TECHNICAL WORKING PAPER SERIES

# A LA RECHERCHE DES MOMENTS PERDUS: COVARIANCE MODELS FOR UNBALANCED PANELS WITH ENDOGENOUS DEATH

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Technical Working Paper No. 180

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 1995

Abowd acknowledges financial support from the National Science Foundation (SBR 91-11186) and the Ministère de la Recherche et la Technologie. This paper was begun while Abowd was a visiting researcher at INSEE. Preliminary versions of the paper were presented at the 1992 Bristol econometrics seminar, at the Malinvaud seminar (Paris), at ESEM 1992 in Bruxelles, at HEC (Paris) and in Louvain la Neuve. We thank participants for helpful comments, in particular, S. Gregoir, G. Laroque, T. Magnac, A. Monfort, G. Rabault, and M. Wells. All errors are ours. The data used in this paper are confidential but the authors' access is not exclusive. Other researchers interested in using these data should contact the Centre de Recherche en Economie et Statistique, ENSAE, 15 bd Gabriel Péri, 92244 Malakoff Cedex, France. This paper is part of NBER's research programs in Labor Studies and Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Technical Working Paper #180
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# A LA RECHERCHE DES MOMENTS PERDUS: COVARIANCE MODELS FOR UNBALANCED PANELS WITH ENDOGENOUS DEATH


## ABSTRACT

We develop a model for decomposing the covariance structure of panel data on firms into a part due to permanent heterogeneity, a part due to differential histories with unknown ages, and a part due to the evolution of economic shocks to the firm. Our model allows for the endogenous death of firms and correctly handles the problems arising from the estimation of this death process. We implement this model on an unbalanced longitudinal sample of French firms which have both known and unknown ages and histories. For firms with unknown birthdates, we find that the structural autocorrelation in employment, compensation and capital is dominated by the part due to initial heterogeneity and random growth rates. Serial correlation in the periodic shocks is less important. For these firms, profitability, value-added and indebtedness have processes in which the heterogeneity components are less important. Firms with known birthdates and histories (which are younger than the censored firms) have autocorrelation structures dominated by the heterogeneity.



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## 1 Introduction

A diverse and growing literature on the use of longitudinal statistical methods to model the decisions of individuals, households, and businesses has developed within economics. Within this literature there is a tendancy to apply many of the same techniques to data on individuals and businesses. Because the statistical tools should reflect the important features of the economic decision unit modeled, failure to adapt dynamic statistical techniques to these units minimizes some important differences between the various types of economic entities and retards the use of techniques that are more appropriate for one analysis unit vis-a-vis another. We can identify three important sources of variation among businesses that appear in a given sample. First, in the initial year of the sample, firms that are already in business are the survivors of the economic process under study. Second, firms that die in the course of the sample are the victims of the same economic process. Finally, the differences among active firms in a sample may be due to heterogeneous initial conditions, survivorship, or heterogeneous histories. These same sources of variance are important for individuals. Although the age of the individual is generally known, the date of entry into the labor force or a spell of unemployment may not be known. Thus the researcher must model the survival process or the participation decision (in the case of employment models) and develop techniques that are robust to the economic decisions of the individuals during the life of the longitudinal sample. With respect to individual unemployment histories, for example, the literature on duration modelling does exactly this.

The economic analysis of firm level decisions has not, in general, been modeled with the same attention to the endogeneity of the survivor process (but see Meghir and Sanders 1987, Ridder 1990a and 1990b, Theodossiou 1993 and Corres and Ioannides (1994) for other models that do address this problem). No doubt, one reason for the relative scarcity of business unit models that address the statistical process underlying dynamic samples of firms is the fact that such data are relatively scarce. Consequently, researchers have used the data they have found.[1] With the emergence of scientific samples

[1]This criticism applies, in particular, to samples of firms from private information services like COMPUSTAT and Dunn and Bradstreet in the United States and IFS in Great Britain. The researchers have no control

of firms it is now possible to specify statistical models that are appropriate for a given group of sampled enterprises and to assess the consequences of various survival processes on the properties of those models.

There is a renewed interest among economists in the micro-economic analysis of samples of firms or establishments (see Davis and Haltiwanger, 1990, and Dunne, Roberts and Samuelson, 1989, for examples). Statistical problems abound in these studies because of the difficulty of modeling samples with heterogeneous surviving firms (at the initial sample date), entry of new firms, and exit of existing firms.

The problem is most evident if we consider the evolution of a firm's employment or capital stock. These variables represent the accumulated effects of historical decisions by the firm, the evolution of the economic environment and the noise associated with the measurement process. At any given time, the sample of currently active firms includes only those entities that have survived the economic contingencies for at least one year. The non-surviving firms have been censored from the analysis. Uncritical application of balanced panel techniques could produce very misleading statistical inferences if the censored firms were those that most poorly managed the shocks they suffered. A balanced panel systematically excludes short-lived economic entities–those for which we expect the effects of shocks to be most evident and those for which the consequences of public policy interventions are also potentially very important. A sample based on currently active firms (which need not be balanced) captures the variations associated with the birth of a firm but misses the effects of deaths. A sample based on historically active firms (in a given year) captures the death but not the birth effects. Most of these problems have been recognized by the statistical agencies that survey enterprises for the purpose of constructing economic time series like the National Income and Product Accounts in the most developed countries. Commercial databases, on the other hand, rarely specify the sampling procedure for their data. Finally, recent scientific use of both national and commercial firm-level data has not, in general, discussed the relation between the underlying sampling process of the antecedent data and the resulting analysis data set. In this paper we use data from the French "Echantillon d'entreprises" (see Corbel 1990), which has a specified scientific sampling structure that

over the sampling structure of the data and do not, in general, model the survivor process.

permits us to disentangle the effects of firm births and deaths from the statistical and economic processes governing the evolution of the variables in the surveys.

In the first section of this paper we describe the methods used for constructing a representative dynamic sample of French firms. Next, we describe a class of variance components models that decompose the evolution of firm-level variables into several components. We consider the cases of exogenous and endogenous death for firms with complete, known histories and for firms with unknown and potentially heterogeneous histories prior to the start of the sample. For firms with known histories, our models decompose the mean and variance of firm-level variables into components due to initial heterogeneity, heterogeneous growth rates, macro-economic shocks and micro-economic shocks. For firms with unknown, heterogeneous histories we add two additional components due to: heterogeneous initial conditions and heterogeneous evolutions of the micro-economic shocks. We show how to estimate these components when the birth and death of firms is exogenous. Then, we show how the endogeneity of firm deaths affects the statistical structure of these estimates, in particular the crucial identifying role of the known histories. Finally we implement a model with exogenous births and endogenous deaths to explain the evolution of employment, wages, profitability, capital, and other economic outcomes.

## 2 The Construction of the Data

### 2.1 A Dynamically Representative Sample

We started with a representative unbalanced longitudinal sample of French firms from the INSEE échantillon d'entreprises 1978-1988 (Corbel 1990). As constructed by the division of economic studies, the 1988 version of this sample includes 21,642 firms. The sample is cross-sectionally representative in each of the eleven years; that is, the age structure of the sampled firms is representative of the age structure in the population for each year. Although a complete description of the sampling plan can be found elsewhere, there are several aspects of the design which must be understood in order to interpret the statistical models below. The enterprise universe consists of every enterprise that responded to the BIC (Bénéfices industriels et commerciaux) in any year between 1978 and 1988. In the primary sample year, 1986, every entreprise that completed the BIC in 1986 was at risk to be sampled with probabilities that depended upon the size of the entreprise in 1986 and the sector of economic activity. Firms with 500 or more employees were sampled with probability one. Firms with less than 20 employees were not sampled.[2] Firms of intermediate sizes were sampled with probabilities between $\frac{1}{30}$ and 1 according to the size and sector. Every firm that existed in 1986 was then removed from the universe. Complementary samples were then constructed for each of the other years using the same sampling weights as in 1986. After a complementary sample was drawn for a particular year all firms at risk to be sampled that year were eliminated from the universe.

In principle, the BIC is exhaustive, however many enterprises have at least one year of missing data between the first and last years they appear in the BIC. From the initial sample of 21,642 firms, 8.2% were discarded for having two missing years between the first and last year they appear. An aditional 14.9% of firms, having one year of missing data in BIC, were used in the dynamic sample with the missing year imputed (using the procedure described in Appendix E). Finally, 16,645 enterprises had complete data and were used without imputation. The enterprises that appear in the dynamic sample represent 91.8% (unweighted) of the firms in the échantillon d'entreprises and 90.4% (weighted[3]). The number of observations (firm-years) is 129,447 of which 2.5% were imputed. We eliminated all data from large governmental entreprises (the SNCF, RATP, la Poste, Air France, Electricité de France, Gaz de France, governmental savings banks, etc.) and all data from a group of extremely anomalous entreprises.[4] All told, we eliminated data for 53 entreprises, including the governmental firms.

Table 1 shows the statistical structure of the firms used in our analyses, overall and by year of entry into the sample. Pairs of rows in the table show, on the top line, the number of employees represented by the analysis firms (in thousands) and, on the bottom line, the number of actual firms. There are a total of 19,520 unique firms. The increase in the number of firms between 1978 and 1979 and the decrease in the number of firms between 1987 and

[2]Firms with less than 20 employees (full- or part-time) were not required to complete the BIC until 1984.

[3]The weights come from the sampling plan of the échantillon d'entreprises, described above.

[4]These firms had negative assets or some other gross accounting anomaly.

1988 are artifacts of the decision not to impute missing data before the first year in which a sample firm had BIC data or after the last year. Otherwise, the fluctuation in the number of firms represents observed fluctuation in the number of firms in the population at risk to be sampled. The much less important fluctuations in employment between 1978 and 1979 and between 1987 and 1988 show that the firms with missing first and last years of data are almost exclusively very small entreprises. Firms that enter the sample in 1978 have a heterogeneous age structure; however, firms that enter from 1979 to 1988 have a known age structure, except for the relatively minor problem of missing BIC data in the initial year. We use as the estimated survival rates of firms with a given year of entry into the sample the ratio of the number firms still present in a given year to the number present in the birth year.[5]

## 2.2 Definition of Variables

Although there are many different financial and operational variables available in our analysis file, we focus in this paper on total employment, several compensation measures, capital, profitability and debt. From the BIC, we used total employment at December 31$^{st}$ (effectif), total employer labor cost (frais de personnel y.c. charges patronales), total gross payroll (masse salariale), value-added (valeur ajoutée brute des coûts de facteurs), gross operating profit (excédent brut d'exploitation), total assets (actif) and total debt (dettes). From the Enquête annuelle des entreprises (EAE) we used total compensation costs (remuneration). The variables were taken directly from the BIC and EAE with no redefinitions and converted to 1980 francs using the 1980 base consumer price index (for value added, gross operating profit and all compensation variables) and the industry capital price index for total assets.[6]

[5]For example, for firms born in 1979 the estimated survival rate to 1980 is 83%, which is the ratio of 1,015 to 1,222.

[6]See Abowd and Kramarz 1993 for additional descriptions of these variables.

## 2.3 Summary Statistics

Table 2 presents summary statistics for each of the 8 variables analyzed in this paper for firms that first appear in the sample in 1978. These firms have unknown birth dates. Table 3 presents summary statistics for each variable for firms that enter the sample in 1979.[7] In our statistical summaries and models, total employment, total labor costs, total payroll, and total capital were analyzed in logarithms while value added, gross operating profits and debt were analyzed as ratios to total capital. For the sample of firms first observed in 1978, the autocorrelation structure is shown at the bottom of Table 4. For the sample of firms entering between 1979 and 1988, the autocorrelation structure is shown at the bottom of Table 5.

The cohort of firms observed for the first time in 1978 exhibits certain regularities for each variable. In these surviving firms, there is an upward trend in employment, labor costs (all measures) and total assets. Profitability and indebtedness are constant (or slightly declining). Finally, value added (as a percent of capital) has a strong negative trend. The variances of the variables measured in logarithms (employment, compensation, total capital) increase for the surviving firms whereas the variances of the ratio variables (profitability and value-added) decline, except for indebtedness. The logarithmic variables also display very strong serial correlation (much stronger than the autocorrelation that would be due to a simple autoregressive model with a parameter of 0.95). The ratio variables, however, display much weaker serial correlation.

