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equilibria with a high likelihood of disasters. When the probability of a disaster is high enough, the
coordination game becomes like a prisoner's dilemma situation in which the high-output equilibrium
disappears because the portfolios that agents choose induce them to produce less. We derive an upper
bound on the disaster probability, we derive asset pricing implications including the disaster premium,
and we study the effect on stock prices of news shocks to beliefs.

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Abstract

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1   Introduction

Our paper combines coordination games and trade in sunspot-contingent
assets. An equilibrium is a distribution over outcomes such as output, con-
sumption and investment. In a coordination game with multiple Pareto-
ranked equilibria, an outcome can be chosen by an extrinsic device such as
a sunspot. The sunspot is a public signal that correlates players' actions.
The mapping between sunspots and equilibrium outcomes is in most models

exogenous, as is the distribution of the sunspot and, hence, the distribution of outcomes.

In our model, the mapping from sunspots to equilibrium play is endogenous. This is done through a prior stage in which players, before taking a real action, trade securities that pay contingent on the realization of a sunspot. The choice of which securities to trade reflects the Nash equilibrium beliefs that determine the mapping between a sunspot state and an action profile and, hence, the probabilities with which outcomes arise. Trade results in commitments to transfers that then shape equilibrium investment decisions in the post-trade subgame.

The model has two types of agents, rich and poor, and they differ in their attitudes towards aggregate risk. This motivates financial trade between these two groups. The coordination sub-game that follows the trade has two equilibria – "adverse" and "favorable.". Aggregate output, consumption and investment are low in the adverse sub-game equilibrium. Thus, both types face aggregate risk and each type is better off in the favorable equilibrium. The poor purchase insurance against the adverse outcome, which means the rich are paid when the outcome is favorable. The higher the likelihood of the adverse outcome is believed to be, the larger are the transfers and, in particular, the larger is the transfer to the rich after the favorable outcome. But agents have a concave utility of consumption, and if the transfer is large enough the rich are not willing to invest. The favorable outcome then fails to be an equilibrium in the sub-game. That is, if the probability of the adverse outcome is high enough, trading in the sunspot-contingent assets transforms the coordination sub-game into a prisoner's dilemma game with a unique outcome. The set of equilibrium distributions shrinks as the less favorable distributions of outcomes are no longer equilibria.

Adding the asset-trading stage thus changes the "macroeconomic" sub-game itself by bounding from above the probability of the adverse, in our context disastrous, outcome. The end result, nevertheless, is that financial markets *raise* welfare – by eliminating equilibria that place too high a probability on the adverse outcome, financial trade in fact eliminates low-welfare equilibria. Financial markets also raise welfare through the usual channel, when the distribution of outcomes as summarized by π^L is given, agents can trade income over states.

Figure 1 illustrates both positive effects of finance on real outcomes. It plots the welfare of the rich and poor agents in the economy with and without financial trade. If the financial markets are closed both the favorable

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and the adverse outcomes are possible, and the welfare of the two types is shown by the pairs of dots on each vertical axis. As the probability, π^L, of the bad outcome rises, agents' welfare declines. Opening financial markets improves everyone's welfare conditional on π^L, and also invalidates equilibria that feature a high probability of the low outcome. Financial markets thus eliminate the low-welfare equilibria; the one exception is the deterministic low outcome associated with the welfare levels B_1 and B_2 in the Figure; no sunspot-contingent trades can arise here because sunspots play no role when $\pi^L = 0$.
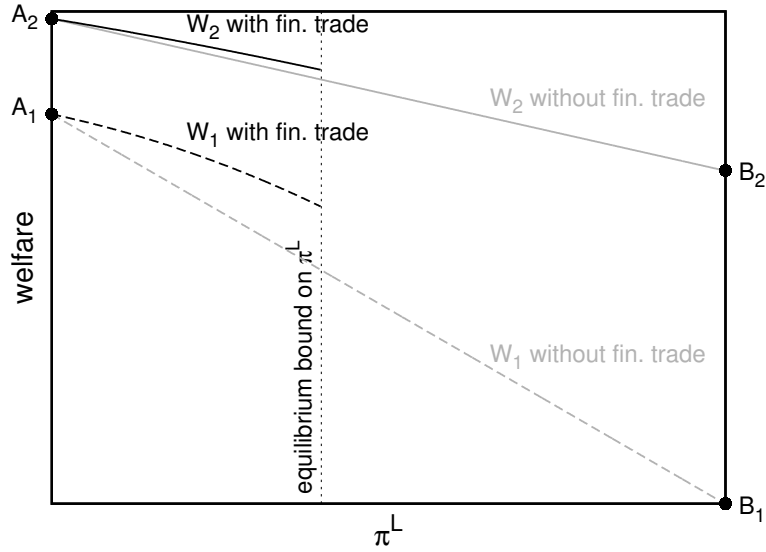
Figure 1: Welfare with and without trade in the financial markets. At $\pi^L \in \{0, 1\}$, points A_1, A_2, B_1, B_2, three is no trade in financial assets and the welfare is the same across the two regimes.

When the sunspot shock realization selects a subgame equilibrium, then individual expectations of aggregate investment and productivity, individual decisions, and the level of aggregate output are uniquely determined. Thus our results would remain unchanged if the financial assets paid not contingent on a sunspot realization but contingent on aggregate output, as is the case with an aggregate stock market index such as the S&P 500. We show that

trade in equity and in a safe asset – not "exotic" assets – suffices for our results to obtain.

The asset-pricing implications relate to the disaster-risk literature. We interpret coordination failures as disasters and in our model they are the only cause of disasters. Our asset prices display a disaster premium that reflects both the size of a disaster and its probability. The disaster size is 0.29 and the disaster probability is only 2%; both are similar to the estimates in Barro (2006). As the probability of disasters rises, the premium grows. However, a higher than 2% probability of a disaster is not sustainable as trading in the financial markets changes the set of possible equilibria. Thus, we provide a theory of disaster risk. In probable contrast to wars and natural catastrophes, we find that the size of disasters and their frequency are positively correlated across equilibria: The larger the disaster, the higher is the likelihood that it can occur.

The stock market index is an early warning indicator of disasters. Had the agents received a signal about the sunspot realization, before the trade in sunspot-contingent assets, this information would weigh down both the stock market index and the expected output.

The remaining related literature divides roughly into three parts: The sequential service constraint and the possibility of bank runs, coordination-games such as speculative attacks, and coordination games as explaining business cycles.

Bank runs.—The results in the bank-run literature differ in two ways. First, a set of papers study how sunspot-non-contingent bank-deposit contracts depend on exogenously specified sunspot probabilities, e.g. Cooper-Ross (1998) and Peck and Shell (2003). Another strand studies how government intervention affects the set of equilibria – Keister (2014). Second, Freeman (1988) and Bental, Eckstein and Peled (1991) have studied bank contracts that allow dependence of a deposit contract on the sunspot.[1] Goldstein and Pauzner (2005) derive a unique probability of a bank-run using the arguments from the global games literature.

Speculative attacks.—We endogenize the disaster probability by allowing trade in the competitive financial markets where assets pay on the realization

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[1]A technical difference is that such contingent contracts can specify payments on individual actions, even off-equilibrium actions. Examples would be penalties on early withdrawals. By contrast, the Arrow securities studied in our are contingent on the aggregate variables/events.

of a publicly observed signal. In a different model with a public signal, Atkeson (2000) reached a conclusion opposite to ours. He argued that trading on assets would raise the number of equilibria by aggregating information that is dispersed among agents. Thus prices would serve as a coordinating device restoring multiplicity. There is no private information in our work and financial markets play a role by changing incentives rather than information sets.

Business cycles.— The research on coordination failures as causes of the real business cycle; Cooper and John (1988), Benhabib and Farmer (1999). In their work the set of equilibria is exogenously specified and they do not consider trading in sunspot-contingent assets.

Our paper is tangentially related to the literature on sunspot-driven asset market cycles as in Lagos and Zhang (2013) and Benhabib and Wang (2014). In these models multiple equilibria arise because of frictions in the financial markets. In Lagos and Zhang it is search and Benhabib and Wang it is market exclusion.

Plan of paper.—We begin with a model without financial markets. We then show how capital markets restrict the equilibrium set. We then look at asset pricing, the disaster premium and the effects of news shocks as manifested through changes in asset prices.

2   The model

Consider a production economy with two types of individuals lasting one period.

Endowments.—Type i receives endowment z_i, with $0 < z_1 < z_2$. The fraction of type i agents is f_i.

Preferences.—Utility depends on consumption $c \geq 0$ and investment $x \in \{0, 1\}$:

$$U(c) - \kappa x, \tag{1}$$

where κ is the utility cost of investment.[2]

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[2]We use this specification for analytical convenience. Ideally, one would consider a game with two stages and individuals ranking allocations using $v(c_0) + u(c_1)$. Because c_0 is the difference between the fixed endowment z_0 and investment the utility can be written as $v(z_0 - x) + u(c_1)$. Our specification obtains if we set $v(c) = \kappa c$.

We also assume that $U''' > 0$. This is sufficient to insure that it will be poor individuals that will be insured by the rich and not vice versa.

Production.—Let

$$\bar{x} = \sum f_i x_i$$

denote aggregate investment. We restrict our attention to symmetric pure strategy equilibria in which all agents of one type invest the same amount. As a function of own investment x and aggregate investment $\bar{x}$, an agent's output is

$$y(x, \bar{x}) = (\alpha + \bar{x})x.$$

Aggregate output is zero when $\bar{x} = 0$, and $1 + \alpha$ when $\bar{x} = 1$.

Consumption.—Consumption occurs after production has taken place and after assets and obligations are settled. If financial markets are closed, an agent consumes his endowment z and his output y which are his only sources of income. That is, $c = z + y(x, \bar{x}) > 0$. If financial markets are open, consumption also includes asset payoffs.

