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OFFERINGS, EXTINGUISHMENTS, AND OUTSTANDINGS

AGGREGATE statistics on offerings, extinguishments, and outstandings may be used in several ways. Their most direct use is in the analysis of the volume of investment instruments available to investors. If holdings of other-than-household investor groups are deducted, estimates of the savings held as corporate bond investments by the household sector may be obtained. Within the nonfinancial corporate sector of the economy, the debt series reflect the ability and willingness of various obligor groups to tap the market for capital funds, and when related to the series on interest payments developed in Chapter 6, they provide a basis for measuring the burden of the funded debt once this market has been tapped.

From the standpoint of general economic analysis these data should be particularly useful in studying the interrelationships of debt formation, asset formation, and the level of economic activity. Under different hypotheses, debt financing is assumed to be associated with the tempo of business activity, with interest rates, and with security prices. In order to test such hypotheses it is necessary to trace accurately and over a long period the principal movements occurring in corporate funded debt.

The first section after the chapter summary will deal with movements in total outstandings of straight bonds and with the major industrial and size components of the total. Later sections will trace the movements in offerings, extinguishments, and net changes in outstandings, and investigate the interrelationships of the series.

SUMMARY OF FINDINGS

The analysis will clarify certain problems that are encountered in efforts to control the bond market through monetary measures. Some credit markets—instalment sales credit is an example—have been susceptible of monetary control through regulation of the total volume of new credit granted (analogous to total bond offerings as defined in our study) because the volume of credit repaid is relatively steady. But in the bond field the volume of refundings and repayments is large and highly variable. Extremely low interest rates in the thirties, for example, induced a large volume of refunding activity in the bond market yet had little influence on the net volume of bond financing. Although bond offerings reached near-record levels in that period, they were exceeded by bond extinguishments, with the result that outstandings turned down. In 1918, on the other hand, bond offerings were unusually low but extinguishments were even lower, so that outstandings increased. In short, in the bond market the total volume of offerings, because it includes refundings, is a poor guide to the net amount of credit originating. Whether a series covering only offerings for new-money purposes (offerings excluding refundings) may be a more reliable guide will be examined in the next chapter.

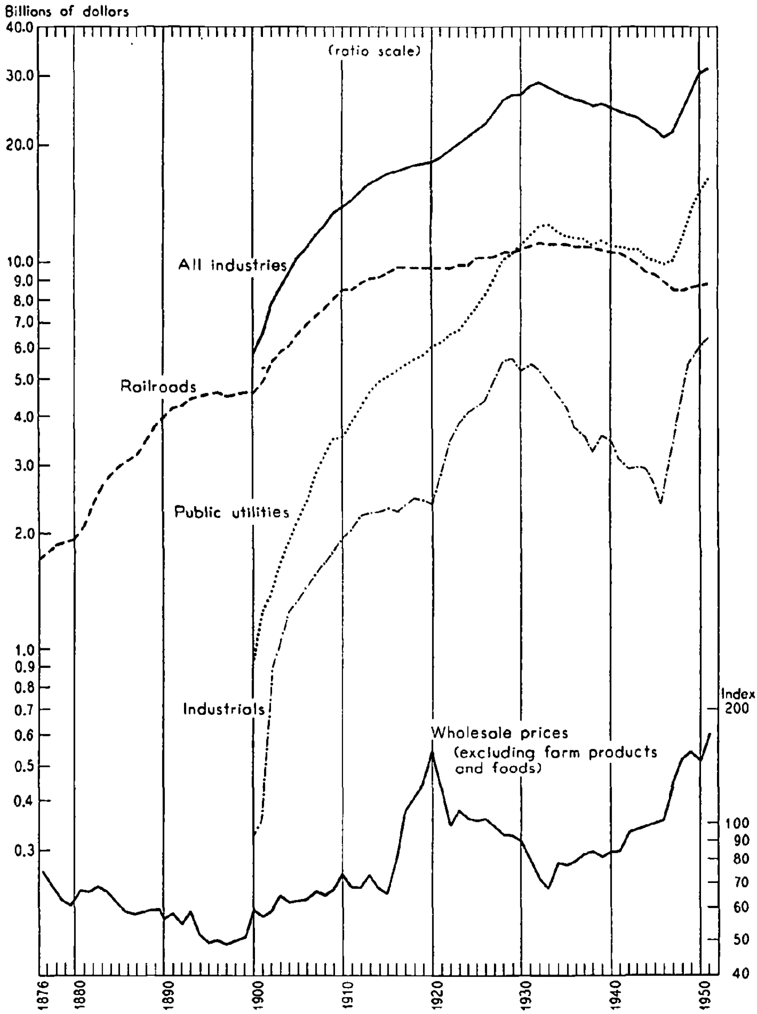
Here we are able to throw some light on another practical problem, that which besets the investor in attempting to space his portfolio so that cash funds will flow in as needed. It is found, in the first place, that maturity dates are not at all a trustworthy indication of actual expiration dates; and in the second place, that the actual repayment of bonds has tended over the business cycle to coincide with the flow of other funds toward the investor, while, conversely, the life spans of bonds have tended to lengthen at a time when the money market was already tight. We also find a progressive shortening of both the nominal and the actual life spans of bonds in the period studied.

Outstandings

Table A-2 presents dollar figures for outstandings of straight bonds classified by major industry group, and Chart 2 traces the annual movements of the series.

As the chart shows, aggregate outstandings of straight bonds increased continuously from $5.9 billion par amount in 1900 to a peak of $29.0 billion in 1932, the average annual increment being $0.7 billion. Except for a single rise in 1938, they then decreased in every year to $22.8 billion at the beginning of 1944, the average annual decrement being $0.5 billion. Our special tabulations on outstandings terminate on the latter date, but rougher estimates for later years, whose construction will be described in Chapter 6, are also shown on the chart. They indicate that the decline prob-

CHART 2—Corporate Bond Outstandings and Index of Wholesale Prices, 1876-1951



Equal vertical distances represent equal ratios of change.

Outstandings include straight issues only, January figures, par amount; from Table A-2. The price index, 1913-51, covers wholesale prices of commodities other than farm products and foods (Bureau of Labor Statistics, January prices, 1926 = 100). For earlier years this index was estimated from movements in the BLS "all commodities" index.

ably continued to a trough of about $21 billion at the beginning of 1946. Total outstandings then climbed rapidly, reaching about $32 billion at the beginning of 1951.

From the evidence it is clear that outstandings exhibit negligible conformity with short cycles in business activity. The only statistics available before 1900 cover railroads back to 1876.[1] They show a continuous rise up to 1896 and a minor trough in the following year. Since we know that rail debt was 79 percent of the total in 1900, there is a suggestion that outstandings of the combined industries may also have reached a minor trough around 1897. If so, the series on total outstandings described only one complete swing upward and downward during a period of fifty years. This single expansion and contraction swing does not, of course, constitute evidence of a long cycle, which implies recurrence.

Industry and Size Distribution of Outstandings

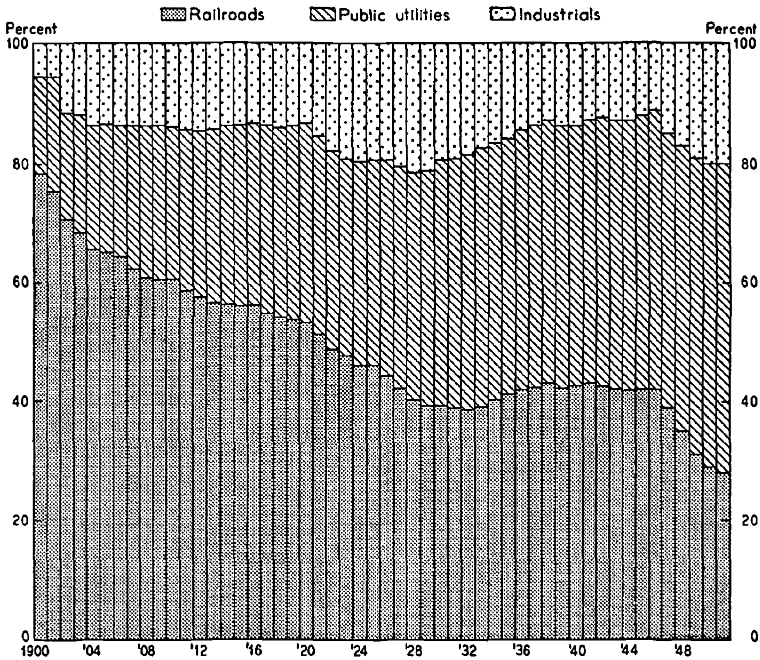
Chart 3 presents the percentage distribution of annual outstandings by major industry group, and may be used with the absolute figures shown in Chart 2 to trace the principal movements in the composition of outstandings since 1900.

The market for corporate bonds was predominantly a rail market for the greater part of the period covered by our basic series. To a considerable extent our present rail system was already "in being" at the beginning of the century and funded debt was already large: rail bonds amounted to $4.7 billion, or 79 percent of total outstandings, at that time. While in absolute amount the volume of rail bond outstandings rose rather steadily after the turn of the century to its maximum of $11.3 billion in 1932, during the same period it fell just as steadily in relation to the volume of other issues, so that by 1932 it stood at only 39 percent of the total. Long-term rail debt then contracted mildly to $9.5 billion in 1944 (42 percent of the total).

[1] Pre-1900 estimates for railroads were obtained by chaining aggregate balance sheet data on funded debt as given in Poor's *Manual of Railroads* for 1900 to our 1900 estimates of outstandings. Poor's data were chained to our straight bond estimate to obtain the figures shown in Chart 2 and Table A-2, and to the estimate for issues of all types to obtain those given in Table A-1. Although balance sheet data include some duplicating debt, it is roughly eliminated by the chaining process.

The dominant position of rail bonds in the corporate bond market of the past is underscored by the fact that the rails accounted for more than 50 percent of the total par amount of all issues outstanding at all times prior to 1922, and were not surpassed in absolute amount by either of the other major industry groups until 1929. The estimate for January 1, 1951, however, puts rail debt at $9.0 billion, only 28 percent of total outstandings.

CHART 3—Percentage Share of Major Industry Groups in Corporate Bond Outstandings, 1900-1951



Based on Table A-2; straight bonds, January figures, par amount.

The history of utility outstandings is in marked contrast to that of the rails. Funded debt of the utility group stood at only $0.9 billion in 1900 (16 percent of total outstandings) but increased rapidly with the development of the electric light and power industry to a maximum of $12.5 billion in 1933 (44 percent of the total). It then declined mildly to $10.4 billion in 1944 (45 percent of the total). Thus the relative importance of utility outstandings increased steadily over the whole period; and in absolute volume, utility bonds have been the dominant group in the bond market since 1928. Estimates for recent years indicate that the mild decline in utility outstandings that began in 1933 was abruptly reversed after World War II. Utility outstandings at the beginning of 1951 were 32 percent above their prewar (1933) peak and currently account for about half of corporate bond outstandings.

Except during the late twenties and the recent period of credit expansion, industrial bonds have not occupied a position of major importance in the corporate bond market. They stood at only $0.3 billion in 1900 (5 percent of total outstandings), increased rapidly to $1.0 billion in 1903 (12 percent of the total), and then grew somewhat irregularly—note the interruption in 1919-20—to a peak of $5.7 billion in 1929 (21 percent of total outstandings). During the depression and war years industrial outstandings contracted somewhat more sharply than did the outstandings of the other two industry groups, reaching a low of $2.4 billion by 1946 (11 percent of total outstandings). Both in absolute and in relative terms, however, the postwar expansion has been rapid: industrial outstandings nearly trebled after 1946, so that by 1951 they constituted 20 percent of total outstandings. Of the $32 billion of straight corporate bonds outstanding at the beginning of 1951 approximately half were utilities; the rails accounted for only slightly more of the remainder than the industrials.

It is interesting to speculate on the significance of the postwar rise in funded debt, which may seem moderate or excessive depending upon one's point of view. The postwar rise in outstandings appears moderate when compared with the rise in the general price level (Chart 2). It also appears moderate when viewed in relation to the volume of funds obtained by corporations from nondebt sources during the same period. Thus, undistributed corporate profits for 1948 alone aggregated $12.8 billion, in this one year exceeding the entire increase in funded debt between 1945 and 1951.[2] On the other hand, corporate debt is large when viewed against the background of its recent past; and the debt is

[2] Department of Commerce estimate of undistributed corporate profits (domestic) as stated in *Survey of Current Business*, July 1950, Table 7, p. 11.

still growing. From the investors' point of view, the most important question is whether corporate earnings will suffice to sustain the fixed charges arising out of this expanding volume of funded debt, or whether, as after the twenties, the rapid debt growth will be followed by heavy corporate defaults.

