Challenges in Designing Electricity Spot Markets

Mete Şeref Ahunbay   Martin Bichler   Johannes Knörr*

October 11, 2023

Abstract

Electricity market design has drawn significant attention due to their size, their importance for the economy, and the intricacies in their design. Non-convexities in the valuation and cost functions play a central role and they make the allocation and pricing problems difficult. In addition, these markets need to consider power flows on the electricity network, which is different from many other markets. Increasing levels of renewable energy sources such as wind and solar lead to new requirements. In particular, the volatility of renewable supply requires attention. We discuss some of the central issues in the design of electricity spot markets and possible solutions that were discussed in the recent literature.

1 Introduction

Electricity markets globally, especially in Europe and extensive regions of the United States, have transitioned from being monopolistic to adopting a competitive wholesale market structure, a shift that primarily occurred in the 1990s. Presently, short-term electricity purchases in these jurisdictions are facilitated through power exchanges. These platforms are integral in establishing core pricing signals for both over-the-counter transactions and futures markets, as outlined by Shah and Chatterjee (2020). In the United States, notable examples of these power exchanges encompass the California Independent System Operator (CAISO), the Electric Reliability Council of Texas (ERCOT), the Midwest ISO (MISO), the New York ISO (NYISO), and the ISO New England (ISO-NE). These markets typically facilitate the trading of hourly products for the subsequent day in the day-ahead markets.

Typically, on day-ahead markets hourly products for the next day are traded. After the day-ahead markets, the market operators use real-time markets in the United States (or intraday markets in Europe) to deal with changes in supply and demand that are closer to the actual dispatch. We will distinguish these types of *electricity spot markets* from *futures markets* where participants can hedge against longer-term price risks.

Spot markets are significant in size. In 2020, European coupled day-ahead markets alone cleared 1,530 TWh in 27 countries with average prices between 30 and 40 EUR/MWh (NEMO Committee 2021). Similarly, the cost of serving load amounted to $8.9 billion in the Californian market, covering 26,000 circuit miles, roughly 1,000 power plants, a population of 30 million, and about 9,700 pricing nodes (California ISO 2018, 2021). While many aspects of electricity market design are similar to other commodity markets, a few features stand out.

*Technical University of Munich, School of Computation, Information and Technology, Department of Computer Science. E-mail: `mete.ahunbay/bichler/knoerr@cit.tum.de`

- First, demand and supply need to be balanced at all times to guarantee a stable electricity grid. This holds at least for the real-time market.
- Second, demand and supply meet on the nodes of the power grid, and excess supply can be shipped to neighbouring nodes via transmission lines.
- Third, electricity markets are characterized by non-convex preferences. For example, electricity suppliers often incur fixed costs for starting up and running their generators. On the demand side, industrial customers typically need a certain volume of electricity over consecutive hours to finish production or maintain energy-intensive services. Such consumption profiles can sometimes be shifted over time, but the profiles themselves must not be altered. Non-convexities in the preferences of this sort lead to non-convex optimization problems that need to be solved in order to determine the efficient or welfare-maximizing dispatch and prices. We will use the term *non-convex markets* in what follows.

The landscape of power systems is undergoing a transformation due to the influx of renewable energy sources (RES). A significant portion of this new capacity is attributed to Variable Energy Resources (VER) like solar and wind power. These energy sources, characterized by their inherent variability and unpredictability, necessitate the incorporation of flexible demand to ensure grid stability and reliability (Reihani et al. 2016). Demand response is the most immediately available way of increasing demand flexibility and the cheaper option compared to storage technologies today (EU 2016). For example, industrial processes for the production of pulp and paper are able to provide demand response with a duration of up to three hours without any notice time (EU 2016). Still, this flexibility comes at a cost and bidders want lower prices if they provide more flexibility.

