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INTERNATIONAL RISK SHARING AND THE CHOICE OF EXCHANGE RATE REGIME

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Abstract

This paper examines the argument that the fixed exchange rate regime should be preferred to the flexible rate regime because the former allows risk sharing across countries while the latter does not. The analysis is performed in a two-country overlapping generations model, where markets are incomplete under either exchange regime. In this second best world, it is demonstrated that the ability to share risk across countries in the fixed rate regime does not necessarily lead to higher welfare than the inability to share risk in the flexible rate regime.

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1. Introduction

One argument which has been advanced in favor of the fixed exchange rate regime over the flexible rate regime is that the former allows risk sharing across countries (See Mundell [1973]). This is not necessarily welfare improving, however, when the set of markets are not complete (in an Arrow-Debreu sense) in either regime.

This paper makes use of a two-country overlapping generation model similar to that in Karekan and Wallace [1981], and outlined in Section 2. Individuals in one country face an exogenous uncertainty in their endowments, while individuals in the other country receive a nonrandom endowment. The flexible rate regime, discussed in Section 3, permits no risk sharing. The fixed rate regime, on the other hand, forces agents in the country with nonrandom endowments to share the risk of agents in the country with random endowments, as explained in Section 4. Explicit welfare comparison in Section 5 shows that the choice of the exchange rate regime rests on the degree of relative risk aversion. Some concluding remarks are offered in the final section.

2. The Model and Notation

There are two countries, denoted A and B, and one nonstorable consumption good. All agents are identical,[1/] live for two periods, and receive an endowment of the good only in the first period. Each agent is indexed by a vector (i, t, h), where i is the country of residency, t the current date, and h his age. A young person is denoted by $h = 1$, and an old person by $h = 2$. There are N^a agents in each generation residing in country A, and N^b in country B.

[1/]The case where preferences are different across countries is considered later. It does not alter any results of the paper.

When young, an agent of generation t in country i receives $w^i(t)$ units of the consumption good, which depends on the state of nature $z(t)$. For simplicity, there are two states of nature, denoted by "one" and "two", occuring with probability q and $(1 - q)$ respectively. Each draw of $z(t)$ is stochastically independent of any other draw. An A-young gets w_1^a units in state one, and w_2^a units in state two. Without loss of generality, assume $w_2^a > w_1^a$. Each B-young gets the same endowment in both states, i.e., $w_1^b = w_2^b$.

He consumes $c^i(t, 1)$ units when young, selling the rest for $m^i(t)$ units of real balance. Money is the only store of value. Each country issues its own currency, the nominal quantities of each are fixed at M^i. An agent is allowed to hold only money of his own country.[2] When old, he purchases $c^i(t + 1, 2)$ units of consumption, leaving no bequest.

The problem faced by each young person is to maximize his expected lifetime utility, which is separable in the two periods:[3]

$$\text{(2.1)} \qquad \text{maximize} \quad U(c^i(t, 1)) + E[V(c^i(t + 1, 2))]$$

$$\text{subject to} \qquad c^i(t, 1) = w^i(t) - m^i(t)$$

$$c^i(t + 1, 2) = m^i(t)p^i(t)/p^i(t + 1)$$

$$c^i(t, 1),\ c^i(t + 1, 2),\ m^i(t) \geqq 0$$

The agents know the current prices, $p^a(t)$ and $p^b(t)$, and the distribution of prices in the next period. E[ ] is the expectations operator.

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[2] Other regimes, allowing agents to hold money from both countries are considered in Karekan and Wallace. The equilibria under these regimes all lead to a constant exchange rate, and can be identified with the "fixed exchange rate regime" of this paper.

[3] This is not essential, but simplifies the mathematics.

