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3 Risk and Required Returns on Debt and Equity

Zvi Bodie, Alex Kane, and Robert McDonald

One of the most striking developments in the United States capital markets during the past decade has been an enormous increase in the riskiness of long-term bonds and other fixed income securities. This has stemmed in part from increased inflation uncertainty and in part from fundamental shifts in Federal Reserve policies. In this paper we measure this phenomenon and explore its implications for the returns required by investors on these debt instruments and the equity securities which can substitute for them in wealth portfolios. We believe that our results help to explain why real interest rates on long-term bonds have been so high in recent years.

The data set used in the paper also enables us to address a different, although somewhat related, issue in the study of United States financial markets: Why, despite the apparent increase in inflation risk in the recent past, has no private market for indexed bonds developed in the United States?

The paper is organized as follows. We first discuss alternative approaches to estimating risk premiums on debt and equity securities and then explain our approach, which is based on modern portfolio theory. We apply our model to data on common stocks and on United States government bonds of eight different maturities to estimate risk premiums for the period 1973–83. Next we estimate the risk premium on Treasury bills relative to a hypothetical riskless real rate of interest over that same period. Finally, we discuss the implications of our model for the question why no private market for index-linked bonds exists in the United States.

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3.1 Estimating Risk Premiums

All investors, be they individual or institutional investors seeking to allocate funds or a nonfinancial business entity seeking to acquire funds, are faced with choosing the proportions of debt and equity in their portfolio. Making this decision requires knowing both the expected rates of return on assets and the riskiness of those assets. A basic tenet of financial economies is that in equilibrium, expected returns on the various traded assets will reflect the perceived risk inherent in them. In a capital market dominated by risk-averse investors, the riskier the asset the higher the premium it will have to bear over the riskless rate of interest. This premium is usually called the "risk premium," and in general it is unobservable.

In order to make their investment and financing decisions, all parties involved must quantify their beliefs regarding the relative magnitudes of these risk premiums. Traditionally there have been two generic approaches to estimating risk premiums: (1) statistical estimation based on ex post data and (2) estimation based on economic models of security price formation coupled with forecasts of the fundamental variables in the models (e.g., earnings forecasts for stocks, the yield curve for bonds). Most analysts have used some combination of these two approaches.

Often one or even both of these approaches results in estimated risk premiums for some assets which violate one's criterion of "reasonableness," based on the perceived risk of those assets. For example, in many studies, past rates of return are used to compute means, variances, and covariances. The means are then taken as measures of expected future returns and the variances and covariances as measures of risk. Unfortunately, at times some of the risk premiums implied by the estimated means bear a relationship to the estimated risk measures, which contradicts the theory underlying the study.

Our approach is to use past data solely to obtain risk measures and then to compute the corresponding risk premiums implied by theory. Our reason for ignoring the estimated means is that, in order to get a reliable estimate of the mean of a stochastic time series, it is necessary to observe it over a long span of time. If the mean is changing over the period of observation, reliable estimation is virtually impossible. Variances and covariances, however, can be measured fairly accurately over much shorter sample periods.[1]

We view our approach as a supplement to other, more traditional methods of estimating risk premiums and think that it can be used to check the reasonableness of the estimates which they provide.

1. See Merton (1980) for a full discussion of this point.

3.2 The Model

The theoretical model we employ is a modified version of the capital asset pricing model, which has become the standard financial model of capital market equilibrium over the past two decades and has gained widespread acceptance within the financial industry under the name Modern Portfolio Theory (MPT). The fundamental insight of this model is that the riskiness of an individual asset is not its volatility or riskiness considered in isolation but rather its contribution to the risk of a portfolio of assets. The model has most frequently been used in the past to explain the structure of required returns on common stocks, but it applies just as well to all other traded assets, including bonds.

This theory implies that in equilibrium, the risk premium on any traded asset can be expressed as the product of two terms:

(1) risk premium on asset i =
average degree of risk aversion of market participants ×
covariance of asset i with the market portfolio.

The first term, the average degree of risk aversion,[2] is the same for all assets, and thus at any point in time is simply a constant of proportionality. The second term, the covariance with the market portfolio,[3] is thus the critical determinant of differences in risk premiums across asset categories.

