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WORKINGS OF A CITY: LOCATION, EDUCATION, AND PRODUCTION

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ABSTRACT

We examine the implications of local externalities in human capital investment for the size and composition of the productive labor force. The model links residential choice, skills acquisition, and production in a city composed of several communities. Peer effects induce self-segregation by occupation, whereas efficiency may require identical communities. Even when some asymmetry is optimal, equilibrium segregation can cause entire "ghettos" to drop out of the labor force. Underemployment is more extensive, the easier it is for high-skill workers to isolate themselves from others. When perfect segregation is feasible, individual incentives to pursue it are self-defeating, and lead instead to a shutdown of the productive sector.

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Introduction

Most cities are segregated along income and occupational lines. People with high skill, high wage jobs live together in certain select areas or suburbs, and those with low skill, low wage jobs, or no job at all, reside in different parts. This separation is sustained essentially by differentials in the price of housing between the two types of communities.

This situation, which has become accentuated in recent years, is often deplored on grounds of inequity (e.g. Reich, (1991). But there is also an implicit claim of inefficiency: occupational or income segregation is said to deprive some communities of the chance to acquire even modest levels of skills, and thereby to adversely impact the overall quality of the labor force. Moreover, it is argued, this underutilization of human resources has or will eventually have negative repercussions on the standard of living of the high skill class itself -although through which channels is rarely articulated.

This paper attempts to address some of these issues, by formalizing the links between residential choice, occupational choice, and production in a city composed of several communities. In doing so, it relates two strands of work: local public finance on the one hand, and the macroeconomic literature on human capital and skill acquisition on the other. With the former it shares a concern for the impact of community composition (peer group effects) on the provision of public goods, particularly education. With the latter it shares the aim to explain endogenous, self-replicating distributions of skills and incomes.

The local public finance and club theory literature has emphasized the way in which peer effects mitigate the standard Tietbout (1956) efficiency motive for agents to coalesce into communities of homogeneous preferences. Berglas (1976) originally showed how complementarity in production skills interacts with heterogeneity of tastes, resulting in mixed communities. Brueckner and

Lee (1989) and Scotchmer (1991) demonstrate that peer effects have similar consequences, and characterize the conditions under which competitive developpers can bring about the efficient outcome, by means of type-specific taxes or membership fees. Arnott and Rowse (1987) compute the optimal school composition for various forms of peer effects. Also focusing on education, De Bartolome (1990) analyzes the inefficiency which results when discriminatory fees are not available, and shows how a a central planner can use a tax on educational expenditures as a second-best instrument. Finally, Schwab and Oates (1990) provide related results in a more general framework.

The present work is closely related to these papers, in particular to Berglas (1976) and De Bartolome (1990). But it also departs from this literature in several important ways. First, it examines the interaction of local and global externalities, respectively peer group effects at the community level and competition in a city-wide labor market. Most fundamentally, it rejects the standard assumption of an exogenously given distribution of agents with different abilities or tastes. Instead, the overall distribution of types (professional occupations) is determined in equilibrium, together with the composition of local communities.[1]

This reflects a conviction that the distribution of "abilities" or "skills" in the (national) population should be explained rather than assumed, because of its central importance not only for local community composition, but also for macroeconomic productivity and growth. Moreover, empirical studies show that among the young's characteristics most relevant for their own and their peers' achievements, endogenous family attributes such as parents' education, occupation and income play a prominent role. So for a model of human capital with peer effects to be internally consistent, the young's distribution of abilities (the new input into the education process) must reflect the distribution of skills acquired by their parents (the previous output of the education process). What must be explained are self-replicating distributions of skills, and the factors which shape them.

It is through this preoccupation that the paper relates to recent work on the determination of the level and distribution of income. For instance in Loury (1981), Gal-Or and Zeira (1989) or Banerjee and Newman (1990),(1991) the acquisition of human capital or skills is an investment which is impeded by credit market imperfections. Bequests therefore link together the occupational choices and corresponding incomes of successive generations; in the long-run, a steady-state, self-sustaining distribution of skills and income is reached. While privately purchased inputs to education and wealth constraints are undoubtedly important considerations, this literature ignores the public finance aspects of the problem (save for Perotti (1990) who introduces voting on taxes used to finance educational loans), and even more the peer group effects which appear very important empirically (Summers and Wolfe (1977); Henderson, Mieskowsky and Sauvageau (1978); Dynarski, Schwab and Zampelli (1989)). It is on these local externalities that we focus. This emphasis on location also relates the paper to Krugman (1991) and Thomas (1990), who examine the regional specialization of firms; the underlying externality is different, however, and operates there through market size.

We focus directly on steady-state occupational and residential distributions, by constructing a simple simultaneous choice model where: (i) identical agents choose whether to become high-skill workers, low skill workers, or to remain outside the labor force; (ii) they also decide in which part of the city to live; (iii) the acquisition of the two types of skills is subject to asymmetric externalities which operate at the local level. The equilibria of this game are precisely the steady-states of an overlapping generations model where altruistic parents decide where to locate, knowing that their children's human capital decisions will be affected by the occupational makeup of the community's adult population.

This approach allows us to examine three issues which lie outside the scope of previous models. The first is how self-segregation affects the composition of the entire labor force. Second, we formalize the idea that high-skill workers' incentives to segregate themselves into homogeneous communities may deprive the other communities from the ability to sustain even low-skill employment, thereby turning them into unproductive "ghettos". Finally, we show that this flight may in fact be self-defeating, and hurt the would-be high-skill as much as the would-be low-skill.

The paper also represents a departure from the standard approach to non-convexities or complementarities in macroeconomic models. The usual focus is on the potential multiplicity of symmetric, Pareto ranked equilibria: say, no agent invests due to coordination failure, or all do so. We explore instead the role of asymmetries, leading to an endogenous partitioning of the set of agents and raising new issues of interactions within and between subgroups.

The model abstracts from several of the issues which are prominent in the local public finance and human capital literatures, such as inter-community competition or liquidity constraints; we therefore discuss future extensions at the end of the paper. But these omissions also make the model quite simple, and most effectively bring to light the main effects of interest.

Section I presents and discusses the model. Section II examines the integrated city, a useful reference point. Section III characterizes the segregation which arises in a dual city, and its consequences for labor force composition and efficiency. Section IV extends the main argument to a large number of communities, and links the extent of segregation to that of unproductive "ghettos". Section V concludes.

I. THE MODEL

1. People: There is a large number N of identical individuals. Each of them is endowed with one unit of labor, which can be used to participate in the production of a single, numeraire good. Each agent has the choice between three occupations:

(a) remaining outside the (industrial) labor force, e.g. being unemployed or engaging in home production only. This yields a utility level v.

(b) becoming a low-skill worker. This requires exerting an effort level C_L, but allows him to earn the low-skill wage w_L.

(c) becoming a high-skill worker. This requires exerting effort $C_H > C_L$, but allows him to earn the high-skill wage $w_H > w_L$.

The determinants of w_H, w_L, C_H and C_L are examined below. The high and low skill occupations could also be called white collar and blue collar, or managerial and line production jobs. For simplicity, utility is additively separable in income and effort:

$$U^i = w^i - C^i + \pi^i - r^i \tag{1}$$

where w^i and C^i are the wage and effort level corresponding to the occupational choice of agent i, r^i the rent which he pays, and π^i any additional income which he might receive (from land ownership and home production); for an unemployed worker, $w^i = C^i = 0$, $\pi^i = v$.

2. The city: These agents can either live in the city or remain outside -in the countryside or other cities; the latter choice yields the reservation utility v corresponding to being out of the labor force. The city is separated in two, say by a river. On each side (East and West) is a total number N/2 of indivisible land plots, suitable for occupation by one

individual or family. Each of these plots belongs to a landowner who will receive a competitively determined rent, in the numeraire good. Landowners can be viewed as a separate group of N agents who consume but do not work, and have utility given by (1). Equivalently, each of the N workers could own a plot and receive its rent.