We consider actions of agents only in stationary equilibria. A stationary equilibrium is one in which the same prices, p_z^a and p_z^b, occur whenever the state of nature is z. It is completely characterized by the ratios of prices in the two states, $\phi^a = p_1^a/p_2^a$ and $\phi^b = p_1^b/p_2^b$.4/

An individual born in state one will solve the first-order condition:

$$(2.2) \qquad U'(w_1^i - m_1^i) = qV'(m_1^i) + (1 - q)V'(m_1^i\phi^i)\phi^i, \quad i = a, b$$

and it is straightforward to show that the optimal real balance, $m_1^i(\phi^i)$, is a decreasing function if $R < 1$, a constant function if $R = 1$, and an increasing function if $R > 1$, where R is the degree of relative risk aversion of the second period utility:

$$(2.3) \qquad R(x) = -xV''(x)/V'(x)$$

as defined in Pratt [1964]. His expected lifetime welfare is:

$$(2.4) \qquad w_1^i(\phi^i) = U(w_1^i - m_1^i(\phi^i)) + qV(m_1^i(\phi^i)) + (1 - q)V(m_1^i(\phi^i)\phi^i), \quad i = a, b \quad .$$

An individual born in state two will solve the first-order condition:

$$(2.5) \qquad U'(w_2^i - m_2^i)\phi^i = qV'(m_2^i/\phi^i) + (1 - q)V'(m_2^i)\phi^i, \quad i = a, b \quad .$$

The optimal real balance, $m_2^i(\phi^i)$, is an increasing function if $R < 1$, a constant function if $R = 1$, and a decreasing function if $R > 1$. His expected lifetime welfare is:

$$(2.6) \qquad W_2^i(\phi^i) = U(w_2^i - m_2^i(\phi^i)) + qV(m_2^i(\phi^i)/\phi^i) + (1 - q)V(m_2^i(\phi^i)), \quad i = a, b \quad .$$

4/ ϕ^i is a measure of uncertainty of prices. A rise in ϕ^i increases the mean and variance of prices.

From (2.4) and (2.6), we can calculate the expected lifetime welfare of an unborn individual:

$$W^i(\phi^i) = W_1^i(\phi^i) + (1 - q)W_2^i(\phi^i) \tag{2.7}$$

$W^i(\phi^i)$ is a convex function when $R < 1$, a constant function when $R = 1$, and a concave function when $R > 1$. This fact is crucial in the comparison of welfare in the two exchange rate regimes.

3. Equilibrium under the Flexible Rate Regime

Each country has its own central bank. In the flexible rate regime, the sole function of the central banks is to distribute money to the old at time $t = 0$. They do not intervene in the exchange market. The trade balance in the two countries must therefore be zero, since agents are not allowed to hold the other country's money. Hence prices are random in A, and nonrandom in B.

In country A, the goods market at time t clears when the money market clears:

$$N^a p^a(t)m^a(t) = N^a p^a(t - 1)m^a(t - 1) = M^a \quad . \tag{3.1}$$

The stationary equilibrium is a price pair (p_1^a, p_2^a), which satisfies:

$$p_z^a = M^a/[N^a m_z^a], \quad z = 1, 2 \tag{3.2}$$

where m_1^a and m_2^a are the optimal real balances in the two states. We can show that $m_2^a > m_1^a$, and $w_a^a - m_2^a > w_1^a - m_2^a$, i.e., A-young born in state "two" will save and consume more. So the stationary equilibrium price ratio $\phi^a = p_1^a/p_2^a = m_2^a/m_1^a$ is greater than unity.

In country B, there is no uncertainty in the stationary equilibrium. Hence prices and real balances are constant:

(3.3) $$m_1^b = m_2^b$$

(3.4) $$p_1^b = p_2^b = M^b/[N^b m_1^b] \quad .$$

The equilibrium price ratio is:

(3.5) $$\phi^b = 1 \quad .$$

The equilibrium exchange rate in state z is given by the "law of one price":[5]

(3.6) $$S_z = p_z^a/p_z^b, \quad z = 1,\ 2$$

which is the B-currency price in terms of A-currency. The crucial result is that the exchange rate is different in the two states:

(3.7) $$S_1/S_2 = \phi^a > 1 \quad .$$

4. Equilibrium under the Fixed Rate Regime

In the fixed rate regime, the central banks cooperatively fix the exchange rate at S forever, and are willing to trade any amount of currencies at this rate. This fixes the world private stock of money at $M = M^a + SM^b$, whose composition is determined by demand conditions. The world goods market clears when the money market clears:

(4.1) $$\pi(t)\left[N^a m^a(t) + N^b m^b(t)\right] = \pi(t-1)\left[N^a m^a(t-1) + N^b m^b(t-1)\right] = M$$

where $\pi(t)$ is the price of consumption in units of A-currency.