Table A-6, from which Chart 4 is drawn, gives par amount figures, annually, for outstandings of straight bonds classified by large and small issues. For purposes of economic analysis this simple breakdown is of limited use, since the $5 million limit separating the large from the small issues has varied in significance with the general growth and price trends in our society since 1900. Its purpose was the purely technical one of selecting the 10 percent sample of small issues, and its chief usefulness is in checking the accuracy of certain of our estimates that are based on sample data (cf. page 37). Detailed size breakdowns and measures of size inequality, which require longer treatment than would be convenient here, are reserved to a later monograph.

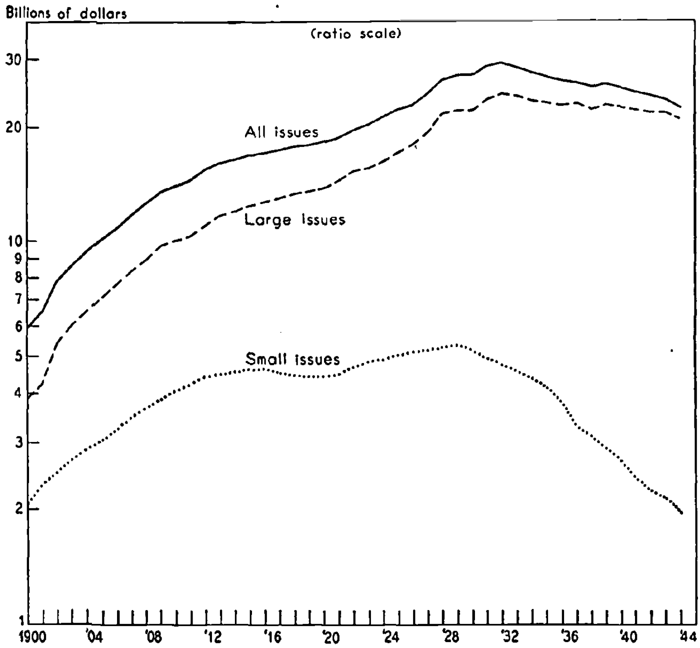
Chart 4 nevertheless throws some light on the changing fortunes of large and small issues. Comparison of trends in their graphs indicates a relative displacement of small issues by large issues. This displacement, which has been continuous since 1900, was accelerated after 1929 with the abrupt fall in the absolute volume of small issues outstanding. The emerging importance of large issues is a development of some significance in the financial history of the country, and will be touched on again at several points.

Growth and Retardation Rates

The movements of the various series shown on Charts 2 and 4 may be summarized and compared most readily by means of growth and retardation rates. When graphs are drawn, as here, to ratio scale, with equal vertical distances implying equal percentage rates of growth, any series growing at a constant rate will trace out a linear path. It will be noted that the paths of the various series on outstandings are roughly linear over the successive short periods 1900-1909, 1909-20, 1920-32, 1932-46, and 1946-51, industrials constituting the principal exception. It follows that constant growth rates describe the behavior of the series over these short periods with reasonable accuracy in most cases. Constant short-period growth rates are presented in Table 2 up to January 1, 1944, the terminal date of our basic estimates.

Over the full period 1900-1944 the various series on outstandings appear to be concave to the time axis and to describe a

CHART 4—Corporate Bond Outstandings by Size of Issue, 1900-1944



From Table A-6; straight bonds, January figures, par amount.

rough parabola. Abstracting from short-run deviations from trend, this suggests (1) that the growth rate of each series decreased with time and (2) that it decreased at a roughly constant rate. The annual rate of decline in growth rate is called the retardation rate; and when it is constant, the path of a series on a ratio chart has a parabolic shape. If, for example, a series is growing at some particular point in time at the rate of 10 percent per annum and has a constant retardation rate of 1 percent per annum, its growth rate in succeeding years is 9 percent, 8 percent, and so on. Table 2 presents, in addition to the short-period growth rates described above, estimates of the growth rates at the central year (1922) and estimates of constant retardation rates.[3]

For each series the highest short-period growth rate occurred in the first decade of the century. The rates of growth slackened off over the period 1909-20, and then, except in the case of rails and small issues, increased again in the twenties. It should be

[3] The theory behind the estimates presented in Table 2 may be summarized symbolically as follows: let

$$\begin{aligned} t &= \text{time} \\ n &= \text{length of period} \\ y(t) &= \text{outstandings} \\ r &= \text{growth rate} \\ \dot{r} &= \text{retardation rate} \end{aligned}$$

Then for a series with constant growth rate and with time measured from the initial year of the period,

$$(1) \qquad y(n) = y(0)(1+r)^n$$

$$r = \left(\frac{y(n)}{y(0)}\right)^{1/n} - 1$$

Constant short-period growth rates were computed by means of Eq. (1).

For a series describing a parabolic pattern when plotted on ratio scale,

$$\log y(t) = \log K + (\log a)t + (\tfrac{1}{2} \log b)t^2$$

$$y(t) = Ka^t b^{t^2/2}$$

Since the growth rate at time t is defined as the relative change in outstandings per unit interval of time,

$$\begin{aligned} r(t) &= \frac{1}{y} \cdot \frac{dy}{dt} \\ &= \log a + (\log b)t \end{aligned}$$

Placing the time origin at the central year ($t = 0$ in 1922)

$$(2) \qquad \begin{aligned} r(0) &= \log a \\ &\sim a - 1, \text{ for small } r \end{aligned}$$

(The symbol $\sim$ indicates approximate equality.)

Differentiation of the growth rate gives

$$(3) \qquad \begin{aligned} \dot{r}(t) &= \log b \\ &\sim b - 1, \text{ for small } \dot{r} \end{aligned}$$

From (3) it is evident that $\dot{r}$ is constant.

The parameters a and b were estimated for each series for the period 1900-1944 by the method of least squares and substituted in Eqs. (2) and (3) to obtain the central-year growth rate and the retardation rate.

For a discussion of growth and retardation rates and their applications to the analysis of economic time series, see *Production Trends in the United States Since 1870*, by Arthur F. Burns (National Bureau of Economic Research, 1934).

TABLE 2—Annual Growth and Retardation Rates for Corporate Bond Outstandings, 1900-1944

PERIOD	*All industries*	*Railroads*	*Public utilities*	*Industrials*	*Large issues*	*Small issues*
			Short-period growth rates			
1900-1909	9.5%	6.4%	15.6%	21.0%	10.6%	7.2%
1909-1920	2.8	1.6	5.1	2.6	3.3	1.3
1920-1932	4.0	1.3	6.1	6.9	4.9	0.7
1932-1944	−2.0	−1.4	−1.5	−4.8	−1.3	−7.2
			Long-period central-year growth rates (1922)			
1900-1944	2.9	1.4	5.0	3.7	3.6	0.0
			Long-period retardation rates			
1900-1944	0.222	0.153	0.311	0.466	0.212	0.373

Based on annual data, Tables A-2 and A-6.

noted that the growth rates for rails and small issues also accelerated in the twenties when compared with their growth rates calculated over periods beginning a few years later than 1909 (say 1912). Growth rates were uniformly negative in the thirties; that is, all series declined. In each of the short periods identified in Table 2 the growth rates for large issues were higher than for small, a finding that confirms the earlier observation of the increasing importance of large issues in the total of all issues outstanding.

The last two lines in the table show that the rails had the lowest central-year growth rate and the lowest retardation rate. In other words, the growth of rail debt was quite mild up to its peak in 1932, and the decline was equally mild thereafter. As is indicated by their high retardation rate, industrials were the most volatile group. Industrial debt expanded rapidly up to 1929, and the contraction was sharp in the thirties and early forties. The volatility of industrial outstandings is confirmed by recent experience, industrials showing the most rapid growth of any group in the expansion after World War II. Utilities had the highest central-year growth rate and a moderate rate of retardation. That is to say, utility outstandings continued to grow for several years after the other series had turned down, and the subsequent rate of contraction was about as mild as for the rails.

The growth rate for small issues was virtually zero at the central year (1922), and the retardation rate was fairly high. We may infer, then, that growth rates for small issues are roughly symmetrical about 1922, and that the steepest rates of growth and decline occurred at the initial and terminal years of the period studied. Outstandings of large issues not only started at a much higher absolute level in 1900 than did the small issues, but had a higher central-year growth rate and a lower rate of retardation. It follows that they grew at all times at a more rapid rate than the small issues and have occupied a position of increasing importance in the market.

Average Size of Outstanding Issues

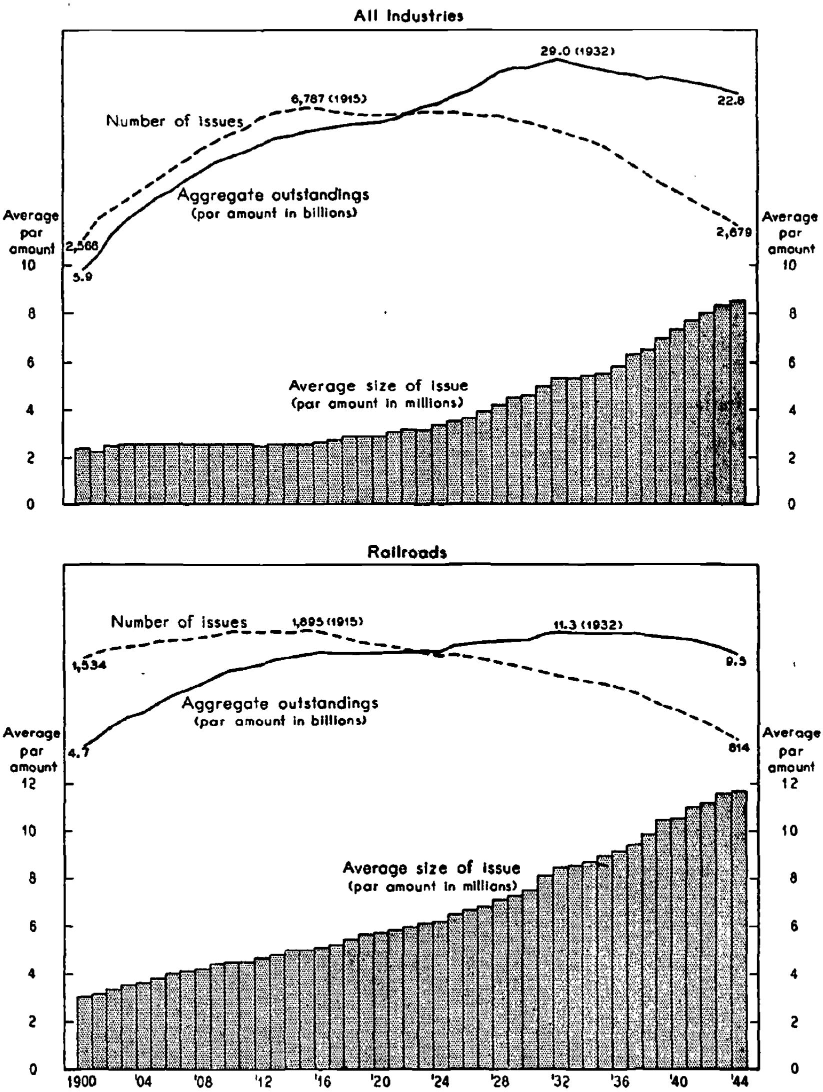
Chart 5 illustrates the dramatic increase in average size of outstanding issues that has occurred since 1900, and particularly since 1923. For all industries combined, the average size of issue was $2.3 million in 1900 and increased slowly to $3.1 million at the beginning of 1923. The upward movement then accelerated, so that by January 1, 1944 the average size reached $8.5 million, more than two and a half times what it had been twenty years before.