Aggregate shocks.—The model has no intrinsic shocks. There is an extrinsic variable called a "sunspot." We depart from the literature in that we have more sunspot realizations than there are equilibria. In fact, the sunspot can take on a continuum of values, as does temperature for example. The distribution of the sunspot variable is exogenous, but the mapping between sunspots and equilibrium play is endogenous.

The mechanism that endogenizes the mapping is agents' selection of what portfolio to trade, and the resulting beliefs concerning equilibrium play. As the agents choose which portfolio they want to trade among themselves, they will endogenize the probabilities with which the equilibria are selected. That is, they will endogenize the mapping between the sunspots and equilibrium play. We now define our terms more precisely.

Sunspots.—A sunspot is an exogenous random variable s that is uniformly distributed on $[0, 1]$ or, more formally, has Lebesgue measure $\mu(s)$ over the Borel subsets of $[0, 1]$. When financial markets are open, securities pay as a function of s. We start with the setting in which there are no financial markets.

The space of sunspot realizations is rich enough that it can be transformed into any other space of realizations. One can generate two conceptually different types of financial markets. One is for securities that pay depending on

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some other extrinsic random variable taking on values in some set other than $[0, 1]$. But, this can be shown to be equivalent to trading assets contingent on realizations in $[0, 1]$; one simply needs to change the probabilities associated with the new set of realizations.

Another market type, more relevant empirically, is for securities that pay based on outcomes that depend, at least in part, on actions that agents take, outcomes such as aggregate output. Our methods apply to such cases as well, as we explain in Section 3.

2.1 Equilibrium without financial markets

When financial markets are closed investment, x, is the only action. We then can talk interchangeably about equilibria and aggregate outcomes (as summarized below by $\bar{x}$). An agent's action can depend on his endowment, z, and on the sunspot, s. When the equilibrium is symmetric an agent's strategy is a function $x : \{z_1, z_2\} \times [0, 1] \rightarrow \{0, 1\}$.

Nash Equilibrium with no assets.—A Nash equilibrium is a function x such that for all $(z, s) \in \{z_1, z_2\} \times [0, 1]$,

$$x(z, s) \in \arg \max_{x \in \{0,1\}} \{U(z + y[x, \bar{x}(s)]) - \kappa x\} \tag{2}$$

where

$$\bar{x}(s) = \sum_{i=1}^{2} f_i x(z_i, s) \tag{3}$$

the following equilibria may arise at a particular sunspot realization s:

Equilibrium "L".—In the first type of equilibrium $x(z, s) = \bar{x}(s) = 0$ for all z. No individual invests. We call this a "low" equilibrium, or equilibrium L. For this to be an equilibrium we need the following two conditions:

$$U(z_1) \geqslant U(z_1 + \alpha) - \kappa,$$
$$U(z_2) \geqslant U(z_2 + \alpha) - \kappa.$$

That is, if $\bar{x}$ is zero, the reward to investing is just α, and each type should prefer not to invest. Because U is concave it is sufficient that the poor are *not* willing to invest:

$$U(z_1 + \alpha) - U(z_1) \leqslant \kappa. \tag{4}$$

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Equilibrium "H".—At the other extreme, everyone invests and $x\,(z,s) = \bar{x}\,(s) = 1$. We call this a "high" equilibrium, or equilibrium H. For this equilibrium to exist we need the following two conditions:

$$U(z_1 + \alpha + 1) - \kappa \geq U(z_1),$$
$$U(z_2 + \alpha + 1) - \kappa \geq U(z_2).$$

Again, because U is concave it is sufficient that the rich *are* willing to invest:

$$U(z_2 + \alpha + 1) - U(z_2) \geqslant \kappa. \tag{5}$$

In equilibria H and L, every agent takes the same action – either every agent invests or no agent does. There generally are, however, other equilibria and some of these are symmetric pure strategy equilibria, some not. In all these equilibria some agents invest while others do not.

Equilibrium M.—In this equilibrium only the poor invest and $\bar{x} = f_1$. We call this a "middle" equilibrium, or equilibrium M. For this equilibrium to exist we need the following two conditions:

$$U(z_1 + \alpha + f_1) - \kappa \geqslant U(z_1),$$
$$U(z_2 + \alpha + f_1) - \kappa < U(z_2).$$

Neither condition implies the other. This is also a symmetric equilibrium. Note that the conditions guaranteeing equilibria H and L do not involve the f_i, the conditions involving the existence of equilibrium M do depend on the f_i.

Asymmetric equilibria.—In these types of games the number of equilibria is generically odd. This means that when L and H both exist (see Proposition 1), there will also be a third equilibrium. This third equilibrium will either by asymmetric so that a fraction of agents of some type play $x = 1$ while the remainder play $x = 0$, or it will be the symmetric equilibrium M.

We shall assume that equilibrium M and the asymmetric equilibria are never chosen. If they sometimes were chosen, the number of cases proliferates, but nothing conceptually new is added. Thus we only admit L and H as possibilities. Sometimes only L exists, sometimes only H, and sometimes both H and L do.

Next, we define the parameter set under which both H and L exist, which is the set of parameters for which (4) and (5) both hold:

Definition 1. *Let* $\mathcal{P}_{aut} = \{(z_1, z_2, \kappa) : U(z_2 + \alpha + 1) - U(z_2) \geqslant \kappa \geqslant U(z_1 + \alpha) - U(z_1)\}.$

Roughly speaking, if α is high relative to κ, equilibrium L does not exist, and if α is low relative to κ, equilibrium H does not exist. If neither extreme obtains, L and H both exist. The set $\mathcal{P}_{aut}$ is always non-empty. To see this fix α. Then for any $z > 0$ we have $(\kappa, z_1, z_2) = (U(z+\alpha+0.5)-U(z), z, z) \in \mathcal{P}_{aut}$. That is there is a set, with a non-empty interior, where both the low and the high equilibria exist. Intuitively, endowment z_1 must not be too low as then type-1 individuals would always invest and the L equilibrium would not exist. Endowment z_2 must not be too high as then type-2 individuals would never invest and the H equilibrium would not exist.

Proposition 1. *Let* $U(c) = \ln c$. *Then there exists a non-empty set of parameters* $(z_1, z_2, \alpha, \kappa)$ *such that equilibria L and H both exist.*

Let $\delta = 1/(e^\kappa - 1) \approx 1/\kappa$. Then in a special case with logarithmic preferences we have:
$$\mathcal{P}_{ut} = \{(z_1, z_2, \alpha, \delta) : \alpha\delta \leqslant z_1 \leqslant z_2 \leqslant (\alpha + 1)\delta\}. \tag{6}$$

Figure 2 summarizes our findings. Region L(H) denotes the set of endowments for which only the L(H) equilibrium exists. Our main interest is in region H+L that consists of endowments such that both the L and the H equilibria exist. In what follows we study conditions under which this set persists when allow individuals to trade financial securities contingent on sunspots and the sunspots will be correlated with the type of equilibrium that is played at the production stage. The unmarked top left corner is where neither of the two equilibria exists.[3]

2.1.1 Equilibrium selection without financial markets

Let $\mathbf{L} \subset [0, 1]$ be the set of s realizations that lead to equilibrium L, and $\mathbf{H} = [0, 1] \setminus \mathbf{L}$ the set of s realizations that lead to equilibrium H. Define the

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[3]In this region there exist equilibria with $\bar{x} \in (0, 1)$. In such equilibria a fraction of individuals of the type 1 invests while others do not.
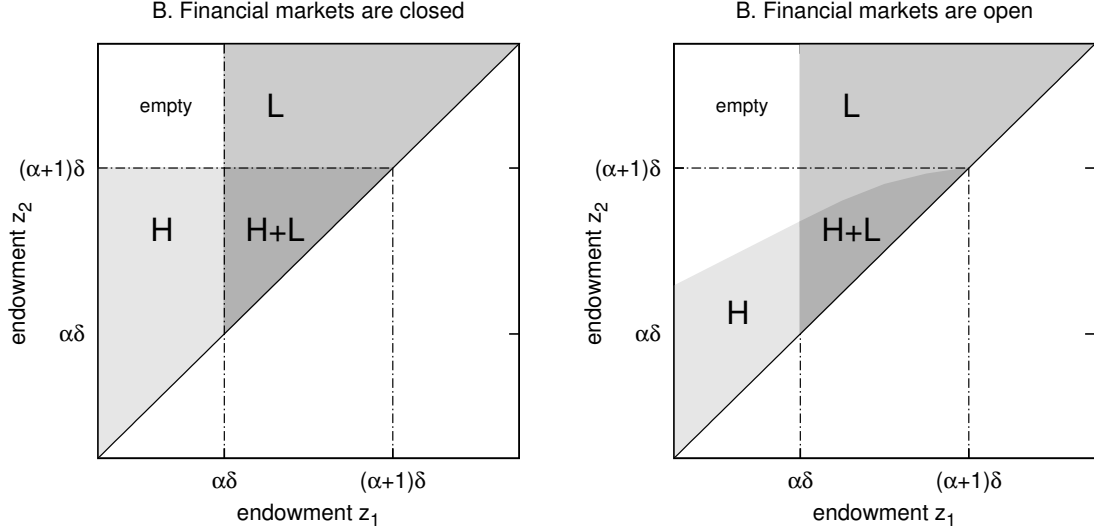
Figure 2: Equilibrium map

equilibrium indicator $\omega(s) \in \{L, H\}$ as follows:

$$\omega = \begin{cases} L & \text{if} \quad s \in \mathbf{L} \\ H & \text{if} \quad s \in \mathbf{H} \end{cases} \tag{7}$$

Thus the probabilities of the two equilibria being played are

$$\Pr(\omega = L) = \pi^{\mathrm{L}} \quad \text{and} \tag{8}$$
$$\Pr(\omega = H) = \pi^{\mathrm{H}}, \tag{9}$$

where
$$\pi^{\mathrm{L}} \equiv \mu(\mathbf{L}) \quad \text{and} \quad \pi^{\mathrm{H}} \equiv \mu(\mathbf{H}) = 1 - \pi^{\mathrm{L}}. \tag{10}$$

Any pair $(\pi^{\mathrm{H}}, \pi^{\mathrm{L}})$ of non-negative numbers summing to unity is admissible when there are no assets. With assets in the model, however, that is no longer true.

