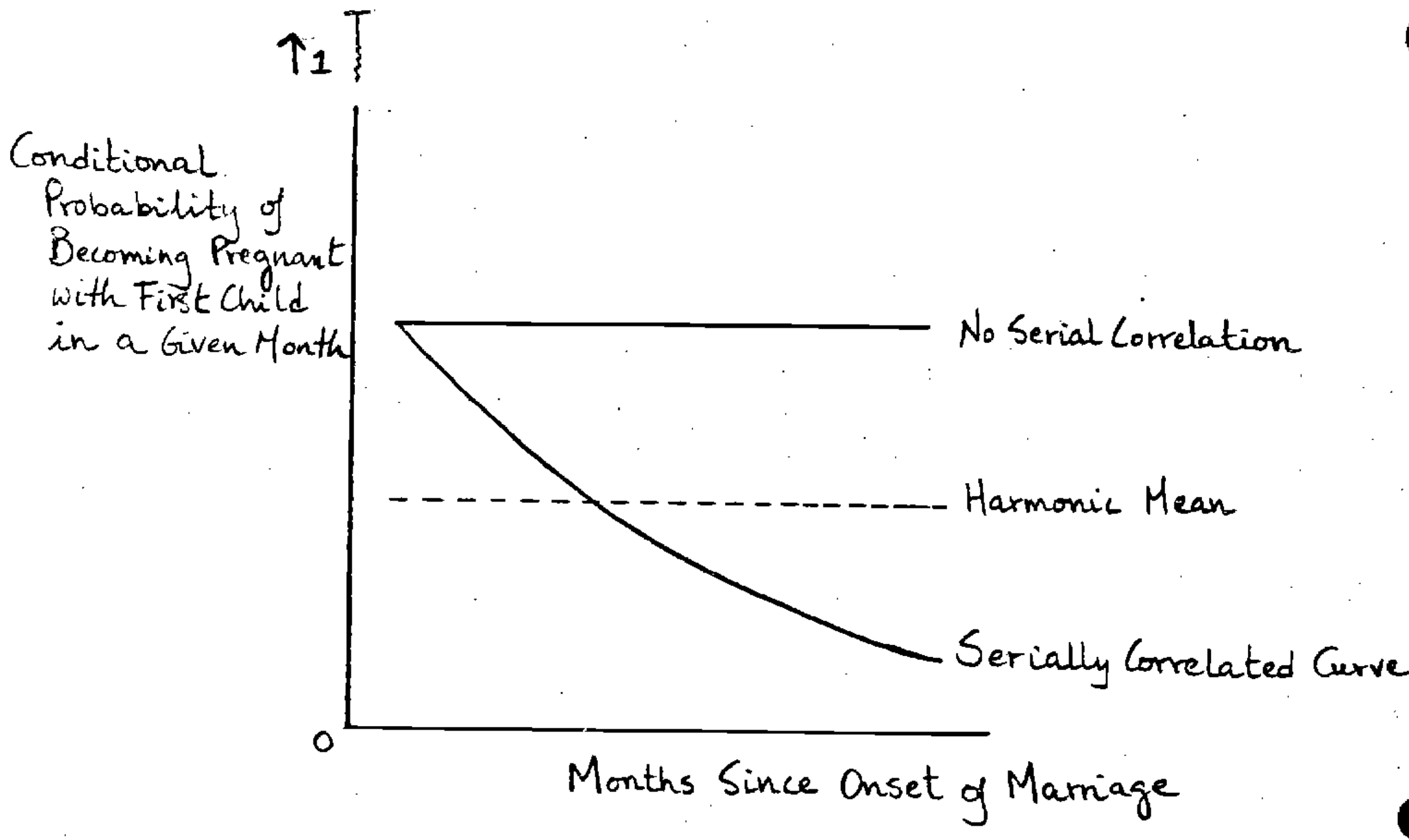


**Figure 5**

The cited convexity result implies that

$$\ln \int_{-\infty}^{\infty}[\Pr(S>\alpha_o+\alpha_1 E|\varepsilon)]^j h(\varepsilon)d\varepsilon \leq \frac{1}{2}\ln \int_{-\infty}^{\infty}[\Pr(S>\alpha_o+\alpha_1 E|\varepsilon]^{j+1} h(\varepsilon)d\varepsilon$$

$$+ \frac{1}{2}\ln \int_{-\infty}^{\infty}[\Pr(S>\alpha_o+\alpha_1 E|\varepsilon)]^{j-1} h(\varepsilon)d\varepsilon$$

Multiplying both sides by 2, and exponentiating, the numerator of expression (26) is seen to be negative, thus proving that successive conditional probabilities decline.

