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ABSTRACT

Why do we see nominal contracts in the presence of price level risk? To answer this question, this paper studies an overlapping generations model in which the equilibrium contract form is optimal, given the contracts elsewhere in the economy. Nominal contracts turn out to be optimal in the presence of aggregate price level risk under two circumstances. First, if individuals have the same constant degree of relative risk aversion. The reason is that in this case nominal contracts (eventually coupled with equity contracts) lead to optimal risk sharing. Second, nominal contracts can be optimal, even if the first condition is not met, if the repayment of contracts is subject to a binding cash in advance constraint. The reason is that a contingent contract, while reducing purchasing power risk, also increases the cash flow risk. Under a binding cash in advance constraint on the repayment of contracts, this second risk is costly, and it is minimized by a nominal contract. Finally, the paper also identifies some symmetry conditions under which nominal contracts are optimal even in the presence of relative price risk.

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1. Introduction

One of the greatest economic puzzles in an age of widely varying, random rates of inflation is the persistent use of nominal contracts, that is, of promises of a future payment of a prespecified, uncontingent sum of fiat money. The goal of this paper is to suggest reasons why such contracts might represent the optimal contract form in a large class of environments.

The starting point of our analysis is the observation that every individual is generally a party to several contracts with several other individuals. Hence in this paper we study a general equilibrium model in which the equilibrium contract form is optimal given the contract form eleswhere in the economy. We consider two kinds of shocks. One induces a relative price shock, the other an aggregate price level shock. We show that there are reasonable economic environments in which contracts contingent on either shock are not superior to nominal contracts.

When there are aggregate shocks, what matters is how alternative contract forms share the aggregate risk among the contracting parties. If all contracts are nominal, then aggregate risk is shared in proportion to net wealth. When individuals have the same degree of constant relative risk aversion, this is exactly what optimal risk sharing requires.

Departures from constant relative risk aversion imply that there are gains from having contracts contingent on aggregate shocks. But these gains are generally a second order of magnitude, because in general, optimal risk sharing still requires wealthier individuals to bear more risk, though not necessarily in proportion to their wealth. Hence, small costs of incorporating contingencies may restore optimality of fixed nominal contracts, even if individuals differ in their risk attitudes.

When contracts are payable in fiat money, contingencies that reduce the uncertainty of final real wealth generally increase the uncertainty of cash flows. Hence, even if the state of the world is costlessly observable, contingent contracts payable in fiat money entail a cost: it is the cost of holding enough cash to meet the maximum contingent payment specified by the contract. We show that for this reason fixed nominal contracts may be optimal even if there are aggregate shocks observable at no cost and individuals have different attitudes towards risk.

Nominal (uncontingent) contracts remain optimal even in the presence of relative price shocks, unless the contracting parties differ from each other in certain important ways. Specifically, in the paper we show that nominal contracts are optimal if: (i) individuals do not know with certainty which commodities they will want to consume in the future; (ii) they are ex-ante identical, in the sense of being exposed to the same kind of preference uncertainty; and (iii) preference shocks are not observed by other individuals. The third condition implies that contracts cannot be contingent on actual preferences. Moreover, since everyone wants to consume the same expected basket of commodities, individuals cannot insure each other against variation in the relative price between that basket and some other. Naturally, any cost of incorporating contingencies, including the cost due to the extra variability of cash flows, reinforces the optimality of nominal contracts in this case as well.

Finally, when nominal contracts are not optimal in the presence of either relative or aggregate risk, there is a combination of contingent and nominal contracts that turns out to be optimal. These more complicated contingent contracts can be interpreted as equity or future contracts or explicit insurance.

We formalize our reasoning in a fully specified, general equilibrium model of money and debt. In this model, fiat money serves as a medium of exchange, a means of payment and a unit of account. In particular, the interaction of agent preferences and the physical environment implies that i) fiat money has value as a medium of exchange among those who cannot trade directly, even if its rate of return is dominated by that of another asset; ii) people borrow and lend, choosing to specify fiat money as the means of payment by which IOUs are settled; and iii) in several non-trivial circumstances, fiat money serves as the unit of account, i.e., IOUs promising a fixed nominal sum of fiat money are an optimal contract form. We wish to stress that these are all implications, not assumptions, of the model. No feasible, mutually advantageous contracts or markets are arbitrarily ruled out and no demand for any asset or contract is imposed on the model.

There is a large literature that asks why nominal uncontingent contracts are so widespread. Gottfries (1990) stresses the role of labor market imperfections, but his analysis lacks explicit microfoundations and his results hinge on the assumption that there is an unidentified cost of writing contingent contracts. Cooper (1988) also focuses on the labor market in a model with microfoundations, but in his paper nominal contracts are generally not optimal, even if firms are risk neutral, and the contract only provides risk sharing and has no allocative role. (In our paper, by contrast, everybody is risk averse and IOU contracts serve both a risk-sharing and an allocative role.) Azariadis and Cooper (1985) show that uncontingent contracts may provide optimal risk sharing, but here too, firms are risk neutral and in addition, the optimal contract is not nominal (in the sense that it is not payable in fiat money and that it specifies an uncontingent real wage). Finally Smith (1985) studies an overlapping generations economy in which nominal contracts are a device for sorting out different types of workers; but in more general environments other sorting devices are likely to be available and optimal.

The paper outline is as follows. Section 2 lays out the model of an overlapping generations economy with spatially separated agents born in each generation. Section 3 proves the optimality of nominal contracts when there are only relative price shocks and future preferences are unknown. The general properties of an equilibrium with nominal contracts are described in section 4. In section 5, we show that nominal contracts lead to optimal sharing of aggregate risk if individuals have the same risk preferences. Section 6 proves that a binding cash-in-advance constraint on the settlement of private debt reinforces the optimality of nominal contracts. Finally, section 7 contains some concluding remarks.

2. The Model

This section describes the economic environment. A growing population is distributed over a large, even number I of contiguous islands. The islands are located in a circle and are numbered consecutively in a clockwise direction around the circle according to the variable i, $i=1,2...I$. There is a separate market in each island; no centralized "inter-island" market exists. Households, each comprised of two partners, live two periods. A new generation is born every period. Each household is endowed with one unit of non-storable output when young and nothing when old.

