NBER WORKING PAPER SERIES

# WHEN IS A POSITIVE INCOME TAX OPTIMAL?

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Working Paper No. 631

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

February 1981

The research reported here is part of the NBER's research program in Taxation. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #631
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# When is a Positive Income Tax Optimal?

## ABSTRACT

When will the optimal mix of a constant income tax with a constant consumption tax involve a positive income tax? The assumptions of the model in which this question is asked include (1) identical individuals with coincident lifetimes who work in every period; (2) initial endowments of physical capital; (3) fixed government expenditures; and (4) government borrowing (or lending) that goes to zero when the world ends. In a model like this, we can ignore the transition problem. If we allow the constant tax on income from capital and the constant tax on wage income to be at different rates, we can ask a further question. When will the optimal mix of all three taxes (including the consumption tax) involve a positive tax on either income from capital or wage income?

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## INTRODUCTION

Current theoretical models of the choice between an income tax and a consumption tax suggest that the optimal mix of these two taxes depends in general on the elasticities of labor supply and saving as the mix of taxes is changed.[1] These models generally involve only two periods of life, with labor in the first period and retirement in the second period. Extending them to cases where labor is supplied in many periods seems to make a consumption tax more likely to dominate an income tax.[2] In fact, the optimal tax on income from capital may become negative.[3]

Assuming that people live and work for many periods is interesting not only because individual lifetimes may be long, but also because the bequest motive may cause people to act as if they have lives spanning many generations.[4] The limiting case is to assume that lives are infinite.

If we are to assume that lives are infinite, then there is no need to assume that generations overlap.[5] Each person will start out with an endowment of physical capital. In an overlapping generations model, the usual assumption is that the young acquire physical capital by saving out of their wage income, but they do not start out with any physical capital.

The simplest infinite lives model is one in which everyone is identical. If everyone is identical, then a change in tax policies will not cause any transfers from one group to another. Thus we can ignore any transition problems.[6]

We can start with a finite lives model in which everyone is identical, and then let the lives become longer and longer. Since people know that the life of the human race is not truly infinite, this is sounder for both economic and mathematical reasons than assuming literally infinite lives.

We will assume a world of certainty in which people know everything about the future and behave optimally. We will limit our attention to consumption and income taxes that are constant over time, partly because we hope to find a tax policy that is time consistent.[7] Policies involving planned changes in tax rates (for efficiency reasons) do not seem to receive serious political consideration. I believe that any policy involving constant tax rates (at least if the rates are positive or zero) would be politically feasible. Policies involving lump sum taxes or temporarily high taxes on income from capital or taxes that shift from capital to labor over time do not seem politically feasible. Assuming that tax rates are constant does not, however, ensure that the best mix of taxes will be time consistent. The best mix of constant taxes may depend on the level of the capital stock or on the amount of public debt outstanding, even when we assume infinite lives.

Since everyone is identical, it seems natural to assume zero population growth. Taxes are used to pay for given government programs; there is no reason to use taxes to redistribute wealth. Since tax rates are constant through time, we must assume that the government can borrow or lend, but we will assume that government debt must be zero when the world ends.

We will assume that the consumption tax is proportional, at a rate that is the same for all kinds of consumption. The income tax will also be proportional, and will be applied initially at the same rate to wage income and income from capital. This allows us to ignore the effect of a change in the tax rate on income from capital on the present value of wage income.[8] It also avoids the effects of differential taxation of income from investments in human and physical capital which would occur in an expanded model including such investments.[9] This assumption means that we do not allow the use of added leisure to make investments in human capital, which would effectively eliminate the tax on investments in human capital.[10]

Assuming that individuals are identical and have indefinitely long lives seems less restrictive when we are looking at taxes that affect saving than it does in other contexts. The essence of the problem is the long run. If some individuals care only about the current generation, a society might give their opinions little weight in making decisions that affect its growth and possibly its survival.

I have not yet obtained any results for the infinite lives problem. I don't even fully understand the two period version of the problem, so that is the version we will start with.