The traditional literature on electricity market pricing typically relies on the assumption of price-inelastic demand, but this assumption is unlikely to hold in the future (Herrero et al. 2020). It is expected that in the future we will see more price-responsive demand (Hytowitz et al. 2020). However, such price-sensitive bids for flexible demand make market design more challenging. First, new bid formats lead to additional non-convexities which can even increase the make-whole payments needed with currently used pricing rules. Second, prices that are individually rational and clear the market at the efficient dispatch cannot always be budget balanced anymore (Bichler et al. 2022). Overall, we move to an economy with many thousands of small generators and a more price-sensitive demand side that actively bids in electricity markets and offers flexibility to cope with variability in the supply (IRENA 2019, Hytowitz et al. 2020).

Such changes in the market have led to renewed interest in the design of electricity markets. We start out discussing standard notions of market equilibrium, before we survey the literature on electricity market pricing and some of the current research challenges.

2 Competitive Equilibrium on Markets

The literature on electricity markets draws on insights from general equilibrium theory. The Arrow-Debreu model shows that under convex preferences, perfect competition, and demand independence, there must be a set of competitive equilibrium prices (Arrow and Debreu 1954, McKenzie 1959, Gale 1963, Kaneko 1976). The results derived from the Arrow-Debreu model led to the well-known welfare theorems, representing important arguments for markets to be used as efficient or welfare-maximizing means to allocate scarce resources such as electricity. The first theorem states that any Walrasian equilibrium leads to a Pareto efficient allocation of resources. The second theorem states that any efficient allocation can be attained by a Walrasian equilibrium under the Arrow-Debreu model assumptions (Mas-Colell et al. 1995). Walrasian equilibrium prices are such that there is a single price for each product (i.e., prices for a package are *linear*) and this is the same price for all participants (i.e., *anonymous* prices with no price differentiation). The question is, whether we can achieve such properties on electricity markets.

One feature of electricity markets is that they are based on a network with possibly thousands of spatially distributed nodes connected via transmission lines. To put it differently, we have a set of coupled markets and need to equalize supply and demand at each node.

Definition 1 (Ahunbay et al. (2023b)). *A* ***coupled market*** *consists of a set of goods M, a set of transmission network parameters F and a set of market participants $L = B \cup S \cup R$, partitioned into the set of buyers B, set of sellers S and the set of transmission operators R. Each market participant $\ell \in L$ has preferences over bundles in $\mathbb{R}^{M \cup F}$, i.e. each buyer b has a valuation function $v_b : \mathbb{R}^{M \cup F} \to \overline{\mathbb{R}}$, each seller s has a cost function $c_s : \mathbb{R}^{M \cup F} \to \overline{\mathbb{R}}$, and each transmission operator r has a cost function $d_r : \mathbb{R}^{M \cup F} \to \overline{\mathbb{R}}$.*

Thus, we also need to price transmission lines in such coupled markets appropriately (Lété et al. 2022). For example, if a transmission line is congested, the price on two adjacent nodes should differ. Similarly, prices on all nodes should be identical if there is no network congestion. So, do the Welfare Theorems hold in coupled markets connected via transmission networks? Fortunately, the answer to this question is affirmative:

Theorem 2.1 (The Welfare Theorems for Coupled and Convex Markets, Ahunbay et al. (2023b)). *Let price vector $p^* \in \mathbb{R}^{M \cup F}$ and the allocation $(z_\ell)^*_{\ell \in L}$ be a Walrasian equilibrium, then this allocation maximizes social welfare. Conversely, if $(z_\ell)^*_{\ell \in L}$ is a welfare-maximizing allocation, then it can be supported by a Walrasian price vector p^* that forms a Walrasian equilibrium.*

However, this theorem as well as the original welfare theorems assume convex preferences. As we discussed earlier, electricity markets have non-convex preferences and complex constraints. It is well known that only restricted types of valuations (e.g., substitutes valuations or unimodular demand types) allow for convex allocation problems and Walrasian equilibria (Kim 1986, Bikhchandani and Mamer 1997, Leme 2017, Baldwin and Klemperer 2019). Thus, we cannot expect Walrasian equilibria to exist on electricity spot markets.