The population of newborn households on each island is random. An island can either be "large" or "small". A <u>large</u> island receives a number N_t of newborn households in period t; a <u>small</u> island receives a number θN_t of newborn households, where $\theta<1$. Whether an island is large or small in period t is determined by the realization of the random variable ω_t. For simplicity, ω_t can only take two values, 1 and 2, with equal probability. If $\omega_t=1$, then all islands for which i is even are large in period t, and all islands for which i is odd are small. If $\omega_t=2$, then the opposite is true. Because the total number of islands, I, is assumed to be even, the total population size does not depend on the realization of ω_t.

Each island produces a different commodity. This difference matters because the preferences of each household depend on its location on the circle. When young, a household born in island i wants only to consume the commodity produced on island i+1. Except for their location, all young households are identical.[1]

When old, each household moves to some other island, whose commodity it wants to consume.[2] When young, the household does not yet know where it will move when old. Its destination when old depends on a preference shock that it will experience in the second and last period of its life. Each old individual has the same probability of moving to an odd or an even island, and the same number of old consumers moves to each island. The realization of the preference shock is private information and cannot be learned by others.

The only relevant difference between islands is whether they are large (L) or small (S); for this reason, we can write the expected utility function of an old household without reference to odd or even locations and to his preference shock, and we only need to distinguish between households born in a small or large island. The expected utility of a household born in period t on an island of type k, for k=S,L is:

$$U(x_t^k) + \frac{1}{2}\sum_{\omega=1}^{2} V\left(c_{t+1}^k(\omega_{t+1})\right) = U(x_t^k) + \frac{1}{2}V(c_{t+1}^k(1)) + \frac{1}{2}V(c_{t+1}^k(2)) \tag{2.1}$$

1 Townsend (1987) studies a related model with spatially separated individuals in an overlapping generations economy. However in his model, unlike here, neighboring individuals only meet once in their lifetime, and hence cannot write IOU contracts among themselves.

2 The random assignment of agents to other islands follows the models of Townsend (1989) and Mitsui and Watanabe (1990).

where x_t^k and c_{t+1}^k denote its consumption when young and old, respectively, and where U(·) and V(·) are twice-continuously differentiable, strictly increasing, and strictly concave functions. Notice that 1/2 is the probability of ending up on either type of island when old.

Each period is split into two sub-periods. In the first sub-period everybody observes the realization of the shock ω and one partner of each household (young and old) starting on island i travels to island i+1.[3] In the second sub-period the travelling partner returns, old households move to another island, and consumption takes place. This structure of travel and the specified preferences generate the following trading pattern for a household born on island i. The pattern is outlined here and charted in Figure 1.

1) youth

i.) first sub-period

–each household is split into two units, a buyer and a seller;

the buyer travels to island i+1 and makes purchases by issuing IOUs payable next period;

the seller remains on island i and sells his commodity against IOUs receivable in the next period to the young buyers coming from island i-1.

ii.) second sub-period

–the partners reunite at the home island and consume the commodity purchased from their neighbor;

–the home commodity is sold for fiat money to the arriving old households.

2) old age

i.) first sub-period

–the household is again split into two, a debt collector and a debt repayer;

the debt repayer travels to island i+1 and repays the household's IOUs with fiat money;

the debt collector remains on its island and collects the fiat money repayment of the IOUs of island i-1.

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3 The division of a household into a partner who travels and one who stays is adapted from a model attributed to Lucas by Townsend (1980).

Old age, cont'd

ii.) second sub-period
 –the two partners reunite and travel to some other island;
 -the household purchases the commodity of the destination island with fiat money, then consumes it.

In the initial period (period 0) there is a generation of households of size N_0, equally distributed among all islands, that simply wish to maximize consumption on the island where they are located. This initial old generation cannot trade until the second sub-period of the initial period. It has no endowment of goods but owns a total of N_0M_0 units of fiat money on each island, implying an initial aggregate money supply equal to $M_0 \equiv IN_0M_0$.

The aggregate money supply is assumed to grow at the constant (gross) rate $z=M_t/M_{t-1}$. Changes in the stock of fiat money are used to finance government purchases to be spent in equal amounts in each island. Aggregate population grows at the constant rate $n=N_t/N_{t-1}$. Shocks to population growth and to money supply growth are studied in section 5 below.

Let b^k and a^k denote the nominal value of the IOU issued and accepted respectively by a young household born in an island of type k, and let m^k be the quantity of money that he holds, for k=L,S. Then we can write his budget constraints when young as

$$b_t^k \geq p_t^h x_t^k \qquad h,\ k=L,S \quad , \quad h \neq k \qquad (2.2a)$$

$$p_t^k \geq m_t^k + a_t^k \qquad k=L,S \qquad (2.2b)$$

where p^h and p^k are the money prices of the goods sold in an island of type h and k respectively.

When old, the consumer faces the budget constraint:

$$p_{t+1}(\omega_{t+1})\, c^k_{t+1} \leq m^k_t + R^h_{t+1}(\omega_{t+1})a^k_t - R^k_{t+1}(\omega_{t+1})b^k_t \equiv F^k_{t+1}(\omega_{t+1})\ , \quad h,k=S,L\ ,\ h\neq k \qquad (2.3)$$

where $R^k_{t+1}(\omega_{t+1})$, $R^h_{t+1}(\omega_{t+1})$ are the possibly state-contingent, gross rates of return on the IOUs issued by individuals born in islands of type k and h respectively, $F_{t+1}(\omega_{t+1})$ denotes net nominal financial wealth, and $p_{t+1}(\omega_{t+1})$ is the price faced by the consumer when old, which depends on whether he ends up on a large or small island, and hence on the realizations of the state ω_{t+1} (as well as of his preference shock which we omit here to simplify notation). Note that since the preference shock is private information, individuals cannot write IOUs contingent on the realization of these shocks. The question of whether equilibrium contracts will be contingent on the shock ω_{t+1} which determines which islands are large and which are small is addressed in the next section.

