# Online Appendix

Exchange Rates, Interest Rates, and Gradual Portfolio Adjustment

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This Online Appendix has three sections. Section A develops a full general equilibrium model that leads to the same real exchange rate equation as in the paper. Section B develops the algebra associated with the model with long terms bonds in Section 4 of the paper. Section C provides empirical evidence on U.S. international equity portfolios that is consistent with delayed portfolio adjustment.

# A General Equilibrium Model

We first describe the model, then the first-order conditions, followed by linearization and the solution.

## A.1 Model

There are two countries. Each country has a continuum of agents on the interval [0,1]. There are overlapping generations, with agents living two periods. Agents make decisions about consumption, portfolio allocation and price setting. When young, agent $i$ in the Home country produces Home good $i$. Output is equal to labor $L_t(i)$ supplied by the agent. The good is sold in both countries:

$$L_t(i) = x_{Ht}(i) + x^*_{Ht}(i) \tag{A.1}$$

$x_{Ht}(i)$ and $x^*_{Ht}(i)$ are the quantities sold in the Home and Foreign countries. The revenue from selling the good, measured in the Home currency, is

$$Y_t(i) = P_{Ht}(i)x_{Ht}(i) + S_t P^*_{Ht}(i)x^*_{Ht}(i) \tag{A.2}$$

Here $P_{Ht}(i)$ is the price of Home good $i$ in the Home market in the Home currency and $P^*_{Ht}(i)$ is the price of Home good $i$ in the Foreign market in the Foreign currency. The nominal exchange rate $S_t$ is measured as Home currency per unit of the Foreign currency.

The assets of agent $i$ are equal to saving when young:

$$A^y_t(i) = Y_t(i) - P_t C^y_t(i) - tax_t \tag{A.3}$$

Here $P_t$ is the consumer price index and $C_t^y(i)$ is a consumption index (defined below) when young at time $t$. $tax_t$ is a nominal lump sum tax. The assets are invested in Home and Foreign bonds and the returns are consumed at time $t+1$:

$$C_{t+1}^o(i) = R_{t+1}^p(i)\frac{A_t^y}{P_t} \tag{A.4}$$

where the portfolio return is

$$R_{t+1}^p(i) = \left[z_t(i)\frac{S_{t+1}}{S_t}e^{i_t^*}e^{-\tau} + (1-z_t(i))e^{i_t}\right]\frac{P_t}{P_{t+1}} + T_{t+1} \tag{A.5}$$

Here $z_t(i)$ is the fraction invested in the Foreign bond and $1-z_t(i)$ is the fraction invested in the Home bond. The nominal interest rate is $i_t$ for the Home bond and $i_t^*$ for the Foreign bond. There is a cost $\tau$ of investment abroad that is reimbursed through a lump sum $T_{t+1}$. (A.5) corresponds to equation (2) in the paper.

Agent $i$ in the Home country maximizes

$$\begin{aligned}&\omega C_t^y(i) + ln(C_{t+1}^{o,CE}(i)) - \phi L_t(i)^\eta - 0.5\psi(z_t(i) - z_{t-1}(i))^2 \\ &-0.5\alpha\nu(P_{Ht}(i) - P_{H,t-1}(i))^2 - 0.5(1-\alpha)\nu(P_{Ht}^*(i) - P_{H,t-1}^*(i))^2\end{aligned} \tag{A.6}$$

Here $C_{t+1}^{o,CE}(i)$ is the certainty equivalent of consumption $C_{t+1}^o(i)$ when old, defined as

$$C_{t+1}^{o,CE}(i) = \left[E_t\left(C_{t+1}^o(i)\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{A.7}$$

There is a cost of both portfolio adjustment and price adjustment. The cost of portfolio adjustment is the same as in the paper and depends on the parameter $\psi$. The cost of price adjustment depends on $\alpha\nu$ when sold in the Home country and $(1-\alpha)\nu$ when sold in the Foreign country. Here $\alpha$ is the fraction spent on domestic goods. For given prices, agents $i$ will supply the labor $L_t(i)$ needed to produce enough of the good to fullfil all demand by Home and Foreign agents.