The market portfolio is by definition composed of all existing assets in the economy, each held in proportion to its relative outstanding supply. The covariance of any asset with the market portfolio is the sum of two factors: (1) the relative supply of that security times its own variance and (2) a weighted sum of its covariances with all other assets.

To facilitate our understanding of the empirical results to follow, let us examine how these factors work for the case of three categories of assets: stocks, bonds, and bills. Let us assume that the market portfolio consists of 65% stocks, 10% bonds, and 25% bills and that bills are riskless. The risk premiums would then be

risk premium on stocks = risk aversion × (.65 variance of stocks +
.1 covariance between stocks and bonds)

risk premium on bonds = risk aversion × (.1 variance of bonds +
.65 covariance between stocks and bonds).

2. The measure of risk aversion referred to is Pratt's coefficient of relative risk aversion. The higher the value of this coefficient, the greater the compensation an investor requires to bear a given degree of risk. For a complete explanation see Bodie et al. (1985).

3. The covariance referred to here is the covariance between the real rate of return on security i and the real rate of return on the market portfolio.

In the results presented in the next section we will refer to the correlation coefficient between asset returns rather than the covariance. The correlation coefficient is a more familiar measure of comovement in returns, and it is related to covariance by the following formula:

covariance between stocks and bonds = correlation coefficient × standard deviation of stocks × standard deviation of bonds.

The standard deviation is the square root of variance.

In the following section we estimate what the risk premiums implied by this model were over the period 1973–83 on 10 categories of financial assets: common stocks, bills, and United States government bonds of eight different maturity classes. Our measure of a bond's life is duration rather than the conventional measure of maturity. Duration is a weighted average of the times until each payment (coupon and principal) made by the bond, and may be used to compare consistently bonds with very different payment streams, for which maturity may provide a misleading comparison. A bond's price volatility is also more directly related to duration than to maturity.[4]

We follow the usual practice of computing the risk premiums on stocks and bonds relative to the rate of return on bills. Since bills are not completely riskless in real terms, this requires a slight modification in equation (1). The modification is as follows:

(1′) risk premium on asset *i* relative to bills =
risk aversion × (covariance of asset *i* with the market portfolio
− covariance of bills with the market portfolio).

3.3 Implied Risk Premiums on Stocks and Bonds

Consider a representative investor who goes to the market on the first day of each month and adjusts his portfolio according to his current views on the risk-return profile of different types of investments. We assume that the investor has the choice of the 10 different categories of financial assets mentioned at the end of the previous section, stocks, bills, and bonds of eight different durations.

4. Duration is defined by Macaulay (1938). The distinction between maturity and duration is important, because the duration of bonds of a given maturity shortened considerably in the late 1970s. For coupon bonds and mortgages, duration is always less than maturity. The difference between maturity and duration for ordinary coupon bonds and mortgages is greater, the longer the final maturity and the higher the level of interest rates. In 1953, the average maturity of the bonds in our 8-year duration portfolio was just under 9 years; in 1981, the average maturity was 23 years. This variation calls into question the appropriateness of a bond-return series with a constant maturity of 20 years, such as the one tabulated by Ibbotson and Sinquefield (1982).

The 10 different types of assets have been ordered according to their usual degree of riskiness, with 1-month Treasury bills carrying the least risk and (a diversified portfolio of common) stocks the most. In this context, "risky" refers to unexpected changes in security prices during the 1-month holding period. Changes in the yield curve will lead to capital gains or losses on fixed interest debt, and owners of common stocks will also be unable to predict with great accuracy the value of their holdings 1 month in the future.

In our estimation using United States data we assume that investors look back on the most recent 24 months when they estimate the volatilities of the different returns. We hold constant the degree of risk aversion[5] at 3.5 and the relative weights of the different categories in the market portfolio at roughly 65% stocks, 10% bonds, and 25% bills.[6] All change in the risk premiums over time is therefore coming from changes in the variances and correlations among asset categories.

An Appendix details our data sources and the precise procedure used for generating our time series, but it is possible to explain the general procedure in a fairly straightforward manner: Each month we use the model to forecast expected returns for the next month. Actual returns on stocks and bonds in any period will deviate from these forecasts. Investors will observe these forecast errors and use this information to improve upon their estimates of the variances and correlations attached to the 10 categories of risky investments. They do this by using data for the most recent 24 months. As we proceed through time, the estimates change and so too do the risk premiums required in order to compensate for the perceived risk.