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[5] We assume that there are no barriers to trade, i.e., no transport costs, tariffs, quotas, etc. Therefore goods arbitrage ensures the validity of the "law of one price."

The stationary equilibrium under this regime is a price pair (π_1, π_2) which satisfies:

$$\pi_z = M/[N^a\mu_z^a + N^b\mu_z^b], \quad z = 1, 2 \tag{4.2}$$

where μ_1^a, μ_2^a, μ_1^b, and μ_2^b are the optimal real balances in the two states. The equilibrium price ratio, $\phi^* = \pi_1/\pi_2$, is the fixed point of $[N^a m_2^a(\phi) + N^b m_2^b(\phi)]/[N^a m_1^a(\phi) + N^b m_1^b(\phi)]$. It can be shown that ϕ^* is between $\phi^b = 1$ and $\phi^a > 1$. (The proof is outlined in Appendix 1.)

There are several interesting points about this equilibrium. One, the real balances and the price ratio are independent of the money supplies and the (fixed) exchange rate. Second, the larger the population of A relative to that of B, the closer ϕ^* is to $\phi^a > 1$; conversely, the smaller the population of A relative to that of B, the closer ϕ^* is to $\phi^b = 1$.

Third, A-young born in state two desires to hold a higher real balance than those born in state one. In fact, they hold a higher nominal balance: $\pi_1\mu_2^a > \pi_1\mu_1^a$. (A proof is furnished in Appendix 1.)

Four, the nominal trade balance is zero on average, while the real trade balance is positive (negative) for country A (B) on average. This comes from the previous observation. There are three cases: (a) If the state of nature was "one" last period and "two" this period, the current A-young will want to accumulate a higher nominal balance than the current A-old. They sell some of their endowment to B-old, running a trade surplus. (b) Conversely, if the state of nature was "two" last period and "one" this period, A will run a trade deficit of an equal nominal amount. (c) When the same state of nature occurs consecutively, the trade balance in the second period is zero. Since the events (a) and (b) occur with equal probability, the nominal trade balance is zero on average. But the real trade balance is nonzero, since A tends to run a surplus when the price of consumption is low, and a deficit when it is high, which implies that A expects to run a real trade surplus.

Intuitively, the fixed regime unifies the currency and goods markets, forcing B-residents to share the endowment risks of A-residents. In a good state, i.e., $z = 2$, A "gives" B consumption by running a trade surplus. In a bad state, i.e., $z = 1$, A "takes" consumption from B by running a trade deficit. This risk sharing arrangement increases the mean consumption in B, i.e., A expects to run a real trade surplus.

5. Comparison of Exchange Rate Regimes

First, we compare the distribution of random variables under the two regimes. From the point of view of country B, the fixed rate regime increases the price variance, because the disturbance is external to country B. On the other hand, the fixed rate regime reduces the price variance for country A, since the disturbance is internal to country A. It is also easy to show that the mean and variance of aggregate consumption are lower (higher) in country A (B) under the fixed rate regime. This agrees with the usual findings, as in Fischer [1977].

Second, we compare the expected lifetime welfare of an unborn agent, $W^i(\phi)$, between the two regimes. As noted in Section 2, $W^i(\phi)$ depends on