This phenomenon is depicted in Figure 5. The slope of the curve for the case of serial correlation is negative as shown, but the precise shape of the curve is only suggestive. A simple estimation method, such as logit or probit, applied to data on fertility outcomes imposes the constraint of constancy on conditional probabilities. It is intuitively obvious, and formally correct, that if persistence is important, but neglected in forming parameter estimates, a time trend that does not belong in the model might nonetheless prove statistically significant.

Since serial correlation in ordinary regression models does not lead to bias in coefficient estimates, it is important to motivate why it leads to bias in our case. To show what is involved, consider specializing the simple model further so that there are only two education classes. Suppose, in particular, that E assumes the value of zero or 1 corresponding to low or high levels of education. For each education class, we may estimate a monthly probability of becoming pregnant P (i) where i=0 for low education women and i=1 for high education women. Given a functional form for the distribution of the $S_t$, $t = 1, \ldots \infty$, we may solve P(i) uniquely for $\alpha_o^*$ and $\alpha_1^*$, so that a comparison of direct estimates of the P(i) for the two education groups will give direct information on $\alpha_o^*$ and $\alpha_1^*$.

Suppose estimates of P(i) are formed neglecting serial correlation. This may be done in several ways all of which lead to the same estimate. One way is to partition the data on length of time to first pregnancy by educational level, and estimate the average interval for each education class. The inverse of these two averages leads to estimates of the monthly probability of pregnancy assuming that serial correlation is absent. A second, and equivalent approach, is to maximize the sample likelihood for each education class.[23]

Note that these estimators are the correct maximum likelihood estimators assuming no serial correlation. The procedure yields consistent estimators of mean lengths of duration to first pregnancy even in samples with serial correlation since the population mean is the same for all observations and Khinchine's theorem readily applies.[24] However, in the presence of serial correlation, the mean length of duration is not simply related to any measure of direct interest. In fact, the inverse of the mean duration estimates the harmonic mean of the probabilities

$$Pr(S<\alpha_o+\alpha_1 E|\varepsilon)$$

over all value of $\varepsilon$ . To see this, note that the mean duration to first pregnancy is simply

$$\int_{-\infty}^{\infty} \frac{1}{Pr(S < \alpha_o + \alpha_1 E|\varepsilon)} h(\varepsilon)d\varepsilon$$

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[23] Thus, in constructing this function, if a highly educated woman goes $\ell-1$ months without pregnancy and becomes pregnant in the $\ell$th month, the probability of this event is

$$(1 - P(1))^{\ell-1}P(1) .$$

Similarly, if the sample period is T months, a highly educated woman never gets pregnant with probability $(1-P(1))^T$. Producting these probabilities associated with observed events, we reach the probability of the sample outcomes. Choosing value of P(1) which maximizes this probability yields maximum likelihood estimates of P(1). Defining $N_\ell$ as the number of women who become pregnant in month $\ell$ ,

$$\mathcal{L}(1) = [P(1)]^{N_1}[(1-P(1)P(1)]^{N_2}[(1-P(1)^2P(1)]^{N_3} \ldots [(1-P(1)]^{N_T}$$

where $N_T$ is the number of women who do not become pregnant in the sample observation period. Thus maximizing $\mathcal{L}(1)$, or equivalently in $\mathcal{L}(1)$, the estimator for P(1) is clearly

$$\hat{P}(1) = 1/(\sum_{i=1}^{T} \frac{N_i i}{N}) , \text{ where } N = \sum_{i=1}^{T} N_i$$

i.e. the inverse of the average interval.

[24] For a statement and proof of Khinchine's theorem see C.R. Rao (1965), p. 92.

so that the inverse of this is the harmonic mean

$$\left[\int_{-\infty}^{\infty} [\Pr(S < \alpha_o + \alpha_1 E|\varepsilon)]^{-1} h(\varepsilon)d\varepsilon\right]^{-1} .$$

We seek estimates of the arithmetic mean

$$\int_{-\infty}^{\infty} \Pr(S < \alpha_o + \alpha_1 E|\varepsilon)h(\varepsilon)d\varepsilon$$

for each group (E=1 or 0) to estimate the effect of education on the probabilities of birth. Since in general the difference in arithmetic means is different from the difference in harmonic means, estimators based on the harmonic means will be biased, although it is not possible in general to sign the bias. The same argument applies if other explanatory variables apart from education are included as well.

