

Optimal Portfolio Choice with Fat Tails

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Abstract

The recent financial crisis has highlighted the importance of modeling and managing extreme risk, especially retirement savings. Virtually all standard optimal stock-bond portfolio allocation models, however, assume that risk is normally distributed (bell shape). In reality, stock market risk exhibits "fat tails." Allowing for "fat tails" can add considerable computational complexity to standard optimization framework, which is already quite complicated. This paper demonstrates how to model fat tails using a g-and-h distribution that allows for skewness and kurtosis of arbitrary degree. Unlike alternative extreme value and other coupla approaches, the g-and-h distribution has a well defined pdf, is smooth and satisfies certain regularity conditions that allow for tractable integration. It also appears to fit the data the best. We hope that our modeling approach will open the door for more realistic modeling of retirement income risk in the future. Our own SSA grant proposal for next year will extend the current research by adding a greater degree of fiscal policy institutions that materially can affect saving for retirement.

Journal of Economic Literature Classification Number: G11.

1 Introduction

The decision of consumption and portfolio allocation over the lifecycle is encountered by every investor. There is a rich literature exploring consumption and saving behavior in a lifecycle model in which individual maximizes expected discounted utility. The early works by Samuelson (1969) and Merton (1971) implied that if markets are complete, then labor income can be capitalized and traded using the financial markets securities so that idiosyncratic labor income risks can be fully insured. However, human capital is a non-tradable asset due to the moral hazard problem. An individual who has sold a claim against future labor income can always stop working (Campbell and Viceira 2002, Heaton and Lucas 1996). Therefore human capital risk is uninsurable under incomplete markets.

Many papers studied the effects of uninsurable human capital risk on consumption and portfolio choice. Bodie, Merton and Samuelson (1992) allowed endogenous labor supply and showed that labor flexibility induces the individual to assume greater risks. Deaton (1991) developed an infinite horizon model with borrowing constraints and concluded that assets act like a buffer stock protecting consumption against income uncertainty. Viceira (2001) allows risky labor income to be correlated with stock returns. Similar to Deaton (1991), Carroll (1992, 1997) also looked at buffer stock saving behavior and introduced both permanent and transitory labor income shocks in the model. Koo (1998) considered risky labor income in a continuous approach. Cocco, Gomes and Maenhout (2005) developed a quantitative and realistically calibrated model. Campbell, Cocco, Gomes and Maenhout (2000) extended the work by CGM (2005) to explore alternative retirement savings systems and fixed costs of equity market participation.

However, it is well known that the standard expected-utility model cannot closely match the empirical evidence on portfolio allocation. Several papers turn to alternative utility forms such as Epstein-Zin-Weil (EZW) utility functions (Epstein and Zin (1989), Weil (1990)), which disentangles risk aversion and elasticity of intertemporal substitution. Gomes and Michaelides (2005) show that with an EZW preference, a fixed stock market entry cost, and moderate heterogeneity in risk aversion, their model can match stock market participation rates and asset allocations. Michaelides, Gomes and Polkovnichenko (2006) solve for optimal savings in the presence of tax-deferred retirement accounts using EZW utility.

All the previous literature, however, assume a normal distribution when

they model risky assets. This conventional assumption in the finance literature has been increasingly challenged in recent years. A variety of empirical studies show that financial assets are skewed and fat-tailed (Mandelbrot (1963), Fama (1965), Hols et al. (1991)). Several other distributional forms have been explored (Mandelbrot (1963), Fama (1965), Praetz (1972), Blatberg and Gonedes (1974), Kon (1984)). Tukey (1977) introduced the g-and-h distribution, which is a transformation of the standard normal distribution. The power of this distribution is its ability to fit almost all parametric distributions up to fourth moments. Therefore, g-and-h distribution seems to be superior to the conventional normal distribution in modelling equity returns.

The main contribution of this paper is to combine both generalized utility forms (Epstein-Zin-Weil) and generalized return distribution (g-and-h distribution) and solve for the optimal consumption and portfolio choice in the presence of uninsurable labor income risk and liquidity constraint. We present several sensitivity analysis to measure the importance of the model parameters. We also match our model with empirical mean stock/bond ratio and median wealth/income ratio and show that with the introduction of skewness and elongation parameters we are able to match the empirical data fairly closely.

The paper is organized as follows. Section 2 describes the model setup. Section 3 discusses the calibration of the model parameters. Section 4 presents the simulated optimal consumption and portfolio allocation rules and several sensitivity analysis. Section 5 shows the estimation results that match the empirical evidence. Section 6 concludes.

2 Model

2.1 Preferences

The individual enters the job market at age τ , works for $K - \tau$ years, retires at time K , and lives a maximum of T periods. For simplicity, we assume exogenous labor supply so that K is exogenous, and the individual has no bequest motive. Mortality is captured by p_t , the probability that the individual is alive at time $t + 1$, conditional on being alive at time t . Households have Epstein-Zin-Weil utility functions (Epstein and Zin (1989), Weil (1990)), which allow us to disentangle risk aversion and elasticity of intertemporal substitution. Therefore at time t , the household i 's preferences are given by

the following recursion:

$$V_{i,t} = \{(1 - \beta)C_t^{1-1/\psi} + \beta p_t(E_t[V_{t+1}^{1-\gamma}])^{\frac{1-1/\psi}{1-\gamma}}\}^{\frac{1}{1-1/\psi}} \quad (1)$$

where C_t is consumption at time t , β is the discount rate, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution. This utility reduces to the standard power utility when $\gamma = 1/\psi$.

2.2 Labor Income

Let $Y_{i,t}$ denote the individual i 's labor income at time t . We assume a deterministic labor income profile with a probability of unemployment u before retirement and no labor income after retirement:

$$\begin{aligned} Y_{i,t} &= \begin{cases} \exp(f(t, Z_{i,t})) & \text{with probability } 1 - u \\ 0 & \text{with probability } u \end{cases} \text{ for } t < K \\ &= 0 & \text{for } t \geq K \end{aligned} \quad (2)$$

where $f(t, Z_t)$ is a deterministic function of age t and household's characteristics Z_t .

