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FINITE LIFETIMES AND GROWTH

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ABSTRACT

The recent literature on endogenous growth models has emphasized the effect that the rate of return has on the capital accumulation decisions and, consequently, on the growth rate of the economy. In most cases the basic model is a variant of the representative agent growth model. The key feature of the infinitely lived agent model is that "substitution effects" dominate, that is, in order to induce individuals to accumulate capital all that is required is a sufficiently high rate of return.

In this paper we explore the long run behavior in a model with finite lifetimes-a version of Diamond's overlapping generations model. Because individuals do not live forever (although the economy does) their level of income as well as the rate of return determine the rate of accumulation. Specifically, we show that for all one sector convex technologies the equilibrium limiting growth rate of the economy is zero. In this setting capital income taxation can have paradoxical effects; it is shown that if the proceeds are used to redistribute income to the young it is possible to have a positive long run growth rate. The effect of the tax rate on the growth rate is not monotonic: for small tax rates the effect is positive, while for sufficiently high rates it is negative. Additionally, income redistribution to the young will normally have positive effects upon the long run growth rate.

We then study a two sector growth model and show conditions under which the laissez faire equilibrium displays long run growth. Intuitively, the key property is that the existence of a sector producing investment goods makes it possible that, along a growth path, the relative price of capital decreases sufficiently fast and allows the young to purchase ever increasing quantities of capital.

Finally, we show that in an overlapping generations setting, a one sector model can generate growth if the technology displays a nonconvexity, as this is similar to the effect of a decrease in the price of capital: it prevents the ratio of the value of capital and the level of wealth of the young from exceeding one.

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1. Introduction

In recent years considerable attention has been given to the analysis of conditions on preferences and technologies that generate equilibrium growth. The natural first step is to study the behavior of the economy under the assumption of perfect markets and no government intervention. This, however, is only of limited interest and it can be thought of as a logical prerequisite to the analysis of the effects of market imperfections and government policies. It seems that a set of more interesting questions are those related to the identification and welfare properties of the type of policies that can affect the growth rate.

Initial work in this area was done using versions of the infinitely lived representative agent model. In this setting it is possible to provide some intuition about the crucial forces behind the growth process. Consider a simple intertemporal consumption/saving problem. Specifically, suppose that a household has to decide its level of future consumption relative to the present. If the interest rate is sufficiently high (today's price of future consumption is low), then it will choose to save today and to have a level of future consumption that exceeds that of current consumption. Of course, if this happens period after period the household's profile of consumption over time is increasing, that is, individual consumption is growing. If we add consumption across many infinitely lived households we obtain that *aggregate* consumption is growing. Then, at least from the point of view of a household, the *engine* of growth is a high interest rate. Of course, this perspective is not very useful as we need to understand the conditions which lead to a high interest rate. The economy's equilibrium interest rate is not independent of the technology and, to a first approximation, it must equal the marginal product of capital. Then it is possible to describe the true engine of growth as a sufficiently high marginal product of capital.

The differences among recent models of endogenous growth can be traced to the specific details that prevent the marginal product of capital from decreasing to a level that is so low that the resulting interest rate does not induce families to save in excess of the amount of depreciation. Romer (1986) assumes that there are increasing returns in the aggregate technology. If the degree of increasing returns is sufficiently high, the marginal product of capital not only does not decrease (as in the standard Solowian model of growth) but it increases over time. Lucas (1988) emphasizes a nonconvexity in the production of human capital that is sufficient to generate growth. Stokey (1988) concentrates on the study of the potential growth effects of learning by doing (another nonconvexity). The convex case has been analyzed by Rebelo (1987) and Jones and Manuelli (1990).[1]

What are the policy implications of these models? In all of them taxation of capital (including human capital) income reduces the growth rate. In some cases, the competitive laissez faire equilibrium is not optimal (when there are nonconvexities), and it is necessary for governments to subsidize investment. In the case of convex technologies the standard welfare results are applicable: a competitive equilibrium is Pareto optimal, and every Pareto optimal allocation can be supported as a competitive equilbrium.

Useful models must, of necessity, be quite abstract. There are many interesting details that cannot be incorporated because they render the model intractable. It is then essential to explore how robust the growth results are to the specifications studied. In particular, one that is important from the point of view of economic policy is the idea of a high interest rate (or marginal product of capital) as the *engine* of growth. Is it then reasonable, from the point of view of a policy maker interested in understanding the effects of alternative policies on the growth rate, to restrict attention to the impact of any given policy on the rate of return on saving (or accumulation of human capital), ignoring, for example, the possible redistributive or income effects? For the class of infinitely lived representative agent models the answer to this question is yes; to affect the growth rate it suffices to influence the (relevant) rate of return: substitution effects dominate long run behavior.

Is this a robust implication? It is possible to argue that the class of infinitely lived models is too narrow to accomodate some forms of heterogeneity. In particular, it is impossible, in the standard setting, to understand the effect of intergenerational income redistribution on growth. Is there room for a growth effect of redistribution? Can intergenerational issues bring a new perspective to our understanding of the link between policy and growth? To address these questions the natural class of alternative models to explore includes economies in which individuals are alive for finitely many periods but the economy lasts forever. Why is it that in the context of overlapping generations a high interest rate need not guarantee growth? At the individual level a high interest rate ensures that *individual* consumption grows over time. Agggregate consumption, however, is the sum of consumption of individuals of different age. It is then possible that this aggregate remains bounded even though the interest rate is arbitrarily high.

In fact, in this paper we show that for the class of one sector growth models with convex technologies it is not possible to find an equilibrium with positive asymptotic growth. The basic intuition is that the young generation does not have sufficient income to purchase a sufficiently large stock of capital from the old generation. Therefore, the limiting growth rate of the economy is zero for all interest rates. The limiting factor is not an unwillingness to save (the high interest rate guarantees this) but, instead, that total saving cannot increase as fast as the capital stock. This is not an issue in the infinitely lived model as it is the same family that buys and sells capital in very period.

In the face of this general no growth result for the overlapping generations model, it is of interest to study the changes to the basic model and the class of policies that can restore the existence of equilibrium growth. The possibilities that we explore fall into three broad categories: first, modify the basic structure of the model so that finitely lived individuals will behave, in fact, as infinitely lived agents. More precisely, one interpretation of the infinite horizon model due to Barro (1974) is that it is an overlapping generations model in which parents care about the utility of their children. This reinterpretation restores the growth result in the special case of fully transferable capital. However, taking the notion of finite lifetimes seriously, forces us to explore the differences between the way society as a whole and families individually accumulate human capital.

The second possible remedy is to transfer income to the young. In particular, this possibility suggests the existence of a role for some redistributive policy as an engine of growth. Consider, for example, a program of transfers to the young financed by income taxes. It is not clear then what the effect of higher taxes will be. There are two opposite forces: a higher tax rate reduces the rate of return and, through the standard substitution effect, tends to reduce saving and capital accumulation. There is a second effect as the proceeds of the income tax can be given to the potential savers. This income effect increases the demand for saving (capital) and must have a positive impact on the growth rate. In fact it is possible to characterize the properties of tax and spending policies that can turn an economy which has a no growth laissez faire equilibrium into an economy that has a competitive (interventionist) equilibrium that displays long run growth. This adds a different dimension to the known interactions between government policies and growth that arises because the relevant planning horizon for one individual does not coincide with the economy's planning horizon.

A third possible deviation from the standard one sector model that has, in principle, the potential for generating equilibrium growth is to modify it so that the ever increasing amounts of capital that are required along any growth path become affordable. One case is in which the price of capital is allowed to decrease. We do this by studying a two sector model. We show that under some conditions there is positive asymptotic growth in equilibrium, but that the condition cannot be summarized in terms of the interest rate. In terms of its policy implications, it is necessary to consider the rate at which the price of capital decreases along the equilibrium path and not simply the level of this price.

Another strategy to make the capital stock affordable is to guarantee that, along any growth path, the real wage increases at a sufficiently high rate. One model that delivers this result is a one sector growth model with a nonconvex technology, i.e., increasing returns to scale have the potential to generate growth. We show that this is indeed the case in a model where the economy wide capital stock feeds back into individual production functions. It is then not surprising that most of the research on endogenous growth using overlapping generations models has adopted some form of nonconvexity as the basic *engine* of growth.[2]

To summarize, the study of endogenous growth in a setting in which individuals have finite lifetimes although the economy is open ended adds to our understanding of the growth process in at least two fundamental senses. First, it shows that a high interest rate is not sufficient to induce growth. Second, it establishes that redistributive policies financed by income taxes *can* increase the long run growth rate. Although much more work is needed in this area, these first results suggest a potentially large payoff in terms of our basic understanding of the mechanics of growth and the effects of government policies.

The paper is organized as follows. Section 2 presents the basic one sector infinite horizon model and discusses the effects of taxation. Section 3 analyzes an overlapping generations version of the model and shows the impossibility of equilibrium growth. Section 4 demonstrates that redistributive policies financed by income taxes can generate equilibrium growth. In section 5 we study a two sector growth model and we establish sufficient conditions for it to display growth in equilibrium. In Section 6 we introduce a nonconvexity in the one sector model of section 3 and prove that if the degree of aggregate nonconvexity is sufficiently high growth is possible. Section 7 offers some concluding comments.

2. Infinite Horizons, Income Taxation and Redistribution

In this section we present a standard one sector infinitely lived agent model of economic growth. The economy we study is a special case of the class of models analyzed in Jones and Manuelli (1990). It is in the tradition of convex models of long run growth of Rebelo (1987) and King and Rebelo (1990) and it is a good vehicle to illustrate the interplay between conditions on technologies and preferences that result in asymptotic equilibrium growth. Moreover, it is relatively simple to investigate the effect of distortionary taxes and income redistribution on the aggregate behavior of the economy.

Each household has preferences defined over infinite sequences of consumption given by,

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \quad , \quad 0 < \beta < 1, \tag{1}$$

where u is increasing and strictly concave. In the recursive version, we consider households selling labor, human capital and physical capital in spot markets, purchasing consumption and investment goods and, if necessary, bonds to effect intertemporal transfers of wealth.