2.2   Equilibrium with financial markets

Arrow securities.—An Arrow security is in zero net supply, and pays a unit of consumption in a particular sunspot state s, and zero otherwise, and its

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price is $Q(s)$. There is a continuum of such securities, one for each s. Now an agent of type z has an additional set of actions consisting of the number of securities, $N(z, s)$ to hold as claims to consumption in state s. This adds for each agent a trading strategy $N : \{z_1, z_2\} \times [0, 1] \to R$. Market clearing then requires that for each $s \in [0, 1]$

$$\sum_{i=1}^{2} f_i N(z_i, s) = 0 \tag{11}$$

Budget constraint.—$N(z, \cdot)$ is agent z's *portfolio*. An agent trades before he receives his endowments and before he receives the output that he will have produced with the investment that he has made. His endowment is not contractible and his trades must therefore net out to zero. For a type-z agent, the portfolio $N(z, \cdot)$ must then satisfy[4]

$$\int_0^1 Q(s) N(z, s) \, d\mu(s) = 0. \tag{12}$$

Nash Equilibrium with financial markets.—It consists of three functions, $Q : [0, 1] \to R_{++}$, and $(x, N) : \{z_1, z_2\} \times [0, 1] \to \{0, 1\} \times R$ such that (11) holds and such that for all $(z, s) \in \{z_1, z_2\} \times [0, 1]$,

$$N(z, s) = \arg \max_{N(\cdot)} \int_0^1 \max_{x \in \{0,1\}} \left[U(z + y[x, \bar{x}(s)] + N(z, s)) - \kappa x \right] d\mu(s) \tag{13}$$

subject to (12), and such that investment is optimal *ex post*:

$$x(z, s) = \arg \max_{x \in \{0,1\}} \left\{ U(z + y[x, \bar{x}(s)] + N(z, s)) - \kappa x \right\}, \tag{14}$$

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[4]It must be true that $N(z, s) = c(z, s) - z - y(s)$ for each type. The corresponding inter-temporal budget constraint then is:

$$\int_0^1 Q(s) c(z, s) ds = z + \int_0^1 Q(s) y(s) ds.$$

Our first stage budget constraint does not include z. The alternative budget constraint formulation is: $\int_0^1 Q(s) N(z, s) \, ds = z$. It implies the same intertemporal budget constraint, because then $N(z, s) = c(z, s) - y(s)$, and, so, leaves the solution unchanged. This would not be true if the individuals had to make their portfolio decisions before they knew their type z. In this case there would be incentives to insure against the risk of being a type 1.

where $\bar{x}(s)$ is given in (3). Strictly speaking, equilibrium consists of 4 functions, $(Q(s), \bar{x}(s), x(z,s), N(z,s))$ satisfying (3), (11), (13) and (14).

Simple portfolios.—A simple portfolio of Arrow securities is an allocation that places equal weights on all those securities in which an agent is long and equal weights on those in which he is short. That is, for any subset $A \subseteq [0,1]$, a simple portfolio A implies the holdings

$$N(z,s) = \begin{cases} N_A & \text{for } s \in A \\ N_{\tilde{A}} & \text{for } s \in \tilde{A} \end{cases}.$$

A simple portfolio thus places equal weights on the securities $s \in A$, and an equal weight on securities with $s \in \tilde{A}$, so that (12) reads

$$N_A \int_A Q^s d\mu(s) = -N_{\tilde{A}} \int_{\tilde{A}} Q^s d\mu(s). \tag{15}$$

Other, unequally-weighted bundles are also possible, but deviations to such portfolios will not raise any agent's utility as we shall show later. We shall adopt the convention that $N_A \geq 0$ and $N_{\tilde{A}} \leq 0$, i.e., we shall label A for the set of securities that are assets in portfolio A, with the remainder being liabilities. Then a portfolio is fully described by the pair (A, N_A). Given this pair we then infer $N_{\tilde{A}}$ from the budget constraint. We shall refer to a portfolio as "portfolio A." An agent can trade portfolio A at any scale indexed by N_A.

Portfolio payoffs.—Let $w(A)$ denote the payoff of portfolio A. Then

$$w(A) = \begin{cases} N_A & \text{if } s \in A \\ N_{\tilde{A}} & \text{if } s \in \tilde{A} \end{cases}. \tag{16}$$

Probability of a positive payoff for portfolio A is denoted by π^A:

$$\pi^A \equiv \mu(A). \tag{17}$$

Portfolio choices.—This choice is made after the agents have discovered the z_i that they will be receiving prior to consumption. There are only two types of agents indexed by their endowments, rich and poor, only two portfolios will be chosen in equilibrium. Let A denote the portfolio chosen

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by the poor. The rich will take the other side of each s-security trade, and so the rich choose portfolio $\tilde{A}$.

This equilibrium selection is consistent with trades in that the poor wish to receive income if outcome L arises, and they pay the rich if outcome H arises which occurs because the preferences we assume have the property that $U''' > 0$.

Figure 3 illustrates a portfolio of a poor agent who is long on securities $s \in A$, and short on securities $s \in \tilde{A}$.
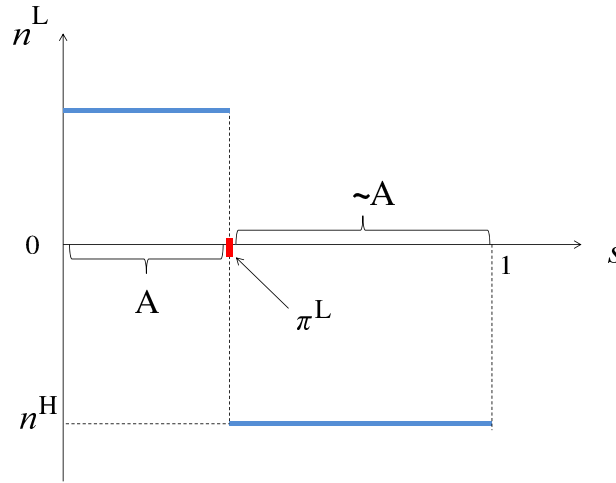
Figure 3: Portfolio of a low-endowment individual (z_1)

Trading strategies as functions of belief formation over $\bar{x}$.—Nash equilibrium beliefs are over the profile of others' actions in state s. In particular, the profile in question is the function $x(z,s)$. An agent cares only about the per-capita action of others, $\bar{x}(s)$, which is the following function of the sunspot:

$$\bar{x}(s) = \begin{cases} 0 & \text{if } s \in \mathbf{L} \Leftrightarrow \omega = \text{ L} \\ 1 & \text{if } s \in \mathbf{H} \Leftrightarrow \omega = \text{ H} \end{cases},$$

The financial-markets-open game *de facto* introduces just two additional actions, namely

(i) which portfolio A to trade, and

(ii) what quantity N_A to trade.

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c) Sufficiency requires that neither agent type wants to deviate to a different portfolio, i.e., to a set $A \neq \mathbf{L}$. What the agent wants is insurance. Given the beliefs specified above, however, his production income depends on ω alone. At the equilibrium portfolio, the same is true for his asset income. In other words, for the poor agent, asset income is perfectly negatively correlated with his production income, whereas for the rich, asset income is perfectly positively correlated. We show that because $U''' > 0$, the poor are priced out of claims in states $s \in H$ and the rich are priced out of claims in state $s \in L$.

Trading equilibrium.—An equilibrium entails simple portfolios for all agents. They are of the form

$$A = \mathbf{L} \quad \text{and} \quad \sim A = \mathbf{H} \quad \text{for the poor,}$$
$$A = \mathbf{H} \quad \text{and} \quad \sim A = \mathbf{L} \quad \text{for the rich.} \tag{18}$$

That is, the disaster states $s \in \mathbf{L}$ entail transfers to the poor, whereas states $s \in \mathbf{H}$ entail transfers to the rich.

Once A is given, all securities $s \in \mathbf{L}$ will have the same price that we shall denote by Q^{L}, and all securities $s \in \mathbf{H}$ will have the same price that we shall denote by Q^{H}. Then

$$q^{\mathrm{L}} = \pi^{\mathrm{L}} Q^{\mathrm{L}} \quad \text{and} \quad q^{\mathrm{H}} = \pi^{\mathrm{H}} Q^{\mathrm{H}}.$$

For the equal-weighted assets and equal-weighted liabilities portfolios we shall now use the notation

$$N(z, s) \equiv \left\{ \begin{array}{l} n_z^{\mathrm{L}} \ \textit{if} \ s \in \mathbf{L} \\ n_z^{\mathrm{H}} \ \textit{if} \ s \in \mathbf{H} \end{array} \right. .$$

In that case these new definitions and (15) imply that type-z agents' asset trades must satisfy the following budget constraint

$$q^{\mathrm{L}} n_z^{\mathrm{L}} + q^{\mathrm{H}} n_z^{\mathrm{H}} = 0.$$

We assume that the portfolio choices are made simultaneously and non-cooperatively. Each security trades at the price q^{L} if $s \in \mathbf{L}$ or q^{H} if $s \in \mathbf{H}$. A trading equilibrium is then indexed by $\mathbf{L}$, and associated with these equilibria is a "disaster probability" π^{L}, defined in (17). Not all $\pi^{\mathrm{L}} \in [0, 1]$ are equilibria, as we shall see, but generally a continuum exists.