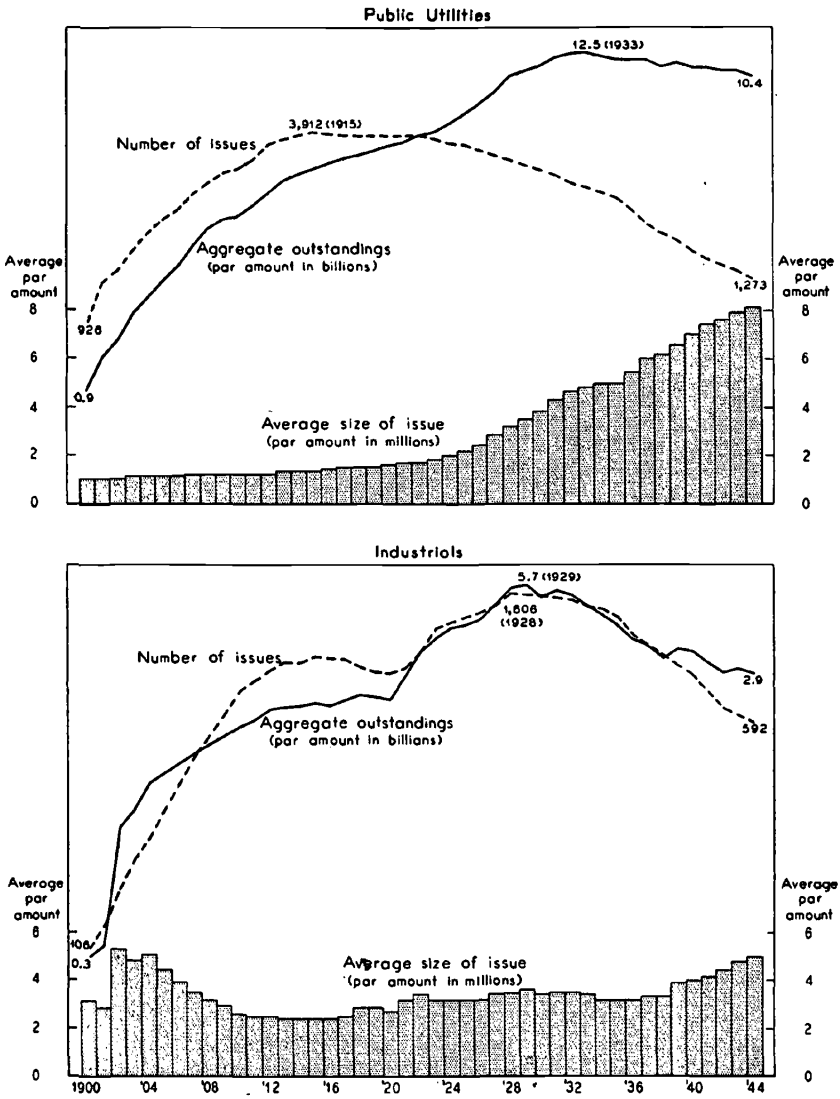
The increase was largely accounted for by bonds in the public utility group, their average size in 1944 being four times what it was in 1924. The average par amount of the rail issues outstanding has always exceeded that of any other group, with the exception of industrials in a few of the early years. Moreover, the average size of the rail issues has increased most regularly over the years, growing by approximately constant increments of $200,000 per year.

The anatomy of the changes occurring in the average par amount of issues outstanding is revealed by comparing the movements in the series for number of issues and aggregate par amount of outstandings in Chart 5. The average par amounts were derived by dividing the aggregate par amounts of issues outstanding (Table A-2) by the number of issues (Table A-7). The two latter series are plotted on ratio scale. So long as they move parallel to each other (that is, so long as number and aggregate par amount change at the same rate), the average remains unchanged. The average rises if the number series increases more slowly (or decreases more rapidly) than aggregate par amount, and with converse changes the average falls.[4] For the two large groups, rails and utilities, the peak in the number of issues outstanding occurred somewhat earlier than the peak in aggregate par amount of outstandings, and the number of issues declined more rapidly from its high. Thus there was a general upward movement in average size of issue over the period studied. In the case of industrials the two series entering into the average moved alike, and the average therefore exhibits little trend.

[4] Since these averages are for outstanding issues, they are affected by the average size both of new offerings and of old issues extinguished. In some periods, as is illustrated by the case of rail bonds since 1940, the dominant factor causing an increase in average size of outstanding issues has been the small size of old issues repaid or refunded into larger issues. It follows that the average size of outstanding issues bears an uncertain relation to the average size of new offerings. (Readers interested in analyzing the latter may do so by comparing the series on number of offerings in Table A-9 with the par amounts of offerings in Table A-2.)

CHART 5—Number, and Aggregate and Average Par Amount, of Corporate Bond Issues Outstanding, 1900-1944





Based on Tables A-2 and A-7; straight bonds, January figures.

Number and aggregate amount of bond issues outstanding are plotted on ratio scale; average size of issue is plotted on arithmetic scale.

In each industry group, the principal cause of the increase in average size of issue can usually be traced back to a prior consolidation of obligors. The large railroad systems—most of which had already been formed by 1900—were combinations of a large number of small roads, each of which generally carried its own underlying issues. The systems, as distinct from the operating units, were financed in large part by "general and refunding" issues secured by blanket mortgages junior to the divisional issues. The indentures of the refunding issues usually provided that the divisional bonds were to be pledged on retirement as security under the blanket mortgages. These refunding operations were carried out gradually over the period studied, and the average size of outstanding rail issues steadily rose.

Both the number and the aggregate par amount of public utility issues increased at about the same rate up to the mid-twenties, and accordingly there was little change in average size. The subsequent increase in average size of issue is attributable in part to consolidations of street railways and of gas and electric companies into larger operating systems, and in part to the growing number of public utility holding companies, which frequently financed themselves by large public bond issues, transferring the funds thus acquired through "downstream" loans to the operating companies. Many small street railways not entering larger systems were scrapped, and others were taken over by local municipalities, events which also contributed to the rapid rise in average size of issue of privately owned public utilities.

The indecisive movements in the average size of industrial issues are linked with changes in the tempo of industrial consolidations and in the use of stock and bond instruments in external financing. Relatively few industrial bonds were outstanding in the early years of the century, and these were dominated by a few large issues. The movement toward industrial combination was well under way by 1900. The United States Steel Corporation was founded in 1901 and in the same year borrowed large sums through the sale of its 5's of 1951 (series A through F), borrowing again in 1903 through the sale of its Collateral Trust Sinking Fund 5's of 1963. Large amounts were also raised in 1901 by the Consolidated Tobacco Company (merged in 1904 with the American Tobacco Company) by sale of its Collateral

Trust 4's of 1951. Such concerns largely accounted for the abrupt rise in average size of industrial issues between 1901 and 1904.

Around 1904, smaller corporations began to enter the bond market, with the result that the number of issues rose more rapidly than the aggregate par amount, and the average size of issue fell. Changes in the capital structure of the Tobacco Trust contributed further to the decline in average size of issue. When Consolidated Tobacco Company was merged with American Tobacco in 1904, holders of the old bonds received 50 percent in new bonds and the remainder in preferred stock. With the dissolution of the Tobacco Trust in 1912, bondholders of American Tobacco received 50 percent in cash and the remainder in four smaller issues of P. Lorillard and Liggett and Myers. The large U.S. Steel issues of 1901 and 1903 were also retired rapidly but were not finally extinguished until 1929.

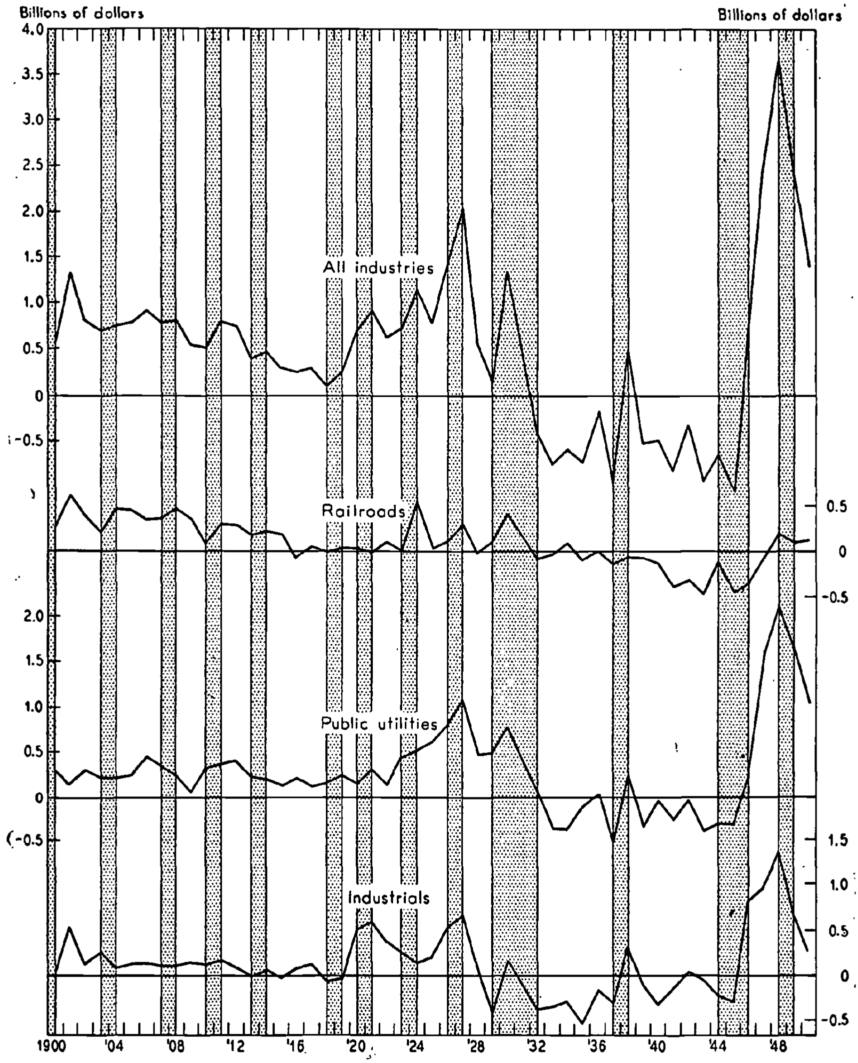
The general expansion in corporate funded debt in the twenties was not reflected in the industrial averages. Although a substantial number of new issues appeared at that time, most were of moderate size, the larger corporations preferring to finance themselves by sale of stock rather than bonds. The average size of issue for industrials did not rise appreciably until the late thirties, when many issues were called and consolidated into larger refunding issues.[5]

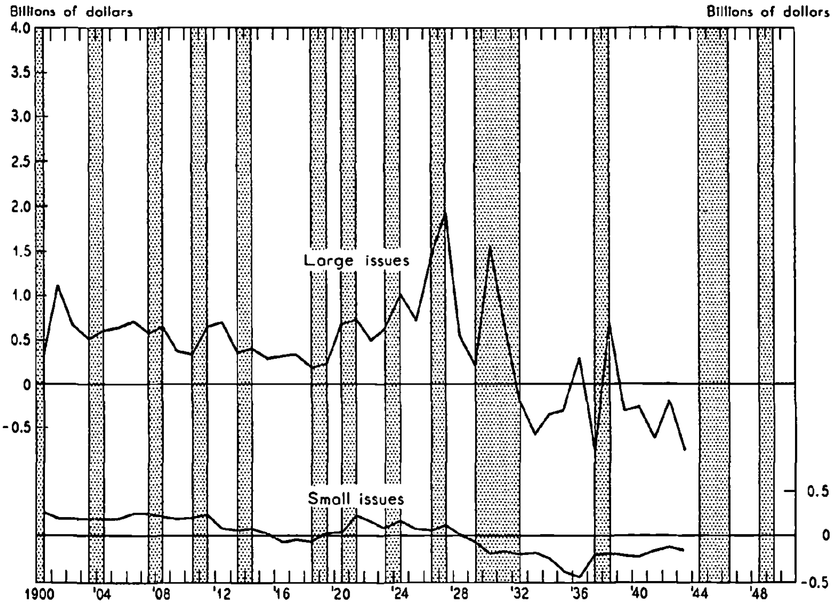
Net Changes in Outstandings

By definition, annual net changes in outstandings, which are presented for straight bonds in Table A-2 and plotted on Chart 6, are differences between outstandings at the beginning and the end of each calendar year. As the table indicates, they were obtained by subtracting the annual par-amount total of extinguishments from that of offerings, and may therefore be interpreted as the net volume of new bond financing by the corporate sector of the economy. Just as it is necessary to distinguish between the aggregate value of the physical assets of the economy (i.e. national wealth) and the net increment to this wealth (in-

[5] The rise occurred after the enactment of the Securities Act of 1933. This act had the effect of adding the cost of registration to other flotation costs, and may thus have worked against the flotation of small issues. On the other hand, the underwriters' spread appears to have declined markedly since the passage of the act. See Chapter 6, footnote 1.

CHART 6—Net Changes in Corporate Bond Outstandings, 1900-1950





From Tables A-2 and A-6; straight bonds, yearly totals, par amount.

Shaded areas, representing contractions in general business activity, and white areas, representing expansions, are based on the National Bureau's business cycle chronology; cf. Arthur F. Burns and Wesley C. Mitchell, Measuring Business Cycles (National Bureau of Economic Research, 1946), p. 78.

vestment), so we must distinguish here between the aggregate volume of claims on the national wealth in the form of corporate bonds (i.e. corporate bond outstandings) and the increments to these claims (net changes in outstandings).

From the standpoint of dynamic analysis the net change in outstandings is a highly significant variable, although for certain types of problems it may be less relevant than other series to be considered later. In an analysis of the gross volume of funds available for capital formation that were obtained through the bond market, for example, we would be most interested in a series covering new-money offerings (cf. Chapter 3). But when the net amount available to business corporations for that purpose is in question, the net change in outstandings is a more pertinent variable.