3. Externalities in occupational choice: We assume that agents choosing to become high-skill workers confer positive externalities on others within the same community (East or West) who pursue the same occupation, as well as on those who become low-skill workers. Formally, C_H and C_L are decreasing functions of the fraction x of individuals in the community who are investing in high skills.[2]

For instance in a high school, the more students work hard with the aim of going to college, the less the individual effort required by any of them; but also, the less the effort required to finish high-school by a student who will not be going to college. Or it could be, as in Banerjee and Besley (1990), that a higher proportion of hard-workers, by making grades more informative of individual ability, increases everyone's incentive to work hard. Another example, now involving adults, is that of social networks (Montgommery (1990)): knowing an established worker -especially at the managerial level- whose recommendation could "get you in" decreases the costs of getting any type of job. That person can also serve as a role model. Finally, an alternative interpretation is that unemployed and low-skill workers (or a given fraction of them) impose negative externalities on their community (e.g. disruptive influence, crime). These last three effects are consistent with Crane's (1991) findings that high-school drop-out and teen-age pregnancy rates are significantly affected by the proportion of adults holding managerial or professional jobs in the neighborhood.

We shall make the important assumption -which is strongly suggested by these examples- that the externality decreases the cost of acquiring high skills more than it decreases the cost of becoming low-skilled. In other words, those who decide to invest in high skills are more sensitive to the occupational choice of their peers than those who opt for low skills.[3]

Assumption A1: The cost functions $C_H(x)$ and $C_L(x)$ are both decreasing, with $\Delta C(x) = C_H(x) - C_L(x)$ positive and decreasing in x.

This sorting condition, reflected by a steeper slope in Figure 1, will cause self-segregation of worker types, since those choosing high skills will be willing to bid more for land in a community with more of their own (endogenous) types.[4] Since workers with high skills earn higher wages, we see that yet another important interpretation of Assumption 1 is that of a pecuniary or fiscal externality: the model's basic properties would remain unchanged if the argument of C_H and C_L were the community's average labor income instead of x.

Standard education production functions include three types of variables: individual characteristics, purchased inputs, and peer effects. Our technology highlights the third kind, and to some extent embodies the second one in reduced form.[5] It provides the simplest setting in which to demonstrate the impact of local externalities on the makeup and productivity of the labor force. In practice, idiosyncratic factors are also important (regressions for educational achievement have large unexplained residuals), and will interact with the effects which we isolate here.

Under assumption A1, the more people choose the high-skill occupation, the lower the costs $C_H(x)$ and $C_L(x)$ for those in their community. Intuitively, one would still expect a community's total educational cost

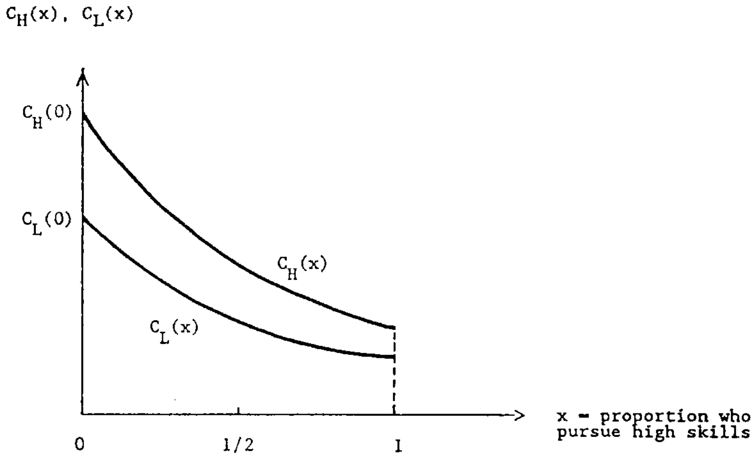


Figure 1: The costs of skill acquisition

(2) $\Phi(x) = x.C_H(x) + (1-x).C_L(x)$

to increase with the proportion of workers acquiring high skills, at least past a certain level. But our results do not require any such assumption.

4. Production: In contrast to skill acquisition, which takes place at the community level, production takes place at the city-wide level -for instance, in an island on the river. Sharing common labor and product markets is in fact what makes communities part of the same metropolis, rather than mere disjointed cities. This interaction of local and global externalities is at the heart of the novel effects identified in this paper.[6]

Workers from both communities are employed in competitive firms which produce the numeraire good, using a constant returns to scale technology $F(H,L) = \theta.f(H,L)$, with $\partial^2 f/\partial H^2 < 0$, $\partial^2 f/\partial L^2 < 0$, $\partial^2 f/\partial H \partial L > 0$, and θ a productivity parameter. The wages $w_H = \partial F(H,L)/\partial H$ and $w_L = \partial F(H,L)/\partial L$ earned in each occupation therefore do not depend on where the worker lives, but only on the city-wide proportion H/L of the two types of labor.

To ensure that both kinds of labor are present in equilibrium, we assume that the net incentive to become a high-skill rather than a low-skill worker is positive when there are very few high-skill workers, and vice-versa:

Assumption A2: $\lim_{x \to 0} [(w_H - w_L)(x,1-x) - (C_H - C_L)(x)] > 0$

$$\lim_{x \to 1} [(w_H - w_L)(x,1-x)] < \min\{0, \Phi'(1)\}$$

This requires the two types of labor not be perfect substitutes in production. We shall denote the wage differential as $\Delta w(H,L) = (w_H - w_L)(H,L)$. It must be positive in any equilibrium, since acquiring high skills takes greater effort no matter what community one lives in. Therefore, the overall ratio H/L of high to low skill workers is always less than $\bar{\rho}$, defined as the unique solution to $\partial f(\tilde{\rho},1)/\partial H = \partial f(\tilde{\rho},1)/\partial L$; $\tilde{\rho}$ is the factor ratio which maximizes the city's gross output.

Finally, while we model local externalities as affecting the cost side of human capital investment, we could assume instead that they operate on the benefit side, as follows: (i) becoming a low-skill worker is costless, but acquiring high-skills requires a fixed effort $C > 0$; (ii) in a community where a fraction x of the workers have high skills, each of them derives $e_H(x)$ efficiency units of labor from his endowment, while each low-skill worker derives $e_L(x)$; (iii) $e_H(x)$ and $e_L(x)$ are both increasing, but $e_H(x)$ has a higher elasticity. This alternative model has very similar properties to the formulation described above; the latter is somewhat more convenient because of the separability between wages and effort.[7]

5. Overlapping generations and dynamic externalities

For some of the externalities which Assumption A1 is meant to capture, particularly those related to education, the simultaneity of occupational choice, peer effects and residential choice may seem unrealistic. It is mostly adults who determine the quality and resources of the communities in which children make their investments in education. To see that our analysis encompasses such intergenerational effects, consider the following model.

Individuals live two periods. When young, they choose a level of human capital investment, putting in effort levels 0, $C_L(x_{jt})$, or $C_H(x_{jt})$, where x_{jt} is the proportion of high-skills adults in their native community j. When adult, they reap the corresponding returns: 0, $w_L(H_{t+1},L_{t+1})$, or $w_H(H_{t+1},L_{t+1})$, where H_{t+1}, L_{t+1} are the total numbers who chose high and low skills when young. As adults they also decide in which community k to raise their offspring. Being altruistic, they care (additively) not only about the rent r_{kt}, but also about their child's intertemporal utility, and in particular on his or her educational opportunity set, which depend on x_{kt+1}.

It should be clear that the steady-states of this dynamic model coincide, up to a discount factor on wages and rents, with the equilibria of our model.

II - INTEGRATED CITY EQUILIBRIUM

We start by looking for an equilibrium where both communities are identical. This is a useful benchmark, for two reasons. First, it corresponds exactly to the case of a city which is integrated, i.e. composed of a single community or sharing group. Second, in a subdivided city it remains an (unstable) equilibrium, where location plays no role. So by using it as a reference point with which to compare the (stable) asymmetric equilibrium, one can better isolate the specific effects of mobility and self-segregation.

1. Equilibrium: Let us first consider the case where all agents are employed. With identical communities (or a single global community), they must all be indifferent between the two occupations. So if $\hat{x}$ is the high-skill fraction of the labor force:

$$(3) \qquad (w_H - w_L)(\hat{x}, 1-\hat{x}) = (C_H - C_L)(\hat{x})$$

Since multiple equilibria arising from the complementarities described in assumption A1 are not the focus of our interest, we shall rule them out:

Assumption A3: $(w_H - w_L)(x, 1-x) - (C_H - C_L)(x)$ is decreasing in x.