2.3 Financial Assets

At each time t the individual can invest a portion α_t of his liquid wealth in the following two assets:

- 1) A risk-free asset (bond) with gross real return R ;
- 2) A risky asset (stock) with return:

$$R_t - R = \mu + \eta_t \quad (3)$$

where μ is the deterministic equity premium and η_t is an independently and identically distributed (i.i.d) equity return shock which follows a g-and-h distribution.

The g-and-h distribution, introduced by Tukey (1977) is a transformation of the standard normal distribution. In particular, if Z is a random variable that follows the standard normal distribution, then X follows g-and-h distribution if it can be written as:

$$X_{g,h}(Z) = a + b \frac{e^{gZ} - 1}{g} e^{hZ^2/2} \quad (4)$$

where a,b,g,h represent location, scale, skewness, and elongation respectively. When g=0, the g-and-h distribution reduces to $X_{0,h}(Z) = a + bZe^{hZ^2/2}$, known as the h-distribution. Similarly, when h=0, the g-and-h distribution reduces to $X_{g,0}(Z) = a + b\frac{e^{gZ} - 1}{g}$, known as the g-distribution. When g=h=0, g-and-h distribution reduces to normal distribution with mean a and standard deviation b. The g-and-h distribution has been studied by Martinez and Iglewicz (1984) and Hoaglin et al. (1985) and used by Badri-nath and Chatterjee (1988, 1991), Mills (1995), Dutta and Babbel (2002, 2005), Dutta and Perry (2007) to model equity returns, interest rates and interest rates options and operational risks. The power of this distribution is its ability to fit almost all parametric distributions up to fourth moments. Therefore, g-and-h distribution seems to be superior to the conventional normal distribution in modelling equity returns that are known to be skewed and heavy-tailed.

2.4 Optimization problem

In each period t, the individual i has initial wealth $W_{i,t}$. He receives a labor income $Y_{i,t}$ if he is employed and no income if he is unemployed or retired. He spends h_t of his labor income for housing. He then has a "cash-on-hand"

$$X_{i,t} = W_{i,t} + (1 - h_t)Y_{i,t} \quad (5)$$

where he chooses consumption $C_{i,t}$ and portfolio share in equity $\alpha_{i,t}$. The recursive relation of wealth is given by:

$$W_{i,t} = R_{i,t}(X_{i,t-1} - C_{i,t-1}) \quad (6)$$

where $R_{i,t}$ is the gross return on the individual's portfolio from $t - 1$ to t :

$$R_{i,t} = \alpha_{i,t-1}R_t + (1 - \alpha_{i,t-1})R = R + \alpha_{i,t-1}(\mu + \eta_t) \quad (7)$$

We use subscript t for the portfolio return because it is not in the information set when the individual makes decision at $t - 1$.

The two control variables are $\{C_{i,t}, \alpha_{i,t}\}$. The two state variables are $\{t, X_{i,t}\}$.

Throughout the individual’s lifetime, we assume that consumption and portfolio weight in risk-free and risky assets are all nonnegative. In other words, we prohibit the individual from borrowing and short-selling:

$$0 \leq C_{i,t} \leq X_{i,t} \tag{8}$$

$$0 \leq \alpha_{i,t} \leq 1 \tag{9}$$

In summary, the individual maximizes equation (1) subject to constraints (5) to (9).

2.5 Model Solution

The optimization problem cannot be solved analytically. Therefore we use backward induction to derive the optimal decision rules for this dynamic program. In other words, the Bellman equation is solved from year T back to year τ . The continuous state space is discretized using unevenly spaced grids, with more points used at lower levels of wealth. The value function $V_{i,t}(X_{i,t})$ is interpolated between grid points of the state variable using Schumaker shape-preserving quadratic splines. The expectation over the g-and-h distributed equity shock η_t is approximated using a transformation of Gauss-Hermite quadrature numerical integration by discretizing the shock into several nodes. Maximization at each grid point is performed using the Nelder-Mead simplex optimization.

3 Calibration

3.1 Preferences

We follow the calibration by Campbell et al. (2000) and assume that the individual with a college degree enters the job market at age 22. The age of retirement is 65. The individual dies with probability one at age 100. The discount factor $\beta = 0.95$. In the benchmark case, the coefficient of relative risk aversion is set to $\gamma = 4$ and the elasticity of intertemporal substitution (IES) is set to $\psi = 0.5$. We will report results for different values of RRA and IES as well. We use the mortality tables in the National Vital Statistics Reports from National Center for Health Statistics to determine the conditional survival probabilities.

3.2 Labor Income

The deterministic labor income profile is taken from Cocco, Gomes and Maenhout (2005), who used Panel Study of Income Dynamics (PSID) to estimate labor income as a function of age, family size, marital status for different education groups. We estimate the probability of unemployment using U.S. unemployment rate data from the Bureau of Labor Statistics. The mean unemployment rate from January 1948 to July 2009 is 5.63%, while the median is 5.5%. Therefore we use a probability of unemployment $u = 0.056$. Table 1 and Figure 1 show the deterministic labor income profile.