Further suppose that there operate financial markets that trade portfolios paying one unit of consumption good conditional on the realization of ω. Security L (H) pays one unit if and only if state $\omega = L$ ($\omega = H$) realizes. Security ω is traded at price q^ω and the trade occurs before endowments are delivered. We let n_z^ω to denote quantity of securities ω purchased by a type-z individual.

The financial market clearing conditions for securities L and H are:

$$f_1 n_1^L + f_2 n_2^L = 0, \tag{19a}$$
$$f_1 n_1^H + f_2 n_2^H = 0. \tag{19b}$$

where we write $n_i = n_{z_i}$ to keep notation short.

The goods-market clearing conditions are:

$$f_1 c_1^\omega + f_2 c_2^\omega = Y^\omega, \quad \omega \in \{L, H\},$$

where Y^ω denotes the aggregate output in state ω:

$$Y^\omega \equiv \begin{cases} \alpha + 1 & if \quad \omega = H \\ 0 & if \quad \omega = L \end{cases} . \tag{20}$$

endow. z_1, z_2 assigned   portfolios (n_i^L, n_i^H) chosen ω realized   production   security payoffs consumption
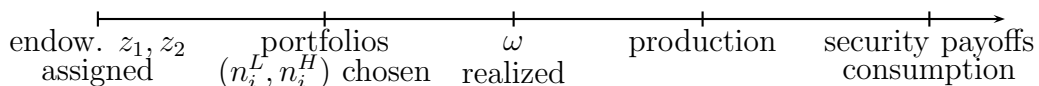
Figure 4: Timing of events

The first-order necessary conditions imply that for each individual the marginal rate of substitution between consumption in the states H and L equals the relative price:

$$\frac{U'(z + \alpha + 1 + n_z^H)}{U'(z + n_z^L)} = \frac{\pi^L}{1 - \pi^L} \cdot \frac{q^H}{q^L}, \quad z \in \{z_1, z_2\}. \tag{21}$$

The above implies that the marginal rate of substitution is the same across individuals: This is a standard risk-sharing result that obtains here because markets are complete.

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To understand portfolio decisions of the two types consider the case when the financial markets are closed. While a low-endowment type-1 individual has lower utility in every state his relative marginal value of consumption is higher in the low outcome:

$$\frac{U'(z_1)}{U'(z_2)} > \frac{U'(z_1 + \alpha + 1)}{U'(z_2 + \alpha + 1)}.$$ (22)

A sufficient condition for the above to hold is a decreasing absolute risk aversion that, in turn, is true if $U'''(c) > 0$.[5] So, we expect the low-endowment type to purchase securities that pay in state $\omega = L$, $n_1^L \geqslant 0$, and sell securities that pay in state $\omega = H$ ($n_1^H \leqslant 0$). This intuition will be used to derive sufficient conditions for existence of equilibria.

2.3 Optimal portfolios with logarithmic utility

With $U(c) = \ln(c)$ equation (21) simplifies to:

$$\frac{z_1 + n_1^L}{z_1 + \alpha + 1 + n_1^H} = \frac{z_2 + n_2^L}{z_2 + \alpha + 1 + n_2^H} = \frac{\pi^L}{\pi^H} \frac{q^H}{q^L},$$

and implies:

$$\frac{q^H}{q^L} = \frac{\pi^H}{\pi^L} \frac{\bar{z}}{\bar{z} + \alpha + 1}.$$

Using the budget constraints and the market clearing conditions allows us solving for the optimal portfolios:[6]

$$n_2^L = -\pi^H f_1 \Delta_z \frac{\alpha + 1}{\bar{z} + \alpha + 1}, \quad n_1^L = \pi^H f_2 \Delta_z \frac{\alpha + 1}{\bar{z} + \alpha + 1},$$ (23a)

$$n_2^H = \pi^L f_1 \Delta_z \frac{\alpha + 1}{\bar{z}}, \quad n_1^H = -\pi^L f_2 \Delta_z \frac{\alpha + 1}{\bar{z}}.$$ (23b)

with $\Delta_z = z_2 - z_1$. Notice that $n_2^H > 0$ as conjectured.

At the optimal portfolios agents achieve perfect insurance across the two outcomes. By this we mean that consumption of each type is a fixed fraction of the total good supply. This implies that consumption of any type in

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[5]That is $-u''(c)/u'(c)$ must be decreasing.

[6]We use market clearing conditions to determine optimal purchases of securities by type-1 individuals: $n_1^\omega = -(f_2/f_1)n_2^\omega$, $\omega \in \{L, H\}$.

outcome L is smaller than in outcome H.[7] Then notice that $c_2^H > c_1^H$ because with the financial markets open the low-endowment type-1 repays the other type in outcome H. And in outcome L, consumption ordering is implied by the endowment ordering:

$$c_2^L - c_1^L = z_2 + n_2^L - z_1 - n_1^L = \Delta_z - \pi^H \Delta_z \frac{\alpha + 1}{\bar{z} + \alpha + 1} > 0.$$

We state this result formally as we shall refer to it later.

Lemma 2. $c_z^L < c_z^H, \forall z$ and there is no "consumption leapfrogging": $c_1^\omega < c_2^\omega, \omega = L, H$.

The effect of group size.— If each individual from a larger low-endowment type saved one unit then individuals in the other, smaller, group would receive more than one unit. For this reason, the payment to the high-endowment individuals in outcome H is rather large. But a large payment, as is shown later, may destroy outcome H as an equilibrium in the subgame. That is, we expect the financial markets to have a strong effect on the set of possible equilibria when there is a sizable group of endowment-poor individuals. In societies with a small fraction of poor individuals, opening the financial markets is unlikely to affect the set of equilibria. Yet, in the latter case significant improvement in risk-sharing across outcomes can be achieved. This is true because it costs little for the populous high-endowment group to insure a small group of poor. Formally, $|c_1^H - c_1^L|$ decreases as f_2 increases. Note that low-endowment individuals demand insurance, and high-endowment individuals are willing to provide it, regardless of the group proportions (f_1, f_2). The size of the two groups matters for its effect on the financial market clearing – that is ability of one group to satisfy demands of the other.

2.4   The equilibrium set in the post-trade subgame

Next we study how the set of subgame equilibria changes as a function of the transfers. We will later map the feasible outcomes to the underlying probabilities which affect the transfers.

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[7]This can be also proven directly. For type 1 we have $n_2^L > 0 > n_2^H$. Yet, because $f_2 \Delta_z < \bar{z}$, we get:

$$c_1^H - c_1^L = (\alpha + 1) \left[1 - \pi^L f_2 \Delta_z / \bar{z} - \pi^H f_2 \Delta_z / (\bar{z} + \alpha + 1) \right] \geqslant 0.$$

For type 2 the claim is trivial because $c_2^L = z_2 + n_2^L \leqslant z_2 < z_2 + 1 + \alpha \leqslant z_2 + 1 + \alpha + n_2^H = c_2^H$.

Suppose that without the financial markets only the low subgame equilibrium exists. We now ask if it is possible that after the financial markets open both subgame equilibria would survive. In the next section we ask if any of the equilibria could be destroyed.

Region L in figure 2: Suppose that when there are no financial markets only the L equilibrium exists: $z_2 \geqslant z_1 \geqslant \alpha\delta, z_2 \geqslant (\alpha+1)\delta$. When the financial markets are open the H and L equilibria exist if:

$$z_1 + n_1^{\mathrm{L}} \geqslant \alpha\delta, \tag{24a}$$

$$z_2 + n_2^{L} \geqslant \alpha\delta, \tag{24b}$$

$$(\alpha + 1)\delta \geqslant z_1 + n_1^{\mathrm{H}}, \tag{24c}$$

$$(\alpha + 1)\delta \geqslant z_2 + n_2^{H}. \tag{24d}$$

The first inequality always holds because $\alpha\delta \leqslant z_1, 0 \leqslant n_1^{\mathrm{L}}$. The second inequality must be checked. The third inequality always holds because $z_1 \leqslant (\alpha + 1)\delta, n_1^{\mathrm{H}} \leqslant 0$. The fourth inequality cannot hold because $z_2 \geqslant (\alpha + 1)\delta$ and $n_2^{H} \geqslant 0$. So, the H equilibrium cannot be created.

Region H in figure 2: Suppose that when there are no financial markets only the H equilibrium exists: $z_1 \leqslant z_2 \leqslant (\alpha + 1)\delta, z_1 \leqslant \alpha\delta$. When the financial markets are open the H and L equilibria exist if the inequalities in (24) hold. The first and the second inequality could hold. But the third inequality cannot hold because $z_1 \leqslant (1 + \alpha)\delta, n_1^{\mathrm{H}} \leqslant 0$. So, the L equilibrium cannot be created either. We state these results in the following proposition.

Proposition 3. *Opening financial markets cannot create the H (L) equilibrium if only the L (H) equilibrium existed under financial autarky.*

We now ask if equilibria can be destroyed. Case 1(2) below studies if opening the financial markets can destroy the H (L) equilibrium if the two equilibria existed under financial autarky.

Region H+L in figure 2, case 1: Suppose that the H and L equilibria exist: $(\alpha + 1)\delta \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta$. When the financial markets are open only the H equilibrium exists if:

$$\alpha\delta \geqslant z_1 + n_1^{\mathrm{L}}, \ or \ \alpha\delta \geqslant z_2 + n_2^{L}, \tag{25a}$$

$$(\alpha + 1)\delta \geqslant z_1 + n_1^{\mathrm{H}}, \tag{25b}$$

$$(\alpha + 1)\delta \geqslant z_2 + n_2^{H}. \tag{25c}$$

The third inequality in the above system cannot hold because $z_1 \leqslant (\alpha + 1)\delta$ and $n_1^{\mathrm{H}} \leqslant 0$.