This condition holds if productivity θ is high enough; (3) then has a unique solution, which constitutes an equilibrium of the integrated city if agents find work preferable to unemployment:

Assumption A4: $\hat{r} = w_L(\hat{x},1-\hat{x}) - C_L(\hat{x}) - v > 0$.

Clearly $\hat{x}$ is an increasing function of productivity θ; moreover, $\hat{r}$ increases in $\hat{x}$, so A4 is satisfied if technology is productive enough. Finally, the land market clears when workers are indifferent between living in or out of the city.

Proposition 1: Under assumptions A1-A4, there is a unique symmetric, or integrated, full employment equilibrium. A fraction $\hat{x}$ of agents acquire a high level of skills, where: $\Delta w(\hat{x},1-\hat{x}) = \Delta C(\hat{x})$; the land rent is $\hat{r}$.

At the other extreme from this full employment equilibrium, there may exist a trivial equilibrium where no one works, or equivalently where the city fails to materialize. In this case there also exists an unstable equilibrium with partial unemployment in between.[8] Both these equilibria generate zero surplus or rents; they represent coordination failures in an integrated city, and have nothing to do with location. We shall ignore them from here on.

Assumptions A1 to A4 will be assumed to hold from here on; in particular they will be implicit in all propositions.

2. Efficiency: The equilibrium of Proposition 1 is characterized by the standard under-investment problem, as individuals acquiring high skills are not rewarded for the benefits which they confer on others. Indeed, denoting aggregate surplus by $V(x) = F(x,1-x) - \Phi(x)$, we have:

$$(4) \qquad V'(\hat{x}) = -\hat{x}.C_H'(\hat{x}) - (1-\hat{x}).C_L'(\hat{x}) > 0.$$

This inefficiency in the workforce's composition does not describe anything new; nor is it related to location. The more interesting issue, to which we now turn, is whether it is improved or worsened in the segregated equilibrium.

III - SEGREGATED EQUILIBRIUM

We now turn to the case of a dual city, where neighborhood effects generate occupational segregation. Our purpose is not to account for all segregation or rent differentials: the model abstracts from many other sources, such as heterogeneity in tastes over public services, transportation costs, or land consumption. It concentrates instead on how residential and human capital decisions shape one another, and on their long-run implications for the surplus generated by a city.

A. Equilibrium conditions

An equilibrium consists of wages, community compositions and land rents which clear both labor and land markets: firms maximize profits, and no agent wants to change occupations, communities, or leave the city.

Formally, denote the proportions of high skill, low skill and unemployed workers in community $j = 1,2$ as $(x_j, y_j, 1-x_j-y_j)$; the other community is denoted by $-j$. The rents are r_j and r_{-j}. The total high-skill workforce is then $H = x_1 + x_2$, and the low-skill workforce $L = y_1 + y_2$. In a non-trivial equilibrium, we must have for each j:

(a) Occupational choice:

* if $x_j = y_j = 0$, $w_H(H,L) - C_H(0) \leq v$, $w_L(H,L) - C_L(0) \leq v$.

* if $y_j > x_j = 0$, $w_H(H,L) - C_H(0) \leq v \leq w_L(H,L) - C_L(0)$.

* if $x_j > y_j = 0$, $w_H(H,L) - C_H(x_j) \geq v \geq w_L(H,L) - C_L(x_j)$.

* if $x_j, y_j > 0$, $w_H(H,L) - C_H(x_j) = w_L(H,L) - C_L(x_j) \geq v$, with equality if $x_j + y_j < 1$.

(b) Mobility:

* $r_j = \max\,(\, w_H(H,L) - C_H(x_j) - v,\ w_L(H,L) - C_L(x_j) - v,\ 0\,)$

This last set of conditions states that r_j is the maximum of the rents or surpluses which workers in each of the three occupations are willing to bid for living in community j.[9] Equivalently, these are the rents which make people indifferent between living in any community and living outside the city (hence also indifferent between communities). Note that if community j is not fully employed, or equivalently, is partially empty ($x_j + y_j < 1$), then $r_j = 0$.

Rather than present a taxonomy of all possible equilibrium configurations, we shall ignore parameter values which allow unemployed and low-skilled workers to coexist in a community ($0 < y_j < 1-x_j$), and focus instead on two particularly interesting polar cases. The first one (Section III.B) obtains when in equilibrium, everyone still wants to work. The size of the labor force is therefore not an issue, but only its skill composition, and how efficiently this mix is achieved through the geographical allocation of workers. The second case (Section III.D) is that where a completely unemployed "ghetto" appears, constituting a downright waste of productive resources.

B. Equilibrium with full employment

To ensure that low skill work is always preferred to unemployment, the required effort must not be too large, even in a community deprived of the externalities conferred by workers acquiring high skills. This requires a stronger version of A4:

Assumption A5: $w_L(\hat{x}, 2-\hat{x}) > C_L(0) + v$.

This condition holds if technology is productive enough, as long as $C_L(0) < +\infty$. Next, to rule out an uninteresting multiplicity of equilibria, we shall need to strenghten assumption A3 to:

Assumption A6: $(F_H-F_L)(x_1+x_2, 2-x_1-x_2) - (C_H-C_L)(x_1)$ is decreasing in x_1, for all x_2 in $[0,1]$.

This expression is the net incentive to switch from low to high skills, when the skilled labor force already includes a fraction x_1 of one's own community, plus a fraction x_2 of the other community.[10]

We are now ready to characterize the (non-trivial) equilibria of the subdivided city. We first assume, then verify, that all agents want to work. We identify equilibria which are mirror images of one another when community 1 is relabelled as community 2 and vice-versa. Community 1 (the West side) can therefore be taken to always have the most high-skill workers: $x_1 \geq x_2$.

Consider first the case where both sides are mixed: $0 < x_2 \leq x_1 < 1$. This requires $\Delta C(x_1) = \Delta w(H,L) = \Delta C(x_2)$, so $x_1 = x_2 = \hat{x}$, by A1 and A4. The only solution is the symmetric allocation characterized in Section II, and ilustrated on the left panel of Figure 2. Since communities are identical, so are rents, and there is no reason to move; this situation is still an equilibrium. It is, however, unstable: as soon as x_1 becomes slightly larger than x_2, community 1 becomes more attractive to all workers; its rent increases, with the land going to the highest bidders. But the latter are clearly the workers who want to invest in high skills, since they value moving to community 1 more than the others do, i.e. $C_H(x_2) - C_H(x_1) > C_L(x_2) - C_L(x_1)$. So additional skilled workers move to the West side, making it even more attractive and increasing $r_1 - r_2$ further, until the stable, segregated equilibrium is reached:[11]

Proposition 2: Assume A5 and A6. There is a unique stable equilibrium, which involves maximal concentration of high-skill workers:

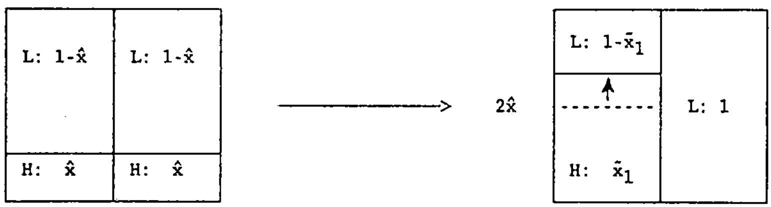
(i) If $\hat{x} \leq 1/2$, all high-skill workers are concentrated in community 1, and their total number is increased by mobility: $\tilde{x}_1 > 2.\hat{x}$. Rents are:

$$0 < \tilde{r}_2 = w_L(\tilde{x}_1, 2-\tilde{x}_1) - C_L(0) < w_L(\tilde{x}_1, 2-\tilde{x}_1) - C_L(\tilde{x}_1) = \tilde{r}_1.$$