Table 1: Deterministic Labor Income Profile

Coefficient of characteristic variables	No high school	High school	College
Constant	2.6275	2.7004	2.3831
Marital Status	0.4008	0.4437	0.4831
Family Size	-0.0176	-0.0236	-0.0228
Coefficient of age dummies			
Constant	-2.1361	-2.1700	-4.3148
Age	0.1684	0.1682	0.3194
Age ² /10	-0.0353	-0.0323	-0.0577
Age ³ /100	0.0023	0.0020	0.0033

3.3 Housing Expenditures

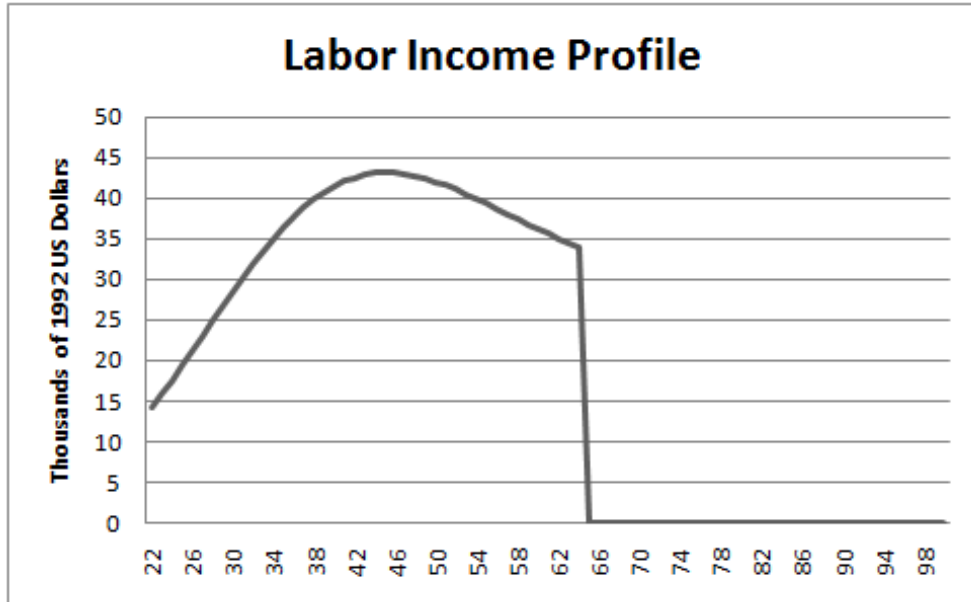
The share of labor income for housing expenditures h_t is taken from Gomes and Michaelides (2005), who compute the ratio of annual mortgage payments and rent payments to annual labor income using data from the Panel Study of Income Dynamics (PSID) from 1976 to 1993. The age effects are identified by regressing the ratio on a constant, a cubic polynomial of age and time dummies. We assume no housing expenditures after age 65. The regression coefficients are:

$$\widehat{h}_t = 0.703998 - 0.0352276 \times age + 0.0007205 \times age^2 - 0.0000049 \times age^3 \quad (10)$$

3.4 Financial Assets

We use a risk-free rate of 2% and a risky equity premium of μ equal to 4%. The standard deviation of risky asset return shock σ_η is set to be its historical

Figure 1: Deterministic labor income profile



value of 0.157. In the benchmark case, we set $g = h = 0$ to show results for normal distribution. We consider different values of g and h later on in the paper to account for skewness and kurtosis in the observed data on US equity. Badrinath and Chatterjee (1988) estimated g and h parameters for daily and monthly returns on the CRSP equal-weighted and value-weighted market portfolios. The median g ranges from -0.034 to 0.042, while the median h ranges from 0.091 to 0.146. Therefore we report results for both positive and negative g values and positive h values.

Table 2 summarizes the parameter values in the benchmark case.

Table 2: parameter values in the benchmark case

Parameter	Value
Beginning of career (τ)	22
Retirement age (K)	65
Maximum age (T)	100
Discount rate (β)	0.95
Risk aversion (γ)	4
Elasticity of intertemporal substitution (ψ)	0.5
Riskfree rate (R)	1.02
Risk premium (μ)	0.04
Prob. of unemployment (u)	0.056
Skewness parameter (g)	0
Elongation parameter (h)	0

4 Results

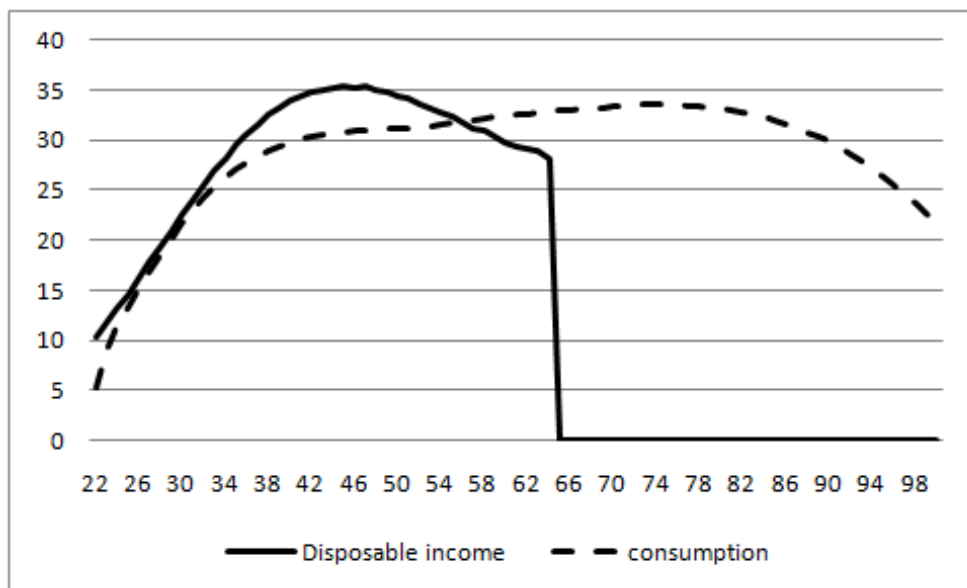
After solving for the optimal policy functions, we simulate the model over 10,000 households and compute the average household behavior. We first report the average outcomes using the parameters in the benchmark case. Then we discuss the importance of model parameters for these results.

4.1 Benchmark case

Figure 2 plots the simulated disposable income (after housing expenditures) and consumption profiles. The results are generally similar to the previous papers on lifecycle models (Coco, Gomes and Maenhout (CGM) (2005), Gomes and Michaelides (2005)). Consumption displays hump-shape over the life time as a result of precautionary saving and impatience. Households are liquidity constrained during the first decade of their working lives. As disposable income increases, households start to accumulate wealth and consumption becomes smooth during middle age. As households get older after retirement, the effective impatience increases due to increasing mortality risk. Therefore consumption decreases gradually toward the end of life.

