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GOVERNMENT DEBT AND PRIVATE LEVERAGE: AN EXTENSION OF THE MILLER THEOREM

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ABSTRACT

This paper shows how government financing decisions can influence the corporate decision to use debt or equity finance. In particular, it is shown that an increase in the stock of taxable government debt reduces the equilibrium quantity of corporate debt, and that an increase in the stock of tax-free government debt reduces the equilibrium quantity of corporate equity. The effects of inflation rate and tax rate changes are also considered.

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"GOVERNMENT DEBT AND PRIVATE LEVERAGE:
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INTRODUCTION

This paper shows how government borrowing decisions can influence the corporate decision to use debt or equity finance. In particular, it is shown that an increase in taxable government debt reduces the equilibrium quantity of corporate debt, and that an increase in tax-free debt reduces the equilibrium quantity of corporate equity. This provides a possible explanation for the post-war increase in the corporate debt-to-equity ratio, and may explain the constancy of the debt-to-GNP ratio noted by Friedman (1981).

It is well-documented that U.S. corporations have made increasing use of debt finance in the post-World War II years [Ciccolo (1981), Gordon and Malkiel (1981), and Holland and Myers (1978)]. The reason for this rise in the corporate debt-equity ratio is not well documented.

The most compelling explanations are micro-economic. For example, Gordon and Malkiel argue that corporate leverage is determined by a trade-off between bankruptcy cost and a tax advantage to debt, and that a burgeoning belief in the stability of the economy lowered the estimated probability of bankruptcy, and hence raised optimal leverage. Friedman (1981) notes that the aggregate nonfinancial debt-to-GNP ratio has been roughly constant the last thirty years, but does not explain why corporations have increased their use of debt over this period.

In view of the typical micro-economic approach to corporate leverage decisions, the seminal paper by Miller (1977) takes an ingenious twist in showing that the aggregate leverage ratio may be determinate even if leverage at the level of the individual firm is not determinate. The Miller model thus raises the possibility that macro-phenomena may have something to do with the post-war increase in corporate leverage. In addition, because the Miller model explains aggregate leverage, it offers some hope of explaining the constancy of the debt-to-GNP ratio. If the wealth-to-GNP ratio is approximately constant, then Friedman may merely have measured a constant debt-to-wealth ratio.

The basic point of Miller's paper is that the tax treatment of both the suppliers and demanders of assets affect the equilibrium configuration of asset supplies. For example, interest payments on debt are a tax-deductible expense for corporations, while payments to equity holders are not. Considering only the supplier, tax laws favor debt over equity. On the other hand, debt returns are taxed more heavily than equity returns at the personal level, so considering only the demand side, tax laws favor equity over debt.

This paper shows that in a model like Miller's, an exogenous increase in government debt will reduce the (endogenously determined) supply of similarly taxed, privately-supplied assets.[1] This result will hold as long as the distribution of wealth across tax brackets is approximately unchanged by the increase in government debt.

This argument is presented analytically in Section 1, where it is assumed that equity is riskless. The argument is not particularly sensitive to assumptions about whether government debt is or is not net wealth. Much of this paper is devoted to analyzing the above argument in a model where it is assumed that there is risky equity, and three riskless assets: tax-free municipal debt, federal debt, and corporate debt. There is a progressive income tax, with a lower tax on equity income than on debt income, and a corporate income tax. There are no bankruptcy or other costs to issuing debt.[2] The principal conclusions are that

1) An increase in taxable government debt always reduces the supply of corporate debt but has an uncertain effect on the quantity of equity; an increase in tax-free government debt reduces the supply of corporate equity but has an uncertain effect on corporate debt.

2) Changes in the inflation rate have no effect on the equilibrium aggregate debt-to-wealth or corporate debt-to-equity ratio, despite the taxation and deductibility of nominal rather than real interest payments. This assumes that there are no costs to debt finance. If there are bankruptcy or other debt costs, increases in the inflation rate raise leverage.

Section 2 introduces the model under uncertainty, and Section 3 derives the comparative static results discussed above. Section 4 explores the implications of government debt not being net wealth, and shows that government debt issues can have real effects in this model even if future tax payments are capitalized. Section 5 discusses one implausible predicion of the model and Section 6 concludes.

Before presenting the argument explicitly, it will prove useful to examine some historical time series which bear on the theory. The argument is that increases in taxable government debt are associated with reductions in the private supply of taxable debt.

The first column of Table 1 presents the market value of taxable debt outstanding as a fraction of total wealth.[3] This series is not completely constant during the post-war years, but it appears reasonably constant. This series is more striking when examined in conjunction with Column 2, in which the federal debt-to-wealth ratio is displayed. Despite a substantial reduction in federal indebtedness, the total taxable debt-to-wealth ratio in the seventies was at about the same level as in the fifties.

Figure 1 plots the ratio of taxable debt--excluding government debt--to wealth (the rising curve) against the ratio of taxable government debt to wealth (the falling line). Again the inverse relationship is dramatic.

Figure 2 depicts the market-value-of-debt to firm-value ratio for the business sector as a whole. This has risen throughout the postwar period. It is not perfectly correlated with the debt-to-wealth ratio in Figure 1 because the latter includes household issues of debt, and also because the firm value-to-wealth ratio has changed over the period.[4] It appears clear that Figure 1 is potentially useful in explaining Figure 2.

1. EQUILIBRIUM UNDER CERTAINTY

This section develops Miller's model under certainty and demonstrates that an increase in taxable government debt lowers the corporate debt to equity ratio. The purpose is heuristic, so the model is as simple as possible. There are three assets: equity, which is untaxed, and corporate and government debt, both of which are taxed. There is a progressive tax at the rate u_i on interest income, where i indexes the tax bracket. Within each tax bracket, there are n_i identical individuals, each with wealth W_i. Corporate income is taxed at the rate θ, and interest payments are a tax-deductible expense. It is assumed that $u_{min} < \theta < u_{max}$.

Because all assets are riskless, individuals choose among assets solely on the basis of after-tax return. This model is exactly that of Miller (1977), but with the trivial addition of government debt as an additional asset.

1.1 Demand Equilibrium

Let α be the return on equity, and let r be the gross-of-tax return on debt. It is easy to see that individuals will hold equity if $\alpha > (1 - u_i)r$

and they will hold debt if the inequality is reversed. Define u^*, the marginal tax bracket, by

$$\alpha = (1 - u^*)r \tag{1}$$

In equilibrium, the quantity of assets supplied must equal that demanded:

$$E = \sum_{u>u^*} n_i W_i$$

$$F + B = \sum_{u<u^*} n_i W_i = D(u^*) \; ; \qquad D' > 0 \tag{2}$$

where E is the market value of equity, F is the market value of federal government debt, and B is the market value of corporate debt. The notation $u > u^*$ means that the summation is taken over all investors in tax brackets greater than u^*.

1.2 Supply Equilibrium

Take the quantity of government debt as given. The advantage to a firm of using debt instead of equity finance is given by $\alpha - r(1 - \theta)$. Firms are indifferent between debt and equity finance only if

$$0 = \alpha - r(1 - \theta) = r(1 - u^*) - r(1 - \theta) = r(\theta - u^*) \tag{3}$$

Miller's important insight was that if $u^* < \theta$, firms will find debt finance cheaper and they will issue more debt and use the proceeds to retire equity. From (2), we can see that if E falls and F + B rises, u^* will rise. Thus there is a natural mechanism ensuring that $u^* = \theta$

The equilibrium conditions (2) and (3) are graphed in Figure 3, where the vertical line at $\theta = u^*$ represents (3), and the line OB represents the demand for debt as a function of u^*, which is given by (2). Equilibrium occurs at point A, where $B = D^* - F$.

1.3 Changes in Government Debt

Suppose now that the government issues additional debt, increasing F to F'. In general, this will change interest rates and savings behavior, as investors absorb the debt issue in their portfolios. The important point, however, is that as long as investors all change their savings behavior in the same way, so that W_i/W_j is not changed, then the debt-to-total wealth ratio will not be changed. By direct calculation,

$$\frac{B + F}{E + B + F} = \frac{\sum\limits_{u<u^*} n_i W_i}{\sum\limits_{u} n_i W_i} \qquad (4)$$

and we know that u* is unchanged, since by (3) it must equal the corporate tax rate, θ. Thus, as long as the distribution of wealth across tax brackets is unchanged, the debt-to-wealth ratio is unchanged. If the government debt-to-wealth ratio rises, the corporate debt-to-wealth ratio falls. This is a consequence of maximizing behavior by both investors and firms.