The previous assumptions about timing have a straightforward but important implication for the nature of an equilibrium. Namely, only fiat money will be accepted in payment for an IOU. The reason is that the old, who are scattered among the islands, need fiat money to carry out their consumption purchases when old. In particular, because of the spatial separation between islands, a payable IOU cannot be settled by offering in exchange an IOU issued by some other island. Hence, the settlement of an IOU is subject to a physically imposed cash-in-advance constraint, which can be written as:

$$R^k_{t+1}(\omega_{t+1})\, b^k_t \leq m^k_t \quad , \quad k=L,S\ . \qquad (2.4)$$

The equilibrium conditions in the securities markets of each island are:

$$\theta a_t^S = b_t^L \tag{2.5a}$$

$$a_t^L = \theta b_t^S \ . \tag{2.5b}$$

In writing (2.5) we have used the fact that the young population in a small island is a fraction θ of the population in a large island.

Since at the start of any period fiat money is held only by the old, and since by assumption the old are drawn equally from all islands, the money supply in each island is N_0M_t . With aggregate population growing at the (gross) rate n and with money supply growing at the (gross) rate z, the equilibrium condition in the money market of each island is

$$\theta m_t^S = M_0\,(z/n)^t \tag{2.6a}$$

$$m_t^L = M_0\,(z/n)^t \tag{2.6b}$$

An implication of the market clearing conditions (2.5) and (2.6) is that the aggregate nominal wealth of the old must equal the total stock of fiat money

$$[\theta F^S(\omega_{t+1}) + F^L(\omega_{t+1})]\; I\, N_t/2 = \overline{M}_t = IN_0M_t$$

or (2.7)

$$\theta F^S(\omega_{t+1}) + F^L(\omega_{t+1}) = 2M_0(z/n)^t \ .$$

We can now define a rational expectations competitive equilibrium with optimal contracts (hereafter, simply "equilibrium") as a sequence of the vector $[R^S_{t+1}, p_t^S, b_t^S, a_t^S, m_t^S, x_t^S, c_t^S, R^L_{t+1}, p_t^L, b_t^L, a_t^L, m_t^L, x_t^L, c_t^S]$ such that i) young households choose money balances, IOUs payable and receivable, and consumption to maximize expected utility taking prices and interest rates as given; ii) IOUs take a form such that it is not possible to increase the expected utility of a member of any generation t without reducing the expected utility of another member of that generation; iii) each household maximizes expected utility basing its decisions on the probability distribution actually generated by the equilibrium; iv) markets in IOUs and fiat money clear.

3. The Equilibrium Contract If There Is No Aggregate Risk

In this section we discuss under what conditions nominal IOU contracts are optimal if the only sources of randomness are relative price risk and shocks to preferences. Essentially these conditions identify when there is no insurable individual risk in this economy. Hence, the purpose of the section is mainly to illustrate a method of analysis and to clarify the properties of the model, rather than to derive any general and novel result. Section 5 extends the analysis to the more interesting case of aggregate price level shocks.

An IOU contract is a promise to pay R^j_{t+1}, j=L,S, units of fiat money tomorrow, for each IOU issued today. If R^j_{t+1} is not contingent on the realization of any shock, then we say that the IOU is a "nominal contract," since it is a promise to pay a fixed amount of money tomorrow, irrespective of the state of the world. The rate of return R^j_{t+1} cannot be contingent on the preference shock when old, since the realization of this shock is private information. The remaining question is whether a contract for which R^j_{t+1} is not contingent on the realization of ω_{t+1} is optimal, in the sense that it is not possible to increase the expected utility of a member of any generation t without reducing the expected utility of another member of that same generation. The answer is contained in the following:

Proposition 1: If aggregate population growth and money supply growth are not random, then the nominal contract is an optimal IOU contract.

The proof is straightforward. An "optimal contract" between the households of neighboring islands maximizes the expected utility of those born on a small island for a given level of expected utility of those born on a large island, subject to the constraints and the equilibrium conditions outlined in the previous section. Consider first an equilibrium in which the cash-in-advance constraints (2.4) are not binding. Then, combining (2.1) and (2.3), and noting that every household has a probability of 1/2 of ending up in a large or small island, we can characterize an optimal contract as a choice of $F^S(\omega)$, $F^L(\omega)$ for each ω to maximize

$$E_\omega\left\{\tfrac{1}{2}V\left[\frac{F^S(\omega)}{p^S}\right] + \tfrac{1}{2}V\left[\frac{F^S(\omega)}{p^L}\right] + \lambda\left[V^* - \left[\tfrac{1}{2}V\left(\frac{F^L(\omega)}{p^S}\right) + \tfrac{1}{2}V\left(\frac{F^L(\omega)}{p^L}\right)\right]\right]\right\} \tag{3.1}$$

subject to $\theta F^S(\omega) + F^L(\omega) = 2M_0(z/n)^t$, for ω=1,2 and where E_ω is the expectations operator with respect to ω. Notice that we have used the symmetry of the model to write p^S and p^L as independent of ω.

The resulting first order conditions may be written as

$$V'\left[\frac{F^S(\omega)}{p^S}\right]/p^S + V'\left[\frac{F^S(\omega)}{p^L}\right]/p^L = \theta\lambda\left\{V'\left[\frac{F^L(\omega)}{p^S}\right]/p^S + V'\left[\frac{F^L(\omega)}{p^L}\right]/p^L\right\} \tag{3.2}$$

for ω=1,2 and for every t.

This condition is satisfied by $F^S(\omega)=F^S$ and $F^L(\omega)=F^L$ for all realizations of ω. From the definitions of $F^S(\omega)$ and $F^L(\omega)$ in (2.3), we see that they are not contingent on ω if R^S and R^L are not contingent on ω. Hence a nominal contract is optimal.

If the cash-in-advance constraint (2.4) is binding, then *a fortiori* a nominal contract is optimal, since a binding cash-in-advance constraint makes it more difficult to reshuffle cash between borrowers and lenders through contingent rates of return. The formal proof is a bit more complicated, and is provided in section 6 below as a proof to Proposition 4.

The intuition underlying Proposition 1 is also straightforward. Suppose that R^j_{t+1} is not contingent on ω_{t+1}, and consider the expected utility of an old individual, conditional on the realization of ω_{t+1}, but not on that of the preference shock. By assumption every old individual has the same probability of travelling to an odd or an even island. But then, whether odd islands are small and even islands are large, or vice versa, is irrelevant: the realization of ω_{t+1} does not affect this expected utility. Hence, an IOU contract contingent on ω_{t+1} alone cannot achieve any relevant risk sharing among individuals born in contiguous islands. A contract contingent on both ω_{t+1} and the destinations of the old (the preference shock) could. But such a contract is ruled out by an incentive compatibility condition, since the destinations of the old are not publicly observable ex-post.[4] Therefore, given this incentive constraint, a nominal IOU contract is optimal.