Figure 3 presents the simulated wealth profile. Wealth accumulation remains very low during the early years and increases steeply as households receive more income. Households start to decumulate wealth after retirement as they receive no social security. Households in general save more than the results in CGM probably because the introduction of the probability of losing

Figure 2: Simulated income and consumption profiles



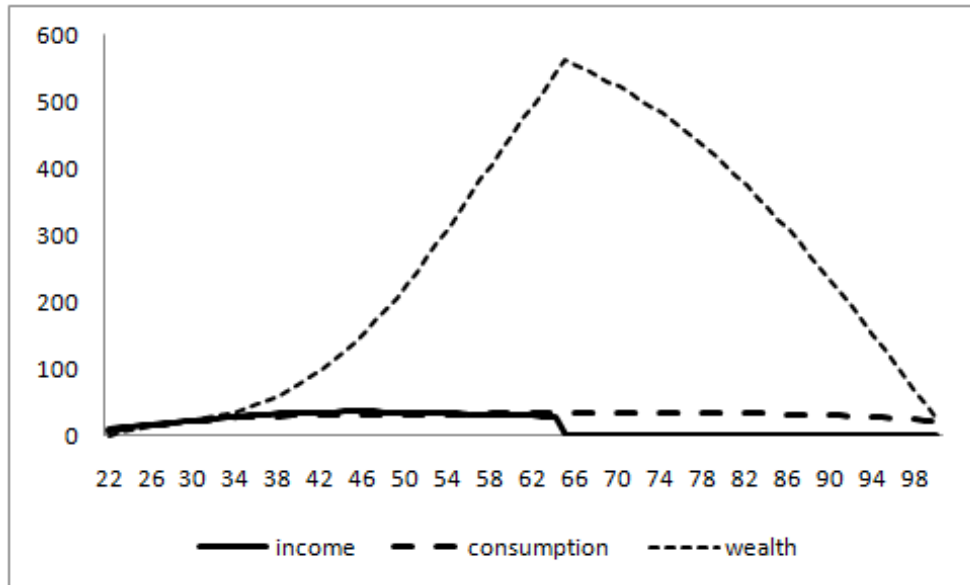
job induces more precautionary saving.

Figure 4 describes the simulated stock allocation. Households invest almost fully in stock during their young ages and early part of their middle life. Starting from around age 50, equity share gradually falls as households approach their retirement. After retirement, stock allocation becomes quite stable at around 53% of their financial assets in stock.

4.2 Sensitivity Analysis

In this section we solve for the optimal consumption and portfolio rules for different values of risk aversion, elasticity of intertemporal substitution, skewness and elongation and analyze the sensitivity of our results to these parameters.

Figure 3: Simulated income, wealth, and consumption profiles



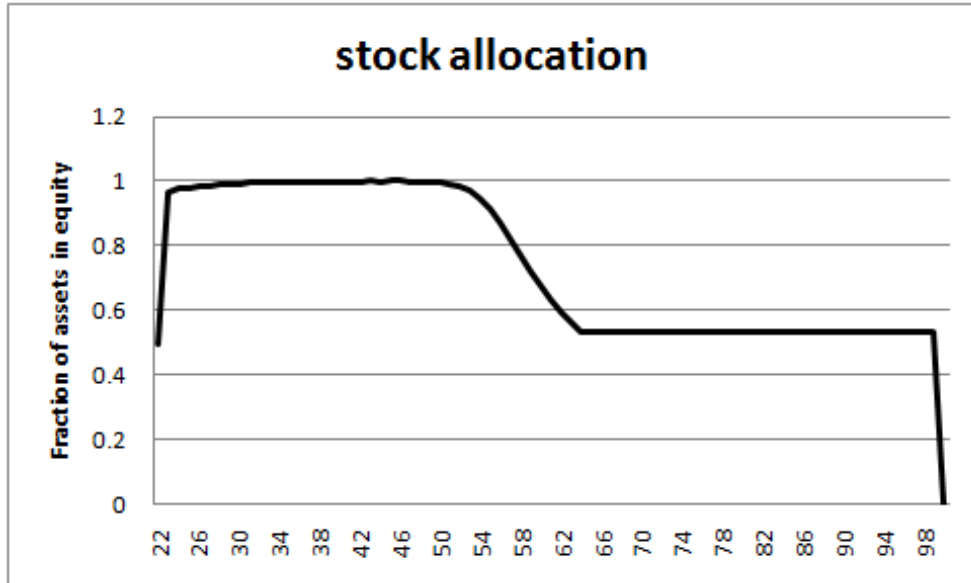
4.2.1 Risk Aversion

Figure 5 shows the simulated stock allocation for coefficient of relative risk aversion of 3 and 5, compared with the benchmark case 4. In early years, portfolio shares in equity are still very close to 1. However, the stock allocation for more risk-averse households ($\gamma = 5$) lowers substantially starting from middle age and remains at around 42% after retirement. This is intuitive because more risk-averse households invest more cautiously in the stock market. In contrast, less risk-averse households ($\gamma = 3$) invest substantially more in equity. The portfolio share in equity remains as high as 70% even after retirement, compared to a 53% in our benchmark case.

