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Chapter Author: RobertJ. Willis

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Economic Theory of Fertility Behavior

Robert J. Willis

Graduate School, City University of New York, and National Bureau of Economic Research

Until the past decade or so, economists tended to believe that the determinants of fertility are largely noneconomic or, at least, that the analysis of fertility is outside the scope of economic theory.[1] In part, these beliefs

This is a substantially revised version of a paper of the same title, which, in its earlier form, is the first chapter of my University of Washington Ph.D. dissertation. I have accrued a substantial debt to too many individuals who have contributed to the evolution of the ideas in this paper to acknowledge them all individually. I would like especially to thank John Floyd, who first suggested to me that economic analysis might be applied to population, Jon Rasmussen, who contributed importantly to the mathematical development of the paper, and Warren Sanderson, whose contribution to my thinking on fertility behavior is so ubiquitous that I can only reluctantly absolve him from responsibility for any errors or inadequacies in this paper. My debt to Yoram Ben-Porath will be apparent from his paper in this book. T. W. Schultz, Gary Becker, and H. Gregg Lewis made valuable suggestions for improving the final draft of this paper. I would also like to acknowledge the excellent research and programming assistance of C. Ates Dagli. Work on this paper was supported by a Ford Foundation grant to the National Bureau of Economic Research, for study of the economics of population. Earlier, I was aided by a Ford Foundation dissertation fellowship and by Wesleyan University.

[1] Becker (1960) argued that fertility could be analyzed within an economic framework. He emphasized the connection between income and fertility which he believed to be positive under conditions in which birth control knowledge was equalized across income classes. He also distinguished, importantly, between the cost and quality of children, arguing that the latter but not the former is subject to parental choice. This distinction remains a matter of controversy. Efforts were made in the sixties to investigate empirically the relationship between income and fertility over time and cross-sectionally, with mixed results in the sense that income did not seem to have a consistent positive or negative effect on fertility, nor did the magnitude of the effect of income seem to be large (see, e.g., Adelman 1963; Freedman 1963; Silver 1965, 1966; Freedman and Coombs 1966a, 1966b; Friedlander and Silver 1967; Easterlin 1968, 1969). Mincer (1963) shifted the emphasis of the economic approach from income effects to the effects on fertility of variation in the cost of children by showing that the opportunity cost of the wife's time as measured by the wife's wage rate was negatively related to fertility. The allocation of time between home and market and within the home was also discussed by Mincer (1962*a*), and a formal theory of time allocation was provided by Becker (1965). Becker's theory of time allocation has

were fostered because the neo-Malthusian proposition that increases in income tend to stimulate fertility conflicted with the facts that income growth has been accompanied by secular decline of fertility and that family income is inversely associated with cross-section fertility differentials in the industrialized countries.[2]

More fundamentally, economists have neglected fertility behavior because it has been difficult to incorporate it rigorously into the traditional theory of consumer choice. Recent extensions of economic theory to cope with human capital, allocation of time, and nonmarket household behavior now make possible the analysis of fertility as well as other traditionally demographic, sociological, and bio-medical aspects of behavior such as marriage, divorce, birth control, child-rearing practices, schooling, and health along with more conventional economic variables such as income, consumption, saving, and labor-force behavior within a unified choice-theoretic framework.

This framework might be called the "economic theory of the family." In it, the family is treated as a complex social institution in which the interdependent and overlapping life-cycle behavior of family members and the family unit as a whole is determined by the interaction of the preferences and capacities of its members with the social and economic environment they face currently and expect to face in the future. Clearly, no single tractable, intelligible, or testable model of the full range of family life-cycle behavior is yet feasible. At best, the present state of the economic theory of the family provides a framework within which a large class of models may be developed and their implications tested against one another as well as against hypotheses derived from other, more comprehensive theoretical frameworks.

I present a static economic theory of lifetime marital fertility within the context of this new economic approach to family behavior. The theoretical model is developed under a set of restrictive assumptions designed to make it analytically tractable and capable of yielding implications which may be tested with individual data on the number of children born to recent cohorts of American women who have completed their fertility. The model also has implications for child "quality," which is

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heavily influenced models of fertility, child quality, and related aspects of household behavior by T. P. Schultz (1969), Nerlove and Schultz (1970), and Sanderson and Willis (1971), as well as a number of unpublished studies. Empirical tests in these studies lend support to Mincer's finding that increases in the cost of the wife's time tend to reduce fertility while the effect of income is more problematic.

2 After World War II many of these countries experienced an upswing in fertility which, in the United States, is known as the postwar "baby boom." The U.S. birth rate peaked in 1957 and has followed a declining path since then to a current level lower than the previous minimum reach in the 1930s. The inverse association of income and cross-section fertility differentials also changed in the postwar period to a more U-shaped pattern in which the fertility of the middle-income classes tends to be lower than that of either the lowest or highest classes.

defined as a function of the resources parents devote to each child, and for the wife's lifetime market earnings capacity and labor supply.

1. Fertility as a Form of Economic Behavior

A number of characteristics of fertility behavior have made it difficult to analyze fertility within a choice-theoretic framework: (1) Childbearing and child rearing are nonmarket activities in which there are few transaction prices to provide information to the outside observer about the cost of children to suppliers or the value of children to suppliers. Parents are both demanders and suppliers of children. (2) Children and competing household activities both require the expenditure of parental time in addition to money. (3) The parental obligation to a child tends to be a long-term one, extending, sometimes, beyond the parents' lifetimes. (4) The wide variation of parental expenditures of time and money on bearing and rearing children observed from family to family and culture to culture suggests that the parents' obligations to children do not entail an exogenously determined program of expenditures per child. Rather, within the scope of laws and mores, parents may exercise considerable discretion in their expenditures in an attempt to shape the characteristics and activities of children in accordance with parental desires. The concept of the cost of children contains unavoidable ambiguity unless discretionary expenditures on what may be termed "child quality" are explicitly included in the analysis. (5) The motives for having children may include both the direct satisfaction children are expected to provide their parents and the indirect satisfaction they may render by working in the household or family business or by remitting money income to their parents. Thus, fertility is motivated by consumption, saving, or investment considerations. (6) Parents cannot exercise direct control over the number and timing of children they will bear and rear to maturity. A couple may only attempt to influence the monthly probability of conception and, given conception, the probability that a pregnancy will terminate in a live birth. Similarly, the probability of survival of a child will depend on choices made by parents as well as on environmental conditions outside their control. Imperfect fertility control and child mortality and morbidity pose additional constraints on family fertility behavior and add further dimensions to family choice. (7) Finally, there are difficulties in defining the appropriate unit of analysis. Decisions to have children and decisions concerning age at marriage and the characteristics and preferences of the marital partners are closely intertwined. Moreover, as children mature, they may have an independent effect on family decisions.

The principal problem to be resolved in analyzing fertility as a form of economic behavior is how to define conceptually satisfactory measures of the costs and satisfactions of children to their parents in a manner consistent with the distinctive characteristics of fertility behavior just listed. The required measure of cost is one that corresponds to the concept of opportunity cost, that is, the value to parents of the opportunities foregone in having an additional child; and the required concept of satisfaction is one that specifies the characteristics of children that give rise to utility.

In order to focus on these problems in the simplest possible context, the theoretical model presented here abstracts from the sequential and stochastic nature of the family's economic and demographic life cycle. A one-period comparative static framework will be used in which a husband and wife of given ages and characteristics are considered to adopt, at the outset of marriage, a utility-maximizing lifetime plan for childbearing, for expenditures of time and money on children, and for other sources of parental satisfaction not related to children. The utility function being maximized reflects the tastes and preferences of all family members as they are taken into account by the husband and wife, who are assumed to make all family decisions. The couple will be assumed to have perfect and costless control over their fertility and to possess perfect foresight concerning all relevant demographic and economic variables over the course of their marriage, so that the lifetime plan adopted ex ante at marriage coincides with ex post observations of their completed fertility.

The new approach to consumer theory suggested by Lancaster (1966) and especially the pioneering work of Becker, Mincer, and others on the allocation of time and human capital provide a theoretical framework within which the costs and satisfactions of children to their parents may be formulated in a more satisfactory way than is possible within the conventional theory of consumer choice.[3] Becker (1965) and Lancaster (1966) argue that a family's utility is not received directly from its consumption of market goods or leisure, as it is in conventional models. Instead, Becker assumes that the family combines time supplied by family members with goods and services purchased in the market to produce within the household the more basic "commodities" which are the true objects of utility. For example, rather than assume that medical care is purchased because it yields satisfaction directly, it may be assumed that medical care along with other purchased goods and services, the individual's own time, and the time of other family members combine to produce the commodity "good health," which is the actual source of utility.[4]

In general, family utility is considered to be a function of a vector of nonmarketable, home-produced commodities such as good health, entertainment, nutrition, and, as I will suggest, satisfaction from children.

[3] For a survey of the application of these ideas to a variety of problems in the field of human resources, see T. W. Schultz (1972*a*).

[4] See Grossman (1972*a*) for an application of this analysis to the demand for health. He analyzes health both as as a consumption and as an investment commodity.

This utility function, whose properties reflect the family's tastes or preferences, is defined by Becker on the n vector of commodities Z and may be written as

$$U(Z) \qquad Z = (Z_i) \qquad i = 1, \ldots, n. \tag{1}$$

It is assumed that the family will behave as if it attempts to maximize (1) subject to its limited capacity to produce Z_i.

The validity of this assumption has been discussed by Samuelson (1956). In general terms, a family may be regarded as a collection of individuals whose common welfare is a function of the utility of each of its v members so that in place of (1) we may write a Bergson-Samuelson "family welfare function" of the form

$$W = W(U^1, U^2, \ldots, U^v), \tag{2}$$

where the $U^j (j = 1, \ldots, v)$ is the level of utility of family member j. Assuming that the family attempts to maximize W, that $U^j = U^j(Z_{ij})$, and that

$$Z_i = \sum_j Z_{ij},$$

Samuelson proved that the family will behave as if it were an individual attempting to maximize (1). The condition $U^j = U^j(Z_{ij})$ implies that an individual family member's utility is independent of the level of utility of any other family member, and the condition

$$Z_i = \sum_j Z_{ij}$$

means that an additional unit of Z_i allocated to family member j must be subtracted from the consumption of other family members. Thus, Samuelson's proof that a family may be treated as if it were an individual maximizing a utility function such as (1) assumes no interdependency in utility among family members and no jointness in consumption.

But these are precisely the factors most responsible for the existence of the family as the predominant social institution in which individuals live. As Samuelson wrote (1956, pp. 9–10):

> Where the family is concerned the phenomenon of altruism inevitably raises its head: if we can speak at all of the indifference curves of any one member, we must admit that his tastes and marginal rates of substitution are contaminated by the goods that other members consume. These Veblen-Duesenberry external consumption effects are the essence of family life. They require us to build up an interpersonal theory that sounds more like welfare economics than like positive demand analysis. Such problems of home economics are, abstractly conceived, exactly

of the same logical character as the general problem of government and social welfare.

Samuelson's emphasis can be reversed, however. The family exists as an institution because, given altruism and the nonmarket mechanisms by which it is able to allocate commodities and welfare among its members, it has both the incentive and the capacity to resolve allocative problems involving public goods, externalities, and the like that in impersonal markets inevitably lead to market imperfections. The capacity of the family to resolve these problems efficiently provides a basis for a positive theory of family behavior, because, given efficient allocation, the family will tend to respond systematically to changes in the position or shape of the constraints it faces. Consequently, for many purposes, it may be assumed, as it is in this paper, that the family behaves as if it is attempting to maximize a utility function of the type in (1).

It is assumed in Becker's model that each of the commodities Z_i is produced according to a household production function with inputs of an m vector of market goods and services, x_i, and vector of time inputs, t_i, of the v family members. The set of household production functions, f^i, may be written

$$\begin{aligned} Z_i &= f^i(t_i,x_i), \quad t_i \geqslant 0, x_i \geqslant 0, \\ t_i &= (t_{ij}), \quad j = 1, \ldots, v, \\ x_i &= (x_{ik}), \quad k = 1, \ldots, m. \end{aligned} \tag{3}$$

The properties of these household production functions may be said to be determined by the state of the family's consumption technology in exactly the same sense that the properties of conventional production functions of firms are said to be determined by the state of standard production technology.

It is natural, within this framework, to consider those characteristics of children that provide satisfaction (or dissatisfaction) to their parents as commodities produced with time and goods according to household production functions. The relevant dimensions of child characteristics and the processes by which parents may alter them are inherently complex matters about which there is much ignorance (especially among parents). Fortunately, the traditions of economics permit the economic actors to solve the difficult problems.

In this spirit, it is assumed that a couple may choose to bear up to a maximum of $\bar{N}$ children and that the vector of utility-generating characteristics of a given child may be aggregated into the commodity Q_i, which will be called the "quality" of the ith child. Each child's quality is produced according to a household production function of the form

$$Q_i = f^i(t_i, x_i), \qquad i = 1, \ldots, \bar{N}, \tag{4}$$

where t_i and x_i are, respectively, vectors of purchased goods and family members' time devoted to the ith child.[5]

It is assumed that the marginal products of time and goods in the production of Q_i are positive but diminishing and that the ith child is born if $Q_i > 0$ and is averted if $Q_i = 0$, where the index i indicates the order of birth. The production functions for child quality imply that the parents may increase the satisfaction they derive from a given child by increasing the resources devoted to the child and that a given level of child quality may be obtained with alternative combinations of time and goods. The efficient or least-cost input combination will depend on the relative prices of time of individual family members and on the relative prices of market goods and services.

As specified in (4), the production functions for child quality need not be the same across birth orders. For example, the "technology" of producing child quality in the first child may differ from that appropriate to the second child because of interactions between the children, because parents may apply lessons learned from the first child to rearing the second child, or because some inputs are jointly productive for both children. Heterogeneity in technology implies that for a given set of input prices, the marginal costs of augmenting the Q_i may differ and that there will tend to be changes in their relative costs with respect to changes in relative input prices. This formulation also allows the Q_i to be imperfect substitutes or complements for one another in consumption, assuming that they enter as separate arguments into the family utility function. The effect of changes in the prices of time and goods or in the total resources of the family would, in general, result in changes in the input mix used to produce child quality, changes in the relative and absolute levels of quality of each child, and, most important from the standpoint of this study, changes in the number of children born.

The advantages of this rather general and flexible formulation of the problem of child quality are, for the purposes at hand, outweighed by the disadvantages of its analytical complexity and the lack of data necessary to place appropriate restrictions on the production functions for child quality or to test the implications of intuitively plausible restrictions. An analytically simpler specification which will prove to have testable implications is derived, given the following assumptions: (1) the production

[5] This concept of child quality need bear no connection with an outsider's judgment of the physical, intellectual, or personal characteristics of higher-quality compared with lower-quality children; it merely reflects the parents' judgment about the optimal quantity of resources to be devoted to each child. This disclaimer does not rule out the possibility that parents do affect the characteristics of their children or that child characteristics are unrelated to child quality. Indeed, an investigation of the connection between resources devoted to children and child characteristics would seem amply justified on empirical grounds (see, e.g., Wray 1971) and in terms of its relevance for the implications of policies designed to affect fertility. Some work in this direction has been done already by De Tray (1972*a*).

functions for child quality, f^i, are linearly homogeneous and identical, (2) there is no joint production of child quality,[6] and (3) parents choose an equal level of child quality for each child born.

Under these assumptions, the production function for the quality per child, Q, may be written as the linearly homogeneous function

$$Q = f(t_c/N, x_c/N), \tag{5}$$

where t_c and x_c are, respectively, the vectors of the total amount of time and goods devoted to all children during the parents' lifetime and N is the total number of children born, so that t_c/N and x_c/N are the amount of time and goods devoted to each child. Multiplying (5) by N, we may write

$$C = NQ = f(t_c, x_c), \tag{6}$$

where C, the total amount of child quality, will be called "child services."[7] It is assumed that N and Q enter as separate arguments into the family utility function.

In addition to utility derived from the number and quality of children, parents derive satisfaction from many other sources. These other sources of satisfaction which are unrelated to the number and quality of children will be expressed as the aggregate commodity, S, which is assumed to be produced according to the following linearly homogeneous household production function:

$$S = g(t_s, x_s), \tag{7}$$

where t_s and x_s are, respectively, vectors of time and goods devoted to S production. It is assumed that inputs to S do not jointly produce child quality. It should be noted that S embodies all sources of satisfaction to the husband and wife other than those arising from their children. Thus, the family utility function, which is written as

$$U = U(N, Q, S), \tag{8}$$

is a function of the number and quality of children as well as the parents' other sources of satisfaction.

The level of utility the family may achieve is limited by its capacity to produce C ($= NQ$) and S. Given its state of consumption technology as embodied in the properties of household production functions, the pro-

[6] There is joint production if a unit of time or goods devoted to the production of Q_i simultaneously increases or decreases the output of Q_j ($j \neq i$). See Grossman (1971) for a theoretical analysis of joint production in the household.