To simplify, we explicitly use different prices for human capital and raw labor although we can combine them to define compensation per hour. In this view compensation depends, in a nonlinear way, on the level of human capital and number of hours worked by each individual. Let q_{ht} and q_{kt} be the rental prices of human and physical capital respectively, and w_t the real wage, all measured in units of consumption at time t. Let r_t be the real interest rate and τ_h, τ_k the tax rates on labor/human capital income and capital income. The sequential budget constraint is given by

$$c_t + x_{ht} + x_{kt} + \tau_h q_{ht} h_t + \tau_h w_t n_t + \tau_k q_{kt} k_t + b_{t+1} \leq q_{ht} h_t + w_t n_t + q_{kt} k_t + (1 + (1 - \tau_k) r_t) b_t + T_t$$

where c_t is consumption at time t, x_{ht} and x_{kt} are investment in human and physical capital, n_t is the number of hours worked and b_{t+1} is the stock of one period bonds purchased at time t. T_t is a transfer from the government that the household considers as lump sum.

Each household faces a set of constraints that govern capital accumulation. These are given by,

$$(2) \qquad \begin{cases} k_{t+1} &= (1-\delta) k_t + x_{kt} \\ h_{t+1} &= (1-\delta) h_t + x_{ht} \end{cases} .$$

Finally, firms use physical and human capital and labor to produce consumption and investment goods. Given that we assume a homogeneous of degree one technology, it is with no loss of generality that we describe the production side in terms of a single representative firm.

Profits are given by,

$$(3) \qquad c_t + x_{ht} + x_{kt} - w_t n_t - q_{ht} h_t - q_{kt} k_t,$$

and they are maximized subject to,

$$(4) \qquad c_t + x_{ht} + x_{kt} \leq F(k_t, h_t, n_t),$$

where F is a concave, increasing in each argument, homogeneous of degree one and differentiable production function.

An equilibrium is a sequence of prices $(q_{ht}, q_{kt}, w_t, r_t)$ and an allocation $(c_t, x_{ht}, x_{kt}, k_t, h_t)$ such that:

(i) Given the sequence of prices, the allocation solves the individual household maximization problem.

(ii) Given the sequence of prices, the allocation also solves the representative firm profit maximization problem.

(iii) The allocation is feasible.

(iv) $T_t = \tau_h (q_{ht} h_t + w_t n_t) + \tau_k q_{kt} k_t$

All conditions are fairly standard. The last one simply says that the government runs a balanced budget in the sense that transfers equal tax revenues in each period.[3] In the definition of equilibrium we have not explicitly allowed for heterogeneity across households to keep the notation simple. It is straightforward to extend the definition to account for heterogeneity in preferences, endowments and transfers.

Let us first consider whether there is equilibrium growth in the long run. From the first order condition of the households utility maximization problem it follows that consumption satisfies,

$$u'(c_t) = u'(c_{t+1})\beta(1+(1-\tau_k)r_{t+1}). \tag{5}$$

To guarantee $c_{t+1} > c_t$ it is necessary and sufficient to have $\beta(1+(1-\tau_k)r_{t+1}) > 1$. If this discounted after tax interest rate remains uniformly bounded away from one, e.g., $\beta(1+(1-\tau_k)r_t) \geq 1+a > 1$, then consumption grows without bound and it converges to infinity.

The "key" feature of this economy is then if the technology is productive enough to guarantee a sufficiently high rate of return (high r_t) and the tax rates are not so high as to make the relevant after tax rate of return unattractive.

A standard no arbitrage argument shows that,

$$(1-\tau_k)r_{t+1} = (1-\tau_k)q_{kt+1} - \delta = (1-\tau_h)q_{ht+1} - \delta \tag{6}$$

From the firm's first order conditions, it follows that the rental prices of physical and human capital are,

$$\begin{cases} q_{kt+1} &= \frac{\partial F}{\partial k}(k_{t+1}, h_{t+1}, n_{t+1}) \\ \\ q_{ht+1} &= \frac{\partial F}{\partial h}(k_{t+1}, h_{t+1}, n_{t+1}) \end{cases} . \tag{7}$$

Thus, we can rewrite the condition that guarantees growth as,

$$\beta[1+(1-\tau_k)r_{t+1}] = \beta[(1-\tau_k)\frac{\partial F}{\partial k}(k_{t+1}, h_{t+1}, n_{t+1}) + 1 - \delta] \geq 1 + a > 1. \tag{8}$$

Then, from the technology side growth is fueled by a high marginal product of capital (and human capital). In Jones and Manuelli (1990) general restrictions on preferences and technologies under which the growth condition (8) is satisfied are discussed. To understand the interaction between taxes and technology we concentrate on a specific example. We assume,

$$F(k,h,n) = A_0 k^\alpha h^{1-\alpha} + A_1 k^{\phi_1} h^{\phi_2} n^{1-\phi_1-\phi_2},$$

$$0 < \alpha < 1 \ , \ 0 < \phi_i < 1 \ , \ \phi_1 + \phi_2 < 1 \ , \ A_i > 0,$$

We also assume that the utility function is of the constant relative risk aversion class

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} \ , & \sigma > 0 \quad \sigma \neq 1 \\ \ell nc \ , & \sigma = 1 \end{cases}$$

When A_1/A_0 is close to zero or along any growth path where k and h go to infinity we can approximate the no arbitrage condition by,

$$(1-\tau_k)\alpha(k/h)^{\alpha-1} = (1-\tau_h)(1-\alpha)(k/h)^{\alpha},$$

or,

$$k/h = ((1-\tau_k)\alpha)/((1-\tau_h)(1-\alpha)). \tag{9}$$

Then, as might be expected, the long run physical to human capital ratio is determined by their *effective* relative marginal products, i.e., the coefficients on the Cobb-Douglas production function times one minus the relevant tax rate. The higher the raw average product of capital (α) or the lower the tax rate on capital income (τ_k) the higher the ratio of physical to human capital. Note that the distortionary effect of regular income and capital income taxation captured in (9) is purely static. If $\tau_k = \tau_h$ the equilibrium physical to human capital ratio would coincide with the optimal ratio as given by the solution of the planner's problem. This is the familiar effect of distortionary taxes: they induce incorrect proportions.

There is an additional dynamic effect associated with taxes. To see this consider the growth condition (8) evaluated at the ratio (9). It is given by,

$$\beta[(1-\tau_k)\frac{\partial F}{\partial k} + 1 - \delta] \geq \beta[A_0((1-\tau_k)\alpha)^{\alpha}((1-\tau_h)(1-\alpha))^{1-\alpha} + 1 - \delta] > 1 \ .$$

In the no tax case the relevant growth condition is,

$$\beta[A_0\alpha^{\alpha}(1-\alpha)^{1-\alpha} + 1 - \delta] > 1$$

If this last condition is satisfied it is possible to show that the asymptotic growth rate of consumption in a laissez faire equilibrium, γ, is given by,

$$\gamma = \{\beta[A_0\alpha^{\alpha}(1-\alpha)^{1-\alpha} + 1 - \delta]\}^{\frac{1}{\sigma}} > 1 \ .$$

In this economy a higher level of productivity (high A_0) or a low discount factor (high β) result in higher growth, while a small intertemporal elasticity of substitution (high σ) reduces the growth rate.

Consider next the case $\tau_k = \tau_h = \tau$. The equilibrium growth rate of consumption is,

$$\gamma_\tau = (\beta[A_0(1-\tau)\alpha^\alpha(1-\alpha)^{1-\alpha} + 1 - \delta])^{\frac{1}{\sigma}}, \tag{10}$$

if $\gamma_\tau > 1$. Assume that this is the case. Then, an increase in income taxes is equivalent to a decrease in the marginal product of the two types of capital and this reduces the equilibrium growth rate. The intuition for this result is as follows: an increase in τ increases the amount of consumption that must be given up today in order to obtain one (after tax) unit of consumption next period; it is this increase in the relative price of future consumption that reduces the demand for it and, consequently, the growth rate. As before, the essential component of the effect of taxes is given by the substitution effect: relative prices determine equilibrium growth rates. This public finance exercise generates no income effects as individuals receive from the government the value of their taxes in the form of lump sum transfers.

If γ_τ as given by (10) is less than one, it is possible to show that the economy converges to a steady state. It is possible to think of this case of zero growth as the consequence of high income taxes because, even if γ is greater than one, a high enough tax rate will turn an economy that has strictly positive limiting growth rates into an economy that has a steady state.

From now on we concentrate on the case in which there is equilibrium growth. With differential taxes on capital and labor income, the long run growth rate of consumption is given by,

$$\bar{\gamma}_\tau = \{\beta[A_0((1-\tau_k)\alpha)^\alpha((1-\tau_h)(1-\alpha))^{1-\alpha} + 1 - \delta]\}^{\frac{1}{\sigma}}$$

It is illustrative to consider the effect on long term growth of eliminating income taxation. Consider, as the base case, $\alpha = .3$ and $\delta = .10$. These numbers are consistent with capital's share in GNP and depreciation studies. If we assume a current level of income taxes of 30% ($\tau_h = .3$) and of capital income taxes of 20% ($\tau_k = .2$) (the rationale for using a lower rate is that some capital income can be spread over the life cycle to reduce the effective tax rate and that some capital goods, e.g., housing are taxed at very low rates) we have that the before tax rate of return is,

$$r = [A_0((1-\tau_k)\alpha)^\alpha((1-\tau_h)(1-\alpha))^{1-\alpha} - \delta]/(1-\tau_k)$$

If we impose $r = .06$ we can use the above expression to obtain an estimate of A_0. We find that A_0 must be .37. If we assume $\sigma = 2$, and using the historical growth rate of consumption of per capita of 2%, it follows that $\beta = .99$. To illustrate the effect of taxation on the long run growth of the economy consider an extreme fiscal reform. Specifically, suppose the government reduces the tax rates on physical and human capital to zero and raises lump sum taxes to obtain the same amount of revenue. What is the effect on the long run growth rate? Using the parameter values that make the tax distorted model roughly consistent with the U.S. evidence we find a sizable impact of that fiscal reform: the growth rate of consumption increases from 2% to 4.6%. Additionally, the ratio of human to physical capital also increases around 15%. Therefore, although the basic data and parameter values are, at best, tentative, the prediction is that the elimination of all forms of income taxation will cause the growth rate of consumption to increase substantially.

This prediction of a sizable growth effect of lower taxation is common to many models of endogenous growth. It appears in the work of Rebelo (1987), King and Rebelo, (1990) and can be computed to be large in simple versions of the externality model of Romer (1986). It is also sizable in a version of Easterly's (1989) distortion model where the range of growth rates varies from 5% to almost 10%. Empirically, Chapman and Lutter (1989) find that increases in the relative price of capital are associated with lower growth rates. Of course, it is possible to consider taxes as one element that increases the relative price of capital, although Chapman and Lutter emphasize monopoly power in the capital goods sector. Lucas (1989) finds that the growth effects of following an optimal tax policy (a zero limiting tax rate on capital income) are small. In his model the accumulation technology is very different from that defined in (2), and this may account for the difference in estimated growth effects.

