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DEEP STRUCTURAL EXCAVATION?
A CRITIQUE OF EULER EQUATION METHODS

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Abstract

Rational expectations theory instructs empirical researchers to uncover the values of 'deep' structural parameters of preferences and technology rather than the parameters of decision rules that confound these structural parameters with those of forecasting equations. This paper reevaluates one method of identifying and estimating such deep parameters, recently advanced by Hansen and Singleton, that uses intertemporal efficiency expressions (Euler equations) and basic properties of expectations to produce orthogonality conditions that permit parameter estimation and hypothesis testing. These methods promise the applied researcher substantial freedom, as it is apparently not necessary to specify the details of dynamic general equilibrium to study the behavior of a particular market participant. In this paper, we demonstrate that this freedom is illusory. That is, if there are shifts in agents' objectives which are not directly observed by the econometrician, then Euler equation methods encounter serious identification and estimation difficulties. For these difficulties to be overcome the econometrician must have prior knowledge concerning variables that are exogenous to the agent under study, as in conventional simultaneous equations theory.

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1. Introduction

In current macroeconomic theories, the decisions of economic agents are viewed as solutions to dynamic choice problems under uncertainty. The focus on dynamic rational expectations models rules out applications of some conventional econometric methods for reasons detailed by Lucas [14], thereby spawning a rapidly growing field of econometric research.[1] The goal of this new econometrics is to estimate 'deep' parameters of preferences and technology that will be structural, i.e., invariant to a particular class of interventions.

One important line of this research--culminating in Hansen and Singleton [9]--develops methods of parameter estimation and hypothesis testing from two basic properties of dynamic choice under rational expectations.[2] First, efficient decisions under uncertainty require that infinitesimal marginal changes in current actions yield no expected gains or losses. Second, rational expectations implies that realized gains or losses differ from expected levels by a random variable that has zero mean and is uncorrelated with any information available at the decision date. These two properties jointly imply substantial restrictions on data that can be used to estimate

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[1] See Lucas and Sargent [18] for a good overview of the economic and econometric theory developed in the initial stage of the rational expectations revolution.

[2] The initial recognition that expectation errors under rational expectations should be uncorrelated with available information seems to have arisen in "efficient markets" tests in finance (e.g., Fama [2]). In a macroeconomic context Kennan [11], Hayashi [10], and Goodfriend [3] use econometric strategies that are best viewed as estimating linear stochastic Euler equations for, respectively, dynamic factor demand, consumption, and cash balances. Nonlinear Euler equations have been estimated by Hall [5,6], Hansen and Singleton [], and Mankiw, Rotemburg, and Summers [19]. Kennan [11], Goodfriend [10], and Mankiw, Rotemburg, and Summers [19] provide discussions of the consequences of inexact specification or objective function "shocks" in their particular contexts, but do not discuss the general difficulties with which we are concerned here.

parameters and test hypotheses.

This econometric approach is alluring on theoretical and empirical grounds. Theoretically, the intertemporal efficiency conditions resulting from dynamic optimization--mathematically, discrete time stochastic Euler equations--embody the main content of a specific economic theory of individual behavior in a compact and elegant manner. On the other hand, applied researchers investigating one component of a general equilibrium system are reticent to have results depend too much on detailed auxiliary hypotheses, i.e., one does not wish to produce a full scale stochastic representation of the economy to study a particular element such as labor supply. Empirically, the methods put forward by Hansen and Singleton [9] promise this sort of freedom, since the Euler equations must hold for a wide class of circumstances that individual agents view as beyond their control.[3]

Yet, from the perspective of both rational expectations theory and the standard econometric theory of simultaneous systems, it seems surprising that such freedom is feasible. In this paper, we demonstrate that it is not generally so, but rather requires the strong a priori belief that the researcher has exactly specified and can accurately observe all of the relevant variables that shift agents' objectives. Once these strong assumptions are dropped, the researcher must impose other prior restrictions