[7] A multiplicative treatment of quality-quantity relationships similar to that specified in eq. (6) is given, in the general case, by Theil (1952) and, for children, by Becker (1960). Neither, however, used the concept of a household production function as a basis for the relationship, nor did they consider time inputs. The more complex specification involving separate production functions for each child was adopted by Sanderson in a life-cycle model of fertility described in Sanderson and Willis (1971, pp. 34–42).

ductive capacity of the family is limited by its lifetime supplies of time and goods. The model is further simplified by assuming (1) that only the husband and wife contribute market earnings to family income,[8] (2) that only the wife's time is productive at home, and (3) that the structure of relative market prices remains fixed, so that the Hicks composite commodity theorem may be used to justify treating goods inputs as an aggregate good, x, with a price index, p.

Under these assumptions, the family's input of purchased goods is limited by its lifetime money income (or money wealth),

$$Y = px, \tag{9}$$

which, in turn, is equal to the sum of its nonlabor wealth, V, and the lifetime market earnings of the husband and wife. Since the husband's time is assumed unproductive at home, he will have an incentive to work "full time" in the market during marriage. His lifetime earnings and the family's nonlabor wealth together will be called the husband's lifetime income or wealth, H, and will be treated as an exogenous variable. The family's lifetime income and expenditure equation may be written as

$$Y = H + wL = px, \tag{10}$$

where w is the average hourly market wage received by the wife and L is the number of hours she works in the labor market during marriage. The amount of the wife's time available for home production, t, is equal to her life-span after marriage, T, minus (marital) lifetime hours of market work, L. Thus, the time constraint may be written

$$T = t + L, \tag{11}$$

where T is considered exogenous.

Since joint production of C and S is ruled out, it follows that a unit of goods or the wife's time devoted to C production must be subtracted from S production so that

$$x = x_c + x_s \tag{12}$$

and

$$t = t_c + t_x = \rho_c x_c + \rho_s x_s, \tag{13}$$

where x_c and t_c are inputs of goods and time to children, x_s and t_s are inputs of goods and time to S, and $\rho_c = t_c/x_c$ and $\rho_s = t_s x_s$ are, respectively, the time intensities of C and S production.

The structure of the model is completed by considering the determinants

[8] It is assumed, in other words, that children do not remit any money income to their parents. The lifetime earnings of children, which depend partly on the time and goods devoted to them by their parents, may be considered as a component of child quality (see De Tray 1972*a*).

of the wife's lifetime market earnings. Her average market wage, w (defined as equal to her lifetime earnings divided by her lifetime hours of work) is determined by an earnings (capacity) function of the form

$$w = w(L, \kappa), \tag{14}$$

where κ is a shift parameter which is assumed to increase w. The earnings function may be regarded as a reduced-form equation embodying the solution to the wife's optimal program of human capital accumulation for each possible level of her lifetime labor supply, L.[9] The parameter κ represents her initial stock of human capital at the outset of marriage (and an associated exogenous time rate of depreciation on that stock during marriage). The dependence of the wife's average wage on her lifetime labor supply reflects the interaction of the supply and demand for postmarital investments in human capital. Generally, it may be expected that $\partial w/\partial L = w_L$ will be positive. Thus, the return from any given postmarital investment which increases the wife's market earnings capacity increases with the number of hours she plans to work over her lifetime, so that the larger L is, the more likely investment is to be undertaken and the higher the wife's average wage will be.[10] An additional factor that would tend to make w_L positive is that "learning by doing" forestalls depreciation of the initial human capital stock or leads to its augmentation.[11] Since the wife's lifetime labor supply, L, is subject to choice, it follows that her average wage, w, is an endogenous variable in the model.[12]

[9] Compared with males, relatively little is known about the effects of investment in human capital on the market (or nonmarket) productivity of females, particularly in the post school and postmarital phases of the life cycle. Empirically, the life-cycle profile of male wage rates, particularly better-educated males, tends to rise quite steeply with age while the life-cycle profile of married women's wages tend to be much flatter, even for college-educated women. Strong theoretical and empirical support has been offered in favor of the hypothesis that the rising profile of male earnings is a function of labor market experience and investment in human capital, against the alternative hypothesis that it is merely caused by a maturation process (see, especially, the theoretical and empirical work of Mincer, particularly his 1970*a* paper and 1974*b* book and the theoretical work of Ben-Porath [1967] and Becker [1964, 1967]). The relative flatness of married female wage profiles is consistent with the human-capital interpretation of life-cycle wage rates because of their much lower levels of lifetime labor-force participation, because their periods of greatest participation are relatively late in the life cycle after childbearing, and because, with rapid growth of female labor-force participation across cohorts, cross-section age profiles are a progressively downward-biased approximation to the life-cycle profile of women of a given cohort. An interesting piece of evidence supporting this view is Fuchs's (1971) finding that the wage profile of single (never married) women, who have much higher participation rates than married women, has a shape closer to that of males than to that of married females. Whatever its past importance, it is likely that continued growth in the married female labor-force attachment will make postmarital investment in human capital of increasing importance in the future.

[10] See Lindsay (1971) for further discussion of a static, lifetime model of human capital and labor supply.

[11] Michael and Lazear (1971) emphasize this point, using the terminology "negative user cost" instead of learning by doing.

[12] The husband's earnings capacity could be similarly discussed. However, since his

The family's capacity to obtain satisfaction from the number and quality of its children and from S is limited by its consumption technology, its endowments of wife's time and nonlabor income, and the earnings capacities of the husband and wife. The constraint on the family's production and consumption of commodities may be written in implicit form as the production-possibility function

$$\phi\,(NQ, S, H, \kappa, T) = 0. \tag{15}$$

This function may be interpreted as follows. For preassigned levels of the exogenous variables H, κ, and T and for a preassigned output level of S, the production-possibility function gives the maximum attainable output of NQ $(= C)$.

The assumption that the production-possibility function is the relevant constraint on family behavior implies that the family allocates its resources optimally. The family allocates its resources between C and S production by choosing vectors of wife's time inputs (t^*_c, t^*_s) and market goods (x^*_c, x^*_s) corresponding to a commodity vector (N^*, Q^*, S^*) satisfying (15), that maximize the output of C for any given output of S. Like a small economy in a world of many countries, the family need not be self-sufficient. Although commodities N, Q, and S are nontradeable, the family does "export" the time of the husband and wife to the labor market and, in return, it "imports" goods from the market at terms of trade determined by market prices for labor and goods and by earnings capacities of husband and wife. Thus, the family must also choose the optimal supplies of wife's home time, t^*, and market goods, x^*, by choosing her optimal lifetime labor supply, L^*.[13] Finally, the optimal commodity vector (N^*, Q^*, S^*) is the one that maximizes the family utility function (8).

It is well known from the principle of duality that the optimal physical allocation of commodities and factors implies an optimal set of shadow prices which reflect the marginal opportunity costs of commodities and factors in consumption and production. The value of the family's real lifetime consumption or "full wealth," to use Becker's term, evaluated in terms of shadow prices of commodities is

$$I = \pi_c NQ + \pi_s S = \pi_c C + \pi_s S, \tag{16}$$

where I is full wealth and π_c and π_s are the shadow prices of C and S. In order to measure real rather than nominal full wealth, S is chosen as the numeraire commodity and π_s, its shadow price, is set equal to unity.

The duality between optimal consumption and production of N, Q, and S and the optimal shadow prices, π_c and π_s, is seen readily by noting that

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lifetime hours of work are assumed to be exogenous, it follows that his lifetime program of human-capital accumulation and lifetime income are also exogenous.

[13] Recall that the husband's time is assumed to be unproductive at home, so that it is "exported" to the labor market at any positive price.

the one-stage process of maximizing utility subject to the production-possibility function is equivalent to the two-stage process of (1) minimizing the "expenditure" of full wealth subject to production possibilities and (2) maximizing utility subject to the (minimum) full wealth constraint. By considering the optimization process in two stages rather than one, it is possible to analyze the supply and demand sides of family behavior separately even though they are determined simultaneously. The linkage between supply and demand is given by the family's full wealth, I, and the shadow price of children, π_c. In the second stage, the family demands for N, Q, and S may be derived as functions of I and π_c, and in the first stage, I and π_c may be expressed as functions of the exogenous variables H, κ, and T. For example, the effect of a change in husband's lifetime income on fertility may be analyzed by its effect on I and π_c and, in turn, the effect of these changes in I and π_c on demand for N.

My analysis of the theoretical structure will follow this two-stage scheme. Demand functions for N, Q, and S will then be derived subject to the full wealth constraint (sec. 2), followed by analysis of the properties of the production-possibility function and its implications for full wealth and the opportunity cost of children (sec. 3). In section 4, I examine the family's desired fertility and wife's lifetime labor supply in full (general) equilibrium of supply and demand. Finally, the empirical implications of the model will be considered and some tests of these implications with U.S. census data reported.

2. Fertility Demand and the Demand for Child Quality

The family's demand functions for number of children, child quality, and S may be written as follows:

$$N = N(I, \pi_c, \pi_s), \tag{17}$$

$$Q = Q(I, \pi_c, \pi_s), \tag{18}$$

$$S = S(I, \pi_c, \pi_s), \tag{19}$$

and, since $C = NQ$,

$$C = C(I, \pi_c, \pi_s) \quad = N(I, \pi_c, \pi_s)\, Q(I, \pi_c, \pi_s), \tag{20}$$

where the family's full wealth, I, and the shadow price of children, π_c, are treated as parameters and the shadow price of the parents' standard of living, π_s, is treated as a numeraire and set equal to 1. These demand functions are derived in Part A of the Mathematical Appendix by maximizing the family utility function (8) subject to its full-wealth constraint (16).

The first-order conditions for utility maximization in (A1) of the Mathematical Appendix may be solved for the Lagrange multiplier, $-\lambda$, to obtain the marginal equalities

$$-\lambda = \frac{U_N}{\pi_c Q} = \frac{U_Q}{\pi_c N} = \frac{U_S}{\pi_s}. \tag{21}$$

The interpretation of (21) is that the family equates the ratios of the marginal utilities of the number of children, quality per child, and parents' standard of living to its respective marginal costs, where $\pi_c Q = p_N$ is the marginal cost of an additional child of given quality, $\pi_c N = p_Q$ is the marginal cost of raising the quality per child given the number of children, and $\pi_s = 1$ is the marginal cost of the parents' standard of living. Thus, parents not only balance the satisfactions they receive from their children against those received from all other sources not related to children (satisfactions from S), but they also must decide whether to augment their satisfaction from children at the "extensive" margin by having another child or at the "intensive" margin by adding to the quality of a given number of children.

Because of the multiplicative relationship between the number and quality of children, there are interesting differences between the properties of the demand functions for N and Q and the properties of more conventional demand functions in which the commodities enter the budget constraint additively.[14] Let the wealth elasticities ϵ_N, ϵ_Q, ϵ_C, and ϵ_S be defined as the percentage change in N, Q, C, or S demanded per percentage increase in I, holding π_c constant. In the Mathematical Appendix, it is shown that

$$\alpha(\epsilon_N + \epsilon_Q) + (1 - \alpha)\epsilon_S = 1 \tag{A7}$$

and

$$\epsilon_C = \epsilon_N + \epsilon_Q, \tag{A8}$$

where $\alpha = \pi_c NQ/I$ is the share of I devoted to children. Becker (1960) speculated that all of these wealth elasticities are positive but that the quality elasticity, ϵ_Q, is likely to be substantially larger than the quantity elasticity, ϵ_N, because high-income families are observed to have only slightly larger or even smaller numbers of children than low-income families, but they tend to spend much more on each child. This conjecture,

[14] The discussion here will focus on the "observed" wealth and price elasticities of demand for N, Q, and S as distinguished from the "true" elasticities discussed herein by Becker and Lewis. The difference between the observed and true elasticities stems from the measure of wealth implied by the multiplicative model and the conventional measure implied by an additive model. To take the simplest case, suppose children are the only source of wealth, so that $I = \pi_c NQ$. Conventionally, a doubling of N and Q would be said to double full wealth and the wealth elasticities of N and Q would each be equal to unity. However, in order to double N and Q, I must be quadrupled so that the equiproportionate increase in N and Q resulting from a quadrupling of I results in observed wealth elasticities of one-half. I am grateful to Becker and Lewis for permitting me to see their notes on quality-quantity models and for helpful discussions which have greatly improved the treatment of quality and quantity in this paper. Any errors remaining are mine.

he pointed out, is also consistent with evidence that the quality elasticity of demand for consumer durables, such as automobiles, is generally found to be substantially higher than the quantity elasticity.

A theoretical basis for substantial differences in quality and quantity elasticities, given some propensity for tastes to be biased toward N or Q, lies in the fact that relative costs of N and Q do not remain constant when π_c is held constant unless N and Q change equiproportionately. Thus, the relative marginal costs of N and Q are $p_N/p_Q = \pi_c Q/\pi_c N = Q/N$. Assuming tastes are biased relatively toward Q, increases in I will tend to increase Q/N, thereby increasing the cost of numbers relative to quality of children, which induces a substitution effect toward Q and away from N. This is illustrated in figure 1, in which the initial equilibrium in the (N, Q)-plane is achieved at the tangency of the indifference curve, U_0, and the rectangular hyperbola, $C_0 = NQ$, at point a. Assume that an increase in income leads the family to increase its output of C to C_1 and that its new choice of N and Q is to the left of the ray Oad at point c. Had the relative costs of N and Q remained the same as they were at a, the new equilibrium point, remaining on the new indifference curve, U_1, would be at b rather than c, with a smaller consumption of Q and a larger consumption of N. It is even possible, in this case, for the wealth elasticity of number of children, ϵ_N, to be negative without N being, in the conventional sense, an inferior commodity.

The substitution effects on N and Q caused by changes in π_c, holding utility constant, also differ from those in demand functions derived from an additive constraint, because the marginal costs of N and Q, $p_N = Q\pi_c$ and $P_Q = N\pi_c$, cannot be varied independently.

In the Mathematical Appendix, it is shown in equation (A14) that $\eta_N + \eta_Q = \eta_C < 0$, where η_N, η_Q, and η_C are, respectively, the compensated

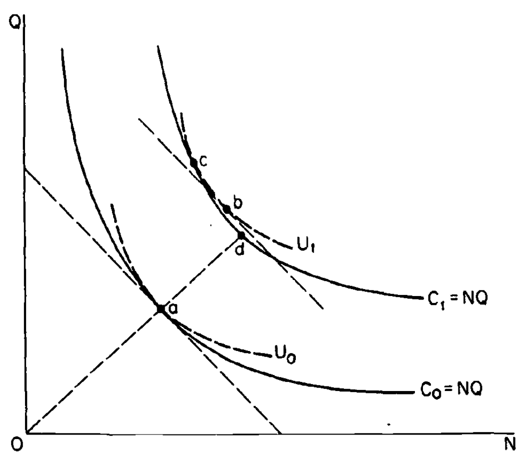

FIG. 1

substitution elasticities of N, Q, and C with respect to π_c.[15] It is possible, therefore, for an increase in π_c to increase either N or Q but not both. In equations (A19)–(A21) it is shown that η_N and η_Q may be expressed as follows under the pretense that p_N and p_Q may be varied independently:

$$\eta_N = \eta_{NN} + \eta_{NQ}, \eta_{NN} < 0;$$

$$\eta_Q = \eta_{QQ} + \eta_{NQ}, \eta_{QQ} < 0;$$

and

$$\eta_C = \eta_{NN} + 2\eta_{NQ} + \eta_{QQ} < 0,$$

where η_{NN} and η_{QQ} are own-substitution elasticities which measure the percentage change in N (or Q) caused by a given percentage change in p_N (or p_Q), holding utility, p_Q (or p_N), and p_S constant and where $\eta_{NQ} = \eta_{QN}$ are cross-substitution elasticities which measure the percentage change in N (or Q) caused by a given percentage change in p_Q (or p_N), also holding utility and the other prices constant. As η_{NQ} is positive or negative, N and Q are said to be substitutes or complements.

The signs of η_N and η_Q depend on whether N and Q are substitutes or complements and, if they are substitutes, on the relative magnitudes of the negative own effects, η_{NN} and η_{QQ}, and the positive cross effect, η_{NQ}. Since the issues to be resolved in order to derive hypotheses about the signs of η_N and η_Q depend on the nature of family tastes, economic theory as such has little to say. Some clues, however, may be found by judging the implications of alternative assumptions about η_{NN}, η_{QQ}, and η_{NQ} for the relationships among N, Q, and S in terms of their sociological and intuitive plausibility.

For example, if N and Q are complements, both η_N and η_Q would be negative. This would imply that an increase in π_c would reduce both N and Q and would increase S. This implication would be objectionable to those persons including Duesenberry (1960) and Blake (1968) who have argued that parents either cannot or will not choose child quality independently of their own standard of living. They argue, in effect, that Q and S should usually move in the same direction. The thrust of this argument may be accommodated by assuming that N and Q are substitutes and that η_{QQ} and η_{NQ} are roughly equal in absolute magnitude so that η_Q is near zero or even positive, which would imply, of course, that η_N is negative. This also implies that substitution in consumption between N and S is easier than between Q and S and, indeed, that Q and S are complements if η_Q is positive. This also seems consistent with the sociological argument.[16] These sociological considerations suggest the hypotheses that η_N is negative and that η_Q is small in magnitude and possibly positive.