Figure 3.1 illustrates the results of this procedure in computing the standard deviations for stock and long-term bond returns. It shows the standard deviation of the forecast errors in the monthly rate of return on our diversified portfolio of common stocks and in the monthly rate of return on long-term government bonds. Assuming no change in correlations, the pattern of risk premiums should follow the pattern of standard deviations. Figure 3.2 shows the corresponding risk premiums on stocks and on long-term bonds. We emphasize that these risk premia are computed relative to the return on T-bills.

The most striking aspect of these figures is the dramatic increase in the volatility of long-term bonds starting in the last quarter of 1979, coinciding with a basic change in Federal Reserve operating procedure. Between

5. For a discussion of why 3.5 is a reasonable number to use for the degree of risk aversion, see Bodie et al. (1985).

6. The relative weights of assets in the market portfolio changed over the period 1973–83, but it is shown in table AIV of Bodie et al. (1984) that the changes had only a small effect on risk premiums.

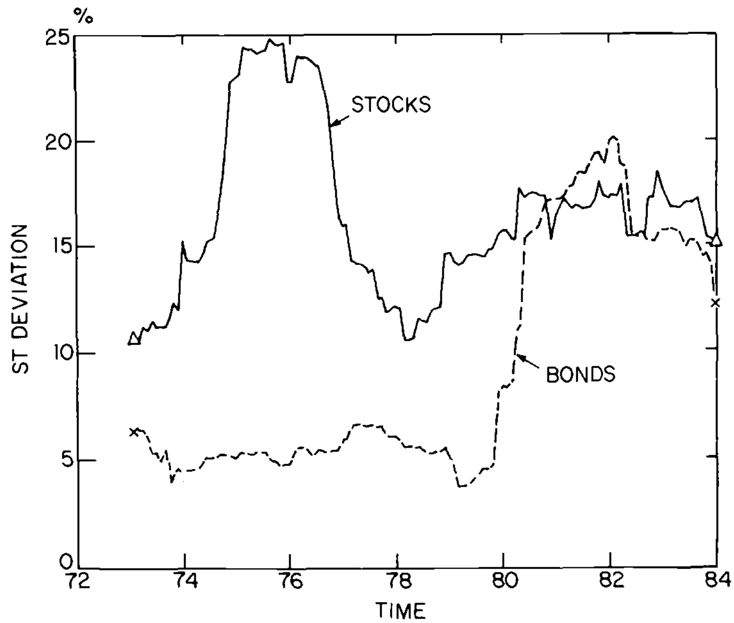

Fig. 3.1 Standard deviation of real rates of return on stocks and bonds, 1973–84

October 1979 and August 1982 the Fed was targeting monetary aggregates rather than interest rates, and as a result created a considerable increase in bond price volatility.

The volatility of stock prices peaked at about 22% in the period from 1975 to 1977. It subsequently declined, and has fluctuated in a range from 12% to 17% since 1979. In the period 1981–83 the volatility of long-term bond prices actually exceeded that of stocks, although by the end of the period they were about equal.

The risk premiums on stocks and bonds in figure 3.2 closely follow the time profiles of standard deviations shown in figure 3.1. But despite the rough equality in their standard deviations during 1982–83, the risk premium on stocks is roughly double that on long-term bonds. This is primarily because stocks constitute a much larger proportion of the market portfolio than do long-term bonds.[7]

7. To be more precise, the covariance of stocks with the market portfolio is approximately .65 × variance of stocks + .1 × covariance between stocks and bonds, while the covariance of bonds with the market portfolio is approximately .1 × variance of bonds + .65 × covariance between stocks and bonds. The covariance between stocks and bonds is much smaller than the variance of stocks or bonds. Even though the variance of bonds and stocks were roughly equal in 1982–83, the covariance of stocks with the market (and therefore the risk premium on stocks) was greater because the variance term has a weight of .65 for stocks vs. .1 for bonds.