We should note that the symmetry of the model plays a crucial role in the proof of this proposition. Suppose for instance that individuals born in an odd island have a probability greater than 1/2 of going to an odd island when old, and conversely that individuals born in an even island are more likely to go to an even island when old. It is easy to show in this case that individuals born in contiguous islands wish to insure each other against the relative price shock by writing IOU contracts contingent on the

[4] Alternatively, we could have made the simpler but more restrictive asumption that the destination when old is learned only upon arrival (i.e., after the IOU's are paid).

realization of ω. Hence nominal contracts are no longer optimal, even if the preference shock is unobservable.

The general lesson to be drawn from this section is that contingent contracts can provide insurance against relative price risk only if individuals are sufficiently different from each other in an ex-ante sense. Which relative price risk one needs to insure against is often ex-ante unknown to the contracting parties. There are many consumption decisions, like going to a movie versus going out for dinner, which are difficult to predict in advance. These decisions are determined by random events that are private information, and which therefore cannot be incorporated in any contingent contract. If individuals are subject to the same uncertainty about their future preferences, so that everybody is ex-ante identical, then nominal contracts are optimal. If instead the contracting parties assign different probabilities to alternative future consumption baskets, then contracts contingent on relative price shocks are optimal. Naturally, in this case a combination of a nominal contract with an explicit insurance contract (or a future contract) would also be optimal.

4. Properties of the Equilibrium

This section outlines some general properties of the equilibrium, including the valuation of the equilibrium contracts. We retain the assumption that there is no aggregate risk, so that nominal contracts are optimal and $R^k_{t+1}(\omega_{t+1})=R^k_{t+1}$. Under this assumption, a young household born at time t in an island of type k, k=S,L, maximizes:

$$U\,(b_t^k/p_t^h) + \tfrac{1}{2}V(F_{t+1}^k/p_{t+1}^k) + \tfrac{1}{2}\,V(F_{t+1}^k/p_{t+1}^h) \quad , \quad h{=}S,L \quad , \quad h{\neq}k \tag{4.1}$$

by choice of m_t^k, b_t^k and a_t^k, subject to (2.2) – (2.4) .

The first order conditions are:

$$U'(b_t^k/p_t^h) = R_{t+1}^k R_{t+1}^h\ \pi_{t+1}^h\left[V'(F_{t+1}^k/p_{t+1}^k)\,\frac{p_{t+1}^h}{p_{t+1}^k} + V'(F_{t+1}^k/p_{t+1}^h)\right] \tag{4.2a}$$

$$(R_{t+1}^h - 1)\left[V'(F_{t+1}^k/p_{t+1}^k)\,\frac{1}{p_{t+1}^k} + V'(F_{t+1}^k/p_{t+1}^h)\,\frac{1}{p_{t+1}^h}\right] = 2\mu_t^k\,\frac{1}{p_t} \tag{4.2b}$$

where $\pi_{t+1}^h \equiv p_t^h/p_{t+1}^h$ is the inverse of the (gross) inflation rate in the price of the good produced in an island of type h, and μ_t^k is the Lagrange multiplier of the cash-in-advance constraint (2.4), written in real terms. Thus, not surprisingly, if the cash-in-advance constraint does not bind for the consumers born in island k (if $\mu_t^k{=}0$), then the (gross) lending rate in that island equals unity ($R_{t+1}^h{=}1$). And conversely, if the cash-in-advance constraint binds (if $\mu_t^k>0$), then IOUs receivable earn a positive rate of return ($R_{t+1}^h{>}1$) even though fiat money is valued.[5]

Section 1 of the appendix characterizes a stationary equilibrium, namely a constant equilibrium allocation supported by constant relative prices, interest rates and inflation rate. Let q^k be the real money balances demanded by the young born in an island of type k in such equilibrium. Then the clearing of the money market in each type of island at t requires

$$\theta p_t^S q^S = p_t^L q^L = \overline{M}_t/N_t \equiv M_0(z/n)^t \quad , \quad k{=}S,L \tag{4.3}$$

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[5] A cash discount would be equivalent to $R_{t+1}^h > 1$.

Imposing (4.3) for periods t and $t+1$ and taking ratios of the demand and supply for money in both periods, we obtain that the inverse of the inflation rate in the stationary equilibrium is $\pi \equiv p_t/p_{t+1} = n/z$ for all t.

It is proved in the appendix that in a neighborhood of $\theta=1$, the Lagrange multiplier μ^k is non-increasing in π, and strictly decreasing if $\mu^k>0$. Intuitively, as the inflation rate rises (as π drops), individuals try to reduce their holdings of real cash balances. At some point the cash-in-advance constraint starts to bind, and when that happens nominal interest rates on the IOU contracts rise above unity. As inflation keeps rising, the cash-in-advance constraint becomes more and more binding, and μ^k increases.[6]

5. Equilibrium Contracts With Aggregate Randomness

Aggregate output shocks We now discuss the desirability of nominal contracts extends when there is randomness in aggregate output or in the fiat money stock. As an example of an economy with randomness in aggregate output, suppose that the (gross) rate of population growth, $n_t = N_t/N_{t-1}$, is an always positive i.i.d. random variable, while the stock of fiat money grows at the constant rate z, like in the previous sections. In this section we consider equilibria in which the cash-in-advance constraint is not binding.

[6] In this economy, a liquidity crunch (i.e., a more binding cash-in-advance constraint) is associated with higher (and not lower) real money balances. Intuitively, as the cash-in-advance constraint becomes more binding, young households reduce their demand of consumption loans. The equilibrium counterpart is that more goods are sold for cash (to the old) and less for credit (to the young) so that real money balances increase. This feature of the model, which is not entirely implausible, is due to the fact that the cash-in-advance constraint binds the repayment of consumption loans, rather than the purchase of consumer goods, as in the familiar models of Lucas (1980) and Svensson (1983). We conjecture that, with a labor-leisure choice when young, the real money balances will no longer always increase with expected inflation, even though μ^k would.

