NBER TECHNICAL WORKING PAPER SERIES

RATIONAL FRENZIES AND CRASHES

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Technical Working Paper No. 112

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 1991

Bulow is also affiliated with the National Bureau of Economic Research. This research was partially funded by the National Science Foundation. This paper is part of NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

Most markets clear through a sequence of sales rather than through a Walrasian auctioneer. Because buyers can decide between buying now or later, rather than only now or never, buyers' current "willingness to pay" is much more sensitive to price than is the demand curve. A consequence is that markets will be extremely sensitive to new information, leading to both "frenzies," where demand feeds upon itself, and "crashes," where price drops discontinuously. Although no buyer's independent reservation value reveals much about overall demand, a small increase in one such value can cause a large increase or decrease in average price.

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1. Introduction

Few markets are as efficient as the world's largest stock markets. Yet even in these markets prices are extremely volatile. The Crash of 1987 is often cited as Exhibit A in the case against both market efficiency and rationality.

Why do many economists consider volatile prices a sign of poorly performing markets? The intuition probably stems from the basic Walrasian model of market clearing. In that model, a little bit of new information will have only a small effect on the equilibrium price. But real markets do not use a Walrasian auctioneer to clear: information about supply and demand is transmitted through actual sales. Therefore buyers must act strategically, and not just offer to buy whenever the price falls below their reservation level. They must decide at each moment whether the current price is a sufficiently good bargain, or whether they should hold off in the hope that prices will fall further.

We build a simple model of market clearing with rational potential buyers, each of whom wishes to purchase one unit of a good, and a single profit-maximizing seller with a fixed supply. In the interests of transparency and tractability we make buyers' valuations independent and identically distributed, so finding out one person's value tells you nothing about anyone else's. All market participants are risk neutral.

We assume that a seller begins by asking for a high price, which is then lowered over time. Buyers may purchase at the price named at any moment. Price rises, and current demand is not satisfied, only if the number of buyers who jump in simultaneously at a given price exceeds the number of units that a seller has left.[1]

Our results are strikingly different from those that come from the standard story.

First, demand will "feed on itself." A purchase by just one buyer can *and should* make many previously reluctant buyers eager to buy at the same price. This second

[1] This "tie-breaking" assumption is unimportant. See Section 6.

wave of buyers at a given price will quite often be followed by more waves at the same price. We call such buying waves "frenzies."

Second, a frenzy can quickly peter out. While many customers may pay a given price at one moment, it can become common knowledge that no buyer will pay anything close to the same amount the next. When this happens, the price must fall discontinuously and we have a "crash." It is not unusual for the price to crash even when a crash is virtually certain to be followed, after the very next sale, by another frenzy.

Third, small differences in the underlying demand structure can cause large, unpredictable changes in the price path. The demand curve, in terms of buyer reservation values, can be unambiguously higher in one market than in another with the same supply, while seller revenue can be much lower.

Why does allowing traders to choose whether to buy now or later lead to large frenzies and crashes? Because when a buyer has the option of deferring purchase, his current "willingness to pay" may be much more closely related to his expectation of the market clearing price than to his reservation value. The implication is that even though a market's demand curve may be inelastic, the willingness to pay curve may be almost flat in the relevant (upper) range. As a consequence, even the small amount of new information that is revealed by the purchase of just one buyer changes many bidders' views of the current price from slightly too high to sufficiently attractive. Therefore the initial purchase should lead to a frenzy of activity.

Because frenzies potentially involve large numbers of buyers, they reveal a great deal of information about demand. In particular if the number of bidders who participate in a frenzy is much below expectations, then all bidders reduce their expectations of the market clearing price and therefore their current willingness to pay. No further trading can then occur until after the asking price has crashed.

Our model may most closely parallel the way U.S. underwriters sell new securities. An initial price is maintained or supported until either an issue sells out or it becomes apparent that there will be insufficient demand. For example, in the then-largest corporate bond issue in history, Salomon Brothers in 1979 was lead underwriter for, and sought to market, \$1 billion of IBM securities. After several days, approximately \$300 million of bonds remained unsold. Finally, the asking price was reduced by roughly \$45 per \$1,000 bond—a huge drop for virtually default-free securities.

Some commodities and currency markets have exhibited similar characteristics. For example, in the mid-1980s, tin producers attempted to maintain minimum prices in the face of falling demand. When it became apparent that the minimum was no longer viable, price had to be cut sharply.

In the housing market, declines in demand are sometimes met with sticky asking prices, leading to a reduction in sales until either demand picks up or prices fall. Many stock market analysts look at volume as a predictor of a stock's future price changes. And investors often place limit orders to *sell* when markets decline.

In all these cases prices are moved by transactions, with potential buyers incorporating the information from these transactions into their bidding strategies. As a consequence, demand at any price can change quickly and prices can react sharply. These are the characteristics that we develop and explain.[2]

2. The Model

A seller has K units of a good for sale. There are $K + L$ risk neutral potential buyers, each having a reservation value for a single unit of the good. These reser-

[2] Joseph Stiglitz suggests our model may have macroeconomic implications. Assume, for example, a market will open at a fixed date (e.g., Europe in 1992, or a drug when its patent expires). Then sinking an investment at an earlier date corresponds to paying a higher (present-value) price to enter the market. Our model explains that even if potential entrants have very different expected values of participating in the market, we should expect any first investment to be immediately followed by a frenzy of further investment activity, then, quite often, by a period in which it is common knowledge that no investment will occur, then another frenzy of activity, and so on.

vation values are independent and identically distributed, drawn from the common distribution $F(v)$, with $F(\underline{V}) = 0$, $F(\overline{V}) = 1$, and $F(v)$ strictly increasing and atomless on $[\underline{V}, \overline{V}]$, with density function $f(v)$. The seller and other buyers do not observe a buyer's v, but know it is drawn from $F(v)$. A buyer with value v obtains surplus $v - p$ if he obtains a unit at price p, regardless of when he buys.