[15] This result was first proved by Theil (1952).

[16] The following relationships hold between own elasticities and cross-elasticities of N with Q and S and of Q with N and S:

There is also an interesting implication of this analysis for the demographic impact of changes in the cost of fertility control. Willis (1971) shows that the (shadow) marginal cost of fertility control, π_f, defined as the additional fertility-control cost incurred per birth averted, acts as a per unit subsidy to childbearing. That is, a couple may reduce their fertility control costs by π_f by having an additional birth so that the full marginal cost of an additional child becomes $p_N = \pi_c Q - \pi_f$. This implies that a decrease in π_f caused by, say, an improvement in birth control technology will increase p_N, leaving p_Q and p_S unaffected. Thus, the effect of a change in π_f is

$$\frac{\partial N}{\partial \pi_f} = \frac{\partial N}{\partial p_N}\frac{\partial p_N}{\partial \pi_f},$$

which, in elasticity form, is

$$\eta_{Nf} = -\eta_{NN}\left(\frac{\pi_f}{\pi_c Q - \pi_f}\right).$$

If η_{NQ} is positive and η_N is negative, it follows that $|\eta_{NN}| > |\eta_N|$, so that a given percentage increase in p_N caused by an improvement in fertility control will cause a larger decrease in fertility than an equal change caused by an increase in π_c. Moreover, the effect of a given decrease in π_f will be larger the smaller $\pi_c Q$ is. While this may suggest that the demographic impact of improved birth control technology would be greatest among low-income groups, such a conclusion is unwarranted until it is known how the change in technology affects π_f in different income groups. It is beyond my scope here to explore the determinants of fertility-control costs which doubtless include a large element of psychic cost in addition to costs in time and money. Thus, for present purposes, let us revert to our former assumption of perfect and costless fertility control.

The results of the preceding analysis of the demand side of the theoretical model of fertility behavior suggest the hypothesis that the observed wealth elasticity of demand for the number of children, ϵ_N, is likely to be small and possibly negative even though N is not an inferior commodity in the conventional sense. The other main hypothesis, which resulted from a

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$$\gamma(\eta_{NN} + \eta_{NQ}) + (1-\gamma)\,\eta_{NS} = 0,$$

$$\gamma(\eta_{QQ} + \eta_{QN}) + (1-\gamma)\,\eta_{QS} = 0,$$

where γ is the share of full wealth devoted to children and η_{NS} and η_{QS} are, respectively, the cross-elasticities of N and Q with respect to p_N and p_Q. Assume that $\eta_Q = \eta_{QQ} + \eta_{QN} = 0$. It follows that η_{QS} must also equal zero, indicating that there is no substitutability between Q and S. If η_Q is positive, it follows that Q and S are complements, since η_{NS} must then be positive. Since, in either of these cases, $\eta_N = \eta_{NN} + \eta_{NQ}$ must be negative, it follows that η_{NS} is positive, so that N and S are substitutes. I am indebted to Lewis for pointing out these implications to me.

combination of economic analysis and sociological considerations, is that the compensated effect of an increase in π_c, the opportunity cost of child services in terms of parents' standard of living, will tend to reduce fertility so that η_N is expected to be negative. The next step is to analyze the supply side of the model in order to derive the relationship between I and π_c and the exogenous variables H, κ, and T.

3. The Supply of Child Services and the Allocation of Time

The family's capacity to produce and consume N, Q, and S is given by its production-possibility function (eq. [15]). If (15) is the effective constraint on family behavior, the implication is that the supplies of wife's home time and market goods are allocated efficiently between the production of C $(=NQ)$ and S, and that, through the family's choice of the level of the wife's lifetime labor supply, an optimal mix of the supplies of time and goods is achieved.

The duality between optimal allocation of resources and optimal shadow prices may be exploited to facilitate the analysis of the production-possibility function. It was noted earlier that the family may be considered to minimize the cost of its consumption by minimizing its "expenditures" of full wealth subject to the constraint of its production-possibilities function. The condition for minimum cost is

$$-\frac{dS}{dC} = \pi_c/\pi_s = \pi_c = \phi_c/\phi_s,$$

where $-(\partial S/\partial C)$ is the opportunity cost of an additional unit of C in terms of the amount of S foregone, ϕ_c/ϕ_s is the marginal rate of transformation along the production-possibility function, and $\pi_c/\pi_s = \pi_c$ is equal to the marginal rate of substitution in consumption between S and C.

Efficient allocation of family resources depends on the fulfillment of certain marginal productivity conditions. Because the production functions for C and S, (6) and (7), are assumed to be linearly homogeneous functions of inputs of wife's time and market goods, the marginal products of these two factors are functions solely of the input ratios or time intensities, $\rho_c = t_c/x_c$ and $\rho_s = t_s/x_s$. Thus, (6) and (7) may be written as $C = x_c F(\rho_c)$ and $S = x_s G(\rho_s)$, where their first derivatives, F' and G', are positive and their second derivatives, F'' and G'' are negative, indicating that the marginal product of wife's time is positive but diminishing.

Production within the home will be optimized when the value of the marginal product (VMP) of each factor is equal to its shadow price and when the ratio of the marginal products of the two factors in the production of each commodity is equal to the ratio of shadow factor prices. These conditions permit the addition of three equations to the model. The shadow price of the wife's time is equal to the *VMP* of her time in the production of C and S:

$$\hat{w} = \pi_c F' = G'; \tag{22}$$

the *VMP* of market goods is

$$\hat{p} = \pi_c(F - \rho_c F') = G - \rho_s G' = 1; \tag{23}$$

and the ratios of the marginal products of time and goods are equal to the ratio of the shadow factor prices,

$$\frac{\hat{w}}{\hat{p}} = \hat{w} = \frac{F'}{F - \rho_c F'} = \frac{G'}{G - \rho_c G'}, \tag{24}$$

where S is taken to be the numeraire commodity, so that $\pi_s = 1$, and the market and shadow prices of market goods, p and $\hat{p}$, are also set equal to 1.

Since $\pi_s = \hat{p} = p = 1$, the family's full wealth and the opportunity cost of children, π_c, are measured in dollars of constant purchasing power and the wife's shadow price of time, $\hat{w}$, is measured in dollars per hour, the same unit in which her market wage is measured. The wife's price of time, $\hat{w}$, plays a central role in the allocation of her time between home and market, in the allocation of resources within the home, and through its relationship with π_c, in the division of consumption among the number of children, child quality, and other sources of satisfaction.

The linkage between $\hat{w}$ and π_c is provided by the famous Stolper-Samuelson theorem, which states that there will be a one-to-one monotonic correspondence between factor and commodity prices in a two-factor (time and goods), two-commodity (children and standard of living) general-equilibrium system if the following conditions are met: (*a*) the commodity production functions are linearly homogeneous, (*b*) the factor intensities (factor-input ratios) of the two commodities differ, and (*c*) the sign of the factor-intensity ordering is invariant over all possible values of the factor-price ratio (see Stolper and Samuelson 1941). If condition (*c*) fails to hold and there exist so-called factor-intensity reversals, the monotonicity of the correspondence between factor and product prices will be destroyed. In this case, the Stolper-Samuelson theorem will hold locally, with the sign of the change in factor prices caused by a change in product prices (or vice versa) depending on the difference in factor intensities prevailing in that locality.[17]

In terms of the present model, (*a*) has already been assumed to be true and (*c*) will be satisfied if $\rho_c \neq \rho_s$ for all possible values of $\hat{w}$. The Stolper-Samuelson theorem implies that the elasticity of π_c with respect to $\hat{w}$ is

[17] See Samuelson (1949) for a discussion of factor-intensity reversals.

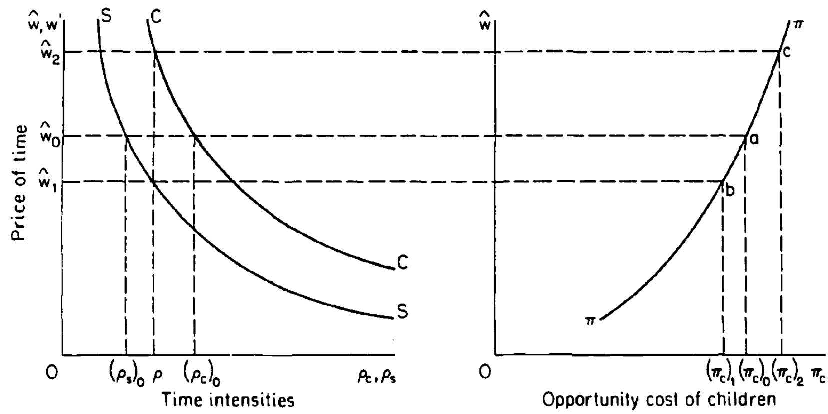

FIG. 2

$e = k_c\,[(\rho_c - \rho_s)/\rho_c]$, where $k_c = \hat{w}t_c/(\hat{w}t_c + \hat{p}x_c)$ is the share of wife's time cost in the total cost of children and $(\rho_c - \rho_s)/\rho_c$ is the percentage difference in the time intensities of C and S.[18] Since it is the product of two terms, each smaller than 1 in absolute value, e must also be smaller than unity in absolute value. On the basis of evidence cited later, it is assumed that children are more time intensive than S (i.e., $\rho_c > \rho_s$), so that e is positive and increases in $\hat{w}$ will tend to increase π_c.

The relationships between factor intensities, ρ_c and ρ_s, and the shadow factor and commodity prices, $\hat{w}$ and π_c, may be illustrated by a diagram introduced by Samuelson (1949). In the left panel of figure 2, the "substitution curve," CC, plots the locus of points satisfying $\hat{w} = F'/(F - \rho_c F')$ in (24). Curve CC slopes downward because, as the wife's price of time rises, the family switches to progressively less time-intensive methods of producing C, so that ρ_c falls as $\hat{w}$ increases. The elasticity of CC is equal to σ_c, the elasticity of substitution of the C production function. If time and goods must be used in fixed proportion ($\sigma_c = 0$), CC is vertical; if t_c and x_c are perfect substitutes ($\sigma_c = \infty$), CC is horizontal; and if they are imperfect substitutes ($0 < \sigma_c < \infty$), CC will slope downward as drawn.[19]

[18] This elasticity is derived by forming a simultaneous system from (22)–(24) in which π_c, ρ_c, and ρ_s are endogenous and $\hat{w}$ is treated as exogenous. Totally differentiating this system, the solution

$$\frac{\partial \pi_c}{\partial \hat{w}} = \rho_c - \rho_s$$

is obtained, which is written in elasticity form in the text.

[19] It should be emphasized that t_c is the wife's own time and that purchased time such as that of a babysitter is a component of purchased market goods, x_c. Since CC is not horizontal, it is implied that purchased time is an imperfect substitute for a

A similar substitution curve for S production, satisfying $\hat{w} = G'/(G - \rho_s G')$ in (24), is indicated by the curve SS. It is everywhere drawn to the left of CC to conform to the assumption that children are more time intensive than S, and its elasticity is equal to σ_s. The diagram is completed with the Stolper-Samuelson relationship, drawn as the inelastic, positively sloped curve, $\pi\pi$, in the right panel of figure 2. It slopes upward because of the assumption that $\rho_c > \rho_s$. Since (24) requires that $\hat{w}$ be equal in C and S production, it can be seen that each level of the price of time (e.g., $\hat{w}_0$) corresponds to a pair of optimal time intensities [e.g., $(\rho_c)_0$ and $(\rho_s)_0$] and to a given value of the opportunity cost of children [e.g., $(\pi_c)_0$].[20]

The allocation of the wife's time between home production ($t = t_c + t_s$) and market work ($L = T - t$) depends on the opportunity cost of an additional hour of market work in terms of the value of home production foregone. Assume, for the moment, that the wife may work any number of hours during marriage at a constant wage rate, w', and that her price of time when she does no market work ($L = 0$) is $\hat{w}_0$ in figure 2. If $w' < \hat{w}_0$, it will be optimal for the wife to supply no market labor, because the dollar value of commodity production sacrificed by withdrawing an hour of her time from direct input into home production exceeds the gain from the added goods input obtained from her additional market earnings. If $w' > \hat{w}_0$, optimality requires that she supply labor to the market until ρ_c and ρ_s are reduced sufficiently to raise her price of time at home to equality with her market wage. Thus, the weak inequality

$$\hat{w} \geqslant w' \tag{25}$$

may be added to the model.

It was argued earlier that the wife's lifetime market earnings capacity depends on her initial stock of human capital at the outset of marriage, κ, and on the additional human capital she accumulates during marriage through postmarital investment, which, in turn, is an increasing function of her lifetime labor supply, L, so that her lifetime earnings are $wL = w(L, \kappa)\ L$, where w is her average lifetime wage and $w(L, \kappa)$ is her earnings (capacity) function specified in (14). The value of an additional hour of work during marriage, which is called the marginal wage rate, w', is given by the first derivative of wL with respect to L as follows:

wife's time in raising children. It should also be pointed out that k_c, the share of wife's time in the total cost of children, will increase with $\hat{w}$ if $\sigma_c < 1$, will decrease if $\sigma_c > 1$, and will remain constant if $\sigma_c = 1$. Analogous remarks also pertain to S production.

[20] Time-intensity reversals, which would disrupt the one-to-one correspondence between π_c and $\hat{w}$ and change the sign of e, may be visualized in fig. 1 by imagining CC and SS to intersect at one or more points. Given that $\rho_c > \rho_s$ at, say, the median level of $\hat{w}$ in a population, the likelihood of a reversal depends on the magnitude of $\rho_c - \rho_s$, the difference between σ_c and σ_s, and the range of variation of $\hat{w}$ in the population. If there is one reversal (intersection), it will occur below the median $\hat{w}$ if $\sigma_c < \sigma_s$ and above the median if $\sigma_c > \sigma_s$; π_c will be negatively related to $\hat{w}$ for sufficiently low $\hat{w}$ in the former case and for sufficiently high $\hat{w}$ in the latter case.

$$w' = w + w_L L, \qquad (26)$$

where w_L reflects the increased earnings capacity the wife finds it worthwhile to acquire when her lifetime labor supply increases. Since she will stop investing if increasing costs or diminishing returns are sufficient to reduce her average wage with further investment, w_L must be greater than or equal to zero.[21]

Optimal allocation of time between home and market requires the wife to adjust her lifetime labor supply so as to equate her price of time, $\hat{w}$, and her marginal wage, w'. This implies that her price of time will tend to exceed her average wage and, more importantly, that the price of time will tend to be an endogenous variable, dependent on the choice of L. The change in w' with respect to L is $w'' = 2w_L + w_{LL}L$. For low levels of the labor supply, w'' will be positive and w' will be an increasing function of L. If the average wage has a maximum, however, the marginal wage will reach a maximum before this point and begin declining while the average is still rising. Thus, w'' may become zero or negative for sufficiently large values of L.

The production-possibility function (15) is determined by the simultaneous solution of the set of equations embodying the household production functions for C and S, the time and goods constraints, the wife's earnings function, and the conditions for efficient allocation of time and goods within the home and efficient allocation of the wife's time between home production and market work. Gathering these equations together and renumbering them for convenience, we can determine the family's production-possibility function by the following 10 equations and one weak inequality:

$$C = x_c F(\rho_c), \qquad (27.1)$$

$$S = x_s G(\rho_s), \qquad (27.2)$$

$$T = L + \rho_c x_c + \rho_s x_s, \qquad (27.3)$$

$$px = x = H + wL, \qquad (27.4)$$

$$x = x_c + x_s, \qquad (27.5)$$

$$\pi_c F' = G' = \hat{w}, \qquad (27.6)$$

$$\pi_c(F - \rho_c F') = G - \rho_s G' = p = 1, \qquad (27.7)$$

$$\hat{w} = \hat{w}/\hat{p} = \frac{F'}{F - \rho_c F'} = \frac{G'}{G - \rho_s G'}, \qquad (27.8)$$

[21] It is possible for w_L to be negative if physical and mental strain from long hours of work reduce the average wage associated with a given stock of human capital. Such positive "user costs" are the converse of the negative user costs stemming from learning by doing which were mentioned earlier (n. 11 above).

$$w = w\ (L, \kappa) \tag{27.9}$$

$$w' = w + w_L L, \tag{27.10}$$

$$\hat{w} \geqslant w'. \tag{27.11}$$

The endogenous variables of the model are: C = output of child services, S = output of all other sources of satisfaction, L = lifetime labor supply of wife after marriage, x = total quantity of market goods, x_c = goods used in C, x_s = goods used in S, $\rho_c = t_c/x_c$ = time goods ratio in C production, $\rho_s = t_s/x_s$ = time goods ratio in S production, w = wife's average lifetime market wage, w' = wife's marginal lifetime market wage, $\hat{w}$ = wife's shadow price of time or home wage, and $\pi_c = \pi_c/\pi_s$ = marginal opportunity cost or shadow price of child services.

The exogenous variables are: H = the husband's lifetime income, κ = the wife's stock of human capital at the outset of marriage, and T = the life-span of the wife after marriage.