Figure 2: Full employment equilibrium

Figure 2.a: $\hat{x} < 1/2$

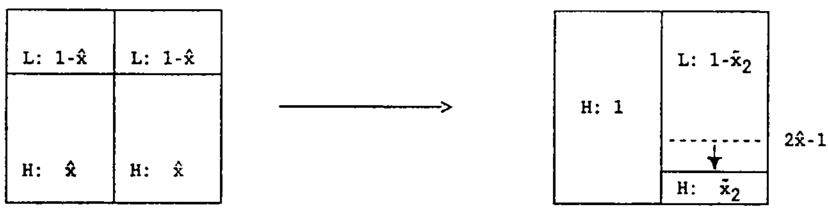
Integrated equilibrium (unstable) | Segregated equilibrium (stable)



$r_1 = r_2 = \hat{r}$ | $r_1 - r_2 = C_L(0) - C_L(\tilde{x}_1)$

Figure 2.b: $\hat{x} > 1/2$

Integrated equilibrium (unstable) | Segregated equilibrium (stable)



$r_1 = r_2 = \hat{r}$ | $r_1 - r_2 = C_H(\tilde{x}_2) - C_H(1)$

(ii) If $\hat{x} > 1/2$, all low-skill workers are concentrated in community 2, and the total number of high-skill workers is decreased by mobility: $\tilde{x}_1 + \tilde{x}_2 = 1 + \tilde{x}_2 < 2.\hat{x}$. Rents are:

$$0 < \tilde{r}_2 = w_H(1+\tilde{x}_2, 1-\tilde{x}_2) - C_H(\tilde{x}_2) < w_H(1+\tilde{x}_2, 1-\tilde{x}_2) - C_H(1) = \tilde{r}_1.$$

(iii) There is a range of parameter values for which the equilibrium involves complete separation, i.e. $\tilde{x}_1 = 1$, $\tilde{x}_2 = 0$.

Proof: in appendix.

The intuition for these results can be most simply understood as follows. Starting from the unstable symmetric configuration (with proportions $\hat{x}$ and $1-\hat{x}$ of high and low-skill workers on each side), a perturbation leads more and more high-skill workers to gather in community 1, until either: (i) all of them have regrouped there ($2.\hat{x} \leq 1$, Figure 2.a); (ii) or they have filled up the West side completely and some of them have to live on the East side, with the low-skill individuals ($2.\hat{x} > 1$, Figure 2.b). In either case, this is not yet the equilibrium, because these migrations affect the incentives for occupational choice. Since x_1 has increased, it is now easier to attain high skills in community 1. Hence in the first case, some marginal agents in community 1 will make that choice, and we end up with $\tilde{x}_1 > 2.\hat{x}$ high-skill workers. In the second case, community 1 is already full with high-skill types; the marginal agent is in community 2, which has experienced a drain of high-skill workers. Since the cost differential $\Delta C(x_2)$ has increased, less workers will become highly skilled, until the wage differential $\Delta w(x_1+x_2, 2-x_1-x_2)$ has risen enough to compensate for the increase in ΔC. Thus the city-wide population of skilled workers must decline: $1 + \tilde{x}_2 < 2.\hat{x}$.

When $\hat{x}$ is around 1/2, the two effects described above leads to a completely segregated equilibrium, i.e. $\tilde{x}_1 = 1$, $\tilde{x}_2 = 0$.[12] This is because acquiring high-skills is very easy for the last few workers in a community where almost everyone else is doing it, but very difficult for the first few workers in a community where no one else is doing it.

Finally, it is shown in appendix that assumption A6 precludes any stable equilibrium with unemployment, except of course the "empty city" equilibrium where no one works.

C. Efficiency

We now examine the efficiency properties of the market outcome, by comparing it to a planner's solution. An allocation is a Pareto optimum if and only if it maximizes the utility of the representative agent (including landowners), i.e. the per capita surplus V of output over educational or training costs (effort):[13]

$$(5)\qquad 2.V(x_1,y_1,x_2,y_2) = F(x_1+x_2,\ y_1+y_2) - x_1.C_H(x_1) - y_1.C_L(x_1) - x_2.C_H(x_2) - y_2.C_L(x_2) + v.(2-x_1-y_1-x_2-y_2)$$

over x_i, y_i and $1-x_i-y_i$ in [0,1], $i = 1,2$. We can also write:

$$(6)\qquad V - v = \frac{1}{2}\sum_j x_j.(F_H - C_H(x_j) - v) + y_j.(F_L - C_L(x_j) - v)$$

so that V - v is the sum of all land rents. We focus on the case where the planner wants the whole labor force to work.

Proposition 3: If productivity θ is high enough, the planner chooses full employment.

Proof: in appendix.

The intuition is that a less than fully employed community does not produce any surplus; then if θ is high enough, the surplus from the other community is shown to be less than $\hat{f}$, which can be generated by a fully employed city. Hence, for a productive enough economy, the planner's problem simplifies to choosing the skill composition of each community's labor force, and her objective function to:

$$V(x_1,x_2) = F(x_1+x_2,\ 2-x_1-x_2) - \Phi(x_1) - \Phi(x_2) \tag{7}$$

The problem is separable into finding the geographical allocation which minimizes the cost of achieving any given proportion x/(2-x) of high-skill workers, and then finding the value of this ratio which maximizes surplus:

$$\underset{0\le x\le 1}{\text{Max}} \{\ 2.F(x,1-x) - \min\ \{\Phi(x_1) + \Phi(x_2) \mid x_1 + x_2 = x\}\ \}$$

Whether it is optimal to bunch N.x/2 high-skill workers together or spread them out evenly depends on whether Φ is concave or convex. This in turn depends on the interplay of two effects. On one hand is the greater sensitivity of high-skill workers to community composition, which tends to make Φ concave (and produces bunching in equilibrium). On the other, the benefits confered to either type by a marginal high-skill worker in the community may be decreasing. For instance, Henderson, Mieskowsky and Sauvageau (1978) find significant concavity in the effects of mean class ability on a student's educational achievement; Dynarski, Schwab and Zampelli (1989) find that greater income dispersion, ceteris paribus, raises a school district's average test performance. The case where the agglomeration of high-skill workers involves private economies but social diseconomies is thus of particular interest (Section III.D, however, will relax this assumption):

Assumption A7: A community's educational costs $\Phi(x) = x.C_H(x) + (1-x).C_L(x)$ are convex in the proportion of individuals acquiring high skills.

Assumption A7 requires that there be sufficient curvature in peer effects, compared to the differential sensitivity effect of assumption A1: it means that $C_H(x) - C_L(x) + x.C_H'(x) + (1-x).C_L'(x)$ is increasing in x, or:

$$x.C_H''(x) + (1-x).C_L''(x) > -2.(C_H - C_L)'(x) \quad \text{for all } x.$$[14]

Proposition 4: If productivity is high enough and A7 holds, the planner's problem has a unique solution, which is symmetric: in each community, a fraction x* of individuals acquire high-skill levels, where:

$$(8) \qquad (F_H-F_L)(x^*,1-x^*) - (C_H-C_L)(x^*) = x^*.C_H'(x^*) + (1-x^*).C_L'(x^*) < 0.$$

The rest of the population acquires low skills, so all are employed.

Proof: in appendix.

The overall proportion of high-skill workers is of course chosen so that the net social benefit of training an additional one is zero. As to their geographical distribution, the reason the planner wants it evenly spread is that a marginal worker choosing high skills is much more beneficial in a community where there are few like him than in one where they are already abundant: although $-C_H'(x) > -C_L'(x)$ for all x, $-C_H'(x_1) < -C_L'(x_2)$ when x_1 is sufficiently larger than x_2.[15]

A simple example makes this transparent: let technology be Leontieff, requiring both types of labor in unit proportions. The only issue is then the cost of training this labor force. With all high-skill workers on one side and all those with low skill on the other, it is $C_H(1) + C_L(0)$ (times N/2); if both sides have the same mix of skills, it is $C_H(1/2) + C_L(1/2)$. This arrangement is more efficient if $C_L(0) - C_L(1/2) > C_H(1/2) - C_H(1)$, meaning that $C_L(x)$ is steeper at lower values of x than $C_H(x)$ at high values.

We can now summarize how self-segregation affects welfare:

(i) if Φ is convex, it creates an additional inefficiency with respect to the symmetric equilibrium, namely an excessively high cost of educating the labor force.