1.4 Comments

i) This analysis has ignored the issues which arise when government debt is not net wealth. That interesting case is treated in Section 4.

ii) In writing the equilibrium conditions (2) we implicitly imposed a no short-sale constraint on investors. If short-sales were allowed, there would be no equilibrium because high tax-bracket investors would issue debt, deduct the interest payments, and use the proceeds to buy equity, earning a positive return on a zero net investment. This is known as tax arbitrage, and would be pursued infinitely by investors. In a model like this, it is necessary to assume (as does Miller) that only corporations can borrow.[5]

iii) When there is municipal debt, the analysis is similar, except that municipal debt becomes a perfect substitute for equity, since both are untaxed. Auerbach and King (1981) analyze the certainty case with taxable equity, taxable debt, and tax-free debt. They show that in equilibrium, there will be three clienteles, with investors in the top tax brackets holding only tax-free debt, investors in middle brackets holding only equity, and low bracket investors holding only taxable debt.

2. EQUILIBRIUM WITH UNCERTAINTY

This section presents a model similar in spirit to that in the previous section, but with more realistic assumptions. Specifically

i) Equity returns are assumed to be risky and taxed at the rate $c_i < u_i$.

ii) There is perfectly anticipated inflation at the rate π, and the tax system is not indexed.

iii) There is tax-free municipal debt, M, with a real safe rate of return, ρ.

iv) All investors may short-sell equity. Investors in high tax brackets may not sell taxable debt, and those in low tax brackets may not sell tax-free debt.

It is still assumed that corporate and federal debt are riskless and that they receive the same tax treatment. Note that there is no safe asset which receives the same tax treatment as equity. Thus, Auerbach and King's (1981) spanning condition does not hold. Markets are incomplete in the sense of Taggart (1980).

2.1 The Model

To simplify the expostion, assume that there is a single, price-taking corporation. The analysis is concerned only with investor demands for risky as opposed to riskless assets. The composition of demand across risky assets is an interesting question in its own right, but it is a subsidiary issue in this paper.[6]

We will introduce uncertainty by using the model of intertemporal consumption and portfolio allocation introduced by Merton (1971). This has the dual advantages of allowing investors to have arbitrary utility functions, and yet resulting in a mean-variance model of asset pricing. In addition, the intertemporal nature of the problem imposes multiple sources of uncertainty on the investor, and Merton's continuous-time asset-pricing model provides a convenient way to handle this analytically. It is assumed that the corporation generates a gross real cash flow, the dynamic behavior of which is given by the stochastic process[7]

$$dg = Gdt + \sigma_g dz \qquad (5)$$

where G is the instantaneous conditional mean cash flow, σ_g^2 is the instantaneous conditional variance of cash flow, and dz is a Wiener process.

Bondholders are paid a nominal return of $r + \pi$, which is a tax-deductible expense for the corporation. The real, after-tax cost of debt finance is thus

$$(r + \pi)(1 - \theta) - \pi = r(1 - \theta) - \theta\pi \qquad (6)$$

The real gross cash flow dg is taxed at the rate θ. After taxes, the net real return available to equity holders is given by

$$\begin{aligned} dE &= (1 - \theta)dC - (r(1 - \theta) - \theta\pi)Bdt + v'dz_e \qquad (7) \\ &= \alpha Edt + yEdz + vEdz_e \end{aligned}$$

where

$$\alpha = \frac{(1 - \theta)G - (r(1 - \theta) - \theta\pi)B}{E}$$

$$y = \frac{(1 - \theta)\sigma_g}{E} \quad ; \quad v = \frac{v'}{E}$$

and B is the quantity of debt issued by the firm. The dz_e term in (7) allows for the possibility of capital gains and losses on equity for reasons other than unexpected current cash flow. Note that α, y, and v depend on leverage. In addition, α depends on the inflation rate as well as the gross-of-tax real bond rate, r.

We can also calculate the expected real after-tax returns to investors on debt and equity. The after-tax real return on equity is $(1 - c_i)\alpha - c_i\pi$ and that on taxable debt is $(1 - u_i)r - u_i\pi$, while that on municipal debt is simply ρ, with no inflation distortion because the returns are untaxed.

The safe assets are distinguishable only by tax treatment. Investors will never hold both taxable and tax-free debt, unless they are in the marginal tax bracket u^*, defined by

$$(1 - u^*)r - u^*\pi = \rho \tag{8}$$

Investors with tax rate u^* are indifferent between taxable and tax-free debt. Investors for whom $u > u^*$ will hold municipal debt, and those for whom $u < u^*$ will hold taxable debt. A change in r will generally require a change in ρ if (8) is to be satisfied.

To ensure that an equilibrium exists, we must constrain high-bracket investors not to issue taxable debt, and we constrain low-bracket investors not to issue tax-free debt. There will be no other constraints, however; all investors are free to short-sell equity and the one kind of debt they are permitted to hold.

2.2 Asset Demand

Following Merton (1971), the investor chooses ω, the fraction of his portfolio invested in risky assets, so as to maximize

$$E_t \int_t^\infty e^{-\lambda(s-t)} u(C_t)ds \qquad (9)$$

subject to

$$dW = W[(\alpha^* - r^*)\omega + r^*]dt - Cdt + (1-c_i)\omega W(ydz + vdz_e)$$

where C is consumption, W is the investor's wealth, λ is the rate of time preference, α^* and r^* are the after-tax, real expected asset rates of return, and E_t is the expectation as of time t.

There are other sources of uncertainty besides equity returns, however. If the government issues additional debt (taxable or tax-exempt) it is simultaneously endowing future tax liabilities (to be denoted L_i) upon investors. In general, this also induces a change in equilibrium interest rates. The rational investor will take into account this uncertainty, and will treat L_i and r as additional state variables when choosing an optimal consumption path. I assume that L_i and r are random and evolve according to

$$dL_i = a_i dt + b_i dz$$

$$dr = cdt + fdz_r$$

With three state variables--W, L, and r--it can be shown (Merton, 1971) that maximizing (9) is equivalent to maximizing

$$I = e^{-\lambda t}u(C_t) + J_t + J_W[W((\alpha^* - r^*)\omega + r^*) - C] \qquad (10)$$

$$+ J_r c + J_L a + \frac{1}{2} J_{rr} f^2 + \frac{1}{2} J_{LL} b^2 + \frac{1}{2} J_{WW} \omega^2 \sigma^2 W^2 (1-c_i)^2$$

$$+ J_{LW} \omega \sigma W b R_{LW} (1-c_i) + J_{rW} \omega \sigma W f R_{rW} (1-c_i) + J_{Lr} f b R_{Lr}$$

where $J_t = \frac{\partial J}{\partial t}$, etc., $\sigma^2 = y^2 + v^2 + 2yv\text{Cov}(z,z_e)$, R_{ij} is the correlation coefficient between the unexpected changes in i and j,

$$J(t, W) = E_t \int_t^{\infty} e^{-\lambda(s-t)} \tilde{u}(C_s)ds$$

and $\tilde{C}_s$ is the expected consumption path which maximizes (9), conditional on information at time t. Equation (10) is really just the investors Bellman function as approximated by a Taylor expansion.

The first-order conditions for optimal C and ω are

$$\frac{\partial I}{\partial C} = u' - J_W = 0 \tag{11a}$$

$$\frac{\partial I}{\partial \omega} = J_W W(\alpha^* - r^*) + J_{WW}\omega\sigma^2 W_i^2(1-c_i)^2 + (J_{WL}\sigma W b R_{WL} + J_{Wr}\sigma W f R_{Wr})(1-c_i) \tag{11b}$$

which may be rewritten[8]

$$\omega W = [A_i \frac{1}{\sigma^2}(\alpha^* - r^*) - (\frac{J_{WL} b\ R_{WL}}{J_{WW}\ \sigma} + \frac{J_{Wr} f\ R_{Wr}}{J_{WW}\sigma})(1-c_i)](1-c_i)^{-2} \tag{12}$$

where

$$A_i = -\frac{J_W}{J_{WW}}$$

is the inverse of the investor's coefficient of absolute risk aversion. The terms J_{WL} and J_{Wr} can be interpreted by differentiating (11a):

$$J_{WL} = u'' \frac{dC}{dL} \quad ; \quad J_{Wr} = u'' \frac{dC}{dr}$$

Since an increase in future tax liabilities constitutes a reduction in current net wealth, it is reasonable to suppose that $\frac{dC}{dL} = -\mu\frac{dC}{dW}$, where μ ($0<\mu<1$) is the fraction of true tax liabilities which are perceived by the investor. Thus $J_{WL} = -\mu J_{WW}$. J_{Wr} can be positive or negative. For our purposes, it will be necessary to assume either that $\frac{dC}{dr}$ is zero, or that J_{Wr}/J_{WW} is insensitive to changes in W, L, and r. With this assumption, most of the analysis to follow will be as if wealth were the only state variable.