For the cohorts of firms whose birthdates are observed (firms entering the sample in the years 1979-1988), the basic patterns in the means are similar. Surviving firms show in upward trend in employment, compensation costs and total assets, constant profitability and indebtedness, and declining value-added. Like the sample of survivor firms, the variances of employment and the compensation cost measures rise over time but the effect is much stronger for firms observed from birth. The variances of profitability and value added (both in ratios to capital) decline while the variance of indebtedness is constant. Autocorrelation patterns are similar to the firms with unknown birth dates.

[7] Sample statistics for firms that enter the sample in the years 1980-1988 are materially similar to those shown in Table 3.

## 3 A Statistical Model for Unbalanced Panels

We are interested in modeling the evolution of firm-level variables like employment, labor costs and profitability, which display considerable autocorrelation when measured in levels, logarithms or ratios. This autocorrelation could be due to permanent differences in the means of the variables, permanent differences in their growth rates, censoring of dead firms, autocorrelation of macro-economic shocks and autocorrelation of micro-economic shocks. In this section we show how to decompose the mean, variance and autocovariances of these firm-level measurements into five components related to level, growth rate, death censoring, macro-economic shocks and micro-economic shocks. Our covariance decomposition permits us to partition the observed autocovariance into parts due to each of these factors. We demonstrate the identification that arises from knowledge of the age of the firms. We then show how the birth and death of firms affects estimates of the contributions of these components to the overall covariance structure and we propose a general technique for analyzing these covariances that allows for very general birth and death rates of the firms. Our model shows the importance of being able to condition on firm age and the effects of a heterogeneous (unknown) age structure on the resulting estimates.

### 3.1 The Moments of a Variable When Death is Exogenous

Consider a firm-level random variable $y_{jt}$ for firm $j$ and date $t$ born at $b_j$ (possibly censored by the start of the sample) and dead at $d_j$ (possibly censored by the end of the sample). Time is measured relative to an arbitrary date $t = 1$, which corresponds to the first year of a longitudinal sample of the firms. Firms born on, or before, $t = 1$ have birth dates $b_j$ that are unknown. The sample runs from $t = 1$ to $T$. Firms that die on or after the end of the sample $t = T$ have, therefore, death dates that are unknown. We describe now the stochastic process that governs $y_{jt}$.

For the first year of a firm's life let:

$$y_{jb_j} = \mu_j + \beta_j + \delta_{b_j} + \epsilon_{jb_j}$$

where $\mu_j$ is a random variable with the mean $\mu$ and variance $\sigma_\mu^2$; $\beta_j$ is a random variable with mean $\beta$, variance $\sigma_\beta^2$ and covariance $\sigma_{\mu\beta}$ with $\mu_j$; $\delta_t$ is a random variable common to all firms with mean $\delta$ and variance $\sigma_\delta$; [8] $\epsilon_{jt}$ is a random variable with mean 0 for all $j$ and $t$, variances $\sigma^2_{\epsilon_t}$ for all $j$, and serial covariances of 0 at all leads and lags for all $j$ and $t$; and, finally, all other unspecified covariances among these components of $y_{jt}$ are 0. In the second year of the firm's life we have

$$y_{jb_j+1} = \mu_j + 2\beta_j + \delta_{b_j+1} + \epsilon_{jb_j+1} + \rho(y_{jb_j} - \mu_j - \beta_j - \delta_{b_j})$$

where the term $(y_{jb_j} - \mu_j - \beta_j - \delta_{b_j})$ is simply $\epsilon_{jb_j}$ and $\rho$ is the parameter of an autoregressive process whose innovations are the $\epsilon_{jt}$s.

The date of birth of firm $j$, $b_j$, is also a random variable. In what follows, we suppose that $b_j$ is independent of the random process $y_{jt}$ and, therefore, is strictly exogenous with respect to $y_{jt}$. In this subsection, we also assume that the date of death, $d_j$, is strictly exogenous with respect to $y_{jt}$. The next subsection contains a specification in which the date of death is endogenous.

We now introduce the distinction between firms with known age ($b_j > 1$) and firms with unknown age. First, note that age is also a random variable. Computations on firms where age is known, that is with birth years from 1979 to 1988 in our sample, yield:

$$y_{jt} = \mu_j + (t - b_j + 1)\beta_j + \delta_t + \sum_{k=b_j}^{t} \rho^{t-k}\epsilon_{jk} \tag{1a}$$

for

$$t = b_j, ..., d_j \quad b_j > 1$$

where $d_j$ is the date of the firm's death, possibly censored by the end of the sample and the age of the firm is $t - b_j + 1$.[9] When the age of the firm is unknown we obtain:

$$y_{jt} = \mu_j + (t - b_j + 1)\beta_j + \delta_t + \sum_{k=b_j}^{0} \rho^{t-k}\epsilon_{jk} + \sum_{k=1}^{t} \rho^{t-k}\epsilon_{jk}$$

[8] In all estimation we condition on $\delta_t$

[9] The date $t = 1$ indicates the first period in which firms were sampled. On this date, our sample includes both firms that were born in year 1 ($b_j = 1$) and firms born before year 1 ($b_j < 1$) with no indication as to which firms were actually born before year 1. Thus, only firms with $b_j > 1$ have known birthdates and we will use $b_j \leq 1$ to indicate firms with unknown birthdates.

or

$$y_{jt} = \mu_{jt}^0 + t\beta_j + \delta_t + \sum_{k=1}^{t} \rho^{t-k}\epsilon_{jk} \qquad (1b)$$

for

$$t = 1, ..., d_j$$

where

$$\mu_{jt}^0 \equiv \mu_j + (-b_j + 1)\beta_j + \rho^t \sum_{k=b_j}^{0} \rho^{-k}\epsilon_{jk} \equiv \mu_j^0 + \rho^t \nu_j^0$$

$$\mu_j^0 \equiv \mu_j + (-b_j + 1)\beta_j$$

and

$$\nu_j^0 \equiv \sum_{k=b_j}^{0} \rho^{-k}\epsilon_{jk}$$

In order to compute the theoretical moments of the variable $y_{jt}$, we must make some additional assumptions. In particular, because our panel is unbalanced and includes no holes (i.e., no missing years once a firm is sampled), we make explicit the relations among the processes of birth, death and $y_{jt}$. Since birth is strictly exogenous with respect to the $\mu_j$, $\beta_j$, $\delta_t$, and $\epsilon_{jt}$ processes, the conditional distribution of these random variables, given $b_j$, does not depend upon $b_j$ and, thus, is identical to the unconditional distribution described above. Similarly, in this section we treat the date of death as exogenous; hence, the conditional distribution of these random variables given $d_j$ is also identical to the unconditional distribution described above. We now derive the first and second moments of $y_{jt}$ for the case of strictly exogenous birth and death.

Let us consider all firms at date $t$ for which we know the date of birth and who have one year of age, $a_{jt} = 1$. Then, we can write:[10]

$$E[y_{jt} \mid a_{jt} = 1] = \mu + \beta + \delta_t \equiv y_t^1$$

The hypothesis that the expectation of $\epsilon_{jt}$ is zero conditional on age is always true for one year old firms because, with exogenous birth, the value of the

[10] All expectations are taken with respect to the distribution of the random variable over firms. Thus, the $j$ subscript is omitted from the expectation functional $E_j$ in all formulas.

firm-specific error term does not affect the probability that a firm will come into existence (and therefore be at risk to be sampled). Note that, because we always observe a firm in its first year of life, there would be no endogenous censoring bias for data on the firm's initial year of operation even if death were endogenous. Since in the present subsection, death, like birth, is exogenous, for all firms with the same known age, $a_{jt} = a$, at date $t$ the mean of $y_{jt}$ is:

$$E[y_{jt} \mid a_{jt} = a] = \mu + a\beta + \delta_t \equiv y_t^a$$

where, again, the expectation is taken with respect to the distribution of firms $j$ conditional on age. Using these conditional means and maintaining the assumptions of exogenous birth and death, one can compute the conditional covariance structure of our process:

$$E[(y_{jt} - y_t^a)y_{js} \mid a_{jt} = a, s \leq t] =$$

$$\sigma_\mu^2 + a(s - t + a)\sigma_\beta^2 + (s - t + 2a)\sigma_{\mu\beta} + \sum_{k=t-a+1}^{s} \rho^{t+s-2k}\sigma_{\epsilon_k}^2$$

These formulas can be extended directly to the case where the date of birth of the firm is unknown. Recall that when age is not known, the only changes in the formulas are the term $\mu_{jt}^0 = \mu_j^0 + \rho^t\nu_j^0$, which replaces $\mu_j$, and the substitution of $t$ for $a_{jt}$. Therefore, the required conditional expectations for the unknown birth date are:

$$E[y_{jt} \mid b_j \leq 1] = \mu_t^0 + t\beta + \delta_t \equiv y_t$$

where

$$\mu_t^0 \equiv E[\mu_{jt}^0 \mid b_j \leq 1] = \mu + (1 - E[b_j \mid b_j \leq 1])\beta$$

and

$$E[(y_{jt} - y_t)y_{js} \mid b_j \leq 1, s \leq t] =$$

$$\sigma_{\mu^0}^2(s,t) + (s + t)\sigma_{\mu^0\beta} + ts\sigma_\beta^2 + \sum_{k=1}^{s} \rho^{t+s-2k}\sigma_{\epsilon_k}^2$$

where $\sigma_{\mu^0}^2(s,t)$ is the conditional variance of $\mu_j^0$, given $b_j \leq 1$, $\sigma_{\mu^0\beta}$ is the conditional covariance between $\mu_j^0$ and $\beta_j$, given $b_j \leq 1$, and other terms retain their original definitions. Finally, we note that the components of $\sigma_{\mu^0}^2(s,t)$ and $\sigma_{\mu^0\beta}$ are:

$$\sigma_{\mu^0}^2(s,t) = \sigma_\mu^2 + 2\zeta_1\sigma_{\mu\beta} + \zeta_2\sigma_\beta^2 + \beta^2 V[(1 - b_j) \mid b_j \leq 1]$$

$$+\rho^{(s+t)}E[\sum_{k=b_j}^{0}\sum_{l=b_j}^{0}\rho^{-k-l}\epsilon_{jk}\epsilon_{jl} \mid b_j \leq 1]$$

and

$$\sigma_{\mu^0\beta} = \zeta_1\sigma_\beta^2 + \sigma_{\mu\beta}$$

where

$$\zeta_1 = E[(-b_j + 1) \mid b_j \leq 1]$$

and

$$\zeta_2 = E[(-b_j + 1)^2 \mid b_j \leq 1]$$

We can restate the conditional variance of $\mu^0$, given $b_j \leq 1$, in a more compact form as:

$$\sigma_{\mu^0}^2(s,t) = \sigma_{\mu^0}^2 + \rho^{(s+t)}\sigma_{\nu^0}^2$$

where

$$\sigma_{\mu^0}^2 \equiv \sigma_\mu^2 + 2\zeta_1\sigma_{\mu\beta} + \zeta_2\sigma_\beta^2 + \beta^2V[(1-b_j) \mid b_j \leq 1]$$

and

$$\sigma_{\nu^0}^2 \equiv E[\sum_{k=b_j}^{0}\sum_{l=b_j}^{0}\rho^{-k-l}\epsilon_{jk}\epsilon_{jl} \mid b_j \leq 1]$$

### 3.2 The Moments of a Variable When Censored by Death

Let us now consider the case of an arbitrary variable $y_{jt}$ for a firm $j$ born at $b_j$ (possibly censored by the beginning of the survey) and dead at $d_j$ (possibly censored by the last year of the survey) where death is not independent of $y_{jt}$. Some of the components of $y_{jt}$ will, therefore, be correlated with the date of death $d_j$. The formulas for the theoretical moments of $y_{jt}$ will, therefore, differ from those computed in the previous subsection. Unless the process governing $d_j$ is explicitly related to $y_{jt}$, the conditional means, variances and covariances are impossible to model directly. Our strategy is to provide very general sufficient conditions under which it becomes possible to estimate the parameters of the above model when firms may die from an unspecified endogenous death process.