The following market and shadow prices are set equal to unity: p = the market price of goods, $\hat{p}$ = the shadow price of goods, and π_s = the shadow price of S.

The system (27.1)–(27.11) constitutes a household-level general-equilibrium system whose properties depend on whether or not the wife works in the labor market. If she does work (i.e., $L > 0$), $\hat{w} = w'$ in (27.11) and the general-equilibrium system is composed of 11 equations, (27.1)–(27.11), in 12 unknowns (C, S, L, x, x_c, x_s, ρ_c, ρ_s, w, w', $\hat{w}$, and π_c). If the wife does not work (i.e., $L = 0$), (27.11) becomes the strong inequality $\hat{w} > w'$ and equations (27.9) and (27.10) become irrelevant. In this case, the system is composed of eight equations, (27.1)–(27.8), in nine unknowns (C, S, x, x_c, x_s, ρ_c, ρ_s, $\hat{w}$, and π_c). In either case, the number of unknowns exceeds the number of equations by one. This degree of freedom represents the scope of family choice: given the family's choice of the level of C (or S), the level of S (or C) and the equilibrium values of the other endogenous variables will be determined simultaneously. The system is closed in the full model by the family's choice of the utility-maximizing combination of C ($= NQ$) and S. For now, the system may be closed by treating S as a parameter in order to focus attention on the supply side of the model.

The two general-equilibrium systems, (27.1)–(27.11) if the wife works and (27.1)–(27.8) if she does not work, may be solved for the equilibrium values of each of the endogenous variables as functions of the exogenous variables of the system: H, κ, and T if the wife works and H and T if she does not. The properties of these "reduced form" equations depend on the properties of the "structural" equations of the general-equilibrium systems, particularly the properties of the household production functions for C and S in (27.1) and (27.2) and the earnings-capacity function of the wife in (27.9).

Among the reduced-form equations, the most important are the production-possibility functions which constrain the family's consumption possibilities and subject to which the family attempts to maximize its utility. If the value of the wife's time at home exceeds her marginal market wage so that she does no market work, the production-possibility function may be written in implicit form as

$$C = K(S, H, T) \tag{28}$$

and will be called the K-type constraint. If the wife's marginal market wage is sufficient to cause her to enter the labor market, the production-possibility function may be written in implicit form as

$$C = J(S, H, \kappa, T) \tag{29}$$

and will be called the J-type constraint. Taken together, the K- and J-type constraints constitute the constraint on household consumption and production possibilities over all possible values of the exogenous variables H, κ, and T. The full constraint may be written

$$0 = \phi(NQ, S, H, \kappa, T) = \begin{cases} -C + K(S, H, T);\ \hat{w} > w' \\ -C + J(S, H, \kappa, T);\ \hat{w} = w' \end{cases}. \tag{30}$$

The condition for the family to face a K-type constraint (i.e., $\hat{w} > w'$) is also the condition for the wife to remain out of the labor force (i.e., $L = 0$); conversely, the condition for the family to face a J-type constraint (i.e., $\hat{w} = w'$) is also the condition for the wife to participate in the labor force (i.e., $L > 0$). Thus, the properties of the constraint faced by the family are determined simultaneously with the wife's labor-force participation decision. Since $\hat{w}$ and w' are also endogenous variables, it follows that the participation decision will be a function of the exogenous variables H, κ, and T and the family's choice of S (and C).

Let us define the wife's lifetime labor-force participation function as

$$R = R(S, H, \kappa, T), \tag{31}$$

where $R = 0$ if $\hat{w} > w'$ and $R = 1$ if $\hat{w} = w'$. If the wife is out of the labor force, her price of time, $\hat{w}$, is determined by values of S, H, and T, and at $L = 0$, the value of her (potential) marginal wage rate, w', is simply an increasing function of her initial human capital stock, κ. Consequently, for given values of S, H, and T, the larger κ is, the more likely it will be that $R = 1$ and that the family will face a J-type constraint. It will be shown shortly that $\hat{w}$ is an increasing function of H, a decreasing function of T, and, provided that children are more time intensive than S, a decreasing function of S. It follows that R is more likely to equal 1 and that the family is more likely to face a J-type constraint, *ceteris paribus*, the smaller H is and the larger T and S are. If R is interpreted as a continuous cumulative probability function (e.g., $0 \leqslant R \leqslant 1$) rather than as a dichotomous step function, this discussion implies the following signs for the partial derivatives of R: $R_S > 0$, $R_H < 0$, $R_\kappa > 0$, and $R_T > 0$.

The shape of the production-possibility function ϕ in (30) and the manner in which its shape and position change when the exogenous variables H, κ, and T change depend on whether the wife participates in the labor market and, if she does work, on whether her marginal market wage is a constant or varies with her lifetime labor supply because of postmarital investment in human capital. The slope of ϕ at any given point, $\partial C/\partial S = -(1/\pi_c)$, measures the reciprocal of the opportunity cost of children.[22] The shape of ϕ in the neighborhood of a given point depends on the sign of the second partial derivative, $\phi_{SS} = (1/\pi_c)^2/(\partial\pi_c/\partial S)$, which measures the change in the opportunity cost of children, π_c, given a change in S along the production-possibility curve. As ϕ_{SS} is negative, zero, or positive, the curve will be concave, linear, or convex to the origin. The amount by which ϕ shifts, given a change in H, κ, or T, is measured by the first partial derivatives, ϕ_H, ϕ_κ, and ϕ_T, and the change in its slope (i.e., the change in π_c) by the second partials, ϕ_{SH}, $\phi_{S\kappa}$, and ϕ_{ST}.

These first and second partial derivatives of ϕ are derived in Parts B and C of the Mathematical Appendix for three forms of the production-possibility function: (i) the K-type constraint (wife does not work); (ii) the J^*-type constraint (wife works at a constant marginal wage, w'); and (iii) the J-type constraint (wife works and her marginal wage, w', varies with her lifetime labor supply, L). The shapes of the K-, J^*-, and J-type constraints are determined, respectively, by the signs of the second partial derivatives, K_{SS}, J^*_{SS}, and J_{SS}, in (C9), (C14), and (C18) in the Mathematical Appendix. In what follows, these results are interpreted diagrammatically.

If the wife does not work, the family's total supplies of goods and time are fixed (i.e., $x = H$, $t = T$) and the K-type production-possibility curve has the conventional "bowed-out" shape of $K_0a'K_0$ in figure 3 if it is assumed that the time intensities, ρ_c and ρ_s, are not equal. The reason for the bowed-out shape and its connection with the opportunity cost of children, the wife's price of time, and her labor supply may be explained with the aid of figure 2.

Since the slope of $K_0a'K_0$ at any given point is equal to $-\pi_c$, it can be seen that the upper and lower limits of π_c correspond, respectively, to the slope of the constraint at the maximum output of C and at the maximum output of S. Let $\rho = t/x = T/H$ (in fig. 2) be the ratio of the total supplies of time and goods corresponding to the fixed-resource endowment underlying $K_0a'K_0$ in figure 3. Each point on $K_0a'K_0$ must satisfy $\rho = k\rho_c + (1-k)\rho_s$, where $k = x_c/x$ is the share of goods devoted to children. The output of C along $K_0a'K_0$ ranges from zero when $k = 0$ and $\rho = \rho_s$

[22] For ease of interpretation, the production-possibility curves in fig. 3 have been drawn with C on the horizontal axis so that their slopes are $\partial S/\partial C = -\pi_c$.

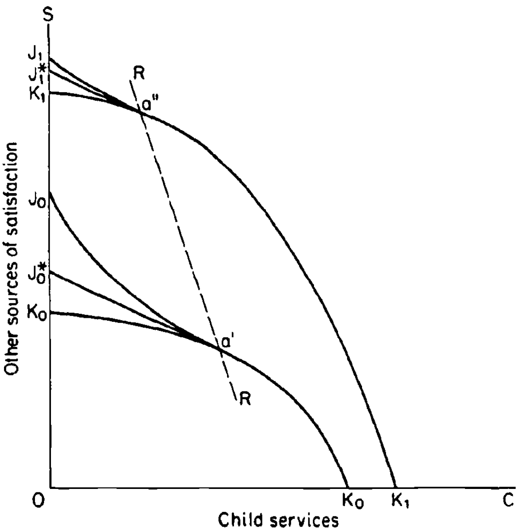

FIG. 3

to a maximum when $k = 1$ and $\rho = \rho_c$. If $\rho_c > \rho_s$, as is assumed in figure 2, the price of time ranges from a minimum of $\hat{w}$, when $\rho = \rho_s$, to a maximum of $\hat{w}_2$, when $\rho = \rho_c$, because relatively more goods and less time are released from S production than can be reabsorbed in increasing C production at existing levels of ρ_c and ρ_s. Consequently, both ρ_c and ρ_s must fall, implying that $\hat{w}$ must rise as C increases and, because the cost of time is more important in C than in S, that the opportunity cost of children must increase as well from a minimum of $(\pi_c)_1$ to a maximum of $(\pi_c)_2$. Had it been assumed, instead, that $\rho_c < \rho_s$, the price of time would decrease as the output of C increased, but π_c would still increase because the cost of goods would be relatively more important in C than in S.

While the hypothesis that children are relatively time intensive has some intuitive appeal, its most important support is its consistency with the negative relationship between the number of children in the household and the labor-force participation rates and hours of work of married women which has been found in many empirical studies of female labor-force behavior (e.g., Mincer 1962*a;* Cain 1966; Bowen and Finegan 1969). The wife's labor-force participation decision depends on whether her marginal wage, w', exceeds her price of time, $\hat{w}$, at $L = 0$. From figure 2, it can be seen that the wife will never work if w' is less than the lower limit of the price of time, $\hat{w}_1$; that she will always do some market work if w' exceeds the upper limit of the price of time, $\hat{w}_2$; and that if w' falls within these limits, the participation decision depends on the value of $\hat{w}$ implied by the family's choice of C and S. If children were assumed to be relatively goods intensive, this would imply, *ceteris paribus,* that childless women would have the lowest participation rates and that, as C increases and $\hat{w}$ declines, participation rates would rise. Of course, these implications are counterfactual. If, conversely, children are assumed to be relatively time intensive, the predicted negative relationship between C $(=NQ)$ and the wife's labor supply is consistent with the negative relationship between N and the labor supply if an increase in N is not wholly offset by a decrease in Q.[23]

The form of the constraint faced by the family depends on the wife's labor-force participation decision. If she does participate, the shape of the J-type constraint depends on the relationship between her marginal market wage, w', and her lifetime labor supply, L, implied by the properties of her lifetime earnings function (14) and on the negative relationship between C and L implied by the assumption that children are relatively time intensive. The family faces a K-type constraint over the full range of its production possibilities if the wife's marginal market wage is less than her minimum price of time corresponding to $C = 0$ and $\rho = \rho_s$; it faces a J^*- or J-type constraint over the full range if w' exceeds her maximum $\hat{w}$ when $S = 0$ and $\rho = \rho_c$, and it faces a mixed constraint if $\hat{w}_{\min} < w' < \hat{w}_{\max}$.

Two mixed constraints are illustrated in figure 3 by $K_0a'J^*_0$ and $K_0a'J_0$, each of which is tangent to the K-type constraint $K_0a'K_0$ at point a'. Point a' in figure 3 is assumed to correspond to point a in figure 2 so that the common slope of the three production-possibility curves at a' is assumed to equal $(\pi_c)_0$ at point a. It is implied from figure 2 that the time/goods ratios correspond to points a' and are $(\rho_c)_0$ and $(\rho_s)_0$ and that the wife's price of time is $\hat{w}_0$. Point a' is a point of mutual tangency if it is assumed that point a corresponds to the threshold value of the wife's labor-force participation function R in (31) such that (i) the wife is not working $(L = 0)$ and (ii) her price of time is equal to her marginal market wage $(\hat{w}_0 = w'_0)$.

If the family chooses relatively large outputs of C to the right of a', the price of time rises above w'_0, the wife remains out of the labor force, and the family faces a K-type constraint over the segment $a'K_0$. If it

[23] Two results of recent research on married women's labor supply bear on the model of this paper. Leibowitz (1972) has found that additional young children cause more highly educated women to withdraw from the labor force more than do women with less education. One explanation for this might be that the more highly educated women have children of higher quality, so that a given increase in N causes a larger increase in C for them than for the less educated women. Smith (1972*a*), using a different body of data and a somewhat different empirical technique, was unable to obtain Leibowitz's result. The second result, obtained by Smith and by Landsberger (1971), among others, is that, *ceteris paribus*, the negative effect of additional children on the wife's labor supply is attenuated as the children age, becoming possibly positive for teen-age children. This suggests the hypothesis that children become relatively less time intensive as they age, eventually becoming relatively goods intensive. While this hypothesis cannot be incorporated into the static, lifetime framework of the present model, it was incorporated by Warren Sanderson into a dynamic life-cycle model of fertility behavior described in Sanderson and Willis (1971).

chooses relatively small outputs of C to the left of a', the price of time tends to fall below w'_0, which leads the wife to withdraw time from home production in order to supply labor to the market. If her marginal wage remains constant at w'_0, the opportunity cost of children must also remain constant at $(\pi_c)_0$, so that the constraint is the linear segment $a'J^*_0$ in figure 3. The amount by which $a'J^*_0$ lies above $K_0a'K_0$ and to the left of a' in figure 3 represents the "gains from trade" to the family from "exporting" the wife's time to the labor market in order to "import" additional market goods.

Postmarital investment in human capital opens the possibility of additional gains from trade through improvements in the wife's lifetime earnings capacity as illustrated by the "bowed-in" segment of the constraint, $a'J_0$, to the left of point a' in figure 3. As the output of C decreases to the left of point a', the wife's optimal labor supply and optimal investment in human capital both increase, causing w' to rise above w'_0. As long as w' is an increasing function of L (i.e., $w'' = 2w_L + w_{LL}L > 0$), the price of time and the opportunity cost of children will decrease as the output of C increases, causing the J-type constraint to be convex to the origin as it is in figure 3. At a sufficiently high level of the labor supply, it is possible that w' will reach a maximum and begin to decrease with further increases in L (i.e., $w'' \leqslant 0$). In this case, not depicted in figure 3, the J-type constraint will have an inflection point and become concave to the origin as C becomes sufficiently small.

The family's desired levels of fertility, child quality, and the parents' standard of living depend on the interaction of the family's tastes and on the constraints that it faces. More precisely, the family's optimal consumption of N, Q, and S and the production of C $(=NQ)$ and S are determined simultaneously by maximizing utility subject to the production-possibility constraint. The solution of this household-level general-equilibrium system implies, simultaneously, solutions for the wife's optimal labor supply and postmarital investment in human capital and optimal lifetime allocations of her home time and market goods between C and S production.

The solution of this household-level general-equilibrium system may be expressed as a set of reduced-form equations relating the optimal quantity of each of the endogenous variables of the model to the values of the exogenous variables. This set of equations consists of (1) a set of final consumption demand functions for N, Q, C $(=NQ)$, and S, (2) a set of derived input demand functions for t, t_c, t_s, x, x_c, and x_s, (3) a derived labor supply function for L, and (4) a threshold function for the wife's labor-force participation, R. Each of these is a function of the exogenous variables H, κ, and T.

The hypotheses about family behavior that can be derived from this model are embodied in the properties of these reduced-form demand and supply functions. These properties depend, in turn, on the form of the constraint faced by the family and on the hypotheses about the nature of the family, its goals, and the capacities of its members which are embedded in the specification of the utility function, household production functions, and earnings functions.

According to the model, changes in family behavior are in response to changes in the family's lifetime economic circumstances resulting from a change in the husband's lifetime income, H, the wife's initial stock of human capital, κ, or her lifespan after marriage, T. Changes in these variables (1) change the family's full wealth, I, by shifting the production-possibility curve, (2) change the opportunity cost of children, π_c, by changing its slope, and (3) if the wife's labor-force status is altered, change the form of the constraint the family faces from a K- to a J^*- or J-type constraint or vice versa. It follows that the effect of a change in a given exogenous variable on each endogenous variable may be resolved into the sum of a wealth effect caused by a change in I and a substitution effect caused by a change in π_c. The effect of changes in the exogenous variables on I and π_c depends on the form of the production-possibility function in the neighborhood of the initial equilibrium. Because of this, the form of the demand and supply functions depends on the value of the wife's labor-force participation function, R, which determines whether the family faces a K-type or a J^*- or J-type constraint. Since R is also determined by the values of the exogenous variables, the participation function is an integral part of the demand and supply functions.

In the next section, the empirical hypotheses for family fertility behavior implied by the model will be developed in detail. The same line of analysis may also be used to obtain the empirical implications of the model for other aspects of family behavior, such as the demand for child quality or the derived demand for time or goods inputs to children.