2.3 Asset Supplies

The purpose of this section is to derive an expression for the value of the firm, which will enable us to see how the Miller equilibrium is characterized under uncertainty. Equation (12) tells us the investor's dollar demand for the risky asset. We will now use (12) to solve explicitly for the market value of the firm; this will permit us to solve for the equilibrium u*, given the assumption that firms maximize their market value. By aggregating across investors, we obtain the equilibrium condition comparable to (2) in the certainty case:

$$E = \sum_u n_i \omega_i W_i$$

$$= \sum_u n_i A_i \frac{1}{\sigma^2(1-c_i)^2}(\alpha_i^* - r_i^*) + \sum_u n_i D_i \frac{1}{(1-c_i)\sigma} \qquad (13)$$

where

$$D_i = -\frac{J_{WL} b\, R_{WL}}{J_{WW}} - \frac{J_{Wr} f\, R_{Wr}}{J_{WW}} = \mu b R_{WL} - \frac{J_{Wr} f\, R_{Wr}}{J_{WW}}$$

Now multiply (13) by $\sigma^2 E$, and recall that $\sigma^2 E^2 = E^2(y^2 + v^2 + 2yv\mathrm{Cov}(z,z_e))$ is a constant. For ease of notation, let $\sigma^2 E^2 = \gamma^2$. Substituting in the definitions of α (equation (7)) and ρ (equation (8)) at the same time gives

$$\gamma^2 = \sum_u N_i\,((1 - c_i)[(1 - \theta)G - (r(1 - \theta) - \theta\pi)B] - c_i \pi E) \qquad (14)$$

$$- [\sum_{u<u^*} N_i\,((1 - u_i)r - u_i\pi) + \sum_{u>u^*} N_i \rho]E + \sum_u \frac{n_i D_i}{(1-c_i)}\gamma$$

where $N_i = n_i A_i (1 - c_i)^{-2}$.

Solving (14) for E yields

$$E = \Delta[(1 - \theta)G - (r(1 - \theta) - \theta\pi)B] - \Delta[\gamma^2 - \gamma \sum_u \frac{n_i D_i}{(1-c_i)}][\sum_u N_i(1-c_i)]^{-1} \tag{15}$$

where

$$\Delta = \frac{\sum_u N_i(1 - c_i)}{\sum_{u<u*} N_i[(1 - u_i)r - (u_i - c_i)\pi] + \sum_{u>u*} N_i[\rho + c_i\pi]} \tag{16}$$

The market value of the firm is thus

$$E + B = \Delta[(1 - \theta)G - (\gamma^2 - \sum \gamma \frac{n_i D_i}{(1-c_i)})(\sum_u N_i(1-c_i))^{-1}] \tag{17}$$

$$+ B(1 - \Delta [r(1 - \theta) - \theta\pi])$$

Note that if all taxes and inflation are zero, and if wealth is the only state variable in the investor's maximization problem, then (17) becomes

$$E + B = \frac{G - \gamma^2 (\sum n_i A_i)^{-1}}{r} \tag{17'}$$

which is the usual expression for the value of the firm in a mean-variance model [Jensen (1972)].[9] For future reference, note that the value of a firm behaves like a consol when the interest rate changes.

In a model such as this, with distorting taxes, investors usually will not be unanimous about the choice of financial policy. Auerbach and King (1981) show that if the firm's financial policy has any effect on investor's opportunity sets--as it will unless investors can undo on personal account any action the firm takes--then investors will disagree about financial policy. Nevertheless, I assume that the (competitive) firm will choose financial policy so as to maximize its market value.[10]

The expression multiplying B in (17) is crucial to the analysis. (17) implies that the firm will be indifferent about financial policy only if

$$\Delta[(1 - \theta)r - \theta\pi] = 1 \tag{18}$$

By using the definition of Δ and collecting terms in π and r, (18) can be rewritten to show that

$$0 = (r + \pi)[(1 - \theta) \sum_{u} N_i(1 - c_i) - \sum_{u<u^*} N_i(1 - u_i) - \sum_{u>u^*} N_i(1 - u^*)] \tag{19}$$

By choosing u* to set the bracketed expression equal to zero, the right-hand side expression is set to zero, independently of the level of the inflation rate. This demonstrates that changes in the inflation rate leave unaffected the condition for the firm to be in financial equilibrium. Since financial equilibrium is unaffected by inflation rate changes, I henceforth assume that $\pi = 0$.

2.4 Market Equilibrium

Regardless of the inflation rate, (19) is satisfied when[11]

$$\frac{(1 - \theta) \sum_{u} N_i(1 - c_i)}{\sum_{u<u^*} N_i (1 - u_i) + (1 - u^*) \sum_{u>u^*} N_i} = 1 \tag{20}$$

It is easy to see that, given A_i and n_i, and hence N_i there will be only one u* for which (20) holds. (The expression on the left-hand side is strictly increasing in u*.) The interpretation of (20) is clearer if we assume that investors have constant relative risk aversion.[12] Then $A_i = RW_i$, where R is the inverse of the relative risk aversion coefficient. (20) then becomes

$$\frac{(1 - \theta) \sum_{u} n_i \frac{W_i(1 - c_i)}{(1 - c_i)^2}}{\sum_{u<u*} \frac{n_i W_i (1 - u_i)}{(1 - c_i)^2} + \sum_{u>u*} \frac{n_i W_i (1 - u*)}{(1 - c_i)^2}} = 1 \qquad (20')$$

Equilibrium obtains only when a particular fraction of investors prefers tax-free to taxable debt. Note that, as in the certainty case, this fraction is fixed, provided that the wealth shares of investors in different tax brackets remains fixed.

It is necessary that c_i be small relative to u_i for there to exist a u* which solves (20). It is also necessary that θ not be "large," since (20) requires that a weighted average of personal tax rates equal the corporate tax rate. If $c_i = 0$ for all i, a $u* > \theta$ will solve (20). If $c_i = u_i$, there will be no solution. For this reason, I will assume that c_i is small, to guarantee that a u* exists to solve (20).[13]

Assuming that u* does exist, I will now show that competitive firm behavior leads to (20) being satisfied. The intuition is the following: if u* is too low, then "too many" investors are holding municipal debt. From (17), firms will have an incentive to issue more debt, and retire equity. As leverage increases, investors are satisfied to hold smaller amounts of equity in their portfolio, since the variability of a dollar of equity increases with leverage. As investors hold more debt and less equity in their portfolios, the supply of municipal debt is absorbed by a smaller fraction of investors, and u* rises in order to maintain equilibrium. This process stops when u* has risen by enough to satisfy (20), so that firms are indifferent towards further leverage changes.

This process can be depicted graphically. As a first step, I will show that the amount of equity demanded by those who also hold municipal debt is an increasing function of the total amount of equity issued by the firm. Let E^H denote the amount of equity held by high tax-bracket investors (those for whom $u_i > u^*$). From (12), we have

$$E^H = \sum_{u>u^*} N_i(\alpha(1 - c_i) - r(1 - u^*)) \frac{1}{\sigma^2} + \sum_{u>u^*} \frac{n_i D_i}{(1-c_i)\sigma}$$

$$= \frac{E}{\gamma^2} \sum_{u>u^*} N_i \quad [(1 - \theta)(1 - c_i)[G - r(B + E)] \tag{21}$$

$$- rE(1 - u^* - (1 - \theta)(1 - c_i))] + E \sum_{u>u^*} \frac{n_i D_i}{(1-c_i)}$$

Differentiating (21) gives

$$\frac{\partial E^H}{\partial E} = \frac{E^H}{E} - \frac{E}{\gamma^2} \sum_{u>u^*} N_i \quad [r(1 - \theta)(1 - c_i) \frac{\partial(B + E)}{\partial E}$$

$$+ r((1 - u^*) - (1 - \theta)(1 - c_i))] \tag{22}$$

In the vicinity of equilibrium, $\frac{\partial(B + E)}{\partial E} = 0$, and if c_i is small,

$$\sum_{u>u^*} N_i \quad [1 - u^* - (1 - \theta)(1 - c_i)] \leq 0 \tag{23}$$

since $u^* > \theta$ even if $c_i = 0$, and u^* becomes larger as c_i/u_i increases (see footnote 13). Thus, $\frac{\partial E^H}{\partial E} > 0$.