Consider first that the sample starts at the same known date, $t = b$ for all firms $j$. Although the firms in the sample are observed from $t = b$, some may die within the period $b, ..., T-1$ while others survive to $T$ and beyond. Let $\underline{y}_j' = (y_{jb}, ..., y_{jT})$ represent the vector of potential observations. For some firms, only a part of this vector is observed. For instance, if $d_j = d$ is the date of death of firm $j$, then $(y_{jb}, ..., y_{jd})$ is observable but the attrition of this firm is endogenous because $d$ is related to the history $y_{jb}, ..., y_{jd}$. After death, we assume that the process continues in a latent form; in other words that $y_{jd+1}, ..., y_{jT}$ follow equation $(1a)$ but are unobservable.[11]

Thus, for the vector composed of both observed and latent observations:

$$\begin{aligned} \underline{y}_j &= \mu_j \underline{e} + \beta_j \underline{a} + \underline{\delta} + (I - \rho N)^{-1} \underline{\varepsilon}_j \\ E\underline{y}_j &= \mu \underline{e} + \beta \underline{a} + \underline{\delta} \equiv \underline{m} \\ V\underline{y}_j &= \sigma_\mu^2 \underline{ee}' + \sigma_\beta^2 \underline{aa}' + \sigma_{\mu\beta}(\underline{ea}' + \underline{ae}') + [I - \rho N]^{-1}(Diag)(\sigma_t^2)[I - \rho N']^{-1} \equiv \Sigma \end{aligned} \tag{2}$$

where

$$\underline{e}' = (1, ..., 1), \underline{a}' = (1, 2, ..., T - b + 1), \underline{\varepsilon}_j' = (\varepsilon_{jb}, ..., \varepsilon_{jT})$$

and

$$N = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & & 0 & 0 \\ 0 & 1 & & & \vdots \\ \vdots & & & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix}$$

is the "lag" matrix. When the date of birth is unknown ($b \leq 1$), equation (2) must be modified to include the effect due to $\nu_j^0$. More importantly, all the parameters for firms with unknown birth dates differ from those of firms with known birth dates due to the survival bias: we do not observe firms that have died before the sample starts.

We now turn our attention to the attrition mechanism. For a cohort of firms born at date $b$, possibly before date $t = 1$, attrition from the sample is based upon the history of $y$: $y_{jb}, ..., y_{jd}$ and a specific random effect $\alpha_{jbd}$. More precisely we assume that there is a function $H_{bd}$, which may have dimension larger than 1, such that the attrition rule at date $d$ is represented by $H_{bd}(y_{jb}, ..., y_{jd}; \alpha_{jbd}) \geq 0$. The set $\{b, d\}$ of firms born in $b$ and dead in $d$

[11] To make our dating conventions as clear as possible, the firm is born in $b$, and we observe its outcomes for that year. The firm dies in $d$ and, again, we observe its outcomes in that year. For the years $d + 1$ and beyond the process $y_{jt}$ is latent.

is defined as:

$$\{b,d\} = \{j \mid H_{bb}(y_{jb};\alpha_{jbb}) < 0 \text{ and } H_{bb+1}(y_{jb},y_{j,b+1};\alpha_{jb,b+1}) < 0 \text{ and}$$

$$\text{... and } H_{bd}(y_{jb},...,y_{j0},y_{j1},...,y_{jd};\alpha_{jbd}) \geq 0\}$$

which can be summarized as $G_{bd}(y_{jb},...,y_{jd};\alpha_{jbd}) \geq 0$. Our representation of the attrition mechanism, based on the continuing economic activity of the firm until the date $d$ and a random individual effect $\alpha_{jbd}$, is very general. Notice that even if death is determined only by $y_{jd}$, the function $G_{bd}$, because it includes the survival from date $b$ to $d-1$, must incorporate the whole process $y_{jt}$.

In this type of problem, the classical approach, a Tobit perhaps, provides an explicit specification of the death process. However, there is no obvious function representing such a process. To see why, note that firms can die for at least two reasons: they are not profitable enough or they are too profitable. In the first case, economic activity cannot go on for the obvious reasons. In the second case, the firm may disappear because of a takeover. This makes even the two-limit Tobit representation ackward. Furthermore, the estimation procedure for a Tobit is particularly difficult to implement. Correlation of residuals through time (due to individual random effects and/or autocorrelation of micro-economic shocks) requires simulation procedures that are extremely time-consuming given the large size of the sample.

We propose the following alternative to explicit modeling of the death process. Recall that:

$$\sum_d Prob(j \in \{b,d\})E[h(y_{jb},...,y_{jT}) \mid j \in \{b,d\}] = E[h(y_{jb},...,y_{jT}) \mid j \in \{b\}]$$

where the set $\{b\}$ is the collection of all firms born in year $b$ and $h(.)$ is any function of the random vector $\underline{y}_j$. We use this property to model the first and second order moments of the $\underline{y}_j$ process. Instead of a complete model for the death date, $d_j$, we must supply a model for the moments of the unobserved (latent) part of the $y_{jt}$ process i.e. for each date $t$ between $d+1$ and $T$, conditional on death at date $d$. Hence, we are "à la recherche des moments perdus" (looking for the lost moments). As we show below, a variety of death processes are consistent with our general formulas for the expectations of functions of the latent part of $\underline{y}_j$ We treat the lost moments as missing data induced by the death process. We show, that under certain conditions on the conditional distributions of the observable part of $y_{jt}$, the latent part and the variable $\alpha_{jbd}$, the parameters of the process described by equation (2) can be recovered.[12]

We begin with the following lemma:

**Lemma 1** *Let three random variables $(x, y, z)$ with p.d.f. $f(x, y, z)$ and with conditional p.d.f. of $z$ given $(x, y)$, $k(z \mid x, y)$, equal to the conditional p.d.f. of $z$ given $y$, $l(z \mid y)$. Then,*

$$E[h(y,z)|g(x,y) \geq 0] = E[E(h(y,z)|y)|g(x,y) \geq 0]$$

**Proof:** see Appendix A.

To see why this lemma is useful, consider the case where the birth date $b$ is known and suppose the death date is death $d$. Let $y$ and $z$ in the lemma be, respectively, the observed and unobserved (after death) parts of $\underline{y}_j$, i.e. $y_{jb}, ..., y_{jd}$ and $y_{jd+1}, ...y_{jT}$, respectively. Furthermore, suppose that $x$ is the random variable related to the death process, i.e.$\alpha_{jbd}$. Thus, if

$$k(y_{jd+1}, ..., y_{jT} \mid \alpha_{jbd}; y_{jb}, ..., y_{jd}) = l(y_{jd+1}, ..., y_{jT} \mid y_{jb}, ..., y_{jd}) \qquad (3)$$

where $k$ and $l$ are defined as in the above lemma; then, for any measurable function $h$,

$$E[h(y_{jb}, ..., y_{jT}) \mid G_{bd}(y_{jb}, ..., y_{jT}; \alpha_{jbd}) \geq 0] =$$

$$E(E[h(y_{jb}, ..., y_{jT}) \mid (y_{jb}, ..., y_{jT})] \mid G_{bd}(y_{jb}, ..., y_{jd}; \alpha_{jbd}) \geq 0)$$

Now if we assume that every random variable used in forming the $y_{jt}$ process is jointly normal, we can recover the lost moments as a function of the observed moments of the censored process $y_{jt}$ and the parameters of the model.

**Remark:** If our model were Markov, property (3) would automatically be satisfied. However, in our model an observation at date $t$ depends upon the observation at date $t-1$ but also upon the random effects $\mu_j$ and $\beta_j$ as well as $\varepsilon_{jt}$. Because of the structure of equation (2), however, the terms in

[12]Our statistical model resembles the idea of "ignorable" missing data models, as defined by Little and Rubin (1987) and applied to panel data models in Verbeek and Nijman (1992) but is more general.

$\varepsilon_{jt}$ are not problematic, since $\varepsilon_{jt}$ is orthogonal to $d_j$ for $t > d_j$. To apply the property (3), we require that:

$$k(\mu_j, \beta_j \mid \alpha_{jbd}; y_{jb}, ..., y_{jd}) = l(\mu_j, \beta_j \mid y_{jb}, ..., y_{jd})$$

which is satisfied, for example, if $\mu_j$ and $\beta_j$ enter the attrition rule $G(..)$ only through $y_{jt}$.[13] Two simple examples clarify the role of property (3) in the identification of our missing moments.

**Example 1**

$$y_{j1} = \mu_j + \varepsilon_{j1}$$

$$y_{j2} = \mu_j + \varepsilon_{j2} + \rho\varepsilon_{j1}$$

$$\alpha_{j11} = \mu_j + \tau_j$$

where $\tau_j$ is independent of $\mu_j, \varepsilon_{j1}$ and $\varepsilon_{j2}$. Let

$$G_{11}(y_{j1}; \alpha_{j11}) \equiv y_{j1} - \alpha_{j11} \geq 0$$

indicate the survival of firm j to period 2. Property (3) is not satisfied in this model because

$$k(y_{j1}, y_{j2} \mid \alpha_{j11}, y_{j1}) \neq l(y_{j1}, y_{j2} \mid y_{j1}).$$

**Example 2**

$$y_{j1} = \mu_j + \varepsilon_{j1}$$

$$y_{j2} = \mu_j + \varepsilon_{j2} + \rho\varepsilon_{j1}$$

$$\alpha_{j11} = \tau_j$$

where $\tau_j$ is independent of $\mu_j, \varepsilon_{j1}$ and $\varepsilon_{j2}$. Let

$$G_{11}(y_{j1}; \alpha_{j11}) \equiv y_{j1} - \alpha_{j11} \geq 0$$

indicate the survival of firm j to period 2. Property (3) is satisfied in this model because

$$k(y_{j1}, y_{j2} \mid \alpha_{j11}, y_{j1}) = l(y_{j1}, y_{j2} \mid y_{j1}).$$

Example (1) corresponds to the case where private information, say of the managers, about the firm's $\mu_j$ enters the decision to shutdown or merge.

[13]See Verbeek and Nijman (1992) and Meghir-Saunders (1987) for a discussion of closely related issues.

Example (2) corresponds to the case where only public or uncorrelated information about the firm enters the decision to shutdown or merge. Note that for neither example (1) nor (2) is the death process ignorable in the sense of Little and Rubin, because $f(y_{j1}, y_{j2}) \neq g(y_{j1}, y_{j2} \mid G_{11} \geq 0)$, where $f(.)$ is the joint density of $y_{j1}, y_{j2}$ and $g(.)$ is the conditional density of $y_{j1}, y_{j2}$ given $G_{11} \geq 0$.

Suppose now that the birth date $b$ is not observed, i.e. the firm appears in the sample at $t = 1$, so $b \leq 1$. Since the attrition rule $G_{bd} \geq 0$ includes the full history of $y_{jt}$, in particular, the part from the unobserved dates $b, ..., 0$, we require some hypothesis concerning $y_{jb}, ..., y_{j0}$. To apply the property (3) in a way that allows us to recover the lost moments of firms with unknown birthdates, as a function of the observables and of the parameters from equation (2), we would have to assume that:

$$k(y_{j,d+1}, ..., y_{jT} \mid \alpha_{jbd}; y_{jb}, ..., y_{jd}) = l(y_{j,d+1}, ..., y_{jT} \mid y_{j1}, ..., y_{jd})$$

Again, a first order Markov model would satisfy this property. But, due to the presence of $\mu_j$ and $\beta_j$, the property does not hold for the process described by equation (2). Furthermore, even if this property held, the joint normality assumption that we need below would not be satisfied since $b_j$ appears in the formulas for the $\mu_j^0$ and $\nu_j^0$. Conditional on the birth date, these variables are normal as soon as all the variables in the $y_{jt}$ process are normal. But, given the randomness of the birth process, when $b_j$ is not observed, these variables are mixtures of normal and, therefore, not normal. In conclusion, when the date of birth is not known, there is not a property similar to property (3) that permits identification of the parameters of the random variables in equation (2). In all of our subsequent statistical analyses, we consider firms with unknown birth dates separately from those with known dates of birth.