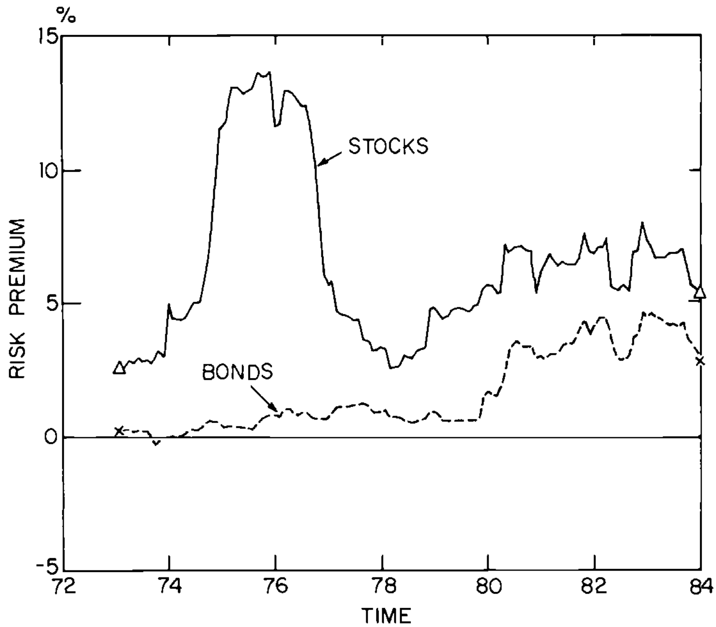

Fig. 3.2 Risk premiums on stocks and bonds, 1973–84

Note that while the volatility of long-term bonds has been falling more or less steadily since the end of 1981, the risk premium has not. After falling precipitously at the end of 1981 and the beginning of 1982, it climbed back to its previous high before starting to come down again in 1983. In order to understand this seemingly odd behavior of the risk premium on bonds we must look at the correlation between bond and stock returns.

Figure 3.3 shows the behavior of the correlation coefficient between bonds and stocks. It rose steeply between the beginning of 1981 and the end of 1982, which served to counteract the effect of a declining bond price volatility on the risk premium on bonds.[8]

Table 3.1 shows the risk premiums on all asset categories other than bills at three different points in time. Focusing on the December 1983 column, we see that the risk premiums on bonds rise more or less uniformly with maturity class. Starting with .53% for the shortest, it rises to 3% for duration 6 and then levels off. It should be emphasized that these risk pre-

8. We can express the covariance of bonds with the market portfolio approximately as

.1 × variance of bonds + .65 × (correlation between stocks and bonds × standard deviation of stocks × standard deviation of bonds)

A rise in the correlation between stocks and bonds can offset a decline in the variance of bonds and keep the covariance of bonds with the market (and therefore the risk premium on bonds) from falling.

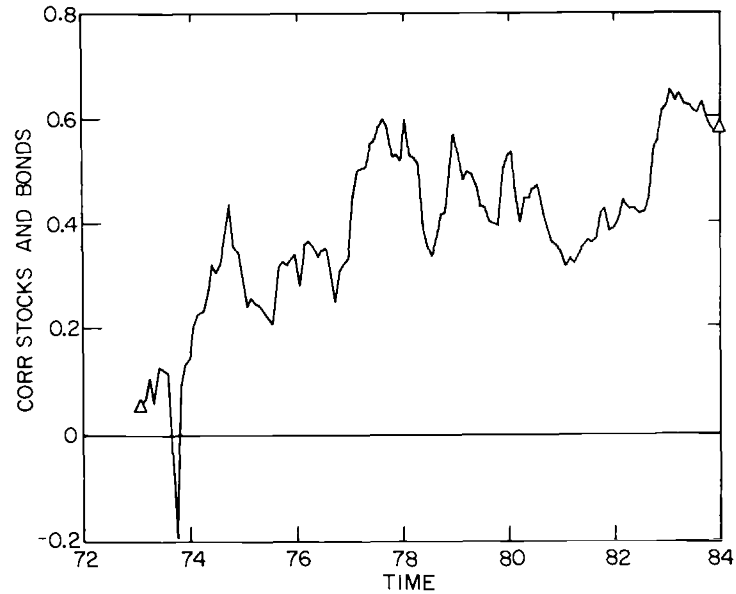

Fig. 3.3 Correlation between the real rates of return on stocks and bonds, 1973-84