Figure 4 shows how equilibrium is determined. Lower case letters represent the asset-to-total wealth ratio, i.e., $m = M/W$, etc. where $W = M + F + B + E$. Also let

$$w^* = \frac{\sum_{u>u^*} N_i}{\sum_{u} N_i} \tag{24}$$

and remember that w^* is fixed independently of changes in the composition of assets. The d(e) schedule is a graph of

$$d(e) = \overline{m} + e^h(e); \qquad e^{h'} > 0$$

We saw that increases in the quantity of equity raise the demand for equity by those who also hold municipal debt; this is why the d(e) schedule is upward-sloping. If the system were at point A, u* would be too high, and firms would have an incentive to issue additional equity, from (17). This increase in the supply of equity leads to additional equity demand by municipal debt holders, who in turn sell some of their municipal debt. This lowers u*, and leads to a greater fraction of investors holding municipal debt. The process stops at A'.

2.5 Effects of Tax Rate Changes

The equilibrium condition (20) allows us to infer immediately what will be the effects of tax rate changes on financial equilibrium. Any general tax rate change will induce an offsetting change in equilibrium u* in order for (20) to be satisfied. Previous arguments have shown that if u* rises, corporate leverage rises, and if u* falls, corporate leverage falls.

Therefore, increases in the corporate tax rate θ or in the equity tax rate c_i will raise private leverage, while increases in the personal tax rate or ordinary income will lower private leverage. While it is common to speak of u* as the marginal tax bracket, changes in rates in inframarginal tax brackets will affect u*. The Miller model under certainty predicts that changes in only the maximum tax rate will leave financial equilibrium unaffected; under uncertainty this is no longer true.

3. COMPARATIVE STATICS

I will consider two comparative static experiments: a change in the quantity of taxable government debt outstanding (municipal debt may be treated analogously) and a change in the scale of the firm. I will assume that w^* remains unchanged, i.e., that the distribution of wealth across tax brackets is constant, and that government debt behaves like a consol in response to changes in the interest rate. I will also assume throughout that consumption is unaffected by interest rate changes, i.e. that J_{rW} is zero.

To perform the comparative static calculations it is necessary to make an assumption about the effect of government debt issues on saving. To understand the importance of this, it is necessary to digress briefly on the implications of whether or not investors perceive future tax liabilities.

When the government uses debt finance instead of tax finance to pay for current expenditures, it obligates itself to collect taxes in the future to pay off the debt. With perfect capital markets and no distorting taxes, the present value of these future tax collections is equal to the value of the current debt issue. The current value of an investor's wealth is lowered by expected future tax liabilities. If individuals are aware of these future tax liabilities, Barro's (1974) argument implies that in equilibrium, investors will purchase the newly-issued government debt in proportion to their endowment of future tax liabilities.[14] The increase in government debt is absorbed entirely through new saving; given a level of government spending, gross aggregate wealth will rise by the amount of the debt issue, and each individual's net wealth will be unchanged by the use of debt finance in lieu of tax finance. In effect, when capital markets are perfect, investors are indifferent between paying taxes now and paying taxes later. This implies that interest rates are unchanged by a new issue of government debt.

Suppose on the other hand that investors do not perceive future tax liabilities. When the government issues additional debt the price of bonds falls because the increased demand for bonds (if any) is insufficient to meet the increase in supply. The interest rate increase lowers the price of assets generally. The most extreme case would occur when wealth was unchanged in the new equilibrium. The increase in debt would have then been completely absorbed through an increase in interest rates.

I will use δ to measure the extent to which aggregate wealth rises when there is an increase in the stock of (taxable or tax-free) government debt.[15] $\delta = 0$ signifies that aggregate wealth is unchanged, and that the increase in debt has been absorbed into the economy through an increase in interest rates. $\delta = 1$ signifies that the debt issue has been absorbed through an increase in saving, and that interest rates have not changed. The use of δ is a device which makes use of Barro's elegant insights and avoids complicating the analysis needlessly.

It is assumed throughout that tax rates are unchanged. The increase in government debt can be viewed as resulting from increased government spending or as due to a reduction in a lump-sum tax, but not from a change in the personal or corporate tax rates. Money is excluded from this model, so the possibility of an inflation tax as a third means of finance is ignored.

The main results are that an increase in the stock of taxable government debt always reduces corporate debt, but has uncertain effects on the quantity of equity. There is ambiguity only when the market value of the firm changes due to an interest rate increase, i.e., when $\delta < 1$. When $\delta = 1$, a reduction in corporate debt implies a corresponding increase in equity. An increase in the stock of tax-free debt reduces the stock of equity, but has an uncertain effect on the stock of corporate debt. (Again, this is ambiguous only when $\delta < 1$.)

Before performing the comparative static calculations, I will explain the strategy to be used. Suppose that the government issues dF in new taxable debt. Private saving will change, and total wealth will increase by a fraction δ of the government debt issue. Thus, $dW = \delta dF$.[16] Extensive use will be made of the condition that the total demand for assets by investors in high tax brackets must equal the total wealth of high-bracket investors, i.e. that

$$E^H(E,r) + M(r) = w^*W$$

Given r, W, M, and w*, and remembering that E^H is an increasing function of E, this equation implies an equilibrium quantity of total equity. If we totally differentiate this equation, we obtain

$$w^* \frac{dW}{dF} = w^*\delta = \frac{dM}{dF} + \frac{dE^H}{dF} = (\frac{\partial E^H}{\partial r} + \frac{dM}{dr}) \frac{dr}{dF} + \frac{\partial E^H}{\partial E} \frac{dE}{dF}$$

This may be solved for dE/dF to give

$$\frac{dE}{dF} = (\frac{\partial E^H}{\partial E})^{-1} (w^*\delta - (\frac{\partial E^H}{\partial r} + \frac{dM}{dr}) \frac{dr}{dF}) \qquad (25)$$

Equation (25) can be used to examine the question of how equilibrium supplies of equity and debt change when the Federal government issues more debt. The only problem in evaluating (25) is calculating $\frac{dr}{dF}$, which can be shown to equal[17]

$$\frac{dr}{dF} = \frac{(1 - \delta)r}{V + M} \qquad (26)$$

We are now in a position to calculate (25).

Finally, in what follows we will make use of the fact (from (21)) that

$$\frac{\partial E^H}{\partial r} = \frac{E}{r} (\frac{\partial E^H}{\partial E} - \frac{E^H}{E}) \qquad (27)$$

This expression is positive, as may be seen by evaluating the elasticity of E^H with respect to E, using (22). Doing so yields

$$\frac{\partial E^H}{\partial E}\frac{E}{E^H} = 1 + \frac{E}{E^H}[\cdot]$$

where the bracketed term is positive. Hence the elasticity is greater than one.

3.1 Increase in Taxable Government Debt[18]

With (25), we are now in a position to ask what happens to the quantity of corporate equity and debt when there is an increase in the stock of taxable government debt.

A. The Quantity of Corporate Debt

An increase in the quantity of taxable government debt reduces the equilibrium quantity of corporate debt.

To show this, we will compute

$$\frac{dB}{dF} = \frac{dV}{dF} - \frac{dE}{dF} \tag{28}$$

Inserting (25) into (28), we obtain

$$\frac{dB}{dF} = \frac{dV}{dr}\frac{dr}{dF} - \left(\frac{\partial E^H}{\partial E}\right)^{-1} \left(w^*\delta - \left(\frac{dM}{dr} + \frac{\partial E^H}{\partial r}\right)\frac{dr}{dF}\right) \tag{29}$$

and substituting and rewriting, it can be shown that the right-hand side of (29) is equivalent to

$$-\left(\frac{\partial E^H}{\partial E}\right)^{-1} \left[(1 - \delta)\frac{\partial E^H}{\partial E}\left(1 - \frac{M+E}{V+M}\right) + w^*\delta + \frac{(M+E^H)(1-\delta)}{V+M}\right] \tag{30}$$

Every term within the square brackets is positive. Hence, increases in the quantity of taxable government debt always reduce the equilibrium quantity of corporate debt.

