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7 Optimal Investment Strategies for University Endowment Funds

Robert C. Merton

7.1 Introduction

To examine the question of optimal investment strategies for university endowment funds, one must of course address the issue of the objective function by which optimality is to be measured. My impression is that practicing money managers essentially sidestep the issue by focusing on generically efficient risk-return objective functions for investment which are just as applicable to individuals or nonacademic institutions as they are to universities. Perhaps the most common objective of this type is mean-variance efficiency for the portfolio's allocations. Black (1976) provides a deeper approach along those lines that takes account of tax and other institutional factors, including certain types of nonendowment assets held by institutions. The Ford Foundation study of 1969 gave some early practical (if ex post, somewhat untimely) guidance for investment allocations.

Much of the academic literature (which is not copious) seems to focus on appropriate spending policy for endowment, taking as given that the objective for endowment is to provide a perpetual level flow of expected real income (cf. Eisner 1974; Litvack, Malkiel, and Quandt 1974; Nichols 1974; Tobin 1974). Ennis and Williamson (1976) present a history of spending patterns by universities and a discussion of various spending rules adopted. They also discuss the interaction between spending and investment policies. Fama and Jensen (1985) discuss the role of nonprofit institutions as part of a general analysis of organizational forms and investment objective functions, but they do not address the functions of endowment in such institutions.

In contrast, Hansmann (1990) provides a focused and comprehensive re-

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view of the various possible roles for a university's endowment. Despite the broad coverage of possibilities ranging from tax incentives to promoting intergenerational equity, he is unable to find compelling empirical evidence to support any particular combination of objectives. Indeed, he concludes that "prevailing endowment spending rules seem inconsistent with most of these objectives" (p. 39). Hansmann goes on to assert (pp. 39–40):

> It appears, however, that surprisingly little thought has been devoted to the purposes for which endowments are maintained and that, as a consequence, their rate of accumulation and the pattern of spending from their income have been managed without much attention to the ultimate objectives of the institutions that hold them.

The course taken here to address this question is in the middle range: it does not attempt to specify in detail the objective function for the university, but it does derive optimal investment and expenditure policy for endowment in a context which takes account of overall university objectives and the availability of other sources of revenue besides endowment. In that respect, it follows along lines similar to the discussion in Black (1976, 26–28). In addition, our model takes explicit account of the uncertainties surrounding the costs of university activities. As a result, the analysis reveals another (perhaps somewhat latent) purpose for endowment: namely, hedging against unanticipated changes in those costs. Formal trading rules for implementing this hedging function are derived in sections 7.3 and 7.4. However, the paper neither assesses which costs, as an empirical matter, are more important to hedge nor examines the feasibility of hedging those costs using available traded securities. The interested reader should see Brinkman (1981, 1990), Brovender (1974), Nordhaus (1989), and Snyder (1988), where the various costs of universities are described and modeled, both historically and prospectively.

Grinold, Hopkins, and Massy (1978) develop a budget-planning model which also integrates endowment returns with other revenue and expense flows of the university. However, their model differs significantly from the one presented here, perhaps because their focus is on developing policy guidelines for expenditures instead of optimal intertemporal management of endowment.

In the section to follow, we describe the basic insights provided by our analysis and discuss in a qualitative fashion the prescriptions for endowment policy. The formal mathematical model for optimal expenditures and investment that supports those prescriptions is developed in sections 7.3 and 7.4. It is based on a standard intertemporal consumption and portfolio-selection model. Hence, the formal structure of the optimal demand functions is already widely studied in the literature. It is the application of this model to the management of university endowment which is new. For analytical simplicity and clarity, the model is formulated in continuous time. However, it is evident from the work of Constantinides (1989), Long (1974), and Merton (1977) that a discrete-time version of the model would produce similar results.

7.2 Overview of Basic Insights and Prescriptions for Policy

As indicated at the outset, a standard approach to the management of endowment is to treat it as if it were the only asset of the university. A consequence of this approach is that optimal portfolio strategies are focused exclusively on providing an efficient trade-off between risk and expected return. The most commonly used measure of endowment portfolio risk is the variance (or equivalently, standard deviation) of the portfolio's return. As is well known, the returns on all mean-variance efficient portfolios are perfectly correlated. Thus, a further consequence of treating endowment as the only asset is that the optimal endowment portfolios of different universities should have quite similar risky investment allocations, at least as measured by the correlations of the portfolio returns.

Universities, as we all know, do have other assets, both tangible and intangible, many of which are important sources of cash flow. Examples of such sources are gifts, bequests, university business income, and public- and private-sector grants. Taking explicit account of those assets in the determination of the endowment portfolio can cause the optimal composition of that portfolio to deviate significantly from mean-variance efficiency. That is, two universities with similar objectives and endowments of the same size can nevertheless have very different optimal endowment portfolios if their nonendowment sources of cash flow are different.

A procedure for selecting the investments for the endowment portfolio that takes account of nonendowment assets includes the following steps:

1. Estimate the market value that each of the cash flow sources would have if it were a traded asset. Also determine the investment risk characteristics that each of those assets would have as a traded asset.

2. Compute the *total wealth* or net worth of the university by adding the capitalized values of all the cash flow sources to the value of the endowment.

3. Determine the optimal portfolio allocation among traded assets, using the university's total wealth as a base. That is, treat both endowment and cash flow–source assets as if they could be traded.

4. Using the risk characteristics determined in step 1, estimate the "implicit" investment in each traded-asset category that the university has as the result of owning the nonendowment (cash flow–source) assets. Subtract those implicit investment amounts from the optimal portfolio allocations computed in step 3, to determine the optimal "explicit" investment in each traded asset, which is the actual optimal investment allocation for the endowment portfolio.

As a simple illustration, consider a university with $400 million in endowment assets and a single nonendowment cash flow source. Suppose that the only traded assets are stocks and cash. Suppose further that the university estimates in step 1 that the capitalized value of the cash flow source is $200 million, with risk characteristics equivalent to holding $100 million in stock and \$100 million in cash. Thus, the total wealth of the university in step 2 is (400 + 200 =) \$600 million. Suppose that from standard portfolio-selection techniques, the optimal fractional allocation in step 3 is .6 in stocks and .4 in cash, or \$360 million and \$240 million, respectively. From the hypothesized risk characteristics in step 1, the university already has an (implicit) investment of \$100 million in stocks from its nonendowment cash flow source. Therefore, we have in step 4 that the optimal amount for the endowment portfolio to invest in stocks is \$260 million, the difference between the \$360 million optimal total investment in stocks and the \$100 million implicit part. Similarly, the optimal amount of endowment invested in cash equals (240 − 100 =) \$140 million.

The effect on the composition of the optimal endowment portfolio induced by differences in the size of nonendowment assets can be decomposed into two parts: the wealth effect and the substitution effect. To illustrate the wealth effect, consider two universities with identical preference functions and the same size endowments, but one has nonendowment assets and the other does not. If, as is perhaps reasonable to suppose, the preference function common to each exhibits decreasing absolute risk aversion, then the university with the nonendowment assets (and hence larger net worth) will prefer to have a larger total investment in risky assets. So a university with a \$400 million endowment as its only asset would be expected to choose a dollar exposure to stocks that is smaller than the \$360 million chosen in our simple example by a university with the same size endowment and a nonendowment asset valued at \$200 million. Such behavior is consistent with the belief that wealthier universities can "afford" to take larger risks with their investments. Thus, if the average risk of the nonendowment assets is the same as the risk of the endowment-only university's portfolio, then the university with those assets will optimally invest more of its endowment in risky assets.

The substitution effect on the endowment portfolio is caused by the substitution of nonendowment asset holdings for endowment asset holdings. To illustrate, consider again our simple example of a university with a \$400 million endowment and a \$200 million nonendowment asset. However, suppose that the risk characteristics of the asset are changed so that it is equivalent to holding \$200 million in stocks and no cash. Now, in step 4, the optimal amount for the endowment portfolio to invest in stocks is \$160 million, the difference between the \$360 million optimal total investment in stocks and the \$200 million implicit part represented by the nonendowment asset. The optimal amount of endowment invested in cash rises to (240 − 0 =) \$240 million. If instead the risk characteristics of the asset had changed in the other direction to an equivalent holding of \$0 in stocks and \$200 million in cash, the optimal composition of the endowment portfolio would be (360 − 0 =) \$360 million in stocks and (240 − 200 =) \$40 million in cash.

Note that the changes in risk characteristics do not change the optimal deployment of *total* net worth (\$360 million in stocks and \$240 million in cash).

However, the nonendowment assets are not carried in the endowment portfolio. Hence, different risk characteristics for those assets do change the amount of substitution they provide for stocks and cash in the endowment portfolio. Thus, the composition of the endowment portfolio will be affected in both the scale and fractional allocations among assets.

With the basic concept of the substitution effect established, we now apply it in some examples to illustrate its implications for endowment investment policy. Consider a university that on a regular basis receives donations from alums. Clearly, the cash flows from future contributions are an asset of the university, albeit an intangible one. Suppose that the actual amount of gift giving is known to be quite sensitive to the performance of the general stock market. That is, when the market does well, gifts are high; when it does poorly, gifts are low. Through this gift-giving process, the university thus has a "shadow" investment in the stock market. Hence, all else the same, it should hold a smaller portion of its endowment in stocks than would another university with smaller amounts of such market-sensitive gift giving.

The same principle applies to more specific asset classes. If an important part of gifts to a school that specializes in science and engineering comes from entrepreneur alums, then the school de facto has a large investment in venture capital and high-tech companies, and it should therefore invest less of its endowment funds in those areas. Indeed, if a donor is expected to give a large block of a particular stock, then the optimal explicit holding of that stock in the endowment can be negative. Of course, an actual short position may not be truly optimal if such short sales offend the donor. That the school should optimally invest less of its endowment in the science and technology areas where its faculty and students have special expertise may seem a bit paradoxical. But the paradox is resolved by the principle of diversification once the endowment is recognized as representing only a part of the assets of the university.

The same analysis and conclusion apply if alum wealth concentrations are in a different class of assets, such as real estate instead of shares of stock. Moreover, much the same story also applies if we were to change the example by substituting government and corporate grants for donations and gift giving as the sources of cash flows. That is, the magnitudes of such grant support for engineering and applied science may well be positively correlated with the financial performance of companies in high-tech industries. If so, then the prospect of future cash flows to the university from the grants creates a shadow investment in those companies.