Equilibria with binding cash-in-advance constraints are studied in the next section.

The stationary distribution of growth rates of population ensures the existence of stationary equilibrium like the one described in the previous section. In particular, repeating the argument of the previous section, the inverse of the inflation rate between t and t+1 is:

$$\pi_t^k \equiv p_t^k/p_{t+1}^k = \frac{M_t/q^k N_t}{M_{t+1}/q^k N_{t+1}} = n_{t+1}/z. \tag{5.1}$$

In addition, the equilibrium relative price between the commodities sold in large and small islands is unaffected by the aggregate shock, since by (4.3) it is given by: $p_t^L/p_t^S=\theta q^S/q^L$ for all t. Thus, the risk to the value of fiat money posed by the aggregate population shocks strikes money-holders belonging to the same generation in the same way, irrespective of where they are born. A large realization of n_{t+1} makes households from both large and small islands of generation t better off by increasing the value of their real money balances, while for a small realization of n_{t+1} both members of generation t are worse off. Therefore this aggregate risk cannot be insured away, but it can only be shared between households born in different islands and belonging to the same generation.

If all islands are alike, (if θ=1), then all members of the same generation are identical. In this case optimal risk sharing requires that everybody faces exactly the same risk. Since every individual is a party to two opposite contracts, optimal risk sharing imposes only the general requirement that exactly the same contingencies be incorporated in every contract. When this requirement is satisfied, the effect of the aggregate shock on the two IOU contracts written by every individual offset each other exactly; the aggregate shock then only affects individual welfare by changing the purchasing power of real money balances, and since everybody within a generation is identical, this is the same for all.

Thus, nominal contracts are optimal, as are many other contracts.

When islands differ in size (if $\theta<1$), however, individuals born in different islands have net financial wealth of different size. Nominal IOU contracts, in this case, expose the parties to a risk exactly proportional to their net financial wealth. Whether this form of risk sharing is optimal or not depends on how risk aversion changes with wealth. As shown in the following proposition, if individual preferences exhibit the same degree of constant relative risk aversion, then risk should be borne in proportion to wealth, and nominal contracts are indeed optimal:

Proposition 2: If all households exhibit the same degree of constant relative risk aversion, fixed nominal contracts are optimal even in the presence of shocks to the aggregate rate of population growth.

Proof: As before, an optimal contract maximizes the expected utility of those born on a small island for a given expected utility of those born on a large island. Repeating the procedure outlined in the proof of Proposition 1, we can write the first order condition of this maximum problem as:

$$\begin{aligned} &V'\left[\frac{F^S(\omega,n)}{p^S(n)}\right]/p^S(n) + \left[\frac{F^S(\omega,n)}{p^L(n)}\right]/p^L(n) \\ &= \theta\lambda\left\{V'\left[\frac{F^L(\omega,n)}{p^S(n)}\right]/p^S(n) + V'\left[\frac{F^L(\omega,n)}{p^L(n)}\right]/p^L(n)\right\} \end{aligned} \tag{5.2}$$

where now $F^k(\cdot)$ and p^k are also contingent on the realization of the population shock, n. With a constant relative risk aversion utility function, $V(c)=\frac{c^{1-\alpha}-1}{1-\alpha}$, $\alpha>0$, the optimality condition (5.2) becomes:

$$\frac{F^S(\omega,n)^{-\alpha}}{P^S(n)^{(1-\alpha)}} + \frac{F^S(\omega,n)^{-\alpha}}{P^L(n)^{(1-\alpha)}} = \theta\lambda\left[\frac{F^L(\omega,n)^{-\alpha}}{P^S(n)^{(1-\alpha)}} + \frac{F^L(\omega,n)^{-\alpha}}{P^L(n)^{(1-\alpha)}}\right] \tag{5.3}$$

which simplifies to

$$\frac{F^S(\omega,n)}{F^L(\omega,n)} = \theta\lambda^{(-1/\alpha)} \tag{5.4}$$

Condition (5.4) is met for $F^S(\omega,n)=F^S$ and $F^L(\omega,n)=F^L$, the case of nominal contracts. Q.E.D.

Equation (5.4) reveals that optimal risk sharing under constant relative risk aversion requires that the ratio between the wealth of two members of the same generation born in different types of islands must be the same for all realizations of the aggregate shock. Because some fraction of a household's wealth already lies in its fiat money balances, the simplest way to achieve a proportionate exposure to aggregate risk is to denominate all wealth (net IOUs as well as fiat money) in nominal terms. Hence the optimality of a system of nominal contracts.

Monetary shocks: Suppose now that the rate of growth of fiat money is a serially uncorrelated random variable, that has a time t realization we denote as z_t. We immediately have:

Proposition 3: If all households exhibit the same degree of constant relative risk aversion, fixed nominal contracts are optimal even in the presence of shocks to the rate of growth of fiat money.

To understand this proposition, it is simply necessary to see that from (5.1), shocks to z have the same but opposite effects on the inflation rate as shocks to n. A formal proof of Proposition 3 would therefore follow the steps of Proposition 2.

Risk sharing with real assets Our model is a bit special in its insistence that an optimal system of contracts requires that all contracts be fixed in nominal returns. This results from the model's assumption that fiat money is the only source of outside wealth of the old. Suppose instead that households will receive fixed endowments of real goods when old.[7] If these endowments are not exactly proportionate to a household's equilibrium net wealth, nominal debt alone will no longer proportionately expose households to aggregate risk. However, the simple combination of nominal debt and equity in real endowments could now be used to let each household hold real and net nominal assets in the same proportion—thus exposing their portfolios to the same proportionate risk.