The problem of interpreting the contribution of the net change in outstandings to business expansions and contractions is complicated by several factors. For one thing, the corporate bond market is only one among several sources of funds for asset expansion, all of which should be examined jointly in a complete analysis. Thus, under certain circumstances a given increment to bond outstandings may be used to finance a net addition of equal amount to the community's capital stock, while under other circumstances it may be largely offset by a release of funds to other markets.

Aside from such offsetting changes a second problem arises in evaluating the effect of a change in outstandings on the aggregate demand for goods and services. To state whether an expanding debt *always* implies a net increase in aggregate demand is not possible on purely theoretical grounds. The issue turns on whether or not the funds borrowed (or repaid) are transferred from idle to active hands. In practice, however, funds are usually borrowed to be used and are seldom loaned if the lender has more pressing commitments. Moreover, our data suggest that repayments on bonded debt are usually made prior to maturity and at the option of the borrower, which implies a voluntary release of funds. In many cases, therefore, large increments in outstandings may be interpreted as contributing to general business expansion, and decrements as contributing to business contraction.

There is the further question of whether the bond market contributes to business expansion only when outstandings are growing (when the net change is positive), or only when the net change itself is growing (when the change in the net change is positive). Haberler, in his analysis of consumer credit, takes the first view, interpreting an increase in outstandings as a factor of expansion and a decrease as a factor of contraction. Abramovitz, on the other hand, in his study of manufacturers' inventories, gives special attention to fluctuations in inventory investment (analogous to changes in the net change).[6] Personally

[6] Readers interested in the pros and cons of this question are referred to Gottfried Haberler's *Consumer Instalment Credit and Economic Fluctuations* (National Bureau of Economic Research, Financial Research Program,

the writer inclines toward the latter point of view. But under either interpretation it is still important to know at which stage of the business cycle the net change is largest and at which stage it is smallest. We shall therefore concentrate on movements in the net change in bond outstandings, with occasional reference, also, to its absolute level.

Working with annual data for major industry and size groups, we first examine the secular drifts in the net changes in outstandings and then consider briefly the behavior of these series over business cycles. A more detailed cyclical analysis based on monthly data will be presented in Chapter 4.

Trends in Net Changes

Except for the sharp upward movement between 1918 and 1927, the general picture obtained from Chart 6 is one of downward drift through 1945. Thereafter the net changes shot upward to a record high in 1948. Although there has been some abatement in the incremental changes since then, they were still abnormally high in 1949 and 1950.

For the combined industries, net changes were positive up to 1932 as funded debt expanded and were generally negative after that time as funded debt contracted. Through 1912, corporations in the three major industry groups were about equally responsible for the general expansion in funded debt; but during the period spanning World War I only the utilities were persistent borrowers. Thus, industrial corporations repaid funded debt on balance in each of the years 1913, 1915, 1918, and 1919, and the rails did so in 1916 and 1918. Funded debt expanded by unusually large amounts in 1920-27 for industrial corporations and in 1923-27 for utilities. Railroad debt expanded in this period only in the trough years of general business activity 1924 and 1927, and again in the contraction year 1930.

For utilities and industrials 1927 was a peak year of net new borrowing; for rails it was a minor peak. In 1928-29, however, all three groups turned from bond to stock financing. The railroads reduced funded debt only slightly in 1928. The decline in industrial debt in 1929 was larger than that in any year but

1942), particularly pp. 69-80, and Moses Abramovitz's *Inventories and Business Cycles* (National Bureau of Economic Research, 1950), particularly pp. 330-31.

1935. Utility debt continued to expand, but by somewhat smaller increments than in earlier years.

Throughout the thirties and up to the close of World War II, contraction in funded debt was general; afterward all major industry groups returned to the captial market to obtain funds for reconversion and expansion. In these developments the railroads were generally laggards. As is well known, rail earnings were extremely low in the thirties, and the roads were unable to free themselves of their excessive debt burdens until the period of higher earnings ushered in by World War II. They did not again become net borrowers until 1948, two years later than the other two industry groups. Moreover, utilities and industrial corporations have been appreciably heavier net borrowers than the railroads throughout the postwar period.

As Chart 6 indicates, the absolute amplitude of variation of the net change in outstandings for large issues was slightly smaller through 1943 than for all issues, but the year-to-year movements of the two series were essentially the same. Small issues had a much smaller amplitude than all issues and large issues. The trend of the net changes for small issues, persistently downward over the period 1900-1943, illustrates a point mentioned earlier, that there was first a slowing up in the rate of growth of small issues outstanding, followed by an actual decline.

Annual Movements in Net Changes over Business Cycles

Corporate bond outstandings, we have seen, show no evidence of short-run cyclical movements. In the net change in outstandings, however, marked cyclical swings will appear.

To reveal whether movements in net changes in outstandings are associated with general business activity, peak and trough years have been plotted on Chart 6 from the National Bureau's chronology of reference-cycle turning points, and reference contractions have been shaded.[7] The annual data on net changes in outstandings, though in some respects inferior for cyclical analysis to the monthly or quarterly data, nevertheless exhibit definite inverse conformity with business cycles. Thus, out of

[7] Cf. Arthur F. Burns and Wesley C. Mitchell, *Measuring Business Cycles* (National Bureau of Economic Research, 1946), p. 78.

twenty possible comparisons at reference-cycle turns through the trough in 1938 we can detect fifteen inverse coincidences in the combined-industries series (counting the near-trough in 1923 as an inverse coincidence), a number far too large to be accounted for solely on the basis of chance.[8]

The cyclical conformity of a series may be measured most conveniently by use of the National Bureau's conformity indexes, which will be described more fully in Chapter 4. Conformity indexes for the annual series are presented in Table 3 for expansion and contraction phases and for the full cycle. A negative expansion index indicates that the series has generally moved downward during expansion phases of business cycles, while a negative contraction index shows that it has generally moved upward during contraction phases. Thus, the almost entirely negative results in the first two columns of the table suggest that the various series on net changes in outstandings are inverted with respect to business cycles, a conclusion that will be borne out by the more detailed analysis of Chapter 4.

TABLE 3—Conformity Indexes for Net Changes in Outstandings of Corporate Bonds: Ten Reference Cycles 1900-1938

	Expansion	*Contraction*	*Full cycle*
All industries	−20	−80	−68
Railroads	−60	−60	−68
Public utilities	−20	−40	−26
Industrials	+20	−40	−5
Large issues	−40	−80	−68
Small issues	−40	−60	−58

Based on annual data, Tables A-2 and A-6; these indexes do not take account of possible leads or lags at reference-cycle turning points.

Expansion indexes for a series with a strong downward trend may be negative even though the short-run movements of the

[8] Our basic series cover only ten complete reference cycles, 1900-1938, being extended by rough estimates to later years only in the case of outstandings and the net changes in outstandings. For the sake of intercomparability between the cyclical analyses of various series, the later information available for net changes is not utilized in this connection; but see footnote 9.

series are totally unrelated to general business expansions; similarly, contraction indexes for such a series may be positive. The full-cycle conformity indexes in Table 3 are computed so as to adjust for the disturbing influences of trend. They are negative in all cases, and confirm the impression of definite inverse conformity.[9]

The evidence of the annual par-amount data suggests that the corporate bond market performs a contracyclic function, supplying, on balance, an increasing volume of funds to the corporate sector during general business contractions and a diminishing volume during general business expansions. Detailed study of this important suggestion must be delayed until monthly data can be examined and cash series developed (Chapters 4 and 6). It is interesting to note in passing, however, that the exceptional behavior of the net-change series in 1932 provides some evidence to the effect that the corporate bond market is typically a market of last resort. After the 1929 collapse of the stock market, outstandings at first expanded in the usual contracyclic way. But the deteriorating credit position of domestic railroad and industrial concerns, coupled with international monetary disturbances, led to a bad break in the bond market in September 1931.[10] The market could not therefore perform its typical contracyclic function at the trough in 1932, and the virtual credit blockage that developed in that year no doubt contributed to the severity of the ensuing depression. Conversely, the atypical behavior of the net-change series in 1947-48 may have been caused in part by the weakness of the stock market in that period. Later analysis

[9] Monthly and quarterly data to be examined in Chapter 4 suggest that the net changes for the all-industry totals typically lead reference turns by a few months, but this does not show up in the annual data. The absolute values of most of the indexes of Table 3 are reduced slightly when recomputed to cover the two later cycles 1938-49, since the net changes conform inversely to the first of them and positively to the second.

[10] A financial crisis had occurred in Austria in May 1931, with the failure of the Credit-Anstalt, and the disturbance quickly spread to New York via the Berlin and London markets. England's suspension of gold payments in September was the occasion for a general liquidation of foreign security holdings in New York and adverse gold movements. To protect the dollar the Federal Reserve banks raised the discount and bill rates, thus contributing further to the strain on the domestic bond market.

of the relationship between stock and bond financing will strongly suggest that conclusion.

Offerings and Extinguishments

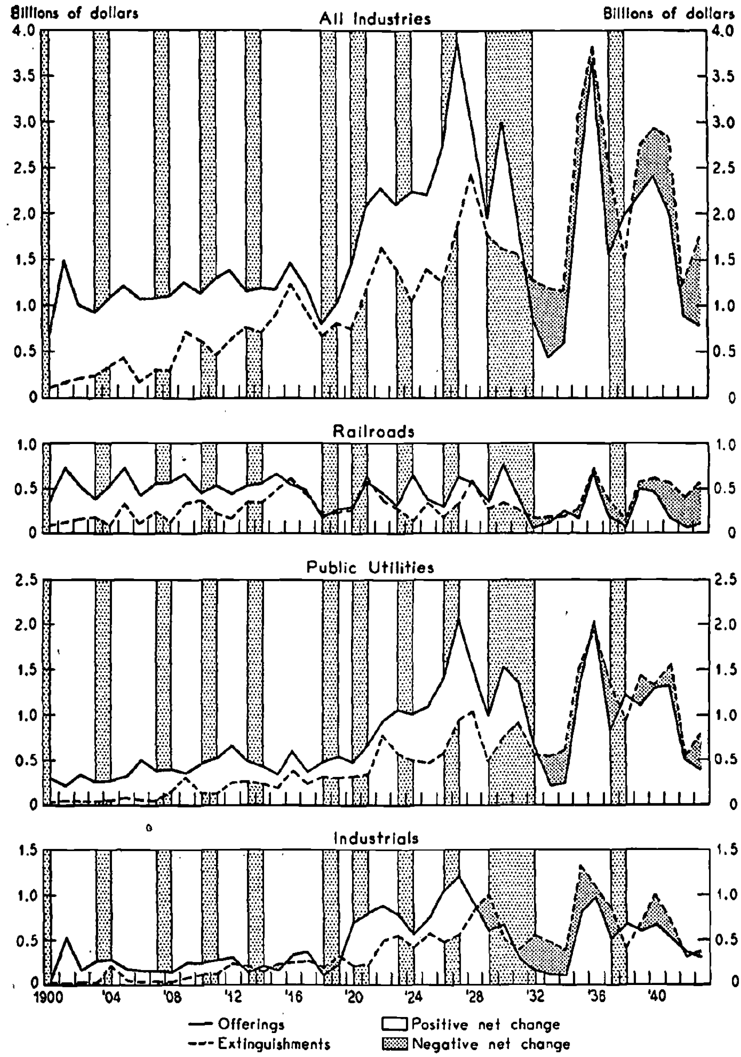
Annual par-amount estimates of straight bond offerings and extinguishments—the two components of the net change in outstandings—are presented in Table A-2 and are plotted in Chart 7. These basic estimates are global in the sense that they cover the total volume of new bond instruments created and of old instruments extinguished. Total offerings include both cash and noncash transactions, and bonds offered to refund other issues as well as those offered for new-money purposes. The extinguishment series also include cash and noncash transactions, bonds refunded by other capital issues, and actual debt repayments. Analysis of the constituent series belongs to the following chapter. Of the gross estimates of offerings and extinguishments now to be examined it should be noted that they are mutually consistent: together they completely determine the net change in outstandings.

Secular and Cyclical Movements in the Annual Series on Offerings and Extinguishments

Secular and cyclical movements in straight bond offerings and extinguishments may be analyzed from Chart 7, on which are also plotted the National Bureau's reference-cycle turning points. The net change in outstandings (i.e. the difference between offerings and extinguishments) is shown as a white area between the two graphs when positive and as a shaded area when negative. As we have seen, until about 1932 offerings exceeded extinguishments and the debt grew. After that, extinguishments usually exceeded offerings, the net change in outstandings turned negative, and the volume of funded debt declined.