The focus of our analysis is on optimal asset allocation for the endowment portfolio. However, the nature and size of a university's nonendowment assets significantly influence optimal policy for spending endowment. As shown in section 7.4, for a given overall expenditure rate as a fraction of the university's total net worth, the optimal spending rate out of endowment will vary, depending on the fraction of net worth represented by nonendowment assets, the expected growth rate of cash flows generated by those assets, and capitalization rates. Hence, neglecting those other assets will generally bias the optimal expenditure policy for endowment.

In addition to taking account of nonendowment assets, our analysis differs from the norm because it takes account of the uncertainty surrounding the costs of the various activities such as education, research, and knowledge storage that define the purpose of the university. The breakdown of activities can of course be considerably more refined. For instance, one activity could be the education of a full-tuition–paying undergraduate, and a second could be the education of an undergraduate who receives financial aid. The unit (net) cost of the former is the unit cost of providing the education less the tuition received, and the unit cost of the latter is this cost plus the financial aid given. As formally demonstrated in section 7.3, an important function of endowment investments is to hedge against unanticipated changes in the costs of university activities.

Consider, for example, the decision as to how much (if any) of the university's endowment to invest in local residential real estate. From a standard mean-variance efficiency analysis, it is unlikely that any material portion of the endowment should be invested in this asset class. However, consider the cost structure faced by the university for providing teaching and research. Perhaps the single largest component is faculty salaries. Universities of the same type and quality compete for faculty from the same pools. To be competitive, they must offer a similar standard of living. Probably the largest part of the differences among universities in the cost of providing this same standard of living is local housing costs. The university that invests in local residential housing hedges itself against this future cost uncertainty by acquiring an asset whose value is higher than expected when the differential cost of faculty salaries is higher than expected. This same asset may also provide a hedge against unanticipated higher costs of off-campus housing for students that would in turn require more financial aid if the university is to compete for the best students. Note that this prescription of targeted investment in very specific real estate assets to hedge against an unanticipated rise in a particular university's costs of faculty salaries and student aid should not be confused with the often-stated (but empirically questionable) assertion that investments in real estate generally are a good hedge against inflation. See Bodie (1976, 1982) for empirical analysis of the optimal assets for hedging against general inflation.

Similar arguments could be used to justify targeted investment of endowment in various commodities such as oil as hedges against unanticipated changes in energy costs. Uncertainty about those costs is especially significant for universities located in extreme climates and for universities with major laboratories and medical facilities that consume large quantities of energy.

The hedging role for endowment can cause optimal investment positions that are in the opposite direction from the position dictated by the substitution effects of nonendowment assets. For example, consider a specialized institute of biology that receives grants from biotech companies and gifts from financially successful alums. As already explained, such an institute has a large shadow investment in biotech stocks, and it should therefore underweight (perhaps to zero) its endowment investments in such stocks. Suppose, however, that the institute believes the cost of keeping top faculty will rise by considerably more than tuition or grants in the event that there is a strong demand for such scientists outside academe. Then it may be optimal to invest a portion of its endowment in biotech stocks to hedge this cost, even though those stocks' returns are highly correlated with alum gifts and industry grants.

As demonstrated in section 7.3, the hedging role for endowment derived here is formally valid as long as there are traded securities with returns that have nonzero correlations with unanticipated changes in the activity costs. However, the practical significance for this role turns on the magnitude of the correlations. As illustrated in Bodie's (1976, 1982) work on hedging against inflation, it is often difficult to construct portfolios (using only standard types of traded securities) that are highly correlated with changes in the prices of specific goods and services. Nevertheless, the enormous strides in financial engineering over the last decade have greatly expanded the opportunities for custom financial contracting at reasonable costs. As we move into the twenty-first century, it will become increasingly more common for the financial services industry to offer its customers private contracts or securities that allow efficient hedging when the return properties of publicly traded securities are inadequate. That is, implementation of the quantitative strategies prescribed in sections 7.3 and 7.4 will become increasingly more practical for universities and other endowment institutions. See Merton (1990b, chap. 14; 1990c, 264–69) for a prospective view on financial innovation and the development of custom financial contracting.

There are of course a variety of issues involving endowment management that have not been addressed but could be within the context of our model. One such issue is the decision whether to invest endowment in specific-purpose real assets such as dormitories and laboratories instead of financial (or general-purpose physical) assets. The returns on those real assets are likely to be strongly correlated with the costs of particular university activities, and thereby the assets form a good hedge against unexpected rises in those costs. However, because the real-asset investments are specialized and largely irreversible, shifting the asset mix toward such investments reduces flexibility for the university. That is, with financial assets, the university has more options as to what it can do in the future. In future research, I plan to analyze this choice problem more formally by using contingent-claims analysis to value the trade-off between greater flexibility in selecting future activities and lower costs in producing a given set of activities.

Another issue not explicitly examined is the impact long-term, fixed liabilities such as faculty tenure contracts have on the management of endowment.

Our formal model of sections 7.3 and 7.4 that uses contingent-claims analysis (CCA) can handle this extension. See McDonald (1974) and Merton (1985) for CCA-type models for valuing tenure and other wage guarantee contracts.

In summary, the paper explores two classes of reasons why optimal endowment investment policy and expenditure policy can vary significantly among universities. The analysis suggests that trustees and others who judge the prudence and performance of policies by comparisons across institutions should take account of differences in both the mix of activities of the institutions and the capitalized values of their nonendowment sources of cash flows.

The overview completed, we now turn to the development of the mathematical model for the process and the derivation of the quantitative rules for implementation.

7.3 The Model

The functions or purposes of the university are assumed to be a collection of activities or outputs such as education, training, research, and storage of knowledge. We further assume that the intensities of those activities can be quantified and that a preference ordering exists for ranking alternative intertemporal programs. In particular, the criterion function for this ranking can be written as

$$(1) \qquad \max E_0\left[\int_0^\infty \bar{U}(Q_1, \ldots, Q_m, t)dt\right],$$

where $Q_j(t)$ denotes the quantity of activity j per unit time undertaken at time t, $j = 1, \ldots, m$; the preference function $\bar{U}$ is assumed to be strictly concave in $(Q_1, \ldots, Q_m)$; and E_t denotes the expectation operator, conditional on knowing all relevant information as of time t. This preference ordering satisfies the classic von Neumann–Morgenstern axioms of choice, exhibits positive risk aversion, and includes survival (of the institution) as a possible objective. The infinite time horizon structure in (1) implies only that there need not be a definite date when the university will liquidate. As shown in Merton (1990b, 149–51, 609–11), $\bar{U}$ can reflect the mortality characteristics of an uncertain liquidation date.

The intertemporally additive and independent preference structure in (1) can be generalized to include nonadditivity, habit formation, and other path-dependent effects on preferences, along the lines of Bergman (1985), Constantinides (1990), Detemple and Zapatero (1989), Duffie and Epstein (1992), Hindy and Huang (1992), Sundaresan (1989), and Svensson (1989). However, as shown in Merton (1990b, 207–9), those more realistic preference functions do not materially affect the optimal portfolio demand functions. Moreover, just as Grossman and Laroque (1990) show for transactions costs in consumption, so it can be shown here that imposing adjustment costs for changing the levels of university activities does not alter the structure of the portfolio demand functions. Hence, because the focus of the paper is on optimal investment (rather than on optimal expenditure) strategies, we assume no adjustment costs for activities and retain the additive independent preference specification to provide analytical simplicity.

Let $S_j(t)$ denote the (net) cost to the university of providing one unit of activity j at time t, $j = 1, \ldots, m$. For example, if $j = 1$ denotes the activity of having full-tuition–paying undergraduates, then S_1 would be the unit cost of providing the education minus the tuition received. If $j = 2$ denotes the activity of having undergraduates who receive financial aid, the unit cost S_2 would equal S_1 plus the financial aid given. In general, all costs and receipts such as tuition that are directly linked to the quantities of specific activities undertaken are put into the activity costs or prices, (S_j). As will be described, fixed costs and sources of positive cash flows to the university that do not depend directly on the activity quantities are handled separately. As in Merton (1990b, 202, 499), we assume that the dynamics for these costs are described by the stochastic differential equations: for $S \equiv (S_1, \ldots, S_m)$,

$$dS_j = f_j(S,t)S_j dt + g_j(S,t)S_j dq_j \,, j = 1, \ldots, m \,, \tag{2}$$

where f_j is the instantaneous expected rate of growth in S_j, g_j is the instantaneous standard deviation of the growth rate, and dq_j is a Wiener process with the instantaneous correlation coefficient between dq_i and dq_j given by υ_{ij}, $i, j = 1, \ldots, m$. f_j and g_j are such that $dS_j \geq 0$ for $S_j = 0$, which ensures that $S_j(t) \geq 0$. Especially since (S) has components that depend on tuition, financial aid, and other variables over which the university has some control, one would expect that the dynamic path for those costs would be at least partially endogenous and controllable by the university, even though competition among universities would limit the degree of controllability. However, as specified, (2) is an exogenous process, not controlled by the university. Alternatively, it can be viewed as the "reduced-form" process for S after optimization over nonportfolio choice variables.

The university is assumed to have N nonendowment sources of cash flows, which we denote by $Y_k(t)dt$ for the kth source at time t. As noted in section 7.2, examples of such sources are gifts, bequests, university business income, and public- and private-sector grants. It can also be used to capture transfer pricing for the use of buildings and other university-specific assets where Y_k is the rental rate and this rental fee appears as an offsetting charge in the (S_j) for the appropriate university activities. The dynamics for these cash flows are modeled by, for $Y \equiv (Y_1, \ldots, Y_N)$,

$$dY_k = \mu_k(Y,S,t)Y_k dt + \delta_k(Y,S,t)Y_k d\varepsilon_k \,, \tag{3}$$

where μ_k and δ_k depend at most on the current levels of the cash flows and the unit costs of university activities and $d\varepsilon_k$ is a Wiener process, $k = 1, \ldots, N$. Equation (3) can also be used to take account of fixed costs or liabilities of the university such as faculty tenure commitments, by letting $Y_k < 0$ to reflect a cash outflow. However, the focus here is on assets only, and therefore we assume that μ_k and δ_k are such that $dY_k \geq 0$ for $Y_k = 0$, which implies that $Y_k(t) \geq 0$ for all t.