(ii) its impact on the underinvestment problem ($\hat{x} < x^*$) is more ambiguous: it brings about improvement in communities where high-skill workers concentrate, but deterioration in those which they desert. The net effect depends on whether the community which remains mixed just after segregation occurs has experienced an inflow or an outflow of high-skill workers: $\tilde{x}_1 + \tilde{x}_2 \gtrless 2.\hat{x}$ as $2.\hat{x}-1 \gtrless \hat{x}$, or $\hat{x} \gtrless 1/2$.

The results for $\hat{x} < 1/2$ have one noteworthy consequence; even when the first-best is symmetric, imposing symmetry (say, of adults) without at the same time subsidizing high skills (for children) may actually lower welfare.

D. Unemployment equilibrium

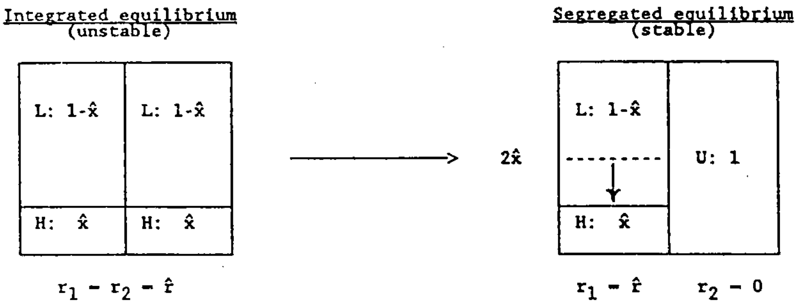
We now turn to a case where the attempt by the high-skill group to separate itself has more drastic consequences than before; instead of only affecting the composition of the labor force, it also reduces its size, resulting in idle human resources. Moreover, this attempt will be shown to be self-defeating, in a sense specified below.

We do not make any convexity assumption on educational costs Φ, but simply replace assumption A5 by:

Assumption A8: $w_L(\tilde{\rho},1) < C_L(0) + v$.

This means that it is difficult (but not necessarily impossible) to acquire low skills without exposure to some high-skill workers in the community. Recall that $\tilde{\rho}$ is an upper bound on the factor ratio H/L in any equilibrium, so that $w_L(\tilde{\rho},1) - w_H(\tilde{\rho},1)$ is an upper bound on the low-skill wage.

Figure 3: **Unemployment equilibrium**



We now derive the equilibrium. Let us start again from the unstable symmetric equilibrium, assuming for now that $\hat{x} < 1/2$; see Figure 3. When the $N.\hat{x}/2$ high-skill workers of the East side move to the West side, it becomes very difficult for those who remain in or come to the East side to acquire even a low level of skills; the cost exceeds the reward, so they drop out of the productive sector. But this is not the end of the story: since the number of low-skill workers has been drastically reduced, their wage rises. This reduces the incentive for workers in community 1 to become skilled; hence less of them do so, until the labor market clears. Since the labor market is now reduced to community 1, constant returns to scale imply that in equilibrium H/L must be the same as it was in the integrated equilibrium, i.e. $\hat{x}/(1-\hat{x})$; see Figure 3 again. The end result is a halving of production and surplus, despite the fact that the initial proportions $(2.\hat{x}, 1-2.\hat{x})$ in community 1 made acquiring high skills relatively easy.

Proposition 5: Assume A5 and A8. Then:

(i) If $\hat{x} \leq 1/2$, there is a unique stable equilibrium. Mobility reduces the productive labor force by half, with its skill composition unchanged: community 1 still has proportions $\hat{x}$ and $1-\hat{x}$ of high and low-skill workers, but all of community 2 remain unproductive. Rents are: $0 = r_2 < \hat{r} = r_1$.

(ii) If $\hat{x} > 1/2$, this allocation remains a stable equilibrium. The only other possible stable equilibrium is the full-employment allocation derived in Proposition 2 (ii). It is indeed an equilibrium if and only if $w_L(1+\tilde{x}_2, 1-\tilde{x}_2) \geq C_L(\tilde{x}_2) + v$. In all cases, the total number of high-skill workers decreases.

Proof: in appendix.

The result in the case $\hat{x} \leq 1/2$ stands in sharp contrast to the situation of Proposition 2, where workers in community 2 remained employed with low-skills. There, concentrating the high-skill workers allowed an increase in their number H, and a decrease in their wage; it now leads to a reduction in their number, with unchanged wage. The result is also very different from what would obtain in a model with exogenously given types of agents, such as Berglas (1976), De Bartolome (1990) or Schwab and Oates (1990).

Intuitively, each skilled worker who moves or stays away from community 2 contributes to deprive all his peers (and himself) from a complementary input, namely low-skill production workers; this in turn reduces the demand for his services. We shall come back to this "self-defeating" flight in Section IV.

The second case in Proposition 5 is also quite intuitive. Consider the full employment allocation $\tilde{x}_1 = 1$, $\tilde{x}_2$ solving $\Delta w(1+\tilde{x}_2, 1-\tilde{x}_2) = \Delta C(\tilde{x}_2)$, as described in Proposition 2 (ii). It is a stable equilibrium if $\tilde{x}_2$ is high enough that $w_L(1+\tilde{x}_2, 1-\tilde{x}_2) \geq C_L(\tilde{x}_2) + v$.[16] If not, full-employment is not sustainable and some agents switch from low-skill work to inactivity.[17] By reducing the high-low wage gap, this induces others to switch from high to low skills; this in turn makes acquiring low-skills more difficult, and this unravelling continues until community 2 ends up completely unemployed. See Figure 3 again.

Note finally that: (i) in contrast to the full employment case, H always declines; (ii) when $\hat{x} \leq 1/2$ or $w_L(1+\tilde{x}_2), 1-\tilde{x}_2)) < C_L(\tilde{x}_2)) + v$, mobility worsens the inefficiency of the integrated allocation, whether or not Φ is convex. For instance when Φ is concave, the planner would also implement an asymmetric allocation; but unlike the market, she would provide workers in community 2 with sufficient incentives to remain in the labor force in spite of the high effort required.

IV - THE EXTENT OF MIXING AND THE EXTENT OF PRODUCTION

The preceding sections derived and compared the equilibrium of an integrated city and that of a dual city. The assumption of two communities was convenient but somewhat arbitrary. Moreover, the issue of whether the whole high-skill labor force of the integrated equilibrium could regroup into a single community ($\hat{x} \gtrless 1/2$) played an important role in shaping the segregated equilibrium.

To show that the basic insights are quite robust, we derive in this section a related result which holds for any number of communities, and is invariant to the value of $\hat{x}$. Moreover, it makes strikingly clear that the degree to which a city "works" (in both senses of the term) may be inversely related to the feasibility of segregation.

Suppose that the city is divided into m communities of equal size N/m.[18] The parameter m measures how effectively groups of agents can segregate themselves from others; it may reflect technological constraints such as a minimum efficient community size resulting from fixed costs, or institutional ones such as school districting or as zoning laws. We shall apply the reasoning of Section III.D to this more finely partitioned city.

We start on Figure 4.a from the symmetric, full employment equilibrium, with $\hat{x}$.N/m high-skill workers in each community. As usual, it is unstable since high-skill-workers will attempt to regroup into homogenous communities. Clearly, a stable equilibrium can have at most one mixed community, i.e. with a proportion $0 < x_j < 1$ of high-skill workers. Moreover, any community without high skills is fully unemployed: since H/L is still bounded by $\bar{\rho}$, investing in low skills pays at most $w_L(\bar{\rho},1) < C_L(0) + v$ (by A8), and is therefore not worth the effort. As a result, there is at most one community

<u>Figure 4</u>: Segregation as a limit on production in a city with many communities

<u>4.a</u>: <u>Integrated equilibrium</u> (unstable)

L: $1-\hat{x}$	L: $1-\hat{x}$	L: $1-\hat{x}$	L: $1-\hat{x}$	L: $1-\hat{x}$	L: $1-\hat{x}$	L: $1-\hat{x}$	L: $1-\hat{x}$
H: $\hat{x}$	H: $\hat{x}$	H: $\hat{x}$	H: $\hat{x}$	H: $\hat{x}$	H: $\hat{x}$	H: $\hat{x}$	H: $\hat{x}$
N/m	N/m	N/m	N/m	N/m	N/m	N/m	N/m

<u>4.b</u>: <u>Segregated equilibrium</u> (stable)

H: 1	H: 1	L: 1-x H: x	U: 1	U: 1	U: 1	U: 1	U: 1
N/m	N/m	N/m	N/m	N/m	N/m	N/m	N/m

containing low-skill workers (and high-skill workers), and $\max(\tilde{\rho}-1, 0)$ communities containing only high-skill workers; see Figure 4.b. We have shown:

Proposition 6: Assume A8. As the ability to segregate, measured by m, increases, the per capita productive labor force, output and surplus in any stable equilibrium remain bounded by:

$$(9)\qquad \begin{array}{ll} L/N < 1/m, & H/N < \tilde{\rho}/m, \\ F(H,L)/N < F(\tilde{\rho},1)/m, & (V-v)/N < (F(\tilde{\rho},1) - C_L(1))/m \end{array}$$

and therefore decline toward zero.