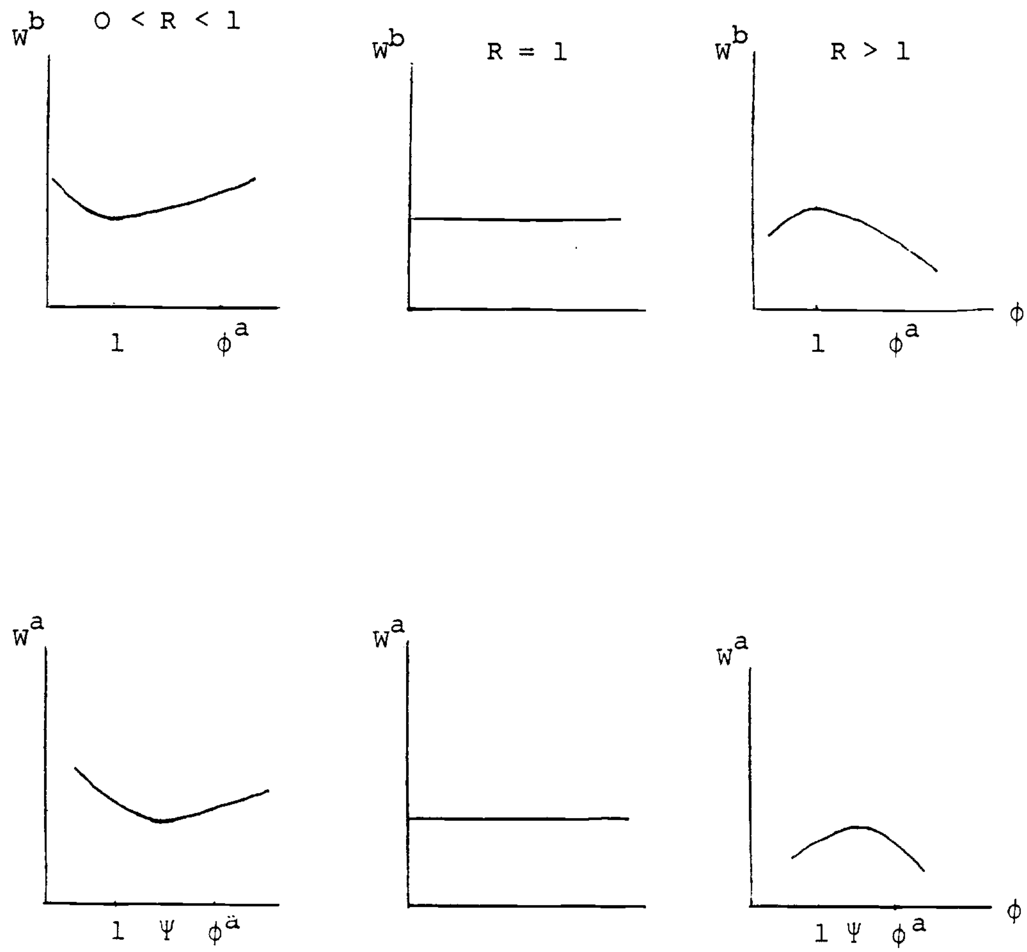

Figure 1

Welfare Comparisons Between Exchange Regimes the degrees of relative risk aversion. The various cases are exhibited in Figure 1, and summarized as follows:

(1) If $R < 1$, $W^i(\phi)$ is convex. $W^a(\phi)$ is decreasing over the interval $(1, \psi)$, for some ψ between 1 and ϕ^a, and increasing over (ψ, ∞). $W^b(\phi)$ is decreasing over $(0, 1)$, and increasing over $(1, \infty)$.

(2) If $R = 1$, $W^i(\phi)$ is constant.

(3) If $R > 1$, $W^i(\phi)$ is concave. $W^a(\phi)$ is increasing over the interval $(1, \psi)$, for some ψ between 1 and ϕ^a, and decreasing over (ψ, ∞). $W^b(\phi)$ is increasing over $(0, 1)$, and decreasing over $(1, \infty)$.

(4) If either $R < 1$ or $R > 1$, $W^a(1) > W^a(\phi^a)$.

(The proofs are in Appendix 2.)

Clearly, B-individuals prefer the fixed regime if $R < 1$. They are indifferent if $R = 1$, and they prefer the flexible rate regime if $R > 1$. These results are simple to explain. For a given increase in both the mean and variance of consumption, B-residents are better off if they are not very risk averse, and worse off if they are very risk averse.

The cases for A-residents are not so clear. If they are highly risk averse, i.e., $R > 1$, they prefer the fixed rate regime, because the reduction in the variance of consumption more than compensates for the reduction in the mean of consumption. If $R = 1$, they are indifferent. But if they are not very risk averse, i.e., $R < 1$, then the case is ambiguous. When A is large, ϕ^* is close to ϕ^a, and they prefer the flexible rate regime. When A is small, ϕ^* is close to $\phi^b = 1$, and they prefer the fixed rate regime.