By inspection of (2) and (3), the dynamics for (Y,S) are jointly Markov. A more realistic model would have μ_k and δ_k depend on both current and historical values of $Q_1, \ldots, Q_m$. For example, if a university has undertaken large amounts of research activities in the past, it may attract more grants and gifts in the future. The university may also affect the future expected cash flows from nonendowment sources by investing now in building up those sources. Thus, the dynamic process for Y should be in part controllable by the university. However, again for analytical simplicity, the Y process is taken as exogenous, because that abstraction does not significantly alter the optimal portfolio demand functions.

If for $k = 1, \ldots, N$, $V_k(t)$ denotes the capitalized value at time t of the stream of future cash flows, $Y_k(\tau)$ for $\tau \geq t$, and if $K(t)$ denotes the value of the endowment at time t, then the net worth or wealth of the university, $W(t)$ is given by

$$W(t) = K(t) + \sum_1^n V_k(t) \; . \tag{4}$$

A model for determining the $V_k(t)$ from the posited cash flow dynamics in (3) is developed in section 7.4.

The endowment of the university is assumed to be invested in traded assets. There are n risky assets and a riskless asset. If $P_j(t)$ denotes the price of the jth risky asset at time t, then the return dynamics for the risky assets are given by, for $j = 1, \ldots, n$,

$$dP_j = \alpha_j P_j dt + \sigma_j P_j dZ_j \; , \tag{5}$$

where α_j is the instantaneous expected return on asset j; σ_j is the instantaneous standard deviation of the return; and dZ_j is a Wiener process. The instantaneous correlation coefficients $(\rho_{ij}, \eta_{kj}, \zeta_{lj})$ are defined by, for $j = 1, \ldots, n$,

$$\begin{aligned} dZ_i dZ_j &= \rho_{ij} dt \;\; , \; i = 1, \ldots, n \\ dq_k dZ_j &= \eta_{kj} dt \;\; , \; k = 1, \ldots, m \\ d\varepsilon_l dZ_j &= \zeta_{lj} dt \;\; , \; l = 1, \ldots, N \; . \end{aligned} \tag{5a}$$

For computational simplicity and to better isolate the special characteristics of endowment management from general portfolio management, we simplify the return dynamics specification and assume that $(\alpha_j, \sigma_j, \rho_{ij})$ are constants over time, $i, j = 1, \ldots, n$. As shown in Merton (1990b, chaps. 4, 5, 6), this assumption of a constant investment opportunity set implies that $[P_j(t + \tau)/P_j(t)]$, $j = 1, \ldots, n$, for $\tau > 0$ are jointly lognormally distributed. The riskless asset earns the interest rate r, which is also constant over time. Optimal portfolio selection for general return dynamics would follow along the lines of Merton (1990a, sec. 7; 1990b, chaps. 5, 15, 16).

To analyze the optimal intertemporal expenditure and portfolio-selection problem for the university, we begin with a further simplified version of the model in which the university's entire net worth is endowment (i.e., $Y_k[t] = V_k(t) = 0$, $k = 1, \ldots, N$ and $W(t) = K[t]$). The budget equation dynamics for $W(t)$ are then given by

$$dW = \{[\sum_1^n w_j(t)(\alpha_j - r) + r]\,W - \sum_1^m Q_k S_k\}dt + \sum_1^n w_j(t) W \sigma_j dZ_j\,, \tag{6}$$

where $w_j(t)$ = the fraction of the university's wealth allocated to risky asset j at time t, $j = 1, \ldots, n$; the fraction allocated to the riskless asset is thus $1 - \Sigma_1^n w_j$. Trustees, donors, and the government are assumed not to impose explicit limitations on investment policy for the endowment, other than general considerations of prudence. In particular, borrowing and short selling are permitted, so the choice for (w_j) is unrestricted. We further posit that spending out of endowment is not restricted, either with respect to overall expenditure or with respect to the specific activities on which it is spent. However, we do impose the feasibility restrictions that total expenditure at time t, $\Sigma_1^m Q_k S_k$, must be nonnegative and zero wealth is an absorbing state (i.e., $W[t] = 0$ implies $W[t + \tau] = 0$ for $\tau > 0$).

At each time t, the university chooses a quantity of activities $(Q_1, \ldots, Q_m)$ and a portfolio allocation of its wealth so as to maximize lifetime utility of the university as specified in (1). Just as for the case of multiple consumption goods analyzed in Breeden (1979), Fischer (1975), and Merton (1990b, 205), so the solution for the optimal program here can be decomposed into two parts. First, at each t, solve for the utility-maximizing quantities of individual activities, $(Q_1, \ldots, Q_m)$, subject to an overall expenditure constraint, $C(t) = \Sigma_1^m Q_k(t) S_k(t)$. Second, solve for the optimal level of overall expenditures at time t and the optimal portfolio allocation of endowment.

The first part is essentially the static activity-choice problem with no uncertainty

$$\max_{(Q_1, \ldots, Q_m)} \bar{U}(Q_1, \ldots, Q_m, t) \tag{7}$$

subject to $C(t) = \Sigma_1^m Q_k S_k(t)$. The first-order conditions for the optimal activity bundle $(Q_1^*, \ldots, Q_m^*)$ are given by, for $S_k(t) = S_k$,

$$\frac{\bar{U}_k(Q_1^*, \ldots, Q_m^*, t)}{S_k} = \frac{\bar{U}_j(Q_1^*, \ldots, Q_m^*, t)}{S_j} \quad k,j = 1, \ldots, m \tag{8}$$

with $C(t) = \Sigma_1^m Q_k^* S_k$, where subscripts on $\bar{U}$ denote partial derivatives (i.e., $\bar{U}_k \equiv \partial \bar{U}/\partial Q_k$). It follows from (8) that the optimal quantities can be written as $Q_k^* = Q_k^*[C(t), S(t), t]$, $k = 1, \ldots, m$.

Define the indirect utility function U by $U[C(t), S(t), t] \equiv \bar{U}(Q_1^*, \ldots, Q_m^*, t)$. By substituting U for $\bar{U}$, we can rewrite (1) as

$$\max E_0\left\{\int_0^{\infty} U[C(t),S(t),t]dt\right\}, \tag{9}$$

where the "max" in (9) is over the intertemporal expenditure path $[C(t)]$ and portfolio allocations $[w_j(t)]$. Thus, the original optimization problem is transformed into a single-expenditure choice problem with "state-dependent" utility (where the "states" are the relative costs or prices of the various activities). Once the optimal total expenditure rules, $[C^*(t)]$, are determined, the optimal expenditures on individual activities are determined by (8) with $C^*(t) = \Sigma_1^m Q_k^* S_k$.

The solution of (9) follows by applying stochastic dynamic programming as in Merton (1990b, chaps. 4, 5, 6). Define the Bellman, or derived-utility, function J by

$$J(W,S,t) \equiv \max E_t\left\{\int_t^{\infty} U[C(\tau),S(\tau),\tau]d\tau\right\}$$

conditional on $W(t) = W$ and $S(t) = S$. From Merton (1990a, 555; 1990b, 181, 202), J will satisfy

$$\begin{aligned} 0 = \max_{(C,w)} \Big[& U(C,S,t) + \lambda C + J_t + J_W\{[\sum_1^n w_j(\alpha_j - r) + r]W - C\} \\ & + \sum_1^m J_i f_i S_i + \frac{1}{2} J_{WW} \sum_1^n \sum_1^n w_i w_j \sigma_{ij} W^2 \\ & + \sum_1^m \sum_1^n J_{iW} w_j W g_i S_i \sigma_j \eta_{ij} + \frac{1}{2} \sum_1^m \sum_1^m J_{ij} g_i S_i g_j S_j \nu_{ij} \Big] \end{aligned} \tag{10}$$

subject to $J(0,S,t) = \int_t^{\infty} \bar{U}(0, \ldots ,0,\tau)d\tau$, where subscripts on J denote partial derivatives with respect to W, t, and S_i, $i = 1, \ldots ,m$ and $\sigma_{ij} \equiv \rho_{ij}\sigma_i\sigma_j$, the instantaneous covariance between the return on security i and j. λ is a Kuhn-Tucker multiplier reflecting the nonnegativity constraint on C, and at the optimum it will satisfy $\lambda^* C^* = 0$. The first-order conditions derived from (10) are

$$0 = U_C(C^*,S,t) + \lambda^* - J_W(W,S,t) \tag{11a}$$

and

$$\begin{aligned} 0 = {} & J_W(\alpha_i - r) + J_{WW}\sum_1^n w_j^* W \sigma_{ij} \\ & + \sum_1^m J_{kW} g_k S_k \sigma_i \eta_{ki}, \quad i = 1, \ldots ,n, \end{aligned} \tag{11b}$$

where $C^* = C^*(W,S,t)$ and $w_i^* = w_i^*(W,S,t)$ are the optimal expenditure, and portfolio rules expressed as functions of the state variables and subscripts on U denote partial derivatives.

From (11a), the optimal expenditure rule is given by

$$(12)\quad \begin{aligned} U_C(C^*,S,t) &= J_W(W,S,t) \text{ for } C^* > 0 \\ \lambda^* &= \max\,[0, J_W(W,S,t) - U_C(0,S,t)] \end{aligned}$$

From (11b), the optimal portfolio allocation can be written as

$$(13)\quad w_i^* W = Ab_i + \sum_1^m H_k h_{ki}\ ,\ i = 1, \ldots, n\ ,$$

where $b_i \equiv \Sigma_1^n v_{ij}(\alpha_j - r)$; $h_{ki} \equiv \Sigma_1^n \sigma_j g_k S_k \eta_{kj} v_{ij}$; v_{ij} is the ij element of the inverse of the instantaneous variance-covariance matrix of returns (σ_{ij}); $A \equiv -J_W/J_{WW}$ (the reciprocal of absolute risk aversion of the derived-utility function); and $H_k \equiv -J_{kW}/J_{WW}$, $k = 1, \ldots, m$. A and H_k depend on the individual university's intertemporal preferences for expenditures and its current net worth. However, b_i and h_{ki} are determined entirely by the dynamic structures for the asset price returns and the unit costs of the various activities undertaken by universities. Hence, those parameters are the same for all universities, independent of their preferences or endowment size.