The seller begins the trading process by asking a high price, say $\overline{V}$, which is then lowered continuously until all units are sold. Buyers may purchase at any time. We will see, however, that after one customer buys, others may be willing to pay the same price. We specify, therefore, that whenever one or more sales are made, all other customers are asked (simultaneously and independently) if they want to buy. If the number of customers who now wish to purchase does not exceed remaining supply, then sales are made to these buyers, and all remaining customers are asked again.[3] When no more customers wish to buy, the seller again lowers the price continuously until another customer buys. If at any time the number of demanders exceeds supply, then we assume the remaining units are distributed among these demanders using the original procedure.[4] (We show later that using an alternative tie-breaking mechanism, e.g., a lottery, would not importantly affect the results.) Our analysis is restricted to symmetric equilibria.

We call multiple rounds of sales at a single price a *frenzy.*

A *crash* occurs if, after trade takes place at a price, it becomes common knowledge that no further purchases will be made until the price has fallen to some given strictly lower level.

[3] We will see that there may, in fact, be several "rounds" of sales at a single price.

[4] That is, the seller returns the price to $\overline{V}$ and proceeds as before. If an additional iteration of the original procedure results in no reduction in the number of customers who are "tied," the units can be assigned by lottery at the final price after one further iteration. In equilibrium this happens only in the zero-probability event that all these bidders have the same valuation, and this valuation is equal to the final price.

3. General Solution

At any point of the game, we write k for the number of units remaining, $k+\ell$ for the number of bidders remaining, and $\underline{v}$ and $\overline{v}$ for the lowest- and highest-possible-valuation bidders remaining conditional on all bidders having thus far followed their equilibrium strategies.[5]

At any point of the game, let $\epsilon(v)$ be the expected price a bidder with a value of v would pay, contingent on receiving an object, if the remaining goods were allocated according to a standard English auction. That is, when k units remain, $\epsilon(v)$ is the bidder's expectation of the $(k+1)$st highest out of $k+\ell$ remaining values, conditional on that value being below v.[6]

Let $P(v)$ be the bidder's expected price, contingent on obtaining a unit, in our model.

Let p be the current asking price.

Note that at any stage of the game a bidder is more likely to win an object if he offers to buy than if he does not, so his optimal strategy is to offer to buy if and only if his value exceeds some cutoff level. It is straightforward that in a symmetric equilibrium the information publicly revealed about the remaining bidders is always just that their valuations all lie between some lowest- and highest-possible valuations $\underline{v}$ and $\overline{v}$.[7] It therefore follows from elementary statistics that

$$\epsilon(v) = \frac{\int_{\underline{v}}^{v} x\, f(x)\left(F(\overline{v}) - F(x)\right)^{k-1}\left(F(x) - F(\underline{v})\right)^{\ell-1} dx}{\int_{\underline{v}}^{v} f(x)\left(F(\overline{v}) - F(x)\right)^{k-1}\left(F(x) - F(\underline{v})\right)^{\ell-1} dx}. \tag{1}$$

[5] Unless there has been excess demand, $\ell = L$.

[6] In an English auction the price is raised continuously from zero until all but k bidders have dropped out, and these k winners therefore pay the actual $(k+1)$st value.

[7] If at any point there are fewer than k offers, then conditional on all bidders having followed equilibrium strategies, the remaining bidders are now all revealed to have values between the current $\underline{v}$ and the current cutoff level for bidding, $\tilde{v}$. Similarly, if there are more than k offers, then all non-bidders are eliminated and the remaining eligible bidders all have values between $\tilde{v}$ and the current $\overline{v}$. Therefore, the remaining bidders' valuations always remain independently drawn from $F(\cdot)$ conditional on being between some values $\underline{v}$ and $\overline{v}$.

Since it is also straightforward that the highest valuation bidders receive the objects, we can apply the Revenue Equivalence Theorem both to the whole game and to the continuation "subgame" that begins at any point, and so completely solve our model:

Revenue Equivalence Theorem: Assume that each of $k + \ell$ risk-neutral potential buyers has a privately known value independently drawn from a common distribution that is strictly increasing and atomless on the interval $[\underline{v}, \overline{v}]$, for one of k identical objects. Any auction mechanism which has the properties that the objects always go to the buyers with the k highest values, and that any bidders who do not receive a unit get zero surplus, yields the same expected revenue, and results in a buyer with value v making expected payment $\epsilon(v)$ conditional on receiving an object.

This theorem has been developed in different forms by Vickrey (1960), Myerson (1981), and Riley and Samuelson (1981). Our statement follows Myerson (Lemma 3 and Corollary, pp. 64–66), specialized to the case of symmetric bidders but straightforwardly generalized to $k \geq 1$ objects.[8]

Proposition (*Characterization of Equilibrium Bidding Strategies*): At any point in the game, a bidder with valuation v offers to purchase if and only if $\epsilon(v) \geq p$.

Proof: Let $\tilde{v}$ be the valuation of a buyer who would be indifferent to bidding p. Then $P(\tilde{v}) = p$, because such a buyer would either receive a unit immediately at price p or, if there is excess demand at p, surely be outbid. But by the Revenue Equivalence Theorem, $P(\tilde{v}) = \epsilon(\tilde{v})$. Therefore, $v \geq \tilde{v}$ if and only if $\epsilon(v) \geq p$. Q.E.D. ▌

We call $\epsilon(v)$ the "Willingness to Pay" function since the Proposition tells us that this is the price at which a buyer with value v is actually willing to trade

[8] This generalization has been considered by Milgrom and Weber (1982b), Bulow and Roberts (1989), and Maskin and Riley (1989).The theorem (and therefore our results) hold under even weaker assumptions on the distribution.

at a given time. The distinction between this curve and the standard demand curve—which only represents buyers' willingnesses to accept take-it-or-leave-it final offers—is crucial for us.