Consumption is a Cobb Douglas index of Home and Foreign goods:

$$C_t^k(i) = \left(\frac{C_{Ht}^k(i)}{\alpha}\right)^\alpha\left(\frac{C_{Ft}^k(i)}{1-\alpha}\right)^{1-\alpha} \tag{A.8}$$

where $k = y, o$ stands for young and old consumption. $C_{Ht}^k(i)$ and $C_{Ft}^k(i)$ are CES indices of Home and Foreign goods:

$$C_{Ht}^k(i) = \left(\int_0^1 C_{Hjt}^k(i)^{\frac{\mu-1}{\mu}}dj\right)^{\frac{\mu-1}{\mu}} \tag{A.9}$$

$$C_{Ft}^k(i) = \left(\int_0^1 C_{Fjt}^k(i)^{\frac{\mu-1}{\mu}}dj\right)^{\frac{\mu-1}{\mu}} \tag{A.10}$$

where $C^k_{Hjt}(i)$ is consumption of Home good $j$ by Home agent $i$ and $C^k_{Fjt}(i)$ is consumption of Foreign good $j$ by Home agent $i$. Also denote by $P_t$ the overall consumer price index, $P_{Ht}$ the price index of Home goods and $P_{Ft}$ the price index of Foreign goods:

$$P_t = P_{Ht}^{\alpha} P_{Ft}^{1-\alpha} \tag{A.11}$$

$$P_{Ht} = \left( \int_0^1 P_{Ht}(j)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \tag{A.12}$$

$$P_{Ft} = \left( \int_0^1 P_{Ft}(j)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \tag{A.13}$$

The notation is again analogous for Foreign agents. Foreign agent $i$ sells $x_{Ft}(i)$ and $x^*_{Ft}(i)$ in respectively the Home and the Foreign country and

$$L^*_t(i) = x_{Ft}(i) + x^*_{Ft}(i) \tag{A.14}$$

Revenue, measured in the Foreign currency, is

$$Y^*_t(i) = \frac{P_{Ft}(i) x_{Ft}(i)}{S_t} + P^*_{Ft}(i) x^*_{Ft}(i) \tag{A.15}$$

Assets of the young are

$$A^{y,*}_t(i) = Y^*_t(i) - P^*_t C^{y,*}_t(i) - tax^*_t \tag{A.16}$$

Then consumption at $t+1$ is

$$C^{o,*}_{t+1}(i) = R^{p,*}_{t+1}(i) \frac{A^{y,*}_t(i)}{P^*_t} \tag{A.17}$$

with portfolio return

$$R^{p,*}_{t+1}(i) = \left[ z^*_t(i) e^{i^*_t} + (1 - z^*_t(i)) \frac{S_t}{S_{t+1}} e^{i_t} e^{-\tau} \right] \frac{P^*_t}{P^*_{t+1}} + T^*_{t+1} \tag{A.18}$$

The cost $\tau$ of investing in the Home bond is reimbursed through the lump sum $T^*_{t+1}$. $z^*_t(i)$ is the fraction of assets invested in the Foreign bond. Foreign price indices, denoted in the Foreign currency, are

$$P^*_t = (P^*_{Ht})^{1-\alpha} (P^*_{Ft})^{\alpha} \tag{A.19}$$

$$P^*_{Ht} = \left( \int_0^1 (P^*_{Ht}(j))^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \tag{A.20}$$

$$P^*_{Ft} = \left( \int_0^1 (P^*_{Ft}(j))^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \tag{A.21}$$

The real value of the bond supply is assumed to be fixed at $B$ in both countries. The level of $B$ does not matter for what follows. In the paper we set $B = 1$. A constant real bond supply is accomplished through the nominal lump sum tax, $tax_t$ in the Home country in the Home currency and $tax_t^*$ in the Foreign country in the Foreign currency. The real bond supply is constant by assuming that the real value of the tax equals the real interest on the government debt:

$$tax_t = P_t \left( e^{i_{t-1}} \frac{P_{t-1}}{P_t} - 1 \right) B \tag{A.22}$$

$$tax_t^* = P_t^* \left( e^{i_{t-1}^*} \frac{P_{t-1}^*}{P_t^*} - 1 \right) B \tag{A.23}$$

Finally, bond market clearing conditions are

$$\int_0^1 z_t(i) A_t^y(i) di + \int_0^1 z_t^*(i) S_t A_t^{y,*}(i) di = B P_t^* S_t \tag{A.24}$$

$$\int_0^1 (1 - z_t(i)) A_t^y(i) di + \int_0^1 (1 - z_t^*(i)) S_t A_t^{y,*}(i) di = B P_t \tag{A.25}$$

Because of Walras' Law, we only need to impose the first of the two asset market clearing conditions.

## A.2 First Order Conditions

Agents make decisions about consumption, portfolio allocation and prices. The first-order condition for consumption can be obtained by maximizing

$$\omega C_t^y(i) + \frac{1}{1-\gamma} ln E_t \left( R_{t+1}^p(i) (y_t(i) - C_t^y(i) - tax_t/P_t) \right)^{1-\gamma} \tag{A.26}$$

where $y_t(i) = Y_t(i)/P_t$. This gives

$$\omega = \frac{E_t R_{t+1}^p(i)^{1-\gamma} (a_t^y(i))^{-\gamma}}{E_t \left( R_{t+1}^p(i) a_t^y(i) \right)^{1-\gamma}} \tag{A.27}$$

where $a_t^y(i) = A_t^y(i)/P_t$. The solution is

$$a_t^y(i) = \frac{1}{\omega} \tag{A.28}$$

Analogously, for the Foreign country we have $a_t^{y,*} = 1/\omega$. We will assume that $1/\omega = B$, so that real wealth is equal to the real bond supply in both countries.