To provide some economic intuition about the optimal allocation of endowment in (13), consider as a frame of reference the "standard" intertemporal portfolio-selection problem with state-independent utility, $U = U(C(t),t)$. As shown in Merton (1990b, 131–36), given the posited return dynamics in (5), all such investors will hold instantaneously mean-variance efficient portfolios as their optimal portfolios. For $\partial U/\partial S_k \equiv U_k \equiv 0$, $H_k \equiv 0$, $k = 1, \ldots, m$. Hence, in this case, (13) becomes $w_i^* W = Ab_i$, and $w_i^* W / w_j^* W = b_i/b_j$, the same for all investors. This is the well-known result that the relative holdings of risky assets are the same for all mean-variance efficient portfolios. However, the state-dependent preferences for universities induced by the uncertainty surrounding the relative costs of undertaking different desired activities causes the more complex demand structure in (13).

To better understand this differential demand, $w_i^* W - Ab_i = \Sigma_1^m H_k h_{ki}$, it is useful to examine the special case where for each cost S_k there exists an asset whose instantaneous return is perfectly correlated with changes in S_k. By renumbering securities if necessary, choose the convention that $\eta_{kk} = 1$ in (5a), $k = 1, \ldots, m$ $(m < n)$. As shown in Merton (1990b, 203–4), it follows that in this case, $h_{kk} = g_k S_k/\sigma_k$ for $k = 1, \ldots, m$ and $h_{kj} = 0$ for $k \neq j$. Hence, we can rewrite (13) as

$$(14)\quad \begin{aligned} w_i^* W &= Ab_i + \frac{H_i g_i S_i}{\sigma_i} & i &= 1, \ldots, m \\ &= Ab_i & i &= m + 1, \ldots, n\ . \end{aligned}$$

By the strict concavity of U with respect to C, J is strictly concave in W. Hence, $J_{WW} < 0$ and $H_i = -J_{iW}/J_{WW}$ is positively proportional to J_{iW}. Thus, relative to a "normal" investor with state-independent preferences (i.e., $H_i \equiv 0$, $i = 1, \ldots, m$) but the same current level of absolute risk aversion (i.e., $-J_{WW}/J_W$), the university will optimally hold more of asset i if $J_{iW} > 0$ and less if $J_{iW} < 0$, $i = 1, \ldots, m$.

If $J_{iW} > 0$, then at least locally the university's marginal utility (or "need") for wealth or endowment becomes larger if the cost of undertaking activity i increases, and it becomes smaller if this cost decreases. Because the return on asset i is perfectly positively correlated with the cost of activity i, a greater than expected increase in S_i will coincide with a greater than expected return on asset i. By holding more of asset i than a "normal" investor would, the university thus assures itself of a relatively larger endowment in the event that S_i increases and the need for wealth becomes more important. The university, of course, pays for this by accepting a relatively smaller endowment in the event that S_i decreases and wealth is less important. The behavioral description for $J_{iW} < 0$ is just the reverse, because the need for endowment decreases if the cost of activity i increases.

To perhaps help in developing further insights, we use (12) to interpret the differential demand component in (14) in terms of the indirect utility and optimal expenditure functions. By differentiating (12), we have that, for $C^*(W,S,t) > 0$,

$$
\begin{aligned}
J_{WW} &= U_{CC}(C^*,S,t)\,\frac{\partial C^*}{\partial W} \\
J_{kW} &= U_{CC}(C^*,S,t)\,\frac{\partial C^*}{\partial S_k} + U_{Ck}(C^*,S,t) \\
A &= \frac{-U_C(C^*,S,t)}{U_{CC}(C^*,S,t)\,\dfrac{\partial C^*}{\partial W}} \\
H_k &= \frac{\dfrac{-\partial C^*}{\partial S_k}}{\dfrac{\partial C^*}{\partial W}} + \frac{U_{Ck}(C^*,S,t)}{-U_{CC}(C^*,S,t)\,\dfrac{\partial C^*}{\partial W}}
\end{aligned}
\tag{15}
$$

for $k = 1, \ldots, m$. Because $U_{CC} < 0$ and $\partial C^*/\partial W > 0$ for $C^* > 0$, we see that the sign of H_k is determined by the impact of a change in the cost of activity k on two items: the optimal level of total current expenditure and the marginal utility of expenditure. So, for example, if an increase in S_k would cause both a decrease in optimal expenditure ($\partial C^*/\partial S_k < 0$) and an increase in the marginal utility of expenditure ($U_{Ck} > 0$), then, from (15), $H_k > 0$ and the university will optimally hold more of asset k than the corresponding investor with a mean-variance efficient portfolio.

Following (14) causes the university's optimal portfolio to be mean-variance inefficient, and therefore the return on the endowment will have greater volatility than other feasible portfolios with the same expected return. However, the value of the endowment or net worth of the university is not the "end" objective. Instead, it is the "means" by which the ends of a preferred expenditure policy can be implemented. Viewed in terms of the volatility of the time path of *expenditure* (or more precisely, the *marginal utility of expenditure*), the optimal strategy given in (14) is mean-variance efficient (cf. Breeden 1979; Merton 1990b, 487–88). That is, because $\partial C^*/\partial W > 0$, the additional increment in wealth that, by portfolio construction, occurs precisely when S_k increases will tend to offset the negative impact on C^* caused by that increase. There is thus a dampening of the unanticipated fluctuations in expenditure over time. In sum, we see that in addition to investing in assets to achieve an efficient risk-return trade-off in wealth, universities should optimally use their endowment to hedge against unanticipated and unfavorable changes in the costs of the various activities that enter into their direct utility functions.

In closing this section, we note that the interpretation of the demand functions in the general case of (13) follows along the same lines as for the special case of perfect correlation leading to (14). As shown for the general case in Merton (1990a, 558–59; 1990b, 501–2), the differential demands for assets reflect attempts to create portfolios with the maximal feasible correlations between their returns and unanticipated changes in the S_k, $k = 1, \ldots, m$. These maximally correlated portfolios perform the same hedging function as assets $1, \ldots, m$ in the limiting case of perfect correlation analyzed in (14). Furthermore, if other state variables besides the various activities' costs (e.g., changes in the investment opportunity set) enter a university's derived utility function, then a similar structure of differential asset demands to hedge against the unanticipated changes in these variables will also obtain.

7.4 Optimal Endowment Management with Other Sources of Income

In the previous section, we identified hedging of the costs of university activities as a reason for optimally deviating from "efficient" portfolio allocations when endowment is the only means for financing those activities. In this section, we extend the analysis to allow other sources of cash flow to support the activities. To simplify the analysis, we make two additional assumptions. First, we posit that μ_k and δ_k in (3) are constants, which implies that $Y_k(t)/Y_k(0)$ is lognormally distributed, $k = 1, \ldots, N$. Second, we assume that for each k there exists a traded security whose return is instantaneously perfectly correlated with the unanticipated change in Y_k, $k = 1, \ldots, N$. By renumbering if necessary, we use the convention that traded security k is instantaneously perfectly correlated with Y_k. Hence, it follows that $\zeta_{kk} = 1$ in (5a) and

$$d\varepsilon_k = dZ_k\,, \quad k = 1, \ldots, N\,. \tag{16}$$

These two assumptions permit us to derive a closed-form solution for the capitalized values of the cash flows, $[V_k(t)]$, using contingent-claims analysis. As will be shown, those valuation functions are independent of the university's preferences or wealth level.

From (3), (5), and (16) with μ_k and δ_k constant, we have that the cash flows can be written as a function of the traded asset prices as follows, for $k = 1, \ldots, N$,

$$Y_k(t) = Y_k(0)\exp(-\phi_k t)\,[P_k(t)/P_k(0)]^{\beta_k}, \tag{17}$$

where $\phi_k \equiv \beta_k(\alpha_k - \sigma_k^2/2) - (\mu_k - \delta_k^2/2)$ and $\beta_k \equiv \delta_k/\sigma_k$. That (17) obtains can be checked by applying Itô's Lemma. We now derive the capitalized value for Y_k, following Merton (1990a, 562–63; 1990b, 415–19).

Let $F^k(P_k, t)$ be the solution to the partial differential equation, for $0 \leq t \leq T_k$,

$$0 = 1/2\,\sigma_k^2 P_k^2 F_{11}^k + rP_k F_1^k - rF^k + F_2^k + Y_k \tag{18}$$

subject to the boundary conditions:

$$F^k(0,t) = 0 \tag{19a}$$

$$F^k/(P_k)^{\beta_k} \text{ bounded as } P_k \to \infty \tag{19b}$$

$$F^k(P_k,T_k) = 0\,, \tag{19c}$$

where subscripts on F^k in (18) denote partial derivatives with respect to its arguments P_k and t; Y_k is given by (17); and T_k is the last date at which the university receives the cash flows from source k, $k = 1, \ldots, N$. It is a mathematical result that a solution exists to (18)–(19) and that it is unique. Moreover, for $Y_k \geq 0$, $F^k \geq 0$ for all P_k and t.