Proposition 4. *Opening financial markets cannot destroy the L equilibrium if both equilibria existed under financial autarky.*

Region H+L in figure 2, case 2: Suppose that when there are no financial markets the H and L equilibria exist: $(\alpha + 1)\delta \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta$. When the financial markets are open only the L equilibrium exists if:

$$z_1 + n_1^{\mathrm{L}} \geqslant \alpha\delta, \tag{26a}$$

$$z_2 + n_2^{L} \geqslant \alpha\delta, \tag{26b}$$

$$(\alpha + 1)\delta \leqslant z_1 + n_1^{\mathrm{H}}, \ \ or \ (\alpha + 1)\delta \leqslant z_2 + n_2^{H}. \tag{26c}$$

The first inequality always holds. The inequality $(\alpha + 1)\delta \leqslant z_1 + n_1^{\mathrm{H}}$ in the third row cannot hold. So, we need to check if the intersection of $\{z_2 + n_2^{L} \geqslant \alpha\delta, (\alpha + 1)\delta \leqslant z_2 + n_2^{H}\}$ and $\{(\alpha + 1)\delta \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta\}$ is non-empty. This can be easily verified by setting $z_1 < z_2 = (\alpha + 1)\delta$. In this case $n_2^{H} > 0$ and $z_2 + n_2^{H} > (1 + \alpha)\delta$.

Proposition 5. *There exists a non-empty set of parameters such that opening financial markets can destroy the H equilibrium if both equilibria existed under financial autarky.*

2.5 Admissible equilibrium values of π^{L}

Suppose that when the financial markets are closed the L and the H equilibria exist: $(1 + \alpha)\delta \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta$. When the financial markets are open the H and L equilibria exist if:

$$z_1 + n_1^{\mathrm{L}} \geqslant \alpha\delta, \tag{27a}$$

$$z_2 + n_2^{L} \geqslant \alpha\delta, \tag{27b}$$

$$(\alpha + 1)\delta \geqslant z_1 + n_1^{\mathrm{H}}, \tag{27c}$$

$$(\alpha + 1)\delta \geqslant z_2 + n_2^{H}. \tag{27d}$$

The first inequality always holds because $z_1 \geqslant \alpha\delta$ and $n_1^{\mathrm{L}} \geqslant 0$. The second inequality always holds because $|n_2^{L}| < \Delta_z$ and $z_1 \geqslant \alpha\delta$. The third inequality

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always holds because $(\alpha + 1)\delta \geqslant z_1$ and $n_1^{\mathrm{H}} \leqslant 0$. The fourth inequality must be verified. So, both equilibria survive if:[8]

$$(\alpha + 1)\delta \geqslant z_2 + n_2^H \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta. \tag{28}$$

After substituting the formula for n_2^H we obtain:

$$\pi^{\mathrm{L}} \leqslant \frac{(\alpha + 1)\delta - z_2}{(\alpha + 1)f_1\Delta_z/\bar{z}} \equiv \bar{\pi}^L \tag{29}$$

The region where both equilibria exist before and after the financial markets open is plotted in figure 2, panel B. At the upper boundary of the union of the H and the H+L regions, the endowment-rich type 2 is indifferent between investing and not.

Together with the condition for the existence of the two equilibria under financial autarky, $\alpha\delta \leqslant z_1 \leqslant z_2 \leqslant (\alpha + 1)\delta$, inequality (29) is the restriction on equilibrium beliefs and model parameters under which the two equilibria exist regardless of the financial regime. Intuitively, the probability of the L equilibrium, π^{L}, cannot be too high as then the high-endowment type-2 individuals would not invest in the high equilibrium and the latter would cease to exist. This happens because as π^{L} grows the relative price $q^{\mathrm{L}}/q^{\mathrm{H}}$ and n_2^H increase. But when a payoff in any state increases incentives to invest decrease. The restriction on π^{L} could also be vacuous, e.g. when $\Delta_z = 0$, or it could be "prohibitive," e.g. when $z_2 = (\alpha + 1)\delta$.

As explained above, the upper bound on π^{L} stems from the restriction that the high-endowment type-2 agents should support the H equilibrium. The term $(\alpha + 1)\delta - z_2$ is the largest trade that does not destroy type-2's incentives to invest. The term $(\alpha+1)f_1\Delta_z/\bar{z}$ determines the size of the trade, see (23b). If there were no heterogeneity, $\Delta_z/\bar{z}$ is close to zero, then there would be no trade; so, any π^{L} would do. $(\alpha + 1)f_1$ is the additional income earned by the poor when the H equilibrium is selected. The larger it is the stronger are trading motives and, hence, higher chances of destroying the equilibrium. Figure 5 illustrates the relation between $\bar{\pi}^L$ and (α, δ). Notice that as α and/or δ increase the L equilibrium disappears. Similarly, when α and/or δ decrease the H equilibrium disappears. For intermediate value of (α, δ) the figure plots the limit on the probability of the L equilibrium.

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[8]Notice that the inequality $(\alpha + 1)\delta \geqslant z_2$ is redundant. This means that the set of parameters for which the H equilibrium exists shrinks when the financial markets open.

When α and/or δ are high, but not enough to destroy the L equilibrium, the probability of the L equilibrium is unrestricted. In this case the high-endowment type-2 individuals have a substantial "insurance capacity" and provide for the low-endowment individuals while continuing to invest. This area corresponds to the plateau in the figure.
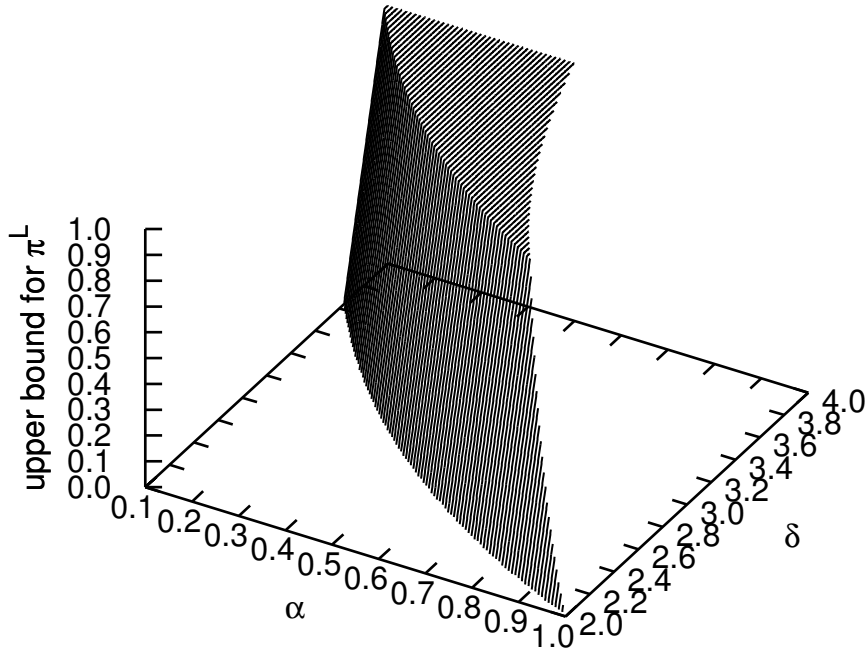

Figure 5: Relation between $\bar{\pi}^L$ and (α, δ).

Observe that the upper bound on π^{L} is linear in δ and hyperbolic in α:

$$\bar{\pi}^L = [f_1 \Delta_z / \bar{z}]^{-1} \left[\delta - \frac{z_2}{\alpha + 1} \right]. \tag{30}$$

It increases with δ as this expands the area where both equilibria are possible. As α increases, two effects are operational. First, it is harder to destroy the H equilibrium: the upper bound on consumption of a type-2 individual increases. Second, trades increase as they are proportional to $(1 + \alpha)$ measuring the increase in the aggregate consumption between the L and the H

equilibrium. However, financial payoffs of any individual cannot not exceed $(1 + \alpha)$, and the first effect dominates.

Lastly, the upper bound on π^L depends on δ. This parameter has no effect on the size of financial trades or equilibrium prices. It also difficult to calibrate. For these reasons, we provide an alternative upper bound that does not involve δ. To this end, note that for equilibrium H to exist we must have $z_1 \geqslant \alpha\delta$. This imposes an upper bound on δ that can, in turn, be used in (30):

$$\bar{\pi}^L \leqslant [f_1 \Delta_z / \bar{z}]^{-1} \left[\frac{z_1}{\alpha} - \frac{z_2}{\alpha + 1} \right]. \tag{31}$$

Size of disasters vs. their frequency.—The size of disasters is governed by α – The larger is α, the more severe is the drop in the aggregate consumption: $C^L / C^H = \bar{z} / (\bar{z} + \alpha + 1)$. If α is taken as a measure of disaster size, then the size and frequency of disasters are positively related: The larger the disaster, the higher is the likelihood that it can occur in equilibrium. Of course, this pertains only to coordination failures; the opposite is probably true of wars and natural catastrophes.

The effect of dispersion of endowments.—Rising inequality, as measured by $\Delta_z / \bar{z}$, reduces the probability of equilibrium L. The more dispersed endowments are the larger are incentives to trade in equilibrium for then the rich value consumption much less than the poor. On the other extreme, when endowments are similar there is little incentives to trade. In this case the set of possible sunspot equilibria is unaffected as $\bar{\pi}^L \geqslant 1$ is not restrictive. When dispersion is small, $\Delta_z / \bar{z} \leqslant [\delta - z_2 / (\alpha + 1)] / f_1$ according to (30), then opening the financial markets has no effect on the probability of equilibrium L. This implies that if a fictitious planner could redistribute endowments across individuals he would not choose an equal distribution. That is increased inequality has a positive welfare effect.

The fact that inequality is beneficial echoes Hopenhayn and Vereshchagina's (2009) finding that gambling can be welfare improving. In their setting there are two technologies – a safe and a risky technology, both offering the same expected return. Because they can abandon a low-return project (exit) entrepreneurs with low wealth choose to invest in risky assets. Thus, gambling is welfare-improving. In our setting the incentives of the rich agents are affected: inequality reduces incentives of the rich agents to work and destroys multiplicity.