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[3] Hansen and Singleton's claims for their method are quite strong: "The procedures we propose do not require a complete, explicit representation of the economic environment and, in particular, do not require strong a priori assumptions about the nature of the forcing variables." (1269). Yet, Hansen and Singleton recognize the difficulties that we stress here: "More generally, our approach to estimation is appropriate for any model that yields implications of the form (2.1) [which is the dynamic first-order condition in the Hansen/Singleton setup] with X [the vector of variables chosen by the agent or viewed as beyond his control] observed. This latter qualification does rule out some models in which the implied Euler equations involve unobservable forcing variables." In this discussion we take the view that the class of models so ruled out is large.

on the nature of the dynamic economic system to identify and estimate parameters of interest. Typically, these will take the form of exclusion restrictions, as in the conventional simultaneous equations model.

The organization of the remainder of the paper is as follows. In section 2, we discuss Euler equations for a representative producer and consumer, illustrating econometric applications of orthogonality conditions based on these equations and rational expectations. In section 3, we illustrate difficulties arising in Euler equation methods when there are shocks to agents' objectives not observed by the econometrician. In an extreme case, an econometrician mistakenly concludes that the intertemporal elasticity of substitution in production is the comparable preference parameter, i.e., intertemporal elasticity of substitution in consumption. Using rates of return determined in dynamic general equilibrium, we illustrate how prior knowledge on excluded variables can produce consistent estimates of some behavioral parameters. Section 4 is a summary and conclusions.

2. Euler Equations and Econometrics

In this section, we provide an informal derivation of intertemporal efficiency conditions for a representative producer and consumer, deferring equilibrium analysis to the appendix. Under suitable conditions, these Euler equations can be used to estimate deep structural parameters of technology and preferences. Our model economy is cast in discrete time (indexed by t).

Producer

The representative producer seeks to maximize the date t market value of his firm, given a (state contingent) discount rate prevailing from t to t+1,

r_{t+1}. The producer simultaneously engages in two activities at date t, supply of final consumption goods (c_t) and reproduction of an intermediate good. The nonstorable consumption good is produced according to a stochastic diminishing returns production function that requires the intermediate good as input (in amount z_t).

$$(2.1) \qquad c_t = \lambda_t z_t^m \quad , \quad 0 < m < 1.$$

The productivity level λ_t follows a strictly positive markov process.

The intermediate good is generated from a stochastic constant-returns-to-scale production process that converts each unit of intermediate good not consumed at date t into $\phi_{t+1} > 0$ units at date t+1. Denoting intermediate good output by y_t, it follows that

$$(2.2) \qquad y_{t+1} = \phi_{t+1} (y_t - z_t).$$

With an appropriately defined state contingent rate of return (see appendix), the firm selects its consumption supply and intermediate goods production so as to maximize the following market value expression (in current commodity units),

$$(2.3) \qquad V_t = c_t + \underset{t}{E}\{V_{t+1}/(1+r_{t+1})\} = \underset{t}{E}\{\sum_{j=0}^{\infty} \rho_{tj}\, c_{t+j}\},$$

where $\rho_{t0} = 1$, $\rho_{t1} = (1+r_{t+1})^{-1}$ and $\rho_{tj} = (1+r_{t+j})\rho_{t,j-1}$ for $j \geq 1$.

An efficient contingency plan for commodity supply and intermediate good production requires that

$$(2.4) \qquad 1 = \underset{t}{E}\{\phi_{t+1}\, z_{t+1}^{m-1}\, \lambda_{t+1}/(1+r_{t+1})\}/\{z_t^{m-1}\, \lambda_t\},$$

where the left-hand side is marginal revenue from date t supply of consumption goods and the right-hand side is the discounted marginal cost of supply. The latter includes three components: the discount rate; the units of date t+1 consumption goods forgone by reducing investment by one unit ($\phi_{t+1} z_{t+1}^{m-1} \lambda_{t+1}$); and the marginal consumption good product of the investment good at date t, $z_t^{m-1} \lambda_t$.