Table 3.1 **Estimated Annual Risk Premiums (percent per year)**

	September 1979	December 1981	December 1983
Bonds:	.17	.77	.53
1	.27	1.47	.97
2	.42	1.67	1.18
3	.43	2.17	1.82
4	.52	2.48	2.15
5	.45	2.40	3.02
6	.41	3.48	2.89
7	.53	4.20	2.85
8			
Stocks	4.93	6.96	5.51

Note: Risk premiums calculated relative to rate of return on Treasury bills. Market weights are those for 1980, with pension fund reserves weighted for long durations.

miums are annualized rates of return in excess of the 1-month bill rate, which are expected to prevail over the next 1-month holding period. They are not yields to maturity. The risk premium on stocks is about 5.5%.

Going across the columns in table 3.1 we see that the risk premium on stocks rose from about 5% in 1979 to 7% in 1981 and then back down to

5.5% in 1983. That same general pattern is evident for all of the bonds too. Duration 8 bonds, for example, rose from about .5% in 1979 to 4% in 1981 and then back down to 3% in 1983. Bonds have clearly not returned as closely to their 1979 levels as have stocks.

Table 3.2 shows the standard deviations and correlation matrices underlying the risk premiums in table 3.1. Note that, as suggested by the behavior of risk premiums in table 3.1, the increase in standard deviation after October 1979 was greater for longer-duration bonds.

Table 3.2 Standard Deviations and Correlations of Real Rates of Return

			Bonds (duration in years)							
	Common stocks	One-Month Bills	1	2	3	4	5	6	7	8
			October 1977 – September 1979							
Standard deviation (% per year)	17.46	1.24	2.26	3.43	4.04	5.25	5.02	4.64	5.38	5.43
Correlation coefficients:										
Stocks		.37	.33	.37	.31	.31	.07	.21	.23	.25
Bills			.69	.64	.56	.58	.37	.31	.40	.36
Bonds: 1				.78	.79	.76	.76	.74	.79	.69
2					.90	.88	.76	.69	.74	.74
3						0.93	.87	.76	.82	.85
4							.84	.74	.78	.80
5								.85	.86	.89
6									.90	.80
7										.86
			January 1980 – December 1981							
Standard deviation	17.55	1.00	5.15	9.60	12.5	14.4	16.9	18.5	19.4	20.4
Correlation coefficients:										
Stocks		.13	.29	.29	.23	.28	.27	.24	.36	.41
Bills			.56	.49	.51	.47	.44	.36	.48	.43
Bonds: 1				.95	.91	.88	.85	.74	.82	.86
2					.95	.94	.90	.83	.88	.90
3						.98	.95	.86	.90	.91
4							.96	.86	.92	.93
5								.89	.93	.90
6									.92	.88
7										.93

Table 3.2 *(continued)*

			Bonds (duration in years)							
	Common stocks	One-Month Bills	1	2	3	4	5	6	7	8
			January 1982 – December 1983							
Standard deviation	15.04	1.08	3.00	4.59	5.85	7.84	9.31	10.63	11.81	11.91
Correlation coefficients:										
Stocks		.17	.45	.50	.48	.54	.54	.67	.55	.57
Bills			.53	.42	.31	.34	.28	.26	.26	.15
Bonds: 1				.95	.91	.88	.87	.82	.80	.73
2					.97	.90	.94	.89	.88	.83
3						.90	.97	.90	.91	.88
4							.92	.91	.90	.90
5								.95	.96	.93
6									.92	.94
7										.96

3.4 The Riskless Real Rate of Interest and the Risk Premium on Treasury Bills

Up to this point we have followed the conventional practice of measuring risk premiums relative to Treasury bills. While 1-month T-bills offer a risk-free nominal rate of return, uncertainty about the rate of inflation over the next month makes their real return risky. This inflation risk, however, is small relative to the risk of unanticipated stock and bond price changes during the month. Furthermore, for holding periods longer than 1 month, a policy of rolling over 30 day T-bills has been shown to offer a relatively stable real rate of return, because nominal bill rates can adjust rapidly to the changing inflation rate.

Nonetheless it is instructive to see what our model implies about the risk premium on T-bills over the 1973–83 period, particularly in the light of the increase in real bill rates in the 1980s. Perhaps we can explain at least part of this increase on the basis of increased covariance with the market portfolio, the way we did with bonds.