### 3.3 Identification of the parameters given the birth date

Assume that $(y_{jb}, ..., y_{jT})$ is a multivariate normal vector. Now, equation (2) allows the computation of conditional moments of each firm $j$ born in year $b \geq 1$ and dead in year $d$ as a function of the moment of the observable period:

$$E[\underline{y}_j \mid j \in \{b, d\}] - \underline{m} = \begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix} (E[y_{1j}^{bd} \mid j \in \{b, d\}) - m_1^{bd})$$

where

$$\underline{m} = \begin{pmatrix} m_1^{bd} \\ m_2^{bd} \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \Sigma_{11}^{bd} & \Sigma_{12}^{bd} \\ \Sigma_{21}^{bd} & \Sigma_{22}^{bd} \end{pmatrix}$$

and $y_{1j}^{bd}$ is the observable part of $\underline{y}_j$, $y_{2j}^{bd}$ is the latent part of $\underline{y}_j$, $m_1^{bd}$ is the first $d-b+1$ components of $\underline{m}$ and $\Sigma_{11}^{bd}$ is the $d-b+1 \times d-b+1$ northwest part of $\Sigma$. Furthermore :

$$V[\underline{y}_j \mid j \in \{b,d\}] =$$

$$\begin{pmatrix} 0 & 0 \\ 0 & \Omega_{bd} \end{pmatrix} + \begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix} V[y_j^{bd} \mid j \in \{b,d\}] \begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix}'$$

where

$$\Omega^{bd} \equiv \Sigma_{22}^{bd} - \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1}\Sigma_{12}^{bd}.$$

The classical properties of the conditional expectation and the conditional variance lead to:

$$\sum_d P(j \in \{b,d\}) \begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix} (E[y_{1j}^{bd} \mid j \in \{b,d\}] - m_1^{bd}) = 0$$

and

$$\sum_d P(j \in \{b,d\}) \begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix}$$

$$\{V[y_{1j}^{bd} \mid j \in \{b,d\}] + \left(E[y_1^{bd} \mid j \in \{b,d\}] - m_1^{bd}\right)\left(E[y_1^{bd} \mid j \in \{b,d\}] - m_1^{bd}\right)' - \Sigma_{11}^{bd}\}$$

$$\begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix}' = 0 \qquad (4)$$

where $P(j \in \{b,d\})$ denotes the probability that $j \in \{b,d\}$. For birth date $b$, this relationship may be written as:

$$M_b(\Delta_0, \lambda_0) = \sum_j P(j \in \{b,d\}) F_{bd}(\Delta_0, E[y_{1j}^{bd} \mid j \in \{b,d\}])$$

where

$$\Delta_0 \equiv [\mu, \beta, \delta_1, ..., \delta_T, \sigma_\mu^2, \sigma_\beta^2, \sigma_{\mu\beta}, \rho, \sigma_{\epsilon_1}^2, \ldots, \sigma_{\epsilon_T}^2]',$$

i.e. all the parameters of interest in the model given by equation (2), and

$$\lambda_0 \equiv \left[P(j \in \{b,1\}), ..., P(j \in \{b,T\}), E[y_{1j}^{b1} \mid j \in \{b,1\}], ..., E[y_{1j}^{b1} \mid j \in \{b,1\}]\right]',$$

i.e. all of the death probabiliies and conditional moments of $\underline{y}_j$.

When the birth date $b$ is not known, we write the same type of expression as in equation (4) and include the variances of $\mu^0$ and $\alpha^0$ in the vector $\Delta_0$. We remind the reader, however, that the parameters in the vector $\Delta_0$ do not have the same interpretation for firms with known and unknown birth dates because the distribution of $\underline{y}_j$ is censored by the unknown birth date.

### 3.4 Estimation by the method of Asymptotic Least Squares

Since $P(j \in \{b,d\})$ can be estimated by the proportion $\hat{\pi}_{bd}$ of firms that die at time $d$, $E[y_j^{bd} \mid j \in \{b,d\}]$ can be estimated by $\bar{y}_{1j}^{bd}$, the average of the observable elements of $\underline{y}_j$, and $V[y_1^{bd} \mid j \in \{b,d\}]$ by $\hat{S}_1^{bd}$, the empirical variance of the observable part of $\underline{y}_j$, we obtain an asymptotic linear model in the terminology of Gouriéroux, Monfort and Trognon (1985). For a given birth cohort $b$, we have:

$$\sum_d \hat{\pi}_{bd} \begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix} (\bar{y}_1^{bd} - m_1^{bd}) = \omega_{b1}$$

$$\sum_d \hat{\pi}_{bd} \begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix} [\hat{S}_1^{bd} + (\bar{y}_1^{bd} - m_1^{bd})(\bar{y}_1^{bd} - m_1^{bd})' - \Sigma_{11}^{bd}] \begin{bmatrix} I_{d-b+1} \\ \Sigma_{21}^{bd}(\Sigma_{11}^{bd})^{-1} \end{bmatrix}' = \omega_{b2} \quad (5)$$

where $m_1^{bd}, \Sigma_{11}^{bd}$, and $\Sigma_{12}^{bd}$ are all functions of the parameter of interest $\Delta_0$ as specified in equation (2) for firms with known birth dates and equivalent expressions, including the parameters $\sigma^2_{\mu_0}$ and $\sigma^2_{\nu_0}$ for firms with unknown birth dates, and the vector $[\omega_{b1}, \omega_{b2}]'$ is the error for cohort $b$ measuring the deviation of equation (5) from its expected value 0. The method of Asymptotic Least Squares (ALS ) can be performed on model (5), giving consistent, asymptotically normal (CAN) estimates of the parameters $\Delta_0$. A more detailed discussion of the implementation of this estimation method is presented in appendix B.

### 3.5 Pooling birth cohorts

We restrict our discussion to firms for which the date of birth is observable. Let $b$ be the date of birth and assume the birth process is exogenous. In this case the relation among the parameters of interest is, $\Delta_0$, and the observable probabilites and conditional moments, $\lambda_0$, is given by:

$$M(\Delta_0, \lambda_0) \equiv [M_2(\Delta_0, \lambda_0), ..., M_T(\Delta_0, \lambda_0)]' = 0$$

Here also, it is possible to obtain CAN estimates using the ALS method. Details of the computaton method are provided in Appendix D.

It is possible to let parameter $\Delta_0$ be specific to one or several cohorts. The equation (5) is specified so that first-step estimates of probabilities and conditional variances for a given cohorts $b$ only enter a single cohort's $M_b$. So, given the independence of first-step estimates between cohorts, the application of ALS methods to pooled cohorts with some common and some specific parameters will lead to the application of ALS separately on the different subsets of cohorts for which parameters are constrained to be the same. Moreover, the corresponding estimated parameters will be independent of one another. Therefore, it is possible to test the identity of the different parameters between subsets of cohorts.

## 4 Estimation Results

Table 4 presents results for all variables for the sample of firms first observed in 1978 (unknown birth dates and pre-sample histories). Although the goodness-of-fit $\chi^2$ statistics are somewhat large, the models fit the data reasonably well (except for the profitability variable) considering that 29 parameters are used for 80 moments. For the payroll, compensation and capital variables, the autocorrelation structure of the uncensored data (displayed in the "fitted" column) is generally quite similar to the raw (but censored) autocorrelation structure. For these variables, the serial correlation coefficient $\rho$ varies between 0.76 and 0.95, which accounts for only a trivial fraction of the structural autocorrelation after the third lag. Persistent autocorrelation due the effects of $\mu_j$ and $\beta_j$ dominate the autocorrelation structure beyond the third lag. The variability of the trend is a more important source of autocorrelation for the compensation variables than for employment or assets. As regards the variables measured in ratios, value added and indebtedness display considerable autocorrelation in the structure, with more of the dampening one would expect from a process in which the serial correlation parameter was important. Profitability, on the other hand, has a very unusual structural serial correlation pattern, dominated by the negative correlation between the random initial condition and trend. The reader is reminded that the model does not fit the profitability variable well. For all variables except value added, the variance of the structural shocks ($\varepsilon$) is roughly constant.

Table 5 presents results for all variables when the birth date is observed (entering the sample from 1979 to 1988). The model has more trouble fitting these moments (24 parameters to fit 275 moments). The structural autocorrelation, however, is similar for all variables (whether measured in logarithms or as ratios): the autocorrelation is dominated by the part due to the random initial conditions and trend after the third lag. Structurally, then, the early history of firm-level employment, compensation, profitability, production and capital growth, seems to be dominated by the permanent differences among the firms and the random trend. There is also more temporal heterogeneity in the structural shock variances among these younger firms.

## 5 Conclusions

We have developed a model for decomposing the covariance structure of panel data on firms into a part due to permanent heterogeneity, a part due to differential histories with unknown ages, and a part due to the evolution of economic shocks to the firm. We have shown how to implement this model on unbalanced longitudinal data for firms which have known and unknown ages and histories. Our model allows the death of firms to be endogenous and correctly handles the problems arising from the estimation of this death process. The model is applied to detailed French enterprise data on employment, payroll, salaries, capital value added, debt and profitability. We find that the structural autocorrelation in these variables is dominated by the the part due to initial heterogenieity and random growth rates. Serial correlation in the periodic shocks is less important.

The results presented here suggest that the technique is feasible and that different firm level measurements may have very different covariance structures. Furthermore, the bias associated with unknown age structures and arbitrarily balanced samples may be important in so far as the models are able to distinguish between permanent heterogeneity and autocorrelation related to differential histories.

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## APPENDIX A

**Lemma 1** *Let three random variables* $(x, y, z)$ *with p.d.f.* $f(x, y, z)$ *and with conditional p.d.f. of* $z$ *given* $(x, y)$, $k(z \mid x, y)$, *equal to the conditional p.d.f. of* $z$ *given* $y$, $l(z \mid y)$. *Then,*

$$E[h(y,z)|g(x,y) \geq 0] = E[E(h(y,z)|y)|g(x,y) \geq 0]$$

**Proof:**

$$\begin{aligned}
& E[h(y,z) \mid g(x,y) \geq 0] \\
= \; & \tfrac{1}{P(g(x,y)\geq 0)} \int\int\int \mathbb{1}[g(x,y) \geq 0] h(y,z) f(x,y,z) dx dy dz \\
= \; & \tfrac{1}{P(g(x,y)\geq 0)} \int\int\int \mathbb{1}[g(x,y) \geq 0] h(y,z) l(z \mid y) f(x,y) dx dy dz \\
= \; & \tfrac{1}{P(g(x,y)\geq 0)} \int\int \mathbb{1}[g(x,y) \geq 0] E[h(y,z) \mid y] f(x,y) dx dy \\
= \; & E\left[E[h(y,z) \mid y] \mid g(x,y) \geq 0\right]
\end{aligned}$$

Q.E.D.

# APPENDIX B

Equation (6) defines a relation between the expectation of the first step estimates ($\lambda_0$) and the parameters of interest ($\Delta_0$) of the form $M_b(\Delta_0, \lambda_0) = 0$, which may be rewritten as:

$$M_b(\Delta_0, P_b, E_b(y_{1j}^{bd})) = \sum_d P_{b,d} F_{b,d}(\Delta_0, E[y_{1j}^{bd} \mid j \in \{b, d\}]) = 0$$

where $\lambda_0 \equiv (P_b, E_b(y_{1j}^{bd}))'$, $P_b$ is the $T - b + 1$ vector of $P_{b,d}$ for $d = b, ..., T$, $P_{b,d}$ is the probability of dying at date $d$ given birth at date $b$ and $E_b(z)$ is the vector of moments of the variable $z$ conditional on death at dates $d = b, ..., T$.

$$E_b(z) = [E(z \mid b, d)]_{d=b,b+1,...,T}$$

Following Gouriéroux, Monfort and Trognon (1985) CAN estimates of $\Delta$ may be obtained from first-step estimates $\hat{\lambda}_n$, based on a sample of size $n$, through the minimization over $\Delta$ of

$$\xi_b(\Delta, \hat{\lambda}_n) = M_b'(\Delta, \hat{\lambda}_n) S_b M_b(\Delta, \hat{\lambda}_n).$$

More precise estimates are obtained with a special choice of $S_b$:

$$S_b = \left[\frac{\partial M_b}{\partial \lambda'}(\Delta_0, \lambda_0) V(\lambda_0) \frac{\partial M_b'}{\partial \lambda}(\Delta_0, \lambda_0)\right]^{-1}$$

where $V(\lambda_0)$ is the asymptotic variance-covariance matrix of $\hat{\lambda}_n$:

$$\sqrt{n}(\hat{\lambda}_n - \lambda_0) \to N(0, V(\lambda_0))$$

Given the result in Appendix C on the distribution of the estimates of the first-step parameters, it is straightforward to compute the optimal weight matrix $S_b$ as:

$$S_b = \left[\sum_d P_{bd}\left(\frac{\partial F_{bd}}{\partial \bar{z}^{d'}} V_{bd} \frac{\partial F_{bd}'}{\partial \bar{z}^d} + F_{bd} F_{bd}'\right)\right]^{-1}$$

where $V_{bd}$ is the variance of the estimation of first and second moments over the population of firms $j \in \{b, d\}$. This leads to an asymptotically normal estimate of $\Delta$:

$$\sqrt{n}(\hat{\Delta}_n - \Delta_0) \to N\left(0, \left[\frac{\partial M_b'}{\partial \Delta}(\Delta_0, \lambda_0) S_b \frac{\partial M_b}{\partial \Delta'}(\Delta_0, \lambda_0)\right]^{-1}\right)$$

As usual, it is possible to test the compatibility of the different restrictions imposed by the model on the first-step estimates. Under this assumption, we have:

$$\xi_b(\hat{\Delta}, \hat{\lambda}_n) \rightarrow \chi^2_{(\dim(M_b)-\dim(\Delta_0))}$$

with

$$T - b + 1 + \frac{(T - b + 1)(T - b + 2)}{2} - 2(T - b + 1) - 4$$

degrees of freedom.