4.2.2 Elasticity of Intertemporal Substitution

A large literature has attempted to estimate the elasticity of intertemporal substitution based on both macroeconomic and microeconomic evidence. This literature, however, has produced mixed results ranging from 0.1 to 2 (Hall (1988), Mulligan (2002), Dynan (1993), Blundell, Browning and Meghir's

Figure 4: Simulated portfolio share in equity

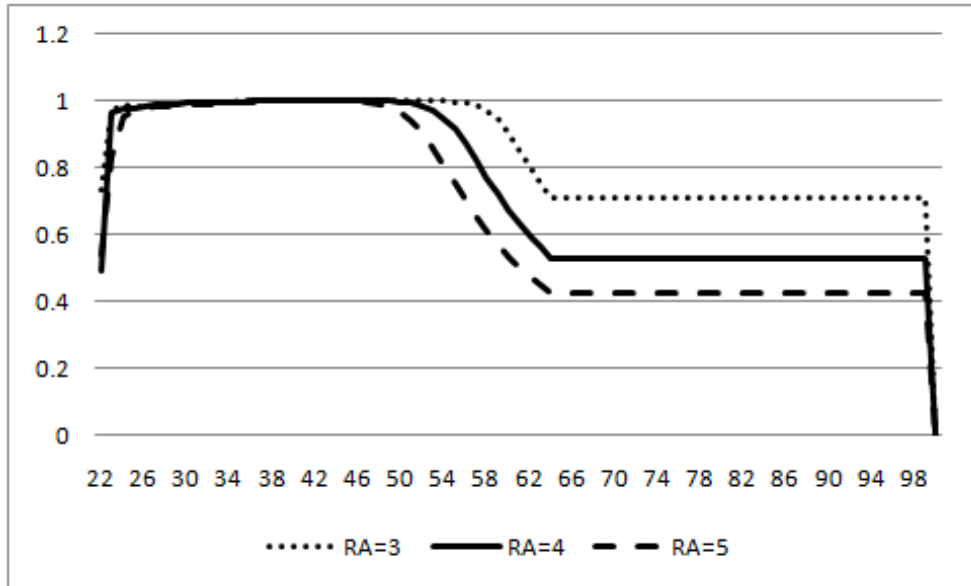


(1994), Gruber (2006)). Therefore we consider a fairly wide range of IES from a low $\psi = 0.2$ to a high $\psi = 1.5$. Figure 6 shows the simulated wealth accumulation for different values of elasticity of intertemporal substitution (IES). A high IES makes the household more willing to substitute intertemporally. In other words, he cares less about consumption smoothing, and therefore saves less for retirement. Similarly, a low IES produces higher wealth accumulation as the household is more concerned with consumption smoothing.

4.2.3 Skewness

An interesting point of this paper is to introduce a non-normal equity shock distribution, namely g-and-h distribution, in the lifecycle model. Thus it is important to analyze how g-and-h distribution affects the optimal consumption and portfolio rules. We first examine the sensitivity of our results to different g values, which govern the skewness of the distribution. Figure 7 presents the simulated stock allocation for both positive ($g=0.1$) and negative ($g=-0.1$) g values. When $g=0.1$, i.e. the distribution is skewed to the

Figure 5: Simulated stock allocation for different coefficients of relative risk aversion

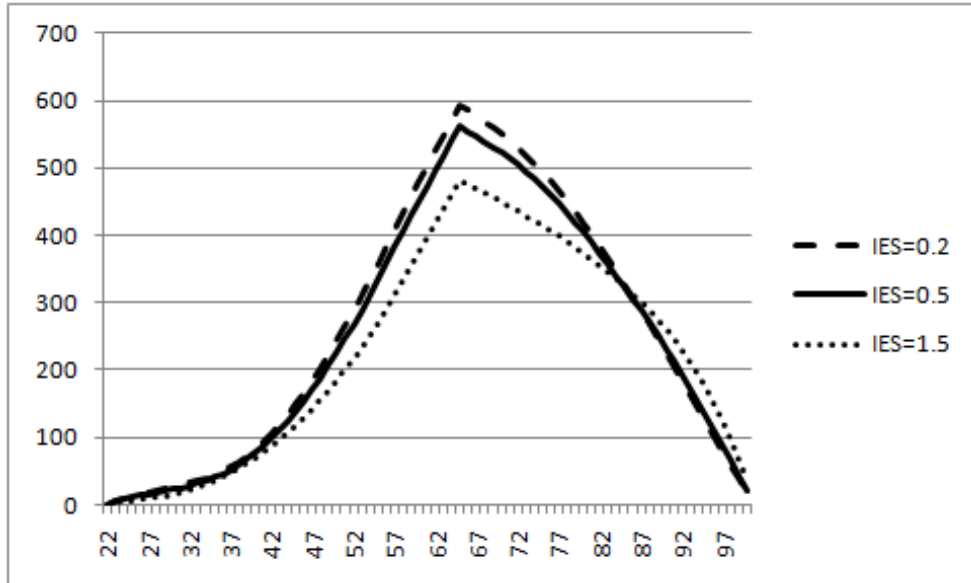


right, households' portfolio share in equity increases compared to what we obtained in the benchmark case. This makes perfect sense because households are willing to invest more in stocks if high stock returns are more likely to realize. When $g=-0.1$, i.e. the equity shock distribution is skewed to the left, households' portfolio share in equity still approaches 1 in early years but is lower later on in life.

4.2.4 Elongation

Now we examine the sensitivity of our results to different h values, which govern the elongation of the equity shock distribution. A higher h value means the distribution is more fat-tailed. In other words, extreme equity returns are more likely to happen. We consider a moderate h value of 0.1, which is close to the estimates obtained by Badrinath and Chatterjee (1988) on market index, as well as a high h value of 0.5. Figure 8 shows the simulated stock allocation for these h values. When $h = 0.1$, portfolio share in equity falls both in the first few years and starting from the middle age. The

Figure 6: Simulated wealth accumulation for different elasticities of intertemporal substitution