For our purposes, it will be convenient to rewrite (2.4) using the intertemporal elasticity of substitution in production (defined as $\mu = m/(1-m)$) and making the substitution $z_t = [c_t/\lambda_t]^{1/m}$,

$$(2.5) \qquad 1 = E_t\{\phi_{t+1}(x_{t+1}^s)^{-1/\mu}[\lambda_{t+1}/\lambda_t]^{(1+\mu)/\mu}/(1+r_{t+1})\},$$

where $x_{t+1}^s = c_{t+1}^s/c_t^s$ is the ratio of consumption supply in adjacent periods.

Consumer

The representative consumer seeks to maximize discounted expected utility,

$$(2.6) \qquad E_t\{\sum_{j=0}^{\infty} \beta^j c_{t+j}^{\alpha} \theta_{t+j}/\alpha\}$$

where $0 < \beta < 1$ and $\alpha \leq 1$. As above, we define the parameter $\sigma \equiv (1/1-\alpha)$, the intertemporal elasticity of substitution in consumption. θ_t is a preference shift variable following a positive markov process.

Our representative consumer receives an income flow from owning the representative firm, c_t^s, and may borrow or lend at the stochastic interest rate, r_{t+1}. Wealth (a_t) thus accumulates according to the stochastic difference equation

$$(2.7) \qquad a_{t+1} = (1+r_{t+1})(a_t + c^s_t - c^d_t),$$

where c^d_t is consumption demand at date t. An efficient contingency plan for consumption demand and asset accumulation requires that

$$(2.8) \qquad 1 = \underset{t}{E}\{(1+r_{t+1})\beta(x^d_{t+1})^{-1/\sigma}(\theta_{t+1}/\theta_t)\},$$

where we have defined $x^d_{t+1} \equiv c^d_{t+1}/c^d_t$.[4] That is, efficient consumption over time equates the expected lifetime gain from postponing a date t unit of consumption, i.e., $\beta(1+r_{t+1})(c^d_{t+1})^{\alpha-1}\theta_{t+1}$, with the marginal utility cost $(c^d_t)^\alpha\theta_t$.

Orthogonality Conditions

Consider forming the expectation errors ε^s_{t+1} and ε^d_{t+1} by replacing the right hand sides of (2.5, 2.8) with realized variables, i.e.,

$$(2.9a) \qquad \varepsilon^s_{t+1} \equiv \{\phi_{t+1}(x^s_{t+1})^{-1/\mu}[\lambda_{t+1}/\lambda_t]^{(1+\mu)/\mu}/(1+r_{t+1}) - 1\}$$

$$(2.9b) \qquad \varepsilon^d_{t+1} \equiv \{\beta(x^d_{t+1})^{-1/\sigma}[\theta_{t+1}/\theta_t](1+r_{t+1}) - 1\}$$

Then, expressions (2.5) and (2.8) have the forms $\underset{t}{E}\{\varepsilon^s_{t+1}\} = 0$ and $\underset{t}{E}\{\varepsilon^d_{t+1}\} = 0$, respectively.

If k_{jt} is an element of the information set which conditions $\underset{t}{E}$, then (2.5) and (2.8) require

$$(2.10) \qquad \underset{t}{E}\{\varepsilon^s_{t+1}\bullet k_{jt}\} = \underset{t}{E}\{\varepsilon^d_{t+1}\bullet k_{jt}\} = 0$$

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[4] If there are many assets, Hall [6] and Hansen and Singleton [9] point out that then a version of (2.8) holds for each asset. For example, if we consider adding a certain one period yield $(1+r_{t+1})$, then it follows that (2.8) holds with $(1+r_{t+1})$ replacing $(1+r_{t+1})$.

Alternatively, converting to an unconditional expectations operator which is more useful for empirical purposes, an iterated expectations argument yields

$$(2.11)\qquad E\{\varepsilon^{s}_{t+1}\bullet k_{jt}\} = E\{\varepsilon^{d}_{t+1}\bullet k_{jt}\} = 0.$$

Expressions (2.10) and (2.11) state that expectation errors are uncorrelated with available information, a basic property of any rational expectations scheme. Hansen and Singleton [9] use such orthogonality conditions as the basis for method of moments estimators of structural parameters. In fact, to illustrate the application of their estimation strategy, they estimate preference parameters analogous to β and σ for a representative household.