Seller Optimality

The Revenue Equivalence Theorem proves that the seller's expected revenue is the same with our mechanism as with any standard auction, for example, an ascending (English) auction, or a sealed-bid auction in which the K highest bidders win and pay the $(K+1)$st bid or the K highest bidders win and pay their own bids. This also implies that provided $F(v)$ is "regular," that is, $\left[v - \frac{1-F(v)}{f(v)}\right]$ is strictly increasing in v, the trading process maximizes the seller's expected revenue over all possible mechanisms that require the sale of all K units.[9] Therefore, assuming regularity, the sales process is consistent with rational behavior by a risk-neutral seller.[10]

Note that in our mechanism we have not allowed the seller to precommit to a reservation price. That is, the seller cannot precommit to a strategy that may leave some output unsold. We think our assumption is the natural one, since maintaining commitment to a reservation price is not sequentially rational. However, nothing of importance in our model would be affected if the seller were able to precommit to a reservation price, r. It is easy to show that in this case, our Proposition is amended so that a bidder offers to purchase if and only if p is less than or equal to the bidder's expectation, conditional on receiving a unit, of the maximum of r and the $(k+1)$st highest remaining value. Frenzies and crashes arise exactly as in our original mechanism. If r is chosen optimally (ex-ante), then the game maximizes

[9] See Myerson (1981) section 5, Maskin and Riley (1989) section 3, or Bulow and Roberts (1989) section V. Bulow and Roberts explain that assuming regularity is equivalent to making the common assumption that a monopolist's marginal revenue is downward sloping.

[10] Because these remarks apply to every continuation "subgame," and because if $F(\cdot)$ is initially regular the distribution of valuations always remains regular, the sales process is also *sequentially* rational, that is, time consistent.

the seller's expected revenue over all selling mechanisms that allow the seller to precommit to a reservation price, provided $F(\cdot)$ is regular.[11,12]

4. Description of the Trading Process

It is straightforward to derive the evolution of the trading process (see Figure 1): using the Proposition we compute, at each stage, those bidders whose willingness to pay, $\epsilon(v)$, exceeds the current asking price, p. At the same time we keep track of how the function $\epsilon(v)$ changes as information is revealed about the parameters k, ℓ, $\underline{v}$, and $\overline{v}$ by the trading to date.

Strictly, the seller begins by asking the price $\overline{V}$. However, no bidder will be willing to pay more than $\epsilon(\overline{v})$, so we can think of the seller as setting $p = \epsilon(\overline{v})$ (step 1).

Price is then lowered continuously, and as long as there is no sale, $\overline{v}$ is continuously revised downward to $\epsilon^{-1}(p)$ (step 2). The first sale will be made to the bidder with the highest actual valuation. Since this bidder knows that conditional on bidding first his valuation must be the highest, the first sale is made at $p = \epsilon(\overline{v})$ when $\overline{v}$ equals this bidder's valuation, and k is then revised to $k-1$ (step 3).[13]

Removing one bidder and one unit by the first sale must increase the price each remaining bidder expects to pay; that is, setting $k = k-1$ increases $\epsilon(v)$ for all v.[14]

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[11] If $F(v)$ is not regular, the monopolist cannot maximize revenue by simply selling to the bidders with the highest values so no standard auction maximizes revenue. In this case the monopolist could gain by precommitting to "crash" the price, and to then hold lotteries, even when crashes do not arise endogenously, so frenzies and crashes seem even more likely to arise.

[12] For the theory of optimal monopoly pricing with demand uncertainty, see Harris and Raviv (1981). For discussion of the case of irregular $F(\cdot)$, see especially Mussa and Rosen (1978).

[13] That is, if v_1 is the actual highest valuation, then using (1), the first sale occurs at $p = \frac{\int_{\underline{v}}^{v_1} x\, f(x)\big(F(v_1)-F(x)\big)^{k-1}\big(F(x)-F(\underline{v})\big)^{\ell-1}\,dx}{\int_{\underline{v}}^{v_1} f(x)\big(F(v_1)-F(x)\big)^{k-1}\big(F(x)-F(\underline{v})\big)^{\ell-1}\,dx}$ which can be simplified to $p = (k+\ell-1)\binom{k+\ell-2}{k-1}\int_{\underline{v}}^{v_1} x\, f(x)\big(F(v_1)-F(x)\big)^{k-1}\big(F(x)-F(\underline{v})\big)^{\ell-1}\,dx \,/\, \big(F(v_1)-F(\underline{v})\big)^{k+\ell-1}$.