Thus far, the analysis has assumed that preferences are identical across countries. All the results will obtain, when we allow preferences to differ

Table 1

Welfare Comparison Between Exchange Regimes

Country A		Country B: $R^b > 1$	Country B: $R^b = 1$	Country B: $R^b < 1$
	$R^a > 1$	(+, -)	(+, 0)	(+, +)
	$R^a = 1$	(0, -)	(0, 0)	(0, +)
	$R^a < 1$	(?, -)	(?, 0)	(?, +)
	*	(-, -)	(-, 0)	(-, +)
	**	(+, -)	(+, 0)	(+, +)

The first entry of the ordered pair pertains to country A, the second entry pertains to B. The following conventions are used:

+: prefer fixed regime

0: indifferent

-: prefer flexible regime

Note: * A is large relative to B

** A is small relative to B.

across countries (but remain identical within each country). Table 1 gives the welfare comparisons. The polar cases are the most interesting ones. For example, $R^a > 1$ and $R^b < 1$ imply that both countries prefer the fixed rate regime. When $R^a < 1$ and $R^b > 1$, and when A is large, both prefer the flexible rate regime. There are also examples where the countries disagree--when $R^a > 1$ and $R^b > 1$, or when $R^a < 1$ and $R^b < 1$ with A large compared to B.

5. Concluding Remarks

When there are no missing markets under either exchange regime, money plays no essential role in the model, and so the allocation of real resources is invariant to the exchange regime.[7] In this case, there is no reason to prefer either regime. On the other hand, when some markets are missing under both regimes, it is not clear that the ability to share risk across countries in the fixed rate regime improves welfare over the flexible rate regime which prevents any risk sharing. This is a standard result in a second best world, and is likely to be robust against all modifications of the model as long as markets remain incomplete in both regimes.

By the same token, there are circumstances in which one exchange regime has a complete set of markets while the other regime does not. We then opt for the regime which attains the first best equilibrium. For example, we can introduce a forward exchange market into our model, thus completing the set of markets in the flexible regime but not in the fixed regime, making the former preferable. Another modification is to include random endowments in country B, such that $w_1^b = w_2^b$, $w_2^a = w_1^b$, and $N^a = N^b$. There will be no aggregate uncertainty in the fixed rate regime, which makes it preferable over the flexible rate regime.

[7] See Lucas [1981].

Appendix 1

Lemma 1: $\phi m_1^b(\phi) \leq m_2^b(\phi)/\phi$ if and only if $\phi \leq 1$.

Proof: Define $h(\phi) = \phi m_1^b(\phi) - m_2^b(\phi)/\phi$. Dropping the superscripts to save notation, we have:

$$h' = \left[1 + \phi m_1'/m_1\right]m_1 + \left[1 - \phi m_2'/m_2\right]\left[m_2/\phi^2\right] \quad .$$

It is easy to show that $\left[1 + \phi m_1'/m_1\right] > 0$, and $\left[1 - \phi m_2'/m_2\right] > 0$. Thus, $h' > 0$. Now $h(1) = 0$. Thus $h(\phi) > 0$ for $\phi > 1$, $h(\phi) < 0$ for $\phi < 1$. Q.E.D.

Corollary 1: $\phi m_1^b(\phi) > m_2^b(\phi)$ when $\phi > 1$.

Lemma 2: $\phi m_1^a(\phi) \leq m_2^a(\phi)/\phi$ if and only if $\phi \leq \psi$, for some ψ in $(1, \phi^a)$.

Proof: Define $h(\phi) = \phi m_1^a(\phi) - m_2^a(\phi)/\phi$. Dropping superscripts to save notation, we have:

$$h' = \left[1 + \phi m_1'/m_1\right]m_1 + \left[1 - \phi m_2'/m_2\right]\left[m_2/\phi^2\right] \quad .$$

Clearly, $h' > 0$, for the same reasons as in Lemma 1. Note that $h(1) < 0$, since $m_1^a(1) < m_2^a(1)$.

Also $h(\phi^a) > 0$, since $\phi^a m_1^a(\phi^a) = m_2^a(\phi^a) > m_2^a(\phi^a)/\phi^a$. Therefore, there exists a unique ψ between 1 and ϕ^a, such that $h(\psi) = 0$.

THEOREM 1: $1 < \phi^* < \phi^a$.