THE PROBLEM

Let us define the following symbols:

| | |
|---|---|
| $\tau_c, \tau_y$ | consumption and income tax rates |
| $w_1, w_2$ | wage per period, periods 1 and 2 |
| $\ell_1, \ell_2$ | leisure (as a fraction) in periods 1 and 2 |
| $r_1, r_2$ | interest per period, periods 1 and 2 |
| $k_1, k_2$ | initial capital per person, periods 1 and 2 |
| $c_1, c_2$ | consumption per person, periods 1 and 2 |
| $x_1, x_2$ | output per person, periods 1 and 2 |

We will use a prime to mean the rate of change of a quantity as we decrease the income tax and increase the consumption tax, assuming that the government's budget constraint is always satisfied. Thus a prime means a "compensated" rate of change.

The first order condition for the government's optimization problem may be written equivalently in any of the following three ways:

(50) $$0 = -(\tau_c + \tau_y)[w_1 \ell_1' + w_2 \ell_2'/(1 + r_2(1 - \tau_y))] + \tau_y(1 + \tau_c) r_2 k_2'/(1 + r_2(1 - \tau_y))$$

(51) $$0 = (\tau_c + \tau_y)[c_1' + c_2'/(1 + r_2(1 - \tau_y))] + \tau_y(1 - \tau_y) r_2 k_2'/(1 + r_2(1 - \tau_y))$$

(52) $$0 = (\tau_c + \tau_y)[x_2' + x_2'/(1 + r_2(1 - \tau_y))] - \tau_c(1 - \tau_y) r_2 k_2'/(1 + r_2(1 - \tau_y))$$

The problem is this: are there restrictions on the utility function, the production function, and the length of the period that will make a positive income tax optimal? If so, what are they? Do similar restrictions ensure a finite positive income tax when the number of periods is increased from two toward infinity, or does the optimal income tax approach zero?

I can find the optimal taxes in two cases: when labor is inelastic, the optimal income tax is zero; and when the length of the second period is zero, the optimal income tax rate is the negative of the consumption tax rate. The first result comes from equation (50), setting both $\ell_1'$ and $\ell_2'$ equal to zero. The second result comes from setting $r_2$ equal to zero in any one of the equations.

The intuition behind the first result is this: when the labor supply is fixed, it will not be distorted by either an income tax or a consumption tax. The income tax distorts saving, while the consumption tax does not. Thus the optimal income tax is zero.

The intuition behind the second result is this: when the length of the second period is zero, it is a one period problem with no saving. The income tax does not distort saving. By making the income tax rate the negative of the consumption tax rate, we eliminate the distortion of labor supply. Since some consumption in the first period is consumption of capital rather than income, this combination of taxes will raise revenue. If it raises enough revenue, it is optimal, because in the one period problem it is nondistorting.

I can imagine many cases in which the optimal income tax rate is between the negative of the consumption tax rate and zero, but I have not been able to imagine any in which the optimal income tax rate is positive. Are there any?

If we allow the constant tax rate on income from capital and the constant tax rate on wage income to differ, the optimal mix of taxes will be a positive consumption tax, a negative wage tax at the same rate, and a zero tax on income from capital when this will provide enough revenue to pay for all government spending.[11] (This mix of taxes is non-distorting.) But suppose the government budget constraint cannot be satisfied by a mix of taxes of this form. When does the optimal mix of these three taxes involve a positive tax on either wage income or income from capital?

## ANALYSIS

### The Individual's Problem

We assume a time separable utility function. Writing $\rho$ for the individual's utility discount factor, the individual's problem is (1), with constraints (2) and (3).

(1)
$$\max_{c_1, \ell_1, c_2, \ell_2} u(c_1, \ell_1) + \rho u(c_2, \ell_2)$$

(2) $$k_2 + d_2 = (k_1 + d_1)(1 + r_1(1 - \tau_y)) + w_1(1 - \ell_1)(1 - \tau_y) - c_1(1 + \tau_c)$$

(3) $$0 = (k_2 + d_2)(1 + r_2(1 - \tau_y)) + w_2(1 - \ell_2)(1 - \tau_y) - c_2(1 + \tau_c)$$

In the constraints, $d_1$ and $d_2$ are the amounts of government debt outstanding at the start of periods 1 and 2.