Thus the "ghetto" effect identified in the preceding section becomes more extensive, the easier it is for those seeking to become high-skill workers to isolate themselves from their low-skill counterparts. Their individual incentives to secede are self-defeating, preventing most of them in equilibrium from achieving the occupation they seek. In the limit where perfect segregation is feasible, its pursuit leads instead to a total shutdown of the productive sector. As usual this is a steady-state outcome, which may be reached only over the course of several generations.

This drastic result is of course a reflection of our very simple model, but it shows most clearly the destructive potential of residential self-segregation in the presence of externalities in human capital investment. Moreover, it is quite robust.

In particular, it does not require that the planner want to achieve equal mixing (she will if educational costs Φ are convex). Nor does the claim of inefficiency even require that the planner want full employment (she will if θ is high enough): the planner can always generate the per capita surplus $\hat{r} > 0$ corresponding to the symmetric, full-employment equilibrium, whereas the laissez-faire surplus becomes arbitrarily small. Of course the loss from self-segregation increases as technology θ becomes more productive.

The result also does not require that it be impossible to acquire low skills when no high-skill workers are around, just costly enough. Finally, it does not require that either type of labor be essential to production, only that the elasticity of substitution σ be finite. For a given m, of course, the size and composition of the sustainable productive sector depend on σ.[19]

In reality, cities contain more than one occupationally mixed community. What matters then is the relative measure of mixed to fully homogeneous communities; this is really how 1/m should be interpreted. Note that individual agents seeking to become high-skill workers will always try to achieve maximal segregation.

V - EXTENSIONS AND RELATED ISSUES

While we focused on a single city, our model of how the labor force is shaped by local externalities extends to the level of the country. Since the stable city equilibrium is unique and gives labor its reservation utility, the national equilibrium simply consists of replications of the representative city. These may embody local variations in factors which impact productive surplus, such as technology or the extent of self-segregation, and result in different land rents.

In a representative city, segregation is sustained in equilibrium by potentially large rent differentials; these in turn can be charged by landowners because community size is inelastic. An upward-sloping supply curve for land in each community, or the possibility of living in a smaller plot, will alleviate segregation and its impact on efficiency. But this will come at the cost of distortions in land utilization, as the more highly skilled community will use more land or have higher population density.

The simple model constructed in this paper abstracts from several other considerations which may be empirically relevant. One is heterogeneity of abilities and tastes; as mentioned earlier, this omission is intentional, to show that one need not appeal to significant innate differences between people to explain how neighborhood effects shape the labor force, or even the existence of unemployed "ghettos". The other omission is dynamics. Our simultaneous choice game yields the steady-states of an overlapping generations model where adults choose location, recognizing that their children's educational opportunities will be affected by community composition; but the convergence paths should also be of interest. Moreover, if residential choice is hampered by wealth constraints or interacts with other inherited characteristics, the long-run equilibrium may depend on the history of the communities.[20]

Another important issue is that of competition between communities and local taxation. Since land rents in each community extract all surplus from the residents, the landowners of communities where high-skill workers congregate are better off; the others should therefore try to bid away the agents who engage in this more valuable occupation, provided they can be identified. Even then, restoring optimality through decentralized taxes and subsidies may be difficult: a pure strategy equilibrium may not exist, because by deviating and offering a little more to those who pursue high-skills, a community can attract a large number of them; moreover, it neglects any impact this might have on the overall labor force. These issues relate to club theory, but the combination of peer effects and imperfect competition has not been examined in the literature; nor has the endogenous distribution of types. Together with dynamics, they will be explored in future work.

NOTES

[1] Loury (1977) represents an intermediate case, where Blacks and Whites have identical innate abilities but the cost of acquiring skills is assumed to depend on the relative income of the two groups.

[2] As illustrated by the examples below, the relevant notion of "community" is the group or area within which this effect operates, and thus depends on the externality under consideration. We make the convenient assumption that each community extends over half the city; it will be relaxed in Section IV.

[3] This assumption may seem at variance with Henderson, Mieskowsy and Sauvageau's (1978) finding that high and low ability students in grade school benefit equally from peer group improvement. However, all that is needed for segregation to emerge is that $C_L' - C_H'$ be even slightly positive. Moreover, one can view A1 as representing the reduced form of some other segregation-inducing effect, such as a differential sensitivity of high and low-skill investments to educational expenditures, in the spirit of De Bartolome (1990).

[4] In Miyao (1978) segregation, i.e. a homogeneous city, results from each group's either disliking the other ("negative intergroup externalities") or liking its own ("positive intragroup externalities"). Here, everyone benefits from high-skill workers, but others seeking high skills benefit most. A1 also differs from De Bartolome (1990), where high-ability types care less about peer effects than low-ability types; on the other hand, they care more about educational expenditures, and this is the force pushing toward segregation. Finally, both previous models involve fixed populations of exogenous types.

[5] If expenditure on education has a higher marginal impact on the cost of high-skill investment than on that of low skill investment, it will be an increasing function of x. The same holds, in steady-state, if children's education has a consumption value which increases with parents' level of

skills or income.

6 Requiring people to work where they reside, as in Berglas (1976), would simply make all communities identical to the integrated city examined in Section II.

7 In this alternative version, labor income is residence-dependent: in community j, $w_H(x_j) = e_H(x_j).\partial F(H,L)/\partial H$, $w_L(x_j) = e_L(x_j).\partial F(H,L)/\partial L$, where $H/N = x_1.e_H(x_1) + x_2.e_H(x_2)$ and $L/N = (1-x_1).e_L(x_1) + (1-x_2).e_L(x_2)$.

8 The zero-employment equilibrium (z.e.e.) occurs when the absence of high-skill workers makes both factors' opportunity cost too high for firms to employ them profitably. Formally, for any pair of wages (ω_H,ω_L), let $\rho(\omega_H,\omega_L)$ denote firms' cost-minimizing factor ratio, and $\lambda(\omega_H,\omega_L)$ the corresponding marginal cost; the z.e.e. exists if and only if $\rho(C_H(0)+v,\ C_L(0)+v) > 1$. In this case there is also a unique partial employment equilibrium (p.e.e.), with x, y, x+y in (0,1) solving:

$$\psi(x,y) = \begin{bmatrix} w_H(x,y) - C_H(x) - v \\ w_L(x,y) - C_L(x) - v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ or: } \begin{bmatrix} \rho(C_H(x)+v,\ C_L(x)+v) = x/y \\ \lambda(C_H(x)+v,\ C_L(x)+v) = 1 \end{bmatrix};$$

the equivalence is by definition of ρ and λ. Indeed, for the full-employment equilibrium, by (3) and (A4): $\lambda(C_H(\hat{x})+v,\ C_L(\hat{x})+v) < \lambda(w_H(\hat{x},1-\hat{x}),\ w_L(\hat{x},1-\hat{x})) = 1$. Therefore, there is a unique $\tilde{x} \in (0,\hat{x})$, such that $\lambda(C_H(\tilde{x})+v,\ C_L(\tilde{x})+v) = 1$. Moreover, $\tilde{y} = \tilde{x}/\rho(C_H(\tilde{x})+v,\ C_L(\tilde{x})+v) < 1-\tilde{x}$, or else $\Delta w(\tilde{x},1-\tilde{x}) \geq \Delta C(\tilde{x})$, so $\tilde{x} \geq \hat{x}$ by (A3), a contradiction. Hence the result. Finally, while the z.e.e. is clearly stable, the p.e.e. is saddlepoint-unstable, since the Jacobian:

$$D\psi(x,y) = \begin{bmatrix} \partial^2F/\partial H^2 - C_H' & \partial^2F/\partial H\partial L \\ \partial^2F/\partial H\partial L - C_L' & \partial^2F/\partial L^2 \end{bmatrix}$$

has a negative determinant.