This raises the question how prices can be computed in the presence of non-convex preferences for indivisible goods and which properties we can hope to achieve compared to Walrasian equilibria. Established market design desiderata are *efficiency* (i.e., maximization of welfare or gains from trade), *individual rationality* (i.e., participants should not make a loss), *budget balance* (i.e., the market operator should not make a loss or a gain), and *envy-freeness* (i.e., participants would not want a different allocation at the prices). These axioms are not only central to economic theory, but are widely adopted and natural design desiderata for practical market design.

If the allocation problem is convex, duality theory and dual variables in convex optimization provides a principled way to determine competitive equilibrium prices that satisfy these desiderata (Bichler et al. 2020). In non-convex markets, it is well known that competitive equilibrium prices might need to be non-linear and personalized and such prices might not even exist (Bikhchandani and Ostroy 2002, Bichler and Waldherr 2019). Thus, in a combinatorial auction or a combinatorial exchange that allows for supply and demand bids on packages of items, each bidder might need to have a different price for the same package (personalized prices), and each package price could differ from the sum of the item prices in this package (non-linear prices). As a simple example, consider a single supplier with an (indivisible) sell bid of 2 MWh for $30, while there is one buyer asking for 1 MWh for at most $10, and another buyer asking for 1 MWh for $28. Linear and anonymous market prices could not be higher than $10/MWh and as such there would be no trade and no gains from trade. With price differentiation, trade would be possible.

However, non-linear and personalized prices would convey little information other than that a bidder lost or won. Besides, if prices should serve as a baseline for derivatives as is the case for options or futures, this is hardly possible with non-linear prices that differ among participants. In other words, anonymity and linearity are important requirements for prices on electricity markets but also in other domains (Bichler et al. 2018). This has led to significant research on pricing in non-convex electricity markets.

3 State-of-the-Practice in Electricity Market Pricing

Let us now introduce the pricing rules used on electricity spot markets today. The literature on this subject is large and we only focus on specific pricing rules that have been implemented in practice. The interested reader is referred to Liberopoulos and Andrianesis (2016) for an excellent survey on the topic.

Electricity spot markets are composed of varying levels of "demand" (load) and matching levels of "supply" (generation). Market participants submit supply and demand bids according to a certain *bid language* that determines the form of the allocation problem (which yields the efficient dispatch) and the pricing rule. Ideally, supply and demand curves at each traded time slice (e.g., hour) determine prices in equilibrium. However, non-convexities in the generators' cost functions have a significant impact on bid languages with ample consequences on the allocation problem (which yields the efficient dispatch) and the pricing rule (Herrero et al. 2020). For example, in markets in the United States (U.S.), generators can express start-up or no-load costs, economies of scale (by means of piecewise-linear cost functions), or minimum-generation requirements. These and other elements of the bid language then translate into non-convex allocation problems (Herrero et al. 2020).

In 2005, the Pennsylvania, Jersey, Maryland Power Pool (PJM) introduced mixed integer programming (MIP) in order to address these non-convexities and to determine the efficient allocation or dispatch (O'Neill et al. 2020). Since 2018, all Independent System Operators (ISOs) in the U.S use MIPs to compute the efficient dispatch instead of the Lagrangian relaxation that was used before. The MIP used to solve the allocation problem on US electricity markets is also known as Direct Current Optimal Power Flow (DCOPF) problem. Dual prices as they are accessible for convex optimization problems are not available in such markets, which led to a fundamental question: How can market prices per hour be computed in such non-convex markets?

One approach followed by European day-ahead markets is to sacrifice efficiency. The EUPHEMIA algorithm that is used to clear European day-ahead markets first solves a welfare maximization allocation problem as a mixed-integer program and then iteratively tries to find linear and anonymous prices that clear the market. If such prices cannot be found, additional constraints are added to the welfare maximization problem (Committee et al. 2020). However, it is unclear how much of the gains from trade are sacrificed this way. Furthermore, this approach inevitably leads to paradoxically rejected bids (Meeus et al. 2009). In particular, there are generators with an ask price that is less than the market price, yet they will not be dispatched. Such prices are also not *envy-free* and hence not a Walrasian equilibrium. We will not further discuss this approach here and focus on market designs as in the U.S. that implement the efficient outcome.