To make this a bit more precise, let X^k and Y^k denote the real stocks of the odd and even island goods owned in equilibrium by an old household of type k and let $P^o(\omega,n)$ and $P^e(\omega,n)$ represent the nominal price of goods on odd and even islands respectively as functions of ω and n. We will continue to let $F^k(\omega,n)$ represent the net nominal wealth (initial money balances plus net nominal IOUs payable in money) of an old island of type k. Total wealth of an old household of type k in nominal terms is now

$$F^k(\omega,n) + P^o(\omega,n)X^k + P^e(\omega,n)Y^k \quad . \tag{5.5}$$

Suppose that households write nominal contracts ($F^k(\omega,n)=F^k$) and exchange equity shares of the two types of endowments so that the ratios of real equity to net nominal wealth are the same positive constant φ_x and φ_y in all household portfolios; i.e.,

$$\frac{X^k}{F^k} = \varphi_x \quad , \qquad \frac{Y^k}{F^k} = \varphi_y \ . \tag{5.6}$$

Then the ratio of the total wealth of small to large island households is

$$\frac{(1+ \ \varphi_x P^o(\omega,n) \ + \ \varphi_y P^e(\omega,n))F^S}{(1+ \ \varphi_x P^o(\omega,n) \ + \ \varphi_y P^e(\omega,n))F^L} = \frac{F^S}{F^L} \tag{5.7}$$

which is a constant for all (ω,n), thus satisfying the requirement for optimal risk sharing.

7 We leave aside the question of how much endowments could be sold to acquire the fiat money desired by the old when they travel. For a simple, if arbitrary, example, suppose that the endowments can be sold by the old to the young but cannot be used to repay consumption loans (so that the cash-in-advance constraint may still bind).

In this way portfolios containing two simple assets, equity and nominal debt, can achieve optimal risk sharing against both aggregate (n) and relative (ω) risk.

Other advantages of nominal contracts If preferences do not exhibit constant relative risk aversion, then nominal contracts no longer provide optimal risk sharing. However, nominal contracts are obviously simpler and easier to enforce. Moreover, the risk sharing offered by contingent contracts can have at best only a second order advantage over nominal contracts since both parties to nominal contracts share aggregate risk in proportion to their wealth. Therefore, it is easy to imagine economies in which some cost of incorporating contingencies will outweigh the benefits.

One potential cost is obvious: the cost of observing or verifying the shocks to population or money stock. Both population and the money stock are aggregate variables; thus they may not be automatically revealed to individuals as would, say, an individual's own endowment of goods. It would be natural to assume then that aggregate variables may be observed by an individual only at some cost. A useful feature of nominal contracts in our model is therefore that they share risk while requiring no information about aggregate variables, in contrast to contracts contingent on the state. Any costs incurred in observing the state represents a deadweight loss to the contracting parties, which may not be offset by the second order benefits of risk sharing.[8]

The next section shows that when cash-in-advance constraints bind, there is yet another cost in incorporating contingencies into contracts.

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[8] In many economies, prices reveal all the information about aggregate variables that individuals require. In our economy, however, individuals must repay their IOUs before the market exchange of the money owned by the arriving old for the endowments of the young. Hence, IOUs must be settled before prices reveal their information about the state of the world. Contracts payable in a fixed quantity of goods also require information about economic aggregates. Because the settling of IOUs requires payment in money, contracts requiring a payment worth a fixed basket of goods must evaluate the price of these goods to determine the money owed. Since the price is not yet directly observable, it must be inferred from information about the population and money stock. Therefore, contracts denominated in fixed real terms are the same as money contracts contingent on the state, and require just as much (potentially costly) information about aggregate variables.

6. Nominal Contracts and Binding Cash–in–Advance Constraints

The propositions stated to this point in the paper have all been restricted to equilibria in which the cash-in-advance constraints do not bind. We delayed our presentation of the case of binding cash-in-advance constraints because nominal contracts are more likely to be optimal in this case.

Since contracts in our model economy must be settled using fiat money, contingent contracts require that agents hold enough fiat money to make the maximum payment specified. A contingent contract with the same expected payment as some noncontingent contract will therefore require the holding of more fiat money. When cash-in-advance constraints bind, there is a utility cost to the holding of additional money balances. If this cost exceeds the benefit of the risk sharing through contingent contracts, a nominal contract will be optimal despite an opportunity for mutually beneficial risk sharing. Our reasoning is developed more formally in the proof of the following proposition.

Proposition 4: There exist economies with aggregate randomness and non-constant relative risk aversion for which nominal contracts are the optimal contract form because of binding cash-in-advance constraints.

Proof: To prove proposition 4, we first characterize the Kuhn–Tucker conditions defining an optimal contract under aggregate uncertainty and binding cash-in-advance constraints. We then present a class of preferences whose deviation from constant relative risk aversion is a continuous function of some parameter γ. We then show that there exist some values of γ such that the conditions for the optimal contract are met when nominal returns on contracts are not contingent on the state.

We take for granted that the optimal contract is independent of ω by the reasoning already explored in Propositions 1 and 2, so that we may concentrate on the implications of the aggregate randomness in n.

To keep the notation as simple as possible, we present the proof of the simple discrete case in which n takes only two values, n_1 and n_2, and takes each value with equal probability. In this two-state case, let R^S and $R^L+\epsilon^L$ denote the nominal rates of return in state 1, and let $R^S+\epsilon^S$ and R^L denote the nominal rates of return in state 2. When cash-in-advance constraints bind, it would make no sense to write contracts that increase the rate of return paid to and by the same island in some state since that would increase the cash balances of both types of islands. For this reason we restrict ϵ^S and ϵ^L to be of the same sign, which we assume is non-negative without loss of generality. Here we are essentially defining n_1 to be the state in which optimal risk sharing requires an extra payment to small island agents.

An optimal contract must then maximize over $\epsilon^S \geq 0$ and $\epsilon^L \geq 0$ a weighted average of agents' expected utilities, constrained by the cash-in-advance constraints. By (2.3) the Lagrangean of this problem in a stationary equilibirum is:

$$\begin{aligned}
&\frac{1}{4}\sum_{j=S,L} V\left[\frac{m^S- R^S b^S+(R^L+\epsilon^L)a^S}{p^j(n_1)}\right] \\
&+\frac{1}{4}\sum_{j=S,L} V\left[\frac{m^S-(R^S+\epsilon^S)b^S+ R^L a^S}{p^j(n_2)}\right] \\
&+\lambda\left\{V^*-\frac{1}{4}\sum_{j=S,L} V\left[\frac{m^L-(R^L+\epsilon^L)b^L+ R^S a^L}{p^j(n_1)}\right]\right. \\
&\left.-\frac{1}{4}\sum_{j=S,L} V\left[\frac{m^L-(R^L b^L+(R^S+\epsilon^S)a^L}{p^j(n_2)}\right]\right\} \\
&+\mu_1\left[m^L-(R^L+\epsilon^L)b^L\right] + \mu_2\left[m^S-(R^S+\epsilon^S)b^S\right] \qquad (5.1)
\end{aligned}$$

where μ_1 and μ_2 are the Lagrange multipliers of the relevent cash-in-advance constraints. Notice that the cash-in-advance constraint binds only for the largest nominal interest rate paid by each person.