Consider a dynamic portfolio strategy in which $F_1^k[P_k(t),t]P_k(t)$ is allocated to traded asset k at time t and $V(t) - F_1^k[P_k(t),t]P_k(t)$ is allocated to the riskless asset, where $V(t)$ is the value of the portfolio at time t. Furthermore, let the portfolio distribute cash (by selling securities if necessary) according to the flow-rate rule

$$D_2(P_k,t) = Y_k(t) \tag{20}$$

as given by (17). Then the dynamics of the portfolio can be written as, for $P_k(t) = P_k$ and $V(t) = V$,

$$dV = F_1^k(P_k,t)dP_k + \{[V - F_1^k(P_k,t)P_k]r - D_2(P_k,t)\}dt\,. \tag{21}$$

Since F^k satisfies (18), it is a twice continuously differentiable function and therefore, by Itô's Lemma, we can write the dynamics for F^k as

$$dF^k = 1/2\,\sigma_k^2\,P_k^2\,F_{11}^k + F_2^k\,dt + F_1^k dP_k\,. \tag{22}$$

But F^k satisfies (18) and hence, $1/2\,\sigma_k^2\,P_k^2\,F_{11}^k + F_2^k = rF^k - rP_kF_1^k - Y_k$. Substituting into (22), we can rewrite it as

$$(23) \qquad dF^k = F_1^k\, dP_k + (rF^k - rP_kF_1^k - Y_k)dt\ .$$

From (21) and (23), we have that

$$(24) \qquad \begin{aligned} dV - dF^k &= (rV - rP_kF_1^k - D_2 - rF^k + rP_kF_1^k + Y_k)dt \\ &= r(V - F^k)dt \end{aligned}$$

because $D_2 = Y_k$. By inspection, (24) is an ordinary differential equation with solution

$$(25) \qquad V(t) - F^k[P_k(t),t] = \{V(0) - F^k[P_k(0),0]\}\exp(rt)\ .$$

Thus, if the initial investment in the portfolio is chosen so that $V(0) = F^k[P_k(0),0]$, then for all t and $P_k(t)$, we have that

$$(26) \qquad V(t) = F^k[P_k(t),t]\ .$$

To ensure that the proposed portfolio strategy is feasible, we must show that its value is always nonnegative for every possible sample path for the price P_k and all t, $0 \leq t \leq T_k$. Because F^k is the solution to (18) and $Y_k \geq 0$, $F^k \geq 0$ for all P_k and t. It follows from (26) that $V(t) \geq 0$ for all P_k and t. We have therefore constructed a feasible dynamic portfolio strategy in traded asset k and the riskless asset that produces the stream of cash flows $Y_k(t)dt$ for $0 \leq t \leq T_k$ and has zero residual value $[V(T_k) = 0]$ at T_k.

Because the derived strategy exactly replicates the stream of cash flows generated by source k, it is economically equivalent to owning the cash flows $Y_k(t)$ for $t \leq T_k$. It follows that the capitalized value of these cash flows satisfies

$$(27) \qquad V_k(t) = F^k[P_k(t),t]$$

for $k = 1, \ldots, N$. Note that by inspection of (18)–(19), F^k, and hence $V_k(t)$, does not depend on either the university's preferences or its net worth. The valuation for source k is thus the same for all universities.

Armed with (27), we now turn to the optimal policy for managing endowment when the university has N nonendowment sources of cash flows. The procedure is the one outlined in section 7.2. To derive the optimal policy, note first that even if those nonendowment sources cannot actually be sold by the university for legal, ethical, moral hazard, or asymmetric information reasons, the university can achieve the *economic* equivalent of a sale by following the "mirror image," or reverse, of the replicating strategy. That is, by (short selling or) taking a $-F_1{}^k[P_k(t),t]P_k$ position in asset k and borrowing $(F^k - F_1{}^kP_k)$ of the riskless asset at each t, the portfolio will generate a positive amount of cash, $F^k(P_k,t)$, available for investment in other assets at time t. The entire liability generated by shorting this portfolio is exactly the negative cash flows, $(-Y_kdt)$, for $t \leq T_k$, because $V_k(T_k) = F^k(P_k, T_k) = 0$. But, since the university receives Y_kdt for $t \leq T_k$ from source k, this short-portfolio liability is entirely offset. Hence, to undertake this strategy beginning at time

t is the economic equivalent of selling cash flow source k for a price of $V_k(t) = F^k[P_k(t),t]$.

As discussed more generally in Merton (1990b, sec. 14.5, esp. 465–67), the optimal portfolio strategy will be as if all N nonendowment assets were sold and the proceeds, together with endowment, invested in the n risky traded assets and the riskless asset. This result obtains because it is feasible to sell (in the economic sense) the nonendowment assets and because all the economic benefits from those assets can be replicated by dynamic trading strategies in the traded assets. Hence, there is neither an economic advantage nor a disadvantage to retaining the nonendowment assets. It follows that the optimal demand for the traded risky assets is given by (13) and the demand for the riskless asset is given by $(1 - \Sigma_1^n w_i^*)W(t)$, where from (4) and (27)

$$W(t) = K(t) + \sum_1^N F^k[P_k(t),t] \; . \tag{28}$$

Because, however, the university has not actually sold the nonendowment assets, the optimal demands given by (13) and (28) include both *implicit* and *explicit* holdings of the traded assets. That is, the university's ownership of nonendowment cash flow source k at time t is equivalent to having an additional net worth of $F^k[P_k(t),t]$, as reflected in (28), *and* to having $F_1^k[P_k(t),t]P_k(t)$ invested in traded asset k and $\{F^k[P_k(t),t] - F_1^k[P_k(t),t]P_k(t)\}$ invested in the riskless asset. Thus, ownership of source k causes implicit investments in traded asset k and the riskless asset. Optimal explicit investment in each traded asset is the position actually observed in the endowment portfolio, and it is equal to the optimal demand given by (13) and (28) minus the implicit investment in that asset resulting from ownership of nonendowment assets. Let $D_i^*(t)$ denote the optimal explicit investment in traded asset i by the university at time t. It follows from (13) that

$$\begin{aligned} D_i^*(t) &= Ab_i + \sum_1^m H_k h_{ki} - F_1^i[P_i(t),t]P_i(t) \; , \quad i = 1, \ldots, N \\ &= Ab_i + \sum_1^m H_k h_{ki} \qquad\qquad\qquad\qquad , \quad i = N+1, \ldots, n \; , \end{aligned} \tag{29}$$

where $W(t)$ used in the evaluation of A and H_k is given by (28). If we number the riskless asset by "$n + 1$," then explicit investment in the riskless asset can be written as

$$\begin{aligned} D^*_{n+1}(t) &= [1 - \sum_1^n w_j^*(t)]W(t) - \sum_1^N \{F^i[P_i(t),t] - F_1^i[P_i(t),t]P_i(t)\} \\ &= K(t) - \sum_1^n D_j^*(t) \; . \end{aligned} \tag{30}$$

By inspection of (29), it is apparent that, in addition to the hedging of activity costs, the existence of nonendowment sources of cash flow will cause further differences between the observed holdings of assets in the optimal endowment portfolio and the mean-variance efficient portfolio of a "standard" investor. Similarly, from (30), the observed mix between risky assets and the riskless asset will differ from the true economic mix.

To explore further the effects of those nonendowment sources of cash flows, we solve the optimal expenditure and portfolio-selection problem for a specific utility function, $\bar{U}$. However, in preparation for that analysis, we first derive explicit formulas for the capitalized values of those sources when $Y_k(t)$ is given by (17). As already noted, there exists a unique solution to (18) and (19). Hence, it is sufficient to simply find a solution. As can be verified by direct substitution into (18), the value of cash flow source k is given by, for $k = 1, \ldots, N$,

$$(31) \quad F^k[P_k(t),t] = Y_k(0)\exp(-\phi_k t)\{1 - \exp[-\theta_k(T_k - t)]\} \frac{\left(\frac{P_k(t)}{P_k(0)}\right)^{\beta_k}}{\theta_k} ,$$

where β_k, ϕ_k are as defined in (17) and

$$(31a) \quad \theta_k \equiv r + \beta_k(\alpha_k - r) - \mu_k .$$

It follows from (31) that, for $k = 1, \ldots, N$,

$$(32) \quad F_1^k[P_k(t),t]P_k(t) = \beta_k F^k[P_k(t),t] ,$$

which implies that the capitalized value of source k has a constant elasticity with respect to the price of traded asset k. Equation (32) also implies that the replicating portfolio strategy is a constant-proportion or rebalancing strategy which allocates fraction β_k of the portfolio to traded asset k and fraction $(1 - \beta_k)$ to the riskless asset. In the case when positive fractions are allocated to both assets (i.e., $[1 - \beta_k] > 0$ and $\beta_k > 0$), then F^k is a strictly concave function of P_k. If $\beta_k > 1$, then F^k is a strictly convex function of P_k, and the replicating portfolio holds traded asset k leveraged by borrowing. In the watershed case of $\beta_k = 1$, F^k is a linear function of P_k, and the replicating portfolio holds traded asset k only.

Using (17) and (27), we can rewrite (31) to express the capitalized value of source k in terms of the current cash flow it generates:

$$(33) \quad V_k(t) = Y_k(t) \frac{1 - \exp[-\theta_k(T_k - t)]}{\theta_k}, \ k = 1, \ldots, N .$$

From (17), (31a), and (32), it is a straightforward application of Itô's Lemma to show that the total expected rate of return for holding source k from t to $t + dt$ is given by

$$(34) \quad \frac{E_t[Y_k(t)dt + dV_k]}{V_k(t)} = (\mu_k + \theta_k)dt = [r + \beta_k(\alpha_k - r)]dt .$$

Thus, if the rights to the cash flows Y_k between t and T_k were sold in the marketplace, the expected rate of return that would be required by investors to bear the risk of these flows is $r + \beta_k(\alpha_k - r)$. Therefore, θ_k equals the required expected rate of return (the capitalization rate) minus the expected rate of growth of the cash flows, μ_k. By inspection of (33), $V_k(t)$ can be expressed by the classic present-value formula for assets with exponentially growing cash flows. For $\theta_k > 0$, the perpetual ($T_k = \infty$) value is $Y_k(t)/\theta_k$, and the limiting "earnings-to-price" ratio, $Y_k(t)/V_k(t)$, is θ_k, a constant. Applying the closed-form solution for F^k, we can by substitution from (27) and (32) into (29) and (30) rewrite the optimal demand functions as

$$\text{(35a)} \qquad \begin{aligned} D_i^*(t) &= Ab_i + \sum_1^m H_k h_{ki} - \beta_i V_i(t)\,, \quad i = 1, \ldots, N \\ &= Ab_i + \sum_1^m H_k h_{ki} \qquad\qquad\quad\;\, , \quad i = N + 1, \ldots, n \end{aligned}$$

and

$$\text{(35b)} \qquad \begin{aligned} D^*_{n+1}(t) &= [1 - \sum_1^n w_j^*(t)]W(t) - \sum_1^N (1 - \beta_i)V_i(t) \\ &= K(t) - A\sum_1^n b_j - \sum_1^m\sum_1^n H_k h_{kj} + \sum_1^N \beta_i V_i(t)\,. \end{aligned}$$