When choosing the optimal portfolio share, Home agents maximize

$$\frac{1}{1-\gamma} ln E_t \left(R^p_{t+1}(i)\right)^{1-\gamma} - 0.5\psi(z_t(i) - z_{t-1}(i))^2 \tag{A.29}$$

The first-order condition is

$$\frac{E_t \left(R^p_{t+1}(i)\right)^{-\gamma} \left(e^{i^*_t + s_{t+1} - s_t - \tau - \pi_{t+1}} - e^{i_t - \pi_{t+1}}\right)}{E_t \left(R^p_{t+1}(i)\right)^{1-\gamma}} - \psi(z_t(i) - z_{t-1}(i)) = 0 \tag{A.30}$$

where $\pi_{t+1} = p_{t+1} - p_t$ is inflation and lower case letters denote logs. The analogous first-order condition for Foreign agents is

$$\frac{E_t \left(R^{p,*}_{t+1}(i)\right)^{-\gamma} \left(e^{i^*_t - \pi^*_{t+1}} - e^{i_t + s_t - s_{t+1} - \tau - \pi^*_{t+1}}\right)}{E_t \left(R^{p,*}_{t+1}(i)\right)^{1-\gamma}} - \psi(z^*_t(i) - z^*_{t-1}(i)) = 0 \tag{A.31}$$

where $\pi^*_{t+1} = p^*_{t+1} - p^*_t$ is inflation in the Foreign country in the Foreign currency.

First-order conditions for goods demand are

$$P_{Ht} C^k_{Ht}(j) = \alpha P_t C^k_t(j) \tag{A.32}$$

$$P_{Ft} C^k_{Ft}(j) = (1-\alpha) P_t C^k_t(j) \tag{A.33}$$

$$C^k_{Hit}(j) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\mu} C^k_{Ht}(j) \tag{A.34}$$

$$C^k_{Fit}(j) = \left(\frac{P_{Ft}(i)}{P_{Ft}}\right)^{-\mu} C^k_{Ft}(j) \tag{A.35}$$

Analogously, for Foreign agents

$$P^*_{Ht} C^{k,*}_{Ht}(j) = (1-\alpha) P^*_t C^{k,*}_t(j) \tag{A.36}$$

$$P^*_{Ft} C^{k,*}_{Ft}(j) = \alpha P^*_t C^{k,*}_t(j) \tag{A.37}$$

$$C^{k,*}_{Hit}(j) = \left(\frac{P^*_{Ht}(i)}{P^*_{Ht}}\right)^{-\mu} C^{k,*}_{Ht}(j) \tag{A.38}$$

$$C^{k,*}_{Fit}(j) = \left(\frac{P^*_{Ft}(i)}{P^*_{Ft}}\right)^{-\mu} C^{k,*}_{Ft}(j) \tag{A.39}$$

Agent $i$ in the Home country then faces the following revenue from Home good $i$:

$$\begin{aligned} y_t(i) = \frac{Y_t(i)}{P_t} &= \alpha \left(P_{Ht}(i)\right)^{1-\mu} P^{\mu-1}_{Ht} \int_0^1 \left(C^y_t(j) + C^o_t(j)\right) dj \\ &+ (1-\alpha) \left(P^*_{Ht}(i)\right)^{1-\mu} \left(P^*_{Ht}\right)^{\mu-1} \frac{S_t P^*_t}{P_t} \int_0^1 \left(C^{y,*}_t(j) + C^{o,*}_t(j)\right) dj \end{aligned} \tag{A.40}$$

Labor supply is equal to the quantity sold in both markets:

$$L_t(i) = \alpha \left(P_{Ht}(i)\right)^{-\mu} P_{Ht}^{\mu-1} P_t \int_0^1 \left(C_t^y(j) + C_t^o(j)\right) dj$$
$$+(1-\alpha) \left(P_{Ht}^*(i)\right)^{-\mu} \left(P_{Ht}^*\right)^{\mu-1} P_t^* \int_0^1 \left(C_t^{y,*}(j) + C_t^{o,*}(j)\right) dj \tag{A.41}$$