2.5.1 Effects of changes in risk-aversion

So far all of our analysis has been done under the assumption of logarithmic preferences that greatly simplified our derivations. However, our results carry over to the case with any CRRA utility function.[9] What role does the risk-aversion play? Figure 6 plots purchases of Arrow security H of the poor type-1 and the rich type-2 individuals as functions of π^L. As the risk-aversion coefficient increases from $\gamma = 1$ to $\gamma = 5$ the position in the Arrow security H of the rich type-2 increases faster. That is the more risk-averse individuals opt for a more equitable allocation that is supported by taking larger portfolio positions. Such a large transfer, however, violates this type's "incentives constraint" – the rich type stops investing and only the L equilibrium survives. Thus, as the level of risk-aversion increases the maximum equilibrium probability π^L decreases.

This has the following implication for asset pricing. As one increases the risk-aversion the risk-premium would increase and the risk-free rate would decrease bringing the model closer to the data. But the upper bound on the probability of disasters would decrease simultaneously, limiting the risk-premium. One should not treat the probability of a disaster as fixed while adjusting the risk-aversion potentially. In other words, the success of the asset-pricing models relying on the disaster risk may be limited.[10]

2.6 The set of equilibria, $\mathcal{A}$

Having established a perfect correlation between asset positions and actions, we may abbreviate the definition of equilibrium as follows: Instead of the

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[9]The ratio of individual consumption and the aggregate supply of goods across the two sub-game equilibria are all the same as in the case with logarithmic preferences. However, the relative price of the two Arrow securities is now a non-linear function of the relative endowment: $q^L/q^H = \pi^L/\pi^H(\bar{z}/(\bar{z} + \alpha + 1))^{-\gamma}$. The optimal investment in the Arrow security L by the poor type-1 individual is:

$$n_2^H = f_1 \Delta_z \frac{\alpha + 1}{\bar{z}} \frac{\pi^L \bar{z}^{1-\gamma}}{\pi^L \bar{z}^{1-\gamma} + \pi^H (\bar{z} + \alpha + 1)^{1-\gamma}}.$$

The upper bound on the probability of the L equilibrium solves the following equation: $(z_2 + n_2^H)^{1-\gamma} = (z_2 + \alpha + 1 + n_2^H)^{1-\gamma} - (1-\gamma)\kappa$. Because n_2^H is an increasing function of π^L it is easy to show that there is a unique solution for the cut-off $\bar{\pi}^L$.

[10]Gourio (2012) and Gabaix (2012) show that incorporating a time-varying severity of disasters improves the model's asset-pricing predictions along several dimensions.
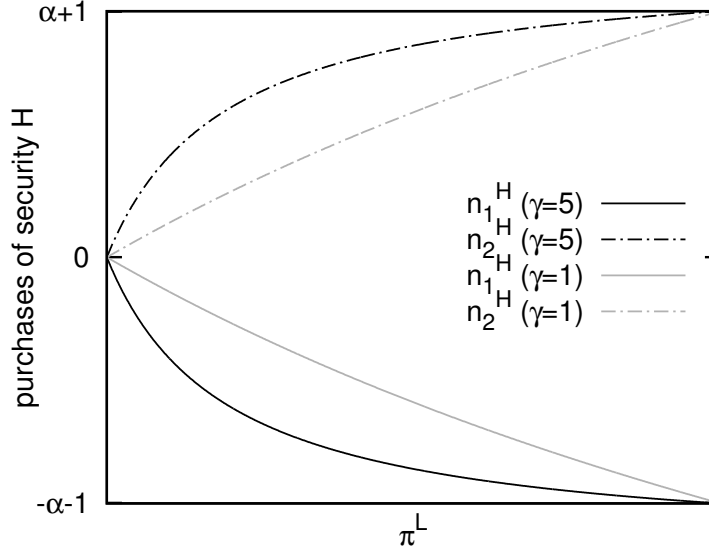
Figure 6: Optimal portfolios for different levels of risk-aversion

objects defined in (13) and (14), we shall refer to equilibrium as the set $A = \mathbf{L}$ of ω values for which agents all set $x = 0$. I.e., it is the set of ω's for which equilibrium L results. The gross asset positions $N(\cdot)$ of the two types of agents then follow straightforwardly.

The equilibrium set $\mathcal{A}$.—The equilibrium is any set of disaster states the measure of which does not exceed $\bar{\pi}^{\mathrm{L}}$. I.e., is the collection of Borel subsets $A \subset [0, 1]$ for which $\pi^A \leq \bar{\pi}^{\mathrm{L}}$. Thus the set of equilibria is the set

$$\mathcal{A} = \left\{ A \in \mathcal{B}\left([0, 1]\right) \mid \int_A d\mu\left(s\right) \leq \bar{\pi}^{\mathrm{L}} \right\}. \tag{32}$$

We have provided only an upper bound on π^{L}. One may ask whether the use of asset trades can narrow things down further if the game were different in some way. We can see two options for narrowing down the set equilibrium π^{L}. One way is to use the theory of the Core in which competition occurs among coalitions, i.e., a theory in which groups of agents can deviate from any outcome. A second way to reduce the number of equilibria is to add stages to the security trading game. Banks could propose securities by sending messages to agents who then would choose where to trade. Using the

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Core equilibrium concept would lead to an open set problem in the coalitions' choice of π^L for the following reason: The upcoming Lemma shows that a smaller value of π^L Pareto dominates a larger, recognizing, of course, that the equilibrium asset prices q^L and q^H depend on π^L. In other words, the equilibria, as indexed by π^L, are Pareto ranked. This is our next result.

2.6.1 Welfare

The utility of a type-i individual is:

$$W_i = \pi^L U(z_i + n_i^L) + (1 - \pi^L) U(z_i + \alpha + 1 + n_i^H). \qquad (33)$$

We will later see that the type-1's portfolio positions (n_1^L, n_1^H) decrease with π^L. Hence, utility of a type-1 individual is strictly decreasing in π^L. The type-2's portfolio positions, on the other hand, increase with π^L. That is, as the probability of L rises, consumption of a type-2 individual increases in both states but his overall utility still falls as H becomes less likely. Lemma 6 shows that W_2 is decreasing in π^L as long as $\pi^L \pi^H f_1 \Delta_z / \bar{z} < 0.5$. This constraint is not vacuous. But it is also not restrictive as it would be satisfied if, for example, $\Delta_z < 2\bar{z}$.

Lemma 6. *If $\pi^L \pi^H f_1 \Delta_z / \bar{z} < 0.5$ then $dW_i / d\pi^L < 0, i = 1, 2$.*

Given this, competition among coalitions would lead them towards the Pareto-optimal outcome. But at $\pi^L = 0$ there can be no trade. We then would be back in a no-financial-asset game that admits both equilibria, L and H.

Alternatively, we may add a prior stage to the security trading game. Banks could propose securities by sending messages to agents who then would choose where to trade. It appears that this could be formulated so as to lead to the same outcome as the Core with the same open set problem. At the moment, then, we cannot shrink $\mathcal{A}$ any further.

2.7 Asset pricing

The same allocations can be implemented by trade in risk-free bonds and equity claims as by trade in sunspot-contingent Arrow securities. The payoff of Arrow security L is the same as payoff of the portfolio consisting of 1 bond and $-\frac{1}{1+\alpha}$ equity claims. Arrow security H is equivalent to $\frac{1}{1+\alpha}$ equity claims.

Suppose now that individuals also receive endowment z_0 in period 0 before types are revealed in period 1. In period 1 type-i individual receives

ω	Equity	Bond	Arrow sec. L	Arrow sec. H
L	0	1	1	0
H	$1+\alpha$	1	0	1

Table 1: Asset payoff matrix

endowment z_i and chooses whether to invest or not as before. In period 0 individuals are offered to buy (equity) claims to the aggregate output Y^ω, and the risk-free bond that pays one unit of consumption regardless of the realized ω. The two assets are traded at prices q^e and q^b that will be determined later.

Timing of events is as follows:
1. Individuals trade bonds and claims to the aggregate output, consume;
2. Individuals learn their type, receive endowments z_i;
3. Individuals trade sunspot-contingent securities, produce and consume.

The period 0 budget constraint is:

$$c_0 + q^e n_0^e + q^b n_0^b = z_0. \tag{34}$$

Since all individuals are symmetric in period 0 we do not use index i. For the same reason purchases of the two assets, equity claim and bond, is zero in equilibrium:

$$n_0^e = n_0^b = 0. \tag{35}$$

So, everyone simply consumes his endowment: $c_0 = z_0$. The two asset prices satisfy the following Euler equations:

$$q^b = \beta E \left[\frac{U'(z_i + n_i^\omega)}{U'(z_0)} 1 \right], \tag{36a}$$

$$q^e = \beta E \left[\frac{U'(z_i + n_i^\omega)}{U'(z_0)} Y^\omega \right], \tag{36b}$$

where the expectation is over types i and states ω. The expected return on bond is:

$$E[R^b] = \frac{1}{q^b} = \frac{U'(z_0)}{\beta \sum_z \sum_\omega f_z U'(z + \alpha + 1 + n_z^\omega)},$$

and the expected return on equity is:

$$E[R^e] = \frac{(\alpha + 1)(1 - \pi^L)}{q^e} = \frac{U'(z_0)}{\beta \sum_z f_z U'(z + \alpha + 1 + n_z^H)},$$

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where the optimal portfolios are:

$$n_1^{\mathrm{H}} = -\pi^{\mathrm{L}} f_2 \Delta_z \frac{\alpha+1}{\bar{z}}, \quad n_2^{H} = \pi^{\mathrm{L}} f_1 \Delta_z \frac{\alpha+1}{\bar{z}}.$$

As the probability π^{L} increases, probability that an equity claim pays decreases. So, the equity claim is valued less and it must offer a higher return. At the borderline case with $\pi^{\mathrm{L}} = 0$ the risk-free bond and the equity claim yield the same return. We state these results in the following proposition.