4. Desired Fertility and Wife's Labor-Force Participation

The theoretical model of family behavior presented in this paper implies the following model of fertility demand:

$$N = \begin{cases} N^0(H, T) & \text{if} \quad R = 0 \\ N^1(H, \kappa, T) & \text{if} \quad R = 1 \end{cases}, \tag{32}$$

$$R = R(S^0(H, T), H, \kappa, T),$$

where N^0 is the family's demand function for number of children if the wife does not work ($R = 0$) and the family faces a K-type demand function; N^1 is the fertility demand function if the wife does work ($R = 1$) and the family faces a J- or J^*-type constraint; and R is the wife's labor-force participation function in which the demand function for S under a K-type constraint, $S^0(H, T)$, has been substituted for S in (31). Before the properties of N^0, N^1, and R are investigated, (32) will be reformulated more suitably for empirical purposes.

Although the theoretical model has concentrated on a single hypothetical family, its empirical implications must be tested with data on a population of families. This population may be considered to be made up of a mixture of families, some proportion of whom face a J^*- or J-type constraint and the remaining proportion of whom face a K-type constraint. Since the demand relationships, N^1 and N^0, between fertility and measures of H, κ, and T are expected to differ in the two groups because of differences in the properties of the J- or J^*- and K-type constraints, it may seem tempting to test for these differences by comparing the estimated relationships from a sample of families containing nonworking wives with those from a sample containing working wives.

Unfortunately, this straightforward procedure is valid only under the highly implausible assumptions that each family has identical tastes and consumption technology, that each wife has an identical earnings function, and that there are no accidental births. Otherwise, the sampling procedure tends to select families by their tastes for children, contraceptive efficacy, and so on because, *ceteris paribus,* those women who have more children are also less likely to work. Moreover, since the labor-force participation function, R, in (32) is a function of H, κ, and T, the degree of selectivity will depend on which portion of (H, κ, T)-space is being considered. For example, women with high potential market wage rates (high levels of κ) may work even though their families desire more children than the average family in the same circumstances, while women with lower κ and the same strong taste for children do not work. Thus, the proportion of families with above-average tastes for children in the sample containing working wives will tend to be positively correlated with κ, *ceteris paribus,* even though the taste for children is uncorrelated with κ in the population as a whole.

An alternative approach is to consider the form of the relationship between N and the exogenous variables H, κ, and T that would be expected on the basis of the model of individual fertility behavior in (32) if the relationship were estimated with data on a sample of families containing both working and nonworking wives. Again, the sample population may be considered to contain a mixture of families, of which some proportion faces a J^*- or J-type constraint and the remaining proportion faces a K-type constraint. The proportion of the two groups in the population as whole are, respectively, $\bar{R}$ and 1-$\bar{R}$, where $\bar{R}$, the percentage of working wives in the population, may be called the average lifetime labor-force participation rate of married women. Since $\bar{R}$ varies with H, κ, and T, the proportions of the mixture of demand functions of each type (N^0 and N^1) will vary across subpopulations whose mean levels of H, κ, and T vary. Thus, the general fertility demand function may be written as a mixture of the two special demand functions as follows:

$$N = N(H, \kappa, T) = \bar{R}N^1(H, \kappa, T) + (1 - \bar{R})\, N^0(H, T) + u, \quad (33)$$

where $\bar{R} = \bar{R}(S^0[H, T], H, \kappa, T)$ is the conditional mean of wives' labor-force participation rates given the values of H, κ, and T, and u is an error term. It is assumed that variations in the parameters of the structural equations of the model (such as the utility function or household production functions) among families in the population are such that u is normally distributed with mean zero and constant variance and is independent of the exogenous variables. If this assumption can be maintained, the "mixture model" in (33) may be used to test the implications of the theoretical model.

The implications of the mixture model depend on the signs of the partial derivatives of its constituent functions: the two demand functions, N^0 and N^1, and the labor-force participation function, $\bar{R}$. The partial derivatives of the demand function are derived rigorously in Part D of the Mathematical Appendix, where it is shown in equation (D22) that these partial derivatives (in elasticity form) may be decomposed into the sum of substitution and wealth effects as follows:

$$\delta_{Ni} = \eta_N e_i + \epsilon_N \gamma_i, \qquad i = H, \kappa, T,$$

where δ_{Ni} is the elasticity of N with respect to the ith exogenous variable: η_N is the compensated elasticity of N with respect to π_c; e_i is the elasticity of π_c with respect to the ith exogenous variable, given the form of the constraint; ϵ_N is the wealth elasticity of N; and γ_i is the elasticity of full wealth with respect to the ith exogenous variable.[24] In section 2, it was argued that η_N is negative and the ϵ_N may be either positive or negative but, in either case, it is likely to be small in magnitude. For convenience in exposition, it will be assumed that ϵ_N is positive. The sign of δ_N under each form of the constraint and the derivatives of $\bar{R}$ will be discussed below.

Assume that the wife is not working, so that the family faces a K-type constraint such as $K_0a'K_0$ in figure 3 and that its initial equilibrium level of number and quality of children and S correspond to the output of C and S at point a'. Because the wife's initial (and postmarital) stock of human capital is assumed to leave her home productivity unaffected, variations in κ do not affect the K-type constraint and, therefore, do not affect desired fertility.[25]

[24] The signs of the compensated substitution elasticities for N, Q, C, and S with respect to each exogenous variable under each type constraint are tabulated in table A1 in the Mathematical Appendix.

[25] This assumption is an obvious candidate for relaxation. From an analytical standpoint, the simplest way to do so is to assume that increases in human capital increase

An increase in the husband's lifetime income, H, increases the family's supply of goods ($x = H$) but leaves the supply of wife's time unaffected ($t = T$), while in T, the wife's life-span after marriage, increases the supply of time without affecting the supply of goods. An important property of the K-type constraint is that an increase in the supply of goods (time) will tend to raise (lower) the opportunity cost of children unless the output of C (S) falls absolutely by a sufficient amount. This property, often called Rybczynski's theorem, follows from the assumption that $\rho_c > \rho_s$.[26] It is illustrated for the case of an increase in the supply of goods caused by an increase in H in figure 3 by the asymmetric outward shift in the K-type constraint from $K_0a'K_0$ *to* $K_1a''K_1$, where the slopes of the two curves at points a' and a'' are equal and the output of C at a'' is smaller than the output of C at a'.

Unless C is sufficiently inferior in consumption (i.e., unless the wealth elasticity of C, ϵ_C, is sufficiently negative), the new equilibrium point on $K_1a''K_1$ must be to the right of point a'' and the slope of the constraint at the new equilibrium must be steeper than it was at the initial equilibrium at a' on $K_0a'K_0$. Since the slope of the constraint equals $-\pi_c$, it follows that an increase in H increases both full wealth, I, and the opportunity cost of children, π_c. Consequently, the wealth effect in favor of fertility is offset by a substitution effect against fertility when the wife does not work so that $N^0{}_H$, the partial derivative of N^0 with respect to H, may be either positive or negative. Since the effect of an increase in T is to reduce π_c, $N^0{}_T$ is unambiguously positive.

If the wife works at a constant marginal wage and the family faces a J^*-type constraint, an increase in H or T simply shifts the constraint out without changing its slope, because the wife adjusts her labor supply in order to keep her price of time, $\hat{w}$, from becoming unequal to her marginal wage, w'. In this case, H and T cause no substitution effects, so that $N^1{}_H$ and $N^1{}_T$ are positive because the wealth elasticity, ϵ_N, is assumed to be positive. An increase in H causes the wife to reduce her labor supply and an increase in T causes her to increase L. If w' is an increasing function of L and the family faces a J-type constraint, it follows that w' and π_c will fall as H increases and rise as T increases. In this case, the substitution

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the wife's productivity neutrally, raising the marginal products of time and goods equally in both C and S production. In this case, because $\hat{w}$ is equal to the ratio of the marginal products in each activity, $\hat{w}$ and π_c would be invariant with respect to κ. An increase in κ would simply shift any of the types of production-possibility functions out homothetically, thereby increasing full wealth and causing wealth effects on consumption. An endless variety of nonneutral effects could be posited which would also affect π_c and $\hat{w}$ in any direction desired. Unless a theory of "bias" is provided, it is not clear that the undoubtedly realistic hypothesis that human capital affects nonmarket productivity has any empirically falsifiable implications.

[26] See Rybczynski (1955). Jones (1965) has shown that the Rybczynski theorem and the Stolper-Samuelson theorem bear a dual relationship to one another.

effect reinforces the wealth effect of H so that $N^1{}_H$ becomes more positive and offsets the wealth effect of T and $N^1{}_T$ becomes ambiguous in sign. The substitution effects caused by H and T are of opposite sign when the wife works and when she does not.[27] An increase in the wife's initial stock of human capital, κ, tends to increase her marginal wage rate, w', and, therefore, to increase the opportunity cost of children, causing a substitution effect against children. Unless her labor supply curve is backward bending, the increase in κ will tend to increase L, which will further increase w' and π_c if the family faces a J-type constraint. The increase in κ also increases full wealth by an amount related to the level of the wife's labor supply. In general, it will be assumed that the positive wealth effect of κ does not offset the negative substitution effect so that $N^1{}_\kappa$ is hypothesized to be negative.

The effects of H, κ, and T on the labor-force participation rates of wives are given by the partial derivatives of $\bar{R}$ in (33). Variation in $\bar{R}$ depends on the relative change of the wife's marginal wage rate, w', and her price of time, $\hat{w}$, both evaluated at $L = 0$ (i.e., when she is not working). The proportion $\bar{R}$ will increase if w' increases, holding $\hat{w}$ constant, or if $\hat{w}$ decreases, holding w' constant. Since an increase in the wife's initial stock of human capital, κ, increases w', $\bar{R}_\kappa$ is positive. It was shown earlier that an increase in H increases π_c when the wife is not working. It follows that it also increases $\hat{w}$ so that R_H is negative. By the same token, increases in T decrease π_c so that R_T is positive.

The empirical implications of the mixture model in (33) follow from the signs of its partial derivatives with respect to H, κ, and T. For theoretical reasons, however, the implications of changes in T will not be tested. Variations in T may be caused by variations either in longevity or in age at marriage, with the latter being the only source which is of practical importance. So far, it has been assumed implicitly that age at marriage is an exogenous variable. Since bearing and rearing children is one of the principal reasons for marriage, this is an untenable assumption. Thus, accidental births sometimes hasten marriage, while averting them may prolong the single state. Moreover, if children are time intensive and time is cheaper earlier in life, those who wish to have more children will have an economic (as well as a biological) incentive to marry earlier. Thus, both accidental births and the taste for children are likely to be negatively correlated with age at marriage, which is contrary to the assumption that the error term, u, is independent of T.

The general form of the mixture model in (33) cannot be estimated. One way to obtain a function that can be estimated is to take a Taylor series expansion of (33) about the mean values of H and κ and estimate

[27] If w' is a decreasing function of L ($w'' < 0$), as it might be among working wives whose optimal labor supplies are large, the substitution effects caused by increases in H and T become negative and positive, respectively.

the coefficients of the resulting polynomial in H and κ as an approximation to (33). A more intuitive but operationally equivalent procedure is to consider each of the constituent functions of (33) (i.e., $\bar{R}$, N^0, and N^1) to be polynomials of degree r, in which case the mixture function will be a polynomial of degree $2r$ whose coefficients will be functions of the coefficients of the constituent functions.

In the simplest plausible case, N^0, N^1, and $\bar{R}$ are each assumed to be linear functions, as follows:

$$N^0(H) = a_0 + a_1 H, \qquad a_1 = N^0{}_H \gtreqless 0$$

$$N^1(H, \kappa) = b_0 + b_1 H + b_2 \kappa, \qquad b_1 = N^1{}_H > 0, b_2 = N^1{}_\kappa < 0,$$

$$\bar{R}(H, \kappa) = c_0 + c_1 H + c_2 \kappa, \qquad c_1 = \bar{R}_H < 0, c_2 = \bar{R}_\kappa > 0.$$

The mixture model is then the following quadratic equation:

$$N(H, \kappa) = d_0 + d_1 H + d_2 \kappa + d_3 H\kappa + d_4 H^2 + d_5 \kappa^2, \tag{34}$$

where

$$d_0 = c_0 b_0 + a_0 (1 - c_0),$$

$$d_1 = c_0(N^1{}_H - N^0{}_H) + (b_0 - a_0)\, \bar{R}_H + N^0{}_H,$$

$$d_2 = c_0 N^1{}_\kappa + (b_0 - a_0)\, \bar{R}_\kappa,$$

$$d_3 = \bar{R}_H N^1{}_\kappa + (N^1{}_H - N^0{}_H)\, \bar{R}_\kappa > 0,$$

$$d_4 = (N^1{}_H - N^0{}_H)\, \bar{R}_H < 0,$$

$$d_5 = \bar{R}_\kappa N^1{}_\kappa < 0.$$

There are no a priori expectations for the signs of the constant term, d_0, or for the coefficients of the first-degree terms, d_1 and d_2, because each involves the constant terms of the constituent equations, a_0, b_0, and c_0, whose signs are not predicted by the theory. The implications of the theory do, however, lead to a priori expectations of the signs of the second-degree coefficients, d_3, d_4, and d_5, as indicated above.

The signs of the coefficients of the squared terms, d_4 and d_5, and of the interaction term, d_3, reflect the differential impact of variations in H and κ on the opportunity cost of children between families in which wives work and families in which wives do not work, together with changes in the proportions of the two types of family caused by variation in H and κ on the labor-force participation rate of married women. The reason that d_5, the coefficient of κ^2, is negative is that increases in κ raise the participation rate ($\bar{R}_\kappa > 0$) and that, among working wives, increases in κ raise the cost of children and depress fertility. An increase in H causes wealth effects in both groups, but it increases π_c among nonworking wives and either lowers

π_c or leaves it unchanged among working wives, so that $N^1{}_H - N^0{}_H$, which measures the algebraic difference between the substitution effects in the two groups, is positive. The reason that d_5, the coefficient of H^2, is negative is that an increase in H reduces participation ($\bar{R}_H < 0$), reducing the proportion of families in which H causes a negative substitution effect and increasing the proportion in which it causes a positive substitution effect. The coefficient of the interaction term, d_3, is positive because increases in H reduce the proportion of families in which κ causes a negative substitution effect (i.e., $\bar{R}_H N^1{}_\kappa > 0$) and because increases in κ reduce the proportion of families in which H causes a substitution effect against fertility (i.e., $\bar{R}_\kappa$ $(N^1{}_H - N^0{}_H > 0)$.

The nonlinearity of the mixture model implies that the effects of changing income and female wage rates on fertility behavior will vary in strength and even in sign with the prevailing levels of income and wage rates. This implication is consistent with the apparent ambiguity of the effect of income on fertility so often noted by students of fertility behavior.

5. Empirical Results

The results of an attempt to test the implications of the mixture model with data on American families from the 1960 census 1/1,000 sample follow. Unfortunately, of the three second-degree coefficients of the mixture model in (34), only d_3, the coefficient of the interaction term, $H\kappa$, may be estimated unless the theoretical variables, H and κ, can be measured empirically up to a linear transformation.[28] Given the unavoidable imprecision of the definition of the theoretical variables, "husband's lifetime income" and "wife's initial stock of human capital," it is difficult to see how one might hope to do better empirically than to measure them up to positive monotonic transformation. Accordingly, the model that will be estimated is the "interaction model"

$$N = d^*{}_0 + d^*{}_1 H + d^*{}_2 \kappa + d^*{}_3 H\kappa + u^*, \tag{35}$$

in which the variables H^2 and κ^2 are omitted. In general, since the truncated model in (35) is a misspecification of the full quadratic model, the coefficients of (35) will not be unbiased estimates of their counterparts in (34).[29]

[28] To see this, let $H = m(y)$, where y is the empirical measure of H and m is a positive monotonic transformation (i.e., its first derivative, m', is positive for all y). Substituting for H in the general fertility demand function, (33), we have $N = N[m(y), \kappa]$. The signs of the first partial derivative with respect to y and the second cross-partial are unchanged by the transformation ($N_y m'$ and $N_{y\kappa} = N_{H\kappa} m'$), but the second partial with respect to y is of arbitrary sign ($N_{yy} = N_{HH} m' + N_H m''$) unless m'' is zero, in which case m is a linear transformation. The same argument holds for empirical measures of κ.