Using that $a_t^y(i) = 1/\omega$, we have $C_{t+1}^o(i) = R_{t+1}^p(i)/\omega$ and $\omega C_t^y(i) = \omega y_t(i) - \omega tax_t/P_t - 1$. The Home agent $i$ therefore sets prices to maximize

$$\omega y_t(i) - \phi L_t(i)^\eta \tag{A.42}$$

Using the expressions for $y_t(i)$ and $L_t(i)$, the first-order conditions for price setting by Home agent $i$ are (after dividing by respectively $\alpha$ and $1-\alpha$):

$$\omega(1-\mu) \left(P_{Ht}(i)\right)^{-\mu} P_{Ht}^{\mu-1} \int_0^1 \left(C_t^y(j) + C_t^o(j)\right) dj$$
$$+\phi\mu\eta L_t(i)^{\eta-1} \left(P_{Ht}(i)\right)^{-\mu-1} P_{Ht}^{\mu-1} P_t \int_0^1 \left(C_t^y(j) + C_t^o(j)\right) dj$$
$$-\nu(P_{Ht}(i) - P_{H,t-1}(i)) = 0 \tag{A.43}$$

and

$$\omega(1-\mu) \left(P_{Ht}^*(i)\right)^{-\mu} \left(P_{Ht}^*\right)^{\mu-1} \frac{S_t P_t^*}{P_t} \int_0^1 \left(C_t^{y,*}(j) + C_t^{o,*}(j)\right) dj$$
$$+\phi\mu\eta L_t(i)^{\eta-1} \left(P_{Ht}^*(i)\right)^{-\mu-1} \left(P_{Ht}^*\right)^{\mu-1} P_t^* \int_0^1 \left(C_t^{y,*}(j) + C_t^{o,*}(j)\right) dj$$
$$-\nu(P_{Ht}^*(i) - P_{H,t-1}^*(i)) = 0 \tag{A.44}$$

Analogous first-order conditions can be derived for Foreign agents. Agent $i$ in the Foreign country faces the following demand:

$$y_t^*(i) = (1-\alpha) \left(P_{Ft}(i)\right)^{1-\mu} P_{Ft}^{\mu-1} \frac{P_t}{S_t P_t^*} \int_0^1 \left(C_t^y(j) + C_t^o(j)\right) dj$$
$$+\alpha \left(P_{Ft}^*(i)\right)^{1-\mu} \left(P_{Ft}^*\right)^{\mu-1} \int_0^1 \left(C_t^{y,*}(j) + C_t^{o,*}(j)\right) dj \tag{A.45}$$

Labor supply is

$$L_t^*(i) = (1-\alpha) \left(P_{Ft}(i)\right)^{-\mu} P_{Ft}^{\mu-1} P_t \int_0^1 \left(C_t^y(j) + C_t^o(j)\right) dj$$
$$+\alpha \left(P_{Ft}^*(i)\right)^{-\mu} \left(P_{Ft}^*\right)^{\mu-1} P_t^* \int_0^1 \left(C_t^{y,*}(j) + C_t^{o,*}(j)\right) dj \tag{A.46}$$

The first order conditions are then

$$\omega(1-\mu)\left(P_{Ft}(i)\right)^{-\mu}P_{Ft}^{\mu-1}\frac{P_t}{S_tP_t^*}\int_0^1\left(C_t^y(j)+C_t^o(j)\right)dj$$
$$+\phi\mu\eta(L_t^*(i))^{\eta-1}\left(P_{Ft}(i)\right)^{-\mu-1}P_{Ft}^{\mu-1}P_t\int_0^1\left(C_t^y(j)+C_t^o(j)\right)dj$$
$$-\nu(P_{Ft}(i)-P_{F,t-1}(i))=0 \tag{A.47}$$

and

$$\omega(1-\mu)\left(P_{Ft}^*(i)\right)^{-\mu}\left(P_{Ft}^*\right)^{\mu-1}\int_0^1\left(C_t^{y,*}(j)+C_t^{o,*}(j)\right)dj$$
$$+\phi\mu\eta(L_t^*(i))^{\eta-1}\left(P_{Ft}^*(i)\right)^{-\mu-1}\left(P_{Ft}^*\right)^{\mu-1}P_t^*\int_0^1\left(C_t^{y,*}(j)+C_t^{o,*}(j)\right)dj$$
$$-\nu(P_{Ft}^*(i)-P_{F,t-1}^*(i))=0 \tag{A.48}$$

## A.3 Linearization

The linearized price indices are

$$p_t=\alpha p_{Ht}+(1-\alpha)p_{Ft} \tag{A.49}$$
$$p_t^*=(1-\alpha)p_{Ht}^*+\alpha p_{Ft}^* \tag{A.50}$$