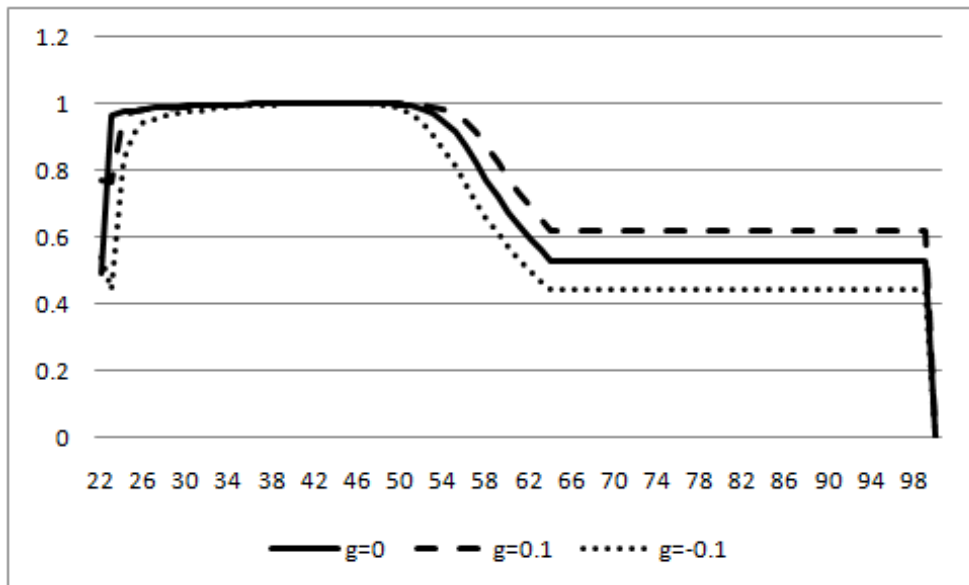


equity share after retirement remains at 41% compared to 53% when there is no elongation. During the early part of the middle age, equity share still approaches 1, but over the lifetime, the time period during which households invest almost fully in stock is substantially shorter. Stock allocation falls even more dramatically for $h = 0.5$. In this case, households never invest fully in stock and the highest equity share over the life time is around 75%. The stock allocation profile displays a nice "hump-shape" which fairly resembles what we observe in the data. (Amerkis and Zeldes (2000), Poterba and Samwick (2001)) Therefore it is shown that elongation has a substantial effect on optimal policy rules in our lifecycle model.

5 Estimation

After we derive the optimal consumption and portfolio rules, we can estimate the parameter values that match the empirical evidence. Specifically, we first hold the coefficient of relative risk aversion (γ) and discount rate (β) constant,

Figure 7: Simulated stock allocation for different g values

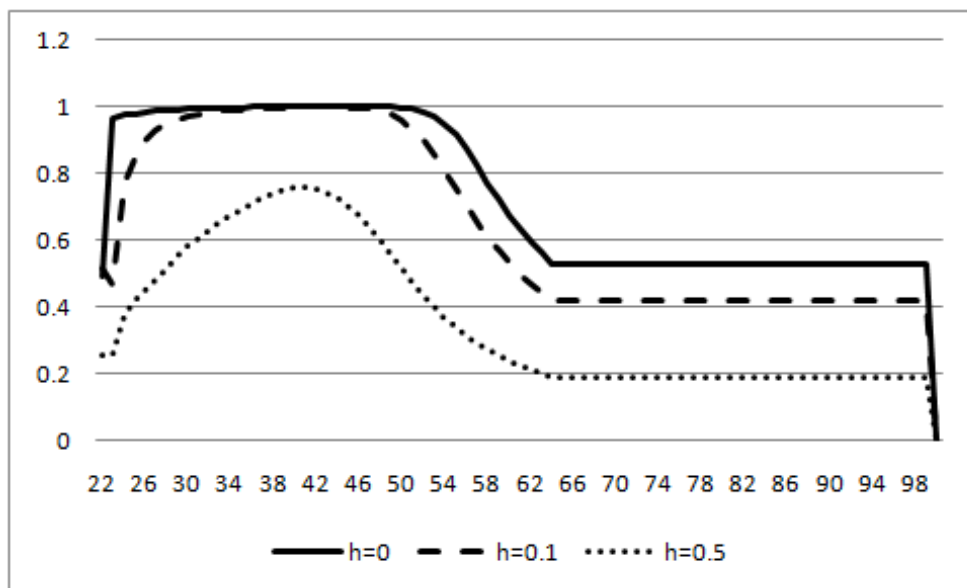


and given these values, we pick the elasticity of intertemporal substitution (ψ) and elongation (h) that match the empirical mean stock/bond ratio and median wealth/income ratio for a given skewness (g). We can then, for each level of γ , draw a line that matches the two empirical data on a skewness-kurtosis (g - h) space. We then ask if there exist any combinations of g and h that we cannot reject empirically.

Bucks, Kennickell Mach, Moore (2009) report an average stock holdings as share of financial assets from the 2007 Survey of Consumer Finances (SCF) of 53.3%. This number converts to a stock/bond ratio of 1.14. Therefore we use a reasonable mean stock/bond ratio of 1.1 as a target. The stock/bond ratio is calculated as the simulated mean of the ratio of total amount of assets held in equity to total amount of assets held in riskfree bond over the lifetime.

A variety of studies have estimated wealth/income ratio from macroeconomic and microeconomic data. Nishiyama and Smetters use the capital-GDP ratio of 2.74 from 2000 data from the Bureau of Economic Analysis (BEA). Gomes and Michaelides (2005)'s estimates of the median wealth-to-labor-income ratio from the 2001 SCF for different age groups range from

Figure 8: Simulated stock allocation for different h values



0.287 for the young group to 2.17 for the middle age group to 7.93 for the old group. We therefore use a reasonable median wealth/income ratio of 2.5 as a target. To calculate the wealth/income ratio, we first obtain the simulated median wealth and income.¹ Then we weight the simulated median wealth and income for each age by population and get the population weighted median wealth/income ratio.