The first order conditions for this problem may be written, using a positive multiplier $\lambda$, as follows:

(4) $$u_1(c_1,\ell_1) = \lambda(1 + r_2(1 - \tau_y))(1 + \tau_c)$$

(5) $$u_2(c_1,\ell_1) = \lambda(1 + r_2(1 - \tau_y))w_1(1 - \tau_y)$$

(6) $$\rho u_1(c_2,\ell_2) = \lambda(1 + \tau_c)$$

(7) $$\rho u_2(c_2,\ell_2) = \lambda w_2(1 - \tau_y)$$

The first order conditons may equivalently be written as follows:

(8) $$u_1(c_1,\ell_1)/u_2(c_1,\ell_1) = (1 + \tau_c)/w_1(1 - \tau_y)$$

(9) $$u_1(c_2,\ell_2)/u_2(c_2,\ell_2) = (1 + \tau_c)/w_2(1 - \tau_y)$$

(10) $$u_1(c_1,\ell_1)/\rho u_1(c_2,\ell_2) = (1 + r_2(1 - \tau_y))$$

(11) $$u_2(c_1,\ell_1)/\rho u_2(c_2,\ell_2) = w_1(1 + r_2(1 - \tau_y))/w_2$$

The second order conditions for the individual's maximization problem are:

(12) $$u_{11} < 0$$

(13) $$u_{22} < 0$$

(14) $$u_{11} - 2u_{12} + u_{22} < 0$$

These conditions will be satisfied whenever the utility function is concave.

The constant-returns-to-scale production function may be written as follows, where $k$ is capital per person. These equations apply separately to each period.

(15) $$x = f(k/(1 - \ell))(1 - \ell)$$

(16) $$x = kr + w(1 - \ell)$$

Let us write $y$ for income per person, which includes interest on any government debt held by the individual.

(17) $$y = x + dr$$

(18) $$y = (k + d)r + w(1 - \ell)$$

The interest per period is the marginal product of capital.

$$r = f'(k/(1 - \ell)) \tag{19}$$

The wage rate may be written in two equivalent ways.

$$w = (x - kr)/(1 - \ell) \tag{20}$$

$$w = f(k/(1 - \ell)) - kr/(1 - \ell) \tag{21}$$

Taking compensated derivatives with respect to an increase in the consumption tax, we have:

$$x' = [f'(k/(1 - \ell))k' + [f'(k/(1 - \ell))k/(1 - \ell) - f(k/(1 - \ell))]\ell' \tag{22}$$

$$x_1' = -w_1\ell_1' \tag{23}$$

$$x_2' = r_2k_2' - w_2\ell_2' \tag{24}$$

$$r' = f''(k/(1 - \ell))[k'/(1 - \ell) + k\ell'/(1 - \ell)^2] \tag{25}$$

$$r_1' = f''(k_1/(1 - \ell_1))k_1\ell_1'/(1 - \ell_1)^2 \tag{26}$$

$$r_2' = f''(k_2/(1 - \ell_2))[k_2'/(1 - \ell_2) + k_2\ell_2'/(1 - \ell_2)^2] \tag{27}$$

$$w' = -kr'/(1 - \ell) \tag{28}$$

The difference between (23) and (24) and the difference between (26) and (27) come from the fact that the capital stock at the start of the first period is given, while the capital stock at the start of the second period depends on the tax rates.

The Government's Problem

The government's optimization problem is (29), with constraints (30) and (31).

(29) $$\max_{\tau_c} \quad u(c_1,\ell_1) + \rho u(c_2,\ell_2)$$

(30) $$d_2 = g_1 + d_1(1 + r_1) - y_1\tau_y - c_1\tau_c$$

(31) $$0 = g_2 + d_2(1 + r_2) - y_2\tau_y - c_2\tau_c$$

In (30) and (31), $g_1$ and $g_2$ are given government expenditures in periods 1 and 2. Note that equation (31) specifies that government debt is zero when the world ends.

The first order condition may be written as follows, where the compensated derivative ensures that the constraints (30) and (31) will both be satisfied. (Actually, (30) may be taken as a definition of $d_2$.)