## APPENDIX C

This appendix proves the properties of the estimates of the first-step parameters. Theses properties are summarized in the following lemma.

**Lemma 2** *Let $z_1$ and $z_2$ be two random vectors observed on a sample of $N$ observations. Let $I$ and $J$ be two subsamples with no common observations. Let*

$$m_{1I} = E(z_1 | \in I), m_{2J} = E(z_2 | \in J), P_I = Prob(\in I) \text{ and } P_J = Prob(\in J)$$

*Then the asymptotic distribution of the estimators:*

$$\hat{m}_{1I} = \frac{1}{N_I} \sum_{j \in I} z_{1j}, \hat{m}_{2J} = \frac{1}{N_J} \sum_{j \in J} z_{2j}, \hat{P}_I = \frac{N_I}{N} \hat{P}_J = \frac{N_J}{N}$$

*is:*

$$\sqrt{N} \left[ \begin{pmatrix} \hat{m}_{1I} \\ \hat{m}_{2J} \\ \hat{P}_I \\ \hat{P}_J \end{pmatrix} - \begin{pmatrix} m_{1I} \\ m_{2J} \\ P_I \\ P_J \end{pmatrix} - \right] N \overrightarrow{\to} \infty$$

$$\mathcal{N} \left[ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} V_I(z_1)/P_I & 0 & 0 & 0 \\ 0 & V_J(z_2)/P_J & 0 & 0 \\ 0 & 0 & P_I(1-P_I) & -P_I P_J \\ 0 & 0 & -P_I P_J & P_J(1-P_J) \end{pmatrix} \right]$$

*where*

$$V_K(z) = E\left[(z - E(z|K))(z - E(z|K))' | \in K\right], K = I, J$$

*and*

$$\hat{V}_K(z) = \frac{1}{N_K} \sum_{j \in K} \left( z_j - \frac{1}{N_K} \sum_{j \in K} z_j \right) \left( z_j - \frac{1}{N_K} \sum_{j \in K} z_j \right)'$$

**Proof :**

Let $I_j$ and $J_j$ be the indicator variables corresponding to subsamples I and J, respectively:

$$\begin{cases} I_j = 1 \text{ if } j \in I, I_j = 0 \text{ otherwise} \\ J_j = 1 \text{ if } j \in J, J_j = 0 \text{ otherwise} \end{cases}$$

We have

$$\hat{m}_{1I} = \frac{\frac{1}{N}\sum_j z_{1i} I_j}{\frac{1}{N}\sum_j I_j}, \hat{m}_{2J} = \frac{\frac{1}{N}\sum_j z_{2j} J_j}{\frac{1}{N}\sum_j I_j}, \hat{P}_I = \frac{1}{N}\sum_j I_j, \hat{P}_J = \frac{1}{N}\sum_j J_j$$

and so:

$$\sqrt{N}\begin{pmatrix} \frac{\frac{1}{N}\sum_j z_{1j} I_j}{\frac{1}{N}\sum_j I_j} - \frac{E(z_1 I)}{E(I)} \\ \frac{\frac{1}{N}\sum_j z_{2j} J_j}{\frac{1}{N}\sum_j J_j} - \frac{E(z_2 J)}{E(J)} \\ \frac{1}{N}\sum_j I_j - E(I) \\ \frac{1}{N}\sum_j J_j - E(J) \end{pmatrix} = \sqrt{N}\begin{pmatrix} \frac{E(I)\frac{1}{N}\sum_j z_{1j} I_j - E(z_1 I)\frac{1}{N}\sum_j I_j}{E(I)\frac{1}{N}\sum_j I_j} \\ \frac{E(J)\frac{1}{N}\sum_j z_{2j} J_j - E(z_2 J)\frac{1}{N}\sum_j J_j}{E(I)\frac{1}{N}\sum_j J_j} \\ \frac{1}{N}\sum_j I_j - E(I) \\ \frac{1}{N}\sum_j J_j - E(J) \end{pmatrix}$$

has the same distribution as:

$$\sqrt{N}\begin{pmatrix} \frac{E(I)\frac{1}{N}\sum_j z_{1j} I_j - E(Z_1 I)\frac{1}{N}\sum_j I_j}{E(I)^2} \\ \frac{E(J)\frac{1}{N}\sum_j z_{2j} J_j - E(Z_2 J)\frac{1}{N}\sum_j J_j}{E(J)^2} \\ \frac{1}{N}\sum_j I_j - E(I) \\ \frac{1}{N}\sum_j J_j - E(J) \end{pmatrix},$$

which is normal with variance $\Sigma = (\sigma_{kl})$. Elements are given by:

$$\sigma_{1,1} = \frac{1}{E^4(I)} E\left[(E(I)z_1 I - E(z_1 I)I)^2\right] = \frac{1}{E^4(I)} E\left[E^2(I)z_1^2 I - E^2(z_1 I)I\right]$$

$$= \frac{1}{E(I)}\left[\frac{E(z_1^2 I)}{E(I)} - \frac{(E(z_1 I))^2}{E(I)}\right] = \frac{1}{P_I} V_I(z_1)$$

because $I^2 = I$ and $E(I) = P_I$. Similarly we have:

$\sigma_{2,2} = \frac{1}{P_J} V_J(z_2)$

$\sigma_{3,3} = P_I(1 - P_I)$

$\sigma_{4,4} = P_J(1 - P_J)$

$\sigma_{3,4} = -P_I P_J$

$\sigma_{1,2} = \frac{1}{E^2(I)E^2(J)} E\left[(E(I)z_1 I - E(z_1 I)I)(E(J)z_2 J - E(z_2 J)J)\right] = 0$

$\sigma_{1,3} = \frac{1}{E^2(I)} E\left[(E(I)z_1 I - E(z_1 I)I)I\right] = \frac{1}{E^2(I)} E\left[E(I)z_1 I - E(z_1 I)I\right] = 0$

and it is obvious that the other terms are zero.

Q.E.D.

Applying the above lemma to a cohort b, we obtain the following distribution for the first-step estimates:

$$\sqrt{N_b}\left[\begin{pmatrix} \hat{\pi}_b \\ \overline{z_b} \end{pmatrix} - \begin{pmatrix} P_b \\ m_b \end{pmatrix}\right] \underset{N_b \to \infty}{\longrightarrow}$$

$$\mathcal{N}\left(0, \begin{pmatrix} \mathrm{Bloc}_{|d=b,\dots,T}(P_b) - P_b P_b' & 0 \\ 0 & \mathrm{Bloc}_{|d=b,\dots,T}\left(\frac{V_{bd}}{P_{bd}}\right) \end{pmatrix}\right)$$

where $\mathrm{Bloc}_{|h=1,\dots,l}(m_h)$ denotes the block diagonal matrix with $m_h$ as its elements and $P_b$ is the $T - b + 1$ vector made of $P_{bd}$ for $d = b, \dots, T$ with $P_{bd}$ being the probability of dying at date $d$ given birth at date $b$.

When we pool the different cohorts together, we assume that the birth date is exogenous. The limit distributions are shown here:

$$\sqrt{N}\left[\begin{pmatrix} \hat{\pi}_b k_b \\ \overline{z_b} \end{pmatrix}_{|b=1,\dots T} - \begin{pmatrix} P_b k_b \\ m_b \end{pmatrix}_{|b=1,\dots T}\right] \underset{N \to \infty}{\longrightarrow}$$

$$\mathcal{N}\left(0, \mathrm{Bloc}_{|b=1,\dots T}\left[\begin{pmatrix} k_b \mathrm{Bloc}_{|d=b,\dots,T}(P_{bd}) - P_b P_b' k_b^2 & 0 \\ 0 & \mathrm{Bloc}_{|d=b,\dots,T}\left(\frac{V_{bd}}{P_{bd} k_b}\right) \end{pmatrix}\right]\right)$$

where $k_b$, assumed to be exogenous, is the proportion of firms born at date $b$. A simple transformation gives:

$$\sqrt{N}\left[\begin{pmatrix}\frac{\hat{\pi}_b}{z_b}\end{pmatrix}_{|b=1,\ldots T} - \begin{pmatrix} P_b \\ m_b \end{pmatrix}_{|b=1,\ldots T}\right] \; \underset{N\to\infty}{\longrightarrow}$$

$$\mathcal{N}\left(0, \mathrm{Bloc}_{|b=1,\ldots T}\left[\frac{1}{k_b}\begin{pmatrix} \mathrm{Bloc}_{|d=b,\ldots,T}(P_{bd}) - P_b P_b' k_b & 0 \\ 0 & \mathrm{Bloc}_{|d=b,\ldots,T}\left(\frac{V_{bd}}{P_{bd}}\right) \end{pmatrix}\right]\right)$$

## APPENDIX D

When cohorts are pooled estimates are obtained through the minimization over $\Delta$ of:

$$M'(\Delta,\hat{\pi},\bar{z})SM(\Delta,\hat{\pi},\bar{z})$$

where the optimal weight matrix $S$ is:

$$S=\left[\frac{\partial M}{\partial\pi\bar{z}'}(\Delta_0,\pi_0,\bar{z}_0)V_{(\pi,\bar{z})}\frac{\partial M'}{\partial\pi\bar{z}}(\Delta_0,\pi_0,\bar{z}_0)\right]^{-1}.$$

Given the expressions for the variance matrices of the full set of first-step parameters derived in Appendix C, it is straightforward to compute this matrix:

$$S^*=\left[\frac{\partial M}{\partial\pi\bar{z}'}(\Delta_0,\pi_0,\bar{z}_0)V_{(\pi,\bar{z})}\frac{\partial M'}{\partial\pi\bar{z}}(\Delta_0,\pi_0,\bar{z}_0)\right]^{-1}=\text{Bloc}_{|b=1,\dots T}[k_bS_b]$$

where

$$S_b=\left[\sum_d P_{b,d}\left(\frac{\partial F_b}{\partial\bar{z}_b'}V_{b,d}\frac{\partial F_b'}{\partial\bar{z}_b}+F_b(\Delta_0,\bar{z}_b)F_b(\Delta_0,\bar{z}_b)'\right)\right]^{-1}$$

is the same matrix defined in Appendix B.

Parameters are obtained by minimization over $\Delta$ of the following objective:

$$\xi(\Delta,\hat{\lambda}_n)=M(\Delta,\pi,\bar{z})S^*M(\Delta,\pi,\bar{z})=\sum_b k_bM_b(\Delta,\pi,\bar{z})S_bM_b(\Delta,\pi,\bar{z})=\sum_b k_b\xi_b(\Delta,\hat{\lambda}_n)$$

which is, therefore, a weighted sum of the objectives corresponding to each cohort. The corresponding estimates have the following precision:

$$\sqrt{N}(\Delta-\Delta_0)\underset{N\to\infty}{\longrightarrow}\mathcal{N}\left(0\left[\frac{\partial M'}{\partial\Delta}S^*\frac{\partial M}{\partial\Delta'}\right]^{-1}\right)$$

where:

$$\left[\frac{\partial M'}{\partial\Delta}S^*\frac{\partial M}{\partial\Delta'}\right]\equiv\sum_b k_b\frac{\partial M_b'}{\partial\Delta}S_b\frac{\partial M_b}{\partial\Delta}$$

The test for the compatibility of the different equations, under the null hypothesis of equation (2), is:

$$\xi(\hat{\Delta},\hat{\lambda}_n)\to\chi^2_{(\dim(M)-\dim(\Delta))}.$$

## APPENDIX E: The Procedure for Imputation of Missing Data

For the 2.5% of firm-year observations that were missing data, we imputed the missing values as follows. Suppose data for $y_t$ is missing. If $t+1$ and $t-1$ variables are available, the basic formula is:

$$y_t = \gamma y_{t+1} + \lambda y_{t-1} + \sum_{j=-1}^{1} \beta_j x_{t+j} + v_t$$

In the case where the following year values, $t+1$, were also missing, the formula becomes:

$$y_t = \lambda^* y_{t-1} + \sum_{j=0}^{1} \beta_j^* x_{t+j} + v_t^*$$

If previous year values, $t-1$, were also missing, the formula is:

$$y_t = \gamma^{**} y_{t+1} + \sum_{j=-1}^{0} \beta_j^{**} x_{t+j} + v_t^{**}$$

The unknown coefficients in these missing data formulas ($\gamma$'s, $\lambda$'s, and $\beta$'s) were estimated by least squares with firm specific intercepts using all firms in the basic DS with nonmissing data for each of the imputed ($y$) and conditioning ($x$) variables. The conditioning variables in all regressions were real value-added and total employment in year $t$. Missing values of variables were imputed as the conditional expectation of $y_t$ given the firm's mean value of $y$, the values of $x$, and the estimated coefficients. This procedure was applied for missing values of all variables.[14]

[14]Missing values of the two conditioning variables, value-added and total employment, were imputed using versions of the above equations that excluded $x_t$ from the model. For these variables, the data from the first and last year a firm is sampled are never missing.