In addition to solving problems of bias, direct estimation of the probabilities allows us to solve the problem of open intervals. If a given sample covers only a portion of a woman's reproductive history, it is likely that some portion of the sample will not conceive. For such women, the probability of this event is easily derived and such data may be pooled in sample likelihood fashion with data from women who conceive. Thus no arbitrary assignment of interval length to nonconceiving women is necessary as would be needed in an ad hoc regression study using interval length between marriage and first birth as the dependent variable.[25]

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[25]Besides avoiding this ad hoc methodology, the procedure suggested in the paper provides an explicit approach to a derivation of theoretically appropriate test statistics, something lacking in the regression approximation approach.

## III. Empirical Results

This section presents estimates of the monthly probability of conception in the first pregnancy interval following conception using the econometric model developed in the preceding section. The data consist of a sample of white non-Catholic women, married once with husband present for 15-19 years from the 1965 Princeton National Fertility Study.[26] The sample of all such women was reduced by eliminating women who reported premarital conceptions or who had missing values for relevant variables. The sample was then divided into two groups, contraceptors and noncontraceptors, on the basis of the woman's response to a question concerning the contraceptive methods she used before her first pregnancy (or in her current interval if she had not had a pregnancy). Summary statistics on the two groups are presented in Table 1 including the means and variances of the three independent variables (wife's education (W), wife's age (A), and husband's predicted income at age 40 (H)) whose influence on the monthly probability of conception is estimated.

Women in each subsample were "followed" for a maximum of 120 months beginning with their first month of marriage. Among the non-contraceptors we estimate the monthly probability of conception in the first pregnancy interval by estimating the parameters of an equation of the form of

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[26] The 1965 National Fertility Study, conducted by Norman B. Ryder and Charles F. Westoff, is a cross-section national probability sample of 5617 U.S. married women which is described in detail in Ryder and Westoff (1971). For our purposes, its most important characteristics are that it records (retrospectively) the date of marriage of the woman, the dates of each pregnancy termination, the use of contraception in each pregnancy interval, and the time of discontinuation of contraception prior to pregnancy in addition to a number of household characteristics such as income and education.

Table 1

Mean and Variance of Independent Variables for Contraceptors and Non-Contraceptors in First Pregnancy Interval After Marriage*

(variance in parentheses)

| | Non-Contraceptors (N = 177) | Contraceptors (N = 246) |
|---|---|---|
| A: Wife's Age at Marriage (in months) | 257.1<br>(2746) | 252.3<br>(1687) |
| W: Wife's Education | 11.2<br>(6.3) | 12.2<br>(4.6) |
| H: Husband's Predicted Income at Age 40 ($1000)** | 7.58<br>(2.5) | 8.17<br>(2.12) |

*Sample: White, non-Catholic women, married once for 15-19 years, no premarital conceptions and no missing values.

**Husband's predicted income is based on an estimated regression relationship between husband's income and his education and experience (i.e., age minus years of schooling minus 6) from data on all white non-Catholic men in the 1965 NFS sample for husbands. The variable H is then imputed for men in the current sample on the basis of the man's education, with age set arbitrarily at 40. Thus, H may be interpreted as a transformation of husband's education or as his permanent income, depending on the reader's preference.

equation (22) in Section II by maximum likelihood methods.[27] That is, using the functional form of the likelihood function implied by equation (22) we estimate parameters which maximize the likelihood of observing the events that occurred in this subsample. These events are (1) that a given woman conceived in month j (j = 1, ..., 120) or (2) that she went 120 months without conceiving. Among the contraceptors, we estimate in similar fashion the monthly probability of conception given that the woman is contracepting. In this case, the events we observe are (1) that a woman conceives in month j while using a contraceptive; (2) that the woman uses a contraceptive for k months without conceiving, at which time she discontinues contraception (this decision is treated as an exogenous event); or (3) she continues using contraception for 120 months and does not conceive.[28]

Parameter estimates for the non-contraceptors are presented in Table 2A and for contraceptors in Table 2B. In each group, we estimated six models which differ in the number of parameters estimated in order to determine the statistical significance of individual parameters or sets

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[27] The methods used are described in Goldfeld and Quandt (1972, Ch. 1). Two algorithms, Powell and GRADX, were used in tandem to ensure that the estimates are stable. That is, in the first stage the parameters of the likelihood function were estimated by the Powell method. These parameters were then given as initial values in a GRADX optimization procedure whose final parameter values are reported in this paper. The computer program, written by C. Ates Dagli and Ralph Shnelvar, is available from the authors on request.