(32) $$0 = u_1(c_1,\ell_1)c_1' + u_2(c_1,\ell_1)\ell_1' + \rho u_1(c_2,\ell_2)c_2' + \rho u_2(c_2,\ell_2)\ell_2'$$

Substituting from the individual's first order conditions (4), (5), (6), and (7), we have:

(33) $$0 = (1 + r_2(1 - \tau_y))[c_1'(1 + \tau_c) + w_1\ell_1'(1 - \tau_y)] + c_2'(1 + \tau_c) + w_2\ell_2'(1 - \tau_y)$$

Rearranging, we have:

$$(34) \qquad 0 = (1 + \tau_c)[c_1'(1 + r_2(1 - \tau_y)) + c_2'] + (1 - \tau_y)[w_1\ell_1'(1 + r_2(1 - \tau_y)) + w_2\ell_2']$$

Substituting from equation (18) into equations (2) and (3), we have:

$$(35) \qquad k_2 + d_2 = k_2 + d_2 + y_1(1 - \tau_y) - c_1(1 + \tau_c)$$

$$(36) \qquad 0 = k_2 + d_2 + y_2(1 - \tau_y) - c_2(1 + \tau_c)$$

Adding (35) and (36), and writing $y*$ and $c*$ for $y_1 + y_2$ and $c_1 + c_2$, we have:

$$(37) \qquad 0 = k_1 + d_1 + y*(1 - \tau_y) - c*(1 + \tau_c)$$

We can combine the individual and government budget constraints as follows. From equations (30), (31), (35), and (36) we have:

$$(38) \qquad k_2 = k_1 - g_1 + x_1 - c_1$$

$$(39) \qquad 0 = k_2 - g_2 + x_2 - c_2$$

Adding (38) and (39), we have:

$$(40) \qquad 0 = k_1 - g* + x* - c*$$

In equation (40), we continue the convention that a star means the sum of the values of the variable over both periods.

Taking compensated derivatives with respect to an increase in the consumption tax of the last three equations, we have:

(41) $$k_2' = x_1' - c_1'$$

(42) $$0 = k_2' + x_2' - c_2'$$

(43) $$x^{*'} = c^{*'}$$

From equations (41) and (42), we have:

(44) $$c_1'(1 + r_2(1 - \tau_y)) + c_2' = x_1'(1 + r_2(1 - \tau_y)) + x_2' - (1 - \tau_y)r_2k_2'$$

From equations (23), (24), and (44), we have:

(45) $$c_1'(1 + r_2(1 - \tau_y)) + c_2' = - w_1\ell_1'(1 + r_2(1 - \tau_y)) - w_2\ell_2' + \tau_y r_2 k_2'$$

From equations (34), (44), and (45), we obtain:

(46) $$0 = - (\tau_c + \tau_y)[w_1\ell_1'(1 + r_2(1 - \tau_y)) + w_2\ell_2'] + \tau_y(1 + \tau_c)r_2k_2'$$

(47) $$0 = (\tau_c + \tau_y)[c_1'(1 + r_2(1 - \tau_y)) + c_2'] + \tau_y(1 - \tau_y)r_2k_2'$$

(48) $$0 = (\tau_c + \tau_y)[x_1'(1 + r_2(1 - \tau_y)) + x_2'] - \tau_c(1 - \tau_y)r_2k_2'$$

From equations (41) and (47), we have:

(49) $$0 = (\tau_c + \tau_y)c^{*\prime} + (1 - \tau_y)r_2(\tau_c c_1' + \tau_y x_1')$$

Dividing equations (46), (47), and (48) by one plus the after-tax interest rate, we have:

(50) $$0 = -(\tau_c + \tau_y)[w_1\ell_1' + w_2\ell_2'/(1 + r_2(1 - \tau_y))] + \tau_y(1 + \tau_c)r_2k_2'/(1 + r_2(1 - \tau_y))$$

(51) $$0 = (\tau_c + \tau_y)[c_1' + c_2'/(1 + r_2(1 - \tau_y))] + \tau_y(1 - \tau_y)r_2k_2'/(1 + r_2(1 - \tau_y))$$

(52) $$0 = (\tau_c + \tau_y)]x_1' + x_2'/(1 + r_2(1 - \tau_y))] - \tau_c(1 - \tau_y)r_2k_2'/(1 + r_2(1 - \tau_y))$$

The $k_2'$ that appears in these three equations represents total added saving, both private and government, as a result of the shift in taxes. Can it be negative in this model?