**Table 1**

**Structure of the Sample of Firms**
**Total Employment and Number of Firms by Date of Entry into Sample**
**(1978-1988, All firms)**

| First Year in Sample | Total Employment in Sample Firms (in thousands, top line) Number of Firms (middle line) Weighted Number of Firms (bottom line) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 | 1988 |
| All | 7,998 | 8,134 | 8,171 | 8,030 | 7,999 | 7,794 | 7,599 | 7,470 | 7,373 | 7,317 | 7,153 |
| | 11,200 | 11,758 | 11,863 | 11,926 | 11,881 | 11,699 | 11,452 | 11,429 | 11,490 | 11,489 | 10,907 |
| | 62,923 | 66,295 | 67,047 | 67,253 | 66,799 | 65,273 | 63,377 | 62,836 | 63,528 | 64,219 | 61,235 |
| 1978 | 7,998 | 7,622 | 7,384 | 6,949 | 6,707 | 6,350 | 6,032 | 5,685 | 5,365 | 5,116 | 4,817 |
| | 11,200 | 10,536 | 10,040 | 9,499 | 9,044 | 8,578 | 8,075 | 7,658 | 7,295 | 6,885 | 6,226 |
| | 62,923 | 57,432 | 53,739 | 49,951 | 46,716 | 43,336 | 39,991 | 37,156 | 34,806 | 32,328 | 28,596 |
| 1979 | | 511 | 457 | 433 | 407 | 380 | 353 | 331 | 320 | 309 | 285 |
| | | 1,222 | 1,015 | 932 | 848 | 771 | 701 | 653 | 613 | 572 | 503 |
| | | 8,863 | 6,803 | 6,076 | 5,232 | 4,599 | 3,992 | 3,550 | 3,208 | 2,866 | 2,392 |
| 1980 | | | 330 | 281 | 267 | 260 | 239 | 231 | 216 | 215 | 211 |
| | | | 808 | 639 | 576 | 519 | 458 | 435 | 398 | 372 | 337 |
| | | | 6,505 | 4,778 | 4,166 | 3,651 | 3,155 | 2,973 | 2,705 | 2,518 | 2,252 |
| 1981 | | | | 367 | 321 | 300 | 273 | 252 | 218 | 209 | 193 |
| | | | | 856 | 682 | 615 | 543 | 506 | 469 | 430 | 367 |
| | | | | 6,448 | 4,826 | 4,210 | 3,552 | 3,237 | 3,000 | 2,674 | 2,285 |
| 1982 | | | | | 296 | 253 | 230 | 209 | 193 | 172 | 156 |
| | | | | | 731 | 592 | 510 | 452 | 420 | 389 | 335 |
| | | | | | 5,859 | 4,381 | 3,758 | 3,199 | 2,914 | 2,669 | 2,273 |
| 1983 | | | | | | 251 | 215 | 205 | 188 | 178 | 167 |
| | | | | | | 624 | 483 | 443 | 396 | 362 | 299 |
| | | | | | | 5,096 | 3,713 | 3,359 | 2,969 | 2,622 | 2,082 |
| 1984 | | | | | | | 257 | 215 | 208 | 184 | 161 |
| | | | | | | | 682 | 525 | 486 | 423 | 354 |
| | | | | | | | 5,216 | 3,754 | 3,406 | 2,890 | 2,334 |
| 1985 | | | | | | | | 343 | 300 | 257 | 225 |
| | | | | | | | | 757 | 600 | 539 | 435 |
| | | | | | | | | 5,608 | 4,206 | 3,714 | 2,933 |
| 1986 | | | | | | | | | 365 | 329 | 290 |
| | | | | | | | | | 813 | 638 | 514 |
| | | | | | | | | | 6,314 | 4,869 | 3,893 |
| 1987 | | | | | | | | | | 349 | 287 |
| | | | | | | | | | | 879 | 589 |
| | | | | | | | | | | 7,069 | 4,497 |
| 1988 | | | | | | | | | | | 362 |
| | | | | | | | | | | | 948 |
| | | | | | | | | | | | 7,698 |

Sources: INSEE: Echantillon d'entreprises, BIC, EAE.

Notes: 1. Total employment is the weighted sum of employment in the sample firms where the weight is the inverse probability of selection into the échantillon d'entreprises (Corbel 1989).

2. Number of firms is the unweighted count of firms in the sample with the indicated characteristics.

3. Weighted number of firms is the sum of the firm weights in the sample with the indicated characteristics.

Table 2

**Summary Statistics for all Variables Used in the Covariance Models**
**Mean, (Standard Deviation), Sample Size for Firms Entering the Sample in 1978**

| 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 | 1988 |
|---|---|---|---|---|---|---|---|---|---|---|
| Logarithm of Employment as of 12/31 (BIC, thousands) | | | | | | | | | | |
| -2.94 | -2.90 | -2.87 | -2.86 | -2.83 | -2.82 | -2.80 | -2.79 | -2.78 | -2.76 | -2.72 |
| (0.93) | (0.93) | (0.94) | (0.95) | (0.95) | (0.95) | (0.96) | (0.96) | (0.96) | (0.97) | (0.98) |
| 11,200 | 10,536 | 10,040 | 9,499 | 9,044 | 8,578 | 8,075 | 7,658 | 7,295 | 6,885 | 6,226 |
| Logarithm of Real Total Labor Cost (BIC, millions F80) | | | | | | | | | | |
| 1.34 | 1.44 | 1.48 | 1.50 | 1.54 | 1.58 | 1.60 | 1.63 | 1.68 | 1.73 | 1.78 |
| (1.04) | (1.02) | (1.02) | (1.03) | (1.03) | (1.03) | (1.06) | (1.04) | (1.06) | (1.04) | (1.06) |
| 11,200 | 10,536 | 10,040 | 9,499 | 9,044 | 8,578 | 8,075 | 7,658 | 7,295 | 6,885 | 6,226 |
| Logarithm of Real Total Payroll (BIC, millions F80) | | | | | | | | | | |
| 1.00 | 1.09 | 1.13 | 1.16 | 1.19 | 1.22 | 1.24 | 1.27 | 1.32 | 1.37 | 1.41 |
| (1.05) | (1.03) | (1.03) | (1.03) | (1.04) | (1.04) | (1.07) | (1.06) | (1.06) | (1.05) | (1.09) |
| 11,200 | 10,536 | 10,040 | 9,499 | 9,044 | 8,578 | 8,075 | 7,658 | 7,295 | 6,885 | 6,226 |
| Logarithm of Real Total Payroll (EAE, millions F80) | | | | | | | | | | |
| 1.19 | 1.19 | 1.20 | 1.18 | 1.20 | 1.21 | 1.21 | 1.24 | 1.28 | 1.32 | 1.37 |
| (1.01) | (1.01) | (1.01) | (1.01) | (1.01) | (1.02) | (1.03) | (1.04) | (1.04) | (1.04) | (1.05) |
| 9,841 | 9,542 | 9,188 | 8,778 | 8,414 | 8,007 | 7,577 | 7,188 | 6,849 | 6,471 | 5,853 |
| Value Added Gross of Factor Costs/Total Assets (BIC, F80/F80) | | | | | | | | | | |
| 1.06 | 1.04 | 1.06 | 1.01 | 1.00 | 0.95 | 0.87 | 0.83 | 0.84 | 0.79 | 0.76 |
| (0.95) | (0.89) | (0.88) | (0.82) | (0.81) | (0.73) | (0.68) | (0.65) | (0.68) | (0.57) | (0.58) |
| 11,200 | 10,536 | 10,040 | 9,499 | 9,044 | 8,578 | 8,075 | 7,658 | 7,295 | 6,885 | 6,226 |
| Gross Operating Profit/Total Assets (BIC F80/F80) | | | | | | | | | | |
| 0.14 | 0.14 | 0.15 | 0.14 | 0.14 | 0.13 | 0.11 | 0.11 | 0.12 | 0.12 | 0.12 |
| (0.39) | (0.26) | (0.28) | (0.25) | (0.30) | (0.22) | (0.17) | (0.16) | (0.16) | (0.14) | (0.18) |
| 11,200 | 10,536 | 10,040 | 9,499 | 9,044 | 8,578 | 8,075 | 7,658 | 7,295 | 6,885 | 6,226 |
| Logarithm of Real Total Assets (BIC, millions F80) | | | | | | | | | | |
| 1.76 | 1.87 | 1.89 | 1.95 | 1.99 | 2.07 | 2.17 | 2.25 | 2.31 | 2.42 | 2.52 |
| (1.43) | (1.42) | (1.42) | (1.43) | (1.44) | (1.45) | (1.46) | (1.47) | (1.49) | (1.48) | (1.50) |
| 11,200 | 10,536 | 10,040 | 9,499 | 9,044 | 8,578 | 8,075 | 7,658 | 7,295 | 6,885 | 6,226 |
| Total Debt/Total Assets (BIC F80/F80) | | | | | | | | | | |
| 0.79 | 0.78 | 0.78 | 0.77 | 0.78 | 0.77 | 0.77 | 0.78 | 0.78 | 0.76 | 0.76 |
| (0.25) | (0.22) | (0.35) | (0.22) | (0.25) | (0.23) | (0.23) | (0.45) | (0.26) | (0.25) | (0.24) |
| 11,200 | 10,536 | 10,040 | 9,499 | 9,044 | 8,578 | 8,075 | 7,658 | 7,295 | 6,885 | 6,226 |