The annual movements in offerings, extinguishments, and net changes in outstandings are irregular, and the relationships among these series are not particularly simple. In Chapters 3 and 4 materials will be presented that show the reasons behind many of these irregular movements, and a few illustrations may be brought in here to illuminate the evidence of the chart. In 1918, for example, offerings for the combined industries were relatively low, because of the restrictions imposed by the Capital Issues

CHART 7—Offerings versus Extinguishments of Corporate Bonds, 1900-1943



From Table A-2; straight bonds, yearly totals, par amount.

Shaded areas represent contractions in general business activity and white areas represent expansions (Burns and Mitchell, op.cit., p. 78).

Committee on new security flotations. But extinguishments were also quite low in that year, considering their general upward trend. The net change in outstandings declined but was still positive, indicating that despite the restriction on borrowing, corporations continued to obtain funds on balance through the corporate bond market.

The drop in offerings in 1928-29 was associated with the rise in the stock market, and the peak in 1930 with the drying up of that source. The decline in offerings and extinguishments during 1932-34 was caused by the disorganization of the securities markets in those years, and the reduced volume of extinguishments was effected principally through a reduction in voluntary refundings. On the other hand, the peaks in both offerings and extinguishments in 1936 and 1939-41 were caused mainly by the large volume of issues voluntarily refunded at the low interest rates prevailing then. Moreover, extinguishments exceeded offerings, and the volume of funded debt declined. The drop in offerings in 1937 reflects the unsettled state of the securities markets in that year. In 1938 bond offerings rose slightly but extinguishments fell, so that business corporations were actually net borrowers of funds.

A close examination of Chart 7 indicates that corporate bond offerings have usually moved inversely with cycles in general business activity and that extinguishments have moved with them directly. Conformity indexes summarizing the behavior of these series over business cycles are presented in Table 4. The general impression obtained is one of moderate to high negative conformity for offerings and of moderate to low positive conformity for extinguishments.[11]

Comparison of the expansion and contraction indexes suggests that the tendency for offerings to conform negatively was strongest over contraction phases of the cycle, whereas the tendency for extinguishments to conform positively was strongest

[11] It is interesting that offerings and extinguishments have opposite conformity to business cycles despite the fact that they are positively correlated with each other (cf. Table 5). The reason for the positive correlation is that they contain a common component, refundings. When refundings are omitted, the residual series (new-money offerings and repayments) in general have opposite conformity to business cycles and are virtually uncorrelated with each other (cf. Tables 13 and 14).

during business expansions. Both tendencies are probably associated with the generally rising trends in the series. The negative full-cycle indexes for offerings indicate that these series usually rose during business contractions, or fell at a less rapid rate than in preceding and succeeding expansions; during business expansions, on the other hand, offerings usually fell, or rose

TABLE 4—Conformity Indexes for Offerings and Extinguishments of Corporate Bonds: Ten Reference Cycles 1900-1938

	Expansion	*Contraction*	*Full cycle*
		Offerings	
All industries	+20	−80	−68
Railroads	−20	−60	−79
Public utilities	+20	−40	−16
Industrials	0	−40	−5
Large issues	−20	−80	−68
Small issues	+20	−20	−16
		Extinguishments	
All industries	+20	+20	+5
Railroads	+40	+40	+16
Public utilities	+40	−20	+5
Industrials	+60	+20	+26
Large issues	+20	+20	+5
Small issues	+80	+60	+68

Based on annual data, Tables A-2 and A-6; these indexes do not take account of possible leads or lags at reference-cycle turning points.

at a less rapid rate than during contractions. Conversely, the positive full-cycle indexes for the extinguishment series suggest that in most cases bond extinguishments fell during business contractions, or rose at a less rapid rate than in expansions; during business expansions, on the other hand, they rose, or fell at a less rapid rate than in contractions.

From the above, it appears that the net change in outstandings, which is the difference between offerings and extinguishments, necessarily has negative conformity; this conclusion agrees with our finding in Table 3. Net new bond financing tends to decline in expansion phases of the cycle (or to rise at a less rapid rate than in contractions), since bond offerings fall at this time while extinguishments rise; and for phases of contraction the converse tendency is shown. These findings are substantially in agreement with the conclusions to be advanced in Chapter 4, where monthly and quarterly data will be used and allowance can be made for "leads" and "lags" in the data.[12]

Influence of Offerings and Extinguishments on Net Changes in Outstandings

The various bond series we have been observing separately, in the main, should also be considered in relation to each other. The basic question of interest here is whether a given volume of new offerings usually produces a substantially similar increase in outstandings or whether the new offerings are usually offset by an approximately equal volume of extinguishments.

This question has received little attention in analyses of debt statistics; yet it has important theoretical and practical implications, particularly as it relates to the influence of monetary policy on the bond market. As is well known, the Federal Reserve System seeks to influence the tempo of economic activity through control of the availability and cost of credit. Its policy includes the use not only of general credit controls of the traditional type but also of selective controls over particular credit markets. Selective controls over the bond market were first applied by the Capital Issues Committee during World War I. Under the Defense Production Act of 1950 a selective system of control was also applied to bond flotations by the National Voluntary Credit Restraint Committee, in cooperation with the Board of Governors of the Federal Reserve System.

Effective control over the volume of any type of credit clearly implies control of outstandings or of the net change in outstand-

[12] Only a rough idea of the timing may be obtained from the annual data. Our analysis suggests that annual bond offerings for the combined industries typically expand from the peak of the business cycle (stage v) to the middle of the following expansion (stage III); but annual conformity indexes computed on that basis show little improvement over the ones given in Table 4, being −40 for expansions, −80 for contractions, and −58 for the full cycle. For extinguishments, on the other hand, the conformity improves markedly. Annual extinguishments typically expand during the first half of business expansions (stages I-III), and conformity indexes computed accordingly are +64, +20, and +60. Both the conformity indexes and the timing are altered slightly when based on monthly or quarterly data (cf. Table 16; the cycle stages are defined on page 135).

ings. General and selective controls exercised by the monetary authority do not affect outstandings (or the net change) directly, but indirectly through the primary markets for new loans—in the case of corporate bonds through the market for offerings. It is therefore important to know whether bond offerings are a good indicator of the net change in outstandings.

In the case of instalment sales credit, new credit granted (analogous to our total bond offerings) is a good indicator of the net change. Because such credit is extinguished in regular monthly instalments, the aggregate volume of extinguishments is more stable than the volume of new loans made.[13] Any alteration in the volume of new credit granted is coupled with an immediate change in the same direction in the volume of loans outstanding. From the statistical point of view, the distinguishing features of markets of this type, in which a new-loan series is an efficient indicator of the net credit change, are (1) that offerings exhibit a greater amplitude of variation than extinguishments, and (2) that the two series are not highly correlated.

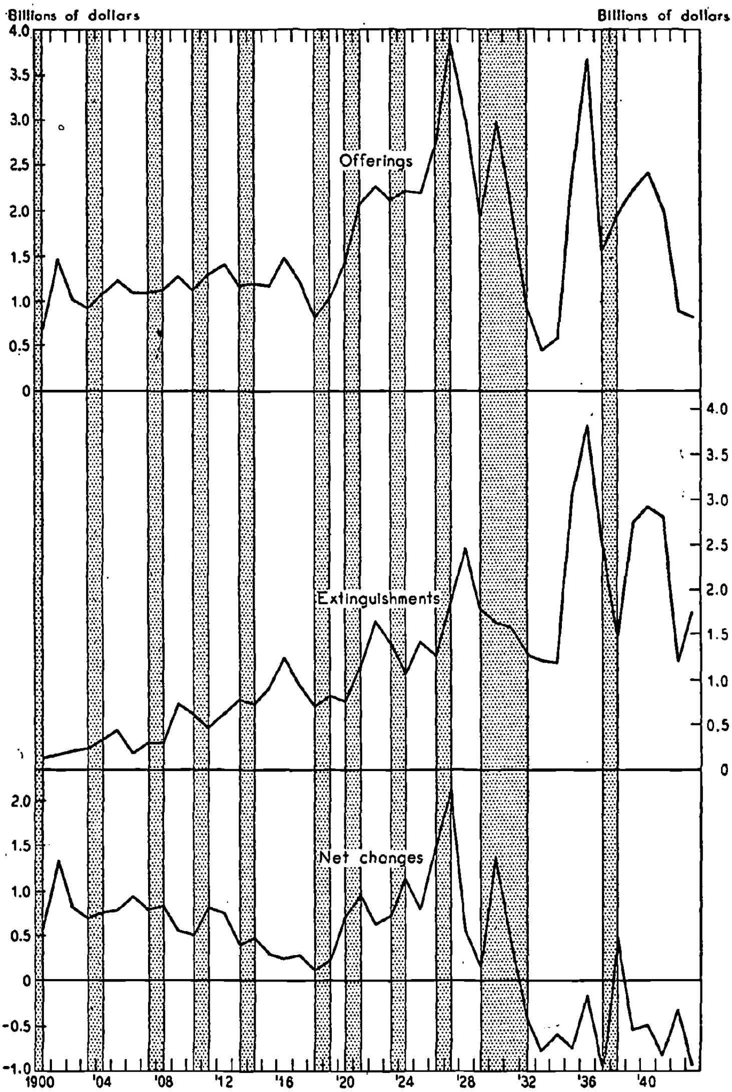
Chart 8 shows that the corporate bond market does not have those characteristics. For all industries combined, total offerings and extinguishments have fluctuated over approximately the same range during the period of study. Moreover, the two series exhibit fairly high correlation, since both of them include bonds offered to refund other bonds. It follows that the year-to-year movements in the net changes in outstandings not only are smaller than the movements in bond offerings but also are not highly correlated with them. Thus a series on gross bond offerings is a rather poor indicator of both trends and cycles in the net change in outstandings.

Table 5 gives the correlation coefficients for the various industry groups between the paired series of offerings, extinguishments, and net changes in outstandings, along with ratios of the variance of offerings to the variance of extinguishments; in general, it supports the conclusions just stated.[14] In each case, the

[13] The aggregate volume of extinguishments owes its stability to the fact that it is, in effect, a moving average of the volume of credit granted in the past.

[14] Variance ratios measure relative variability; a definition of the variance is given in the technical note at the end of this chapter. Of interest principally to mathematical statisticians, the note demonstrates that the behavior

CHART 8—Offerings, Extinguishments, and Net Changes in Outstandings of Corporate Bonds, 1900-1943



From Table A-2; straight bonds, yearly totals, par amount.

Shaded areas represent contractions in general business activity and white areas represent expansions (Burns and Mitchell, op.cit., p. 78).

TABLE 5—Correlation Coefficients and Variance Ratios for Offerings, Extinguishments, and Net Changes in Corporate Bond Outstandings, 1900-1943

	1900-1943				*1900-1907*
	All industries	*Railroads*	*Public utilities*	*Industrials*	*Public utilities*
Correlation Coefficients					
Offerings and extinguishments	+0.67[a]	+0.17	+0.78[a]	+0.63[a]	+0.15
Offerings and net changes	+0.29	+0.71[a]	+0.34	+0.35	+0.98[a]
Extinguishments and net changes	−0.51[a]	−0.57[a]	−0.33	−0.50[a]	−0.06
Ratios of Variances[b]					
Offerings to extinguishments	0.80	1.37	1.01	0.85	21.81[c]

Based on annual data, Table A-2.

[a] A coefficient of this size or larger would be obtained by chance in less than 5 out of 100 trials if drawn from a population in which the variables are uncorrelated.

[b] For a discussion of the variance and variance ratio, see footnote 14 and the statistical note at the end of this chapter.

[c] A variance ratio of this size or larger would be obtained by chance in less than 5 out of 100 trials if drawn from a population in which the variables have equal variances.

range of variation for offerings was approximately the same as for extinguishments during the period 1900-1943 (the ratios of the variances are close to unity). Moreover, the correlation between offerings and extinguishments was fairly high for bonds of the combined industries and for utilities and industrials; hence for these groupings the correlation between offerings and net changes and between extinguishments and net changes was low.

Public utility bonds in the period of early growth, 1900-1907, show behavior very like that typified by the instalment sales credit market, where new credit granted carries over directly to outstandings. Utility offerings and extinguishments appear virtually uncorrelated in those years, with a positive correlation coefficient of only 0.15; and extinguishments were quite stable in relation to offerings, the offerings-extinguishments variance ratio being 21.8. Virtually all of the new flotations of public utility issues therefore contributed to the sharp rise in outstandings over the eight years: the correlation coefficient between offerings and net changes in outstandings was +0.98. These conditions did not hold, however, over the whole period studied, 1900-1943.