The $c_1'$ in equation (51) will be negative when labor supply is fixed, but will be positive when the second period is very short. The expression $c_1' + c_2'/(1 + r_2(1 - \tau_y))$ will be positive in both of those special cases. Will it always be positive in this model?

Can we get more definite answers by assuming a Cobb-Douglas utility function? By going to the limit of an infinite number of periods whose lengths approach zero? Can a positive income tax be optimal in this model? Can a positive wage tax or a positive tax on income from capital be optimal when we allow all three taxes to differ?

FOOTNOTES

*I am grateful to Robert Merton for extensive discussions of this subject, and to Victor Canto, Jonathan Hamilton, Laurence Kotlikoff, and Merton Miller for comments on an earlier draft.

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[1]For example, Feldstein (1978, p. S49) says: "complete elimination of the tax on capital income in favor of a tax on consumption or labor income is optimal only when the structure of individual preferences satisfies a particular separability condition." Bradford (1980, p. 26), discussing the conditions under which zero taxation of the return to saving is optimal, notes: "the key sensitivity affecting this issue is the responsiveness of labor supply, not savings, to the tax rates." King (1980, p. 18), says: "the values of the optimal tax rates depend upon the cross-elasticities of saving with respect to the wage rate and labour supply with respect to the interest rate." Atkinson and Sandmo (1980, p. 539) say: "the tax on savings is more likely to raise welfare, the larger is the compensated elasticity of labour supply $(\sigma_{LL})$ relative to that of future consumption $(-\sigma_{22})$." For a recent discussion of the theory and empirical work on the optimal mix of taxes, see McClure (1980).

[2]Summers (1978, p. 21) says: "since in a realistic life-cycle model savings are very interest elastic, changes in capital taxes have only a small effect on the net interest rate. Thus partial equilibrium analysis by assuming a constant gross interest rate greatly overstates the importance of intertemporal substitution effects . . . The important effect of removing capital taxes on welfare is not captured in partial equilibrium analyses. The large increase in capital which results raises real wages and leads to a larger level of sustainable consumption."

[3]King (1980, p. 42) provides examples of this.

[4]Miller and Upton (1974, pp. 176 - 179) show that the bequest motive can make a consumer act as if he were immortal. Barro (1974, pp. 1098 - 1101) makes the same point.

[5]Atkinson and Sandmo (1980) and King (1980), among others, use models with overlapping generations. Chamley (1980a, 1980b) presents models of optimal taxation in which people have infinite lives. He considers constant tax rates on wages and income from capital in (1980b, pp. 23-26).

[6]Summers (1978, p. 26) discusses the transition problems in an overlapping generations life-cycle model. Auerbach and Kotlikoff (1980) use simulation to look at the transition problems in a life cycle model with overlapping generations and no bequest motive.

[7]Prescott (1977) emphasizes the possible conflict between time consistency and optimality in a dynamic problem. In the model he looks at, the optimal tax policy is not time consistent. He says (p. 21): "assuming expenditures are not too large, only capital income will be taxed in the initial period, as it is supplied inelastically. In subsequent periods, both labor and capital incomes will be taxed. The inconsistency of the optimal solution arises because the optimal tax on labor is zero in the current period and positive in future ones, but eventually future periods become the current one." Kydland and Prescott (1980) discuss the same subject. In their model, the government cannot borrow or lend, so tax rates must be allowed to vary through time (p. 83).

[8]This effect plays a key role in Summers (1978) analysis. When lives are infinite, and we are in steady state, a change in the wage tax and an equal change in the tax on income from capital will leave the present value of future after-tax wage income unchanged.

[9]Driffill and Rosen (1980) look at optimal taxes in a model with investment in both human and physical capital.

[10]This is noted by Boskin (1978, p. 56).

[11]Chamley (1980b, p. 11 and n. 14) makes a similar point.

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