[14] That is, the expected kth highest of $k+\ell-1$ values exceeds the expected

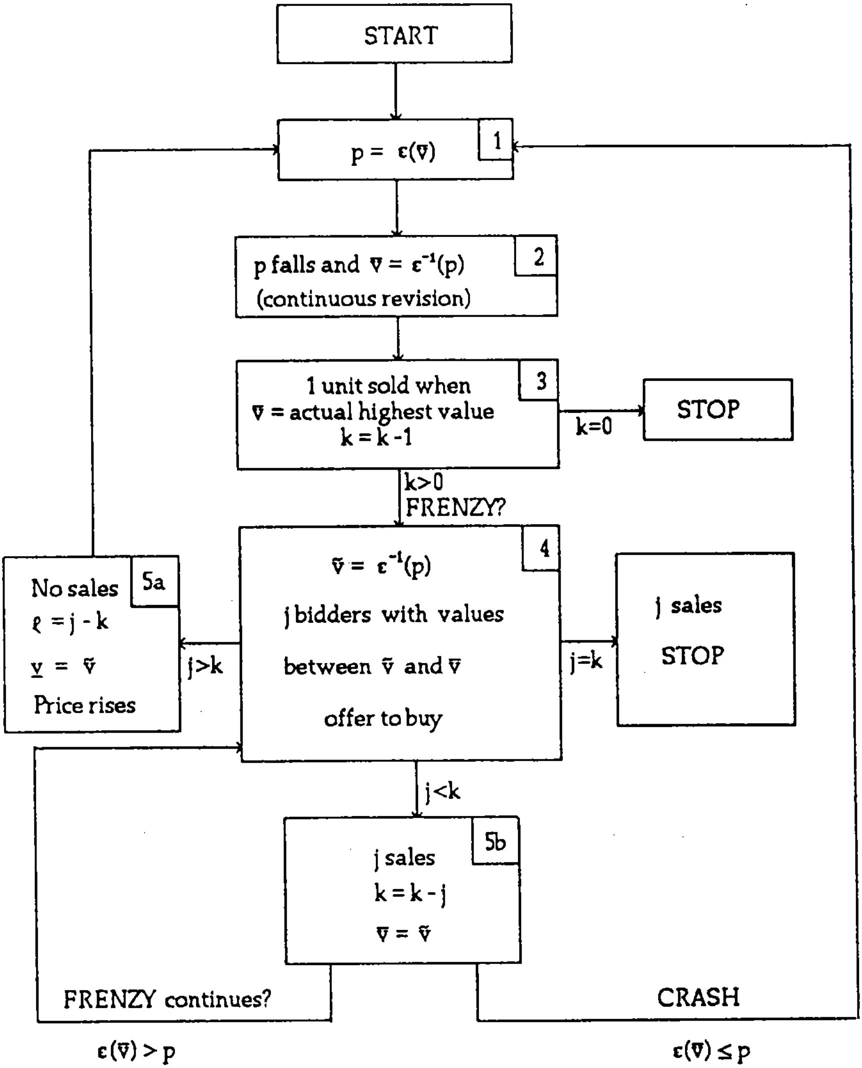


FIGURE 1: EQUILIBRIUM OF TRADING PROCESS

Since immediately previously we had $\epsilon(\overline{v}) = p$, there is now some value $\tilde{v} < \overline{v}$ for which $\epsilon(\tilde{v}) = p$. Our proposition shows that at this time all bidders with values v between $\tilde{v}$ and $\overline{v}$ participate in a frenzy (step 4). We will show in the next section that this may be a large number of bidders.

If the number of bidders j who now offer to buy exceeds k, then there is excess demand, so all players who did not offer to buy are now eliminated from the game. The total number of remaining bidders $k + \ell$ is thus revised to j, that is, $\ell = j - k$, and since all these bidders have values $v \geq \tilde{v}$ (in equilibrium), we reset $\underline{v} = \tilde{v}$ (step 5a). Based on these revisions, a new, higher, $\epsilon(\overline{v})$ is calculated and we return to step 1 to allocate the remaining units among the remaining bidders.

If $j < k$, then all participants in the frenzy are allocated a unit. The number of units remaining is revised to $k = k - j$, but the maximum value of any remaining bidder is revised to $\overline{v} = \tilde{v}$ (step 5b).[15] The first of these revisions raises $\epsilon(v)$ for all v, while the second reduces it. If j is sufficiently low, we then have $\epsilon(\overline{v}) \leq p$ and price must fall to at least $p = \epsilon(\overline{v})$ before there is any chance that another buyer can be attracted; that is, we have a crash (return to step 1). If j is sufficiently large, on the other hand, we have $\epsilon(\overline{v}) > p$ and there is the prospect of a continued frenzy (return to step 4).[16] Note that the larger j is, the lower the new cutoff valuation $\tilde{v} = \epsilon^{-1}(p)$ will now be, that is, the greater the range of valuations that will participate in the next round of the frenzy. In this sense the frenzy "feeds on itself," and a frenzy may run on for several rounds before ending in either excess

$(k + 1)$st highest of $k + \ell$ values or, equivalently, the expected value of the ℓth from the bottom of $k+\ell-1$ values exceeds the expected value of the ℓth from the bottom of $k + \ell$ values.

[15] Strictly, $\overline{v}$ is now the supremum of the remaining values, since we have arbitrarily assumed that indifferent buyers do bid.

[16] It is straightforward to show that if $j = k - 1$, then $\epsilon(\overline{v}) \geq p$, and if $j = 0$, then $\epsilon(\overline{v}) \leq p$, so if a frenzy ends with $k = 1$ and $j = 0$, then $\epsilon(\overline{v}) = p$, and the price falls continuously. For all $k > 1$ both these inequalities are strict, so for $k > 1$ there is both positive probability that the current round of bidding will be immediately followed by a crash, and positive probability that it will be followed by a further frenzy.

demand or a crash.[17]

Illustration: The Uniform Distribution

We can illustrate our analysis by considering the case where buyer values are drawn from a uniform distribution. When the distribution is on $[0,1]$, it is easy to perform the integrations in (1) to obtain

$$\epsilon(v) = \frac{\sum_{j=0}^{k-1} v\binom{k-1}{j}\left(\frac{(-v)^j}{j+\ell+1}\right)}{\sum_{j=0}^{k-1}\binom{k-1}{j}\left(\frac{(-v)^j}{j+\ell}\right)}, \tag{2}$$

and since conditional on the bidding to date, the distribution always remains uniform on $[\underline{v}, \overline{v}]$, $\epsilon(v)$ always remains an affine transformation of (2).[18] For $v = \overline{v}$, $\epsilon(v)$ always has the particularly simple form