Several pricing rules have been suggested aiming to mimic competitive equilibrium prices on such MIP-based electricity markets (Liberopoulos and Andrianesis 2016). Locational marginal pricing (LMP) rules of many ISOs are based on IP pricing (aka. Integer Programming pricing), where the allocation problem is solved to optimality, the integer variables are fixed, and the prices are then derived from the dual variables of the demand-supply constraint of the resulting (convex) linear program (O'Neill et al. 2005). IP pricing computes anonymous and linear prices, but these prices do not constitute competitive equilibrium prices. Some generators might not maximize their individual profits and want to deviate, i.e., switch to a different dispatch at those prices, and IP prices are thus not *envy-free*. The latter is central to the definition of a competitive equilibrium and it leads to stability of the outcome. Importantly, besides a lack of stability, the generators often make a loss at the IP prices, i.e., prices are not even *individually rational*.

A central approach in the academic literature on electricity market pricing is *Convex Hull* (CH) *pricing* (Hogan and Ring 2003, Gribik et al. 2007), which aims to minimize envy and thus implement a stable outcome at the expense of budget balance. We refer to incentives to deviate from the optimal outcome as *lost opportunity costs*, LOCs. They describe the difference between each participant's profits under the welfare-maximizing allocation and the individual profit maxima each participant could obtain given the prices. With CH pricing, the market operator implements the optimal outcome and compensates participants for their GLOCs (violating budget-balance). This way, market participants would not have an incentive to deviate. Extended LMPs (ELMPs) are based on the dual variables from the demand-supply constraint in the LP relaxation of the underlying DCOPF. For some versions of this problem, they implement CH pricing. However, also with ELMP market participants can make a loss and they need to be compentsated.

ISOs use personalized side-payments to address the fact that the public market prices from IP pricing or ELMP are neither envy-free nor individually rational. This effectively differentiates the linear and anonymous market prices from the payments of the market participants, which are then non-linear and personalized. LOCs are differentiated from *make-whole payments* (Schiro et al. 2016). LOCs describe payments that are so high that no generator would want to change its dispatch and envy-freeness is achieved. Such payments may be very large if the market contains non-convexities (Eldridge et al. 2019). However, electricity markets are highly regulated markets and as such there are alternative means to enforce stability other than high lost opportunity cost payments. Actually, most ISOs only pay *make-whole payments* to ensure individual rationality of all generators and stipulate penalties that a generator has to pay if it indeed deviates from the optimal dispatch. In other words, they relax envy-freeness to only individual rationality requirements. We refer to such outcomes as having *penalty-based stability*.

However, even the make-whole payments are a significant concern (Hytowitz et al. 2020). The U.S. Federal Energy Regulatory Commission (FERC) regulates the U.S. wholesale power markets to promote just competition. In 2018, the FERC found that the practices of several ISOs were unjust and ordered them to change their pricing because prices did not accurately reflect the cost of serving load (O'Neill et al. 2019). Make-whole payments are not reflected in the public price signals, and they lead to biased investment signals. This also constitutes a problem for futures markets, where spot market prices serve as the key reference. In addition, the FERC has released several orders and notices about pricing, which argue that "the use of side-payments can undermine the market's ability to send actionable price signals."[1] Similarly, O'Neill et al. (2019) state that "the make-whole payments are not transparent to other market participants and are allocated too broadly to provide correct price incentives for market participants to make efficient entry and exit decisions as well as efficient investments in facilities and equipment." As a consequence, US ISOs continue to search for improvements in the pricing rules.

4 Dealing with Non-Convexities

In what follows, we discuss some of the current developments and research frontiers in electricity spot market design. We will discuss new ways how to deal with non-convexities and approaches to deal with the volatility of supply in future electricity markets.