B. The Quantity of Corporate Equity

An increase in the quantity of taxable government debt has an ambiguous effect on the quantity of corporate equity.

Evaluating (25), we obtain

$$\frac{dE}{dF} = \left(\frac{\partial E^H}{\partial E}\right)^{-1} \left(w^*\delta + \frac{(1-\delta)}{V+M}\right)\left[M - E^H\left(\frac{\partial E^H}{\partial E}\frac{E}{E^H} - 1\right)\right] \tag{25'}$$

In evaluating $\frac{dE^H}{dr}$, using (21), it is helpful to notice that $r(B + E)$ is independent of r when $\pi = 0$.

The sign of (25') is ambiguous, since the elasticity of E^H with respect to E is greater than one. When δ is close to one, $\frac{dE}{dF}$ will be positive. δ close to one implies that the increase that the increase in government debt is absorbed by saving. Consequently, interest rates do not change much, the market value of the firm does not change much, and the unambiguous drop in the quantity of corporate debt implies that there must be a concomitant unambiguous rise in the quantity of corporate equity.

3.2 Increase in Tax-Free Government Debt

Similar results may be derived for changes in the quantity of municipal debt. An increase in municipal debt always "crowds out" equity, but has an ambiguous effect on debt, except when δ is close to 1.

A. The Quantity of Corporate Equity

An increase in the quantity of tax-free government debt reduces the equilibrium quantity of corporate equity.

In the same way that (25) was derived, we can obtain

$$\frac{dE}{dM} = \left(\frac{\partial E^H}{\partial E}\right)^{-1} \left(w^*\delta - 1 - \frac{\partial E^H}{\partial r}\frac{dr}{dM}\right) < 0 \tag{31}$$

The inequality follows since $1 > w^*\delta$ and $\frac{\partial E^H}{\partial r}$ and $\frac{dr}{dM}$ are both positive.

B. The Quantity of Corporate Debt

An increase in the quantity of tax-free government debt has an ambiguous effect on the quantity of corporate debt.

We can calculate

$$\frac{dB}{dM} = \frac{dV}{dM} - \frac{dE}{dM} = -(\frac{V}{r})\frac{dr}{dM} - (\frac{\partial E^H}{\partial E})^{-1}[w^*\delta - 1 - \frac{\partial E^H}{\partial r}\frac{dr}{dM}]$$

Using (27), this becomes

$$\frac{dB}{dM} = (\frac{\partial E^H}{\partial E})^{-1}[1 - w^*\delta - \frac{(1-\delta)(V-E)}{V+F}(\frac{\partial E^H}{\partial E}) - \frac{E^H(1-\delta)}{V+F}]$$

The sign is ambiguous except when $\delta = 1$. In that case, $\frac{dB}{dM} > 0$, which is sensible since when $\delta = 1$, interest rates do not change and the market value of the firm is unaffected by the increase in the stock of tax-free debt. Thus, a decrease in the quantity of equity outstanding will imply an increase in the quantity of debt.

3.3 Changes in the Scale of the Firm

If government debt is net wealth, then a rise in the stock of taxable or tax-free government debt will in the short run increase interest rates. Given the stock of capital, the value of the corporate sector will fall. The corporate sector will in turn reduce investment, and in the long run the capital stock will fall, causing a change in the mean and variance of cash flows. The value of the firm will fall further as the capital stock drops (real crowding out occurs). What is the effect on the quantity of equity and debt as the scale of the firm changes?

Suppose that the value of the firm changes by dV. The equilibrium change in E^H will then be $dE^H = w^*dW = w^*dV$ assuming that interest rates and hence the market values of taxable and tax-free government debt do not change and hence that $dW = dV$. Then,

$$\frac{dE}{dV} = \left(\frac{\partial E^H}{\partial E}\right)^{-1} \frac{dE^H}{dV} \tag{32}$$

$$= \left(\frac{\partial E^H}{\partial E}\right)^{-1} w^* > 0$$

The effect on the quantity of debt is less clear, since

$$\frac{dB}{dV} = 1 - \frac{dE}{dV}$$

(32) can, however, be rewritten as

$$\frac{dE}{dV} = \left(\frac{\partial E^H}{\partial E}\frac{E}{E^H}\right)^{-1} \frac{1 + \frac{M}{E^H}}{1 + \frac{B + M + F}{E}}$$

The elasticity in parentheses is greater than 1. As an empirical matter, the quantity of municipal debt outstanding is small relative to the total of corporate and federal debt outstanding, so it seems likely that $\frac{M}{E^H} < \frac{B + M + F}{E}$ in which case $\frac{dB}{dV} > 0$.

When the capital stock can change what will be the long-run effect on B of a change in outstanding taxable government debt? If $\delta = 1$, changes in government debt have no real effects, so consider the case where $\delta = 0$, and suppose that $\frac{dB}{dV}$ is positive.

The increase in taxable government debt lowers B, from (29). As interest rates have risen, the firm will invest less and for any given interest rate, V will fall. The fall in V lowers B still more. The initial drop in B is reinforced by the later decrease in the scale of the firm. The net long-run effect is greater than would be suggested by calculating (28) alone.

4. WHAT IF GOVERNMENT DEBT IS NOT NET WEALTH?

The implications of the model developed in Sections 2 and 3 hinge on the important assumption that the distribution of wealth across tax brackets is unchanged by changes in the composition and level of government debt. This is a plausible assumption (or at least a reasonable approximation) when government debt is net wealth, since every individual has a fixed rate of time preference, and individuals have identical utility functions. If there are tax clienteles for government debt, however, then the Barro theorem can be false.[19]

If all future tax liabilities are endowed upon those who are in the clientele which holds government debt, the Barro result still holds. In this case, however, the results in Section 4 do not hold--increases in government debt will have no effect on corporate leverage. High bracket investors will have unchanged portfolios, and low bracket investors will completely absorb the increase in government debt.

To examine the other extreme, suppose that all future tax liabilities are endowed upon those who hold only tax-free municipal debt. A possible equilibrium would be for those in high tax brackets to nevertheless hold the new taxable debt, but this would conflict with the principle that the same investor never hold riskless assets with different after-tax returns. Instead, high bracket investors in the aggregate would try to increase their holdings of tax-free debt, and low-bracket investors would be left to absorb the increase in taxable debt. The market return on tax-free debt would fall and that on taxable debt would rise. From (8), u* would rise. Corporations would then retire debt and issue more equity, taking advantage of the relative inexpensiveness of equity. The process would halt eventually, with a new u* greater than the u* that existed in the beginning.[20] The aggregate equity to wealth ratio would rise.

It is possible to show in this model, however, that individuals in inframarginal tax brackets will have their utility changed by changes in corporate leverage. To see this, consider equation (10) in which, for simplicity, wealth is the only state variable. I will show that leverage changes necessarily affect the utility of investors in inframarginal tax brackets.

Equation (11) defines the optimal portfolio weights. Substitute (11) into (10). This gives

$$L = e^{-\lambda t}u(C_t) + J_t + J_W(r^*W_i - C) + J_{WW}W_i(\alpha^* - r^*)\omega_i \qquad (33)$$

Hold consumption and wealth fixed. Utility will then be unchanged with changes in aggregate leverage only if the expression $\omega_i W_i(\alpha^* - r^*)$ is independent of leverage,where ω_i is optimally chosen.[21] This expression may be rewritten

$$\omega_i W_i(\alpha^* - r^*) = \frac{\omega_i W_i}{E}(1 - c_i)(1 - \theta)(G - r(B + E)) +$$

$$+ rE((1 - \theta)(1 - c_i) - (1 - u_i))) \qquad (34)$$

If the investor is in portfolio equilibrium, (12) holds. (12) may be rewritten

$$\frac{\omega_i W_i}{E} \gamma^2 = A_i[(1 - c_i)(1 - \theta)[G - r(B + E)] + rE((1 - \theta)(1 - c_i) - (1 - u_i))] \quad (35)$$

Equation (35) shows that low-bracket investors (for whom $(1 - \theta)(1 - c_i) - (1 - u_i) < 0$) will demand a smaller fraction of total equity as corporate leverage decreases (E rises, for a given B + E). Expression (34) shows that utility is independent of leverage only if the equity share rises as corporate leverage decreases. Thus, I have shown that with a progressive tax system, changes in leverage will in general affect investor utility, even though corporations are indifferent about leverage.[22]

Several comments are in order:

1) The comparative static analysis in Section 4 ignores the kind of effect I have just discussed. Even if future tax liabilities are distributed in such a way that all investors change their wealth proportionally, there can still be real effects operating through the portfolio decision. The case in Section 4 where $\delta = 1$ (increases in government debt are fully absorbed by increased saving) is therefore at best only an approximation to the case where government debt is not net wealth.