[29] The effect of omitting H^2 and κ^2 causes the estimated interaction coefficient, $d^*{}_3$,

Before the empirical results are discussed, another measurement problem must be considered. Any empirical measure of H or κ is likely to measure the "true" variable with a random error. For example, husband's current income is almost certainly an error-ridden measure of the income variable relevant to fertility decisions. Childbearing takes place relatively early in the marital life cycle, and completed fertility and husband's income are usually observed much later. The income variable relevant to childbearing decisions presumably involves the shape and height of the husband's life-cycle income profile as the family expects it to be at the time these decisions are taken. Since the husband's current income is observed long after these decisions have been made, it is likely to be a poor measure of the relevant variable and its regression coefficient a biased estimate (probably toward zero) of the true coefficient.[30]

In an earlier paper, an apparently successful effort was made to alleviate the problem of errors in variables in estimating the interaction model by using weighted cell means as observations where the grouping scheme was chosen in such a way, as, hopefully, to be uncorrelated with the error and correlated with the true measure of husband's lifetime income (see Sanderson and Willis 1971, pp. 35–37 and n. 1). Here, in a different approach to the problem, I use estimates, by occupation, of life-cycle earnings functions of males of a form suggested by Mincer (1970*a*, 1974*b*) to predict the income of husbands as a function of their education, labor market experience, cohort, weeks worked, size of place, and whether or not they reside in the South.[31]

Warren Sanderson and I (1971) attempted to test the interaction model with grouped data from seven independent subsamples consisting of three successive 10-year cohorts (1896–1925) from the 1960 census and four successive 5-year cohorts (1881–1900) from the 1940 census. An interaction regression of the form in (35) was estimated in each subsample, using weighted cell means of husband's income and wife's education as measures of H and κ. In each of the seven regressions, the coefficient of the interaction term, d^*_3, was positive and statistically significant and the coefficients of husband's income and wife's education, d^*_1 and d^*_2, were negative and significant. Rather surprisingly, the estimated coefficients of the inter-

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to be downward biased ($d^*_3 < d_3 > 0$). The reason for this is that H and κ (measured by wife's education) are positively correlated, so that the positive effect on fertility of high levels of H and κ measured by d_3 tends to be offset by the negative effect of high levels of H and κ measured by d_4 and d_5. The bias of d^*_0, d^*_1, and d^*_2 is not certain.

30 See Theil (1971, pp. 607–15) for a discussion of the problem of errors in the variables.

31 These earnings functions were estimated from a pooled sample of married males age 18–65 from the 1960 census 1/1,1000 sample and from the 1967 Survey of Economic Opportunity (SEO). A description of these regressions and their estimated coefficients is available from the author on request.

action model appeared to be sufficiently stable across the cohorts of 1881–85 to 1916–25 for us to suggest "that it may be possible to apply our model to the explanation of trends in cohort fertility as well as to the explanation of cross-section differentials within cohorts" (p. 36).

We were surprised, for it would seem to be beyond the scope of the static theory underlying the interaction model to explain the effect on cohort fertility trends of the complex dynamic changes that took place in the American economy in the period 1881–1965, when these women were born, married, and bore their children. The static theory would appear to be better suited to the explanation of differential fertility within closely adjoining cohorts whose historical experience is held more or less constant by their common years of birth.

In the empirical work reported in this paper, a crude attempt has been made to see more directly whether the interaction model can help explain trends in cohort fertility as well as differential fertility within cohorts. The data consist of a sample of 9,169 white women age 35–64 in 1960, married once, living with husbands, and living in urban areas at the time of the 1960 census (see table 1 for summary statistics of sample). These women, born 1896–1925, are essentially the same women from the 1960 census used in the Sanderson and Willis (1971) regressions just discussed.

A pure trend equation is estimated from the census sample data to provide a benchmark against which to measure the impact of the economic variables in the interaction model. Since this trend was first declining and then rising among the cohorts of 1896–1925, a quadratic trend function is fitted by regressing wife's cohort and cohort squared (the birth cohort of 1925 is set equal to zero) on the reported total number of children born to her. In this regression, reported in line 1 of table 2, the coefficients of both the linear and squared terms are positive and statistically significant.

TABLE 1

SUMMARY STATISTICS ON SAMPLE OF URBAN WHITE WOMEN MARRIED ONCE, LIVING WITH HUSBAND: 1960 CENSUS 1/1,000 SAMPLE (NUMBER OF OBSERVATIONS = 9,169)

Variable	Mean	Standard Deviation
Total number of children born	2.2650	1.7780
Cohort (1925 = 0)	−10.6900	7.4310
Cohort^2	169.4840	191.9660
ED	10.5190	2.8990
H(NOW) ($1,000)	6.7651	5.3936
H(40) ($1,000)	4.2929	2.1407
H(NOW) ED	75.1466	75.2934
H(40) ED	47.8497	31.7309
SMSA (=1, 2, 3, 4)	2.6100	1.6130

TABLE 2

REGRESSIONS ON COMPLETED FERTILITY OF URBAN WHITE WOMEN AGE 35–64, MARRIED ONCE, LIVING WITH HUSBAND: 1960 CENSUS 1/1,000 SAMPLE

Concept H, κ	Cohort	Cohort^2	d^*_1 (H)	d^*_2 (κ)	d^*_3 ($H\kappa$)	SMSA	Constant	R^2
1. Pure trend	.05596	.00150	...	...	...	...	2.60838	.00854
	.00834	.00032						
	(6.65)	(4.66)						
2. H(NOW) ED	.05983	.00132	—.06898	—.14206	.00617	—.08111	4.38947	.04389
	.00824	.00032	.01687	.00991	.00131	.01135		
	(7.09)	(4.13)	(4.09)	(14.31)	(4.71)	(7.15)		
3. H(40) ED	.06004	.00124	—.24836	—.17572	.02023	—.07243	4.83269	.04656
	.00831	.00032	.03381	.01258	.00276	.01173		
	(7.23)	(3.88)	(7.35)	(13.97)	(7.33)	(6.17)		

NOTE.—See table 1 for summary statistics of sample; t-ratios are reported in parentheses.

The minimum level of cohort fertility, as computed from the trend equation, was reached by the cohort of 1906.

Two alternative measures of husband's lifetime income are used in the estimates of the fertility demand equations. The first, H(NOW) (see table 2) is equal to the husband's reported 1959 income. As already discussed, H(NOW) is likely to be an error-ridden measure of husband's lifetime income, particularly since the husbands in the sample range in age from their early thirties to retirement age. It is used for purposes of comparison with the alternative measure, H(40), which is the husband's income at age 40 as predicted on the basis of his occupation, education, labor market experience, cohort, race, residence in the South, and size of urban area from the life-cycle earnings functions described earlier. In addition to its econometric advantages, the use of H(40) has the advantage of permitting the choice of a given point on the life-cycle income profile in order to provide a comparable measure of income for men whose current ages differ considerably.[32] Both variables have the great disadvantage of being ex post measures of income, which may provide a distorted measure of the ex ante expectation of lifetime income on which fertility decisions are based.

The wife's years of schooling, ED, is used to measure κ, her stock of human capital at the outset of marriage. Although all of the effects of education except its effect on the market earnings capacity of the wife have been ruled out by assumption, it is recognized that education may well have a systematic effect on tastes, efficacy of fertility control, or efficiency in household production. This should be borne in mind when the regression results are considered.

The estimates of the interaction model using H(NOW) and H(40) are reported, respectively, in lines 2 and 3 of table 2. In each regression, the variable, Standard Metropolitan Statistical Areas (SMSA), has been added to hold the influence of the size of urban area constant, and the variables "Cohort" and "Cohort^2" have been added so that the effect of the economic variables on cohort fertility trends may be assessed.

The coefficients of husband's income and wife's education, d^*_3, and d^*_2, are negative, and the coefficients of the interaction term, d^*_3, are positive in each regression; all coefficients are statistically significant. The absolute magnitudes of the coefficients involving H(NOW) are considerably smaller in magnitude and have lower t-ratios than the corresponding coefficients of the regression involving H(40).[33] This result supports the belief that

[32] The method of grouping used in Sanderson and Willis (1971) and the auxiliary regression method used here may both be considered as alternative applications of the method of instrumental variables to the problem of errors in the variables. The advantage of the latter method over grouping is that it preserves degrees of freedom and permits the use of a linear combination of a large number of instruments (the regressors in the earnings function); see Malinvaud (1966, p. 606).

[33] Comparisons between the magnitudes of the coefficients involving the two measures of H should be adjusted for differences in their means, which are reported in table 1. Since the mean of H(NOW) is 1.56 time [illegible] ean of H(40), the coefficient

TABLE 3

COMPUTED ELASTICITIES OF FERTILITY WITH RESPECT TO H(40) AND ED FROM REGRESSION 3, TABLE 2

WIFE'S EDUCATION	HUSBAND'S INCOME		
	Mean	High	Low
Mean:			
η_H	−.0674	−.101	−.0338
η_{ED}	−.412	−.181	−.614
High:			
η_H	.0438	.0656	.0219
η_{ED}	−.526	−.270	−.783
Low:			
η_H	−.179	−.268	−.0895
η_{ED}	−.299	−.153	−.445

NOTE. -High husband's income and low husband's income refer to points one standard deviation above and below mean [H(40)], respectively. The same definitions are used for wife's education.

a longer-run, lifetime concept of income is relevant to fertility behavior.

The interaction model helps explain the U-shaped relationship between fertility and cross-section measures of husband's lifetime income that emerged in the United States and some European countries after World War II.[34] The effect of H(40) on fertility is

$$\frac{\partial N}{\partial H(40)} = d^*_1 + d^*_3 \, \mathrm{ED},$$

where d^*_1 is negative and d^*_3 is positive. As the level of wife's education (ED) surpasses about 12 years of schooling, the sign of the "income effect" changes from negative to positive. Thus, in populations or subpopulations in which wife's education levels are low, the effect of income on fertility tends to be negative, and it becomes positive as these levels grow. For effect of variations in the level of H(40) and ED on the elasticities of fertility with respect to husband's income and wife's education, see table 3. These elasticities are computed at the sample mean of H(40) and ED and at one standard deviation above and below the mean for each variable.

The estimates of d^*_1, d^*_2, and d^*_3 using individual data across cohorts are consistent in sign and magnitude with the corresponding estimates within cohorts reported in Sanderson and Willis (1971) and described earlier. Despite this, it appears that my suggestion that the interaction

of H(NOW) is 39 percent of the coefficient of H(40) and the coefficient of H(NOW) ED is 48 percent of the coefficient of H(40) ED after the appropriate adjustment is made.

34 The interaction model also performs quite well with Israeli data, as Ben-Porath reports in this volume.

model may be used to help explain cohort fertility trends is unwarranted, at least when static ex post measures of the exogenous variables are used. Ideally, if the interaction model fully explained cohort fertility trends, the coefficients of the trend terms, "Cohort" and "Cohort^2," would fall to zero. In fact, these coefficients are not substantially changed by the addition of the economic variables.

6. Conclusion

The restrictions placed on the specification of the individual equations of the structural model of fertility behavior that I have presented and on the structure as a whole represent a drastic simplification of the complex interconnections among fertility, family formation, and family behavior. Consideration of these restrictions and the manner in which they may be relaxed or changed suggests that the present model is only one particularly simple member of a large class of economic models of individual fertility behavior which share the common framework of the economic theory of the family. Thus, the static lifetime framework of the present model could be changed to a dynamic life-cycle framework. The assumptions of (1) perfect fertility control could be removed in favor of a theory of imperfect fertility control, (2) exogenous date of marriage and characteristics of marital partners could be replaced by a theory of marriage, and (3) exogenously determined efficiency in household production could be relaxed by applying the theory of investment in human capital to nonmarket efficiency, and so on. Work under way on a number of such models promises to provide a rich source of alternative hypotheses about fertility behavior and, simultaneously, about many other aspects of family behavior.

Recognition that there are potentially many alternative economic models of fertility behavior must influence any assessment of the empirical importance of economic variables on fertility as expressed in the present model. On the basis of evidence presented in this paper, it appears that the interaction model captures an important empirical regularity in the cross-section relationship between fertility and measures of husband's income and wife's education that has become apparent in the emergence of a U-shaped relationship between fertility and income which has been observed in many advanced countries in the past 25 years and which was an incipient relation at the lower levels of income and education prevailing in earlier periods. This empirical regularity is also consistent with the predictions of the theoretical model of fertility demand developed in this paper and must, therefore, be counted as evidence in its favor. To reiterate the position Ben-Porath has taken in his paper in this volume, caution must be exercised in accepting the explanation of fertility behavior provided by this model, because the mechanism by which the empirical regularity is generated need not correspond exactly or even chiefly to the one posited in the theoretical model. If scientific progress consists in large part of the process of rejecting hypotheses, it follows that progress will be impeded if hypotheses are not proposed. It is in this spirit that the hypotheses implied by the theoretical model of this paper are advanced.

Mathematical Appendix

A. Derivation and Properties of Demand Functions Subject to Full-Wealth Constraint

The demand functions for N, Q, and S are derived by maximizing the utility function (eq. [8]) subject to the full wealth constraint (eq. [16]), where I, π_c, and π_s are treated as parameters. Maximizing the Lagrangian expression,

$$U(N, Q, S) + \lambda(\pi_c NQ + \pi_s S - I),$$

where λ is a Lagrange multiplier ($\lambda < 0$), we obtain the following first-order conditions for a maximum:

$$\begin{aligned} U_N + \lambda\pi_c Q &= 0, \\ U_Q + \lambda\pi_c N &= 0, \\ U_s + \lambda\pi_s &= 0, \\ \pi_c NQ + \pi_s S - I &= 0. \end{aligned} \tag{A1}$$

The quantities of N, Q, and S demanded as functions of the parameters I, π_c, and π_s may be obtained by solving (A1) simultaneously. These solutions, expressed in implicit form, are the demand functions for N, Q, S, and C ($=NQ$) in equations (17)–(20).

The properties of these demand functions may be obtained by totally differentiating the first-order conditions (A1) to obtain the following set of simultaneous linear differential equations written in matrix form:

$$\begin{bmatrix} U_{NN} & U_{NQ} + \lambda\pi_c & U_{NS} & Q\pi_c \\ U_{QN} + \lambda\pi_c & U_{QQ} & U_{QS} & N\pi_c \\ U_{SN} & U_{SQ} & U_{SS} & \pi_s \\ Q\pi_c & N\pi_c & \pi_s & 0 \end{bmatrix} \begin{bmatrix} dN \\ dQ \\ dS \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda Q & 0 & 0 \\ -\lambda N & 0 & 0 \\ 0 & -\lambda & 0 \\ -NQ & -S & 1 \end{bmatrix} \begin{bmatrix} d\pi_c \\ d\pi_s \\ dI \end{bmatrix}. \tag{A2}$$

Among the second-order or sufficient conditions for utility maximization are $\Delta < 0$, Δ_{11}, Δ_{22}, and $\Delta_{33} > 0$, where Δ is the determinant of the bordered Hessian matrix on the left in (A2) and Δ_{11}, Δ_{22}, and Δ_{33} are the cofactors of the elements of the principal diagonal.

Holding π_s constant and solving for the differentials dN, dQ, and dS by Cramer's rule, we obtain

$$dN = 1/\Delta[-\lambda(Q\Delta_{11} - N\Delta_{21})d\pi_c - \Delta_{41}(dI - NQd\pi_c)], \tag{A3}$$

$$dQ = 1/\Delta[-\lambda(-Q\Delta_{12} + N\Delta_{22})d\pi_c + \Delta_{42}(dI - NQd\pi_c)], \tag{A4}$$

$$dS = 1/\Delta[-\lambda(Q\Delta_{13} - N\Delta_{23})d\pi_c - \Delta_{43}(dI - NQd\pi_c)], \tag{A5}$$

and, since $C = NQ$ and $dC = QdN + NdQ$,

$$dC = 1/\Delta[-\lambda(Q^2\Delta_{11} + N^2\Delta_{22} - 2NQ\Delta_{12}) + (-Q\Delta_{41} + N\Delta_{42})(dI - NQd\pi_c)]. \quad \text{(A6)}$$

The wealth effects, obtained by setting $d\pi_c$ equal to zero in (A3)–(A6), involve the cofactors $-\Delta_{41}$, Δ_{42}, and Δ_{43}, none of the signs of which are restricted by the second-order conditions. However, the following relationships hold (*a*) the weighted sum of the wealth effects equal unity, and (*b*) the weighted sum of the wealth effects on N and Q equals the wealth effect on C. In elasticity form, these two propositions may be expressed as follows:

$$\gamma(\epsilon_N + \epsilon_Q) + (1 - \gamma)\epsilon_S = 1, \quad \text{(A7)}$$

$$\epsilon_N + \epsilon_Q = \epsilon_C, \quad \text{(A8)}$$

where $\gamma = \pi_c NQ/I$ is the share of full wealth accounted for by expenditures on children and ϵ_N, ϵ_Q, ϵ_S, ϵ_C are, respectively, the wealth elasticities of demand for N, Q, S, and C.