In a related set of papers Barro (1988) and Easterly (1989) explore the connection between taxes and growth when tax revenue is used to produce a government good which, in turn, affects output. In this setting they find a non linear relationship between taxes and growth. The nonlinearity, however, is fundamentally due to the fact that the marginal product of government expenditure is very high when its level is low (taxes close to zero) and, hence, increases in tax rates "produce" more government expenditure which, in turn, increases the growth rate of output. At very high levels, the negative effect on private rates of return more than outweighs the positive impact of additional government spending and growth rates decline with the income tax rate. The mechanism behind these effects is fundamentally different in the sense that the strong link between taxes and expenditures makes it difficult to study in isolation the effect of income taxation.

The model we described assumed the existence of a representative household that is preserved by the government's egalitarian fiscal policy. It is possible to generalize this analysis to the case of similar preferences and discount factors but allow for a transfer policy that redistributes income. Note that this policy induces heterogeneity across households. It does *not*, however, affect the marginal condition (5). Therefore, the growth rate of consumption is similar for all households although the *levels* are changed by the redistributive policy.

In summary, in convex one sector models of endogenous growth the imposition of income taxes has, in general, a negative effect on the growth rate of the economy. This effect, is independent of the amount of income redistribution if all households have the same preferences.

3. The Effect of Finite Lifetimes

In the previous section it was argued that there are some convex technologies and preferences that are consistent with long term growth in a model of infinitely lived agents. In this section we study the long run properties of a model that has the same convex technology but in which individuals live for a finite number of periods. Throughout the section we will assume that there are no bequests.

The somewhat surprising result is that for *all* production and utility functions the laissez faire competitive equilibrium is such that the level of output remains bounded. That is, finite lifetimes have a large effect on the long run behavior of the economy in the class of one sector convex growth models: they preclude growth.

A natural question to ask is why is it that a sufficiently high real interest rates does not induce a positively sloped time profile of consumption? In a sense, it does. At the *individual* level high interest rates convince households to consume more in the future. This, however, does not translate into increasing *aggregate* consumption.

Since aggregate consumption is the sum of individual consumption across different age groups, it is possible to have total consumption bounded. Consider the case in which individuals live for two periods, and let c_i be consumption in the ith period of each individual's life, $i = 1,2$. Then a high interest rate guarantees $c_2 > c_1$. However, at any point in time aggregate consumption, $c_2 + c_1$, is constant.

This argument does not fully capture the growth process, as the level of consumption could increase over time simply because each generation is wealthier. To formalize this, let c_j^t be time j consumption of an individual born at time t. Then, aggregate consumption at time t is $c_t = c_t^t + c_t^{t-1}$, and it is possible for this to grow if we guarantee $c_t^t > c_{t-1}^{t-1}$, that is, if newer generations can consume more in their first period of life than their predecessors. This is possible since each generation's wealth is given by the present value of wages, and wages increase as capital is accumulated. It turns out that this increase is not enough; to generate growth it is necessary for the capital stock to increase faster than wages and this is not possible in a laissez faire competitive equilibrium with a one sector convex technology.

To make the point clear consider a two-period overlapping generations model similar to Samuelson (1958) and Diamond (1965) with only one capital good. A representative individual born at time t solves,

$$\max \quad u^t(c_t^t, c_{t+1}^t)$$

subject to,

$$c_t^t + s_t \leq w_t$$

$$c_{t+1}^t \leq w_{t+1} + (1 + r_{t+1})s_t,$$

$$(c_t^t, c_{t+1}^t) \geq 0 \quad ,$$

where w_t is the wage rate at time t and s_t is the amount saved (if negative this is interpreted as borrowing). We assume no population growth.

The representative firm takes prices as given and maximizes profits given by,

$$\pi_t = y_t - w_t n_t - q_t k_t,$$

subject to,

$$y_t \leq F(k_t, n_t)$$

where F is some constant returns to scale production function.

A standard no arbitrage condition implies that the rate of interest must equal the rate of return on physical assets. That is,

$$q_{t+1} - \delta = r_{t+1}.$$

We can then define an equilibrium as a sequence of prices $\{(w_t, q_t, r_t)\}_{t=0}^{\infty}$ and an allocation $(c_t^t, c_{t+1}^t, y_t, k_t, n_t)$ satisfying:

(i) Given $(w_t, r_{t+1}), (c_t^t, c_{t+1}^t, s_t)$ solves the utility maximization problem of the representative agent of generation $t, t \geq 0$. The initial old consume their wealth.

(ii) Given $(w_t, q_t), (y_t, k_t, n_t)$ solves the profit maximization problem of firm t,

(iii) $q_t - \delta = r_t$

(iv) $s_t = k_{t+1} \geq (1 - \delta)k_t$

The first three conditions are straightforward. The last condition imposes market clearing. It says that total saving by generation t must equal their purchases of "used" capital from generation $t-1$ ($(1-\delta)k_t$) plus their own addition to the future capital stock ($k_{t+1} - (1-\delta)k_t$).

It is now possible to show the no growth result. Consider (iv) and note that $s_t \leq w_t$ as $c_t^t \geq 0$. Then we must have that $k_{t+1} \leq w_t$. Suppose that there is a sequence $\{k_t\}$ that is growing and that it does not converge. Then,

$$\frac{k_{t+1}}{k_t} \leq \frac{w_t}{k_t}$$

However, $w_t = \frac{\partial F}{\partial n}(k_t, n)$ and for all continuous homogeneous of degree one production functions it follows that $\lim_{k\to\infty} \frac{\partial F}{\partial n}(k,n)/k = 0$. Therefore, for all large $k_t, k_{t+1}/k_t < 1$ which, of course, contradicts the growth assumption.

The intuition is simple: although wages are growing (w_t goes to infinity as k_t goes to infinity), their growth rate is always less than the growth rate of capital. In a one sector model the price of capital does not decrease (it is always equal to one) and the young generations do not have sufficient income to purchase the existing capital from the old.

Is this no equilibrium growth result in the presence of a technology that *can* generate growth sufficient to justify government intervention? The answer is no, at least if we restrict government actions to the correction of a suboptimal situation. To make the argument precise, note that in a convex model with a technology that is consistent with growth the gross marginal product of capital (that must equal the interest rate) is uniformly bounded above one. That is, in a no growth equilibrium of the overlapping generations model the equilibrium interest rate exceeds the growth rate. In this case the standard Debreu (1959) argument can be used to prove that the first welfare theorem holds: every competitive equilibrium is Pareto optimal. Therefore, the case for intervention must rest on arguments about the desirability of a given intergenerational distribution of welfare.

Is it possible that the model does not capture some essential aspect of the capital accumulation process? Specifically, it could be argued that an important component of aggregate capital is what could be called societal human capital; that is, knowledge freely available to individuals that they inherit simply because they are born. In order to capture this "external" effect reinterpret k_t in the previous model as human capital. Moreover, assume that if a young individual is born when the aggregate stock of human capital is k, his first period income is $w + (1-\delta)k$, i.e., he receives the return to his raw labor *and* the value of the depreciated human capital. In this case the upper bound on savings is simply,

$$s_t \leq w_t + (1-\delta)k_t.$$

As before, the growth rate of capital is bounded by,

$$\frac{k_{t+1}}{k_t} \leq \frac{\frac{\partial F}{\partial n}(k_t, n)}{k_t} + 1 - \delta.$$

The right hand side converges to $1-\delta < 1$ as k_t goes to infinity. Therefore, even in the case in which the stock of capital is costlessly transferred to the young, their wealth does not grow fast enough to induce them to add to the stock of human capital which, in this interpretation, is the engine of growth.[4]

The basic insight derived from this exercise is that intergenerational income distribution matters.

There is nothing *physical* that prevents these economies from growing, it is the insufficient level of income that precludes the young from accumulating ever increasing amounts of capital.

The analysis of section 2 and the proof of the no growth result suggest possible modifications of the model that will result in long run growth. The first is the existence of bequests. As Barro (1974) showed, an overlapping generations model in which there are bequests is, on the preference side, similar to an infinite horizon model. In this case, the analysis of section 2 gives precise conditions under which there is growth. The analyses of Barro (1974) and Lucas and Stokey (1984) show that intergenerational altruism (and bequests) restores the infinitely lived agent on the preference side. This, however, is not enough. If one of the possible capital stocks is the stock of human capital this reinterpretation of the model assumes that human capital accumulation is conducted at the family level. For many forms of human capital this does not seem to be a reasonable assumption and the technology for intergenerational transmission of human capital needs to be studied more carefully.

Another alternative is to use taxes to redistribute income from the old to the young. This will give the young sufficient resources to acquire the capital stock accumulated by the old. A third avenue is to modify the model so that the ratio of the wage rate ($w(k_t)$) to the value of the capital stock (k_t in our one sector model) does not converge to zero. This will be the case whenever the production function F exhibits the appropriate type of increasing returns to scale. Finally, it is possible to consider a two sector model in which the price of capital p_{kt} is not necessarily one. In such a model, the relevant condition is $w(k_t)/(p_{kt}k_t)$; thus, if p_{kt} decreases fast enough, it is possible for the young to acquire increasing amounts of k_t because its *value* does not grow faster than the wage rate.

In the following sections we explore these possibilities and show the special conditions on preferences, technologies and taxes that must be satisfied for the equilibrium to display long run growth.

4. Finite Lifetimes, Taxation and Growth

In describing the competitive equilibrium with infinitely lived agents we showed that higher income taxes (in the absence of complementary government production of productivity enhancing goods) result in a decrease in the long run growth rate. In that setting, the negative substitution effect associated with the lower rate of return induced by higher taxes reduces saving. Given the infinite horizon, income redistribution plays no role.

When individuals live for finitely many periods and bequests are limited a new dimension is added. The overlapping generations model creates a fundamental heterogeneity that makes the timing of income important. Specifically, as emphasized in section 3, it is possible to interpret the no growth result as arising because the younger generations do not have enough income to purchase an ever increasing stock of capital. This suggests a role for taxation aimed at redistributing income.