9 As in Wheaton (1977), maket rents are the outer enveloppe of bid rents.

10 Given that $\partial^2F/\partial H\partial L > 0$, assumption A3 then holds a fortiori.

11 Following the literature (e.g. Miyao (1978)), the implicit adjustment

process in this paper is one of standard tatonnement; see the proof of Proposition 2. Formalizing a dynamic rational expectations equilibrium with occupational compositions as state variables would complicate the model, but leave steady-state results unchanged. Also, while stability arguments are convenient to focus on a single equilibrium, all the results could be restated in terms of how the set of equilibria is affected by self-segregation.

12 Changes in $\hat{x}$ may reflect changes in productivity θ, or in the cost differential ΔC. In particular, $\hat{x} \gtrless 1/2$ as $\theta.(w_H - w_L)(1,1) \gtrless \Delta C(1/2)$.

13 All agents are ex-ante identical. After choosing their occupations they may be considered different, but utility remains transferable, as income.

14 For instance with $F(H,L) = \theta.H^{\alpha}.L^{1-\alpha}$ and $C_i(x) = c_i.(a+x)^{1-\nu}$, $i = H, L$, $\nu > 1$, this requires: $\nu/2 > (1+a)(c_H/c_L - 1)$.

15 Similar effects are dicussed by De Bartolome (1990) and Schwab and Oates; here they also interact with the overall composistion of the labor force.

16 As usual, between this equilibrium and that with $x_2 = y_2 = 0$, there is then an unstable one where community 2 is partially employed.

17 In particular, if $w_L(\bar{\rho},1) < C_L(2\hat{x}-1) + v$, full employment is not sustainable, since $H/L \leq \bar{\rho}$ and $\bar{x}_2 \leq 2\hat{x} - 1$.

18 Without loss of generality, it will be convenient here to think of agents as forming a continuum with measure N, so that m can take any value.

19 For instance, if $F(H,L) = \theta[\alpha.H^{1-1/\sigma} + (1-\alpha).L^{1-1/\sigma}]^{\sigma/(\sigma-1)}$, then $\bar{\rho} = (\alpha/(1-\alpha))^{\sigma}$. So if $\alpha/(1-\alpha) > 1$, maximal employment is higher -and more skewed toward high skill workers- the higher is σ; but if $\alpha/(1-\alpha) < 1$, a higher elasticity actually contributes to shrinking production.

20 Loury (1977) obtains such a dependence on initial conditions when the relative income of racial groups affects the cost of acquiring skills.

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APPENDIX

Proof of Proposition 2

We first show that the symmetric allocation is unstable. Indeed, it is defined as the zero of the function:

$$\xi(x_1,x_2) = \begin{bmatrix} \Delta w(x_1+x_2, 2-x_1-x_2) - \Delta C_H(x_1) \\ \Delta w(x_1+x_2, 2-x_1-x_2) - \Delta C_L(x_2) \end{bmatrix}$$

whose Jacobian at $(\hat{x},\hat{x})$ is easily seen to admit $-\Delta C'(\hat{x}) > 0$ as an eigenvalue associated to the eigenvector (1,-1). Since (1,-1) corresponds to changes in the geographical allocation of a labor force with constant occupational composition, we can call this instability "locational".

We now turn to segregated equilibria. Recall that by A5, $\psi(x,x') = \Delta w(x+x', 2-x-x') - \Delta C(x)$ is decreasing in x, for all x'.

1. Full-employment equilibria: In the text we showed that the only such equilibrium with $x_1 > 0$, $x_2 > 0$ was the symmetric, unstable allocation. Therefore, in a stable equilibrium, one of the following must hold:

(i) Only community 1 is mixed: $0 = x_2 < x_1 < 1$. Agents in that community must be indifferent between skills, so:

$$\Delta w(x_1, 2-x_1) = \Delta C(x_1), \text{ or } \psi(x_1,0) = 0.$$

The rent differential must make workers choosing low skills indifferent between the two communities, so: $r_1 - r_2 = C_L(0) - C_L(x_1)$. Finally, the level of r_2 is such that all agents obtain v in equilibrium $r_2 = w_L - C_L(x_2) - v$.

(ii) Only community 2 is mixed: $0 < x_2 < x_1 = 1$. Agents in that community must be indifferent between the two skills, so:

$$\Delta w(1+x_2, 1-x_2) = \Delta C(x_2), \text{ or } \psi(x_2,1) = 0$$

Now it is workers choosing high skills who must be indifferent between communities, so: $r_1 - r_2 = C_H(x_2) - C_H(1)$. Again, r_2 is such that all agents obtain utility v.

(iii) Complete segregation: $x_1 = 1$, $x_2 = 0$. For this to be an equilibrium, it must be that:

$$\psi(1,0) = \Delta w(1,0) - \Delta C(1) \geq 0 \geq \Delta w(1,0) - \Delta C(0) = \psi(0,1).$$

Residential indifference requires $r_1 - r_2 = \Delta w(1,0) + C_L(0) - C_H(1)$, with r_2 again such that all agents have utility v.

Having characterized potential full employment equilibria, we now show existence and uniqueness.

Case 1: $\Delta w(1) \leq \Delta C(1/2)$, i.e. $\hat{x} \leq 1/2$. Note that $\psi(2\hat{x},0) = \Delta w(2\hat{x},2-2\hat{x}) - \Delta C(2\hat{x}) = \Delta C(\hat{x}) - \Delta C(2\hat{x}) > 0$. First, since $\Delta w(1) < \Delta C(0)$, $0 > \psi(0,1) > \psi(x,1)$ for all x; hence there can be no equilibrium of type (ii). Two cases are possible:

(a) if $\Delta C(1) \leq \Delta w(1) < \Delta C(1/2)$, then $\psi(x,0) > \psi(1,0) > 0$ for all $x < 1$, and the unique equilibrium is of type (iii), with $\psi(1,0) > \psi(0,1)$.

(b) if $\Delta w(1) < \Delta C(1)$, then $\psi(1,0) < 0 < \psi(2\hat{x},0)$ hence there is a unique $\tilde{x}_1 \in (2\hat{x},1)$ such that $\psi(\tilde{x}_1,0) = 0$, defining a unique equilibrium, of type (i).

Case 2: $\Delta w(1) > \Delta C(1/2)$, i.e. $\hat{x} > 1/2$. Note that $\psi(2\hat{x}-1,1) = \Delta w(2\hat{x},2-2\hat{x}) - \Delta C(2\hat{x}-1) = \Delta C(\hat{x}) - \Delta C(2\hat{x}-1) < 0$. First, since $\Delta w(1) > \Delta C(1)$, $\psi(x,0) \geq \psi(1,0) > 0$ for all x; hence there can be no equilibrium of type (i). Two cases are possible:

(a) if $\Delta C(0) \geq \Delta w(1) > \Delta C(1/2)$, then $\psi(x,1) \leq \psi(0,1) \leq 0$ for all x, and the unique equilibrium is of type (iii), with $\psi(1,0) > \psi(0,1)$.

(b) if $\Delta w(1) > \Delta C(0)$ then $\psi(0,1) > 0 > \psi(2\hat{x}-1,1)$; hence there is a unique $\tilde{x}_2 \in (0,2\hat{x}-1)$ such that $\psi(\tilde{x}_2,1) = 0$. It defines the unique equilibrium, which is of type (ii).