Define $p_{1t}=p_{Ht}-p_{Ft}^*$ and $p_{2t}=p_{Ht}^*-p_{Ft}$. Then

$$\tilde{p}_t=p_t-p_t^*=\alpha p_{1t}-(1-\alpha)p_{2t} \tag{A.51}$$

and the log real exchange rate is

$$q_t=s_t-\tilde{p}_t \tag{A.52}$$

We will use that all agents in the same country (of the same age) will make the same decisions. Therefore $z_t(i)=z_t$, etc.. Using that $A_t^y=BP_t$ and $A_t^{y,*}=BP_t^*$, the Foreign bond market clearing condition is

$$z_tBP_t+z_t^*S_tBP_t^*=BS_tP_t^* \tag{A.53}$$

Dividing by $P_tB$, we have

$$z_t+z_t^*Q_t=Q_t \tag{A.54}$$

where $Q_t=S_tP_t^*/P_t$ is the real exchange rate. This corresponds to equation (9) in the paper. We linearize around a real exchange rate of 1 and $z_t^*=1-\bar{z}$, where $\bar{z}$ is the steady state portfolio share invested in the domestic asset (defined below). This gives

$$z_t^A=0.5(z_t+z_t^*)=0.5+0.5\bar{z}q_t \tag{A.55}$$

This is the same as equation (10) in the paper.

In order to linearize the first-order conditions for price setting, we need to discuss the steady state around which to linearize. There is no steady state price level, which we will normalize to 1. From the first-order condition for price setting we have

$$\bar{L} = \left( \frac{(\mu - 1)\omega}{\mu \phi \eta} \right)^{1/(\eta - 1)} \tag{A.56}$$

Normalize $\phi$ such that $\bar{L} = 1$. Then we also have $\bar{Y} = 1$. From goods market clearing it then follows that $\bar{C}^y + \bar{C}^o = \bar{Y} = 1$. Using this, the first-order conditions for price setting are

$$(p_t - p_{Ht}) + (\eta - 1) l_t - \bar{\nu}(p_{Ht} - p_{H,t-1}) = 0 \tag{A.57}$$

$$(p_t - p^*_{Ht} - s_t) + (\eta - 1) l_t - \bar{\nu}(p^*_{Ht} - p^*_{H,t-1}) = 0 \tag{A.58}$$

where

$$\bar{\nu} = \frac{\nu}{(\mu - 1)\omega} \tag{A.59}$$

We can also write these as

$$p_{Ht} = (1 - \kappa) p_{H,t-1} + \kappa (p_t + (\eta - 1) l_t) \tag{A.60}$$

$$p^*_{Ht} = (1 - \kappa) p^*_{H,t-1} + \kappa (p_t - s_t + (\eta - 1) l_t) \tag{A.61}$$

where

$$\kappa = \frac{1}{1 + \bar{\nu}} \tag{A.62}$$

Analogous first-order conditions for Foreign country price setting are

$$p_{Ft} = (1 - \kappa) p_{F,t-1} + \kappa (p^*_t + s_t + (\eta - 1) l^*_t) \tag{A.63}$$

$$p^*_{Ft} = (1 - \kappa) p^*_{F,t-1} + \kappa (p^*_t + (\eta - 1) l^*_t) \tag{A.64}$$

These first-order conditions imply

$$p_{1t} = (1 - \kappa) p_{1,t-1} + \kappa \left( \tilde{p}_t + (\eta - 1)(l_t - l^*_t) \right) \tag{A.65}$$

$$p_{2t} = (1 - \kappa) p_{2,t-1} + \kappa \left( \tilde{p}_t - 2 s_t + (\eta - 1)(l_t - l^*_t) \right) \tag{A.66}$$

From hereon we will assume that $\eta$ is infinitesimally close to 1. This simplifies as the last term, which depends on $l_t - l^*_t$, drops out. Then

$$\tilde{p}_t = (1 - \kappa) \tilde{p}_{t-1} + (2\alpha - 1) \kappa \tilde{p}_t + 2(1 - \alpha) \kappa s_t \tag{A.67}$$

or

$$\tilde{p}_t = \frac{1-\kappa}{1+(1-2\alpha)\kappa}\tilde{p}_{t-1} + \frac{2(1-\alpha)\kappa}{1+(1-2\alpha)\kappa}s_t \tag{A.68}$$

Define

$$\bar{\kappa} = \frac{2(1-\alpha)\kappa}{1+(1-2\alpha)\kappa} \tag{A.69}$$

Then

$$\tilde{p}_t = (1-\bar{\kappa})\tilde{p}_{t-1} + \bar{\kappa}s_t \tag{A.70}$$

Finally, the Home first-order condition for portfolio choice implies

$$\left(E_t e^{i_t^* + s_{t+1} - s_t - \tau - \pi_{t+1} - \gamma r_{t+1}^p} - E_t e^{i_t - \pi_{t+1} - \gamma r_{t+1}^p}\right) - \psi(z_t - z_{t-1}) = 0 \tag{A.71}$$

Here returns are in deviation from steady state. Take the expectation, using log normality, then linearize. This gives

$$E_t\left(s_{t+1} - s_t + i_t^* - i_t - \tau\right) + 0.5var(s_{t+1}) - cov(s_{t+1}, \pi_{t+1} + \gamma r_{t+1}^p) - \psi(z_t - z_{t-1}) = 0 \tag{A.72}$$