Differentiating by ϵ^L yields the following Kuhn-Tucker conditions defining an optimal contract

$$\sum_{j=S,L} V'\left[\frac{m^S - R^S b^S + (R^L+\epsilon^L)a^S}{p^j(n_1)}\right] / p^j(n_1)$$
$$-\lambda\theta \sum_{j=S,L} V'\left[\frac{m^L - (R^L+\epsilon^L)b^L + R^S a^L}{p^j(n_1)}\right] / p^j(n_1) - \mu_1 \leq 0 \qquad (5.2)$$

with $\epsilon^L=0$ <u>if the strict inequality obtains.</u> Differentiating with respect to ϵ_S reveals a similar condition involving n_2, μ_2 and reversed superscripts L and S.

The first two terms of (5.2) represent the net marginal social benefit of risk sharing in state 1. The marginal cost is represented by μ_1, the marginal utility cost of holding fiat money with a binding cash-in-advance constraint.

We saw in Proposition 2 that there is no benefit to risk sharing when agents have the same relative risk aversion. It follows that as preferences approach constant relative risk aversion, the value of risk sharing goes to zero. The marginal cost of holding money balances (μ) however, does not necessarily approach zero as preferences approach constant relative risk aversion. That is, the cash-in-advance constraint will be binding for a large set of constant relative risk aversion preferences. Therefore, it is easy to envisage preferences yielding a small enough value of risk sharing or a large enough value of μ such that the condition (5.2) holds with a strict inequality. In such a case, the optimal values of ϵ^L and ϵ^S equal zero, implying that the optimal contract is nominal.

It may help at this point to consider the class of preferences given by $V(c)=\frac{(c+\gamma)^{1-\alpha}-1}{1-\alpha}$, for $\alpha>0$. The function and its first derivative are continuous fractions of the parameter γ. Since the function represents constant relative risk aversion when $\gamma=0$, we may therefore consider γ as a measure of the deviation from constant relative risk aversion for preferences in this class. Therefore, the statement that there exists an economy with non-constant relative risk aversion but for which the optimal contract is nominal can be restated as a statement that for any $\mu>0$ we can choose a non-zero γ sufficiently close to 0 such that the first order condition (5.2) with respect to ϵ holds with strict inequality.

Q.E.D.

Summarized, the proof to Proposition 4 relies on two points. First, the benefit of contingent contracts goes to zero as the difference in relative risk aversion of the two parties goes to zero. Second, the cost of adding contingencies to contracts, which is the marginal utility cost of holding money, μ, may well be strictly positive. It follows directly that the costs will exceed the benefits in some neighborhood of constant relative risk aversion. In this neighborhood nominal contracts are optimal.

It should be noted that a similar proof of the optimality of nominal contracts exists if the difference in relative risk aversion is negligible because the ex ante wealth of households on large islands is sufficiently close to that on small islands, for any given utility function. A similar proposition might specify some cost φ of observing the state as the cost of contingent contracts (replacing μ, the cost of holding extra money), along the lines discussed at the end of the previous section.

An interesting feature of the optimal contracts described in the proof to Proposition 4 is that <u>nominal contracts are more likely to be optimal the greater is the</u>

marginal utility of money, μ. As shown in the appendix, μ generally increases as expected inflation rises, implying that, other things being equal, we are more likely to see nominal contracts in times of high expected inflation. This may contribute to explain why nominal contracts and the use of money as a unit of account are not completely abandoned even in times of hyperinflation, when both price level uncertainty and expected inflation are very high. The cost of writing contingencies into contracts payable in fiat money increases with (expected) inflation because of the extra money balances that must be held. During a hyperinflation both expected inflation and price level uncertainty rise, with ambiguous effects on the optimality of nominal contracts.

7. Concluding Remarks

We conclude the paper with a general observation on how the optimality of nominal contracts relates to some fundamental properties of a monetary economy. In the general equilibrium model of the previous pages, fiat money coexists with other assets, it can be dominated in rate of return, and serves as a medium of exchange, as a means of payment and, under general cirumstances, as a unit of account. It is a medium of exchange between agents that belong to different generations, because they meet only once in their lifetime. It is a means of payment because no centralized market exists in which all contracts can be simultaneously cleared (or, equivalently, the velocity of circulation of contracts is not infinite). Thus, when contracts are settled, the creditor demands to be paid in fiat money knowing that he can exchange money for commodities later on. Finally, under the conditions discussed in the previous sections, the terms of the contract are expressed in units of fiat money (i.e., they are fixed nominal contracts). Hence money is also a unit of account.

These three roles of money are linked to each other, and are essential to understanding why nominal contracts may be optimal even neglecting computational or information gathering costs.[9] Money is used as a means of payment precisely because it is also a medium of exchange. And being a means of payment, it is more likely to be used as a unit of account. (i.e., contracts are expressed in fixed nominal terms). The reason is that when money serves as a means of payment and is dominated in rate of return, cash-flow risk is important alongside with purchasing power risk. One way to reduce the cash-flow risk is to have the means of payment also serve as a unit of account.

[9] Niehans (1978), Fama (1983) and White (1984), among others, refer to these computational aspects.