[28] As we noted in footnote 4, p. 9, our data only record whether a woman contracepted in a given pregnancy interval and when and if she discontinued contraception. They do not record when she began contracepting or any other interruptions in contraceptions other than the final decision to discontinue.

of parameters using likelihood ratio tests.[29]

Among these parameters, we have a particular interest in the magnitude of the serial correlation coefficient, $\rho$, its statistical significance and the influence of its inclusion or exclusion from the econometric model on the other parameters of the model (i.e., the constant term, $\alpha_0$, and the coefficients of A, W and H which are, respectively, $\alpha_1$, $\alpha_2$ and $\alpha_3$). Accordingly, we present two estimates of each set of $\alpha$'s in Table 2, one in which $\rho$ is constrained to be zero and one in which $\rho$ is free to assume a nonzero value.

It is easy to see in Table 2 that $\rho$ is positive and statistically significant in every instance.[30] Among the non-contraceptors $\rho = 0.450$ when only the constant term is entered and falls to 0.426 when the wife's age at marriage is held constant, but does not fall any further when wife's education and husband's predicted income are added to the model. Similarly, the estimate of $\rho$ in the contracepting subsample falls from 0.549 to 0.531 when A is held constant and to 0.526 when W and H are also held constant. If we recall that the definition of $\rho$ is the fraction of persistent variance ($\sigma_\varepsilon^2$) in total variance ($\sigma_\varepsilon^2 + \sigma_u^2$), the decrease in $\rho$ is easily understood as showing that the exogenous variable A in the non-contracepting subsample and the variables A, H and W in the contracepting subsample contribute to the persistent component of variation in conception probabilities

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[29] A property of maximum likelihood estimation is that twice the difference in log likelihood between two equations (within set A or set B) is distributed as Chi-square with n degrees of freedom where n is the difference in the number of parameters in the two equations.

[30] Comparing lines (1) and (1') in Table 2, for example, we find that log likelihood rose from -692.71 to -619.50, a difference of 73.21. Twice the difference in log likelihood is 146.4 while the critical value of Chi-square with one degree of freedom at the .95 level is 4.6.

Table 2

Estimates of Parameters of Model for Contraceptors and Non-Contraceptors in First Pregnancy Interval after Marriage

| | Constant ($\alpha_o$) | $\rho$ | Wife's Age at Marriage ($\alpha_1$) | Wife's Education ($\alpha_2$) | Husband's Predicted Income ($\alpha_3$) | $Log_e$ Likelihood |
|---|---|---|---|---|---|---|
| A. Non-Contraceptors | | | | | | |
| (1) | 2.016 | | | | | -692.71 |
| (1') | 1.214 | 0.450 | | | | -619.50 |
| (2) | 1.154 | | 0.0033 | | | -680.42 |
| (2') | 0.172 | 0.426 | 0.0042 | | | -613.36 |
| (3) | 1.022 | | 0.0031 | 0.017 | -0.0033 | -679.80 |
| (3') | 0.132 | 0.426 | 0.0041 | -0.004 | 0.0125 | -613.33 |
| B. Contraceptors | | | | | | |
| (4) | 2.264 | | | | | -336.92 |
| (4') | 1.780 | 0.549 | | | | -319.43 |
| (5) | 1.307 | | 0.0038 | | | -332.42 |
| (5') | 0.646 | 0.531 | 0.0046 | | | -316.32 |
| (6) | 1.072 | | 0.0036 | -0.0016 | 0.0387 | -331.82 |
| (6') | 0.943 | 0.526 | 0.0042 | -0.0068 | 0.0903 | -314.89 |

among women in the two subsamples. The small size of the decrease in $\rho$, however, also shows that the contribution of other factors we have not held constant constitutes the major fraction of persistent variation. This suggests that it is unlikely that the heterogeneity problem can be overcome simply by holding constant a number of observable variables.