3. Identifying and Estimating Demand Parameters

We now consider several potential econometric investigations into our dynamic general equilibrium model. Throughout, we focus on the problem of identifying and estimating the preference parameters β and σ, although the symmetry of the model implies that these results can be applied to production parameters as well. Since observed values of interest rates and quantities will be determined in general equilibrium (see appendix), we drop superscripts indicating supply or demand side consumption quantities.

The state contingent real interest rate (3.1) clears one regime of markets that decentralize our economy's optimal planned allocations (see appendix).

$$(3.1)\qquad (1+r_{t+1}) = \phi_{t+1}(x_{t+1})^{-1/\mu}(\lambda_{t+1}/\lambda_{t})^{(1+\mu)/\mu}.$$

This interest rate is the realized marginal rate of transformation in production and, hence, the firm's Euler equation holds exactly.

No Demand Shifts

Following Hansen and Singleton, we momentarily assume the absence of preference disturbances, setting $\theta_t = \bar{\theta}$ for all t. Further, we assume that an econometrician seeking to estimate preference parameters chooses as instruments a constant and single variable k_t. Now, consider forming the error term η^d_{t+1}, using specified values of the parameter vector, $\hat{\beta}$ and $\hat{\sigma}$, that need not equal the true parameter values β and σ.

$$\begin{aligned}\eta^d_{t+1} &\equiv \{\hat{\beta} x_{t+1}^{-1/\hat{\sigma}}(1+r_{t+1}) -1\} \\ &= \varepsilon^d_{t+1} + \{\hat{\beta} x_{t+1}^{-1/\hat{\sigma}}(1+r_{t+1}) - \beta x_{t+1}^{-1/\sigma}(1+r_{t+1})\}.\end{aligned}$$

The error term η^d_{t+1} consists of the true forecasting error and a "misspecification error" arising from the choice of $(\hat{\beta},\hat{\sigma})$. The econometrician selects estimators $(\hat{\beta},\hat{\sigma})$ guided by the theoretical requirements that $E(\eta^d_{t+1}\bullet 1) = 0$ and $E(\eta^d_{t+1}\bullet k_t) = 0$. Consistent estimators result from requiring the fulfillment of analogous sample orthogonality conditions.[5] That is, theory tells us that $E(\varepsilon^d_{t+1}\bullet 1) = 0$ and $E(\varepsilon^d_{t+1}\bullet k_t) = 0$. Further, the definition of η^d_{t+1} tells us that $\varepsilon^d_{t+1} = \eta^d_{t+1}$ when $\hat{\sigma} = \sigma$ and $\hat{\beta} = \beta$.

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[5] That is, if 1 and k_t are used as instruments, then $\hat{\beta}\{x_{t+1}^{-1/\hat{\sigma}}(1+r_{t+1})\} - 1$ and $\hat{\beta}\{x_{t+1}^{-1/\hat{\sigma}}(1+r_{t+1})k_t\} - k_t$ are averaged and used in a positive semi-definite quadratic form, which is minimized over $(\hat{\beta},\hat{\sigma})$. If one uses more than two 'instrumental variables', the sample orthogonality conditions overidentify the parameters. Using the methods of Hansen [7], the parameter estimates are selected so that the weighted sum of the departures of the sample orthogonality conditions from zero is minimized. Tests of the theory can be constructed by relaxing some or all of the overidentifying restrictions and evaluating the implied improvement in goodness of fit.

No Supply Disturbances

If there are no shifts to production opportunities ($\phi_{t+1} = \phi$ and $\lambda_{t+1} = \lambda_t = \lambda$) but we reinstate preference shocks, then the situation is altered dramatically. Using the solution for $(1+r_{t+1})$ given in (3.1) above, one finds that η^d_{t+1} may be written as

$$(3.2) \qquad \eta^d_{t+1} = \hat{\beta}x_{t+1}^{-1/\hat{\sigma}}(1+r_{t+1}) - 1$$

$$= \{\hat{\beta}\phi x_{t+1}^{-1/\hat{\sigma}} \bullet x_{t+1}^{-1/\mu} - 1\}.$$

Thus, the orthogonality conditions (3.2) will be satisfied by

$$(3.3) \qquad \hat{\beta} = \phi^{-1} \qquad \hat{\sigma} = \mu$$

That is, an econometrician seeking to estimate intertemporal substitution in preferences will actually wind up estimating intertemporal substitution in production. Generally, we would expect to find that this procedure yielded marginal parameter estimates, reflecting a nonlinear combination of supply and demand parameters. The clean results in (3.3) derive from our assumption in (3.1) above that $(1+r_{t+1})$ is the <u>ex post</u> rate of substitution in production.