Proposition 7. *With logarithmic preferences the expected equity premium is a) always non-negative, and b) an increasing function of π^L.*

Proof. By direct differentiation. □

Next, we compute the price of a disaster insurance. The disaster insurance pays one unit of consumption good when the L equilibrium realizes. Notice that the risk-free bond pays $(1,1)$ in the two states and a claim to equity pays $(\alpha+1,0)$. Then a disaster insurance claim generates the same payoff as a portfolio comprised of 1 bond and $-\frac{1}{\alpha+1}$ equity claims. So, in the absence of arbitrage the price of the disaster insurance must be:

$$q^d = q^b - \frac{1}{\alpha+1}q^e = \beta \sum_z f_z \frac{\pi^{\mathrm{L}} U'(z + n_z^{\mathrm{L}})}{U'(z_0)}. \tag{37}$$

endowment. This allows us computing the returns explicitly: equilibrium L.

2.8   The effect of news shocks

The simplest treatment of a news shock is a prior signal ξ on s, drawn from the density $g(\xi \mid s)$. Denote the posterior over s by $\mu(s \mid \xi)$. This in general makes the states not equally likely but the main thing is that the signal changes the disaster probability from $\mu(A)$ to $\mu(A \mid \xi)$.

In order that the previous analysis should apply, however, it is easier to have the new shock leave the likelihood of s unchanged, but to change the designation of which equilibrium is associated with which value of s. We now put a prior distribution ν over $\mathcal{A}$ and, derived from ν, a prior distribution λ over $\left[0, \bar{\pi}^{\mathrm{L}}\right]$. The news consists of an announcement of a particular $A \in \mathcal{A}$

and, hence, an implied value for $\pi^L \in \left[0, \bar{\pi}^L\right]$. The measure ν is an object different from μ; the latter tells us the likelihood of various ω's occurring, whereas ν tells us the likelihood of which combinations of the ω's are to lead to equilibrium L. Thus the measure ν generally will not be Lebesgue measure μ but, rather, can put greater weight on some Borel subsets of $\mathcal{A}$ and less weight on others.

In other words, a news shock is an announcement of the list of $\omega \in [0,1]$ that are to be considered disaster states. If many ω's are announced to be disaster states, then disasters become more likely, and this will affect asset prices as well as asset trading. The list of disaster states will be denoted by A. Suppose that the announced A is drawn randomly from the equilibrium set $\mathcal{A}$ taking $\nu(A)$ as the measure. This implies π^L which is drawn randomly from the set of numbers not exceeding $\bar{\pi}^L$. The prior measure over π^L is λ, where

$$\lambda(\pi^L) = \int_{\mathcal{A}} \mu(A) \, d\nu(A) \tag{38}$$

When A is announced, beliefs shift from ν to a point mass on A or, from λ to a point mass on π^L. This has the interpretation of a belief shock, since it does not affect fundamentals. From now on we shall refer to the news shock as the revelation of a specific value $\pi^L \in \left[0, \bar{\pi}^L\right]$.

Do stock prices lead output?—We ask if q^e is a leading indicator of the aggregate output Y^ω. Conditional on π^L, expected output is $E[Y] = (1 - \pi^L)(\alpha + 1)$. Then before π^L is revealed asset prices are:

$$\tilde{q}^b = \beta \int_0^{\bar{\pi}^L} \frac{\sum_z f_z[\pi^L U'(z + n_z^L) + (1 - \pi^L)U'(z + \alpha + 1 + n_z^H)]}{U'(z_0)} d\lambda(\pi^L), \tag{39a}$$

$$\tilde{q}^e = \beta \int_0^{\bar{\pi}^L} \frac{\sum_z f_z U'(z + \alpha + 1 + n_z^H)}{U'(z_0)} (\alpha + 1)(1 - \pi^L) d\lambda(\pi^L). \tag{39b}$$

The news effect is the difference between the expected price of a portfolio and the realized price after the π^L is revealed:

$$New se \equiv \tilde{q}^e - \beta \sum_z f_z \frac{U'(z + \alpha + 1 + n_z^H)}{U'(z_0)} (1 - \pi^L)(\alpha + 1), \tag{40a}$$

$$New sb \equiv \tilde{q}^b - \beta \sum_z f_z \frac{\pi^L U'(z + n_z^L) + (1 - \pi^L)U'(z + \alpha + 1 + n_z^H)}{U'(z_0)}. \tag{40b}$$

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Because price of equity is a decreasing function of π^{L} it is positively correlated with the expected aggregate output $E[Y]$. So, the stock market index is a leading indicator of output.

The financial market volume[11] is:

$$v = \sum_{w \in \{H,L\}} |f_2 n_2^w| = (1 + \alpha) f_1 f_2 \left\{ \frac{\pi^{\mathrm{L}}}{\bar{z}} + \frac{\pi^{\mathrm{H}}}{\bar{z} + \alpha + 1} \right\}. \tag{41}$$

So, when π^{L} increases the market volume also increases. That is, the trading volume leads the aggregate output.

In a related paper, Angeletos and La'O (2014) also study shocks to beliefs about the actions of others. They do not have multiplicity of equilibria as we do, but they instead have aggregate shocks. The presence of the latter, they show, also allows shocks to beliefs over actions to have real effects.

Is lagged consumption a sufficient statistic for current consumption?— Hall (1978) derived the implication that no variable apart from current consumption should be of any help in predicting future consumption. Hall did find that real disposable income did not help predict aggregate consumption, but that an index of stock prices did help predict it. In our two-period model the question can be posed as follows: Is z_0 a sufficient statistic for predicting y? The answer is "no" since news to π^{L} cannot be reflected in z_0 which is an endowment, and yet low π^{L} is a good news for Y and, hence, for the consumption of all agents. Although the proportions consumed by each type do change with π^{L}, lemma 2 shows that the consumption of each type is higher in equilibrium H than in equilibrium L.

A low realization of π^{L} is also a good news for the equity price, indicating that equity prices can help predict future consumption. Assume that z_0 is a random variable drawn from a known distribution. News then consists of a simultaneous "announcement" of (z_0, π^{L}) that is then followed by trade in the financial markets. It turns out that stock price is also not a sufficient statistic for Y. The level of prices depends on z_0 and therefore one needs to know z_0 in order to be able to predict future consumption. But knowledge of the pair (z_0, q^e) is sufficient to predict future consumption, consistent with what Hall finds empirically. Formally, consider a first-order approximation of q^e around $(z_0, \pi^{\mathrm{L}}) = (E(z_0), 0) : q^e = k_0 + k_z z_0 - k_\pi \pi^{\mathrm{L}}$ where $k_0, k_z, k_\pi > 0$. Expected aggregate consumption is: $E(C^w) = \bar{z} + \alpha + 1 - \pi^{\mathrm{L}}(\alpha + 1)$. Then consider

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[11]A symmetric formula can be defined using positions of a type-1 individual.

the following regression specification relating the expected consumption to the first-stage aggregate consumption z_0 and the equity price q^e: $E(C^w) = \beta^0 + \beta_z z_0 + \beta_q q^e = \beta^0 + \beta_z z_0 + \beta_q(k_0 + k_z z_0 - k_\pi \pi^L)$. One should find significant β_z and β_q. Moreover, β_q should be positive while the coefficient β_z should be negative.[12]

2.9 Example

Heathcote, Storesletten, and Violante (2006, Figure 4) report that an average of the variance of log wages and the variance of log earnings for a 33-year-old worker is 0.33. That is

$$var(z) = f_1 f_2 (ln(z_2) - ln(z_1))^2 = 0.33. \tag{42}$$

With $f_1 = 0.50$ we get $z_2/z_1 = x \equiv exp(2/\sqrt{3}) \approx 3.17$. So, we get: $z_1 = \bar{z}2/(1+x), z_2 = \bar{z}2x/(1+x)$. The restrictions imposed by existence of both equilibria are: $\alpha\delta \leqslant z_1 \leqslant z_2 \leqslant (\alpha+1)\delta$.

We assume $\delta = 3.5$. We choose $\bar{z} = 2.82, \alpha = 0.14$ so that $\bar{z}/(\bar{z}+\alpha+1) \approx 0.71$ as in Barro (2006) and the implied upper bound on π^L is 0.020, similar to Barro's (2006) estimate of 0.017.

We set z_0 so that no growth is expected in the aggregate consumption:

$$z_0 = \bar{z} + (\alpha+1)E[\pi^L]. \tag{43}$$

variable	value	moment
δ	3.50	–
f_1	0.50	Groups of equal size
(z_1, z_2)	(1.32,4.28)	Coefficient of variation for endowment is 0.33
α	0.23	29% loss of output in the L equilibrium
z_0	see (43)	Expected consumption growth is zero

Table 2: Parameters for the numerical example

Table 2 collects all the parameter assumptions. Figure 7 plots returns of the risk-free bond, the equity and the disaster claims. It is assumed that $z_0 = \bar{z} + (a+1)(1-\pi^L)$, that is the expected aggregate consumption is

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[12]Simple coefficient matching gives: $\beta_q = (\alpha+1)/k_\pi > 0, \beta_z = -\beta_q k_0 < 0$.

constant. The vertical line marks the upper bound on the probability of equilibrium L, $\bar{\pi}^L$. When $\pi^L = 0$ then there is only one state of the world – the H equilibrium – and the equity claim and the bond pay the same. When π^L reaches its upper bound 0.02 the return on equity is 0.64% and the risk-free return is -0.23% implying a premium of 0.87%. Despite being relatively small, the premium in the data is about 5%, we would like to emphasize that this premium reflects only the endogenous disaster risk as there are no other sources of uncertainty in the model. As another comparison consider the results in Barro (2006): assuming logarithmic preferences this model predicts only 0.24% premium.[13] The premium and the return on the disaster claim are increasing in π^L. At $\pi^L = 0.04$ the premium is sizeable and measures 1.74%. Finally, the return on equity and the risk-free bond are much higher than that if the disaster claim. The reason for this is that individuals expect a higher consumption growth if equilibrium H realizes. This makes the disaster claim to be very attractive as it pays when consumption is scarce; so, individuals would be willing to purchase it despite the low return that it offers.