Figure 9 plots the combinations of g and h values for different levels of risk aversion that simultaneously match the stock/bond ratio and wealth/income ratio. Each line represents a given level of risk aversion. Along each line, the corresponding g and h value produce the same mean stock/bond ratio and median wealth/income ratio that match the empirical evidence. The lines are upward-sloping, meaning that a higher g requires a higher h . This is intuitive because as stock returns become more skewed to the right, they must be more

¹Wealth is defined as household's liquid wealth at the beginning of the period. Income is defined as household's labor income plus interest income. We exclude home value from wealth, which lowers the wealth/income ratio. But notice that we do not model social security, which increases the wealth/income ratio after retirement. We argue that the two effects potentially cancel with each other overall.

fat-tailed to prevent investors from over-investing in stocks. It is also shown in Figure 9 that as risk aversion goes up, the line shifts down. This is also intuitive as more risk averse investors are less willing to hold stocks. As a result a lower h value is required to maintain the same stock/bond ratio. We can conclude in general that a fairly high level of elongation is required in our model to match the empirical evidence. To see this, for our benchmark value of risk aversion $\gamma = 4$ as an example, the required h value ranges from 0.2 for $g=-0.1$ to 0.38 for $g=0.1$. Even for a higher level of risk aversion $\gamma = 5$ and highly negative skewness $g = -0.1$, an h value of 0.1 is still required. This explains why traditional models assuming normal distribution fail to produce realistic results. We conjecture from the figure that in order for normal distributed stock returns ($g=h=0$) to work, risk aversion needs to be substantially higher, which is often called the "equity premium puzzle".

Figure 9: Estimation results on a g - h space

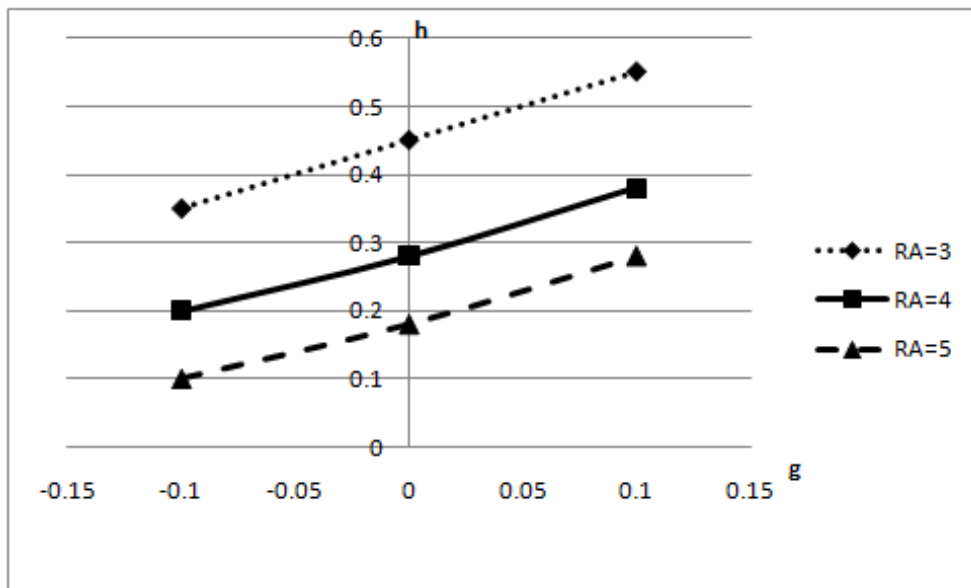


Table 2 presents our estimates of h and IES for different levels of risk aversion and skewness. It is interesting to notice that for all levels of risk aversion and skewness, IES is above one, which is consistent with many empirical estimates. Although macroeconomic estimates of the IES such as Hall

(1988) are well below one, Bansal and Yaron (2004) and Barro (2005) point out that those estimates tend to be biased sharply toward zero. Therefore we believe that any IES between 1 and 2 is reasonable. For each level of risk aversion, IES increases with skewness parameter g . This is intuitive because a more positive skewness induce the investor to save more and invest more in stocks. Therefore IES has to be higher to increase investor's willingness to substitute consumption intertemporally and push down savings. For each level of skewness, IES decreases with risk aversion.

Table 2: Estimates of h and IES
Risk aversion=3

	$g=-0.1$	$g=0$	$g=0.1$
h	0.35	0.45	0.55
IES	1.3	1.5	2.2

	$g=-0.1$	$g=0$	$g=0.1$
h	0.2	0.28	0.38
IES	1.2	1.35	1.7

	$g=-0.1$	$g=0$	$g=0.1$
h	0.1	0.18	0.28
IES	1.15	1.3	1.5

6 Conclusion

In this paper we develop a lifecycle model to solve numerically for the optimal consumption and portfolio rules of households who face uninsurable labor income uncertainty, mortality risk, and borrowing constraints. We incorporate generalized utility forms (Epstein-Zin-Weil utility) and generalized stock return shock distribution (g -and- h distribution). The model generates plausible wealth accumulation and portfolio choice. We also perform several sensitivity analysis to see the importance of our model parameters to the results. The flexibility of our model enables us to match the empirical evidence closely.

References

- [1] Ameriks, John and Zeldes, Stephen P., (2000), "How Do Household Portfolio Shares Vary With Age?". TIAA-CREF Working Paper
- [2] Badrinath, S. G., and Chatterjee, S., (1988), "On Measuring Skewness and Elongation in Common Stock Return Distributions: The Case of the Market Index," *Journal of Business*, 61(4), 451-472
- [3] Badrinath, S. G., and Chatterjee, S., (1991), "A Data-Analytic Look at Skewness and Elongation in Common-Stock-Return Distributions," *Journal of Business and Economic Statistics*, 9(9), 223-233.
- [4] Bansal, Ravi, and Amir Yaron. (2004). "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance*, 59(4): 1481–1509.
- [5] Barro, Robert J. (2005). "Rare Events and the Equity Premium." National Bureau of Economic Research Working Paper 11310.
- [6] Blattberg, R., and Gonedes, N. (1974). A comparison of stable and student distributions as statistical models for stock prices. *Journal of Business* 47 (April): 244-80.
- [7] Blundell, Richard, Martin Browning and Costas Meghir (1994). "Consumer Demand and the Life-Cycle Allocation of Household Expenditures," *Review of Economic Studies*, 61, 57-80.
- [8] Bodie, Z., R. C. Merton, and W. Samuelson. (1991). "Labor supply flexibility and portfolio choice in a life cycle model". *Journal of Economic Dynamics and Control*, 16,427-449.
- [9] Bucks, Brian K; Kennickell, Arthur B; Mach, Traci L; Moore, Kevin B.(2009). "Changes in U.S. Family Finances from 2004 to 2007: Evidence from the Survey of Consumer Finances." *Federal Reserve Bulletin*, vol. 95, 1st Quarter, pp. A1-55.
- [10] Campbell, J. Y., Joao F. Cocco, Francisco J. Gomes, and Pascal J. Maenhout. (2000) "Investing retirement wealth: a life cycle model". In Feldstein, M. and Campbell, J. Y. (eds), *Risk Aspects of Investment-Based Social Security Reform*. Chicago: NBER, University of Chicago Press.