The compensated substitution effects are obtained from (A3)–(A6) by evaluating the partial derivatives of N, Q, S, and C with respect to π_c, holding utility constant by setting $(dI - NQd\pi_c)$ equal to zero. To simplify the interpretation of these effects, the following right-hand expressions will be substituted for their left-hand counterparts in (A5) and (A6): $Q\Delta_{13} - N\Delta_{23} = -\pi_s/\pi_c\, \Delta_{33}$ and $Q^2\Delta_{11} + N^2\Delta_{22} - 2NQ\Delta_{12} = (\pi_s/\pi_c)^2\Delta_{33}$. The compensated substitution effects, written in elasticity form, are

$$\eta_N = (-\lambda\pi_c/\Delta)(Q/N\Delta_{11} - \Delta_{21}), \quad \text{(A9)}$$

$$\eta_Q = (-\lambda\pi_c/\Delta)(N/Q\Delta_{22} - \Delta_{12}), \quad \text{(A10)}$$

$$\eta_S = (\lambda\pi_c/S)(\Delta_{33}/\Delta) > 0, \quad \text{(A11)}$$

$$\eta_C = (-\lambda\pi_s^2/\pi_c C)(\Delta_{33}/\Delta) < 0. \quad \text{(A12)}$$

The second-order conditions imply that η_S is positive and η_C is negative such that

$$\gamma\eta_C + (1 - \gamma)\eta_S = 0, \quad \text{(A13)}$$

where γ is the share of full wealth devoted to children. Since the signs of $\Delta_{12} = \Delta_{21}$ are not restricted by the second-order conditions, the signs of η_N and η_Q are ambiguous. However, they must sum to a negative number, since

$$\eta_N + \eta_Q = \eta_C < 0. \quad \text{(A14)}$$

The conditions for both η_N and η_Q to be negative or for one or the other (but not both) to be positive may be seen by considering the price effects embedded in (A9) and (A10) as if they had been generated by a conventional linear full wealth constraint, $I = p_N N + p_Q Q + p_S S$, which is tangent to the actual nonlinear full wealth constraint, $I = \pi_c NQ + \pi_s S$, at the initial point of equilibrium. The prices in the linear constraint are defined as $p_N = \pi_c Q$, $p_Q = \pi_c N$, and $p_S = \pi_s$.

Although, from their definitions, it is apparent that p_N and p_Q cannot vary independently, we shall pretend for a moment that they do. Under this pretense, the left side of (A2) remains the same and the right side becomes $(-\lambda dp_N -\lambda dp_Q -\lambda dp_S I)'$. The following compensated own-substitution effects, written in elasticity form, are restricted by the second-order conditions to be negative:

$$\frac{\partial N}{\partial p_N}\frac{p_N}{N} = \eta_{NN} = (-\lambda\pi_c Q/\mathrm{N})(\Delta_{11}/\Delta) < 0, \tag{A15}$$

$$\frac{\partial Q}{\partial p_Q}\frac{p_Q}{Q} = \eta_{QQ} = (-\lambda\pi_c N/\mathrm{Q})(\Delta_{22}/\Delta) < 0. \tag{A16}$$

The compensated cross-price effects, in elasticity form, are written as follows:

$$\frac{\partial N}{\partial p_Q}\frac{p_Q}{N} = \eta_{NQ} = \lambda\pi_c(\Delta_{21}/\Delta), \tag{A17}$$

$$\frac{\partial Q}{\partial p_N}\frac{p_N}{\mathrm{Q}} = \eta_{QN} = \lambda\pi_c(\Delta_{12}/\Delta). \tag{A18}$$

Since $\Delta_{21} = \Delta_{12}$, it is clear that $\eta_{NQ} = \eta_{QN}$. If N and Q are substitutes, η_{NQ} is positive, and if they are complements, η_{NQ} is negative.

In fact, a change in π_c affects both p_N and p_Q so that, for example,

$$\frac{\partial N}{\partial \pi_c} = \frac{\partial N}{\partial p_N}\frac{\partial p_N}{\partial \pi_c} + \frac{\partial N}{\partial p_Q}\frac{\partial p_Q}{\partial \pi_c}.$$

Thus, substituting (A15) and (A17) into (A9) and (A16), (A18) into (A10), we have

$$\eta_N = \eta_{NN} + \eta_{NQ}, \tag{A19}$$

$$\eta_Q = \eta_{QQ} + \eta_{QN}, \tag{A20}$$

where, from (A14),

$$\eta_C = \eta_N + \eta_Q = \eta_{NN} + 2\eta_{NQ} + \eta_{QQ} < 0. \tag{A21}$$

B. Partial Derivatives of the K-Type Constraint

The household's production-possibility constraint when the wife does no market work was obtained by implicitly solving the simultaneous equation system (27.1)–(27.8) to obtain the K-type constraint $C = K(S, H, T)$. The partial derivatives of this function may be found by totally differentiating (27.1)–(27.8) to obtain a set of simultaneous linear differential equations that may be solved for the differential dC as a function of the differentials of the independent variables. This task is simpler if appropriate substitutions are made to reduce the number of equations and unknowns to four and if the production functions are written in their general form, $C = f(t_c, x_c)$ and $S = g(t_s, x_s)$. The partial derivatives of these functions are then written as $\partial C/\partial t_c = f_1$; $\partial C/\partial x_c = f_2$; $\partial S/\partial t_s = g_1$; $\partial S/\partial x_s = g_2$; and so on.

The four-equation system to which (27.1)–(27.8) is reduced is

$$-C + f(t_c, x_c) = 0, \tag{B1}$$

$$g(T - t_c, H - x_c) = 0, \tag{B2}$$

$$\pi_c f_1 - g_1 = 0, \tag{B3}$$

$$\pi_c f_2 - g_2 = 0. \tag{B4}$$

Taking the total differential of (B1)–(B4) and writing the result in matrix form, we obtain

$$\begin{bmatrix} -1 & -f_1 & -f_2 & 0 \\ 0 & -g_1 & -g_2 & 0 \\ 0 & \pi_c f_{11} + g_{11} & \pi_c f_{12} + g_{12} & f_1 \\ 0 & \pi_c f_{12} + g_{12} & \pi_c f_{22} + g_{22} & f_2 \end{bmatrix} \begin{bmatrix} dC \\ dt_c \\ dx_o \\ d\pi_c \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -g_2 & -g_1 & 1 \\ g_{12} & g_{11} & 0 \\ g_{22} & g_{21} & 0 \end{bmatrix} \begin{bmatrix} dH \\ dT \\ dS \end{bmatrix}. \tag{B5}$$

Let the determinant of the 4×4 matrix on the left be A and let the cofactor of its element a_{ij} be $A_{ij}(-1)^{i+j}$.

Solving (B5) by Cramer's rule, we can give the first partial derivatives of the K-type constraint as follows:

$$\frac{\partial C}{\partial S} = K_S = -A_{21}/A = -1/\pi_c, \tag{B6}$$

$$\frac{\partial C}{\partial H} = K_H = 1/A(g_2 A_{21} + g_{12} A_{31} - g_{22} A_{41}) = 1/\pi_c, \tag{B7}$$

$$\frac{\partial C}{\partial T} = K_T = 1/A(g_1 A_{21} + g_{11} A_{31} - g_{21} A_{41}) = \hat{w}/\pi_c. \tag{B8}$$

Expressing the partial derivatives of the production functions $f(t_c, x_c)$ and $g(t_s, x_s)$ in terms of simple derivatives of the functions $F(\rho_c)$ and $G(\rho_s)$, we can give the second partial derivatives of the K-type constraint as follows:

$$\frac{\partial^2 C}{\partial S^2} = K_{SS} = (1/\pi_c)^2 \frac{\partial \pi c}{\partial S} = (1/\pi_c)^2 (A_{24}/A)$$

$$= -(1/\pi_c)^2 \frac{\pi_c F''G''(\rho_c - \rho_s)^2}{x_c x_s A} < 0, \tag{B9}$$

$$\frac{\partial^2 C}{\partial S \partial H} = K_{SH} = (1/\pi_c)^2 \frac{\partial \pi_c}{\partial H} = (1/\pi_c)^2$$

$$(g_2 A_{24} - g_{12} A_{34} + g_{22} A_{44})(1/A) \tag{B10}$$

$$= -(1/\pi_c)^2 \frac{\pi_c F''G''(\rho_c - \rho_s)}{x_c x_s A} > 0,$$

$$\frac{\partial^2 C}{\partial S \partial T} = K_{ST} = (1/\pi)^2 \frac{\partial \pi_c}{\partial T} = (1/\pi_c)^2$$

$$(-g_1 A_{24} - g_{11} A_{34} + g_{12} A_{44})(1/A) \qquad \text{(B11)}$$

$$= -(1/\pi_c)^2 \frac{\pi_c \rho_c F'' G'' (\rho_c - \rho_s)}{x_c x_s A} < 0,$$

where

$$A = -\frac{F''}{x_c}(\rho_c \hat{w} + 1)^2 - \frac{G''}{x_s}(\rho_s \hat{w} + 1)^2 > 0.$$

C. *Partial Derivatives of the J-Type Constraint*

The production-possibility function when the wife does some market work is obtained by solving the simultaneous equation system (27.1)–(27.11). The solution is the J-type constraint, which is written in implicit form as $C = J(S, H, \kappa, T)$. Again, the full system may be reduced by substitution to the following system of five equations in five unknowns:

$$-C + f(t_c, x_c) = 0, \qquad \text{(C1)}$$

$$g(T - L - t_c, H + wL - x_c) = S, \qquad \text{(C2)}$$

$$\pi_c f_1 - g_1 = 0, \qquad \text{(C3)}$$

$$\pi_c f_2 - g_2 = 0, \qquad \text{(C4)}$$

$$f_1/f_2 - w' = 0. \qquad \text{(C5)}$$

This system differs from the system (B1)–(B4) underlying the K-type constraint only in the addition of equation (C5) and in the addition of a term involving the labor supply, L, into the arguments of the production function g.

By totally differentiating the (C1)–(C5) system, the following system is obtained:

$$\begin{bmatrix} -1 & f_1 & f_2 & 0 & 0 \\ 0 & -g_1 & -g_2 & 0 & -g_1 + w'g_2 \\ 0 & \pi_c f_{11} + g_{11} & \pi_c f_{12} + g_{12} & f_1 & g_{11} - w'g_{12} \\ 0 & \pi_c f_{12} + g_{12} & \pi_c f_{22} + g_{22} & f_2 & g_{22} - w'g_{22} \\ 0 & \dfrac{f_{11}f_2 - f_{21}f_1}{(f_2)^2} & \dfrac{f_{12}f_2 - f_{22}f_1}{(f_2)^2} & 0 & -w'' \end{bmatrix} \begin{bmatrix} dC \\ dt_c \\ dx_c \\ d\pi_c \\ dL \end{bmatrix} \qquad \text{(C6)}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -g_2 & -g_1 & -w_\kappa L g_2 \\ 0 & g_{12} & g_{11} & w_\kappa L g_{12} \\ 0 & g_{22} & g_{21} & w_\kappa L g_{22} \\ 0 & 0 & 0 & w_\kappa + w_{L\kappa} L \end{bmatrix} \begin{bmatrix} dS \\ dH \\ dT \\ d\kappa \end{bmatrix}.$$

Let the determinant of the matrix on the left in (C6) be B and let the cofactor of its element b_{ij} be $B_{ij}(-1)^{i+j}$.

It may be observed that the cofactor B_{55} of the element w'' is equal to the determinant A in (5), from which it follows that

$$B = B^* - w''A, \tag{C7}$$

where

$$B^* = -\frac{f_{11}f_2 - f_{12}f_1}{(f_2)^2} B_{52} + \frac{f_{12}f_2 - f_{22}f_1}{(f_2)^2} B_{53}.$$

Similarly, it follows that

$$B_{ij}(-1)^{i+j} = (B^*_{ij} - w''A_{ij})(-1)^{i+j}, \tag{C8}$$

where $i, j = 1, \ldots, 4$.

The element w'' $(=2w_L + Lw_{LL})$ is zero if the wife's market wage is independent of her lifetime hours of work. In this case, her market wage, $w' = w(\kappa)$, is a parameter whose value depends solely on her initial stock of human capital, κ, and the partial derivatives of the J-type constraint depend only on B^* and the B^*_{ij}. Let the J-type constraint in this special case be

$$C = J^*(S, H, \kappa, T). \tag{C9}$$

The first partial derivatives of J^* are

$$\frac{\partial C}{\partial S} = J^*_S = -B^*_{21}/B^* = -1/\pi_c; \tag{C10}$$

$$\frac{\partial C}{\partial H} = J^*_H = (1/B^*)(g_2B^*_{21} + g_{12}B^*_{31} - g_{22}B^*_{41}) = 1/\pi_c; \tag{C11}$$

$$\begin{aligned}\frac{\partial C}{\partial \kappa} = J^*_\kappa = (w_\kappa L/B^*)(g_2B^*_{21} + g_{12}B^*_{31} - g_{22}B^*_{41}) \\ + w_\kappa(B^*_{51}/B^*) = w_\kappa L/\pi_c;\end{aligned} \tag{C12}$$

$$\frac{\partial C}{\partial T} = J^*_T = 1/B^*(g_1B^*_{21} + g_{11}B^*_{31} - g_{21}B^*_{41}) = w'/\pi_c. \tag{C13}$$

The second partial derivatives of J^* are

$$\frac{\partial^2 C}{\partial S^2} = J^*_{SS} = (1/\pi_c)^2 \frac{\partial \pi_c}{\partial S} = (1/\pi_c)^2(B^*_{24}/B^*) = 0, \tag{C14}$$

$$\begin{aligned}\frac{\partial^2 C}{\partial S \partial H} &= J^*_{SH} = (1/\pi_c)^2 \frac{\partial \pi_c}{\partial H} \\ &= (1/\pi_c)^2(-g_2B^*_{24} - g_{12}B^*_{34} + g_{22}B^*_{44})(1/B^*) = 0,\end{aligned} \tag{C15}$$

$$\begin{aligned}\frac{\partial^2 C}{\partial S \partial \kappa} = J^*_{S\kappa} = (1/\pi_c)^2 \frac{\partial \pi_c}{\partial \kappa} = (w_\kappa L/\pi^2_c)(-g_2B^*_{24} - g_{12}B^*_{34} \\ + g_{22}B^*_{44})(1/B^*) + w_\kappa(-B^*_{54}/B^*) \\ = (1/\pi_c F)^2 w_\kappa w'(\rho_c - \rho_s) > 0,\end{aligned} \tag{C16}$$

$$\frac{\partial^2 C}{\partial S \partial T} = J^*_{ST} = (1/\pi_c)^2 \frac{\partial \pi_c}{\partial T} = (1/\pi_c)^2(-g_1 B^*_{24} - g_{11}B^*_{34} + g_{22}B^*_{44})(1/B^*) = 0. \tag{C17}$$

If the wife's market wage is affected by her lifetime hours of work because of postmarital investment in human capital (or for any other reason), w'' will not equal zero. In specifying the signs of the following partial derivatives of the J-type constraint, it will be assumed that w'' is positive, as it would tend to be if postmartial investment provided the major source of dependence between w' and L, but the results are also relevant to the converse assumption of negative w''.

The partial derivatives of J may be expressed in terms of the partial derivatives of the K- and J^*-type constraints by utilizing the relationships in (C7) and (C8). The first partial derivatives of J are identical to those of J^* and will not be repeated. The second partials of J are

$$\frac{\partial^2 C}{\partial S^2} = J_{SS} = -w''(A/B)\, K_{SS} > 0, \tag{C18}$$

$$\frac{\partial^2 C}{\partial S \partial H} = J_{SH} = -w''(A/B)\, K_{SH} < 0, \tag{C19}$$

$$\frac{\partial^2 C}{\partial S \partial T} = J_{ST} = -w''(A/B)\, K_{ST} > 0, \tag{C20}$$

$$\frac{\partial^2 C}{\partial S \partial \kappa} = J_{S\kappa} = -w'' w_\kappa L(A/B)\, K_{SH} + (1 + \frac{w_{L\kappa}}{w_\kappa} \mathrm{L})(B^*/B)\, J^*_{S\kappa}. \tag{C21}$$

The sign of $J_{S\kappa}$ is ambiguous because the term on the left involving K_{SH} is negative and the term on the right involving $J^*_{S\kappa}$ is positive.

D. The Derivation and Properties of the Demand Functions for N, Q, *and* S

Let the general production-possibility constraint faced by the family be

$$\Phi(C, S, H, \kappa, T) = 0, \tag{D1}$$

where, if the wife works in the market, $\Phi = -NQ + J(S, H, \kappa, T)$, $R = 1$; and, if she does no market work, $\Phi = -NQ + K(S, H, T)$, $R = 0$. The family is assumed to maximize the Lagrangian function $U(N, Q, S) + \lambda\, \Phi(C, S, H, \kappa, T)$, where λ is a Lagrange multiplier. The first-order conditions are

$$\begin{aligned} U_N + \lambda Q \Phi_c &= 0, \\ U_Q + \lambda N \Phi_c &= 0, \\ U_S + \lambda \Phi_s &= 0, \\ \Phi &= 0. \end{aligned} \tag{D2}$$

The ratio $-\Phi_c/\Phi_s$ is the marginal rate of transformation between S and C along the production-possibility function, and in equilibrium, it is equal to the marginal rate of substitution in consumption, $-\pi_c/\pi_s$. Accordingly, in what follows, let $\Phi_c = \pi_c$ and $\Phi_s = \pi_s(=1)$, where these magnitudes can be interpreted as equilibrium values.