The behavior of railroad bonds over the period 1900-1943 considered as a unit provides an interesting example of a group for which a bond-offerings series is a moderately good index of the net change. Offerings and extinguishments were virtually uncorrelated during the period of study ($r = +0.17$), and both series fluctuated over approximately the same range (the variance ratio was 1.37). For this particular group—and for this group only—the correlation between offerings and net changes over the entire period was fairly high (+0.71). As a result, approximately half of the year-to-year movements in the net change in outstandings for rail bonds were accounted for by new offerings.[15]

To summarize, the general impression obtained from Chart 8 and Table 5 is that the net change in outstandings is not closely

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of the net change in outstandings is completely determined in the statistical sense by the variance ratio for offerings and extinguishments and by the correlation between them, and develops a joint test of significance for the variance ratio and correlation coefficient.

[15] Justification for this statement is given in the technical note at the end of the chapter.

related to a change in total bond offerings, the rail group being the only exception. One reason for this lack of relationship, notable when the bond series are compared with, say, the corresponding series for instalment sales credit, is that corporate bonds have extremely long terms to maturity, whereas consumer loans are relatively short-term. In the case of instalment sales credit, the old loans run off quickly, and a reduction of new loans has an immediate effect on outstandings. Since most bonds mature in the distant future, many years would be required to effect a proportional change in funded debt outstanding. Another reason for the independence of the series is that most bonds, unlike instalment credit loans, have optional maturity features, so that the contractual and actual dates of repayment frequently do not coincide. Thus if gross bond offerings expand in response to favorable market conditions, the proceeds of the offerings may be used largely to call and refund other issues before maturity. In the thirties, for example, the volume of funded debt outstanding actually declined (the net changes were negative) despite official encouragement of bond offerings.

Number of Years from Offering to Extinguishment

The optional maturity feature of corporate bonds also presents a problem for the investor who attempts to space the maturities in his portfolio so that funds will flow in as needed. Since bonds are frequently called at the option of the obligor or are repaid at some date other than maturity because of a default situation, the maturity date is almost completely untrustworthy as a means of determining the timing of repayments in the future.

The investor's problem is complicated further by the fact that his requirements for funds may shift in an unforeseen way with general economic conditions. For example, when the investor is losing funds, the ideal bond portfolio is one with sufficiently large extinguishments to meet the drain on cash. In other times, when funds are flowing on balance toward him, his pressing problem becomes one of keeping the funds at work; that is, of providing investment outlets both for funds released from old obligations and for the net, new acquisitions of cash. But over the period 1900-1943, as we shall see, the actual length of life of corporate bonds—the number of years from offering to extinguishment—has been shortest during general business expansions when funds were flowing toward the large investment intermediaries in excess of the usual amounts. And contrariwise, during periods of general business contraction, when investment funds were flowing toward the intermediaries in less than the usual amounts, the average life of corporate bonds has lengthened.

Median Period from Offering to Maturity and from Offering to Extinguishment

The changes that have occurred in the life spans of corporate bonds may be analyzed directly by computing for each offering the number of years between the dates of offering and of final extinguishment, and by averaging these over different periods. Such an approach to the problem of measurement has the advantage of ease of interpretation but the disadvantage that the mean number of years from offering to extinguishment for a group of bonds cannot be determined until all the bonds offered are finally extinguished. The disadvantage may be overcome in part by employing the median instead of the mean, since the median number of years to extinguishment is determined as soon as 50 percent or more of the bonds offered are extinguished. Yet for short or recent periods the 50 percent point is not always reached. It has been necessary, therefore, to work with fairly large groups of bonds, and to exclude bonds offered after 1939.

Table 6 presents median terms to maturity and to extinguishment (median number of years from offering date to maturity date and from offering date to date of final extinguishment) for offerings of all large issues made during the period 1900-1939, and for those offered in the two subperiods 1900-1929 and 1930-39.[16] These medians are based on the number of offerings and hence give no effect to differences in the size of various offerings. Moreover, they cover only the period from offering to final extinguishment, and give no effect to parts of issues extinguished through sinking fund calls, etc.[17]

[16] Large issues are those whose offerings summed to $5 million or more.

[17] The median term to extinguishment refers to the number of years that bonds were on the books of the obligor, not the books of any particular investment institution. This distinction is important in the case of corporate bonds, which enjoyed an active secondary market during most of the years covered by our records. It is much less important in the case of mortgage loans, which are seldom transferred without the redrafting of the contract

Owing possibly to the higher costs of long-term as compared with short-term borrowing in the thirties, and perhaps also to a growing skepticism on the part of obligors and investors about

TABLE 6—Median Number of Years from Offering to Maturity and from Offering to Extinguishment of Corporate Bonds, 1900-1939

PERIOD OF OFFERING	YEARS TO Maturity	YEARS TO Extinguishment
	All industries	
1900-1939	25.5	14
1900-1929	26.0	16
1930-1939	22.2	7
	Railroads	
1900-1939	33.9	24–25
1900-1929	34.6	27
1930-1939	26.4	12
	Public utilities	
1900-1939	25.6	13
1900-1929	25.7	16
1930-1939	25.1	7
	Industrials	
1900-1939	15.1	8
1900-1929	16.2	9
1930-1939	12.2	5

Based on special tabulations of Corporate Bond Project data supplemented by a search of financial sources for extinguishment data in recent years. The data include only offerings of large straight issues.

the wisdom of extremely long maturities, the terms to maturity of bonds of the combined industries offered in that decade were shorter by 15 percent than the terms characterizing earlier years. The decrease was greatest for the rails and industrials, where the median term fell from 35 to 26 years and from 16 to 12 years

(i.e. the "extinguishment" of the loan). For this reason our figures are roughly comparable with estimates developed in the National Bureau's institutional studies of mortgage loan experience. See, however, C. Lowell Harriss's *History and Policies of the Home Owners' Loan Corporation* (National Bureau of Economic Research, Financial Research Program, 1951), pp. 170-180, for a discussion of the sale, at liquidation of the HOLC, of an appreciable volume of mortgage loans without change of contract.

respectively. The median term remained at about 25 years for utilities.

An even more dramatic change occurred in the actual length of life of bonds offered in the two subperiods. As measured by median term to extinguishment, the life span of offerings shortened from about 16 to 7 years. An equal shortening of median term to extinguishment by roughly 50 percent occurred in each of the major industry groups.[18]

By comparing the median number of years from offering to maturity with the median number of years from offering to extinguishment, we obtain some indication of the unreliability of the maturity date as an indicator of the actual length of life of corporate bonds. For all industries combined, the median number of years over which bonds offered in the period 1900-1939 were actually outstanding was approximately half the contractual term (14 years versus 26 years). For bonds offered in the thirties the actual term was less than one-third the contractual term (7 years versus 22 years). These figures emphasize the difficulties that plague those who attempt to maintain properly spaced portfolios, particularly in periods of falling interest rates such as the thirties and early forties.[19]

Turnover Rates and Their Use in Estimating the Number of Years from Offering to Extinguishment

The annual turnover rate of corporate bonds, as used here, is the ratio, in percentage form, of the par amount of extinguishments during a calendar year to the par amount of all the issues out-

[18] The median number of years from offering to extinguishment would doubtless have been even shorter if it could have included offerings made between 1939 and 1944.

[19] Corporations were able to effect considerable reductions in interest costs during the thirties and early forties by refunding old bonds at lower rates of interest (cf. page 243). The savings, however, were not so large as they might have been if the corporations had been able to make reliable forecasts of the future course of rates of interest. During that period short-term interest rates were consistently below long-term rates; nevertheless corporations initially refunded their old bonds into 22-year maturities on the average, and then, as interest rates continued to fall, repaid or refunded the new obligations in a little over seven years. An initial decision to issue 7-year maturities would have resulted in a further saving in interest costs (approximately one-half percent annually, the average differential between 22-year and 7-year money rates during the thirties).

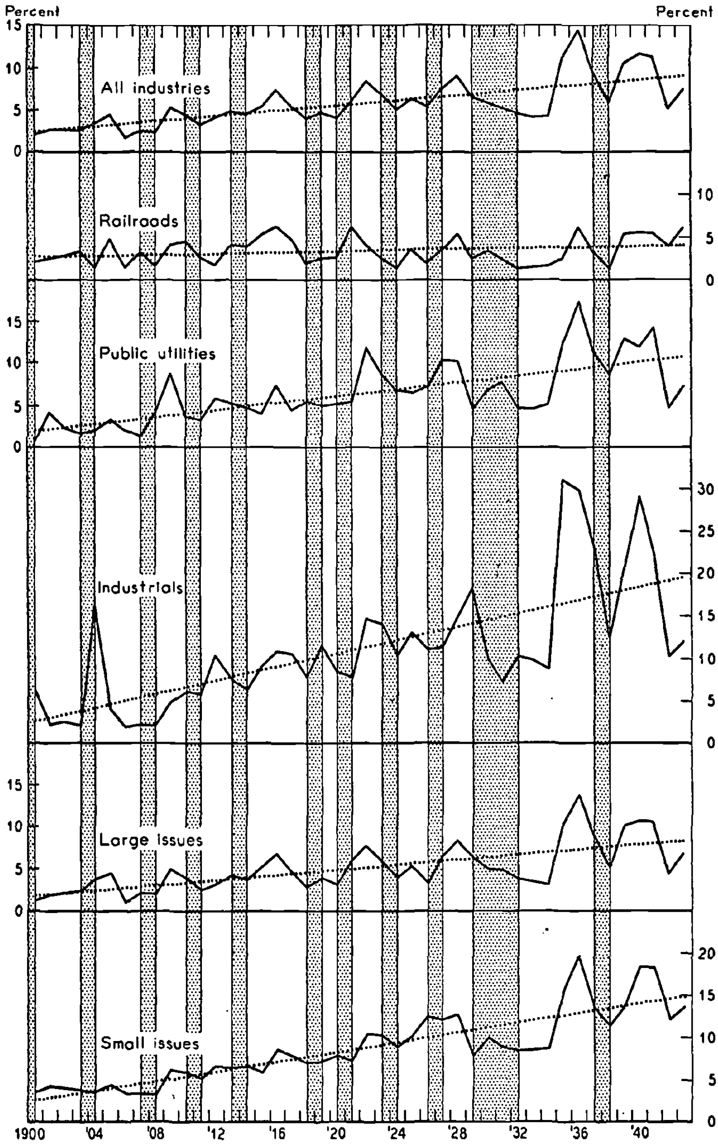
standing at the beginning of that year. The annual rates for straight bonds are shown in Chart 9 and are summarized for a few significant periods in Table 7.

Since outstandings form a smooth series, annual turnover rates are highly correlated with the annual par amount of extinguishments, and the short-run movements in one series may be inferred from the other. The principal advantage of the turnover rates over a series on bond extinguishments is that the reciprocal of these rates throws some light on the length of life of corporate bonds. For example, a turnover rate of 5 percent per annum continuing over a period of several years would show that one-twentieth of the outstandings were being retired in each year, and would imply an "average" length of life of corporate bonds, during the period, of about twenty years. Unlike the medians examined above, the annual turnover rates have the advantage of reflecting extinguishments through sinking fund calls, etc. as well as final extinguishments. From a computational viewpoint, an additional advantage is that the turnover rate is always defined, no matter how short the period over which it is computed. As can be seen from Chart 9, however, turnover rates computed over a period as short as a year are highly unstable. The reciprocal of the annual turnover rate cannot be used to determine the average length of life of corporate bonds unless the rate remains roughly constant for a long period of years. Since the actual rates moved rapidly and erratically, the reciprocals throw light only in a qualitative sense on the broad drift in the length of life of corporate bonds.[20]

Chart 9 indicates an upward and approximately linear drift in

[20] An analogous situation arises in interpreting population statistics. The present U.S. crude death rate (the analogue of our turnover rate) of about ten per 1,000 does not imply an average life expectancy of 100 years, although the continuous decline in this rate does reflect the fact that life expectancy is higher now than in years gone by. The current rate is based on a population with a larger proportion in the lower age brackets than can be expected to reproduce itself in the future. Over short periods, it would theoretically be possible for the crude death rate to drift downward without any change in average life span, solely as a result of a rise in the proportion entering the lower age brackets. The turnover rate on corporate bonds may be influenced similarly, although this is not so likely, owing to the relatively weak association between age of bond and the years remaining to extinguishment.

CHART 9—Annual Turnover Rates for Corporate Bonds, 1900-1943



Based on Tables A-2 and A-6; for straight bands.

Shaded areas represent contractions in general business activity and white areas represent expansions (Burns and Mitchell, op.cit., p. 78).

Trend lines were fitted by the method of least squares.

turnover rates for each industry and size group. The upward drift was pronounced for utilities, industrials, and small issues; it was less pronounced for large issues and the issues of all groups combined, and was smallest for rails. The principal conclusion to be drawn from the chart is that there was a more or less continuous fall in the length of life of corporate bonds over the period studied.