Unobservable Supply and Demand Shocks

Now, we reinstate all the demand and supply shocks $(\theta_t, \lambda_t, \phi_t)$. We assume that the econometrician does not observe the preference (θ_t) or production (λ_t, ϕ_t) shift variables and assumes that θ_t is constant. Then the following 'errors term' will be formed, which now involves a pure expectation error plus a specification error that depends on choice of $(\hat{\beta}, \hat{\sigma})$ and on the preference shocks θ_{t+1}, θ_t not observed by the researcher,

$$(3.4)\qquad \eta^{d}_{t+1} \equiv [\hat{\beta}x_{t+1}^{-1/\hat{\sigma}}(1+r_{t+1}) - 1]$$
$$= \varepsilon^{d}_{t+1} + \{\hat{\beta}x_{t+1}^{-1/\hat{\sigma}} - \beta x_{t+1}^{-1/\sigma}[\theta_{t+1}/\theta_{t}]\}(1+r_{t+1}),$$

where we have again used a circumflex to denote a parameter value chosen by the researcher as opposed to the population value. Using 1 and k_t as instruments and requiring that population orthogonality conditions are satisfied for the error η^{d}_{t+1} yields

$$(3.5a)\qquad E[\eta^{d}_{t+1}\bullet 1] = 0$$
$$\Rightarrow\quad E[\{\hat{\beta}x_{t+1}^{-1/\hat{\sigma}} - \beta x_{t+1}^{-1/\sigma}[\theta_{t+1}/\theta_{t}]\}\ (1+r_{t+1})] = 0$$

$$(3.5b)\qquad E[\eta^{d}_{t+1}\bullet k_{t}] = 0$$
$$\Rightarrow\quad E[\{\hat{\beta}x_{t+1}^{-1/\hat{\sigma}} - \beta x_{t+1}^{-1/\sigma}[\theta_{t+1}/\theta_{t}]\}\ (1+r_{t+1})k_{t}] = 0\ .$$

Generally, parameter estimators $(\hat{\beta},\hat{\sigma})$ based on (3.5a,b) will not be consistent estimators for (β,σ); (3.5a,b) will not typically be satisfied for $\hat{\beta}=\beta$ and $\hat{\sigma}=\sigma$. For example, if one selects--following Hansen and Singleton--the lagged consumption ratio x_t as an instrument, then it follows directly that $(\hat{\beta},\hat{\sigma})$ are not consistent. While some special ad hoc restrictions may produce consistent $(\hat{\beta},\hat{\sigma})$ estimators, e.g., θ_{t+1}/θ_t is independent of $(x_{t+1},(1+r_{t+1}),k_t)$ and $E[\theta_{t+1}/\theta_t] = 1$, they will usually contradict the relations among variables implied by equilibrium behavior.

Observable Supply Shifts

Maintaining the full complement of shocks, suppose that we use a constant plus the lagged productivity shock, λ_t, to form the orthogonality conditions.

$$(3.6)\qquad E\eta^{d}_{t+1}\bullet 1 = 0 \qquad E\eta^{d}_{t+1}\bullet\lambda_{t} = 0.$$