If income is transferred from the old to the young this has two effects: first, by increasing the young's first year income it induces them to save more in order to have a smooth path of consumption over time. This effect was not captured in the argument presented in section 3 as we did not consider optimal savings. The second effect of redistribution is to make affordable to the young the purchase of capital owned by the old. This clearly is necessary to invalidate the reasoning in section 3.

In this section we explore the connection between taxation, income distribution and growth. We start with a two period model with only one capital good that we interpret as physical capital. Individuals pay income taxes at the rate τ. We assume that each individual is endowed with one unit of labor in each period. Then, if first period saving is denoted by s, tax liabilities are τw in the first period of life, and $\tau w + \tau r s$ in the second period. Total government revenue in a given period is $\tau w + \tau w + \tau r s$, and we assume that is transferred back to individuals with the young receiving fraction η of total revenue and the old $1 - \eta$. We denote transfers to the young and the old T_1 and T_2, respectively.[5]

The representative individual born at time t solves,

$$\max \qquad u(c_t^t, c_{t+1}^t)$$

$$\text{subject to } c_t^t + s_t \leq (1-\tau)w_t + T_{1t}$$

$$c_{t+1}^t \leq (1-\tau)w_{t+1} + (1+(1-\tau)r_{t+1})s_t + T_{2t+1} \quad .$$

As in the model of section 3, firms maximize profits and pay no taxes. Given the convex technology, profit maximization ensures that prices are given by the marginal products of capital and labor. Specifically,

$$w_t = \frac{\partial F}{\partial n}(k_t, 2)$$

$$q_t = \frac{\partial F}{\partial k}(k_t, 2).$$

A no arbitrage condition forces the interest rate to be equal to the gross marginal product of capital. If we allow true depreciation to be tax deductible, it follows that no arbitrage requires,

$$1 + (1-\tau)(q_t - \delta) = 1 + (1-\tau)r_t,$$

or,

$$r_t = q_t - \delta.$$

An equilibrium is then defined as in section 3. The only added requirement is that $T_{1t} + T_{2t} = \tau(2w_t) + \tau(\frac{\partial F}{\partial k}(k_t, 2) - \delta)k_t$, i.e., that the government have a balanced budget. We next analyze savings behavior. It is useful to define W^t, wealth of generation t, by,

$$W^t = [((1-\tau)w_t + T_{1t}) + ((1-\tau)w_{t+1} + T_{2t+1})/(1+(1-\tau)r_{t+1})].$$

The utility maximization problem can be expressed as,

$$\max u(c_t^t, c_{t+1}^t),$$

subject to

$$c_t^t + \frac{c_{t+1}^t}{(1+(1-\tau)r_{t+1})} \leq W^t.$$

For a given set of prices and transfers, let θ_t be the fraction of wealth spent on first period consumption. Then, saving of generation t is,

$$s_t = (1-\tau)w_t + T_{1t} - \theta_t W^t.$$

Using the definition of T_{it} and W^t and imposing that aggregate saving equal the capital stock, $s_t = k_{t+1}$, the growth rate of capital must satisfy,

$$\frac{k_{t+1}}{k_t} = [(1-\tau)(1-\theta_t) + (1-\theta_t)\tau\eta 2]\frac{w_t}{k_t} - \frac{\theta_t[(1-\tau)+\tau(1-\eta)2]w_{t+1}}{(1+(1-\tau)r_{t+1})k_t}$$

$$+(1-\theta_t)\tau\eta r_t - \frac{\theta_t(1-\eta)\tau r_{t+1}k_{t+1}}{(1+(1-\tau)r_{t+1})k_t},$$

where we imposed the equilibrium condition that aggregate saving equal the capital stock, $s_t = k_{t+1}$. The first two terms contain elements of the form w_t/k_t. As argued in section 3, $\lim_{k\to\infty} w(k_t)/k_t = 0$, if there is growth in equilibrium. Hence, we can ignore them since we are only interested in the limiting behavior of this economy. The value of θ_t depends on prices and income. Consider the case (we give examples below) in which θ_t converges to some number θ as k_t converges to infinity and r_t converges to r. If we denote by γ the limiting growth rate of capital it follows that, as k goes to infinity, γ must satisfy,

$$\gamma = (1-\theta)\tau\eta r - \frac{\theta(1-\eta)\tau r}{(1+(1-\tau)r)}g,$$

or,

$$\gamma = \max\left\{1, \frac{((1-\theta)\tau\eta r)(1+(1-\tau)r)}{1+(1-\tau)r+\theta(1-\eta)\tau r}\right\}. \tag{11}$$

Then, the model displays long run growth if and only if $\theta > 1$. Equation (11) is not useful to study growth because it depends on θ which is endogenous, being a function of preferences, technology as well as the policy parameters τ, η.

To study the effect of this form of government policies on growth we choose a particular utility function. We assume,

$$u(c_1, c_2) = \frac{c_1^{1-\sigma} - 1}{1-\sigma} + \beta \frac{c_2^{1-\sigma} - 1}{1-\sigma},$$

that is, the two period version of the preferences studied in section 2.

For these preferences, it is straightforward to show that θ_t is given by,

$$\theta_t = \frac{1 + (1-\tau)r_{t+1}}{(1 + (1-\tau)r_{t+1}) + (\beta[1 + (1-\tau)r_{t+1}])^{\frac{1}{\sigma}}}.$$

Using this expression in (11) we obtain,

$$\gamma = \max\left\{1, \frac{(\beta[1 + (1-\tau)r])^{\frac{1}{\sigma}} \tau\eta r}{[1 + (1-\tau)r] + (\beta[1 + (1-\tau)r])^{\frac{1}{\sigma}} + (1-\eta)\tau r}\right\}. \tag{12}$$

Inspection of this equation indicates the effect of some variables on the growth rate γ. First consider the consequence of increasing the productivity of the economy. If we assume, as in Jones and Manuelli (1990) that $\lim_{k \to \infty} F_k(k, 2) = b$, then $r = b - \delta$. Thus the more productive the economy (high b) the higher r and, from (12), the higher γ. The effect of income redistribution can also be determined: an increase in η (the fraction of income that goes to the young) increases γ. This, of course, agrees with the intuition about income distribution affecting growth performance. Similarly, a higher value of β reflects a higher valuation of future consumption and this induces higher growth.

The effect of the tax rate is more difficult to determine. From a purely mechanical point of view it follows from (12) that when $\tau = 0$, there is no growth ($\gamma = 1$). This, of course, is to be expected since, in this case, the model is an example of the general point discussed in section 3. At the other extreme very high taxes ($\tau = \frac{1+r}{r}$) do not generate growth. (Note that the tax rate exceeds one as it falls not only on income from capital but also on its value.) In this case, individuals do not want to save because the government is taxing away all their income. Therefore, the effect of τ on the growth rate (if any) cannot be monotone. There are two opposite forces. Increases in τ reduce the after tax rate of return and, this, in turn tends to a decrease in savings and the growth rate. However, there is an income effect that works in the opposite direction; the higher the tax rate the higher the first period income of the young (because transfers increase) and this tends to increase the growth rate.

Consider, for example, the best case in terms of income distribution from the point of view of the growth rate, i.e., set $\eta = 1$. The denominator in (12) decreases with τ while the behavior of the numerator is more complicated. It increases for low τ, it peaks at some τ^* and then it decreases faster than the denominator. Therefore, for some parameter values, the effect of taxes will be nonlinear, initially increases in τ beyond some threshold will increase growth, but very high taxes will tend to reduce the growth rate. Finally, it follows that a lower degree of intertemporal substitution (high σ) reduces the growth rate if $\beta[1+(1-\tau)r] > 1$, but may have the opposite effect for very high rax rates.

Before we proceed to the numerical analysis it is necessary to determine reasonable bounds for the parameters. A standard difficulty with obtaining numerical results using overlapping generations models is that the notion of a period must be taken seriously. For our two period example we think of a period as lasting 22 years (15 for the three period example). What is a reasonable value of r? We assume that the rate of return r that appears in the overlapping generations model is the annual rate accumulated over 22 (or 15) periods. If the annual rate is r_A before tax then r is given by,

$$r = (1 + r_A)^N - 1,$$

where N is either 22 or 15. For a value of r_A of 6%, the 22-year value of r is 2.6, while the 15 year value is 1.4.

The estimate of β–the discount factor–has to be adjusted accordingly. If β_A is the annual value of beta, it follows that,

$$\beta = \beta_A^N,$$

where, as before, N is either 22 or 15. For the infinite horizon model of section 2 we estimated that a reasonable value for β is .998 (when $\sigma = 2$) or .979 (when $\sigma = 1$). At an annual level these differences are not very large. However, when considering the 15 and 22-year-accumulated values the choice of annual β matters. The 22-year equivalent is either .966 or .625 while the 15-year estimate is .977 or .726, depending on whether the high or low values of β are used.

Finally, consider the interpretation of τ. In the infinite horizon annual model the after tax rate of return is $1+(1-\tau_A)r_A$. Let r be the overlapping generations analog of r (i.e., $r = (1+r)^{22}-1$, in the case of two period lived agents) and τ be the corresponding tax rate. Then τ should be interpreted as the value that solves,

$$1 + (1-\tau)r = [1 + (1-\tau_A)r_A]^N,$$

where N is either 15 or 22. Therefore,

$$\tau = \frac{(1+r) - (1 + (1-\tau_A)r_A)^N}{r}.$$

Table 1 gives τ as a function of (r_A, τ_A) where the subindex distinguishes between $N = 15$ and $N = 22$. The basic message is that the equivalent accumulated tax is higher than the annual value.

Our discussion of reasonable parameter values is meant to be indicative as there is a considerable margin of error in these calculations.

In Figures 1 to 4 we present the effect of changing parameters of the economy and the tax rate on the growth rate for the two period lived economy. Figure 1 shows the value of the net growth rate $(\gamma - 1)$ in an economy with $\beta = .85$, relatively large intertemporal elasticity of substitution $(\sigma = .5)$ and even income distribution $(\eta = .5)$, as a function of the tax rate (τ) and the limiting marginal product of capital (r). As indicated before, r has a positive effect on growth. Also, as conjectured, the effects of taxes on the growth rate are nonlinear: for a given r, taxes that are either too high or too low are inconsistent with growth, and in the region with positive growth the relationship between the tax rate and the growth rate is not monotonic. In particular, there is a Laffer curve in the sense that to attain a given growth rate there are two tax rates that can be used–a high tax and a low tax.