Corresponding to the general maximization problem involving the constraint Φ and to the particular problems involving the K- and J-type constraints, define, respectively, a general fertility demand function,

$$N = N(H, \kappa, T), \tag{D3}$$

a demand function when the wife does no market work,

$$N = N^0(H, T), \qquad R = 0, \tag{D4}$$

and a demand function when the wife does work,

$$N = N^1(H, \kappa, T), \qquad R = 1. \tag{D5}$$

Also define with similar notation general and special demand functions for Q and S

The properties of these demand functions may be examined by totally differentiating the first-order conditions (D2) to obtain

$$\begin{bmatrix} U_{NN} & U_{NQ} + \lambda\pi_c & U_{NS} & Q\pi_c \\ U_{QN} + \lambda\pi_c & U_{QQ} & U_{QS} & N\pi_c \\ U_{SN} & U_{SQ} & U_{SS} + \lambda\Phi_{SS} & \pi_s \\ Q\pi_c & N\pi_c & \pi_s & 0 \end{bmatrix} \begin{bmatrix} dN \\ dQ \\ dS \\ d\lambda \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\lambda\Phi_{SH} & -\lambda\Phi_{S\kappa} & -\lambda\Phi_{ST} \\ -\Phi_H & -\Phi_\kappa & -\Phi_T \end{bmatrix} \begin{bmatrix} dH \\ d\kappa \\ dT \end{bmatrix}. \tag{D6}$$

The second-order conditions for a maximum require that $D < 0$ and D_{11}, D_{22}, and $D_{33} > 0$, where D is the determinant of the bordered Hessian on the left in (D6) and D_{11}, D_{22}, and D_{33} are the cofactors of the elements of the principal diagonal.

Introducing the dummy argument α_i $(i = H, \kappa, T)$, we can solve (D6) by Cramer's rule to obtain the following partial derivatives of the general fertility demand function, (D3):

$$\frac{\partial N}{\partial \alpha_i} = (-\lambda D_{31}/D)\ \Phi_{si} + (-D_{41}/D)(-\Phi_i). \tag{D7}$$

In the conventional manner, the total effect on demand of a change in α_i may be expressed as the sum of a compensated substitution effect and a wealth effect.

The compensated substitution effect is obtained where utility is held constant by setting the total differential of the utility function equal to zero (i.e., $dU = U_N dN + U_Q dQ + U_S dS = 0$). The first-order conditions of (D2) imply that $U_N = U_S(Q\pi_c/\pi_s)$ and $U_Q = U_S(N\pi_c/\pi_c)$, from which it follows, by substitution into dU, that $Q\pi_c dN + N\pi_c dQ + \pi_s dS = 0$ when utility is held constant. The last equation of (D6) is $Q\pi_c dN + N\pi_c dQ + \pi_s dS = -\Phi_i$. Thus, if we set Φ_i equal to zero in (D7) to hold utility constant, the compensated substitution effect is

$$\frac{\partial N}{\partial \alpha_i} = (-\lambda D_{31}/D)\Phi_{si}, \tag{D8}$$

where $\partial N/\partial \alpha_i$ is understood to be evaluated with utility, instead of α_j $(j \neq i)$ being held constant. From the analysis of the second partial derivatives of the production-possibility functions, it is known that $\Phi_{si} = \pi^2{}_c(\partial \pi_c/\partial \alpha_i)$. Consequently, by the chain rule, it follows that

$$\frac{\partial N}{\partial \alpha_i} = \frac{\partial N}{\partial \pi_c}\frac{\partial \pi_c}{\partial \alpha_i},$$

where $\partial N/\partial \pi_c = -\lambda D_{31}/D$. Thus, in elasticity terms, the compensated substitution effect on fertility of a change in α_i, η_{Ni}, is the product of the compensated price elasticity of demand for N, η_N, and the elasticity of π_c with respect to α_i, e_i, where

$$e_i = \frac{\alpha_i}{\pi_c}\frac{\partial \pi_c}{\partial \alpha_i} = \pi_c \alpha_i \Phi_{si}. \tag{D9}$$

An equivalent argument may be made with respect to the compensated substitution effects of a change in α_i on Q and S, so that the following relationships may be expressed:

$$\eta_{Ni} = \eta_N e_i, \tag{D10}$$

$$\eta_{Qi} = \eta_Q e_i, \tag{D11}$$

$$\eta_{Si} = \eta_S e_i, \tag{D12}$$

$$\eta_{Ci} = (\eta_N + \eta_Q)\, e_i. \tag{D13}$$

In section 3, it was found that $\eta_c < 0$ and $\eta_s > 0$ and, less certainly, that $\eta_N < 0$ and $\eta_Q > 0$. Given these signs, the signs of compensated substitution effects will depend on the signs of the e_i, which in turn will depend on which of the α_i (H, κ, or T) is being considered and which type of constraint (K, J^*, or J) the family is facing. The signs of these substitution elasticities are tabulated in table A1.

The wealth effects on fertility are defined as the partial derivatives of N with respect to α_i, holding π_c constant. If we set

$$\Phi_{si} = \pi_c^2 \frac{\partial \pi_c}{\partial \alpha_i}$$

equal to zero in (D1), the wealth effects are

$$\frac{\partial N}{\partial \alpha_i} = (-D_{41}/D)(-\Phi_i). \tag{D14}$$

TABLE A1

SIGNS OF COMPENSATED SUBSTITUTION ELASTICITIES N, Q, C, AND S WITH RESPECT TO H, κ, AND T BY TYPE OF CONSTRAINT

COMPENSATED ELASTICITY	TYPE OF CONSTRAINT: K	J^*	J
Fertility (N):			
η_{NH}	−	0	+
$\eta_{N\kappa}$	0	−	−
η_{NT}	+	0	−
Child quality (Q):			
η_{QH}	+	0	−
$\eta_{Q\kappa}$	0	+	+
η_{QT}	−	0	+
Child services (C):			
η_{CH}	−	0	+
$\eta_{C\kappa}$	0	−	−
η_{CT}	+	0	−
Other sources of satisfaction (S):			
η_{SH}	+	0	−
$\eta_{S\kappa}$	0	+	+
η_{ST}	−	0	+

The change in the family's full wealth, $I = \pi_c C + \pi_s S$, given a change in α_i and holding prices constant, is

$$\frac{\partial I}{\partial \alpha_i} = \frac{\partial I}{\partial C}\frac{\partial C}{\partial \alpha_i},$$

where $\partial I/\partial C = \pi_c$ and $\partial C/\partial \alpha_i = -\Phi_i$. Thus, by the chain rule, the wealth effect may be expressed as

$$\frac{\partial N}{\partial \alpha_i} = -\frac{\partial N}{\partial I}\frac{\partial I}{\partial \alpha_i},$$

where $\partial N/\partial I = -D_{41}/D$ and $\partial I/\partial \alpha_i = -\pi_c \Phi_i$.
Again, an equivalent argument may be made for the wealth effects on Q, C, and S caused by a change in α_i. Thus, the wealth elasticities may be written

$$\epsilon_{Ni} = \epsilon_N \gamma_i, \tag{D15}$$

$$\epsilon_{Qi} = \epsilon_Q \gamma_i, \tag{D16}$$

$$\epsilon_{Ci} = (\epsilon_N + \epsilon_Q)\alpha_i, \tag{D17}$$

$$\epsilon_{Si} = \epsilon_S \gamma_i, \tag{D18}$$

where

$$\gamma_i = \frac{\alpha_i}{I}\frac{\partial I}{\partial \alpha_i} = (-\pi_c \alpha_i / I)\Phi_i$$

is the elasticity of full wealth with respect to α_i and ϵ_N, ϵ_Q, ϵ_C, and ϵ_S are, respectively, the wealth elasticities of demand for fertility, child quality, child services, and other sources of satisfaction. From the first partial derivatives of the K J* and J-type constraints, it is easily seen that the γ_i are simply

$$\gamma_H = H/I, \tag{D19}$$

$$\gamma_K = w_K L/I, \tag{D20}$$

$$\gamma_T = \hat{w}T/I. \tag{D21}$$

The total effect on consumption of a change in γ_i is the sum of the compensated substitution effect and the wealth effect. In elasticity terms, these total elasticities are

$$\delta_N = \eta_N e_i + \epsilon_N \gamma_i, \tag{D22}$$

$$\delta_Q = \eta_Q e_i + \epsilon_N \gamma_i, \tag{D23}$$

$$\delta_C = (\eta_N + \eta_Q) e_i + (\epsilon_N + \epsilon_Q) \gamma_i, \tag{D24}$$

$$\delta_S = \eta_S e_i + \epsilon_S \gamma_i. \tag{D25}$$

Comment

Norman B. Ryder

Office of Population Research, Princeton University

Let me abuse the invitation to discuss the paper by Robert Willis by taking the opportunity to unload a few thoughts on the new home economics and related concerns. Although it is basically presumptuous for an outsider (even one who was once inside) to try to tell a group of professionals what they are really doing or ought to be doing, it may be that noninvolvement in the rituals and routines of the work in question can provide a clarity of vision. If the outcome is mere heresy, it can at least be readily dismissed as the obvious consequence of ignorance.

I think of economics as playing a central and unique role within the complex of the sciences of behavior—central because its area of expertise is the calculus of choice, and choice is ubiquitous and ineluctable in all behavior; unique because its contribution is primarily (maybe exclusively) a deductive one. I think of the principles of economics as ultimately tautological derivations from a branch of applied mathematics. These comments are in no sense denigrating; I perceive demography, which is my lifeblood, to have the same formal characteristics. No act of an individual or group is without an economic dimension, although many classes of action have been underexposed to the risk of an economist's scrutiny because they do not pass through the marketplace. What goes on in the family is an obvious case in point. The subject calls for an expansion of the power and reach of the calculus of choice beyond merely money-valued resources into the economies of time, energy, emotional commitment, and the like.

So economists have entered the home and declared that children can be thought of as purchases by parents and that the time the wife spends on domestic affairs has an opportunity cost. True enough. But to build the new home economics on a solid foundation, so that the other social scientists interested in the family will be forced to pay attention, it is necessary to specify those ways in which the purchase of a child is distinctive from the purchases of those kinds of commodity on which economics has developed its discipline to date—and those ways in which the decision by the wife to divide her time between the world inside and the world outside the home is a peculiarly constrained choice.

I suggest the simple but fundamental proposition that the replacement of a continually aging citizenry by new recruits is much too important to the entire body politic to tolerate untrammeled individual choice to hold sway. On this issue, as on so many others, the society intervenes, in obvious and in subtle ways, to ensure that the outcome, at least in the aggregate, makes sense on the society's behalf. These constraints on choice are what sociologists call norms. Just as no act is devoid of economic content, so no act is devoid of normative content. Norms are not just another discipline's jargon for tastes and preferences; the distinction is crucial between them, because the terms point in entirely different research directions. When tastes and preferences are employed for some purpose more elevating than circular reasoning, they promote research into the properties of individuals, whereas norms are properties of organized groups which individuals pay heed to in their actions to the extent that they have been successfully socialized into membership in the groups. Nor are norms arbitrary in their shape: they are institutionalized solutions to pervasive problems, and if they do not make sense, the group suffers the consequences. Were these norms fixed in time and space, one could readily take them as given (meaning essentially to forget them), but they vary from culture to culture, from subculture to subculture, from class to class, and they vary through time. Only when the time perspective of the economist is very short can they safely be neglected. So thoroughly are they embedded in our lives that they verge on the invisible, and this is one of the major sources of their strength. Yet that creates great research difficulties for the sociologist and provokes great impatience in the nonsociologist. Now no economist would fail to take into account various biological properties which condition behavior, like early dependence or limited reproductive span. Sociocultural properties play the same kind of role.

Willis presents a model within a framework of the economic theory of the family, but he proceeds about this important task by systematically destroying the idea of a family. The family in its skeleton form consists of a flow of person-years through time, encompassing the adult lifetime of one male and one female and the nonadult lifetimes of a varying number of children (including zero). The members of the family are bound to each other by contract, with clear specifications, *inter alia,* of the directions and amounts of flows of resources and services from one to another member. Willis has collapsed time into the instant of initial decision, he has defined the parents as subjects and the children as objects, he has denied the members the right to take satisfaction in the satisfaction of others, he has merged the husband and wife into a single utility function of the individual type—in short, he has solved the problems of family economics by dissolving the family. To give one specific example: there is no characteristic of the child save perhaps the requirement of the expenditure of time as well as other resources that makes the model any different from one concerned with any other purchase. Willis presents a good list of the recalcitrant characteristics of fertility behavior, but the job of coping with those characteristics from an economic standpoint remains undone. Almost the only concession to the family as a concept—and that regrettably an unwittingly sexist position—is to assign the husband to the labor force full time and permit the wife to be assigned to the labor force some proportion of time (from 0 to 1).

I am incompetent to evaluate the merits of the economic model Willis presents, but I feel less abashed by that circumstance than would ordinarily be the case because, as I have suggested, I cannot perceive its special relevance for fertility. But there are some empirical results, and they suggest some observations. Willis examines the determinants of parity for white women age 35–64, married once, husband present, in urban areas in 1960. Why each of these implicit choices was made is unexplained. It seems regrettable to destroy variance by restricting the examination to urban whites; the use of an age limit as low as 35 unfortunately reduces variance still further (because, although fertility beyond age 35 is small in toto, it bulks large as a source of differentials); many interesting kinds of families get short shrift by the restriction to stable unions; and the particular epoch of our history associated with these cohorts is the trough of a cycle, so that the relationships observed may be quite different from observations around a peak or observations independent of cycle altogether.

Willis presents his regression equation in the following form: $P = 4.83269 - 0.24386*I - 0.17572*E + 0.02023*E*I - 0.07243*S$, where P is parity, I is estimated income of husband at age 40, E is education of wife, and S is the size of urban area of residence. This may be reformulated by assigning S its mean value and subsuming it in the constant term (Willis gives us no reason to be interested in S) and by dividing the coefficients of E and I by the coefficient of $E*I$. Collecting terms and doing a little rounding, I obtain $P = 2.5 + 0.02*(8.7 - I)*(12.3 - E)$. The mean of I, which is in thousands of dollars, is 4.3; the mean of E, which is in years of schooling completed, is 10.5. The rephrased regression suggests the presence of threshold values for income and education; it also leads into a favorite theme of some sociologists, that of status inconsistency, since the low parities are produced by combinations of high income and low education, on the one hand, and low income and high education, on the other. The reformulated regression indicates the way in which the partial of P with respect to I depends on E and the partial of P with respect to E depends on I.

The cluster of relationships which leads Willis to his regression are: The proportion of women working varies directly with the education of wife and inversely with the income of husband: for the working woman, fertility varies inversely with education of wife; for the nonworking woman, fertility varies inversely with income of husband. Implicit is the empirical tendency for the education of wife to vary directly with the income of husband. The only one of these relationships with which I find any difficulty is the inverse relationship of fertility with husband's income for the nonworking woman. I suggest that a more plausible version would be that fertility varies inversely with wife's education for the nonworking woman. The point is that I believe the alternative opportunity cost of children rises with the wife's education, whether she is thinking of market or nonmarket pursuits. I recognize that Willis intended her education as a surrogate for her lifetime earning capacity, but the data are blind to the concepts of the theorist, and wife's education means whatever it means, which to me is a lot more than merely earning capacity. Similarly, the dependent variable, completed (or nearly completed) parity, is presumably thought of as a consequence of the initial game plan. I think the results of a generation of fertility research suggest that variations in parity are more likely to reflect variations in the efficacy of fertility regulation than variations in intention. The use of a regression equation to estimate husband's income at age 40 is an interesting innovation. Unfortunately, it has the consequence of erasing from the system one kind of income variable which has been found to affect fertility, that is, the deviation of one's income from what would be expected on the basis of one's occupation, education, and so forth.

Were I designing research on fertility, from an economic standpoint, I think it would be advisable to consider the aspects of the reproductive process which are most clearly discretionary. One of these is the age of the wife (and husband) at birth of the first child. More precisely, that should be the age at birth of the first intended child. There is substantial variance in this. It would appear to be related in obvious ways to current and prospective income, as well as to the education of the wife and her work history, and it is of extraordinary importance demographically (in terms of its consequences for variations in the birth rate, in the short run, and for variations in the ultimate size of the population, in the long run). A second focus would be the decision as to whether or not to have a third child. On that decision hangs the balance between growth or decline in population size. Such was the central concern of the Princeton Fertility Study, by Westoff and others. The yield from their economic inputs was meager, but their staff did not include an economist. A third suggestion is examination of the temporal interdependency of the work history and the procreative history of the wife, because of its potential bearing on the initiation and termination of childbearing as well as on the length of birth interval. Again, it is important to distinguish carefully those acts of procreation which occur by design from those which occur by accident (and are accordingly presumably the focus for another kind of model altogether). The final discretionary point which may deserve increasing attention in the future, although it has largely been proscribed in the past, is the decision as to whether to have children at all (and quite apart from whether or not marriage occurs). The proportion voluntarily infertile seems now to be rising, and the norms defining women's roles are under concerted attack.

While I would be reluctant to dissuade Willis and others from attempts at model building such as the present one—since I work in a theoretically impoverished area and regret it—it does seem to me that some redirection of energies is requisite to the further development of the economic theory of fertility. What seems most needed is information, collected according to the specifications of economists, about the behavior with which their models purport to be concerned. Demographers survived for centuries on official registration and enumeration data, but only in the last few decades have they faced the realization that they have to create their own data to test their own theories. In my judgment, the economic theory of fertility is too important to rely on secondhand data, devised for other purposes, from the U.S. census, or even from our National Fertility Study.