Figure 3.4 shows the behavior of the risk premium implied by equation (1), which says that it should be proportional to the covariance of the real rate of return on bills with the market portfolio. It should be remembered that the risk premium for bills is relative to a hypothetical risk-free real rate of interest. It is clear from figure 3.4 that, if anything, the risk premium on T-bills has declined rather than risen in the 1980s. If we want to understand why real bill rates have risen we must seek other explanations.

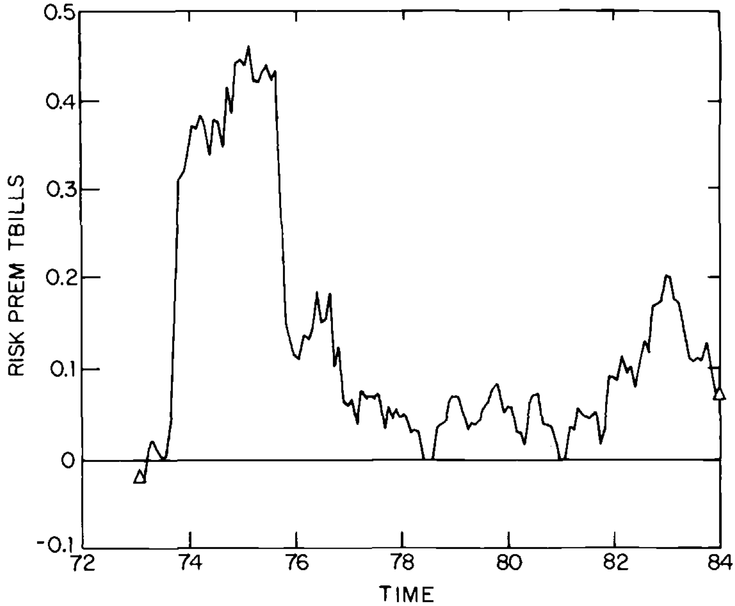

Fig. 3.4 Risk premium on 1-month Treasury bills, 1973-84

3.5 A Private Market for Indexed Bonds

A recurrent theme in both the popular and technical finance literature has been the issue of indexed bonds, that is, bonds whose principal and interest payments are linked to some index of the cost of living and are therefore riskless in real terms. Economists from different sides of the ideological spectrum, like Milton Friedman and James Tobin, find themselves in agreement on the desirability of having the federal government issue such an asset.

Some critics of this idea have argued that if index-linked bonds were a worthwhile innovation, the private capital markets would already have produced them. The data and theoretical model which we employed earlier in this paper can be extended to shed some additional light on this matter.[9]

We computed the value to investors of a private market for indexed bonds in the following way: first, we found the real risk-free rate at which an investor whose degree of risk aversion is equal to the average would be indifferent to holding this riskless asset. (If the asset is privately supplied, it will necessarily be priced so that the average investor is indifferent about holding it.) Then, we asked what one-time dollar payment each investor would be willing to make in order for them to obtain the right to hold the

9. Details behind the calculations in this section are contained in Bodie et al. (1985).

desired amount of the riskless asset for the rest of their lives. This calculation assumes that each investor holds an optimal portfolio, including assets besides the risk-free asset. Table 3.3 shows the amount which an investor with risk aversion greater than the average would be willing to pay, per $10,000 of current wealth, for the opportunity to invest in it.

At an average degree of risk aversion of 3.5, if a market for a riskless real asset could be established costlessly, the market-clearing real interest rate would be about seven basis points below the mean real rate on conventional nominally risk-free bills. Table 3.3 shows how much investors with varying degrees of risk aversion would be willing to pay for the opportunity to invest in a riskless real asset.

The amounts do not appear to be large. The numbers in the first column of table 3.3 show the results obtained using the variances and correlations estimated for the 1982–83 subperiod. The second column shows the results of an experiment in which we made all nominal debt securities twice as risky by doubling their variances and covariances, leaving the variance of stocks unchanged. While the effect is to approximately double the amount at any degree of risk aversion, the magnitudes still seem small.

An additional point about table 3.3 should be noted. The calculation assumes that there is no uncertainty about the future risk-free real rate. Thus, there is no distinction between indexed bills and indexed bonds. The numbers in table 3.3 apply equally to both assets.