Figure 2 illustrates the effect of changing the elasticity of substitution σ. As in the infinite horizon case, larger values of σ (less intertemporal substitution) reduce the growth rate. This, however, is not true for all parameter values. In Figure 3, we show the effect of very high tax rates [the range of τ is in the interval (.95,1)]. In this case it is possible for a decrease in the intertemporal elasticity of substitution to increase the growth rate. The intuition is that when the discounted gross rate of return, $\beta(1 + (1 - \tau)r)$, is less than one (as it must be when τ is high), lower responsiveness to the interest rate (higher σ) *increases* saving. The positive effect on saving is translated into a higher growth rate.

Finally, Figure 4 displays the trade off between income distribution and growth. As indicated, the "best" case (from the point of view of the long run growth rate) is given by a fiscal policy that redistributes income to the young $(\eta = 1)$. Note that there is no simple trade off between taxes and income distribution. Depending on which side of the Laffer-curve the economy is in, an improvement in the redistributive aspects of fiscal policy (lower η) holding the growth rate constant can be accompanied by either an increase (if the system is in the low tax region) or a decrease (if the system is in the high tax region) of the income tax rate.

In principle this exercise need not be confined to two period lived models. It is interesting to extend it to more periods because we expect that in that case income redistribution can have larger effects. In particular, it is necessary to carefully describe how the fraction of transfers received by the middle aged affects their willingness to save.

In order to simplify the calculations we computed a three period example in which the "old" do not receive transfers (this is the "best" case from the point of view of growth). Then η is the fraction of total revenue given to the young and $1 - \eta$ the fraction received by middle aged individuals. In this example θ_1 is the fraction of total wealth spent on first period consumption and θ_2 the fraction spent on second period consumption. The growth rate is then,

$$\gamma = \max\{1, [\hat{\gamma}]\} \text{ where}$$

$$\hat{\gamma} = \Big\{[(1+(1-\tau)r)(1-\theta_1)-(1-\eta)\theta_2]\tau r + \{\,[(1+(1-\tau)r)(1-\theta_1)-(1-\eta)\theta_2]^2(\tau r)^2 - 4[(1+(1-\tau)r)+\theta_1(1-\eta)\tau r]\eta\tau r(1+(1-\tau)r)[\theta_2-(1-\theta_1)(1+(1-\tau)r)]\,\}^{\frac{1}{2}}\Big\} \,/\,(2[(1+(1-\tau)r)+\theta_1(1-\eta)\tau r]\,,$$

where θ_1 and θ_2 are,

$$\theta_1 = \frac{(1+(1-\tau)r)^2}{(1+(1-\tau)r)^2+(1+(1-\tau)r)[\beta(1+(1-\tau)r)]^{\frac{1}{\sigma}}+[\beta(1+(1-\tau)r)]^{\frac{2}{\sigma}}}$$

$$\theta_2 = \frac{[\beta(1+(1-\tau)r)]^{\frac{1}{\sigma}}(1+(1-\tau)r)^2}{(1+(1-\tau)r)^2+(1+(1-\tau)r)[\beta(1+(1-\tau)r)]^{\frac{1}{\sigma}}+[\beta(1+(1-\tau)r)]^{\frac{2}{\sigma}}}$$

The results are presented in Figures 5 to 8. Although there is a strong similarity between the two period lived and the three period lived models, there are some interesting differences as well. In general, it is easier to obtain growth; that is, the regions of the parameter space for which positive growth is feasible are larger. In Figure 5 it is shown that for logarithmic preferences and even if the young do not receive transfers (in this case they go to the middle aged) there is positive growth for $r \geq 6$. Figure 7 shows the effect of changing the intertemporal elasticity of substitution $(\frac{1}{\sigma})$. As in the two period lived case, a decrease in the intertemporal elasticity of substitution reduces, in general, the growth rate given the tax rate. This however, is not the case for all parameter values. Figure 7 displays a "slice" of Figure 6, i.e., it restricts the range of σ to the interval (1,3). It shows that, in some sense, there is a "nonmonotonicity" in the growth rate as a function of σ because, for σ near one, there is an array of tax rates that can generate growth (it excludes $\tau = 1$). For a slightly higher value of σ (σ near 1.4) there is no tax rate (given the desired amount of redistribution) that can sustain growth. However, for even higher values of σ (σ near 1.8) there is a new growth region sustained by very high levels of taxation. This nonmonotonicity of the region of positive asymptotic growth as a function of the elasticity of substitution does not appear in the two period lived example because in that case we need only track the amount of saving that the young individuals demand. In the three period lived model the middle-aged save as well, and an increase in σ can have a different effect on their saving.

The effect of the limited form of income redistribution that we allow (only the young and the middle aged are eligible for transfers) is shown in Figure 8. The parameters are comparable to those in Figure 4. It is clear that the longer horizon makes redistribution to the middle-aged feasible.

These results are by no means exhaustive, and are not intended to give realistic (in a quantitative sense) estimates of the effects of redistributive taxation. We think they are suggestive of the highly nonlinear effects that taxation will have in overlapping generations models. It is interesting to point out that for the reasonable values of the before tax growth rate that we derived from annual estimates (r between 1 and 3), the region of positive growth is small (in the parameter space) or nonexistent. Typically, the values of r that we use in Figures 1 through 8 are higher. This suggests that if low r's are "reasonable" it is crucial to carefully specify the other parameters of the model because the existence of long run growth is not robust to alternative specifications.

To summarize, finite lifetimes and growth are not inconsistent given an active government policy aimed at redistributing income toward the potential savers. However, these policies cannot be justified on efficiency arguments in the context of this model.[6]

5. A Two-Sector Model

As indicated in section 3, the lack of asymptotic growth results because the ratio of the value of labor income ($w(k_t)$) to the value of the capital stock (k_t) decreases without bound as the capital stock increases. Therefore, young individuals with income w_t cannot afford to purchase high levels of capital. This arises because capital is a perfect substitute (in production) with consumption and, hence, its price is fixed.

Consider a two sector economy along the lines of Rebelo (1987) and Chapman and Lutter (1989). Let the sector that produces consumption have a production function given by,

$$c_t \leq A z_t^{\alpha} n_t^{1-\alpha},$$

where z_t is the stock of capital allocated to that sector and n_t is employment. There is a second sector that produces the investment good and, for simplicity, we assume uses only capital. The production function is,

$$x_t \leq b y_t.$$

Of course, in equilibrium $n_t = 2$ and $z_t + y_t = k_t$.

Let q_t be the rental price of capital, w_t the wage rate and p_{kt} the price of capital. Then profit maximization by firms in both sectors results in the following conditions

$$(13) \qquad \begin{cases} q_t &= \alpha A z_t^{\alpha-1} 2^{1-\alpha} = b p_{kt} \\ w_t &= (1-\alpha) A z_t^{\alpha} 2^{-\alpha}. \end{cases}$$

Consider next the value of the total stock of capital at time t. It is simply,

$$p_{kt}k_{t+1} = (1-\delta)p_{kt}k_t + p_{kt}by_t,$$

or,

$$p_{kt}[(1-\delta)k_t + b(k_t - z_t)].$$

Consider an equilibrium in which z_t (the stock of capital allocated to the consumption sector) is a *fixed* fraction ϕ of the total capital stock k_t. In this case we have,

$$q_t = \alpha A\phi^{\alpha-1}2^{1-\alpha}k_t^{\alpha-1}$$

$$w_t = (1-\alpha)A2^{-\alpha}\phi^{\alpha}k_t^{\alpha},$$

and the value of capital is,

$$p_{kt}[(1-\delta)k_t + b(1-\phi)k_t] = \alpha A\phi^{\alpha-1}2^{1-\alpha}k_t^{\alpha}[(1-\delta)+b(1-\phi)]\ .$$

Then the ratio of labor income to the value of end of period capital is given by,

$$\frac{w_t}{p_{kt}k_{t+1}} = \frac{(1-\alpha)A2^{-\alpha}\phi^{\alpha}}{\alpha A\phi^{\alpha-1}2^{1-\alpha}[(1-\delta)+b(1-\phi)]}.$$

This expression is bounded away from zero. Therefore, it is *possible* that by adding a second sector we can restore the positive growth result that characterizes the infinite horizon. We say possible because we need to show that indeed there is an equilibrium in which capital in the consumption sector is – at least asymptotically – some fraction between zero and one of the beginning of the period capital stock. Moreover, we need to model explicitly the savings decision because it is well known that in many models although it is feasible to grow it is not optimal; that is, we now know that the young can afford to purchase the stock of capital and we need to find conditions such that they will choose to do so.

To show that this is indeed possible consider the following example. Assume that individuals have log utility functions. In this case it is standard to show that the demand functions are given by,

$$c_t^t = \frac{1}{1+\beta}[w_t + \frac{w_{t+1}}{1+r_{t+1}}]\ , \qquad t \geq 0$$

$$c_{t+1}^t = \frac{\beta}{1+\beta}[(1+r_{t+1})w_t + w_{t+1}]\ , \qquad t \geq 0$$

$$c_0^{-1} = w_0 + p_{k0}(1-\delta)k_0 + q_0k_0$$

The usual no arbitrage condition requires that the rate of return on financial assets equal the rate of return on capital, i.e.,

$$1 + r_{t+1} = \frac{q_{t+1} + (1-\delta)p_{kt+1}}{p_{kt}} \tag{14}$$

We can then define an equilibrium as an allocation $(c_t^t, c_{t+1}^t, k_t, z_t)$ and prices (q_t, w_t, p_{kt}, r_t) such that:

(i) $c_t^t + c_t^{t-1} = Az_t^\alpha 2^{1-\alpha} = q_t z_t + 2w_t$, where c_{t+j}^t , $j = 0, 1$ solves the utility maximization problem;

(ii) $k_{t+1} = (1-\delta)k_t + b(k_t - z_t)$;

(iii) $z_t \leq k_t$;

(iv) prices satisfy (13) and (14).

Consider (14) when we impose $q_{t+1} = bp_{kt+1}$. We then have,

$$(1 + r_{t+1}) = \frac{q_{t+1}}{q_t}(b + 1 - \delta) .$$

Then,

$$\frac{w_{t+1}}{1 + r_{t+1}} = \frac{(1-\alpha)}{\alpha}\frac{1}{2}q_{t+1}z_{t+1}\frac{q_t}{q_{t+1}(b+1-\delta)} = \frac{1-\alpha}{2\alpha\theta}q_t z_{t+1},$$

where $\theta = b + 1 - \delta$.