4.1 Non-Convexities in Supply and Demand

We have discussed that in non-convex markets, a Walrasian competitive equilibrium does not exist in general (Starr 1969, Bikhchandani and Mamer 1997). Current pricing rules as they were discussed in the previous section violate either envy-freeness or budget-balance or both. One might wonder, whether existing pricing rules used in electricity markets can be improved upon. One way forward is to break down envy-freeness in the context of electricity markets and consider different types of LOCs (Ahunbay et al. 2023b).

Rather than minimizing LOCs as incentives to deviate, hereafter referred to as *Global* LOCs, GLOCs, one can look at so called local lost opportunity costs, LLOCs, and at the specifics that arise from a coupled market. In such a market, we need to price transmission lines appropriately. For example, prices should only differ across a pair of nodes if there is *congestion* in the network and no further power may be transmitted from the node with the higher price to that with a lower price. Violations of this condition, i.e. price differences across uncongested branches, may result in a product revenue shortfall for transmission operators which would require compensation. Convex Hull pricing does not satisfy this requirement (Schiro et al. 2016), but it is important to consider for a pricing rule.

It turns out that *local lost opportunity costs*, describe a way to consider the quality of congestion prices. LLOCs measure incentives to deviate from the optimal allocation *under fixed commitment*.[2] In other words, LLOCs assume that the commitment decisions have been made but that generators can deviate from their assigned volumes in an attempt to improve their payoff. Interestingly, minimizing LLOCs also provides for the desired congestion signals in the network, in the sense that prices reflect the marginal value of additional transmission capacity.

Unfortunately, low GLOCs, LLOCs, and MWPs are conflicting design goals and optimizing only one of these classes comes at the expense of another. If MWPs are minimized, GLOCs can be unreasonably high and congestion signals become distorted leading to high LLOCs. Focusing only on LLOCs can be equally harmful and lead to very high MWPs for some participants. Ahunbay

[1] `https://www.ferc.gov/industries-data/electric/electric-power-markets/energy-price-formation`

[2] Commitment decisions on spot markets determine whether a generator is scheduled to produce electricity during a market time unit (a binary decision variable in the allocation problem), but not the production quantity (in Megawatt hours (MWh)). Commitment decisions occur because many types of generators require a long time to turn on/off.

et al. (2023b) suggest the join pricing rule, which balances trade-offs between MWPs and LLOCs to minimize local incentives of participants to deviate from the efficient outcome as well as incentives to exit the market. Numerical experiments show that prices computed via this join require significantly less MWPs than traditional IP pricing and retain good congestion signals with very low LLOCs at the same time. The approach can be computed efficiently and it requires no fine-tuning of objective weights. In electricity markets, where global incentives to deviate can be enforced by penalizing deviations, the join strikes a desirable balance, and the experiments show that the remaining incentives to deviate are low.

4.2 Non-Convexities from Non-Linear Power Flows

The literature on electricity market pricing so far focused mainly on non-convexities arising from the cost or valuation functions of market participants. The line losses are linearized leading to a mixed-integer linear program, the DCOPF. However, an accurate representation of the transmission network would require the consideration of non-linear line losses. As a result, the allocation problem should be described as a non-linear and non-convex optimization problem referred to as Alternating Current Optimal Power Flow (ACOPF) problem (Molzahn and Hiskens 2019). This ACOPF is computationally intractable for the problem sizes that we observe in real-world electricity markets. This is the main reason why linearized network models as the DCOPF are used for market clearing and pricing. Unfortunately, the optimal solution for DCOPF is generally neither AC-optimal nor AC-feasible. This requires market operators or transmission system operators (TSOs) to adjust the dispatch after the market clearing to reach a physically feasible outcome.