Our explanation for these results is that conventional 1-month bills are a fairly good low-risk alternative to stocks and bonds even for very risk-averse investors. The extra safety of real return provided by an indexed bond is not worth much to them.

These results suggest one possible reason for the nonexistence of index bonds in the United States capital market. Since there would probably be some costs associated with creating a new market for such securities, the benefits would have to exceed those costs. Given the assumptions of our model, in particular the assumption that all market participants have the

Table 3.3 **Willingness to Pay for the Opportunity to Invest in a Real Riskless Asset (Dollars per $10,000 of Wealth)**

	Willingness to Pay	
Coefficient of Relative Risk Aversion	Actual Variances and Covariances (1982–83)	Double All Variances And Covariances But Stocks
3.5	0	0
5	$6.50	$13
6	16	32

same set of price expectations, the benefit from trading in these new securities would have to arise from differences in the degree of risk aversion among investors. If, as table 3.3 suggests, the willingness to pay does not appear to be large over a fairly broad range of risk aversion coefficients, then one should not be surprised at the failure of a private market for index bonds to appear.

3.6 Summary and Conclusions

It appears that there may have been a substantial rise in risk premiums on long-term bonds in the early 1980s as a market response to an increase in bond price volatility and an increased correlation between bond and stock returns. The increase in bond price volatility was sudden and coincided with a shift in Federal Reserve policy to stabilizing monetary aggregates rather than interest rates starting in the last quarter of 1979. Despite a fall in the long-term expected rate of inflation, long-term interest rates may have remained high during this period, at least partially because of this risk premium.

By the end of 1983 the risk premium on bonds had fallen considerably from its peak and was trending downward, reflecting a major decline in bond price volatility. These results suggest that Federal Reserve policy can have a profound effect on the level of long-term interest rates through the effect it has on their variability.

Appendix

We assume that there are 10 classes of assets—stocks, Treasury bills, and nominally risk-free (i.e., government) bonds of duration 1–8 years. We used monthly real rates of return taken from Ibbotson and Sinquefield's Treasury bill series, bond data from the United States government bond file of the Center for Research in Security Prices (CRSP), and stock returns from the New York Stock Exchange monthly CRSP file, adjusting when necessary by the consumer price index excluding the cost of shelter.

Land, residential housing, and consumer durables account for about 40% of household net worth. Unfortunately, there are no reliable rate of return data for a variety of assets that are not literally either stocks or bonds but that are often perceived as substitutes for those assets. Time and demand deposits, for example, are assumed to have the same rate of return as Treasury bills; corporate bonds and municipal bonds are assumed to be like government bonds; and noncorporate equity is assumed to have the same characteristics as equity. In preference to excluding these assets entirely from the market portfolio, we accept these assumptions and include these assets in the appropriate categories for the purpose of determining market weights.

In order to compute the covariance of the return on the market with the return on each asset category, we of course need to know the composition of the market portfolio (the market weights). Theoretically, this should reflect the percentage of household net worth invested in the assets that comprise the market portfolio. We used the *Flow of Funds Sector Balance Sheets* to obtain this breakdown for broad categories of assets for 1976 and 1980. The Treasury Department's *Monthly Statement of the Public Debt* was used to determine the relative quantities of government bonds of different maturities outstanding in those two years; and then maturities were converted into durations.

Table 3.A.1 gives the results of these calculations. The stock category included investment company shares plus other corporate equity plus equity in noncorporate business.

Duration data are not available on pension fund and life insurance reserves, which accounted for almost one-fifth of the financial net worth of households in both 1976 and 1980. We elected to compute asset weights under each of two assumptions—first, that these assets were spread evenly across durations 1–8 (the assumption used to allocate mortgages) and second, that they are predominantly long term. In the second case, we used the sum-of-the-years' digits method to allocate these assets triangularly across durations. The second case is probably more reasonable, inasmuch as pension reserve represents, for households, a long-term nominally

Table 3.A.1 Government Bond Weights

	1976		1980	
Duration	Corresponding Maturity (years)	Weight	Corresponding Maturity (years)	Weight
0	0– .25	.275	0– .25	.219
1	.25– 1.58	.357	.25– 1.60	.342
2	1.58– 2.65	.123	1.60– 2.75	.128
3	2.65– 4.00	.083	2.75– 4.20	.092
4	4.00– 5.45	.037	4.20– 5.75	.032
5	5.45– 7.13	.064	5.75– 7.80	.049
6	7.13– 8.80	.006	7.80–10.90	.031
7	8.80–10.60	.017	10.90–13.70	.021
8	10.60+	.036	13.70+	.097

Note: Flower bonds were omitted from the sample. The maturity date was taken to be first call date if the bond sold at a premium, and maturity otherwise.