Proof: Define $f(\phi) = \dfrac{N^a m_2^a(\phi) + N^b m_2^b(\phi)}{N^a m_1^a(\phi) + N^b m_1^b(\phi)}$. Clearly, $f(1) > 1$, since $m_1^a(1) > m_1^a(1)$, and $m_2^b(1) = m_1^b(1)$. Also, $f(\phi^a) < \phi^a$. This is shown as follows: $m_2^a(\phi^a) = \phi^a m_1^a(\phi^a)$. Hence $f(\phi^a) < \phi^a$ if and only if $m_2^b(\phi^a) < \phi^a m_1^b(\phi^a)$, which is true by Corollary 1 (since $\phi^a > 1$). By continuity, there exists ϕ^* such that $\phi^* = f(\phi^*)$, and $\phi^b = 1 < \phi^* < \phi^a$.

Corollary 2: $\mu_2^a = m_2^a(\phi^*) > \phi^* m_1^a(\phi^*) = \phi^* \mu_1^a$.

Proof: $\phi^*[N^a \mu_1^a + N^b \mu_1^b] = N^a \mu_2^a + N^b \mu_2^b$. So $N^b[\phi^* \mu_1^b - \mu_2^b] = N^a[\mu_2^a - \phi^* \mu_1^a]$. Since $\phi^* > 1$, we know $\phi^* \mu_1^b > \mu_2^b$, from Corollary 1. Hence $\mu_2^a > \phi^* \mu_1^a$.

Appendix 2

Lemma 3: Let $f(x) = xV'(x)$, where $V(\ )$ is increasing and concave. Then $f'(x) > 0$ if $R(x) < 1$, $f(x) = 0$ if $R(x) = 1$, and $f'(x) < 0$ if $R(x) > 1$.

Proof: $f'(x) = V'(x) + xV''(x) = V'(x)\left[1+ \frac{xV''(x)}{V'(x)}\right] = V'(x)[1-R(x)]$. Q.E.D.

Now, define:

$$g_i(\phi) = E[w^i(\phi)] = qw_1^i(\phi) + (1-q)w_2^i(\phi) = qU(w_1^i - m_1^i(\phi)) + (1-q)U(w_2^i - m_2^i(\phi))$$
$$+ q^2V(m_1^i(\phi)) + q(1-q)V(\phi m_1^i(\phi)) + q(1-q)V(m_2^i(\phi)/\phi)$$
$$+ (1-q)^2V(m_2^i(\phi)), \qquad \text{for } i = a, b .$$

Note that $g_i'(\phi) = q(1-q)\{\phi m_1^i(\phi)V'(\phi m_1^i(\phi)) - [m_2^i(\phi)/\phi]V'(m_2^i(\phi)/\phi)\}$. Consider the case for $i = b$. At $\phi = 1$, $g_b'(1) = 0$. Suppose $\phi < 1$. Then $\phi m_1^b(\phi) < m_2^b(\phi)/\phi$, by Lemma 2. Suppose $R < 1$. Then $\phi m_1^b(\phi)V'(\phi m_1^b(\phi)) < [m_2^b(\phi)/\phi]V'(m_2^b(\phi)/\phi)$, by Lemma 4. In other words, $g_b'(\phi) < 0$ for $\phi < 1$. Similarly, $g_b'(\phi) > 0$ for $\phi > 1$. This means that $E[w^b(\phi)]$ is increasing over $(0, 1)$, and decreasing over $(1, \infty)$, reaching a maximum at $\phi = 1$. The other cases for B are shown analogously.

Now consider the case for $i = a$. By Lemma 2 we know there exists ψ between 1 and ϕ^a such that $\psi m_1^a(\psi) = m_2^a(\psi)/\psi$. Hence $g_a'(\psi) = 0$. Suppose $\phi < \psi$. Then $\phi m_1^a(\phi) < m_2^a(\phi)/\phi$, by Lemma 3. Suppose $R > 1$. Then $\phi m_1^a(\phi)V'(\phi m_1^a(\phi)) > [m_2^a(\phi)/\phi]V'(m_2^a(\phi)/\phi)$, by Lemma 4. So, $g_a'(\phi) > 0$ for $\phi < \tilde{\phi}$. Similarly, $g_a'(\phi) < 0$ for $\phi > \tilde{\phi}$. This means $E[w^a(\phi)]$ is decreasing over $(0, \psi)$, reaching a minimum at $\phi = \psi$, and increasing over (ψ, ∞). The other cases for A are shown analogously.

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