The linearized portfolio return, in deviation from steady state, is

$$r_{t+1}^p = z_t(s_{t+1} - s_t + i_t^*) + (1-z_t)i_t - \pi_{t+1} \tag{A.73}$$

Substitution into the first-order condition gives

$$E_t\left(s_{t+1} - s_t + i_t^* - i_t - \tau\right) + (0.5 - \gamma z_t)var(s_{t+1}) - (1-\gamma)cov(s_{t+1}, p_{t+1}) - \psi(z_t - z_{t-1}) = 0 \tag{A.74}$$

The steady state portfolio is

$$\bar{z} = \frac{0.5}{\gamma} - \frac{\tau}{\gamma var(s_{t+1})} + \frac{\gamma - 1}{\gamma}\frac{cov(s_{t+1}, p_{t+1})}{var(s_{t+1})} \tag{A.75}$$

the second moments will be constants in the linear solution. $\tau$ can be set to obtain any level of $\bar{z}$. From hereon we therefore treat $\bar{z}$ as a parameter. In deviation from steady state we have

$$z_t - \bar{z} = \frac{E_t(er_{t+1})}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{t-1} - \bar{z}) \tag{A.76}$$

where $\sigma^2 = var(s_{t+1})$ and $er_{t+1} = s_{t+1} - s_t + i_t^* - i_t$. This corresponds to equation (8) in the paper.

The analogous equation for the Foreign country is

$$z_t^* - \bar{z}^* = \frac{E_t(er_{t+1})}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{t-1}^* - \bar{z}^*) \tag{A.77}$$

where $\bar{z}^* = 1 - \bar{z}$. It follows that

$$z_t^A - 0.5 = \frac{E_t(s_{t+1} - s_t + i_t^D)}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{t-1}^A - 0.5) \tag{A.78}$$

where $i_t^D = i_t^* - i_t$.

## A.4 System of Equations

Based on the results above, we end up with the following system in $s_t$, $\tilde{p}_t$ and $z_t^A$:

$$z_t^A = 0.5 + 0.5\bar{z}q_t \tag{A.79}$$

$$\tilde{p}_t = (1-\bar{\kappa})\tilde{p}_{t-1} + \bar{\kappa}s_t \tag{A.80}$$

$$z_t^A - 0.5 = \frac{E_t(s_{t+1} - s_t + i_t^D)}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{t-1}^A - 0.5) \tag{A.81}$$

Substituting (A.79) into (A.81), we have

$$0.5\bar{z}q_t = \frac{E_t(s_{t+1} - s_t + i_t^D)}{\gamma\sigma^2 + \psi} + 0.5\frac{\psi}{\gamma\sigma^2 + \psi}\bar{z}q_{t-1} \tag{A.82}$$

Define the real interest differential as

$$r_t^D = i_t^D - E_t(\pi_{t+1} - \pi_{t+1}^*) \tag{A.83}$$

Then (A.82) becomes

$$0.5\bar{z}q_t = \frac{E_t(q_{t+1} - q_t + r_t^D)}{\gamma\sigma^2 + \psi} + 0.5\frac{\psi}{\gamma\sigma^2 + \psi}\bar{z}q_{t-1} \tag{A.84}$$

In the paper

$$b = \frac{1-h}{4} = 0.5\bar{z} \tag{A.85}$$

and

$$\theta = 1 + \psi b + \gamma\sigma^2 b \tag{A.86}$$

Using this notation, we can write (A.84) as

$$\theta q_t = E_t q_{t+1} + r_t^D + \psi b q_{t-1} \tag{A.87}$$

This corresponds exactly to equation (11) in the paper.

We assume in the paper that real interest rate follows an AR process:

$$r_t^D = \rho r_{t-1}^D + \varepsilon_t \tag{A.88}$$

The central bank can always choose a monetary policy that leads to this process for the real interest rate. One can think of it as a specific example of a monetary policy rule. The solution for the real exchange rate is determined by (A.87)-(A.88). Using (A.80), we can then also determine relative inflation under this policy rule. (A.80) implies

$$\tilde{p}_t - \tilde{p}_{t-1} = \frac{\bar{\kappa}}{1-\bar{\kappa}}q_t \tag{A.89}$$