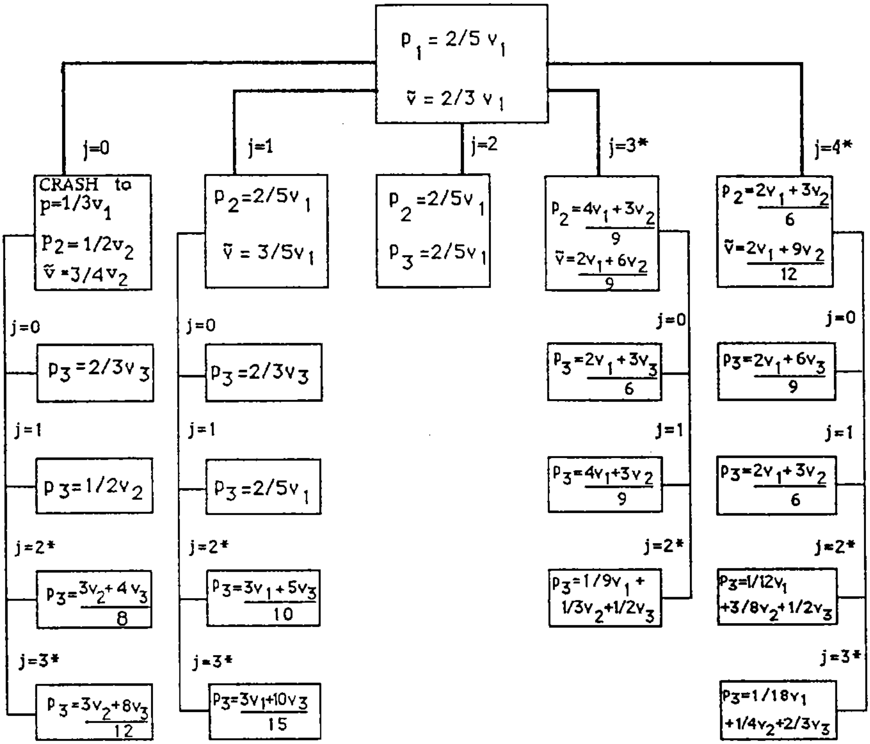
$$\epsilon(\overline{v}) = \left(\frac{k\underline{v} + \ell\overline{v}}{k+\ell}\right) \tag{3}$$

[17] To see that it is common for a frenzy to run on into several rounds, note that after the first unit is sold at any price—that is, before the first round of the frenzy—the price equals the seller's expectation of the $(k+1)$st price, that is, equals the seller's expectation of its revenue per remaining unit. However, the condition for a crash is that the price be not just below the seller's expectation of the $(k+1)$st price, but also below the expectation of the $(k+1)$st price that would be held by a buyer with the highest possible remaining value. That is, for a crash to take place, the information revealed by the numbers of bidders jumping in to buy must be such that the price is now above the per-unit revenue that would be expected by an observer who had both the seller's information and the most optimistic private information possible that any buyer might have. Thus a frenzy runs on into further rounds so long as the aggregate information that is revealed by the earlier rounds of the frenzy since the first sale at this price is either good news for the seller *or* bad-but-not-awful news.

[18] The general form of (2) is

$$\epsilon(v) = \underline{v} + \frac{\sum_{j=0}^{k-1}\binom{k-1}{j}\left(-\left(\frac{v-\underline{v}}{\overline{v}-\underline{v}}\right)\right)^j\left(\frac{v-\underline{v}}{j+\ell+1}\right)}{\sum_{j=0}^{k-1}\binom{k-1}{j}\left(-\left(\frac{v-\underline{v}}{\overline{v}-\underline{v}}\right)\right)^j\left(\frac{1}{j+\ell}\right)},$$

but it is easier to renormalize units and work with the version in the text. See, e.g., next note.



v_i=i^{th} highest actual value

P_i=sale price of i^{th} unit

$\tilde{v}$ =cutoff value for participating in frenzy

j =number of additional bidders at current price

$j>0 \Rightarrow$ frenzy

$^* \Rightarrow$ excess demand

FIGURE 2: FIVE BIDDERS, THREE GOODS

so that, in a crash, or at the beginning of the game, the price crashes to this level and then falls continuously until the first sale actually takes place at $\left(\frac{kv+\ell v_1}{k+\ell}\right)$, in which v_1 is the actual highest valuation.

For example, Figure 2 shows how the trading process evolves when $K+L=5$ bidders with values drawn from a uniform distribution on $[0,\overline{V}]$ compete for $K=3$ units. Using (3), the first unit is sold at $p=.4v_1$. Using (2), all bidders with values of at least $\frac{2}{3}v_1$ now jump in.[19] If none do, the price crashes $16\frac{2}{3}$ percent and on average falls by $33\frac{1}{3}$ percent before another buyer can be found. However, even if only one more buyer does jump in at $p=.4v_1$, there is a 27 percent chance that this second sale will generate demand from at least one more buyer. The overall probability of a frenzy occurring at some point in the game is over 90 percent.

5. Frenzies and Crashes

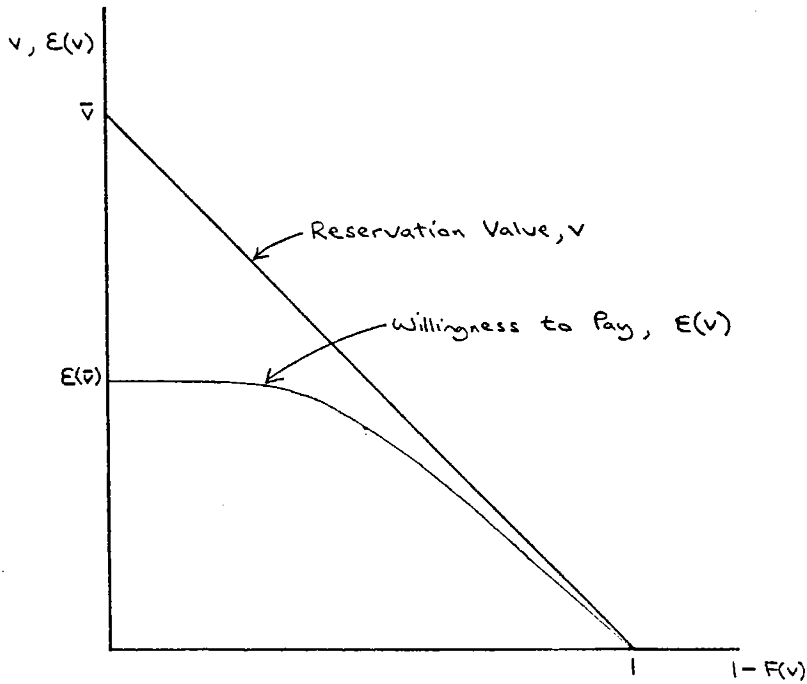
The surprising feature of our trading process is that frenzies and crashes are likely to be very big.