Having derived explicit formulas for the values of nonendowment assets, we turn now to the solution of the optimal portfolio and expenditure problem in the special case where the university's objective function is given by

$$\text{(36)} \qquad \bar{U}(Q_1, \ldots, Q_m, t) = \exp(-\rho t)\sum_1^m \Gamma_j \log Q_j\,,$$

with $\rho > 0$ and $\Gamma_j \geq 0$, $j = 1, \ldots, m$. Without loss of generality, we assume that $\Sigma_1^m \Gamma_i = 1$. From (8), the optimal Q_j satisfy

$$\text{(37)} \qquad Q_j^*(t) = \frac{\Gamma_j C(t)}{S_j(t)}, \; j = 1, \ldots, m\,.$$

From (36) and (37), the indirect utility function can be written as

$$\text{(38)} \qquad U(C,S,t) = \exp(-\rho t)\{\log C - \sum_1^m \Gamma_j[\log S_j - \log(\Gamma_j)]\}\,.$$

It follows from (11a) that the optimal expenditure rule is

$$\text{(39)} \qquad C^*(t) = \exp(-\rho t)\frac{1}{J_W(W,S,t)}\,.$$

It is straightforward to verify by substitution into (10), (11a), and (11b) that

$$\text{(40)} \qquad J(W,S,t) = \frac{1}{\rho}\exp(-\rho t)\log W + I(S,t)$$

for some function $I(S,t)$. By the verification theorem of dynamic programming, satisfaction of (10), (11a), and (11b) is sufficient to ensure that J in (40) is the optimum.

It follows from (40) that $J_{kW} = 0$ and hence that $H_k = 0$ in (13) and (35), $k = 1, \ldots, m$. Therefore, for the log utility specified in (36), there are no differential hedging demands for assets to protect against unanticipated changes in the costs of university activities. The optimal allocation of the university's total net worth is thus instantaneously mean-variance efficient. Noting that $A = -J_W/J_{WW} = W$, we have that (35) can be written in this special case as

$$\text{(41a)}\qquad \begin{aligned} D_i^*(t) &= b_i W - \beta_i V_i(t)\ , \quad i = 1, \ldots, N \\ &= b_i W \qquad\qquad\quad , \quad i = N+1, \ldots, n \end{aligned}$$

and

$$\text{(41b)}\qquad D_{n+1}^*(t) = (1 - \sum_1^n b_j)W - \sum_1^N (1 - \beta_i)V_i(t)\ .$$

By inspection of (41), in the absence of nonendowment assets, the fraction of endowment allocated to risky asset i in the university's optimal portfolio is b_i, $i = 1, \ldots, n$, and the fraction allocated to the riskless asset is $(1 - \Sigma_1^n b_j)$, independent of the level of endowment. If $\chi_i^* \equiv D_i^*(t)/K(t)$ is the optimal fraction of endowment invested in asset i, then from (41) the difference in fractional allocations caused by the nonendowment assets is

$$\text{(42a)}\qquad \begin{aligned} \chi_i^*(t) - b_i &= R(b_i - \beta_i\lambda_i)\ , \quad i = 1, \ldots, N \\ &= Rb_i \qquad\qquad\quad , \quad i = N+1, \ldots, n \end{aligned}$$

and

$$\text{(42b)}\qquad \chi_{n+1}^*(t) - (1 - \sum_1^n b_j) = -R(\sum_1^n b_k - \sum_1^N \beta_k\lambda_k)\ ,$$

where $\lambda_k \equiv V_k(t)/\Sigma_1^N V_i(t)$ is the fraction of the capitalized value of the university's total nonendowment assets contributed by cash flow source k at time t, $k = 1, \ldots, N$, and $R \equiv \Sigma_1^N V_i(t)/K(t)$ is the ratio of the values of the university's nonendowment assets to its endowment assets at time t.

As discussed in section 7.2, the differences in (42) are the result of two effects: (1) the "wealth" effect caused by the difference between the net worth and the endowment of the university and (2) the "substitution" effect caused by the substitution of nonendowment asset holdings for traded asset holdings. Suppose, for concreteness, that the expected returns, variances, and covariances are such that a positive amount of each traded risky asset is held in mean-variance efficient portfolios. Then, $b_i > 0$, $i = 1, \ldots, n$. It follows that the impact of the wealth effect in (42a) and (42b), (Rb_i), is unambiguous: it causes a larger fraction of the optimal endowment portfolio to be allocated to each risky asset and therefore a smaller percentage allocation to the riskless asset. Because $\beta_i \geq 0$ and $\lambda_i > 0$, $i = 1, \ldots, N$, we have that the impact of the substitution effect in (42a) and (42b), $(R\beta_i\lambda_i)$, is also unambiguous: for those traded assets $1, \ldots, N$ for which the nonendowment assets are substitutes, the fractional allocation is smaller; for the traded assets $N + 1, \ldots, n$, the fractional allocation is unchanged; and the allocation to the riskless asset thus increases.

Because the wealth and substitution effects are in opposite directions for $b_k > 0$, whether the optimal endowment portfolio allocates an incrementally larger or smaller fraction to traded asset k depends on whether $b_k > \beta_k\lambda_k$ or $b_k < \beta_k\lambda_k$. $\beta_k\lambda_k$ is the fraction of the total increment to net worth (from nonendowment assets) that is implicitly invested in asset k as the result of owning cash flow source k. If that fraction exceeds the optimal one for total wealth, b_k, then the optimal endowment portfolio will hold less than the mean-variance efficient allocation. Indeed, if $\lambda_k > (1 + R)b_k/(R\beta_k)$, then $\chi_k^*(t) < 0$ and the university would optimally short sell traded asset k in its portfolio. This is more likely to occur when R is large (i.e., nonendowment assets are a large part of university net worth) and λ_k is large (i.e., cash flow source k is a large part of the value of nonendowment assets).

The implications of (42a) and (42b) for optimal endowment fit the intuitions discussed at length in section 7.2. For instance, if a significant amount of gift giving to a particular university depends on the performance of the general stock market, then in effect that university has a "shadow" investment in that market. Hence, all else the same, it should hold a smaller portion of its endowment in stocks than another university with smaller amounts of such market-sensitive gift giving. As noted in section 7.2, much the same substitution-effect story applies to concentrations in other assets, including real estate. The same analysis also follows where grants from firms or the government are likely to be strongly correlated with the financial performance of stocks in the related industries. However, the underweightings in those assets for substitution-effect reasons can be offset by sufficiently strong demands to hedge against costs, as is illustrated by the biotech example in section 7.2.

The analysis leading to (29) and (30) requires that there exist traded securities which are instantaneously perfectly correlated with the changes in $Y_1, \ldots, Y_N$. If this "complete market" assumption is relaxed, then the capitalized values of those nonendowment cash flow sources will no longer be independent of the university's preferences and endowment. However, the impact on endowment investments will be qualitatively similar. This more general case of nonreplicable assets can be analyzed along the lines of Svensson (1988).

We can use our model to examine the impact of nonendowment cash flow sources on optimal expenditure policy. From (39) and (40), we have that the optimal expenditure rule is the constant-proportion-of-net-worth policy

$$C^*(t) = \rho W(t) \tag{43}$$

However, current expenditure *from endowment* will not follow a constant proportion strategy. Optimal expenditure from endowment at time t is $[C^*(t) - \Sigma_1^N Y_k(t)]dt$, which can be either positive or negative (implying net saving from nonendowment cash flow sources). If $s^*(t)$ denotes the optimal expenditure rate as a fraction of endowment ($\equiv [C^*(t) - \Sigma_1^N Y_k(t)]/K(t)$), then from (4) and (43),

$$s^*(t) = \rho + R(t)\,[\rho - y(t)]\,, \tag{44}$$

where $R(t)$ is as defined in (42a) and (42b) and $y(t) \equiv [\Sigma_1^N Y_k(t)]/[\Sigma_1^N V_k(t)]$ is the current yield on the capitalized value of the nonendowment sources of cash flow. In the special case of (33), where the cash flows are all perpetuities (i.e., $T_k = \infty$ and $\theta_k > 0$, $k = 1, \ldots, N$), $V_k(t) = Y_k(t)/\theta_k$ and the current yield on source k is constant and equal to θ_k. In that case, $y(t) = \Sigma_1^N \lambda_k \theta_k$, the value-weighted current yield. From (31a), θ_k will tend to be smaller for assets with higher expected growth rates of cash flow, (μ_k). If on average the current yield on nonendowment assets is less than ρ, then the current spending rate out of endowment will exceed ρ. If the current yield is high so that $y(t) > \rho$, then $s^*(t) < \rho$. Indeed, if $y(t) > \rho(1 + R)/R$, then $s^*(t) < 0$ and optimal total expenditure is less than current cash flow generated by nonendowment sources. Because both $R(t)$ and $\lambda_k(t)$ change over time, we have from (44) that the optimal current expenditure rate from endowment is not a constant, even when expected returns on assets, the interest rate, and the expected rate of growth of nonendowment cash flows are constants.

We can also analyze the dynamics of the mix of the university's net worth between endowment and nonendowment assets. If $\alpha \equiv r + \Sigma_1^n b_i(\alpha_i - r)$ denotes the instantaneous expected rate of return on the growth-optimum, mean-variance efficient portfolio, then, as shown in Merton (1990b, 169–71), the resulting distribution for that portfolio is lognormal with instantaneous expected return $\alpha(> r)$ and instantaneous variance rate equal to $(\alpha - r)$. It follows from (6), (41), and (43) that the dynamics for the university's net worth are such that $W(t)/W(0)$ is lognormally distributed with

$$\begin{aligned} E_0[W(t)] &= W(0)\exp[(\alpha - \rho)t] \\ E_0[\log\frac{W(t)}{W(0)}] &= \left(\frac{\alpha + r}{2} - \rho\right)t \\ \mathrm{Var}[\log\frac{W(t)}{W(0)}] &= (\alpha - r)t\,. \end{aligned} \tag{45}$$

If $X_k(t) \equiv V_k(t)/W(t)$ denotes the fraction of net worth represented by nonendowment cash flow source k, then, because V_k and W are each lognormally distributed, $X_k(t)$ is lognormally distributed, and from (33) and (45)

$$E_0[X_k(t)] = X_k(0)\exp[(\rho - \theta_k)t]$$

$$(46) \qquad E_0\left[\log\frac{X_k(t)}{X_k(0)}\right] = \left[\mu_k + \rho - \frac{(\alpha + r + \delta_k^2)}{2}\right]t$$

$$\mathrm{Var}[\log\frac{X_k(t)}{X_k(0)}] = [\delta_k^2 + \alpha + r - 2(\mu_k + \theta_k)]t$$

for $k = 1, \ldots, N$.