Working with the equilibrium solution derived in the appendix, it is possible to demonstrate that $\beta(\theta_{t+1}/\theta_t)x_{t+1}^{-1/\sigma}(1+r_{t+1}) = \psi_{t+1}/E_t\,\psi_{t+1}$, where ψ_{t+1} is a complicated function of date t+1 disturbances. Then, the specification error in (3.4) may be written as $\{[\hat{\beta}x_{t+1}^{(\hat{\sigma}-\sigma)/\sigma\hat{\sigma}}(\theta_{t+1}/\theta_t)/\beta]-1\}\{\psi_{t+1}/E_t\psi_{t+1}\}$. If λ_t is independent of θ_t, the orthogonality conditions (3.6) will be satisfied by

$$(3.7) \qquad \hat{\beta} = \beta E\psi_{t+1}/E(\theta_t\psi_{t+1}/\theta_{t+1}); \qquad \hat{\sigma} = \sigma.$$

Our knowledge that λ_t is exogenous to the consumer permits consistent estimation of the intertemporal elasticity of substitution in consumption, σ.

A Linear System Analogy

Basically, these results are in accord with the standard analysis of the linear simultaneous equations model. Adding unity to both sides of (2.9a,b), taking logarithms and reorganizing, our model implies that

$$(3.8a) \qquad \log x^d_{t+1} = -\sigma\log\beta - \sigma\log(1+r_{t+1}) - \sigma\log(\theta_{t+1}/\theta_t) - \log(1+\varepsilon^d_{t+1})$$

$$(3.8b) \qquad \log x^s_{t+1} = \mu\log\phi_{t+1} + \mu\log(1+r_{t+1}) - (1+\mu)\log(\lambda_{t+1}/\lambda_t)$$

Note that the term $\log(1+\varepsilon^s_{t+1})$ will be identically zero when the realized rate of return is substituted from (3.1). We assume that $\log(1+\varepsilon^d_{t+1}) \approx \varepsilon^d_{t+1}$.

This system closely mimics the typical textbook example of a linear simultaneous system of market supply and demand, and we now employ it to derive intuitively the identification results of the previous section. From this standpoint of this system, the key prior restriction that permits demand parameter identification is the fact that λ_t enters in supply but not demand. Above, we considered instrumental variables estimation of β,σ with x_t and 1 used as instruments and initially supposed that preferences not subject to disturbances, i.e., $\theta_{t+1} = \theta_t = \theta$. The parameters (β,σ) are identified, and are consistently estimated be x_t depends on λ_t. Second, if the preference shifts $(\log(\theta_{t+1}/\theta_t))$ are part of a composite error term in the demand curve, then this procedure is inconsistent because x_t is correlated with the composite error term. In fact, since the supply curve is exact, one gets supply parameters. Third, if one has information that an observable, exogenous variable $\log\lambda_t$ enters in the supply relation, but not in the demand relation, then the demand parameter σ is identified. The parameter β is confounded with the first moment of $\log(\theta_{t+1}/\theta_t)$. The variable $\log\lambda_t$ can be employed as an instrumental variable to produce a consistent estimator of σ..

5. Conclusions

For the last decade, the Lucas [14] critique has reigned as the guiding principle determining the direction of macroeconomic research. Taken at its most extreme, the critique dictates that each macroeconomic inquiry should begin at the deep levels of choice, the levels of dynamic utility and profit maximization. Only then can the effects of policy innovations be accurately traced through the general equilibrium of the economy. Since macroeconomic results rightly constitute mere curiosities until their empirical implementation is at least conceivable, proponents of the critique's application have launched a search for techniques suitable for estimating deep structural parameters.

One prominent strategy for estimating such parameters, developed in a line of research culminating in Hansen and Singleton [9], employs the intertemporal efficiency conditions of economic agents. In this paper, we have stressed that typical applications of these methods--including the example provided by Hansen and Singleton [9]--achieve identification of behavioral parameters only if there are no shocks to agents' objectives that are not observed by the econometrician. In the felicitous phrase of Sims [22], we take this to be identification via an "incredible" disturbance assumption.

This assumption appears to have its origins in recent theoretical work in macroeconomics. In producing and developing the fully articulated artificial economies advocated in Lucas' [15] methodology, economists have found analytical convenience in introducing disturbances only in the market production sector.[6] (See, e.g., Lucas [16] or Long and Plosser [13]). Since the points made by the builders of such models would likely prove robust to demand disturbances, it is reasonable to ignore them in theoretical work.