Similarly, $w_t = \frac{1-\alpha}{2\alpha}q_t z_t$, and c_{t+j}^t can be written as

$$c_t^t = \frac{1}{1+\beta}\,\frac{1-\alpha}{2\alpha}q_t[z_t + \frac{z_{t+1}}{\theta}] \qquad t = 0, 1, \dots$$
$$c_t^{t-1} = \frac{\beta}{1+\beta}\,\frac{1-\alpha}{2\alpha}q_t[\theta z_{t-1} + z_t] \qquad t = 1, \dots$$

The equilibrium condition in (i) is then,

$$\left\{\frac{1}{1+\beta}\,\frac{1-\alpha}{2\alpha}[z_t + \frac{z_{t+1}}{\theta}] + \frac{\beta}{1+\beta}\,\frac{1-\alpha}{2\alpha}[\theta z_{t-1} + z_t]\right\}q_t = \frac{1}{\alpha}q_t z_t$$

or,

(15) $$\frac{1}{\theta}z_{t+1} + \beta\theta z_{t-1} - \frac{(1+\beta)(1+\alpha)}{1-\alpha}z_t = 0 , \qquad t \geq 1.$$

The first period is special because the initial old demand consumption equal to the total value of their income. The equilibrium condition is,

$$c_0^0 + c_0^{-1} = \frac{1}{\alpha}q_0 z_0,$$

$$\frac{1-\alpha}{(1+\beta)2\alpha}q_0(z_0 + \frac{z_1}{\theta}) + \frac{1-\alpha}{2\alpha}q_0 z_0 + \frac{q_0}{b}(1-\delta)k_0 + q_0 k_0 = \frac{1}{\alpha}q_0 z_0,$$

or,

(16) $$z_0 = \frac{1-\alpha}{\theta[\beta(1+\alpha) + 2\alpha]}z_1 + \frac{2\alpha\theta(1+\beta)}{[\beta(1+\alpha) + 2\alpha]}k_0 .$$

Additionally, the equilibrium must satisfy,

$$k_{t+1} = (1-\delta)k_t + b(k_t - z_t) \tag{17}$$

Thus, an equilibrium is a solution to (15) and (17) that satisfies (16) and $z_t \leq k_t$.

Consider first (15). This is a second order linear difference equation. Its solution is of the form,

$$z_t = a_1\lambda_1^t + a_2\lambda_2^t$$

where λ_1, λ_2 satisfy

$$(1-\lambda_1 L)(1-\lambda_2 L)z_{t+1} = 0,$$

It is simple to show that the largest root, say λ_2, is greater than θ. Then if $a_2 \neq 0$ the long run growth rate of z_t would be $\lambda_2 > \theta$. Because the *maximal* growth rate of k_t is θ (see (17)), this candidate solution violates $z_t \leq k_t$. Thus, we *must* set $a_2 = 0$ in the general solution and z_t is given by,

$$z_t = a_1\lambda_1^t$$

where λ_1, is given by,

$$\lambda_1 = \frac{\theta}{2}\left\{\frac{(1+\beta)(1+\alpha)}{1-\alpha} - \left[\left(\frac{(1+\beta)(1+\alpha)}{1-\alpha}\right)^2 - 4\beta\right]^{\frac{1}{2}}\right\},$$

and a_1 depends on the initial conditions through (16). Note that the growth condition is $\lambda_1 > 1$. This is equivalent to,

$$\beta\theta + \frac{1}{\theta} > \frac{(1+\beta)(1+\alpha)}{1-\alpha}. \tag{18}$$

There are several interesting differences between this overlapping generations version and the corresponding infinitely lived counterpart. First, the growth condition for the infinitely lived model with the same technology is $\beta\theta > 1$. It is clear that this is *necessary* for (20) to hold but not sufficient. That is, it is possible for the overlapping generations version not to grow although the infinite horizon version grows. Second, given the parameter values, if the economy is sufficiently productive in the capital sector (high b which in turn implies high θ) then it will display long run growth. Third, a high average product of capital in the consumption sector (high α) works in the direction of decreasing the region of positive growth. The intuition for this is that the higher the productivity of capital in the consumption goods sector the larger the fraction of total capital allocated to the consumption sector (ϕ_t) and, hence, the lower the rate of accumulation of total capital.

As suspected, the existence of more than one sector makes long run growth feasible. The conditions are not easy to describe because they reflect properties of the capital intensity in each sector. The basic intuition is that if the productivity in the capital sector is sufficiently high, the long run rate of growth of the economy is positive.

6. The Effect of Nonconvexities

Most of the literature on growth with finite horizons has concentrated on the case of non convex technologies. Since the models involved are exclusively one sector models, the results of section 3 indicate that it is *necessary* to assume some form of increasing returns.

Azariadis and Drazen (1988) consider a model with a one sector technology and with physical and human capital. They show that, depending on the exact nature of the nonconvexity, there can be multiple equilibria with some of them exhibiting growth. Bencivenga and Smith (1989) consider a version of Romer's (1986) model with a "liquid" and an "illiquid" technology. They show that financial intermediation, which they model as reducing the fraction of saving that is invested in the low-return liquid technology, can have positive effects on growth. Becker and Murphy (1989) consider the role of specialization in a model of human capital accumulation using a nonconvex technology that is similar to Lucas (1988). Freeman and Polasky (1989) discuss the effects of organizing the production and transmission of human capital in the presence of nonconvexities in the production set. Chou and Shy (1989) show how an externality in the stock of available inputs can generate long run growth. Ehrlich and Lui (1989) study intergenerational transfers (of human capital) and savings when the nonconvexity of the production sets that also resemble that of Lucas (1988). They show that, under some conditions, there is equilibrium growth.

Most of these papers (and the list is by no means exhaustive) are interested in *specific* topics, e.g., financial intermediation, intergenerational transfers, and they make an important contribution to our understanding of the interactions of those factors and growth. However, they all give a similar answer to the following question: What is the *engine* of growth? When this question is interpreted in a strictly technical sense, i.e., when we ignore the specific details that is what makes these models interesting, the answer lies in a nonconvexity in the production set. It seems then useful to show how a standard externality can modify the no growth result.

To make the point we consider the following example. Again, consider the special case where preferences are given by,

$$u(c_t^t, c_{t+1}^t) = \ell n c_t^t + \beta \ell n c_{t+1}^t.$$

The technology is summarized by,

$$c_t^t + c_t^{t-1} + x_t \leq A \bar{k}_t^{\nu} k_t^{\alpha} n^{1-\alpha} z_t,$$

$$k_{t+1} = (1-\delta)k_t + x_t,$$

where $\bar{k}_t^{\nu}$ represents external effects of the aggregate capital stock on the production function of each firm. Of course, in equilibrium $\bar{k}_t = k_t$. If we assume $n_t = 2$, the equilibrium prices of labor and capital services are given by:

$$w_t = (1-\alpha)A2^{-\alpha}k_t^{\xi}$$

$$q_t = \alpha A k_t^{\xi-1} w^{1-\alpha},$$

where $\xi = \alpha + \nu$ and the rate of return on saving must satisfy,

$$1 + r_{t+1} = q_{t+1} + 1 - \delta = \alpha A k_{t+1}^{\xi-1} + 1 - \delta.$$

Total saving by the young of generation t is given by,

$$s_t = \frac{\beta}{1+\beta} w_t - \frac{1}{1+\beta} \frac{w_{t+1}}{1+r_{t+1}}.$$

Using the definitions of w_t and r_t, this equation is equivalent to:

$$(19) \qquad (1+\beta)k_{t+1} + \frac{(1-\alpha)A2^{-\alpha}k_{t+1}^{\xi}}{\alpha A 2^{1-\alpha}k_{t+1}^{\xi-1} + 1 - \delta} = \beta(1-\alpha)A2^{-\alpha}k_t^{\xi}.$$

Given $k_0 \geq 0$, any sequence $\{k_t\}$ satisfying this nonlinear difference equation is an equilibrium capital stock. Given any sequence, the equilibrium prices and consumption levels can be computed using the formulae for factor prices and the demand for consumption function resulting from the maximization problem.

To analyze the equilibrium of this economy let the left hand side of (19) be denoted $H_1(k_{t+1})$, and the right hand side $H_2(k_t)$. consider first the case $\xi \geq 1$ (this says that the degree of increasing returns to scale is sufficiently high, i.e., $\nu \geq 1 - \alpha$). It then follows that,

$$H_1(0) = 0 \quad , \quad H_1'(k) > 0 \quad ,$$

$$\text{and,} \quad H_2(0) = 0, \quad H_2'(k) > 0.$$

We can now argue that, if $\xi > 1$, and k_0 is sufficiently large, the sequence $\{k_t\}$ is increasing and it does not converge. Given that both $H_1(k)$ and $H_2(k)$ are monotone increasing, it suffices to show that, for all large k, $H_1(k) < H_2(k)$. In this case $H_1(k_{t+1}) = H_2(k_t)$ if and only if $k_{t+1} > k_t$.

Fix k, and consider the ratio $H_1(k)/H_2(k)$; it is given by,

$$\frac{H_1(k)}{H_2(k)} = H(k) = \frac{(1+\beta)k}{\beta(1-\alpha)A2^{-\alpha}k^{\xi}} + \frac{1}{\beta[\alpha A 2^{1-\alpha}k^{\xi-1} + 1 - \delta]}.$$

Since $\xi > 1$, it follows that $\lim_{k\to\infty} H(k) = 0$. Therefore, there is some k^* such that for all $k \geq k^*, H(k) \leq 1 - \epsilon$ (for ϵ small and positive), and this establishes the result.

It is interesting to point out that, as it is generally the case in nonconvex models, the long run behavior of the economy is not uniquely determined. Specifically, although $H_1(0) = H_2(0) =$

$0, H_1'(0) = 1+\beta$, while $H_2'(0) = 0$, that is, for small k, $H_1(k) > H_2(k)$. In this region any solution to the difference equation must have $k_{t+1} < k_t$. These economies exhibit then a sort of "trap of underdevelopment" (see Azariadis and Drazen (1988) and Ehrlich and Lui (1989)) in the sense that if the initial per capita capital stock is "too low" not only do they not grow but the equilibrium is such that per capita output decreases over time.[7] In this particular case it is possible to show that the economy also possesses a steady state k_s, so that, in fact, we can distinguish three regions (see Figure 9). If k_0 lies in the interval $(0, k_s)$, the equilibrium is such that output per capita decreases. If $k_0 = k_s$, then $k_t = k_s$, while for $k_0 > k_s$, the sequence k_t goes to infinity, i.e., the economy displays growth.