Sources: INSEE, Echantillon d'entreprises, BIC

**Table 3**

**Summary Statistics for all Variables Used in the Covariance Models**
**Mean, (Standard Deviation), Sample Size for Firms Entering the Sample in 1979**

| 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 | 1988 |
|---|---|---|---|---|---|---|---|---|---|
| Logarithm of Employment as of 12/31 (BIC, thousands) | | | | | | | | | |
| -3.40 | -3.28 | -3.24 | -3.17 | -3.13 | -3.09 | -3.05 | -3.00 | -2.94 | -2.85 |
| (0.74) | (0.77) | (0.79) | (0.81) | (0.83) | (0.86) | (0.88) | (0.90) | (0.93) | (0.96) |
| 1,222 | 1,015 | 932 | 848 | 771 | 701 | 653 | 613 | 572 | 503 |
| Logarithm of Real Total Labor Cost (BIC, millions F80) | | | | | | | | | |
| 0.77 | 1.00 | 1.05 | 1.14 | 1.19 | 1.23 | 1.32 | 1.42 | 1.49 | 1.59 |
| (0.92) | (0.88) | (0.91) | (0.93) | (0.93) | (1.05) | (0.98) | (1.00) | (1.02) | (1.06) |
| 1,222 | 1,015 | 932 | 848 | 771 | 701 | 653 | 613 | 572 | 503 |
| Logarithm of Real Total Payroll (BIC, millions F80) | | | | | | | | | |
| 0.42 | 0.65 | 0.70 | 0.79 | 0.83 | 0.88 | 0.96 | 1.06 | 1.13 | 1.23 |
| (0.96) | (0.89) | (0.92) | (0.94) | (0.94) | (1.05) | (0.98) | (1.00) | (1.02) | (1.05) |
| 1,222 | 1,015 | 932 | 848 | 771 | 701 | 653 | 613 | 572 | 503 |
| Logarithm of Real Total Payroll (EAE, millions F80) | | | | | | | | | |
| 0.84 | 0.81 | 0.79 | 0.80 | 0.82 | 0.85 | 0.89 | 0.98 | 1.05 | 1.16 |
| (0.86) | (0.85) | (0.83) | (0.84) | (0.84) | (0.85) | (0.88) | (0.89) | (0.91) | (0.95) |
| 855 | 795 | 755 | 702 | 642 | 585 | 539 | 504 | 467 | 410 |
| Value Added Gross of Factor Costs/Total Assets (BIC, F80/F80) | | | | | | | | | |
| 1.23 | 1.24 | 1.16 | 1.16 | 1.05 | 0.97 | 0.91 | 0.89 | 0.87 | 0.84 |
| (1.24) | (1.05) | (1.07) | (1.08) | (0.80) | (0.87) | (0.74) | (0.72) | (0.73) | (0.68) |
| 1,222 | 1,015 | 932 | 848 | 771 | 701 | 653 | 613 | 572 | 503 |
| Gross Operating Profit/Total Assets (BIC F80/F80) | | | | | | | | | |
| 0.13 | 0.13 | 0.13 | 0.15 | 0.14 | 0.11 | 0.12 | 0.13 | 0.13 | 0.11 |
| (0.41) | (0.24) | (0.30) | (0.52) | (0.21) | (0.18) | (0.14) | (0.14) | (0.16) | (0.15) |
| 1,222 | 1,015 | 932 | 848 | 771 | 701 | 653 | 613 | 572 | 503 |
| Logarithm of Real Total Assets (BIC, millions F80) | | | | | | | | | |
| 1.08 | 1.29 | 1.39 | 1.50 | 1.63 | 1.78 | 1.92 | 2.06 | 2.17 | 2.30 |
| (1.40) | (1.43) | (1.45) | (1.50) | (1.52) | (1.58) | (1.58) | (1.59) | (1.66) | (1.74) |
| 1,222 | 1,015 | 932 | 848 | 771 | 701 | 653 | 613 | 572 | 503 |
| Total Debt/Total Assets (BIC F80/F80) | | | | | | | | | |
| 0.87 | 0.85 | 0.85 | 0.84 | 0.82 | 0.82 | 0.82 | 0.82 | 0.80 | 0.79 |
| (0.25) | (0.21) | (0.22) | (0.35) | (0.22) | (0.21) | (0.21) | (0.25) | (0.22) | (0.20) |
| 1,222 | 1,015 | 932 | 848 | 771 | 701 | 653 | 613 | 572 | 503 |

Sources: INSEE, Echantillon d'entreprises, BIC.

**Table 4**

**Results of Covariance Model Estimation for Firms Entering the Sample in 1978**

| Parameter | Log(Employment) Estimate | Log(Employment) Std. Error | Log(Total Labor Cost) Estimate | Log(Total Labor Cost) Std. Error | Log(Total Payroll)[a] Estimate | Log(Total Payroll)[a] Std. Error | Log(Total Payroll)[b] Estimate | Log(Total Payroll)[b] Std. Error |
|---|---|---|---|---|---|---|---|---|
| Initial mean ($\mu^0$) | -2.9728 | (0.0749) | 15.5759 | (20.4343) | 10.3946 | (8.9879) | 1.3476 | (0.0654) |
| Trend ($\beta$) | 0.0038 | (0.0127) | -0.6293 | (0.5088) | -0.4356 | (0.2348) | -0.0453 | (0.0108) |
| Initial condition ($\nu^0$) | 0.0318 | (0.0806) | -14.3144 | (20.4331) | -9.4515 | (8.9874) | -0.1564 | (0.0697) |
| Serial correlation ($\rho$) | 0.8005 | (0.0190) | 0.9508 | (0.0391) | 0.9477 | (0.0257) | 0.7607 | (0.0192) |
| Variance of $\mu_j$ ($\sigma^2_{\mu^*}$) | 0.8042 | (0.0280) | 0.2704 | (1.4714) | 0.0267 | (0.8221) | 0.8736 | (0.0289) |
| Variance of $\beta_j$ ($\sigma^2_\beta$) | 0.0035 | (0.0001) | 0.0013 | (0.0010) | 0.0011 | (0.0008) | 0.0007 | (0.0002) |
| Covariance of $\mu_j$, $\beta_j$ | -0.0131 | (0.0022) | 0.0116 | (0.0302) | 0.0199 | (0.0244) | 0.0005 | (0.0016) |
| Variance of $\nu_j$ | 0.0946 | (0.0294) | 0.8491 | (1.2325) | 1.1263 | (0.9613) | 0.2463 | (0.0195) |
| Variance of shocks | | | | | | | | |
| $\varepsilon_{79}$ | 0.0407 | (0.0032) | 0.0322 | (0.0052) | 0.0391 | (0.0055) | 0.0400 | (0.0021) |
| $\varepsilon_{80}$ | 0.0398 | (0.0031) | 0.0467 | (0.0090) | 0.0561 | (0.0085) | 0.0354 | (0.0020) |
| $\varepsilon_{81}$ | 0.0354 | (0.0021) | 0.0447 | (0.0069) | 0.0472 | (0.0056) | 0.0314 | (0.0019) |
| $\varepsilon_{82}$ | 0.0325 | (0.0028) | 0.0364 | (0.0058) | 0.0427 | (0.0057) | 0.0282 | (0.0016) |
| $\varepsilon_{83}$ | 0.0321 | (0.0024) | 0.0295 | (0.0040) | 0.0376 | (0.0040) | 0.0242 | (0.0019) |
| $\varepsilon_{84}$ | 0.0308 | (0.0019) | 0.0550 | (0.0116) | 0.0700 | (0.0138) | 0.0356 | (0.0020) |
| $\varepsilon_{85}$ | 0.0312 | (0.0020) | 0.0336 | (0.0028) | 0.0437 | (0.0065) | 0.0397 | (0.0027) |
| $\varepsilon_{86}$ | 0.0221 | (0.0016) | 0.0310 | (0.0045) | 0.0361 | (0.0048) | 0.0271 | (0.0028) |
| $\varepsilon_{87}$ | 0.0300 | (0.0031) | 0.0347 | (0.0044) | 0.0350 | (0.0048) | 0.0313 | (0.0037) |
| $\varepsilon_{88}$ | 0.0292 | (0.0025) | 0.0480 | (0.0135) | 0.0584 | (0.0300) | 0.0320 | (0.0035) |
| $X^2$ | 191.72 | | 103.35 | | 100.87 | | 123.16 | |
| Degrees of freedom | 51 | | 51 | | 51 | | 51 | |
| P-value | 1.0000 | | 1.0000 | | 1.0000 | | 1.0000 | |

| Serial correlations | Actual | Fitted[c] | Actual | Fitted[c] | Actual | Fitted[c] | Actual | Fitted[c] |
|---|---|---|---|---|---|---|---|---|
| lag 1 | 0.9625 | 0.9772 | 0.9606 | 0.9807 | 0.9603 | 0.9774 | 0.9746 | 0.9792 |
| lag 2 | 0.9411 | 0.9541 | 0.9354 | 0.9602 | 0.9352 | 0.9533 | 0.9570 | 0.9587 |
| lag 3 | 0.9276 | 0.9303 | 0.9209 | 0.9388 | 0.9208 | 0.9283 | 0.9416 | 0.9400 |
| lag 4 | 0.9108 | 0.9054 | 0.8967 | 0.9167 | 0.8977 | 0.9025 | 0.9283 | 0.9235 |
| lag 5 | 0.9053 | 0.8791 | 0.8965 | 0.8942 | 0.8974 | 0.8764 | 0.9203 | 0.9091 |
| lag 6 | 0.8934 | 0.8516 | 0.8547 | 0.8714 | 0.8568 | 0.8503 | 0.9117 | 0.8965 |
| lag 7 | 0.8816 | 0.8229 | 0.8628 | 0.8486 | 0.8540 | 0.8245 | 0.9022 | 0.8855 |
| lag 8 | 0.8742 | 0.7934 | 0.8474 | 0.8259 | 0.8465 | 0.7993 | 0.8942 | 0.8756 |
| lag 9 | 0.8614 | 0.7632 | 0.8483 | 0.8036 | 0.8453 | 0.7749 | 0.8885 | 0.8666 |
| lag 10 | 0.8499 | 0.7327 | 0.8390 | 0.7817 | 0.8177 | 0.7515 | 0.8790 | 0.8581 |

(continued)

**Table 4 (continued)**

**Results of Covariance Model Estimation for Firms Entering the Sample in 1978**

| Parameter | VA/Assets Estimate | VA/Assets Std. Error | Operating Inc/Assets Estimate | Operating Inc/Assets Std. Error | Log(Total Assets) Estimate | Log(Total Assets) Std. Error | Debt/Assets Estimate | Debt/Assets Std. Error |
|---|---|---|---|---|---|---|---|---|
| Initial mean ($\mu^0$) | 0.9771 | (0.0174) | 0.1265 | (0.0089) | 3.1976 | (0.2174) | 0.9114 | (0.0440) |
| Trend ($\beta$) | 0.0254 | (0.0057) | 0.0053 | (0.0033) | -0.2338 | (0.0243) | -0.0190 | (0.0064) |
| Initial condition ($v^0$) | 0.2944 | (0.0661) | 0.0353 | (0.0559) | -1.5538 | (0.2170) | -0.1354 | (0.0462) |
| Serial correlation ($\rho$) | 0.1684 | (0.0153) | 0.1069 | (0.0133) | 0.7742 | (0.0173) | 0.7738 | (0.0248) |
| Variance of $\mu_j$ ($\sigma^2_{\mu^0}$) | 0.7498 | (0.0288) | 0.1128 | (0.0068) | 1.9033 | (0.0462) | 0.0357 | (0.0045) |
| Variance of $\beta_j$ ($\sigma^2_\beta$) | 0.0027 | (0.0004) | 0.0058 | (0.0005) | 0.0027 | (0.0002) | 0.0011 | (0.0001) |
| Covariance of $\mu_j$, $\beta_j$ | -0.0291 | (0.0029) | -0.0257 | (0.0018) | -0.0052 | (0.0021) | -0.0014 | (0.0002) |
| Variance of $v_j$ | 7.1906 | (1.3266) | 2.1693 | (0.5703) | 0.2273 | (0.0233) | 0.0288 | (0.0041) |
| Variance of shocks | | | | | | | | |
| $\varepsilon_{79}$ | 0.1237 | (0.0079) | 0.0217 | (0.0011) | 0.0485 | (0.0018) | 0.0110 | (0.0006) |
| $\varepsilon_{80}$ | 0.1282 | (0.0119) | 0.0549 | (0.0036) | 0.0472 | (0.0024) | 0.0026 | (0.0002) |
| $\varepsilon_{81}$ | 0.1144 | (0.0078) | 0.0418 | (0.0030) | 0.0458 | (0.0021) | 0.0061 | (0.0010) |
| $\varepsilon_{82}$ | 0.1303 | (0.0109) | 0.0445 | (0.0041) | 0.0446 | (0.0027) | 0.0029 | (0.0002) |
| $\varepsilon_{83}$ | 0.1003 | (0.0113) | 0.0295 | (0.0016) | 0.0453 | (0.0023) | 0.0015 | (0.0002) |
| $\varepsilon_{84}$ | 0.0619 | (0.0072) | 0.0052 | (0.0007) | 0.0446 | (0.0028) | 0.0076 | (0.0009) |
| $\varepsilon_{85}$ | 0.0459 | (0.0060) | 0.0082 | (0.0008) | 0.0414 | (0.0027) | 0.0109 | (0.0018) |
| $\varepsilon_{86}$ | 0.0641 | (0.0099) | 0.0052 | (0.0005) | 0.0481 | (0.0031) | 0.0123 | (0.0016) |
| $\varepsilon_{87}$ | 0.0394 | (0.0054) | 0.0064 | (0.0009) | 0.0509 | (0.0038) | 0.0207 | (0.0033) |
| $\varepsilon_{88}$ | 0.0439 | (0.0071) | 0.0128 | (0.0038) | 0.0551 | (0.0045) | 0.0126 | (0.0020) |
| $X^2$ | 194.56 | | 455.23 | | 219.74 | | 167.77 | |
| Degrees of freedom | 51 | | 51 | | 51 | | 51 | |
| P-value | 1.0000 | | 1.0000 | | 1.0000 | | 1.0000 | |