2) The result that changes in the level of government debt have real effects is not surprising since the structure of the model prohibits investors from having "complete access" to capital markets. If all investors could buy and sell unlimited qualtities of all assets,[23] government debt would have no real effects, and it is certain that the model would not yield a plausible equilibrium (see the previous comments about the need for constraints which prohibit tax arbitrage).

5. PLAUSIBILITY OF THE MODEL

In this section I wish to consider a prediction of the model which appears to be false: that the tax rate at which investors are indifferent between taxable and tax-free debt is greater than the corporate tax rate. While this conclusion is troublesome for the model, I will argue that there are mitigating circumstances in practice so that the main conclusions of the model could nevertheless be basically correct.

5.1 The Predicted Marginal Tax Rate

Whether or not there are capital gains taxes, we have seen (see footnote 11) that the model predicts that $u^* > \theta$, i.e., the tax rate at which individuals are indifferent between taxable and tax-free riskless debt exceeds the corporate tax rate. Gordon and Malkiel (1981, Appendix B), however, show that the actual marginal tax bracket is around 25 percent,[24] which is substantially less than the statutory corporate tax rate of 46 percent.

The condition $u^* > \theta$ will be an equilibrium property of any Miller-type model. To understand this result, one can think of firms as being a conduit for equity holders, who do their borrowing through the firm. Because of the interest deductibility, firms can offer a higher interest rate on debt and still find borrowing profitable relative to the cost of issuing equity. In a certainty world, borrowing will be profitable for the firm until the return on debt has been increased by the full amount of the saving due to the interest deduction--only then will the firm be indifferent between debt and equity finance. The model in this paper shows that the same kind of result holds even when there is uncertainty--in fact, uncertainty raises u^* above θ, even when there are no capital gains taxes.

An increase in corporate leverage raises u*, so that if u* in the real world is lower than theory predicts, it can be because firms issue less debt than the theory predicts, which would occur if there were costs to issuing debt. One relevant piece of empirical evidence along these lines is provided by Cordes and Sheffrin (1981) who use Treasury data to show that, on average, for U.S. firms an additional dollar of interest expense results in only $.33 of additional tax deductions. This is without taking account of tax carrybacks and carryforwards, but Cordes and Sheffrin show that inability to use tax credits tends to persist among relatively unprofitable firms. This minimizes the value of the carrybacks and carryforwards.

Another possibility is suggested by the results of Skelton (1980) who finds that for short-term taxable and tax-exempt debt, the marginal personal tax rate is close to the corporate tax rate, while for long-term debt, the marginal personal tax rate is lower. This finding may allow a reconciliation of the empirical finding of Gordon and Malkiel that the marginal personal tax rate is low, and the theoretical prediction of this and other Miller-type models that the marginal personal tax rate should be close to the corporate tax rate.

5.2 Inflation and Leverage Revisited

If there are costs to debt finance, as there must be to reconcile the Gordon and Malkiel finding with the implications of this model, then a change in the inflation rate will have an effect on corporate leverage.

When there are costs to debt finance, the firm's equilibrium condition becomes

$$0 = \frac{d(E + B)}{dB} = \frac{\partial(E + B)}{\partial B} + \frac{\partial(E + B)}{\partial x}\frac{dx}{dB} \tag{36}$$

where $x = x(B)$ represents the dependence of some parameter in the valuation equation on the level of debt, i.e., x is the cost of issuing debt.[23] If $x = 0$, then (20) is still the first-order condition for optimal leverage. If $x \neq 0$, however, then (20) does not hold and the inflation rate does not drop out.

Inflation leaves leverage unchanged when there are no debt costs because the tax advantage to debt finance vanishes in equilibrium. When the inflation rate changes, it affects the after-tax costs to borrowing and returns to lending in the same way, and thus has no effect, because there is no marginal tax advantage to debt.

When there is a cost to debt finance, the marginal tax advantage to debt finance (the negative of the first term in (36)) is equated to the marginal cost of debt finance (the second term in (36)), which is greater than zero. Hence there is a positive tax advantage to debt in equilibrium,[24] and when the inflation rate changes, it lowers the after-tax borrowing cost by more than it raises the after-tax return. Hence the tax advantage to debt for any given interest rate is increased by a rise in the inflation rate, and it follows that if there are costs to debt, firms will use more debt finance if there is an increase in the inflation rate.

6. CONCLUSION

This paper has shown that government financing policy has important implications for private financial decisions. Two important facts about the postwar U.S. economy--the constancy of the debt-to-wealth ratio and the rise in the corporate debt-to-equity ratio--are consistent with the predictions of the model. An important caveat is that the results were derived in a model where financial intermediation is ignored. At the very least, intermediaries have the potential to transform the tax and risk characteristics of the asets they hold, and thus could affect the conclusions of this model in an important way.

The model presented here, if developed more fully, may also provide a useful way to think about several other issues, such as the effect of government debt policy on investment and output, and the commingled effect of inflation and taxes on investment.

With respect to the latter issue, this model has the implication that, despite the deductibility of nominal interest payments on corporate debt, changes in the inflation rate need not affect the corporate after-tax borrowing rate, because there is no tax advantage to debt in equilibrium.

Data Appendix

The following asset stock data, all from the Flow of Funds Sector Balance Sheets (annual, year-end outstandings) were used:

F: Net financial Liabilities of the U.S. government, excluding state and local holdings of federal debt.

M: Net financial liabilities (non-financial sector) of state and local governments, plus holdings of federal debt.

B: Total credit market instruments (includes bonds, loans, mortgages, etc.) issued by non-financial corporate and non-corporate business.

E: Market value of corporate equity, excluding investment fund shares, plus equity value of non-corporate business, valued at replacement cost.

HH: Liabilities issued by households. Mortgages valued at estimated market value; other liabilities at book value.

W: Total wealth (F + M + B + E + Household tangible assets).

π: Annual fourth quarter to fourth quarter percentage change in the personal consumption expenditures deflator.

The long-term debt was all crudely adjusted to market value by assuming that the annual rate of retirement of book debt was four percent; that the weighted average maturity of the debt was 15 years, and that market value equaled book value in 1947.[26] It was assumed (based upon a breakdown in the 1980 Economic Report of the President) that 55 percent of federal government debt and 75 percent of business debt was long-term. Equity was valued at market, and tangible assets (for the household and non-corporate business sectors) were valued at replacement cost.

FOOTNOTES

[1]This "crowding out" is financial, and should not be confused with real crowding out (which may or may not occur), where government purchases of goods and services lead to a reduction in private investment.

[2]The model in this paper is essentially an extension of that in Auerbach and King (1981).

[3]Federal debt is measured here as net financial liabilities of the federal government--thus excluding agency holdings of federal assets--less state and local government holdings of federal liabilities. The idea is to present a measure of the stock of federal libailities which must be absorbed by the private sector. By the same token, Column 3 displays the tax-free government debt-to-wealth ratio, and this is measured as net financial liabilities of state and local governments, plus state and local holdings of federal debt instruments. Again, this is a more accurate measure of the state and local debt which must be absorbed by the private sector than net state and local financial liabilities would be.

[4]Financial intermediaries are conspicuous by their omission in Table 1. It is reasonable to ignore intermediaries and to take seriously the movements between private and government debt in Table 1 provided that we take either of two views of financial intermediaries: that they are a "veil," so that households view the assets held by intermediaries as if the households held them directly (see Friedman (1981) for a discussion of this view); alternatively, we can view intermediaries as independent economic actors, with asset demands that respond to exogenous changes in much the same way as the asset demands of other sectors.

[5]Auerbach and King (1981) contains an exhaustive treatment of the need for borrowing constraints in Miller-type models.