The argument is similar in the borderline case $\xi = 1$ in the sense that for some parameter values there is growth. Specifically, the function $H(k)$ is independent of k and equals,

$$H(k)|_{\xi=1} = \frac{1+\beta}{\beta(1-\alpha)A2^{-\alpha}} + \frac{1}{\beta[\alpha A 2^{1-\alpha} + 1 - \delta]}$$

If this expression is less than one then for all capital stocks there is growth. Conversely, if it is greater than one there is no growth. Note that, for this borderline case a large value of the productivity parameter A guarantees the existence of growth. In this sense the growth condition resembles the corresponding criterion in the one sector model of section 2. The intuition is simple: when the marginal product of capital is not diverging to infinity (as is the case when $\xi > 1$), its value matters, and a large marginal product of capital makes growth more likely.

Finally, consider the case in which the degree of nonconvexity is moderately low, i.e., $\xi < 1$. As before, the functions $H_1(k)$ and $H_2(k)$ are monotone increasing and, hence, it is possible to characterize long run growth in terms of the function $H(k)$ as we did in the $\xi > 1$ case. However, for $\xi < 1, \lim_{k \to \infty} H(k) = \infty$. That is, since $H(k) < 1$ is required for growth, there cannot be positive long run growth in the model.

Although this example is simple, it captures the essence of many overlapping generations growth models with externalities. First, it is possible – if the degree of nonconvexity is sufficiently high – to obtain growth in equilibrium. Second, this long run growth depends on initial conditions and, in general, there are threshold effects. In terms of the example if the economy is "pushed" above k_s (when $\xi > 1$), this starts a process of economic growth.

The key difference between the model in this section and the general treatment in section 3 lies on the behavior of the term $w(k)/k$. There we argued that for all convex production functions, $\lim_{k \to \infty} w(k)/k = 0$ and this, in turn, implies that the young do not have sufficient income (given by $w(k)$) to purchase ever increasing amounts of capital (given by k_t). In the presence of an externality the ratio $w(k)/k$ does not converge to zero. In the example of this section $w(k)/k$ is given by

$$\frac{w(k)}{k} = \beta(1-\alpha)A2^{-\alpha}k^{\xi-1},$$

and with $\xi > 1$ it goes to infinity as k grows, that is, the young generation's wealth grows faster than the capital stock. It is this difference in growth rates that makes equilibrium growth feasible.

7. Concluding Comments

There are several interesting issues that must be confronted by any model in which individuals live for a finite number of periods and that tries to explain long run growth observations.

1) The assumption of finite lifetimes forces us to be precise about the process of knowledge accumulation at the individual and aggregate levels. Specifically, when we model the accumulation of knowledge at the individual level is it reasonable to assume that the individual acquires the best available knowledge? For some examples this is probably a reasonable assumption. However, it seems that generalizations are not appropriate. Consider, for example, the stock of academic human capital. Until about 60 years ago (to make a conservative estimate) it was not uncommon to find academics with a *broad* knowledge of their field. In more recent years there is a tendency to specialize. Indeed, it is often argued that it is impossible (in a sense that it was not 100 years ago) for an individual to know "everything" in a field. We see historians and physicists that become experts in narrowly defined fields. This indicates that, although at the social level, knowledge is not lost, at the individual level the description that investment in human capital simply adds to the existing stock is not the only obvious choice. In particular, a model of human capital differentiation may be more appropriate to describe the evolution of knowledge.

2) In Section 3 we argued that the laissez faire competitive equilibrium in an overlapping generations economy fails to grow because the new generation does not have sufficient income. In Section 4 we showed that an income tax financed redistributive policy can be used to induce equilibrium growth. Although the model is too abstract to deal with specific details, it is reasonable to associate the redistribution to the young as, for example, being in the form of tax financed schooling services. In this sense publicly financed schooling would have a positive effect on growth.

3) The model of Section 4 can be used to study the effect of social security on the growth rate. If we consider the two period lived version and we interpret $1-\eta$ - the fraction of government expenditure received by the old - as social security payments, it is possible to show that the larger this fraction is, the lower the growth rate. The intuition is that an increase in income

in old age reduces saving when young, resulting in a lower rate of capital accumulation and growth. Although in the model higher payments to the old imply necessarily lower payments to the young, it is possible to show that any lump sum tax (levied on both the young and the old) that is used to finance a transfer to the old must have the same depressing effect on the growth rate.

4) The no growth result of Section 3 and the growth induced by taxation and redistribution studied in Section 4 must be considered exercises in positive economics. They have no normative content. Specifically, in the cases in which the technology is consistent with long run growth, the rate of interest (which equals the net of depreciation marginal product of capital) is positive and bounded above zero. In this case the no growth laissez faire competitive equilibrium is Pareto optimal. Therefore, the desirability of policy induced growth must be justified on grounds other than efficiency. It is possible that for some social welfare function, and given that only distortionary taxes are available, that some of the positive growth equilibria of section 4 are optimal. In the context of this model,as well as in many others, a higher growth rate is not necessarily better. To understand why this is the case, consider the situation of the initial old. In general, growth is attained at the expense of lower present consumption, but the old *only* care about present consumption. In the model, as well as in the real world, there are always individuals who directly or indirectly "pay" the cost of growing but do not live long enough to enjoy its benefits. Therefore, in the context of the models we study in this paper, the desirability of growth must be justified by some notion of *social* intergenerational altruism.

5) The research reported in this paper is related to the question of observational equivalence between overlapping generations and infinitely lived models. In a recent series of papers Aiyagari (1985) (1988) argues that there is observational equivalence between infinite horizon and overlapping generations models in the sense that an observer of the behavior of aggregate variables cannot distinguish if the observations are generated by an infinite horizon or by an overlapping generations economy. In particular, Aiyagari shows that under some assumptions the equilibrium path of an infinite horizon model can be replicated by a two period lived overlapping generations economy. One of the key assumptions is that the infinite horizon growth model has a steady state. The argument in Section 3 shows that the assumption that the equilibrium of the infinite horizon economy displays zero growth is *essential* for the class of one sector models: the observational equivalence does not generalize to models of endogenous growth. Blackorby and Russell (1989) extend Aiyagari's analysis in two dimensions. They do not assume that the infinite horizon economy has a steady state and they require that the observational equivalence hold not only for the equilibrium prices but also in a neighborhood of them. They conclude that

for the one good in each period model there is observational equivalence, that is, it is possible to find a two period lived overlapping generations economy that generates the same aggregate excess demand curves. In their construction they consider the income of each generation as a free variable. Although aggregate income is given, its distribution across generations is chosen to satisfy aggregate consistency. Therefore, the overlapping generations equilibrium that replicates the aggregate behavior of the infinitely lived agent model may require some form of government intervention. In fact, the analysis of Section 3 show that this must be the case, i.e., it shows that the results of Blackorby and Russell do not hold if we require that the 2 period lived economy generate (locally) the same aggregate demand curves in a laissez faire equilibrium.

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Notes

1. This list is incomplete. It excludes some earlier treatments of endogenous growth given by Uzawa (1965), Gale and Sutherland (1968) and Shell (1967) (1973), as well as the fast growing literature that links industial organization and growth. The latter is almost exclusively based on models with nonconvexities so that it does not offer any new insights on the *engine* of growth. See, for example, Schmitz (1989) and Muniagurria (1989).

2. See the references in section 6.

3. This view of taxation is similar to the analyses of Jones and Manuelli (1990) and King and Rebelo (1990).

4. Independently, Freeman (1989) studied a model of inventions in which labor can be used to produce long-lived capital and he shows that there is no growth in equilibrium. His model of inventions is a special case of the general class of convex technologies that we explore.

5. There is a large literature on taxation in the framework of two period models that recognized the effect of income redistribution on the equilibrium stock of capital. It includes Diamond (1973), Atkinson and Sandmo (1980). Atkinson and Stiglitz (1980), chapter 8, presents a clear exposition.

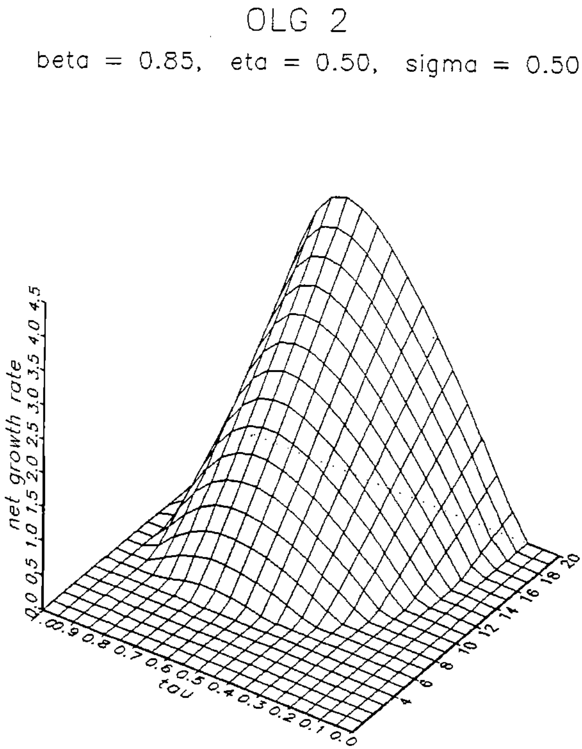
6. In a different context, Manresa (1984) shows that in a model with natural resources the efficient competitive equilibrium in an overlapping generations model is such that aggregate consumption converges to zero although it is technologically feasible for the economy to accumulate enough capital to keep consumption positive. He shows that with income redistribution there are equilibria in which consumption remains bounded away from zero. Both in Manresa's and in this paper redistribution is essential.

7. Another recent paper that obtains a similar result is Easterly (1990). In that paper the interplay between properties of the technology and the law of motion of population growth is essential for the outcome although the production technology is convex.