## A.5 Financial Shocks

While in the paper we focus on interest rate shocks, these account for only a small fraction of actual exchange rate fluctuations. One can introduce financial shocks to the model to account for the observed exchange rate volatility as in the data, in a way similar to Itskhoki and Muhkin (2017). As long as these shocks are uncorrelated with the interest rate shocks, it does not affect the analysis for interest rate shocks. Financial shocks take the form of exogenous portfolio shifts. One easy way to introduce them is by replacing the constant cost $\tau$ of investment abroad with a time-varying cost $\tau_t$ for the Home country and $\tau_t^*$ for the Foreign country. The mean value of this cost is still $\tau$. Then (A.78) becomes

$$z_t^A - 0.5 = \frac{E_t(s_{t+1} - s_t + i_t^D)}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{t-1}^A - 0.5) + h_t \tag{A.90}$$

where

$$h_t = -\frac{0.5}{\gamma\sigma^2 + \psi}(\tau_t - \tau_t^*) \tag{A.91}$$

and (A.87) becomes

$$\theta q_t = E_t q_{t+1} + r_t^D + \psi b q_{t-1} + (\gamma\sigma^2 + \psi)h_t \tag{A.92}$$

The solution for the real exchange rate is then the same as in the paper, with an addtional term associated with financial shocks.

# B Algebra for Section 4 of the Paper

## B.1 Optimal Portfolios

Home agents maximize

$$E_t \frac{C_{t+1}^{1-\gamma}}{1-\gamma} - \frac{1}{4}\psi \sum_{i=1}^{4} (z_{it} - z_{i,t-1})^2 \tag{B.1}$$

subject to

$$\begin{aligned} C_{t+1} = R_t + z_{1t}\left(\frac{Q_{t+1}}{Q_t}R_t^* e^{-\tau} - R_t\right) + z_{2t}\left(\frac{Q_{t+1}}{Q_t}R_{t+1}^{L,*} e^{-\tau_L} - R_t\right) + \\ z_{3t}\left(R_{t+1}^L - R_t\right) + T_{t+1} \end{aligned} \tag{B.2}$$

The aggregate of the cost of investing abroad is reimbursed through $T_{t+1}$, so that in the aggregate

$$C_{t+1} = R_t + z_{1t}\left(\frac{Q_{t+1}}{Q_t}R_t^* - R_t\right) + z_{2t}\left(\frac{Q_{t+1}}{Q_t}R_{t+1}^{L,*} - R_t\right) + z_{3t}\left(R_{t+1}^L - R_t\right) \tag{B.3}$$

Define $\hat{z}$ as a deviation from $z_{4t}$, so for example $\hat{z}_{1t} = z_{1t} - z_{4t}$. First-order conditions for optimal portfolio choice are then

$$E_t C_{t+1}^{-\gamma} \left( \frac{Q_{t+1}}{Q_t} R_t^* e^{-\tau} - R_t \right) = 0.5\psi(\hat{z}_{1t} - \hat{z}_{1,t-1}) \tag{B.4}$$

$$E_t C_{t+1}^{-\gamma} \left( \frac{Q_{t+1}}{Q_t} R_{t+1}^{L,*} e^{-\tau_L} - R_t \right) = 0.5\psi(\hat{z}_{2t} - \hat{z}_{2,t-1}) \tag{B.5}$$

$$E_t C_{t+1}^{-\gamma} \left( R_{t+1}^{L} - R_t \right) = 0.5\psi(\hat{z}_{3t} - \hat{z}_{3,t-1}) \tag{B.6}$$

Denoting logs with lower case letters, define the three excess returns as

$$er_{1,t+1} = q_{t+1} - q_t + r_t^* - r_t \tag{B.7}$$

$$er_{2,t+1} = q_{t+1} - q_t + r_{t+1}^{L,*} - r_t \tag{B.8}$$

$$er_{3,t+1} = r_{t+1}^{L} - r_t \tag{B.9}$$

We can then rewrite the first-order conditions as

$$E_t e^{-\gamma c_{t+1} + er_{1,t+1} - \tau} - E_t e^{-\gamma c_{t+1}} = 0.5\psi(\hat{z}_{1t} - \hat{z}_{1,t-1}) e^{-r_t} \tag{B.10}$$

$$E_t e^{-\gamma c_{t+1} + er_{2,t+1} - \tau_L} - E_t e^{-\gamma c_{t+1}} = 0.5\psi(\hat{z}_{2t} - \hat{z}_{2,t-1}) e^{-r_t} \tag{B.11}$$

$$E_t e^{-\gamma c_{t+1} + er_{3,t+1}} - E_t e^{-\gamma c_{t+1}} = 0.5\psi(\hat{z}_{3t} - \hat{z}_{3,t-1}) e^{-r_t} \tag{B.12}$$

Using log normality of consumption and returns, and approximating $e^x = 1 + x$, we can write this as (also linearizing the right hand side)

$$E_t er_{1,t+1} - \tau + 0.5\sigma_1^2 - \gamma cov(er_{1,t+1}, c_{t+1}) = \frac{0.5\psi}{R}(\hat{z}_{1t} - \hat{z}_{1,t-1}) \tag{B.13}$$

$$E_t er_{2,t+1} - \tau_L + 0.5\sigma_2^2 - \gamma cov(er_{2,t+1}, c_{t+1}) = \frac{0.5\psi}{R}(\hat{z}_{2t} - \hat{z}_{2,t-1}) \tag{B.14}$$

$$E_t er_{3,t+1} + 0.5\sigma_3^2 - \gamma cov(er_{3,t+1}, c_{t+1}) = \frac{0.5\psi}{R}(\hat{z}_{3t} - \hat{z}_{3,t-1}) \tag{B.15}$$

Here $\sigma_i^2 = var(er_{i,t+1})$ and $R$ is the steady state value of the returns.