3 Discussion

Our results extend, in spirit, to models without external effects, and to models with intrinsic shocks. Even exchange economies can have more than one equilibrium. The addition of pre-game trading will generally change the equilibrium set in models even when there are no externalities present.

The literature on global games also features externalities but, unlike our paper, it also features a real shock. In such models uniqueness can sometimes be achieved when agents have private signals about the intrinsic shock. How would our results extend to such games? Instead of writing the output equation as $y = (\alpha + \bar{x})x$, we may alternatively write it as:

$$y = (1 + \alpha\bar{x})x,$$

so that α could represent the return to a currency attack or some other coordination game. Then we could assume that $\alpha \in \{0, 1\}$ is a random variable

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[13] We assume that the bond is risk-free, that is it pays fully even if a disaster occurs. Then, assuming logarithmic preferences, the premium equals approximately $\sigma_c^2 + \pi^d (1 - d)(1/d - 1)$ where σ_c is the consumption growth volatility, π^d is the probability of a disaster, d is the output 'saved' in a disaster state. Setting $\sigma_c = 0$ we are left with the premium component that stems from the disaster risk alone. Setting $\pi^d = \bar{\pi}^L = 0.02$ and $d = 0.71$ we get $\pi^d(1 - d)(1/d - 1) = 0.0024$.
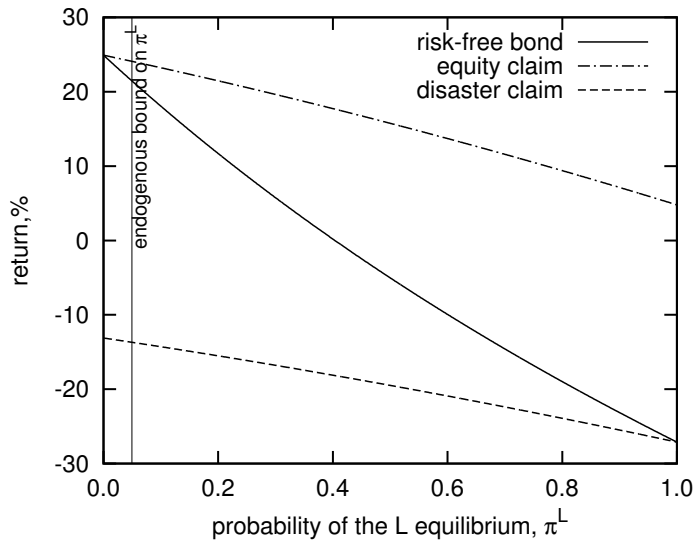
Figure 7: Return on the bond and the claim to the aggregate output.

and that agents do not know the realization of α. It is known that in such situations a little uncertainty can, under certain informational assumptions, lead to a unique equilibrium. This is a different way of restricting the set of equilibria in games that involve intrinsic uncertainty, as Goldstein and Pauzner (2005) have shown in the context of bank-run models. Our model restriction on equilibria applies to such models too, at least when the uncertainty over α is large enough so that the Carlsson and Van Damme (1993) argument cannot eliminate the multiplicity.

Conclusion

In a model in which multiple Pareto-ranked equilibria may arise, we have distinguished between sunspots and the equilibria that result therefrom. By introducing asset trading we have endogenized the mapping from the sunspot to equilibrium play and derived a bound on the probability with which the disaster equilibrium occurs.

We have then used the model to analyze several phenomena, including the effects of shocks to beliefs about the actions of others and how they

manifest themselves in asset prices, and the relation between disaster size and probability on the one hand, and the disaster premium on the other.

Finally, we have shown that asset trading can reduce the incidence of coordination failures while not eliminating them entirely. It also raises welfare, not only through the usual channel by reallocating agents' incomes across states, but also by improving the distribution of aggregate outcomes.

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A   Proof of lemma 6

Proof. The following is true for any utility function:

$$
\frac{dW_q}{d\pi^L} = \underbrace{u(z_1 + n_1^L) - u(z_1 + \alpha + 1 + n_1^H)}_{negative}
$$
$$
+ \underbrace{\pi^L u'(z_1 + n_1^L)\frac{dn_1^L}{d\pi^L} + (1 - \pi^L)u'(z_1 + \alpha + 1 + n_1^H)\frac{dn_1^H}{d\pi^L}}_{both\ terms\ are\ negative} < 0.
$$

Letting $u(c) = ln(c)$ one obtains:

$$\frac{dW_2}{d\pi^L} = \underbrace{u(z_2 + n_2^L) - u(z_2 + \alpha + 1 + n_2^H)}_{negative}$$

$$\underbrace{+ \pi^L u'(z_2 + n_2^L)\frac{dn_2^L}{d\pi^L} + (1 - \pi^L)u'(z_2 + \alpha + 1 + n_2^H)\frac{dn_2^H}{d\pi^L}}_{both\ terms\ are\ positive}$$

$$= u(z_2 + n_2^L) - u(z_2 + \alpha + 1 + n_2^H)$$

$$+ \pi^L\pi^H f_1\Delta_z(\alpha + 1)\left[\frac{u'(z_2 + n_2^L)}{\bar{z} + \alpha + 1} + \frac{u'(z_2 + \alpha + 1 + n_2^H)}{\bar{z}}\right],$$

where the last equality relies on the optimal portfolios derived in 23b. Then by the concavity of u and the fact that $u'(z_2 + \alpha + 1 + n_2^H)/u'(z_2 + n_2^L) = (\bar{z} + \alpha + 1)/\bar{z}$ we get

$$\frac{dW_2}{d\pi^L} \leqslant - u'(z_2 + a + 1 + n_2^H)(\alpha + 1)$$

$$+ \pi^L\pi^H f_1\Delta_z(\alpha + 1)\left[\frac{u'(z_2 + n_2^L)}{\bar{z} + \alpha + 1} + \frac{u'(z_2 + \alpha + 1 + n_2^H)}{\bar{z}}\right]$$

$$= - u'(z_2 + a + 1 + n_2^H)(\alpha + 1) + 2\pi^L\pi^H f_1\Delta_z(\alpha + 1)u'(z_2 + \alpha + 1 + n_2^H)/\bar{z}$$

$$= u'(z_2 + a + 1 + n_2^H)(\alpha + 1)[-1 + 2\pi^L\pi^H f_1\Delta_z/\bar{z}] < 0.$$

$\square$

B Contour plot of $\bar{\pi}^L$

Figure 8 plots contours of $\frac{(\alpha+1)\delta - z_2}{(\alpha+1)f_1\Delta_z/\bar{z}} \equiv \bar{\pi}^L$ for the parameters described in 2. To have multiple equilibria with trading of assets we need $\delta \geq \frac{z_2}{1+\alpha}$. When this holds as an equality, we have $\bar{\pi}^L = 0$, which is the $\bar{\pi}^L = 0$ contour.

C Optimization problem with two periods

The interim expected utility:

$$V_z(\pi^L, n^b, n^e) = \pi^L U(z + n^b + n_z^L) + (1 - \pi^L)[U(z + \alpha + 1 + n_z^H + n^b + (\alpha + 1)n^e) - \kappa].$$
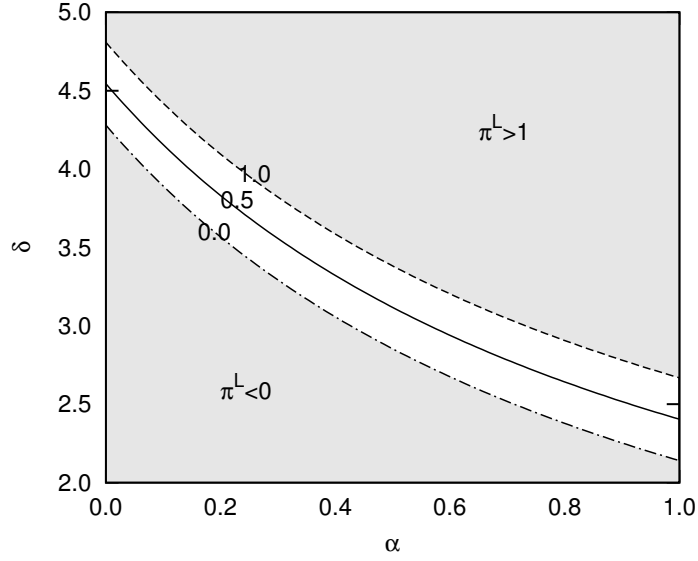
Figure 8: Contours of $\bar{\pi}^L$

The life-time utility

$$\max_{n^b, n^e} U(z_0 - q^b n^b - q^e n^e) + \beta \sum_{z \in \{z_1, z_2\}} f_z V_z(\pi^{\mathrm{L}}, n^b, n^e).$$

The first-order necessary conditions imply that the price of the risk-free bond at $n^b = n^e = 0$ is:

$$q^b = \beta \sum_z f_z \frac{\pi^{\mathrm{L}} U'(z + n_z^{\mathrm{L}}) + (1 - \pi^{\mathrm{L}}) U'(z + \alpha + 1 + n_z^{\mathrm{H}})}{U'(z_0)}. \tag{44}$$

The price of a claim to the aggregate endowment (equity) is:

$$q^e = \beta \sum_z f_z \frac{U'(z + \alpha + 1 + n_z^{\mathrm{H}})}{U'(z_0)} (1 - \pi^{\mathrm{L}})(\alpha + 1). \tag{45}$$