| Serial correlations | Actual | Fitted[c] | Actual | Fitted[c] | Actual | Fitted[c] | Actual | Fitted[c] |
|---|---|---|---|---|---|---|---|---|
| lag 1 | 0.8206 | 0.8414 | 0.2202 | 0.6491 | 0.9837 | 0.9857 | 0.8478 | 0.9057 |
| lag 2 | 0.7842 | 0.7903 | 0.2594 | 0.4477 | 0.9721 | 0.9719 | 0.4751 | 0.8124 |
| lag 3 | 0.7747 | 0.7724 | 0.2371 | 0.1488 | 0.9609 | 0.9588 | 0.6825 | 0.7239 |
| lag 4 | 0.7118 | 0.7580 | 0.1757 | -0.2333 | 0.9510 | 0.9463 | 0.5765 | 0.6431 |
| lag 5 | 0.7122 | 0.7412 | 0.2318 | -0.4984 | 0.9454 | 0.9342 | 0.5676 | 0.5715 |
| lag 6 | 0.7015 | 0.7207 | 0.2764 | -0.6266 | 0.9353 | 0.9223 | 0.5079 | 0.5093 |
| lag 7 | 0.6938 | 0.6963 | 0.2879 | -0.6870 | 0.9263 | 0.9104 | 0.2479 | 0.4560 |
| lag 8 | 0.6348 | 0.6678 | 0.2911 | -0.7179 | 0.9177 | 0.8985 | 0.4108 | 0.4105 |
| lag 9 | 0.6704 | 0.6355 | 0.1960 | -0.7352 | 0.9130 | 0.8863 | 0.3852 | 0.3717 |
| lag 10 | 0.6062 | 0.5996 | 0.1204 | -0.7457 | 0.9046 | 0.8740 | 0.4405 | 0.3385 |

Sources: INSEE, Echantillon d'entreprises, BIC, EAE.

Notes: a. from BIC. b. from EAE. c. accounting for endogenous death. All models include 10 time effects.

## Table 5

**Results of Covariance Model Estimation for Firms Entering the Sample in 1979-88**

| Parameter | Log(Employment) Estimate | Log(Employment) Std. Error | Log(Total Labor Cost) Estimate | Log(Total Labor Cost) Std. Error | Log(Total Payroll)[a] Estimate | Log(Total Payroll)[a] Std. Error | Log(Total Payroll)[b] Estimate | Log(Total Payroll)[b] Std. Error |
|---|---|---|---|---|---|---|---|---|
| Initial mean ($\mu$) | -3.5014 | (0.0107) | 0.6318 | (0.0147) | 0.2706 | (0.0153) | 0.7391 | (0.0150) |
| Trend ($\beta$) | 0.0279 | (0.0018) | 0.0312 | (0.0027) | 0.0318 | (0.0028) | 0.0037 | (0.0023) |
| Serial correlation ($\rho$) | 0.6581 | (0.0313) | 0.6762 | (0.0263) | 0.3506 | (0.0100) | 0.8543 | (0.0244) |
| Variance of $\mu_j$ ($\sigma^2_\mu$) | 0.4194 | (0.0164) | 0.6600 | (0.0217) | 0.6465 | (0.0226) | 0.6177 | (0.0196) |
| Variance of $\beta_j$ ($\sigma^2_\beta$) | 0.0026 | (0.0002) | 0.0077 | (0.0005) | 0.0005 | (0.0000) | 0.0077 | (0.0006) |
| Covariance of $\mu_j$, $\beta_j$ | -0.0105 | (0.0011) | -0.0198 | (0.0017) | -0.0066 | (0.0013) | -0.0224 | (0.0014) |
| Variance of shocks | | | | | | | | |
| $\varepsilon_{80}$ | 0.0265 | (0.0057) | 0.0218 | (0.0038) | 0.0233 | (0.0034) | 0.0485 | (0.0053) |
| $\varepsilon_{81}$ | 0.0317 | (0.0032) | 0.0269 | (0.0024) | 0.0104 | (0.0011) | 0.0430 | (0.0041) |
| $\varepsilon_{82}$ | 0.0278 | (0.0024) | 0.0247 | (0.0021) | 0.0130 | (0.0019) | 0.0384 | (0.0026) |
| $\varepsilon_{83}$ | 0.0220 | (0.0022) | 0.0229 | (0.0020) | 0.0121 | (0.0013) | 0.0336 | (0.0025) |
| $\varepsilon_{84}$ | 0.0240 | (0.0019) | 0.0235 | (0.0015) | 0.0129 | (0.0011) | 0.0336 | (0.0023) |
| $\varepsilon_{85}$ | 0.0212 | (0.0014) | 0.0227 | (0.0016) | 0.0163 | (0.0015) | 0.0319 | (0.0020) |
| $\varepsilon_{86}$ | 0.0176 | (0.0013) | 0.0193 | (0.0013) | 0.0185 | (0.0015) | 0.0341 | (0.0021) |
| $\varepsilon_{87}$ | 0.0190 | (0.0016) | 0.0273 | (0.0020) | 0.0134 | (0.0017) | 0.0335 | (0.0023) |
| $\varepsilon_{88}$ | 0.0179 | (0.0017) | 0.0289 | (0.0025) | 0.0000 | (0.0000) | 0.0358 | (0.0025) |
| $X^2$ | 606.95 | | 982.57 | | 1042.84 | | 532.03 | |
| Degrees of freedom | 251 | | 251 | | 251 | | 251 | |
| P-value | 1.0000 | | 1.0000 | | 1.0000 | | 1.0000 | |

| Serial correlations | Actual | Fitted[c] | Actual | Fitted[c] | Actual | Fitted[c] | Actual | Fitted[c] |
|---|---|---|---|---|---|---|---|---|
| lag 1 | 0.9151 | 0.9693 | 0.8792 | 0.9735 | 0.8831 | 0.9849 | 0.9472 | 0.9634 |
| lag 2 | 0.8856 | 0.9439 | 0.8535 | 0.9439 | 0.8596 | 0.9788 | 0.9071 | 0.9225 |
| lag 3 | 0.8399 | 0.9189 | 0.8231 | 0.9084 | 0.8291 | 0.9751 | 0.8895 | 0.8768 |
| lag 4 | 0.8263 | 0.8916 | 0.7761 | 0.8667 | 0.7832 | 0.9717 | 0.8655 | 0.8270 |
| lag 5 | 0.8251 | 0.8613 | 0.7001 | 0.8201 | 0.7125 | 0.9679 | 0.8586 | 0.7742 |
| lag 6 | 0.8156 | 0.8279 | 0.7483 | 0.7704 | 0.7545 | 0.9634 | 0.8526 | 0.7200 |
| lag 7 | 0.8226 | 0.7919 | 0.7542 | 0.7194 | 0.7595 | 0.9581 | 0.8417 | 0.6659 |
| lag 8 | 0.8042 | 0.7538 | 0.7412 | 0.6690 | 0.7490 | 0.9520 | 0.8335 | 0.6132 |
| lag 9 | 0.7954 | 0.7147 | 0.7108 | 0.6202 | 0.7181 | 0.9451 | 0.8306 | 0.5628 |

(continued)

**Table 5 (continued)**

**Results of Covariance Model Estimation for Firms Entering the Sample in 1979-88**

| | VA/Assets | | Operating Inc/Assets | | Log(Total Assets) | | Debt/Assets | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | Std. Error | Estimate | Std. Error | Estimate | Std. Error | Estimate | Std. Error |
| Initial mean ($\mu$) | 0.9983 | (0.0171) | 0.1265 | (0.0090) | 0.9685 | (0.0220) | 0.8810 | (0.0043) |
| Trend ($\beta$) | -0.0032 | (0.0021) | -0.0158 | (0.0015) | 0.0198 | (0.0043) | -0.0002 | (0.0009) |
| Serial correlation ($\rho$) | 0.2207 | (0.0226) | 0.7071 | (0.0067) | 1.0131 | (0.0053) | 0.6421 | (0.0148) |
| Variance of $\mu_j$ ($\sigma^2_\mu$) | 0.6238 | (0.0259) | 0.0081 | (0.0005) | 1.5944 | (0.0453) | 0.0341 | (0.0020) |
| Variance of $\beta_j$ ($\sigma^2_\beta$) | 0.0027 | (0.0002) | 0.0069 | (0.0004) | 0.0000 | (0.0000) | 0.0012 | (0.0001) |
| Covariance of $\mu_j$, $\beta_j$ | -0.0231 | (0.0015) | -0.0001 | (0.0002) | -0.0261 | (0.0027) | -0.0011 | (0.0002) |
| Variance of shocks | | | | | | | | |
| $\varepsilon_{80}$ | 0.1581 | (0.0234) | 0.0062 | (0.0004) | 0.0982 | (0.0184) | 0.0027 | (0.0002) |
| $\varepsilon_{81}$ | 0.0858 | (0.0098) | 0.0026 | (0.0001) | 0.0927 | (0.0059) | 0.0084 | (0.0010) |
| $\varepsilon_{82}$ | 0.1018 | (0.0107) | 0.0050 | (0.0003) | 0.0843 | (0.0065) | 0.0028 | (0.0002) |
| $\varepsilon_{83}$ | 0.0873 | (0.0092) | 0.0039 | (0.0002) | 0.0732 | (0.0041) | 0.0031 | (0.0002) |
| $\varepsilon_{84}$ | 0.0404 | (0.0027) | 0.0039 | (0.0003) | 0.0669 | (0.0046) | 0.0078 | (0.0006) |
| $\varepsilon_{85}$ | 0.0305 | (0.0028) | 0.0038 | (0.0002) | 0.0483 | (0.0033) | 0.0058 | (0.0004) |
| $\varepsilon_{86}$ | 0.0424 | (0.0041) | 0.0036 | (0.0002) | 0.0564 | (0.0031) | 0.0051 | (0.0004) |
| $\varepsilon_{87}$ | 0.0309 | (0.0031) | 0.0033 | (0.0004) | 0.0456 | (0.0027) | 0.0025 | (0.0001) |
| $\varepsilon_{88}$ | 0.0221 | (0.0030) | 0.0064 | (0.0006) | 0.0538 | (0.0027) | 0.0020 | (0.0004) |
| $X^2$ | 618.94 | | 1088.26 | | 695.74 | | 562.96 | |
| Degrees of freedom | 251 | | 250 | | 250 | | 250 | |
| P-value | 1.0000 | | 1.0000 | | 1.0000 | | 1.0000 | |

| Serial correlations | Actual | Fitted[c] | Actual | Fitted[c] | Actual | Fitted[c] | Actual | Fitted[c] |
|---|---|---|---|---|---|---|---|---|
| lag 1 | 0.8188 | 0.9132 | 0.3717 | 0.7713 | 0.9698 | 0.9765 | 0.6458 | 0.9118 |
| lag 2 | 0.7233 | 0.8864 | 0.3037 | 0.6653 | 0.9499 | 0.9528 | 0.5383 | 0.8377 |
| lag 3 | 0.6320 | 0.8695 | 0.0270 | 0.6073 | 0.9276 | 0.9290 | 0.3637 | 0.7683 |
| lag 4 | 0.6969 | 0.8513 | 0.2985 | 0.5726 | 0.9213 | 0.9051 | 0.5090 | 0.7027 |
| lag 5 | 0.5949 | 0.8290 | 0.0936 | 0.5508 | 0.9038 | 0.8813 | 0.5075 | 0.6421 |
| lag 6 | 0.6989 | 0.8019 | 0.2507 | 0.5362 | 0.9002 | 0.8574 | 0.5601 | 0.5874 |
| lag 7 | 0.6921 | 0.7702 | 0.3159 | 0.5262 | 0.8918 | 0.8336 | 0.4602 | 0.5387 |
| lag 8 | 0.6622 | 0.7341 | 0.2331 | 0.5189 | 0.8880 | 0.8099 | 0.4681 | 0.4958 |
| lag 9 | 0.6561 | 0.6943 | 0.0627 | 0.5135 | 0.8867 | 0.7864 | 0.4707 | 0.4581 |

Sources: INSEE, Echantillon d'entreprises, BIC, EAE.

Notes: a. from BIC. b. from EAE. c. accounting for endogenous death. All models include 9 time effects.