The size of the decrease in $\rho$ caused by the addition of exogenous variables is, of course, related to the statistical significance of these variables. The wife's age at marriage is the only variable to pass a test of statistical significance at conventional levels in either subsample.[31] Wife's education and husband's predicted income are utterly without effect on log likelihood in the non-contracepting subsample (e.g., the change in log likelihood from line (2') to line (3') is 0.03). This is not entirely surprising because the channels through which education and income may affect the monthly probability of conception among non-contraceptors are essentially limited to correlations of these variables with health or coital frequency.

Our theory suggests that we should expect to find a larger impact of income and education on the monthly probability of conception among contraceptors. In this group, variation in conception probabilities is caused by variation in contraceptive efficiency due to differences in the techniques chosen and the care with which a given technique is used as well as variation in natural fecundability. Comparing lines (5') and (6'), we find that the change in log likelihood is not completely trivial (twice

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[31]Twice the change in log likelihood from line (1') to line (2') for noncontraceptors is 12.1 and the corresponding change from line (4') to line (5') for contraceptors is 6.2, both of which exceed the 0.95 confidence level. Wife's age is also significant in the equations in which $\rho$ is constrained to equal zero.

the difference in log likelihood is 2.9), but it falls well below conventional levels of significance.[32]

Estimates of the monthly probability of conception and the effects of changes in exogenous variables on that probability differ substantially depending on whether or not serial correlation is taken into account. In Table 3, we present examples of estimates of levels and changes in the monthly probability of conception among non-contraceptors and contraceptors with and without $\rho$ constrained to equal zero. These estimates are derived from the parameter estimates in Table 2. Before turning to Table 3, it will be helpful to show how the estimates in Table 3 are derived from those in Table 2 and how they are to be interpreted in light of our statistical discussion in Section II.

When $\rho$ is constrained to equal zero, we proved in Section II that the resulting estimate of the monthly probability of conception is an unbiased estimate of the harmonic means of the conception probabilities of the individual women in the sample. Let the harmonic mean be $\tilde{p}$ for noncontraceptors and $\tilde{p}^*$ for contraceptors. If $\rho$ were truly equal to zero (i.e., if all women in a sample had identical conception probabilities), then the harmonic means would equal the arithmetic means of the two groups, $\bar{p}$ and $\bar{p}^*$. If we are interested in measuring the arithmetic means, the difference between $\bar{p}$ and $\tilde{p}$ (or between $\bar{p}^*$ and $\tilde{p}^*$) measures the bias caused by ignoring serial correlation.

In order to make this comparison, it is necessary to evaluate $\bar{p}$ and $\bar{p}^*$

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[32] The critical value of Chi-square with two added parameters is 6.0 at the 0.95 level. Since the critical value with one added parameter is 4.6, it is clear that neither H nor W would be significant if entered alone into the equation.

Table 3

Estimates of Monthly Probability of Conception Derived From Parameter Estimates in Table 2

| | (a) Harmonic Mean ($\tilde{p}$ or $\tilde{p}^*$) with Serial Correlation Ignored ($\rho = 0$) | (b) Arithmetic Mean ($\bar{p}$ or $\bar{p}^*$) with Serial Correlation Allowed ($\rho > 0$) |
|---|---|---|
| A. Model with Constant term only ($\alpha_0$) | | |
| 1. Non-Contraceptors | .022 | .113 |
| 2. Contraceptors | .012 | .038 |
| B. Effect of Wife's Age at Marriage (Model with $\alpha_0$, $\alpha_1$) | | |
| Non-Contraceptors | | |
| 3. Age 20 | .026 | .122 |
| 4. Age 21 | .023 | .111 |
| 5. Age 30 | .010 | .048 |
| Contraceptors | | |
| 6. Age 20 | .013 | .040 |
| 7. Age 21 | .012 | .035 |
| 8. Age 30 | .004 | .010 |
| C. Effect of Wife's Education* (contraceptors only) | | |
| 9. W = 8 | .014 | .046 |
| 10. W = 12 | .015 | .048 |
| 11. W = 16 | .015 | .052 |
| D. Effect of Husband's Predicted** Income (contraceptors only) | | |
| 12. H = 3 | .023 | .097 |
| 13. H = 7 | .015 | .052 |
| 14. H = 10 | .011 | .027 |

*These estimates are obtained from parameter estimates of models with $\alpha_0$, $\alpha_1$, $\alpha_2$, $\alpha_3$ by setting A = 20 and H = 7.