From (46), the fraction of total net worth represented by all sources of nonendowment cash flow, $X(t) \equiv \Sigma_1^N X_k(t) = R(t)/[1 + R(t)]$, is expected to grow or decline depending on whether $\rho > \theta_{min}$ or $\rho < \theta_{min}$ where $\theta_{min} \equiv \min(\theta_k)$, $k = 1, \ldots, N$. In effect, a university with either a high rate of time preference or at least one (perpetual) high-growth nonendowment asset (i.e., with $\rho > \theta_{min}$) is expected to "eat" its endowment. Indeed, it may even go to a "negative" endowment by borrowing against the future cash flows of its nonendowment assets. Whether this expected growth in $X(t)$ is the result of declining expected net worth or rising asset values can be determined from (45). Because $\alpha > r$, if $\rho \leq r$, then both the arithmetic and geometric expected rates of growth for net worth are positive. For $\rho < \theta_{min}$, it follows that $E_0[X(t)] \rightarrow 0$ as $t \rightarrow \infty$. Hence, in the long run of this case, endowment is expected to become the dominant component of the university's net worth. Of course, these "razor's edge" results on growth or decline reflect the perpetual, constant-growth assumptions embedded in nonendowment cash flow behavior. However, this special case does capture the essential elements affecting optimal portfolio allocation and expenditure policies (cf. Tobin 1974).

The formal analysis here assumes that endowment is fungible for other assets and that neither spending nor investment policy are restricted. Such restrictions on endowment could be incorporated, using the same Kuhn-Tucker type analysis used in section 7.3 to take account of the constraint that total expenditure at each point in time is nonnegative. The magnitudes of the Kuhn-Tucker multipliers at the optimum would provide a quantitative assessment of the cost of each such restriction. However, including those restrictions is not likely to materially change the basic insights about hedging and diversification derived in the unrestricted case. The model can also be integrated into a broader one for overall university financial planning. Such integration would permit the evaluation of other nonendowment financial policies such as whether the university should sell forward contracts for tuition.

References

Bergman, Yakov. 1985. Time preference and capital asset pricing models. *Journal of Financial Economics* 14(March):145–59.

Black, Fischer. 1976. The investment policy spectrum: Individuals, endowment funds and pension funds. *Financial Analysts Journal* 32(January–February):23–31.

Bodie, Zvi. 1976. Common stocks as a hedge against inflation. *Journal of Finance* 31(May):459–70.

———. 1982. Investment strategy in an inflationary environment. In *The changing roles of debt and equity in financing U.S. capital formation*, ed. Benjamin Friedman, 47–64. Chicago: University of Chicago Press.

Breeden, Douglas T. 1979. An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics* 7(September):265–96.

Brinkman, Paul T. 1981. Factors affecting instructional costs at major research universities. *Journal of Higher Education* 52(May–June):265–79.

———. 1990. Higher education cost functions. In *The economics of American universities*, ed. Stephen Hoenack and Eileen Collins, 107–28. Albany: State University of New York.

Brovender, Shlomo. 1974. On the economics of a university: Toward the determination of marginal cost of teaching services. *Journal of Political Economy* 82(May–June):657–64.

Constantinides, George. 1989. Theory of valuation: Overview and recent developments. In *Theory of valuation: Frontiers of modern finance theory*, ed. Sudipto Bhattacharya and George Constantinides. Vol. 1, 1–23. Totowa, N.J.: Rowman and Littlefield.

———. 1990. Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy* 98(June):519–43.

Detemple, Jerome B., and Fernando Zapatero. 1989. Optimal consumption-portfolio policies with habit formation. Working Paper. New York: Graduate School of Business, Columbia University.

Duffie, Darrell, and Larry Epstein. 1992. Asset pricing with stochastic differential utility. *Review of Financial Studies* 5:411–36.

Eisner, Robert. 1974. Endowment income, capital gains and inflation accounting: Discussion. *American Economic Review* 64(May):438–41.

Ennis, Richard, and J. Peter Williamson. 1976. Spending policy for educational endowments. Research and Publication Project. New York: Common Fund, January.

Fama, Eugene, and Michael C. Jensen. 1985. Organizational forms and investment decisions. *Journal of Financial Economics* 14(March):101–19.

Fischer, Stanley. 1975. The demand for index bonds. *Journal of Political Economy* 83(June):509–34.

Ford Foundation Advisory Committee on Endowment Management. 1969. Managing educational endowments: Report to the Ford Foundation. New York: Ford Foundation.

Grinold, Richard, David Hopkins, and William Massy. 1978. A model for long-range university budget planning under uncertainty. *Bell Journal of Economics* 9(Autumn):396–420.

Grossman, Sanford, and Guy Laroque. 1990. Asset pricing and optimal portfolio choice in the presence of illiquid consumption goods. *Econometrica* 58(January):25–52.

Hansmann, Henry. 1990. Why do universities have endowments? *Journal of Legal Studies* 19(January):3–42.

Hindy, Ayman, and Chi-fu Huang. 1992. Intertemporal preferences for uncertain consumption: A continuous time approach. *Econometrica* 60(July):781–801.

Hoenack, Stephen, and Eileen Collins, eds. 1990. *The economics of American universities*. Albany: State University of New York Press.

Litvack, James, Burton Malkiel, and Richard Quandt. 1974. A plan for the definition of endowment income. *American Economic Review* 64(May):433–42.
Long, John B. 1974. Stock prices, inflation, and the term structure of interest rates. *Journal of Financial Economics* 1(July):131–70.
McDonald, John. 1974. Faculty tenure as a put option: An economic interpretation. *Social Science Quarterly* 55(September):362–71.
Merton, Robert C. 1977. A reexamination of the capital asset pricing model. In *Risk and return in finance,* ed. Irwin Friend and James Bicksler, Vol. 1, 141–60. Cambridge, Mass.: Ballinger.
———. 1985. Comment: Insurance aspects of pensions. In *Pensions, labor and individual choice,* ed. David A. Wise, 343–56. Chicago: University of Chicago Press.
———. 1990a. Capital market theory and the pricing of financial securities. In *Handbook of monetary economics,* ed. Benjamin Friedman and Frank Hahn, 498–581. Amsterdam: North-Holland.
———. 1990b. *Continuous-time finance.* Oxford: Basil Blackwell.
———. 1990c. The financial system and economic performance. *Journal of Financial Services Research* 4(December):263–300.
Nichols, Donald. 1974. The investment income formula of the American Economic Association. *American Economic Review* 64(May):420–26.
Nordhaus, William. 1989. Risk analysis in economics: An application to university finances. New Haven, Conn.: Cowles Foundation, Yale University, May. Unpublished paper.
Snyder, Thomas. 1988. Recent trends in higher education finance: 1976–77 to 1985–86. In *Higher education administrative costs: Continuing the study,* ed. Thomas Snyder and Eva Galambos, 3–23. Washington, D.C.: Office of Educational Research and Improvement, Department of Education.
Sundaresan, Suresh. 1989. Intertemporally dependent preferences and the volatility of consumption and wealth. *Review of Financial Studies* 2:73–89.
Svensson, Lars. 1988. Portfolio choice and asset pricing with nontraded assets. NBER Working Paper no. 2774. Cambridge, Mass.: National Bureau of Economic Research, November.
———. 1989. Portfolio choice with non-expected utility in continuous time. *Economic Letters* 39(October):313–17.
Tobin, James. 1974. What is permanent endowment income? *American Economic Review* 64(May):427–32.

Comment George M. Constantinides

In addressing the complex problem of optimal investment strategies for university endowment funds, Robert Merton adopts the view that the university is an economic agent maximizing the expected utility of a set of activities subject to a budget constraint. In so doing, he is able to frame the problem in

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the standard microeconomic/finance paradigm and draw heavily on the finance literature to which he is a seminal contributor. Standard finance theory dictates that the university allocate its endowment plus capitalized nonendowment future income in a mean-variance efficient portfolio of financial assets, modified by an overlay of hedging portfolios designed to hedge against unanticipated shifts in the state variables. The extent of hedging depends on the prices and price expectations of financial assets and on the sensitivity of the university's marginal utility of wealth to the shifts of the state variables.

I begin by discussing the university's objective in managing the endowment. Next I review Merton's model and principal results. I then present some generalizations and offer some concluding remarks.

The Objective in Managing the Endowment

The task of managing a university's endowment ought to be placed in the general context of the goals of a university. In addressing the goals of the university, we discuss three questions: (1) What, if any, objective function do market forces impose on the university? (2) What objective function do the university trustees and administration apply in practice? (3) What is the socially desirable objective function of a university? In particular, we consider two paradigms of the university, first as a utility-maximizing agent and second as a profit-maximizing firm.

The Objective Function Imposed by Market Forces

Is it plausible to model the university as an economic agent with a university-wide increasing and concave utility function? Even if we assign a utility function to each one of the agents who make up the university, what devices exist which equalize their marginal rate of substitution and lead to the existence of a university-wide utility function? To some extent, universities compete for students, funding, and the services of faculty, officers, and staff. There are also transfer payments between the undergraduate and graduate divisions of the college, professional schools, and research groups. But I very much doubt that these market mechanisms are adequate to equalize the marginal rate of substitution and give rise to a university-wide utility function.

Is it plausible to model the university as a profit-maximizing firm? A university does not have a clearly defined group of residual claimants. There are diverse groups of claimants to the services of a university which include the past, current, and future generations of students, the faculty, staff, and industry. Furthermore the threat of a takeover or reorganization, which leads corporations toward the goal of profit maximization, does not apply with equal force to universities. I view the university as a nexus of contracts among economic agents which include the state government and legislature in the case of state universities; the alumni, trustees, and officers; a highly individualistic and fragmented faculty; and last but not least a heterogeneous student body with "overlapping generations" features. Rothschild and White, in chapter 1 of this volume, present an insightful discussion of these issues.