Frenzies

Large frenzies occur because the willingness to pay curve $\epsilon(v)$ is very flat; for $k>1$ it is perfectly elastic at $\overline{v}$, regardless of the slope of the demand curve. Therefore a small upwards shift in $\epsilon(v)$, such as that caused by a single purchase, turns bidders with a wide range of reservation values from bystanders into buyers.

Why is $\epsilon(v)$ so flat? Formally, observe that $\epsilon(v)$ is the expectation for a bidder with value v of the $(k+1)$st highest value, conditional on that value being below v; provided v is sufficiently high that the $(k+1)$st value is almost certainly below v, this expectation is almost independent of v. Consider, for example, $k=10$ units, $k+\ell=20$ bidders, and values drawn from a uniform distribution on $[0,\overline{v}]$. A bidder with value $\overline{v}$ would know that his value is highest, so his estimate of the

[19] The easiest way to check this is to renormalize v_1 to 1, so the distribution is now on $[0,1]$ with $k=2$, $\ell=2$, so (2) $\Rightarrow \epsilon(v)=\left[\left(\frac{v}{3}-\frac{v^2}{4}\right)\ /\ \left(\frac{1}{2}-\frac{v}{3}\right)\right]$. The (normalized) price is $p=.4$, so $p=\epsilon(v)\Rightarrow(3v-2)(6-5v)=0$, and the relevant root is $v=\frac{2}{3}$, that is, in the original units, $v=\frac{2}{3}v_1$.



(DRAWN TO SCALE)

FIGURE 3: WILLINGNESS TO PAY

20 bidders compete for 10 objects; values drawn from uniform distribution

$(k+1) = 11^{\text{th}}$ value of the 20 bidders is just his estimate of the 10^{th} value of the other 19 bidders, that is $.5\overline{v}$. However, a bidder with value $.8\overline{v}$ would know that his value also exceeds the actual 11^{th} value with probability .998, so his $\epsilon(v)$ is also very close to the estimate of the 10^{th} value of the other 19 bidders; in fact $\epsilon(.8\overline{v}) = .499\overline{v}$. Figure 3 graphs $\epsilon(v)$ for this example (to scale).

The general point is that when a large number of units remain for sale, a large number of bidders are fairly sure that they are inframarginal. Since, conditional on their being inframarginal, these bidders' expectations of the market clearing price are independent of their own exact values, all these bidders will have almost identical willingnesses to pay.

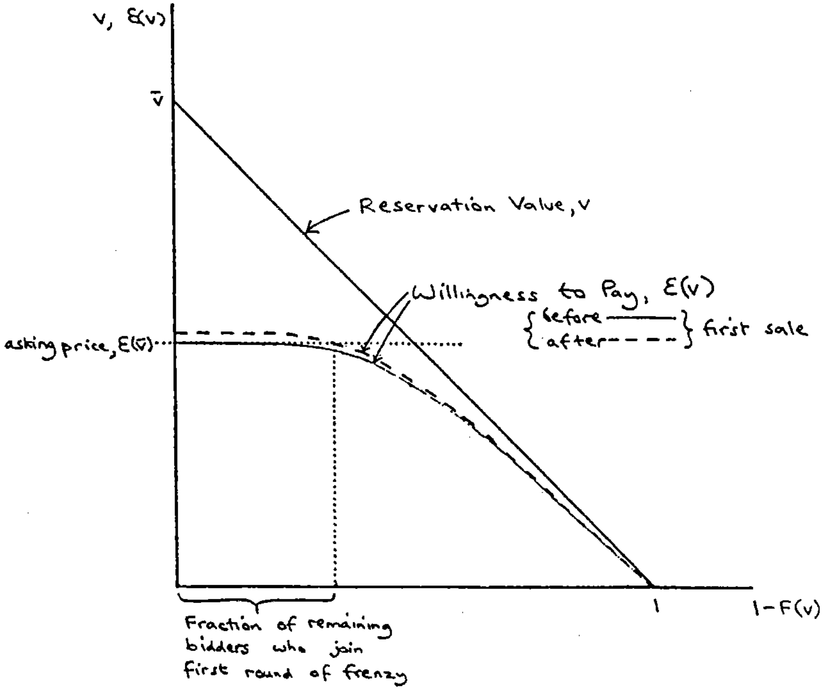
In the example, before any sale is made, the asking price $p = \epsilon(\overline{v}) = .5\overline{v}$. As price falls, so does $\overline{v}$, and the bidders always remain uniformly distributed on $[0, \overline{v}]$, so Figure 3 remains unchanged but with the units on the y-axis scaled appropriately. While the price is falling, it always equals $\epsilon(\overline{v})$, that is, equals the intercept of the willingness to pay curve and the y-axis.[20]

When a sale does occur, it causes a small upwards shift in $\epsilon(v)$ for all remaining bidders. (See Figure 4.) In the example, the estimated market clearing price is now the estimated 10^{th} value of the remaining 19. Thus another bidder with value $\overline{v}$ would now pay the estimated 9^{th} value of the other 18 bidders or $.526\overline{v}$, and a bidder with value $.8\overline{v}$ would, as before, pay almost the same as one with value $\overline{v}$, actually $.525\overline{v}$. Since in fact all bidders with values exceeding $.65\overline{v}$ would now pay more than the current asking price of $.5\overline{v}$, on average 6.6 bidders join the first round of the frenzy.

With larger numbers of units $\epsilon(v)$ is even flatter; with $K = 50$, $K + L = 100$, on average 40 bidders join the first round of the first frenzy.

Finally, another way to see that frenzies must be large is just to ask: Why will

[20] For general $F(\cdot)$, the important features of Figure 3 remain unchanged, that is, $\epsilon(v)$ is always flat at the y-axis and price always equals the intercept of $\epsilon(v)$ with the y-axis, but other details of the Figure will change as price falls.