Appendix

1. The Stationary Equilibrium

Imposing the equilibrium conditions (2.5) and (2.6), equation (4.2a) can be rewritten as:

$$2U'(\theta^k(1-q^h)) = R^k R^h \pi[V'(f^k\pi)q^k + V'(f^k\pi q^k)] \quad , \qquad h,k=S,L \text{ and } h\neq k \qquad (A.1)$$

where $\theta^k=\theta$ if k=L and $\theta^k=1/\theta$ if k=s, q^k denotes real money balances in an island of type k, π is the inverse of the equilibrium inflation rate, d^k is the equilibrium relative price of the good produced in an island of type k in terms of the price of the good produced in h, for h≠k, R^k and R^h are the nominal interest rates, and f^k, the net financial wealth of those born in an island of type k expressed in units of commodity k, is:

$$f^k = R^h(1-q^k) - \theta^k d^h R^k(1-q^h) + q^k \ . \qquad (A.2)$$

If the cash-in-advance constraint is binding for those born in k, then

$$R^k = q^k/[d^h\theta^k(1-q^h)] \qquad (A.3)$$

and (A.2) reduces to $f^k = R^h(1-m^k)$, while the Lagrange multiplier μ^k is obtained from the stationary version of (4.2b):

$$2\mu^k = (R^h-1)\pi[V'(R^h(1-q^k)\pi) + V'(R^h(1-q^k)\pi d^k)d^k] \ . \qquad (A.4)$$

If equations (A.1)–(A.4) have a solution, then a stationary equilibrium exists. For the remainder of this section, we consider the stationary equilibrium in a neighborhood of θ=1. We therefore drop the superscripts k and h, since all islands are alike.

The cash-in-advance constraint (2.4) can be rewritten as:

$$R \leq q/(1-q) \ . \qquad (A.5)$$

When (A.5) is not binding, R=1 and real money balances are determined by (A.1), which simplifies to

$$U'(1-q) - \pi V'(q\pi) = 0 \ . \tag{A.6}$$

Equation (A.6) implicitly defines equilibrium real money balances as a function of π: $q=q^*(\pi)$. Under the assumption that $V(\cdot)$ has a relative risk aversion coefficient less than 1, $q^*_\pi>0$, that is, real money balances decrease as inflation increases (as π drops). This is equivalent to saying that the substitution effect dominates the income effect.

If, on the other hand, (A.5) is binding, then real money balances are determined by

$$U'(1-q)-\pi V'(q\pi)(\frac{q}{1-q})^2 = 0 \tag{A.7}$$

which is obtained by combining (A.1) and (A.5). In this equilibrium, real money balances are again a function of π, $q=Q^*(\pi)$. But here, under the assumption that both $U(\cdot)$ and $V(\cdot)$ have a relative risk aversion coefficient smaller than unity, $Q^*_\pi<0$. That is, higher inflation increases real money balances in equilibrium. The intuition is that when the cash-in-advance constraint binds, higher expected inflation induces households to reduce consumption when young. As everybody does that, sales against IOUs are reduced and sales against cash (and hence real money balances) increase. The key to understanding this result is that the cash-in-advance constraint here binds the repayment of consumption loans, rather than directly the purchase of consumer goods. Finally, note that by (A.5), the interest rate R also rises as π drops, since it moves in the same direction as m.

Since by (A.5) the cash-in-advance constraint is just binding at $q=1/2$, we can summarize the foregoing discussion in the diagrams of Figures 1 and 2, where equilibrium real money balances are shown as a non-monotonic function of the inflation rate and the interest rate is first constant and then rising with $1/\pi$. The threshold inflation rate $1/\pi^*$ such that the cash-in-advance constraint just binds is defined implicitly by the condition obtained from (A.6):

$$U'(1/2) - \pi^* V'(\pi^*/2) = 0 \quad . \tag{A.8}$$

Finally, combining (A.1) and (A.4), the Lagrange multiplier μ on (A.5) can be written as:

$$\mu = \frac{R-1}{R^2} U'(1-q) \quad . \tag{A.9}$$

Thus, for R=1, μ=0. Whereas for R>1, μ is a function of π. Under the same condition mentioned above, that $U(\cdot)$ and $V(\cdot)$ have a relative risk aversion coefficient smaller than unity, and differentiating (A.9) with respect to π, it is possible to show that μ is decreasing in π, and strictly decreasing if $1/\pi > 1/\pi^*$, i.e., the cash-in-advance constraint becomes more binding as inflation increases.

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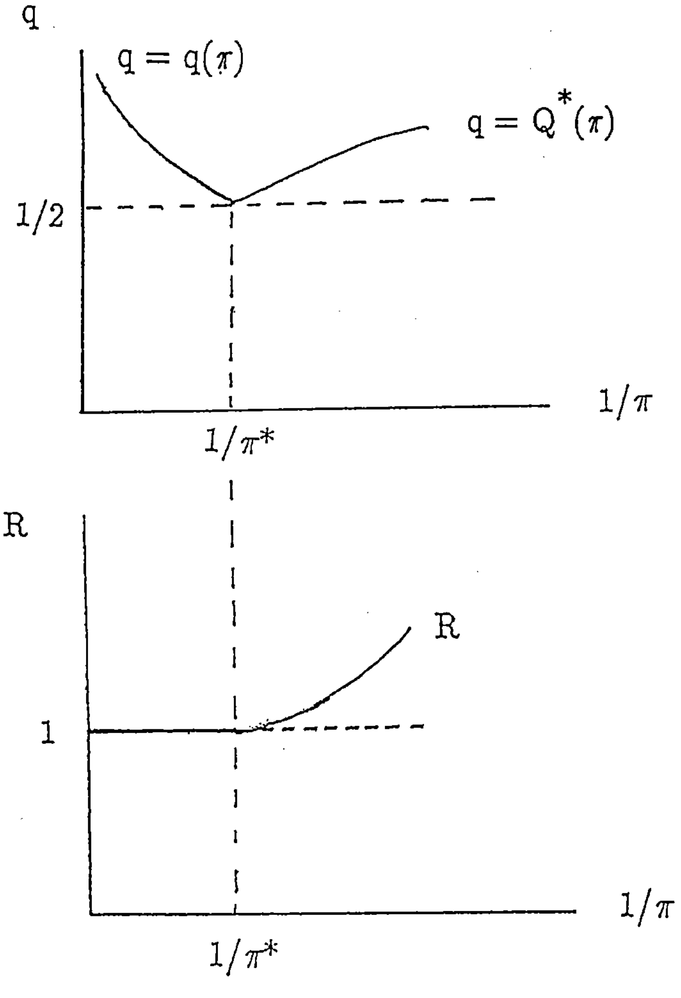
q
q = q(π)
q = Q*(π)
1/2
1/π
1/π*
R
R
1
1/π
1/π*