TABLE 1 ($r_A = .06$)

	τ_A			
	.2	.3	.4	.5
τ_{15}	.27	.39	.50	.60
τ_{22}	.31	.42	.55	.65

Figure 1



OLG 2
beta = 0.85, eta = 0.50, sigma = 0.50
net growth rate
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5
tau
0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
4 6 8 10 12 14 16 18 20

Figure 2

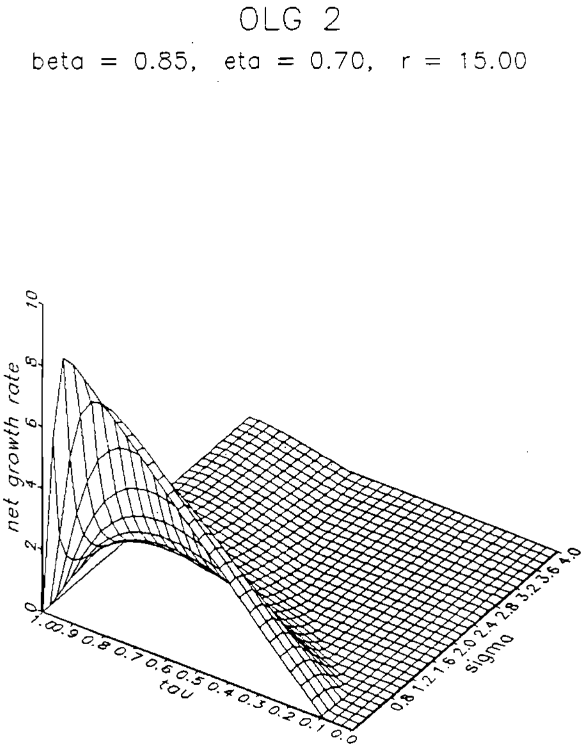

OLG 2
beta = 0.85, eta = 0.70, r = 15.00
net growth rate
0
2
4
6
8
10
tau
sigma

Figure 3

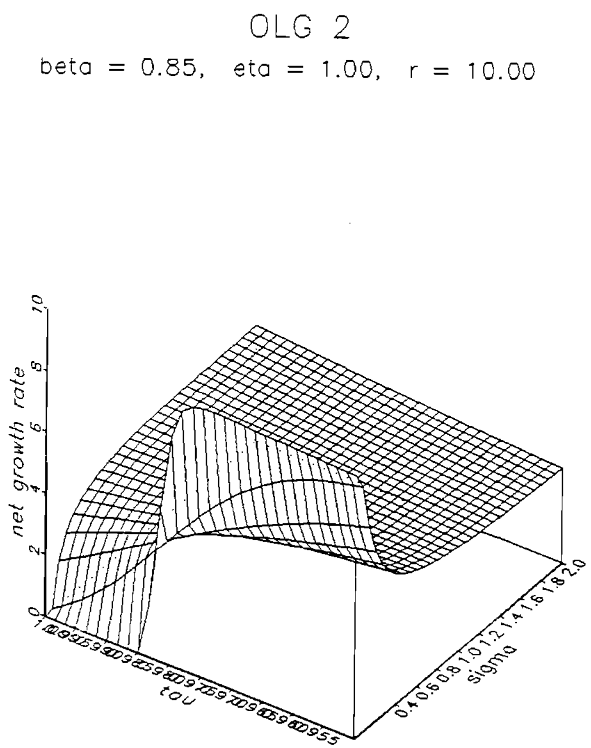
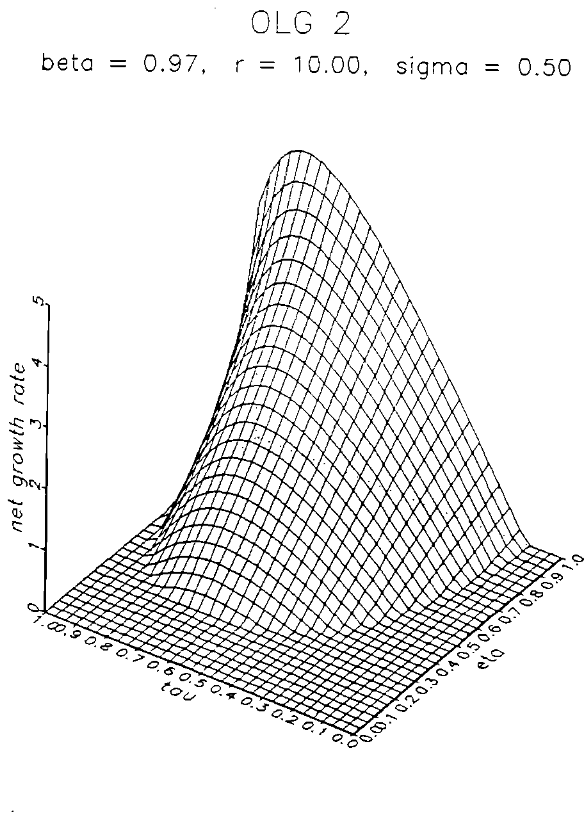

OLG 2
beta = 0.85, eta = 1.00, r = 10.00
net growth rate
0
2
4
6
8
10
tau
sigma
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0

Figure 4



OLG 2
beta = 0.97, r = 10.00, sigma = 0.50
net growth rate
0
1
2
3
4
5
tau
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
eta
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure 5

OLG 3

beta = 0.97, eta = 0.00, sigma = 1.00

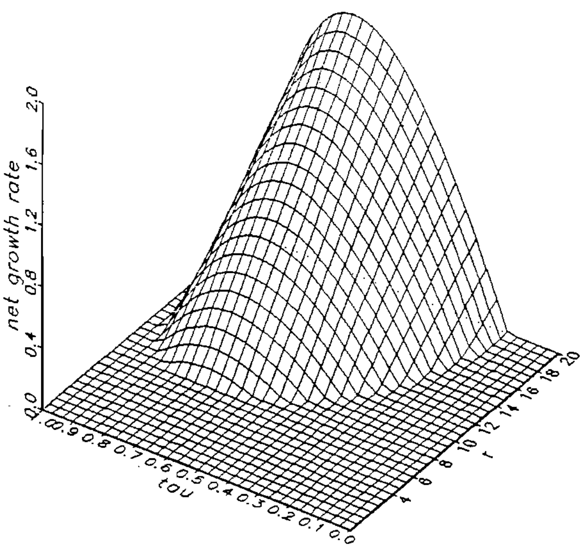


Figure 6

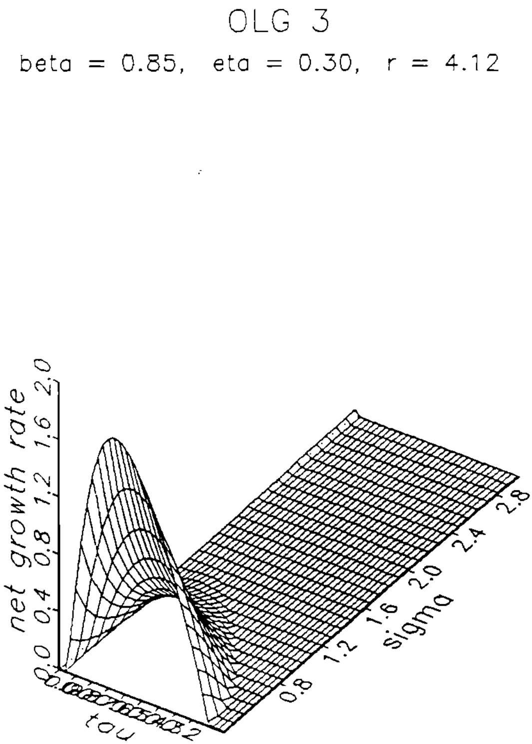

OLG 3
beta = 0.85, eta = 0.30, r = 4.12
net growth rate
0.0
0.4
0.8
1.2
1.6
2.0
tau
sigma
0.8
1.2
1.6
2.0
2.4
2.8

Figure 7

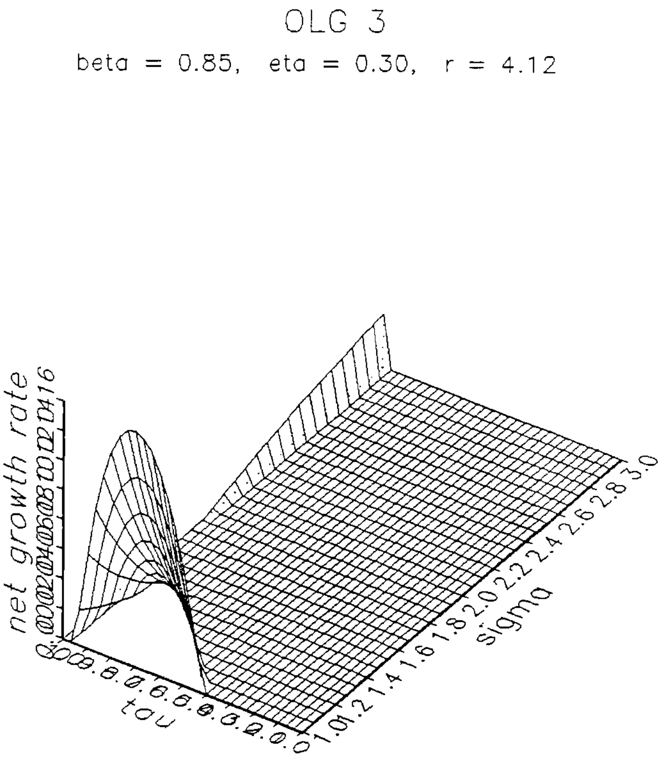


Figure 8

OLG 3

beta = 0.98, r = 5.00, sigma = 0.50

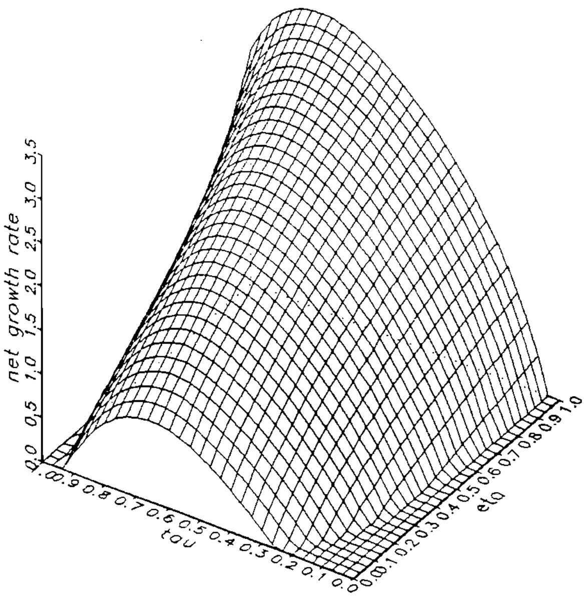


Figure 9