A close examination of the chart indicates that turnover rates, like bond extinguishments, frequently reach peaks during business expansions, a conclusion that will be supported by the detailed analysis of extinguishments in Chapter 4. The implication to be drawn from the rise in turnover rates in periods of general business expansion is that corporate bonds then take on an illusory liquidity—illusory since the turnover rates subsequently fall in late expansion or early contraction phases of the cycle.

Table 7 shows that the average of the annual turnover rates for the entire period 1900-1943 was 5.8 percent for the bonds of all industries combined, and was highest for industrials and lowest for rails. These rates imply a period from offering to extinguishment of 17.2 years for all issues, and of 9.0 years for industrials, 15.4 years for utilities, and 30.3 years for rails. The same order of industrial differences held in both of the subperiods 1900-1934 and 1935-43, but the differences were more marked in the latter period than in the former. In 1935-43 many public utility and industrial issues were refunded at lower interest rates, with an attendant shortening of length of life of issues in these groups. The railroads were not then in a position to take advantage of the lower interest rates, since many of the callable rail bonds were selling at substantial discounts from call price and a large proportion of the remaining bonds were long-term and noncallable.

For various reasons the life span of corporate bonds as implied by the turnover rates is not directly comparable with the median period from offering to extinguishment. For one thing, the reciprocal of a turnover rate should be expected to behave like a mean, and for a distribution such as the one under examination (skewed toward long life spans) it should be larger than the median. For another, computation of the median frequently required the use of extinguishment data for the years 1944-50, whereas the turnover rates are all based on extinguishments for earlier years. The difference in period of coverage introduces a particularly serious discrepancy in the case of rail bonds, which had the longest terms to extinguishment. Despite these and other conceptual differences, the broad conclusions obtained from the

TABLE 7—Average Annual Turnover Rates, and Implied Length of Life of Corporate Bonds, 1900-1943

PERIOD OF EXTINGUISH-MENT	*All industries*	*Rail-roads*	*Public utilities*	*Indus-trials*	*Large issues*	*Small issues*
		Average turnover rates (percent)				
1900-1943	5.8	3.3	6.5	11.1	5.1	8.7
1900-1934	4.8	3.0	5.3	8.5	4.1	7.1
1935-1943	9.7	4.3	11.1	21.1	9.0	15.0
Contraction and trough years	4.6	2.5	5.4	9.0	4.0	7.2
Expansion and peak years	6.2	3.6	6.9	12.0	5.5	9.3
		Implied length of life of bonds (years)				
1900-1943	17.2	30.3	15.4	9.0	19.6	11.5
1900-1934	20.8	33.3	18.9	11.8	24.4	14.1
1935-1943	10.3	23.3	9.0	4.7	11.1	6.7
Contraction and trough years	21.7	40.0	18.5	11.1	25.0	13.9
Expansion and peak years	16.1	27.8	14.5	8.3	18.2	10.8

Based on annual data, Tables A-2 and A-6.

two sets of data are essentially similar. Over the period as a whole there was a shortening in the life span of corporate bonds both as implied by the turnover rates and as observed in the medians. Rail bonds had the lowest turnover rates and longest median period from offering to extinguishment. Industrials had the highest turnover rates and the shortest median period.

TECHNICAL STATISTICAL NOTE ON MARKET STRUCTURE

Economic variables are frequently joined by accounting identities, as in the following examples:

Consumer outlay $\equiv$ Disposable income — Savings

Corporate retained earnings $\equiv$ Income after taxes — Dividends paid

Net change in bond outstandings $\equiv$ Offerings — Extinguishments

Other examples will arise in Chapters 3, 5, and 6. In the analysis of such series the important question is: what effect does each of the component variables (those in the right-hand member of the identity) have on the resultant (the left-hand member)? Thus, in the corporate bond field, we wish to study the influence of offerings and extinguishments on the net change in outstandings.

Classical correlation methods, both multiple and simple, may be inapplicable to such data. When the multiple correlation technique is applied to concurrent data (that is, where each set of observations covers the same interval of time), the multiple and partial correlation coefficients and the regression coefficients are all identically equal to plus or minus one. The multiple correlation technique thus contributes nothing that is not already known a priori to an understanding of the concurrent behavior of variables joined by an accounting identity; it creates meaning in the statistical sense only when the identity is broken by lagging certain of the variables by some specified time interval. To do so, however, changes the nature of the economic problem under investigation (or the theory being tested); it does not help explain relationships among concurrent variables.

Simple correlation techniques sometimes yield more interesting results when applied to accounting identities, but the results are frequently biased or difficult to interpret. Thus, the coefficient of determination (the square of the correlation coefficient) has an unambiguous meaning only in certain limiting cases that are seldom encountered in practice; in addition, the simple regression coefficient is frequently biased, so that this statistic is generally unacceptable.

It is proposed, first, to show that the behavior of variables joined by an identity is completely described by two parameters of the system; second, to illustrate how different types of credit markets may be classified on the basis of the two parameters; and third, to develop a method for testing hypotheses about the behavior of the variables in the identity and for setting up confidence regions for the parameters. The method is quite general: it may be extended to cover all linear accounting identities in which n components have the multivariate normal distribution and to nonlinear identities involving ratios in which the components have the logarithmic normal distribution.

1. *Parameters Characterizing Market Structure*

Accounting identities of the simple type treated here may be expressed in the form

$$X_1 = X_2 - X_3 \tag{1.1}$$

Consider a sample of N observations on the X's, say, O_N: (x_{1a}, x_{2a}, x_{3a}) $a = 1, \ldots, N$, $N > 2$. The sample means $(\bar{x}_i)$, variances (s_{ii}), covariances (s_{ij}), and correlation coefficients (r_{ij}) are defined as follows:

$$N\bar{x}_i = \sum_{a=1}^{N} x_{ia}$$

$$Ns_{ij} = \sum_{a=1}^{N} (x_{ia} - \bar{x}_i)(x_{ja} - \bar{x}_j) \tag{1.2}$$

$$(s_{ii}s_{jj})^{\frac{1}{2}}r_{ij} = s_{ij} \qquad (i, j, = 1, 2, 3)$$

The corresponding population parameters are defined analogously but are written in Greek letters.

By substituting sample values in Eq. (1.1), expressing them in terms of deviations about the sample means, and squaring and summing both members, we obtain the relation

$$s_{11} = s_{22} + s_{33} - 2s_{23} \tag{1.3}$$

Taking expected values in Eq. (1.1) shows that a similar relation holds between the population variances and covariances, viz.,[1]

$$\sigma_{11} = \sigma_{22} + \sigma_{33} - 2\sigma_{23} \tag{1.4}$$

Expressions of the type (1.3) hold for all samples in which the variables are joined by an accounting identity of the type (1.1); expressions of the type (1.4) hold for all populations similarly defined provided only that the population variances and covariances exist. The relationships (1.3) and (1.4) are thus "distribution-free."

It is to be shown first that the behavior of the variables in the accounting identity is completely determined by the ratio of the variances of the two components (X_2 and X_3) and by the correlation between them. Consider the system of equations obtained from (1.1):

$$\begin{aligned} X_1 &\equiv X_2 - X_3 \\ X_2 &\equiv X_1 + X_3 \\ X_3 &\equiv X_2 - X_1 \end{aligned} \tag{1.5}$$

[1] S. S. Wilks, *Mathematical Statistics* (Princeton, 1944), p. 34.

and the corresponding relationships among the variances and covariances:

$$
(1.6)\quad
\begin{array}{llll}
\sigma_{11} - \sigma_{22} - \sigma_{33} & & & +2\sigma_{23} = 0 \\
\sigma_{11} - \sigma_{22} + \sigma_{33} & & +2\sigma_{13} & = 0 \\
\sigma_{11} + \sigma_{22} - \sigma_{33} & -2\sigma_{12} & & = 0
\end{array}
$$

The system (1.6) is homogeneous and linear in the six variances and covariances and is of rank 3. For such a system it can be shown that when the columns of the matrix corresponding to a certain three unknowns constitute a matrix of rank 3, then the remaining three unknowns may be assigned arbitrarily and may be regarded as parameters in terms of which the other three unknowns are determined.[2] An examination of Eqs. (1.6) shows that we can pick any two variances and any covariance and determine a solution. In particular, if we pick σ_{22}, σ_{33}, and σ_{23}, then we can solve for σ_{11}, σ_{12}, and σ_{13}. Let $(\sigma^*_{11}, \sigma^*_{22}, \sigma^*_{33}, \sigma^*_{12}, \sigma^*_{13}, \sigma^*_{23})$ be such a solution; then it is clear that $(c\sigma^*_{11}, \ldots, c\sigma^*_{23})$ is also a solution. Thus, if $c = 1/\sigma^*_{33}$ a unique solution is obtained for $(\sigma^*_{11}/\sigma^*_{33}, \sigma^*_{12}/\sigma^*_{33}, \sigma^*_{13}/\sigma^*_{33})$ in terms of the ratios $(\sigma^*_{22}/\sigma^*_{33}, \sigma^*_{23}/\sigma^*_{33})$. Finally, since $\sigma_{23}/\sigma_{33} \equiv (\sigma_{22}/\sigma_{33})^{\frac{1}{2}} \rho_{23}$, a unique solution of Eqs. (1.6) exists in terms of the assigned parameters $(\sigma^*_{22}/\sigma^*_{33}, \rho^*_{23})$, which was to be proved. It is evident from the relevant definitions and by use of Eqs. (1.6) that the other two correlation coefficients (ρ_{12}, ρ_{13}) and all of the regression coefficients (β_{ij}) are also determined by these parameters. Thus a single variance ratio and correlation coefficient completely determine the behavior of the variables in the accounting identity—that is, their relative variation and covariation. It should be noted that the two parameters of the system considered here are invariant under a linear magnification of the common scale on which X_1, X_2, and X_3 are measured.

System (1.6) shows that the variance of the resultant (σ_{11}) is equal to the sum of the variances of the components $(\sigma_{22} + \sigma_{33})$ plus or minus a factor that depends on the correlation between components. When the components are uncorrelated, $\sigma_{23} = 0$ and $\sigma_{11} = \sigma_{22} + \sigma_{33}$. Substitution of this value of σ_{11} into the second and third equations of (1.6) shows that $\rho^2_{13} = \sigma_{33}/\sigma_{11}$ and $\rho^2_{12} = \sigma_{22}/\sigma_{11}$. It follows that when the components have low correlation, the proportion of the total variation in X_1 attributable to each component is approximately equal to the square of the appropriate correlation coefficient.

When the component series have high correlation, the proportion of the total variation directly attributable to each component usually cannot be determined so simply. However, the intercorrelation may be neglected for practical purposes when the variance of one com-

[2] A. C. Aitken, *Determinants and Matrices* (Edinburgh, 1939), p. 69.

ponent is large in comparison with that of the other, as is shown by expressing the first of the equations in (1.6) in the two forms

$$(1.7)\qquad \begin{aligned} \frac{\sigma_{11}}{\sigma_{33}} &= \frac{\sigma_{22}}{\sigma_{33}} + 1 - 2\left(\frac{\sigma_{22}}{\sigma_{33}}\right)^{\frac{1}{2}} \rho_{23} \\ \frac{\sigma_{11}}{\sigma_{22}} &= 1 + \frac{\sigma_{33}}{\sigma_{22}} - 2\left(\frac{\sigma_{33}}{\sigma_{22}}\right)^{\frac{1}{2}} \end{aligned}$$

It then appears that

$$(1.8)\qquad \begin{aligned} &\lim_{\sigma_{22}/\sigma_{33}\to 0} \frac{\sigma_{11}}{\sigma_{33}} = 1, \text{ and} \\ &\lim_{\sigma_{33}/\sigma_{22}\to 0} \frac{\sigma_{11}}{\sigma_{22}} = 1, \end{aligned}$$

since ρ_{23} is bounded above and below. In other words, when the variance of one component is large in relation to that of the other, the variance of the resultant is approximately equal to the variance of the larger component.

2. *Classification of Hypothetical Credit Markets on the Basis of Two Parameters*

Four hypothetical cases are presented to illustrate how various types of credit markets may be described in terms of the two underlying parameters of the system:

In markets of type A, offerings (new credit granted) and extinguishments are virtually independent (e.g. free of refundings), and extinguishments form a smooth series in relation to offerings. Statistically speaking, the requirements are that in the limit $\rho_{23} = 0$ and $\sigma_{33}/\sigma_{22} = 0$. Eqs. (1.6) and (1.8) show that when these conditions hold, $\sigma_{11}/\sigma_{22} = \rho_{12} = 1$ and $\rho_{13} = 0$. In short, the market is completely characterized by the two parameters σ_{22}/σ_{33} and ρ_{23}. For such markets the net change is perfectly correlated with offerings and is independent of extinguishments: thus a series on new credit granted is a reliable indicator of net credit change. Type A is often approximated in reality. Over short periods, the market for instalment sales credit approaches it. Note also the utility bond market over the period 1900-1907 (Table 5).