[6]Even if all risky assets were taxed at the c_i, investors different tax brackets would hold different risky portfolios unless $(1-u_i)/(1-c_i)$ were constant across tax brackets.

[7]See Fischer (1975) and Merton (1971) for a discussion of this assumption, and for additional references.

[8]If there were several risky assets, and if wealth were the only state variable, the ratio ω_k/ω_j would be given by

$$\frac{(\alpha_k^* - r^*)}{(\alpha_j^* - r^*)} = \frac{\alpha_k - (\frac{1-u_i}{1-c_i})r^*}{\alpha_j - (\frac{1-u_i}{1-c_i})r^*}$$

which in general is not independent of the investor's tax bracket. Hence investors will hold different risky portfolios. The assumption of a single risky asset is not completely general, since there is no mutual fund theorem.

[9]In the usual one-period treatment, G has the interpretation of being profit plus the liquidation value of the firm and r is then replaced by (1 + r). In this model, G is simply profit.

[10]The assumption that the firm is competitive means that when the firm changes its financial policy, it ignores any induced changes in the marginal tax bracket, u*.

[11]When there is only a single capital gains tax rate c and a single income tax rate u, (20) can be written

$$\frac{(1-\theta)(1-c)}{1-u} = 1$$

This corresponds to Miller's (1977) condition for leverage indifference.

[12].The following statements would be true if investors exhibited linear risk tolerance, i.e. if $-J_{WW}/J_W = a + bW$. Constant relative risk aversion is a special case, with a = 0.

[13]If $c_i = 0$, (20) may be rewritten

$$\sum_{u>u^*} N_i\ (u^* - \theta) = \sum_{u<u^*} N_i(\theta - u\ )$$

This can be satisfied only if $u^*>\theta$. If $c_i = \lambda u_i$, the condition becomes

$$\sum_{u>u^*} N_i\ (u^* - \theta - \lambda u_i(1 - \theta)) = \sum_{u<u^*} N_i((1 - \theta)\lambda u_i + \theta - u_i)$$

Increases in λ require increases in u^* to maintain the equality, so it is still the case that $u^*>\theta$ when there are capital gains taxes. For λ near 0, both sides are positive, and the left hand side expression is equivalent to

$$\sum_{u>u^*} N_i\ [(1 - \theta)(1 - c_i) - (1 - u^*)] > 0$$

which is the expression in (20) which I take to be positive.

[14]This is the only equilibrium configuration of debt purchases consistent with the proposition that changes in the level of government debt (given government spending) have no real effects.

[15]δ is obviously related to the parameter μ introduced in Section 2. It will be seen in Section 4 that even if $\mu = 1$, so that all tax liabilities are fully perceived, it is unlikely that $\delta = 1$ exactly. The reasons pertain to distributional (across tax brackets) effects from future tax liabilities, and to tax distortions affecting investors in inframarginal tax brackets.

[16]dF is to be interpreted as the net change in the market value of government debt outstanding, not the gross issue of government debt.

[17]As an identity, we have

$$dW = dF + dV + dM = \delta dF$$

Hence

$$dV + dM = - (1 - \delta)dF$$

The interest rate change required to bring about this change in W is given implicitly by

$$(\frac{dM}{dr} + \frac{dV}{dr})\ \frac{dr}{dF} = - (1 - \delta) \qquad (1n)$$

As noted earlier, the value of the firm behaves like a consol, so that

$$\frac{dV}{dr} = -\frac{V}{r} \quad (2n)$$

and, by assumption,

$$\frac{dM}{dr} = \frac{dM}{d\rho}\frac{d\rho}{dr} = -\frac{M}{\rho}(1 - u^*) = -\frac{M}{r} \quad (3n)$$

since $\rho = (1 - u^*)r$. Substituting (2n) and (3n) into (1n) gives (26).

[18]In the analysis to follow, I will assume for simplicity that investors have constant absolute risk aversion, so that A_i is independent of wealth. Assuming constant relative risk aversion would reinforce the qualitative results.

[19]The argument that Barro's result fails in the presence of non lump-sum taxes is also made by Tobin (1980, Chapter 3).

[20]From (20'), if the relative wealth of high-bracket individuals rises, u^* must rise to reattain equilibrium.

[21]If W_i and C_t are held fixed, then only the last term in (33) can be affected by leverage changes.

[22]From (34) and (35), if all taxes are zero or if $(1 - \theta)(1 - c_i) = 1 - u_i$ for every investor, then a change in leverage will have no effect on utility.

[23]Recall that high bracket investors <u>are</u> allowed to buy and sell unlimited quantities of equity and municipal debt, and low bracket investors can buy and sell unlimited quantities of taxable debt and equity. Government debt has real effects because there are investors in inframarginal tax brackets in this model.

[24]Gordon and Malkiel compare the yields on bond issues by the same firms, with the same indenture provisions, except that one issue is tax exempt and the other taxable. This is as pure a measure of u^* as could be asked.

[25] Debt costs must be a convex function of B in order for (41) to define an optimal leverage position.

[26] This argument was made by DeAngelo and Masulis (1980), who did not, however, explore the implications of inflation for leverage policy.

[27] Appropriate interest rate series were obtained from the Troll Citibank database to use in adjusting the debt to market value. All business debt was adjusted using the average yield to maturity on all ong-term corporate debt outstanding; federal debt using the yield on government bonds with a maturity in excess of ten years; municipal debt using an average yield on high quality municipal issues, and household mortgage debt using a mortgage rate.

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After this paper was written, I learned that Robert A. Taggart, Jr. had independently reached similar conclusions in an unpublished paper.

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Table 1

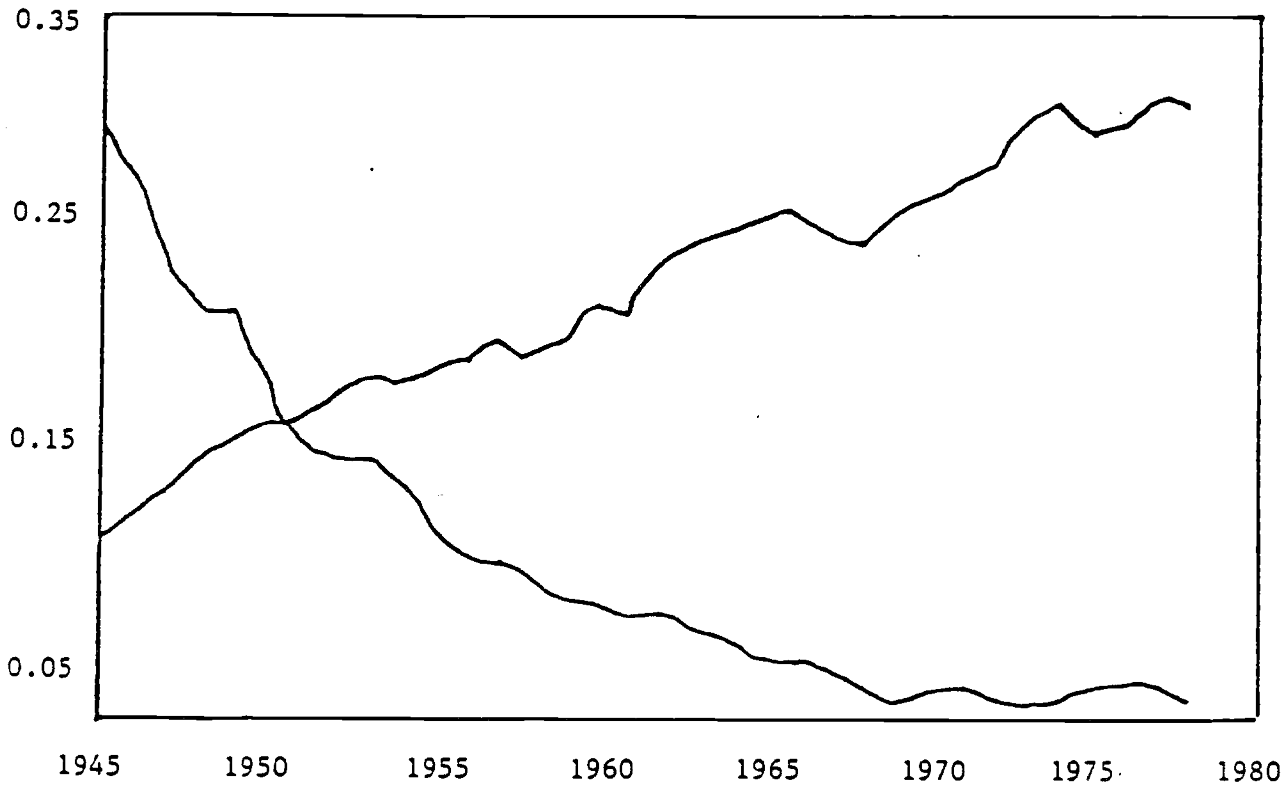
Debt-to-Wealth Ratios for Different Debt Aggregates