2. Unemployment equilibria: We now show that no such (stable) equilibrium exists, except possibly for one where nobody works. Given assumptions A1, high-skill workers will always outbid low-skill workers, and the latter will always outbid unemployed agents, for any available land in community 1. Therefore, the stability of equilibrium requires:

Case 1: If high-skill agents are on both sides, they must fill up community 1 completely; so $x_1 = 1$, $0 < x_2 \leq x_2 + y_2 < 1$. But then $H/L = (1+x_2)/y_2 > (1+x_2)/(1-x_2) > 1$, so $w_L > w_L(1,1) > w_L(\hat{x},2-\hat{x}) > C_L(0) + v > C_L(x_1) + v$ by assumption A6. This means that the unemployed in community 1 would rather acquire low skills, a contradiction.

Case 2: If high-skill agents are in community 1 only and low skill agents in both, then community 1 can have no unemployed agents; so $0 < x_1 = 1-y_1 < 1$, $0 = x_2 < y_2 < 1$. But then $\rho = x_2/(1-x_2+y_1) \in (x_2/(2-x_2), x_2/(1-x_2))$, so $\Delta w(x_2/(1-x_2)) < \Delta C(x_2)$, implying that $x_2 > \hat{x}$. Therefore $w_L(\rho,1) > w_L(x_2,2-x_2) > w_L(\hat{x},2-\hat{x}) > C_L(0) + v$, yielding the same contradiction.

Case 3: If high-skill agents are in community 1 only and low-skill agents in a single community, it must be community 1 also, i.e. $0 < x_1 < x_1 + y_1 < 1$, $x_2 = y_2 = 0$, unless $x_1 = 1$. In the first case, the labor market reduces to community 1, and the occupational instability shown in Footnote 8 for the integrated city holds here as well. The second case must solve:

$$\psi(x_2,y_2) = \begin{bmatrix} w_H(1+x_2,y_2) - C_H(x_2) - v \\ w_L(1+x_2,y_2) - C_L(x_2) - v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The Jacobian $D\psi(x_2,y_2)$ has determinant $C_H' \cdot (-\partial^2 F/\partial L^2) + C_L' \cdot (-\partial^2 F/\partial H \partial L) < 0$. This implies that at any point of intersection of the curves $w_H(1+x,y) - C_H(x) = v$ and $w_L(1+x,y) - C_L(x) = v$, the latter has a higher slope, so there is at most one such solution. Moreover, it must be saddlepoint-unstable. Q.E.D.

Proof of Proposition 3

Recall first that the planner can always get $\hat{r} > v$ by implementing the symmetric equilibrium. Note also from the definition (3) of $\hat{x}$, that as θ increases to $+\infty$, $\hat{x}/(1-\hat{x})$ increases to a limit of $\bar{\rho}$.

In any allocation where $x_j + y_j < 1$, the first-order conditions for (6) show that $w_H - C_H(x_j) = v + x_j.C_H'(x_j) + (1-x_j).C_L'(x_j) < v = w_L - C_L(x_j)$. So unemployment in community 1 implies $V < v$ (recall $x_1 \geq x_2$) and cannot be optimal. Suppose now that there is unemployment in community 2 only. We shall denote $\rho = H/L = (x_1+x_2)/(1-x_1+y_2)$. From (6) and the associated first-order conditions, we have:

$$W < [x_1.(w_H(\rho,1) - C_H(x_1) - v) + y_1.(w_L(\rho,1) - C_L(x_1) - v) + v]/2$$

Case 1: If $x_1 = 1$, then $\rho > 1$, so $W - v < (w_H(\rho,1) - C_H(x_1) - v)/2 < (w_H(1,1) - C_H(1) - v)/2$. But under constant returns to scale, and by definition of $\bar{\rho}$: $(w_H(1,1) + w_L(1,1))/2 = F(1/2,1/2) \leq F(\bar{\rho},1-\bar{\rho}) = w_H(\bar{\rho},1) = w_L(\bar{\rho},1)$. So:

$$\hat{r} - W > w_L(\hat{x},1-\hat{x}) - w_L(\bar{\rho},1) + [w_H(\bar{\rho},1) + C_H(1) - 2.C_H(\hat{x}) - v]/2$$

$$= \theta.[\partial f(\hat{x},1-\hat{x})/\partial L - \partial f(\bar{\rho},1)/\partial L + \partial f(\bar{\rho},1)/\partial L] + [C_H(1) - 2.C_H(\hat{x}) - v]/2$$

Now, as $\theta \to +\infty$, the right hand-side becomes equivalent to $\theta.\partial f(\bar{\rho},1)/\partial L$, hence tends to $+\infty$. So for θ large enough, the allocation cannot be optimal.

Case 2: If $x_1 < 1$, then $\Delta w = \Delta C(x_1) + x_1.C_H'(x_1) + (1-x_1).C_L'(x_1) = \Phi'(x_1)$. Therefore $W - v < (w_L(\rho,1) - C_L(x_1) - v)/2 < (w_L(\rho,1) - C_L(1) - v)/2$, so that:

$$2(\hat{r} - W) > \theta.[2.\partial f/(\hat{x},1-\hat{x})\partial L - \partial f(\rho,1)/\partial L] + C_L(1) - 2.C_L(\hat{x}) - v$$

So for θ large enough, the optimality of W requires $\partial f(\rho,1)/\partial L > 2.\partial f(\hat{x},1-\hat{x})/\partial L \approx 2.\partial f(\bar{\rho},1)/\partial L$. This in turn requires $\rho > \bar{\rho}$, hence $x_1 > \bar{\rho}/(2+\bar{\rho})$ since $\bar{\rho} = (x_1+x_2)/(1-x_1+y_2) \leq 2.x_1/(1-x_1)$. But now for θ large enough, $\Delta w = \theta.[\partial f(\rho,1)/\partial H - \partial f(\rho,1)/\partial L] = \Phi'(x_1)$ requires that $\partial f(\rho,1)/\partial H - \partial f(\rho,1)/\partial L$ be close to zero (as long as Φ' is bounded on $(\tilde{\rho}/(2+\tilde{\rho}),\ 1]$, which we shall assume), i.e. that $\tilde{\rho}$ be close to $\bar{\rho}$. This contradicts $\partial f(\rho,1)/\partial L > 2.\partial f(\tilde{\rho},1)/\partial L$. Q.E.D.

Proof of Proposition 4

With Φ strictly convex, minimizing $\Phi(x_1)+\Phi(x_2)$ over (x_1,x_2) with $x_1 + x_2 = x$ requires $x_1 = x_2 = x/2$, for any $0 \le x \le 1$. The planner's problem thus simplifies to maximizing the strictly convex function $V(x)/2 - F(x,1-x) - \Phi(x)$ over x in [0,1]. We have $V'(x) = \Delta w(x,1-x)-\Delta C(x)-x.C_H'(x)-(1-x).C_L'(x)$, and by A2, $\lim_{x\to 0} V'(x) > 0$, $\lim_{x\to 1} V'(x) = \lim_{x\to 1} [\Delta w(x,1-x)]-\Delta C(1)+C_H'(1) < 0$

Q.E.D.

Proof of Proposition 5

In any equilibrium with production, $H/L < \bar{\rho}$ so $w_L < w_L(\bar{\rho},1)$; assumption A8 then implies that for any community j, if $x_j = 0$ then $y_j = 0$.

If there is a full-employment equilibrium ,it clearly must be the one described in Proposition 2. When $\hat{x} < 1/2$, this requires $x_2 = 0$, hence $y_2 = 0$, a contradiction. When $\hat{x} > 1/2$, it is defined by the unique solution to $\Delta w(1+\tilde{x}_2,1-\tilde{x}_2) = \Delta C(\tilde{x}_2)$, with $0 < \tilde{x}_2 < 2.\hat{x}-1$; this is indeed an equilibrium if $w_L(1+\tilde{x}_2,1-\tilde{x}_2) > C_L(\tilde{x}_2) +v$. If not, there is no full-employment equilibrium. Consider next equilibria with unemployment.

(a) If $x_2 = 0$, then $y_2 = 0$, and only community 1 operates. Under constant returns to scale, its equilibria are those of an integrated city, scaled down to half-size. We thus know (see Footnote 8) that the only stable ones involve either shutdown, or full employment according to $x_1 = 1 - y_1 = \hat{x}$.

(b) If $x_2 > 0$, residential stability requires that $x_1 = 1$. This situation was examined in the proof of Proposition 2 (Case 3) above, where it was shown not to be (occupationally) stable. Q.E.D.