Markets of type B are the reverse of type A in that outstandings are dominated by extinguishments. The situation envisaged is that in which offerings form a smooth series in relation to extinguishments. In the limit $\sigma_{22}/\sigma_{33} = 0$ and $\sigma_{11}/\sigma_{33} = \rho_{13} = 1$. If in addition offerings and extinguishments are uncorrelated ($\rho_{23} = 0$), then $\rho_{12} = 0$. For markets of type B a series on extinguishments is an efficient indicator of the net change in outstandings. Type B is not often approximated in practice; but the behavior of certain contractual savings schemes does approach it. Since the flow into the stock of savings is regular while withdrawals are erratic, the net change is dominated by withdrawals.

Markets of type C occupy an intermediate position between A and B. Here, the variances of offerings and extinguishments are assumed to be of approximately the same magnitude ($\sigma_{22}/\sigma_{33} = 1$) and the series are taken to be independent ($\rho_{23} = 0$). Eqs. (1.6) then show that $\sigma_{11} = 2\sigma_{22} = 2\sigma_{33}$, so that half of the variation in the net change in outstandings is explained by offerings and half by extinguishments. In addition, $\rho_{12} = -\rho_{13} = 0.707$, or $\rho^2_{12} = \rho^2_{13} = 1/2$. For markets of type C, prediction of the net change in outstandings by either offerings or extinguishments separately would be only half as efficient as prediction by the two simultaneously. Table 5 suggests that the railroad bond market is roughly of type C.

Markets of type D are similar to those of type C in that the variance of offerings is of about the same magnitude as the variance of extinguishments ($\sigma_{22}/\sigma_{33} = 1$), and different in that the two series are assumed to have high positive correlation ($\rho_{23} = 1$). By substitution in Eqs. (1.6) we find $\sigma_{11}/\sigma_{33} = \sigma_{11}/\sigma_{22} = \rho_{12} = \rho_{13} = 0$. Clearly the net change in outstandings is constant, so that both offerings and extinguishments are independent of outstandings. Table 5 shows that for the public utility sector of the bond market, 1900-1943, the series on total offerings and extinguishments behave about that way: offerings and extinguishments are highly correlated through refundings; both fluctuate over roughly the same range; and both are virtually independent of the net change in outstandings.

That total offerings and total extinguishments of corporate bonds are positively correlated has been shown in Table 5. To eliminate refundings from each of the series, as will be done in Chapter 3 to obtain series on new-money offerings and repayments, breaks the correlation and reveals an underlying market structure intermediate between situations A and C. Table 14 (page 120) shows that new-money offerings are correlated highly, though by no means perfectly, with the net changes in outstandings and could therefore serve as a rough indicator of the net change.

3. *A Significance Test and a Confidence Region for the Parameters Characterizing Market Structure*

The parameters (σ_{22}/σ_{33}, ρ_{23}) that completely determine market structure are usually not known and must be estimated from sample data. We are then confronted with the problem of testing whether the information provided by the sample is consistent with the hypothesis that the market has a given structure and of laying down a confidence region for the population parameters.

An appropriate criterion for testing whether the market has a given structure can be constructed if we are willing to assume that certain variates in the accounting identity, say X_2 and X_3, have an appropriate probability distribution. In the present investigation of corporate bond financing it has been assumed that the variates have the bivariate normal distribution:

(3.1)

$$P(X_2, X_3) = K \mid \sigma_{ij} \mid^{-\frac{1}{2}} e^{-\frac{1}{2} \sum_{i,j} c_{ij} (X_i - a_i)(X_j - a_j)} \qquad (i, j = 2, 3)$$

where the σ_{ij} are elements of the population covariance matrix Σ, and the c_{ij} are elements of the inverse of this matrix. We wish to test a hypothesis about $(\sigma_{22}/\sigma_{33}, \rho_{23})$ subject to the information provided by S, the sample covariance matrix, under the assumption that Eq. (3.1) holds. The hypothesis (H_0) to be tested may be stated symbolically as follows:

$$(3.2) \qquad H_0 : \left(\frac{\sigma_{22}}{\sigma_{33}}\right) = \left(\frac{\sigma_{22}}{\sigma_{33}}\right)^* ; \quad \rho_{23} = \rho_{23}{}^*$$

We recall from section 1 that such a hypothesis is equivalent to the hypothesis that all of the variance ratios, correlation coefficients, and regression coefficients have specified values. From the fact that

$$\Sigma = \begin{Vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{23} & \sigma_{33} \end{Vmatrix} = \sigma_{33} \begin{Vmatrix} \dfrac{\sigma_{22}}{\sigma_{33}} & \left(\dfrac{\sigma_{22}}{\sigma_{33}}\right)^{\frac{1}{2}} \rho_{23} \\ \left(\dfrac{\sigma_{22}}{\sigma_{33}}\right)^{\frac{1}{2}} \rho_{23} & 1 \end{Vmatrix}$$

it is clear that the hypothesis (3.2) is equivalent to the hypothesis that the population covariance matrix Σ is specified up to an arbitrary scale factor σ_{33}; that is, that Σ is proportional to a given matrix. The hypothesis (3.2) may therefore be expressed in the equivalent form

$$(3.3) \qquad H_0 : \quad \Sigma = c\, \Sigma^*$$

where c is positive but otherwise unspecified and Σ^* is the set of all matrices having the properties $(\sigma_{22}/\sigma_{33}) = (\sigma_{22}/\sigma_{33})^*$, $\rho_{23} = \rho_{23}^*$.

A likelihood ratio criterion to test the hypothesis (3.3) has been developed by John W. Mauchly for the particular case in which Σ^* is the identity matrix I; that is, in which $(\sigma_{22}/\sigma_{33})^* = 1$, and $\rho_{23}^* = 0$.[3] Mauchly's hypothesis may be expressed as

[3] John W. Mauchly, "Significance Test for Sphericity of a Normal n-variate Distribution," *The Annals of Mathematical Statistics*, Vol. 11, No. 2 (June 1940), pp. 204-209; also "A Significance-Test for Ellipticity in the Harmonic Dial," *Terrestrial Magnetism*, Vol. 45 (1940), pp. 145-48. The power of Mauchly's test criterion has been investigated by M. A.

$$H_0 : \Sigma = cI \tag{3.4}$$

where c is positive but otherwise unspecified and $I \equiv \left\| \begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix} \right\|$. The test criterion for the hypothesis (3.4) is

$$L = \frac{2(\lambda_2\lambda_3)^{\frac{1}{2}}}{\lambda_2 + \lambda_3} \tag{3.5}$$

where λ_2 and λ_3 are the two roots of the characteristic equation

$$\phi(\lambda) = | S - \lambda I | = 0 \tag{3.6}$$

that is, the latent roots of S, where S is the sample covariance matrix for X_2 and X_3. Mauchly's test is equivalent to a test that the ellipses of constant probability density are circles (spheres in the n-dimensional case).

From Eq. (3.6), we find

$$\lambda_2 + \lambda_3 = s_{22} + s_{33}$$
$$\lambda_2 \lambda_3 = s_{22}\, s_{33} - s_{23}^2$$

so that Eq. (3.5) may be written in the form

$$L = \frac{2(s_{22}s_{33} - s_{23}{}^2)^{\frac{1}{2}}}{s_{22} + s_{33}} \tag{3.7}$$

Mauchly has determined the moments of the statistic L for the spherical (n-dimensional) case and has derived the exact distribution for the circular (2-dimensional) case. For the 2-dimensional case the moments reduce to

$$M_h(L) = \frac{N-2}{N-2+h}$$

where $M_h(L)$ is the hth moment of L and N is the sample size. Since

$$\begin{aligned} M_h(L) &= \int_0^1 L^h P(L) dL \\ &= (N-2) \int_0^1 L^{N+h-3}\, dL \end{aligned}$$

it is clear that

$$P(L) = (N-2)\, L^{N-3}$$

The distribution is unique since L is a bounded variable.[4]

Girshick, "The Distribution of the Ellipticity Statistic L_e when the Hypothesis is False," *Terrestrial Magnetism*, Vol. 46, No. 4 (1941), pp. 455-57.

[4] Cf. Harald Cramér, *Mathematical Methods of Statistics* (Princeton, 1946), p. 177.

Under the circularity hypothesis (3.4)

$$P(L \leqq L^*) = (N-2)\int_0^{L^*} L^{N-3}\,dL = L^{*(N-2)} = \epsilon$$

In the present analysis of the corporate bond market $N = 44$, $\epsilon = 0.05$, and we reject H_0 when $L \leqq L^* = (0.05)^{1/42}$.

Mauchly's test can be generalized to cover cases in which Σ^* is assumed not equal to I. Since Σ^* is the matrix of a positive definite quadratic form there exists at least one real nonsingular linear transformation on X_2 and X_3 that carries Σ^* over into the identity matrix I. This transformation will carry S over into the matrix $\bar{S}$. For testing any particular hypothesis Σ^* the procedure would then be as follows:

(1) find a matrix A such that $A\Sigma^* A' = I$, where A' is the transpose of A;

(2) determine $ASA' = \bar{S}$, the covariance matrix of the transformed sample values;

(3) calculate the latent roots of $\bar{S}$ from $|\ \bar{S} - \lambda I\ | = 0$, and substitute in Eq. (3.5).

These calculations, while perfectly straightforward, can be avoided once and for all by transforming L into an expression explicitly involving the population variances and covariances. We note that since there is at least one transformation that carries Σ^* over into the circular form (the form having matrix I), there must be an infinite number of them, all similar except for the orientation of the axial system. We can thus perform a second transformation, this one orthogonal, that will carry $\bar{S}$ over into a form $\bar{\bar{S}}$. The latent roots of $\bar{S}$ are unaffected by the second transformation, since they are invariant under all orthogonal transformations.[5] The problem is to find the latent roots of $\bar{\bar{S}}$ (or of $\bar{S}$).

The latent roots can be found most easily by looking upon S as the matrix of a quadratic form in the same variables as the quadratic form whose matrix is Σ^*. Since S and Σ^* may both be taken as positive definite, there exists a real nonsingular linear transformation carrying Σ^* over into I and S into $\begin{Vmatrix} \bar{\bar{\lambda}}_2 & 0 \\ 0 & \bar{\bar{\lambda}}_3 \end{Vmatrix}$ where $\bar{\bar{\lambda}}_2$ and $\bar{\bar{\lambda}}_3$ are the roots of the lambda-equation

$$|\ S - \lambda\Sigma^*\ | = 0 \tag{3.8}$$

and are clearly the desired latent roots of $\bar{\bar{S}}$ (and $\bar{S}$).[6] When the latent roots are determined from Eq. (3.8) and substituted in Eq. (3.5), L reduces to the form

$$L = \frac{2(s_{22}s_{33} - s_{23}{}^2)^{\frac{1}{2}}\,(\sigma_{22}\sigma_{33} - \sigma_{23}{}^2)^{\frac{1}{2}}}{\sigma_{22}s_{33} + \sigma_{33}s_{22} - 2\sigma_{23}s_{23}} \tag{3.9}$$

[5] *Ibid.*, p. 113.

[6] Maxime Bôcher, *Introduction to Higher Algebra* (New York, 1907), p. 171.

By substituting the appropriate values implied by $(\sigma_{22}/\sigma_{33}) = (\sigma_{22}/\sigma_{33})^*$, and $\rho_{23} = \rho_{23}^*$ in Eq. (3.9), we can test any hypothesis of the type (3.3). The test criterion (3.9) reduces to (3.7) in testing for circularity.

A second way of using the criterion (3.9), which proved convenient in the present study of corporate bonds, is to establish a 95 percent confidence region for σ_{22}/σ_{33} and ρ_{23} by substituting $L^* = (0.05)^{1/42}$ for L in the equation and inserting the sample values for s_{22}, s_{33}, and s_{23}. After solving for this region (the region for σ_{22}/σ_{33}, ρ_{23}), the corresponding region for ρ_{12} and ρ_{13} is determined. The confidence-region approach has an advantage over the testing of specific hypothesis in that the confidence region includes all hypotheses that would not be rejected on the basis of the sample data. In the sections of the book that deal with accounting identities, all statements about the significance of deviations of correlation coefficients from zero and of the variance ratio from unity are based on whether or not the assumed values $\rho_{ij} = 0$ and $\sigma_{22}/\sigma_{33} = 1$ fall within the appropriate joint confidence region.