Year	$\frac{B + F + HH}{W}$	$\frac{F}{W}$	$\frac{M}{W}$
1945	.431	.304	.013
1946	.419	.283	.011
1947	.394	.246	.011
1948	.385	.225	.013
1949	.393	.224	.014
1950	.365	.190	.016
1951	.344	.167	.016
1952	.348	.161	.017
1953	.356	.161	.019
1954	.343	.150	.022
1955	.331	.132	.023
1956	.320	.118	.023
1957	.329	.117	.025
1958	.312	.108	.025
1959	.313	.102	.026
1960	.324	.100	.029
1961	.316	.094	.029
1962	.341	.096	.032
1963	.340	.089	.030
1964	.342	.085	.030
1965	.339	.078	.029
1966	.344	.076	.030
1967	.329	.071	.027
1968	.317	.065	.025
1969	.322	.059	.025
1970	.334	.062	.028
1971	.344	.064	.029
1972	.347	.060	.028
1973	.361	.057	.027
1974	.371	.057	.024
1975	.365	.065	.023
1976	.372	.068	.025
1977	.384	.067	.025
1978	.372	.061	.022

Source: Federal Reserve Board of Governors. Data are described in detail in Data Appendix.

Figure 1

Taxable Debt Ratios, 1945-1978



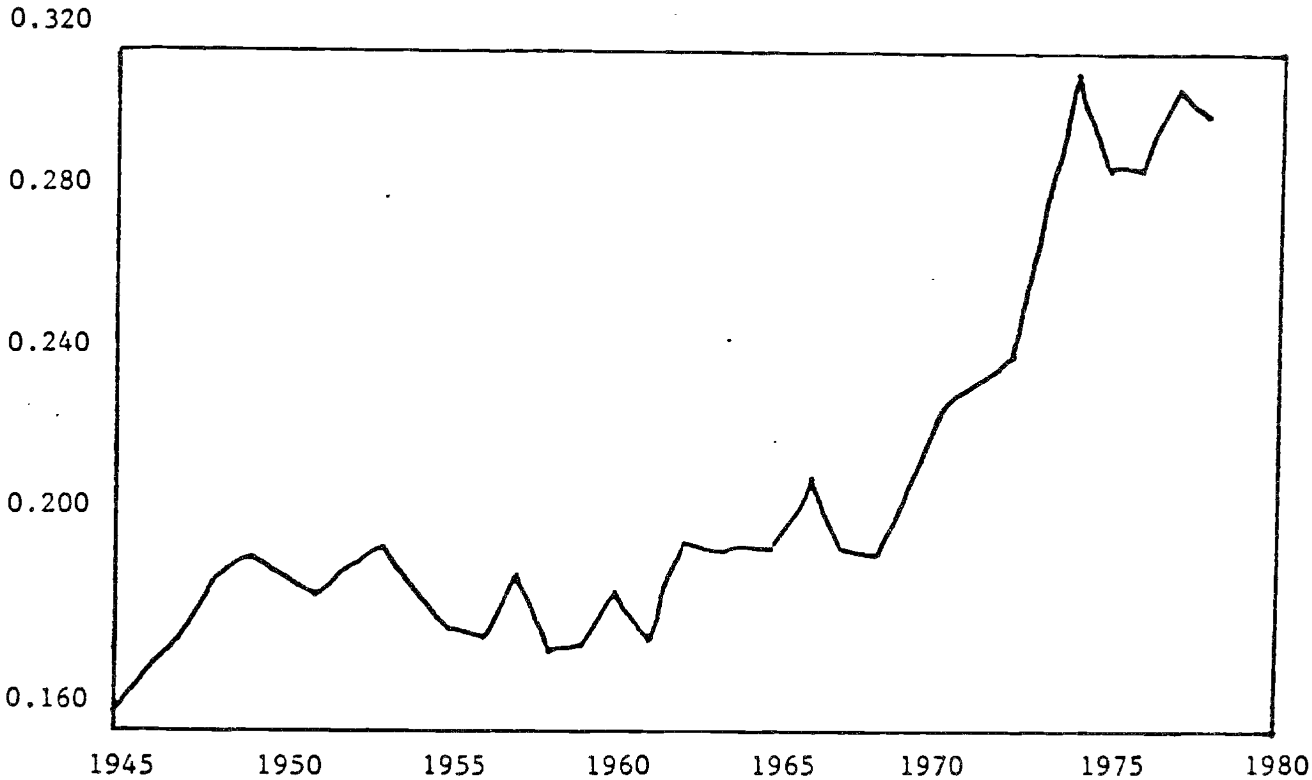
Declining figure: Ratio of net federal liabilities (market value less state and local holdings) to wealth.

Rising figure: Ratio of household plus business liabilities (market value; corporate plus noncorporate (including farm)) to wealth.

Source: Federal Reserve Board of Governors; see appendix for descriptions of data and market-values adjustment.

Figure 2

Debt to Firm Value Ratio, All Business, 1945-1978



Ratio of credit market instruments issued by business (corporate plus non-corporate) divided by sum of corporate equity, corporate debt, and replacement value of tangibles owned by non-corporate business.

Adjustment to market value described in appendix .

Source: Federal Reserve Board of Governors.

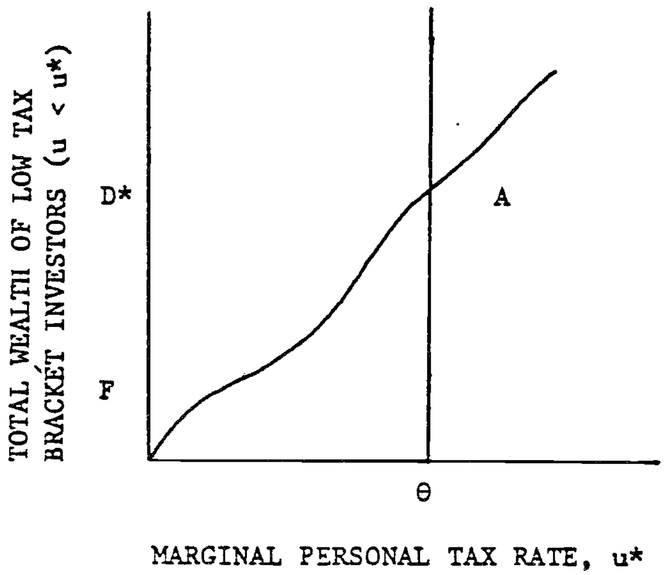


FIGURE 3

Under certainty, financial equilibrium is obtained when the marginal personal tax rate equals the corporate tax rate. The aggregate quantity of debt equals the total wealth of investors in low tax brackets.

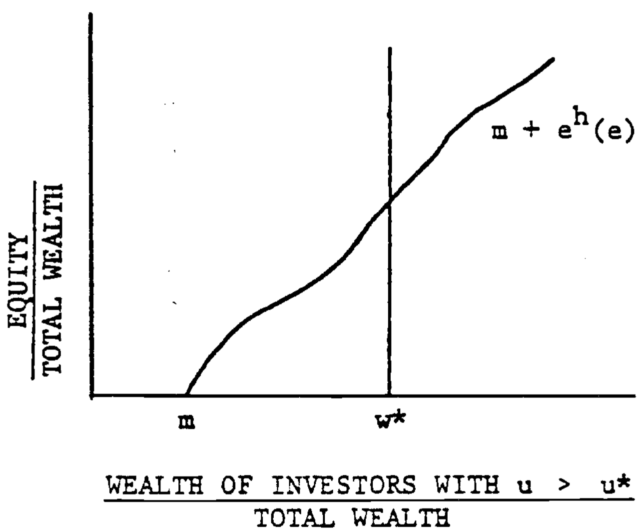


FIGURE 4

Under uncertainty, financial equilibrium is obtained when the aggregate demand for assets by high tax-bracket investors, $m + e^h$, equals the wealth of high tax-bracket investors, w^*.