(DRAWN TO SCALE)

FIGURE 4 : SIZE OF FIRST FRENZY

20 bidders compete for 10 objects; values drawn from uniform distribution

any bidder jump in first? Since a bidder pays the price he bids, why would he not always gain by holding off and bidding second, after someone else has bid first?[21] The reason must be that there is a non-trivial probability that as soon as someone does bid first, there is then immediately excess demand, so no one can guarantee being the second bidder and paying the first bidder's price. And for the probability of immediate excess demand to be non-trivial, the expected number of simultaneous bidders must, of course, be large.

Crashes

The large expected size of the frenzy is precisely what makes big crashes possible. In the last example above, ($K = 50$, $K + L = 100$ and uniformly distributed values) roughly 40 buyers are expected immediately, but the standard deviation is about 5. Finding out that there are only 30 bidders with relatively high values instead of the expected 40 would substantially reduce $\epsilon(v)$ for all remaining bidders, and force the seller to cut asking prices by about one-and-one-half percent for each "missing" purchaser. The probability that the price will fall by more than 10% (15%) after buying stops at the first price is greater than 10% (2%). The key is that the frenzy brings a large block of information into the market at one time, and if that information is unfavorable to the seller, then prices must crash.

"Chaos"

In contrast to conventional models, seller revenue in our mechanism is neither continuous nor monotonically increasing in bidders' reservation values. The reason is that information about demand is revealed in blocks, with bidders with a wide range of values revealing themselves simultaneously, and a change in one bidder's value can dramatically alter the flow of this information. Slightly *higher* bidder value(s) can lead to significantly *lower* revenues.

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[21] Of course it cannot be an equilibrium for no one to bid first. If all bidders entered simultaneously at a price of zero, most of them would have done better to enter a fraction earlier.

Consider a small decrease in v_1, the reservation value of the first buyer. If this buyer pays a little less, then the cutoff value above which bidders will now jump in will also be slightly lower, and an extra bidder may be induced to buy. This extra bidder could make the difference between a continuing frenzy and a crash, so the lower value of the first bidder could benefit the seller. Returning to our example of five bidders uniformly distributed on $[0, 1]$ and competing for three units, imagine that actual bidder values are .95, .62, .60, .45, and .40. The first unit would be sold for .38, a price that is too high to generate any further sales. After a crash, the second and third units would be sold for .31 each. Total seller revenues would equal 1.00. Now reduce all bidder values to .90, .61, .55, .30, and .10. In this case the first sale at .36 will lead to a second, and then a third sale at the same price. Revenues would be 3(.36)=1.08.

6. Rationing Excess Demand

While our mechanism generally requires the seller to meet all demand at the offering price, we do allow the offer to be retracted and the price to rise when there are more immediate bidders than units remaining. This assumption allows us to meet the requirements of the Revenue Equivalence Theorem and to guarantee seller rationality.

An alternative to raising price would be to hold a lottery among the remaining bidders. Making this assumption would preclude our using the Revenue Equivalence Theorem. However, we show in the Appendix that if $F(\cdot)$ is uniform, then under this assumption, at the beginning of the game or whenever the current asking price exceeds $\left(\frac{k\underline{v}+\ell\overline{v}}{k+\ell}\right)$, the price crashes to this level and then falls continuously until the next sale is made at $\left(\frac{k\underline{v}+\ell v_1}{k+\ell}\right)$, in which v_1 is the actual highest remaining valuation. This is exactly as with our original mechanism.

Furthermore, after the first sale is made the mechanism with lotteries always induces a *larger* frenzy than the original mechanism. The intuition is that the marginal buyer has a greater incentive to bid in the current round because by bidding he may earn some surplus even if there is excess demand.

In our example of five bidders competing for three units, all buyers with values greater than $.53v_1$ offer to buy immediately after the first sale, compared with only those above $.67v_1$ in the original trading process.

These larger frenzies also imply potentially larger crashes. For example, if no further sales are made at the first price in the original trading process, price must fall a minimum of 17 percent and an average of 33 percent before the next sale. With the lottery mechanism, price must fall a minimum of 33 percent and an average of 47 percent.

Finally, this discussion probably understates the magnitude of frenzies and crashes when lotteries are used as tie-breakers. We have assumed, as is standard in the auctions literature, that buyers are unable to resell among themselves. This becomes relevant in mechanisms which sometimes allocate units to lower valuation bidders. If resale is in fact possible, then frenzies will be even larger on average (for given k, ℓ, $\underline{v}$, $\overline{v}$, and p) because the marginal bidder has the added incentive that if he wins he may be able to resell for more than his value.[22] The more surplus goes to resellers, the larger frenzies will be.[23]

7. Common Values

The polar alternative to our "private values" assumption is to consider a pure "common values" model. Each bidder, if endowed with the same information, would value a unit at a common price. However, with asymmetric information, those

[22] It remains true, if $F(\cdot)$ is uniform, that the first sale is still at $p = \left(\frac{k\underline{v}+\ell v_1}{k+\ell}\right)$, so the first round of any frenzy is (weakly) larger, for any k and ℓ, than if lotteries are used without resale.

[23] See Appendix. In general, resale will not be efficient if bidders retain private information about their values; see Myerson and Satterthwaite (1983). Efficient resale is possible if, when there is excess demand for $j > k$ units, ownership of k/j of an object is assigned to each bidder—see Cramton, Gibbons, and Klemperer (1987). In this case, the highest valuation bidder enters at the same price as under our original mechanism, for any $F(\cdot)$, but *all* remaining bidders enter in the first frenzy.

bidders who get the most optimistic signals will outbid the others.

We consider a common value model attributable to Myerson (1981). Each of $(K+L)$ risk-neutral symmetric bidders $j = 1, \dots K+L$, obtains a signal, v_j, independently and identically distributed between $\underline{V}$ and $\overline{V}$ according to the common strictly increasing and atomless distribution $F(v)$. The true value of a unit to any bidder is $\sum_{j=1}^{K+L} \frac{v_j}{(K+L)}$. There are K units available.