However, carrying this convenient assumption over to an empirically relevant context is a different matter. Few economists, if any, would agree that such an assumption provides a useful empirical approach. For example, Becker [1] employs stable tastes as a methodological device, but the tastes are over "commodities" or characteristics. Goods are inputs into randomly evolving household production functions which yield commodities. Generally, we believe it is best to view both 'supply' and 'demand' as subject to shocks not observed by the econometrician.

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[6] In fact, Becker [1] and King [12] argue that a coherent explanation of diverse secular and cyclical relationships between hours and wages is best achieved by recognizing secular evolution in the technology of household production.

Once the 'exactness' assumption is dropped, Euler equation methods require that the econometrician produce instrumental variables that are independent of objective function shocks. The conventional theory of linear simultaneous equations models, as extended by Wallis [23] to cover a class of rational expectations models, gives us guidance in this process. But, Sims [22] argues that since all macroeconomic variables materialize through an economy's dynamic general equilibrium, few exogenous variables exist, and economists lack any theoretical basis for eliminating lagged variables from structural relationships or for imposing structure on disturbance covariances. If that is so, then objective function shocks rule out application of Euler equation methods.

Given our above conclusions, an empirical researcher employing Euler equation methods will need to state the nature of prior assumptions on system characteristics that permit a specific set of variables to be used as instruments. Further, since prior restrictions may differ across economists utilizing the results of applied studies, our results reinforce the need for employment of varied instrument lists.

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Appendix

Intertemporal General Equilibrium

In this section, we consider determination of prices and quantities in intertemporal general equilibrium. There are no productive externalities or taxes so that we can use the Lucas and Prescott [17] strategy of solving a social planning problem for quantities. Competitive prices are the relevant derivatives evaluated at optimal quantities.

To keep things analytically manageable, we view the shock vector $(\lambda_t, \theta_t, \phi_t)$ as independently distributed over time although not necessarily as contemporaneously independent. It is easiest to view z_t as the control variable, after 'substituting' production opportunities into preferences. That is, we maximize (3.1) with respect to contingency plans for intermediate good use, $\{z_t\}_{t=0}^{\infty}$,

$$\text{(A.1)} \qquad \underset{t}{E}\{ \sum_{j=0}^{\infty} \beta^j [\lambda_{t+j}^{\alpha} \; \theta_{t+j} \; z_{t+j}^{m\alpha}]/\alpha$$

subject to $y_{t+j+1} = \phi_{t+j}(y_{t+j} - z_{t+j})$. This sort of constant elasticity problem is of a family with those previously studied by Phelps [21] and Hakansson [4], possessing a tractable closed form solution.

As it happens, the decision rule can be directly uncovered from the discrete Euler equation, without actually deriving the value function and other details of the dynamic programming algorithm. An optimal z contingency plan requires that

$$\text{(A.2)} \qquad m \, \lambda_t^{\alpha} \, \theta_t \, z_t^{m\alpha-1} = \beta \, m \underset{t}{E}\{\lambda_{t+1}^{\alpha} \; \theta_{t+1} \; \phi_{t+1} \; z_{t+1}^{m\alpha-1}\}.$$

Using a serendipitous procedure, we guess that the optimal policy takes the form $z_t = A(\theta_t, \lambda_t) y_t$, which actually will hold for any markov structure of $\{\theta_t, \lambda_t, \phi_t\}$. Then, (A.2) becomes

$$\lambda_t^{\alpha}\ \theta_t A(\theta_t, \lambda_t)^{m\alpha - 1}$$

$$= \beta \underset{t}{E}\{\lambda_{t+1}^{\alpha} \theta_{t+1} \phi_{t+1}^{m\alpha} [1 - A(\theta_t, \lambda_t)]^{m\alpha-1}\ A(\theta_{t+1}, \lambda_{t+1})^{m\alpha-1}\}.$$