This issue is magnified by the transition to renewable energy sources (Lété et al. 2022). A decreasing amount of thermal generators can supply reactive power and the consideration of reactive power on the transmission level is therefore crucial (Hemmati et al. 2013, Karmakar and Bhattacharyya 2020). Larrahondo et al. (2021) found that high integration of wind power contributes to the inaccuracies of the DCOPF, e.g., by disregarding reactive power. There have been significant efforts to obtain tighter and more accurate relaxations of the ACOPF problem in an effort to leverage recent advances in convex optimization for real-world markets, which culminated in the ARPA-E Grid Optimization Competition.[3] However, errors in the optimization models due to linearization or simplifying assumptions remain a concern in virtually all electricity markets. For example, the revision of the European Capacity Allocation and Congestion Management (CACM) regulation requires accounting for “linearisation errors” while calculating available capacities for trading (ACER 2021). A key problem of linearized models is the welfare loss arising from a poor approximation of ACOPF.

In recent years, substantial research has been devoted to finding tighter convex relaxations and approximations for the ACOPF problem (Molzahn and Hiskens 2019). This research focuses on the optimality of the solution and computational costs to obtain it. Tighter relaxations of ACOPF could lead to prices that better reflect scarcity in the physical network. However, the magnitude of these improvements is unclear. Bichler and Knörr (2023) study the impact of non-linear convex market clearing models on prices, MWPs and LLOCs. More precisely, they consider the linearized DCOPF, a second-order conic (SOC) relaxation, and a quadratic convex (QC) relaxation (Molzahn and Hiskens 2019). For each of these relaxations, they compute Integer Programming (IP) prices (O’Neill et al. 2005) and Convex Hull (CH) prices (Gribik et al. 2007, MISO 2019, Hua and Baldick

[3]See https://gocompetition.energy.gov/ for further details.

2017) as the two pricing rules primarily used in U.S. markets. The results of show that different power flow models lead to substantial differences in the allocation. This of course has an impact on both welfare and prices.

As one would expect, tighter convex relaxations require substantially less redispatch compared to the standard DCOPF approximation, and MWPs, GLOCs and LLOCs are, on average, lower for the final AC-feasible outcome. Moreover, the tighter convex relaxations lead to higher welfare of the final dispatch. However, welfare gains are not the most important argument for tighter convex relaxations of ACOPF. The numerical experiments show that the prices obtained from DCOPF are often substantially different from those of the non-linear relaxations, leading to biased scarcity signals that distort effective demand response and investment decisions. In contrast, the results of the SOC and QC relaxation, which both model reactive power and line losses more accurately, are almost identical. Prices obtained from DCOPF might be excessively high at some of the nodes, even though there is no congestion in the AC-feasible solution. In other words, the DCOPF leads to unnecessary price peaks at some of the nodes, with prices being multiples of the average price, even though there is no congestion in the physical grid at all. Similarly, we might encounter congestion in the AC-feasible solution that the DC prices of adjacent nodes do not reflect. In a world with 100% renewables, where adequate demand response is even more important, this is a decisive disadvantage of standard DCOPF solutions compared to tighter non-linear relaxations.

4.3 Outlook

While increasing levels of storage suggest that the allocation problem becomes more convex, non-convexities will remain an important concern for a long time. Startup costs of gas turbines, minimum operating levels, minimum run times and down times will also play a role in the foreseeable future. In this short paper, we could only discuss a few of the approaches to address different types of non-convexities on electricity markets. Other ideas revolve around the use of price differentiation or the use of two price vectors (O'Neill et al. 2019, Eldridge et al. 2020, O'Neill et al. 2016, Hytowitz et al. 2017, Milgrom and Watt 2021, Ahunbay et al. 2023a). Also stochastic clearing was suggested in the day-ahead market to better deal with the volatility of supply and demand in markets with large proportions of renewables (Zavala et al. 2017, Uçkun et al. 2015). Changes on electricity markets are not only relevant for market operators, but they also impact generators, industrial and retail consumers alike. How to best design power markets such that they address the challenges of volatile supply of renewables continues to be a challenging research field at the intersection of computer science, economics, and operations research.

Acknowledgments

The financial support from the German Research Foundation (Deutsche Forschungsgemeinschaft, DFG) (BI 1057/10-1) is gratefully acknowledged.

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