**These estimates are obtained from parameter estimates of models with $\alpha_0$, $\alpha_1$, $\alpha_2$, $\alpha_3$ by setting A = 20 and W = 12.

at the beginning of the first month of marriage. The reason for this is that when serial correlation is present, the conditional probability of conception in month j of the subsample of women who have gone j-1 months without conceiving is smaller the larger is j because the most fecund women tend to be selected out of the original sample by conceiving in the early months of the interval.

The derivation of estimates of $\bar{p}$ and $\bar{p}^*$ evaluated at the outset of marriage from the parameter estimates in Table 2 is straightforward. We need only read off the appropriate values from a table of the standard normal integral. If we consider line (1') in Table 2, for example, then $(1 - \bar{p})$, the monthly probability of not conceiving among non-contraceptors in the first month of marriage is

$$1 - \bar{p} = \int_{-\infty}^{\alpha_0 = 1.214} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \, dt = .887$$

so that $\bar{p} = .113$, the value which is entered in line 1(b) in Table 3A. When serial correlation is not allowed, the value of $\alpha_0$ in the upper limit of the integral is 2.016 (see line (1) in Table 2) so that $\tilde{p} = .022$, the value which is entered in line 1(a) of Table 3A. Thus, we see that bias from not considering serial correlation is quite larger. Similarly, in lines 2(b) and 2(a) of Table 3 we see that the arithmetic mean monthly probability of conception among contraceptors is $\bar{p}^* = .038$ and the harmonic mean is $\tilde{p}^* = .012$.

In Table 3B, we evaluate the monthly probability of conception for several values of wife's age at marriage for non-contraceptors and contraceptors with and without $\rho$ constrained to be zero from parameter estimates in lines (2), (2'), (5) and (5') in Table 2. Here, we notice that the effect of increased wife's age is to reduce the probability of conception in both groups and that this negative effect is markedly greater when serial correlation is taken into account. There is some evidence from these estimates that failure to account for serial correlation results in downward biased estimates of contraceptive efficiency, e. Recall from Section I, that we defined the monthly probability of conception while contracepting as $p^* = p(1 - e)$ from which it follows that $e = 1 - p^*/p$. Using the estimates for 20 year old women in lines 3(a) and 6(a) in Table 3 we may compute contraceptive efficiency as $e = 1 - \tilde{p}^*/\tilde{p} = .5$ when $\rho$ is constrained equal to zero, while, from estimates in lines 3(b) and 6(b), we compute $e = 1 - \bar{p}^*/\bar{p} = .67$ when $\rho$ is unconstrained.[33]

In parts C and D of Table 3, we evaluate the ceteris paribus effects of variations in wife's education and husband's predicted income, on the probability of conception among contraceptors using the parameter estimates contained in lines (6) and (6') of Table 3.[34] The most notable features of these estimates are that husband's predicted income appears to have a large negative impact on $\bar{p}^*$ suggesting that higher husband's income is associated with improved contraceptive efficiency while wife's education has, if anything, a slight positive effect on $\bar{p}^*$.

This finding, if it is not simply a result of imprecision in our parameter estimates, is rather surprising because it is wife's education that has been found repeatedly to have a substantial negative impact on completed fertility while husband's predicted income has a weaker, non-monotonic effect

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[33] The absolute values of e should not be taken too seriously because it is quite likely that the natural fecundability, p, of non-contraceptors is lower than that of contraceptors since one of the reasons for not contracepting is subfecundity or sterility. A more complete econometric model would allow for the decision to contracept to be determined simultaneously with the monthly probability of conception in order to reduce or eliminate this selection bias.

[34] It should be emphasized that neither W nor H was statistically significant and, therefore, that little confidence may be placed in the magnitude or signs of their effects.

(see, for example, Willis, 1973). However, it should also be noted that Michael and Willis in their paper at this Conference have found that husband's predicted income has a significantly positive effect on the probability that couples used the highly effective oral contraceptive pill in the period 1960-64 while wife's education had a weaker, non-monotonic effect on this probability. While both our finding and the Michael-Willis finding are based on data from the 1965 National Fertility Study, their samples and ours are independent.[35] Moreover, estimates of completed fertility equations from 1965 NFS data yield very similar results to those estimated by Willis (1973) from 1960 Census data. These apparently contradictory effects of husband's income and wife's education on completed fertility and contraceptive efficiency present a puzzle. Hopefully, future research will determine whether the apparent contradiction is genuine and, if so, how to resolve it.