Because bidder signals are still independent, a version of the Revenue Equivalence Theorem still applies. Specifically, any mechanisms that award units to the bidders with the highest signals and give no surplus to any bidder who does not receive a unit are equivalent in expected revenue and in the expected payment, conditional on receiving an object, of every type of bidder. Therefore, our mechanism still maximizes expected revenue, subject to the constraint that the units must be allocated to the K bidders with the most optimistic signals.

However, because any buyer's decision to purchase raises the valuation of all other bidders, frenzies are even more extensive. As a parallel to our earlier example, assume that five bidders compete for three units and that the signals are drawn from a uniform distribution on [0,1]. In equilibrium, the bidder with the highest signal, v_1, bids when the price falls to $.48v_1$. However, after this initial purchase the expected sales price of the remaining two units becomes $.54v_1$. This contrasts with the private values model, where the initial buyer pays the expected average price.[24]

As a consequence, if the seller *raised* the asking price to $.54v_1$, then on average 1/3 of all remaining buyers would want to purchase immediately, just as in the private values model when price is held constant after the first sale. If, in the common values model, price is kept constant after the first sale at $.48v_1$, then on

[24] Note that in the common value model the bidder with the highest signal makes a profit by paying a below average price, while his *ex post* value is the same as everyone else's. In the private values model the first buyer pays the expected $(K+1)$st value, that is, the expected average price conditional on his value, but gains surplus by having the highest value.

average more than 52 percent of all remaining bidders will jump in immediately.

If, as is the case in the general common value model of Milgrom and Weber (1982a), bidder signals are affiliated, then the Revenue Equivalence Theorem does not apply. Intuitively, one might expect bidders with affiliated signals to exhibit "herd behavior," but analyzing the affiliated case is beyond the scope of this paper.

It seems easiest to imagine crashes and frenzies developing in a common-value or affiliated-signal setting.[25] In our model, frenzies and crashes arise even with independent private values, the most surprising case.

8. Conclusions

We have presented a simple market clearing model, in which almost every first sale at a new price triggers a frenzy of buying. This extra demand may "feed upon itself," attracting more buyers, until demand exceeds supply. Alternatively, the frenzy will end with a crash in which price falls discontinuously.

Relaxing the assumptions of our basic model tends to increase the magnitude of frenzies and crashes. As it is, small changes in bidder values can dramatically change outcomes. An upward shift in all buyer valuations can sharply reduce seller revenue, implying that successive uses of our trading process in similar environments can yield strikingly different results.

Why so? Because in our mechanism, as in real markets, prices are moved sequentially by information drawn from transactions. Buyers must bid strategically because they may get a better bargain if they delay purchase. Simply put, a buyer must not only believe that the current price is below his valuation, but also that now is the time to buy, given the distribution of expected future prices. Even if demand is relatively inelastic, a small amount of new information can stimulate many sales. Bidders revise their willingness to pay depending on how sales compare

[25] Interesting papers which generate "herd behavior" in a common-value setting include Banerjee (1989), Bikhchandani, Hirshleifer, and Welch (1991) and Welch (1990). Gennotte and Leland (1990) explain the 1987 crash by a model in which agents may misinterpret portfolio-insurance trades as containing new information about fundamentals.

with expectations. It is precisely because bidders are rational and strategic that they are so sensitive to this information and adopt behavior that leads to frenzies and crashes.

Appendix

Rationing Excess Demand by Lottery

Write $\pi(v)$ for the probability that a buyer with value v receives a unit, and $S(v)$ for his expected surplus. In equilibrium, no "type" can gain by mimicking another type's strategy, so $S(v^a) = \pi(v^a)(v^a - P(v^a)) \geq \pi(v^b)(v^a - P(v^b)) = S(v^b) + \pi(v^b)(v^a - v^b)$, for all v^a, $v^b \in [\underline{V}, \overline{V}]$. So $S(\cdot)$ has derivative $dS/dv = \pi(v)$ and therefore

$$S(v) = S(\underline{V}) + \int_{\underline{V}}^{v} \pi(x)\, dx.$$

Since K units are sold to $K + L$ potential buyers,

$$\frac{K}{K+L} = \int_{\underline{V}}^{\overline{V}} f(x)\, \pi(x)\, dx$$

in any equilibrium, so if $F(\cdot)$ is uniform and $S(\underline{V}) = 0$, which is guaranteed if the price never falls below $\underline{V}$, we have $S(\overline{V}) = (\frac{K}{K+L})(\overline{V} - \underline{V})$. Since, conditional on the trading to date, the remaining valuations are always uniformly distributed, it follows that at any point in the game a bidder who knows that he has the highest possible remaining valuation, $\overline{v}$, expects surplus of $(\frac{k}{k+\ell})(\overline{v} - \underline{v})$, independent of the allocation mechanism.

Therefore, the price always crashes to $(\frac{k\underline{v}+\ell\overline{v}}{k+\ell})$. Now assume (for contradiction) that the marginal bidder in a frenzy has the same valuation as in the original process. His utility from not bidding would then be the same as before, because each possible number of bidders, j, that may now bid would be as likely as before and would give the same surplus as before (since if there is remaining stock after these bidders' purchases, this buyer has the highest valuation and therefore receives the same surplus as before). However, his utility from bidding is higher with the lottery since it allows him the chance to receive a unit even if there is excess demand. Therefore this bidder would strictly prefer to bid if he thought he was marginal, so the marginal value must be lower, and the frenzy must be larger.

Observe that allowing resale does not affect the argument above that a buyer with the highest possible remaining valuation, $\overline{v}$, expects surplus of $\left(\frac{k\underline{v}+\ell\overline{v}}{k+\ell}\right)$. Therefore the price of the first sale is unaffected. Now assume that the marginal bidder in a frenzy is the same regardless of whether there is resale. Then this bidder's utility from bidding is higher with resale because he is just the type who will gain the most from reselling, but his utility from not bidding is unaffected. This implies a contradiction. Therefore, if a lottery tie-breaker is used, allowing resale will create a larger frenzy.

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