Source: Weights by maturity from *Monthly Statement of the Public Debt*, May 1976, May 1980. Conversion to duration by using data on CRSP government bond files.

fixed claim. Table 3.A.2 reports results for both assumptions; the results reported in the body of the text are premised on the second assumption.

These market weights assume that the liabilities of financial intermediaries are treated as assets by the household sector, a coherent assumption under the finance paradigm. We also computed weights under the alternative assumption that financial intermediaries are a "veil," so that households behave as if they directly hold the assets of intermediaries.

The assumption that financial intermediaries are a veil results in a large redistribution of the weights from short- to long-term assets, while the weight on equity remains at the same level. Although the risk premiums on longer-term bonds rise appreciably, the overall pattern and levels of risk premiums remain much the same. For further details see Bodie et al. (1984).

3.A.1 Computing the Covariance Matrix

We first computed the expected real rate of return on Treasury bills by saying that the expected real yield over the coming month is the current nominal yield on a 30-day Treasury bill, i, less last month's inflation rate, π_{-1}:

$$R_0 = i - \pi_{-1}.$$

To obtain an initial covariance matrix, we used the covariance of the total rates of return for the 24 months before the month in which we started the procedure. In each succeeding month, the most recent unexpected returns were entered into the data matrix, and the previous 24 months of data were used to compute the covariance matrix. Thus, after the first 24

Table 3.A.2 Market Weights for the Household Sector

	Pension Fund Reserves Spread Evenly over Durations 1–8		Pension Fund Reserves Weighted toward High Durations	
	1976	1980	1976	1980
Stocks	.620	.640	.620	.640
Bonds: 0	.264	.262	.264	.262
1	.031	.026	.015	.007
2	.022	.017	.008	.003
3	.016	.013	.008	.005
4	.010	.007	.007	.005
5	.014	.009	.016	.012
6	.006	.007	.014	.016
7	.008	.006	.021	.020
8	.010	.013	.028	.032

months of the procedure, the covariance matrix was computed using only unexpected returns.

The entries in the covariance matrix were computed as

$$V_{ij}(T) = \frac{1}{24} \sum_{t=1}^{24} (R_{i,T-t} - \hat{R}_{i,T-t})(R_{j,T-1} - \hat{R}_{j,T-1}),$$

where R represents the realized real rate of return and R is the estimated expected real rate of return, computed using equation (1) and the previous month's covariance estimate.

An important advantage of this procedure is that the computed expected rates of return for each period are consistent with the model. The only part of this procedure which is ad hoc is the specification of the process generating expected rates of return on Treasury bills. The measure of covariance upon which the theory is based is the covariance of holding period rate of return deviations from expected rates of return. This is precisely what our procedure measures.

References

Bodie, A., Kane, A.; and McDonald, R. 1985. Inflation and the role of bonds in investor portfolios. In *Corporate Capital Structure in the United States*, edited by B. M. Friedman. Chicago: University of Chicago Press (for NBER).

______. 1984. Why haven't nominal rates declined? *Financial Analysts Journal* 40, no. 2 (March-April), 16-27.

Ibbotson, R. G., and Sinquefield, R. A. 1982. *Stocks, Bonds, Bills and Inflation: The Past and the Future*. Charlottesville, Va.: Financial Analysts Research Foundation.

Macaulay, F. R. 1938. *Some Theoretical Problems Suggested by the Movement of Interest Rates, Bond Yields, and Stock Prices in the U.S. since 1856*. New York: National Bureau of Economic Research, 1938.

Merton, R. C. 1980. On estimating the expected return on the market: an exploratory investigation. *Journal of Financial Economics* 8 (December), 323-61.