In turn, this implies that

$$\text{(A.3)} \qquad A(\theta_t, \lambda_t) = (\beta \underset{t}{E} \psi_{t+1})^{\frac{1}{m\alpha-1}} / \{(\beta \underset{t}{E} \psi_{t+1})^{\frac{1}{m\alpha-1}} + (\lambda_t^{\alpha} \theta_t)^{\frac{1}{m\alpha-1}}\}$$

where $\psi_{t+1} = \{\lambda_{t+1}^{\alpha} \theta_{t+1} \phi_{t+1}^{m\alpha}) A(\theta_{t+1}, \lambda_{t+1})^{m\alpha-1}\}$. Under our serial independence assumption $\underset{t}{E}\ \psi_{t+1}$ is constant over time.[7]

As z_t is the 'spending' of the economy's intermediate good stock y_t, the function A() has an intuitive form, in that it is bounded between zero and unity. One can further explore the effects of shocks on A(), although we do not do this in detail here. For example, a positive demand shock (θ_t) has a positive effect on spending $(\partial A / \partial \theta_t) = -A_t(1 - A_t)/\theta_t(m\alpha - 1) > 0$, since $m\alpha < 1$). The effects of λ_t are ambiguous due to offsetting 'income' and 'substitution' effects.

Thus, the stochastic process for equilibrium consumption (c_t) is derived in a straightforward manner from z_t, i.e.,

$$\text{(A.4)} \qquad c_t = \lambda_t\ A(\theta_t, \lambda_t)^m\ y_t^m$$

and intermediate goods output evolves as

$$\text{(A.5)} \qquad y_{t+1} = \phi_{t+1} [1 - A(\theta_t, \lambda_t)] y_t .$$

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[7] The exact solution can be derived by recursive substitution, i.e., eliminating $A(\theta_{t+1}, \lambda_{t+1})$ by iterating on (A.3).

Jointly, these structures imply that $x_{t+1} \equiv c_{t+1}/c_t$ exhibits the following dependence on the shock structure,

$$(A.6) \qquad x_{t+1} = [\lambda_{t+1} A^m_{t+1}(1-A_t)^m \phi^m_{t+1}]/\lambda_t A^m_t$$

$$\equiv x(\lambda_{t+1}, \lambda_t, \theta_{t+1}, \theta_t, \phi_{t+1}),$$

with the function x() cataloging all the relevant dependencies.

Finally, with equilibrium quantity behavior at hand, we can define a state contingent rate of return from the marginal rate of substitution in production

$$(A.7) \qquad (1+r_{t+1}) = \phi_{t+1}(x_{t+1})^{-1/\mu}(\lambda_{t+1}/\lambda_t)^{(1+\mu)/\mu}.$$

It is straightforward to verify that choice of this market structure 'decentralizes' our social planning problem. That is, given (A.7), the firm's Euler equation holds exactly, when it selects the quantity sequence (A.6). Further, the consumer's Euler equation is also satisfied by (A.6), although it is more tedious to demonstrate this.

Since our model is a representative agent framework, there is more than one structure of securities markets that permits the social planning allocation to be decentralized. Choice of (A.7) is analytically convenient for us, as it implies that the firm's Euler equation holds exactly, i.e., the right hand side of (A.5) is unity for all possible realizations of random variables not just in expected value form. That is, ε^d_{t+1} is always identically zero with this security market structure. Our results are robust, however, to other choices of securities markets, including the complete Arrow-Debreu menu. For example, it would be possible for us to solve directly for contingent security prices, ρ_{tj} in 2.3, and to use those to decentralize consumer and producer allocations. Implicitly, these prices yield rates of return. As is typical in dynamic general equilibrium models with closed form solutions, such as Long-Plosser [13] and the present setup, there are particular features of the solution that are not general. In our model, the constant returns-to-scale character of intermediate goods production implies that the model does not possess a stochastic steady state (stationary distribution). That is, intermediate goods output is a logarithmic random walk in (A.5), imparting a nonstationary character to consumption and an absence of dependence of returns on the stock of finished intermediate goods (y_t). Thus, our model would not serve as a particularly useful vehicle for addressing key issues in the modern theory of finance, since such state variables do not shift conditional covariances. Yet, the model is general enough for us to illustrate some important points about Euler equation methods.