The Objective Function Perceived by Universities

A search through the academic literature on the objectives of the university provides valuable insights but falls short of providing answers on what objectives universities do or should adopt. In addressing the investment income formula of the American Economic Association, Nichols (1974) assigns a utility function to the association and outlines the implications of the standard Fisherian analysis. Tobin (1974, 427) formulates the objective of the trustees of an endowed institution as "the guardians of the future against the present. Their task is to preserve equity among generations. . . . In formal terms, the trustees are supposed to have a zero subjective rate of time preference." Tobin neither endorses nor justifies these perceived goals. Litvack, Malkiel, and Quandt (1974) suggest that endowment management should (1) seek to make investment management independent of the spending decisions of the university, (2) protect the real value of the endowment fund, and (3) stabilize spendable income. Finally, Hansmann (1990) surveys a number of possible theories to explain endowment accumulation and explores their strengths and weaknesses. Hansmann is unable to find a plausible and rational explanation for observed endowment accumulation policies.

The lack of consensus on the perceived objectives of universities is hardly surprising. Universities are a diverse group of institutions with heterogeneous objectives. In fact, as we argue next, there is no compelling reason why universities should have uniform objectives.

The Socially Desirable Objectives of a University

I argue that universities provide diverse services to the society, and therefore different universities serve the society best by adopting different objectives.

As my first example, consider a state university system. Its financial collapse has major adverse effects on the current and future generations of students, the academic community, and the economic and cultural life of the state and beyond. It seems socially desirable that this university should follow a prudent policy of diversifying its instructional and research activities and also diversify its endowment and capitalized nonendowment income and hedge it against future contingencies. Merton's paradigm of the university as a utility-maximizing agent has implications which, in this example, are socially desirable.

As my second example, consider a small, research-oriented university located in a rural area where the economic life is dominated by the local specialized industry. Suppose that this university relies heavily on the local industry for research contracts, the supply of students, and endowment income. If we adopt Merton's paradigm, we conclude that the university should diversify its research and instructional activities away from the local industry and should hedge its current and capitalized nonendowment income by selling short, if possible, the stock of the local industry firms. Is this policy socially desirable?

A case can be made that society is best served by the exact opposite policy: specialize the research and instructional services to serve best the demands of the local industry and invest the endowment in the stock of the local industry firms. If the local industry declines into oblivion, so does the university whose social function was to serve this industry. If on the contrary the local industry flourishes, the university is best suited in terms of its specialization and financial strength to serve the local industry.

Which of the two diametrically opposite policies is socially preferred? There is no simple answer. But I hope that the examples illustrate that diversification is not always an obvious social attribute of the university's objective. Furthermore, society is served best by different universities following different, even diametrically opposite, policies.

A Review of the Model and Principal Results

Merton defines the university's objective as the maximization of a von Neumann–Morgenstern, time-separable, and concave utility function of the level of a set of activities $\underline{Q}(t) = [\underline{Q}_1(t), \ldots, \underline{Q}_m(t)]$ as

$$(1) \qquad \max E_o\left\{\sum_{t=0}^{\infty} \bar{U}[\underline{Q}(t), t]\right\}.$$

The price (or net cost) of activity j is $\underline{S}_j(t)$. No distinction is made between the marginal and average cost of an activity; we therefore interpret the supply of activities as perfectly elastic. The vector of activity prices $\underline{S}(t) = [\underline{S}_1(t), \ldots, \underline{S}_m(t)]$ is an exogenous autoregressive process which Merton models as a continuous-time diffusion process.

The endowment capital at the beginning of period t is $K(t)$. The nonendowment income at t is $Y(t)$, an exogenous stochastic process. Merton models $[\underline{S}(t), Y(t)]$ as a diffusion process. Merton assumes that the nonendowment income is spanned by the returns of the financial assets. Then the nonendowment income stream $[Y(t), Y(t + 1), \ldots]$ may be capitalized with value $\hat{Y}(t)$. The university's wealth is defined as $\underline{W}(t) = K(t) + \hat{Y}(t)$, the sum of endowment capital and capitalized present and future nonendowment income.

The expenditure on activities at time t is $\underline{Q}'(t)\underline{S}(t)$, where the prime denotes the transpose. The wealth net of the current expenditure on activities is $\underline{W}(t)$

$- \underline{Q}'(t)\underline{S}(t)$ and is allocated among the financial assets with portfolio weights $\underline{w}(t) = [\underline{w}_1(t), \ldots, \underline{w}_n(t)]$, which sum up to one. The supply of financial assets is perfectly elastic, and the asset returns over one period are denoted by $\underline{R}(t+1) = [\underline{R}_1(t+1), \ldots, \underline{R}_n(t+1)]$. Merton models the joint process of financial asset prices, activity prices, and nonendowment income as a diffusion process. The wealth dynamics is

$$\underline{W}(t+1) = [W(t) - \underline{Q}'(t)\underline{S}(t)]\underline{w}'(t)\underline{R}(t+1)\,. \tag{2}$$

Zero wealth is an absorbing state; that is, $\underline{W}(t) = 0$ implies zero investment in the activities and in the financial assets at all future times.

The control variables are the activity levels $\underline{Q}(t)$ and the portfolio weights $\underline{w}(t)$. Stated formally, the university maximizes the expected utility in (1) by the sequential choice of activity levels and portfolio weights subject to the sequence of budget constraints, (2), and the constraint of nonnegative wealth. Essentially, Merton models the university's problem as the standard intertemporal consumption and investment problem, which has been studied extensively in the finance literature (see Fama and Miller 1972; Ingersoll 1987; Merton 1990).

Merton proceeds along familiar lines to define the indirect utility of consumption as

$$U[C(t), \underline{S}(t), t] = \max_{\underline{Q}(t)} \bar{U}[\underline{Q}(t), t] \tag{3}$$

subject to $\underline{Q}'(t)\underline{S}(t) = C(t)$ and then define the derived utility of wealth as

$$J[\underline{W}(t), \underline{S}(t), t] = \max_{[\underline{w}(t), C(t)]} E_t\left\{\sum_{\tau=t}^{\infty} U[C(\tau), \underline{S}(\tau), \tau]\right\} \tag{4}$$

subject to the budget constraint.

Merton's primary focus is on the portfolio allocation, that is, the control variables $\underline{w}(t)$. In general, the optimal portfolio consists of a mean-variance efficient portfolio of the endowment plus the capitalized nonendowment income, modified by an overlay of hedging portfolios designed to hedge against unanticipated shifts in the state variables. In the special case where the indirect utility of consumption is the sum of the logarithm of consumption and a function of the state variables, a myopic policy is optimal: the university invests the endowment plus the capitalized nonendowment income in a mean-variance efficient portfolio, without an overlay of hedging portfolios.

Generalizations

The University's Production Function

Whether we choose to view the university as a consumer of activities, a profit-maximizing firm, or a nexus of contracts, we should explore the production function of the university. The inputs are the expenditures on teaching, faculty, research, physical plant, and public relations—to mention just a few. The outputs are the activities in Merton's terminology. We should recognize that it takes many years to build a reputation, to attract the best student applicants, to build a superior faculty, or to create a niche in a certain academic field. Therefore, the production function should incorporate adjustment costs and "time to build."

For a minority of activities, such as student aid or visiting faculty, the price per activity unit is exogenous and well defined. But for the majority of university activities, the prices are endogenous. Merton is aware of this and points out that his exogenous price processes of activities can be viewed as "reduced-form" equilibrium price processes. Still it remains unclear whether Merton views these prices as marginal or average. In his budget equation (6), the activity expenditures are the sum of the product of activity levels and prices; therefore, prices are interpreted as average prices. In his first-order equations (8), the same prices play the role of marginal prices. Therefore, the distinction between the marginal and average price of an activity is not drawn. The distinction can be drawn by introducing a production function.

The Intertemporal Complementarity and Substitutability of the Activities in the University's Preferences

Some university activities exhibit strong intertemporal substitutability: a university basks in the glory of a Nobel laureate among its ranks long after the laureate has retired. Other activities exhibit strong complementarities: a university is more disturbed by the lowering of its academic ranking than by the maintenance of a steady but low ranking.

These effects can be modeled in one of two ways. The first is to draw a distinction between university outputs and activities. The university outputs are durable goods which produce a stream of activities over time. In this case, the stream of activities is not directly controllable, and Merton's analysis needs to be modified accordingly.

The second way to model these effects is to define the university's preferences over the outputs rather than over the activity flows from these outputs. But then the preferences are no longer time separable, and we can no longer define a time-separable indirect utility function of consumption as in (3), except in simple cases.

Concluding Remarks

In the context of a simplified, or "reduced-form," model of a university as a utility-maximizing agent, Merton has demonstrated that the basic principles of finance apply and, in particular, endowment funds should be managed according to the principles of diversification and hedging.

I have argued that universities are a diverse group of institutions with heterogeneous functions in the society. Whereas diversification of instructional activities, research activities, and endowment and capitalized nonendowment income may be reasonable and socially desirable objectives of some universities, it is an open question whether these objectives are reasonable and socially desirable for the whole spectrum of universities.

References

Fama, Eugene F., and Merton H. Miller. 1972. *The theory of finance*. New York: Holt, Rinehart and Winston.

Hansmann, Henry. 1990. Why do universities have endowments? *Journal of Legal Studies* 19(January):3–42.

Ingersoll, Jonathan E. 1987. *Theory of financial decision making*, Totowa, N.J.: Rowman and Littlefield.

Litvack, James M., Burton G. Malkiel, and Richard E. Quandt. 1974. A plan for the definition of endowment income. *American Economic Review* 64(May):433–37.

Merton, Robert C. 1990. *Continuous-time finance*. Oxford: Basil Blackwell.

Nichols, Donald A. 1974. Endowment Income, Capital Gains, and Inflation Accounting. *American Economic Review* 64(May):420–26.

Tobin, James. 1974. What is permanent endowment income? *American Economic Review* 64(May):427–432.