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[35] Our sample consists of women whose first birth interval began in 1946-1950, while their sample consists of women who were married, had their first birth or had their second birth in the period 1960-1964.

## Bibliography


Becker, Gary S., "An Economic Analysis of Fertility," Demographic and Economic Change in Developed Countries, National Bureau of Economic Research, 1960.

______________, "A Theory of the Allocation of Time," Economic Journal, Vol. 75, September 1965, pp. 493-517.

Becker, Gary S. and Lewis, H. Gregg, "On the Interaction Between the Quantity and Quality of Children," The Journal of Political Economy, March/April 1973, pp. 279-288.

Ben-Porath, Yoram, "Short Term Fluctuations in Fertility and Economic Activity in Israel," Demography, Vol. 10, No. 2, May 1973, p. 185-204.

Ben-Porath, Yoram and Welch, Finis, Chance, Child Traits, and Choice of Family Size, R-1117-NIH/RF, December 1972, Rand Corporation, Santa Monica, California.

Goldfeld, S.M. and R.E. Quandt, Nonlinear Methods in Econometrics, Amsterdam-London, North-Holland, 1972.

Hardy, G. H., G. Polya and J. E. Littlewood, Inequalities, Second Edition, Cambridge University Press, 1952.

Kendall, Maurice and Stuart, Alan, The Advanced Theory of Statistics, Vol. I, New York, Hafner Publishing, 1969.

Keyfitz, Nathan , "How Birth Control Affects Births," Social Biology, June 1971.

Kiefer, J. and Wolfowitz, J. "Consistency of the Maximum Likelihood Estimator in the Presence of Infinitely Many Incidental Parameters," Annals of Mathematical Statistics, Vol. 27, No. 4, December 1956.

McCabe J. and D. Sibley, "Economic Determinants of Life Cycle Fertility," Yale Economic Growth Center, xerox, October 1973.

Michael, Robert T. and Robert Willis, "Contraception and Fertility: Household Production Under Uncertainty", NBER Working Paper No. 21, December 1973.

Mincer, Jacob and Solomon Polachek, "Family Investments in Human Capital: Earnings of Women", Journal of Political Economy, 1974 (forthcoming).

Pearson, K., "Contributions to the Mathematical Theory of Evolution. Disection of Frequency Curves," Philosophical Transactions of the Royal Society, Series A, Vol. 185, 1894

Perrin, Edward and Mindel C. Sheps, "Human Reproduction: A Stochastic Process," Biometrics, Vol. 20, March 1964, pp. 28-45.

Potter, R.G. and Parker, "Predicting the Time Required to Conceive," Population Studies, 18, 1964, pp. 99-116.

Quandt, R., "New Methods for Estimating Switching Regressions," Journal of the American Statistical Association, June 1972.

Rao, C.R., Linear Statistical Inference and its Applications, Wiley, 1965,

Ryder, Norman B. and Charles F. Westoff, Reproduction in the United States 1965, Princeton: Princeton University Press, 1971.

Sanderson, Warren and Willis, Robert J., "Economic Models of Fertility: Some Examples and Implications," New Directions in Economic Research, 51st Annual Report, National Bureau of Economic Research, New York, September 1971, pp. 32-42.

Schultz, Theodore W. (ed.), "New Economic Approaches to Fertility," National Bureau of Economic Research, printed as a supplement to Journal of Political Economy, Vol. 81, No. 2, Part II (March/April 1973).

Sheps, M. C., "On the Time Required for Contraception," Population Studies 18, 1964: 85-97.

Sheps, M. C. and J. Menken, "On Estimating the Risk of Conception from Censored Data," T.N.E. Greville, Population Dynamics, Academic Press, New York, 1972, pp. 167-200.

Sheps, M. C. and J. A. Menken, Mathematical Models of Conception and Birth, Chicago: University of Chicago Press, 1973

Simon, Julian L., "The Effect of Income and Education Upon Successive Births," mimeo, University of Illinois, 1973.

Willis, Robert J., "A New Approach to the Economic Theory of Fertility Behavior," Journal of Political Economy, Vol. 81, No. 2, Part II, March April 1973, pp. s14-s64.

Zellner, A., "Bayesian and Non-Baysesian Analysis of the Regression Model with Multivariate Student-t Error Terms", May 1973, (Unpublished paper).