- [11] Campbell, John Y. and Luis M. Viceira, (2002). "Strategic Asset Allocation, Portfolio Choice for Long-Term Investors". New York: Oxford University Press.
- [12] Carroll, C. (1992) "Buffer stock saving: some macroeconomic evidence". *Brookings Papers on Economic Activity*, 2:61-135.
- [13] Carroll, C. (1997) "Buffer stock saving and the life cycle/permanent income hypothesis". *Quarterly Journal of Economics*, 112(1):1-56.
- [14] Cocco, Joao F., Francisco J. Gomes, and Pascal J. Maenhout (2005), "Consumption and Portfolio Choice over the Life Cycle." *The Review of Financial Studies*, Vol. 18, No. 2
- [15] Deaton, A. (1991) "Saving and liquidity constraints". *Econometrica*, 59(5): 1221-1248.
- [16] Dutta, K., and Babbel, D., (2002), "On Measuring Skewness and Kurtosis in Short Rate Distributions: The Case of the US Dollar London Inter Bank Offer Rates," Wharton Financial Institutions Center Working Paper.
- [17] Dutta, K., and Babbel, D., (2005), "Extracting Probabilistic Information from the Prices of Interest Rate Options: Tests of Distributional Assumptions," *Journal of Business*, 78 (3), 841-870.
- [18] Dutta, K., and Perry, J., (2007), "A Tale of Tails: An Empirical Analysis of Loss Distribution Models for Estimating Operational Risk Capital," FRB of Boston Working Paper.
- [19] Dynan, Karen (1993). "How Prudent Are Consumers?," *Journal of Political Economy*, 101, 1104-1113.
- [20] Epstein, Larry G., and Stanley E. Zin. (1989). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica*, 57(4): 937-69.
- [21] Fama, E. F. (1965). The behavior of stock prices. *Journal of Business* 38 (January): 34- 105.

- [22] Gomes, Francisco J., and Alexander Michaelides, 2005. "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance*, American Finance Association, vol. 60(2): 869-904.
- [23] Gruber, Jonathan. 2006. "A Tax-Based Estimate of the Elasticity of Intertemporal Substitution." National Bureau of Economic Research Working Paper 11945.
- [24] Hall, Robert (1988). "Intertemporal Substitution in Consumption," *Journal of Political Economy*, 96, 339-57.
- [25] Heaton, J. and Lucas, D., (1996). "Evaluating the effects of incomplete markets on risk sharing and asset pricing". *Journal of Political Economy*, 103(2): 360-399.
- [26] Hoaglin, D.C., Mosteller, F., and Tukey, J.W., (1985), "Exploring Data Tables, Trends, and Shapes," Wiley. New York.
- [27] Kon, S. J. (1984). Models of stock returns a comparison. *Journal of Finance* 36 (March): 149-65.
- [28] Koo, H. K. (1998) "Consumption and portfolio selection with labor income: a continuous time approach". *Mathematical Finance*, 8, 49-65.
- [29] Mandelbrot, B. B. (1963). The variation of certain speculative prices. *Journal of Business* 36 (October): 394-419.
- [30] Martinez, J., and Iglewicz, B., (1984), "Some Properties of the Tukey g-and-h Family of Distributions," *Communications in Statistics-Theory and Methods*, 13(3), 353-369.
- [31] Merton, R. C. (1971). "Optimum consumption and portfolio rules in a continuous-time model". *Journal of Economic Theory*, 3, 373-413.
- [32] Michaelides, Alex, Francisco Gomes and Valery Polkovnichenko, (2006). "Wealth Accumulation and Portfolio Choice with Taxable and Tax-Deferred Accounts," *Computing in Economics and Finance* 2006 23, Society for Computational Economics.
- [33] Mills, T.C., (1995), "Modeling Skewness and Kurtosis in the London Stock Exchange FT-SE Index Return Distributions," *The Statistician* 44 (3), 323-332.

- [34] Mulligan, Casey (2002). "Capital, Interest, and Aggregate Intertemporal Substitution," NBER Working Paper #9373.
- [35] Nishiyama Shinichi and Kent Smetters, (2007). "Does Social Security Privatization Produce Efficiency Gains?," *The Quarterly Journal of Economics*, MIT Press, vol. 122(4), pages 1677-1719, November.
- [36] Poterba, James M. and Andrew Samwick, (2001). "Household Portfolio Allocation over the Life Cycle," NBER Chapters, in: *Aging Issues in the United States and Japan*, pages 65-104 National Bureau of Economic Research, Inc.
- [37] Samuelson, P. A. (1969). "Lifetime portfolio selection by dynamic stochastic programming". *Review of Economics and Statistics*, 51, 239-246.
- [38] Tukey, J. W., (1977), "Modern Techniques in Data Analysis," NSF-sponsored regional research conference at Southeastern Massachusetts University, North Dartmouth, MA.
- [39] Weil, Philippe. (1990). "Nonexpected Utility in Macroeconomics." *Quarterly Journal of Economics*, 105(1): 29-42.