Log-linearizing (B.3), we have

$$c_{t+1} = r_t + z_{1t} er_{1,t+1} + z_{2t} er_{2,t+1} + z_{3t} er_{3,t+1} \tag{B.16}$$

The first-order conditions then become

$$E_t er_{1,t+1} - \tau + 0.5\sigma_1^2 - \gamma z_{1t}\sigma_1^2 - \gamma z_{2t}\sigma_{12} - \gamma z_{3t}\sigma_{13} = \frac{0.5\psi}{R}(\hat{z}_{1t} - \hat{z}_{1,t-1}) \tag{B.17}$$

$$E_t er_{2,t+1} - \tau_L + 0.5\sigma_2^2 - \gamma z_{1t}\sigma_{12} + \gamma z_{2t}\sigma_2^2 - \gamma z_{3t}\sigma_{23} = \frac{0.5\psi}{R}(\hat{z}_{2t} - \hat{z}_{2,t-1}) \tag{B.18}$$

$$E_t er_{3,t+1} + 0.5\sigma_3^2 - \gamma z_{1t}\sigma_{13} - \gamma z_{2t}\sigma_{23} - \gamma z_{3t}\sigma_3^2 = \frac{0.5\psi}{R}(\hat{z}_{3t} - \hat{z}_{3,t-1}) \tag{B.19}$$

Here $\sigma_{ij}$ is the covariance between $er_{i,t+1}$ and $er_{j,t+1}$.

Define $\mathbf{er}_{t+1} = (er_{1,t+1}, er_{2,t+1}, er_{3,t+1})'$ and $\mathbf{z}_t = (z_{1t}, z_{2t}, z_{3t})'$. $\hat{\mathbf{z}}_t$ subtracts $z_{4t}$ from each element of $\mathbf{z}_t$. Then we can write the three first-order conditions for Home agents compactly as

$$E_t \mathbf{er}_{t+1} - \begin{pmatrix} \tau \\ \tau_L \\ 0 \end{pmatrix} + 0.5 diag(\Sigma) - \gamma \Sigma \mathbf{z}_t = \frac{0.5\psi}{R}(\hat{\mathbf{z}}_t - \hat{\mathbf{z}}_{t-1}) \tag{B.20}$$

where $\Sigma$ is the variance of $\mathbf{er}_{t+1}$.

Next consider the Foreign country. We have

$$C^*_{t+1} = R_t e^{-\tau}\frac{Q_t}{Q_{t+1}} + z^*_{1t}\left(R^*_t - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) + z^*_{2t}\left(R^{L,*}_{t+1} - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) + z^*_{3t}\left(R^L_{t+1} e^{-\tau_L}\frac{Q_t}{Q_{t+1}} - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) + T^*_{t+1} \tag{B.21}$$

The cost of investment abroad is reimbursed through $T^*_{t+1}$, so that aggregate Foreign consumption is

$$C^*_{t+1} = R_t \frac{Q_t}{Q_{t+1}} + z^*_{1t}\left(R^*_t - R_t \frac{Q_t}{Q_{t+1}}\right) + z^*_{2t}\left(R^{L,*}_{t+1} - R_t \frac{Q_t}{Q_{t+1}}\right) + z^*_{3t}\left(R^L_{t+1}\frac{Q_t}{Q_{t+1}} - R_t \frac{Q_t}{Q_{t+1}}\right) \tag{B.22}$$

First-order conditions for optimal portfolio choice are

$$E_t (C^*_{t+1})^{-\gamma}\left(R^*_t - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}^*_{1t} - \hat{z}^*_{1,t-1}) \tag{B.23}$$

$$E_t (C^*_{t+1})^{-\gamma}\left(R^{L,*}_{t+1} - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}^*_{2t} - \hat{z}^*_{2,t-1}) \tag{B.24}$$

$$E_t (C^*_{t+1})^{-\gamma}\left(R^L_{t+1} e^{-\tau_L}\frac{Q_t}{Q_{t+1}} - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}^*_{3t} - \hat{z}^*_{3,t-1}) \tag{B.25}$$

where $\hat{z}^*_{it} = z^*_{it} - z^*_{4t}$. We can then rewrite the first-order conditions as

$$E_t e^{-\gamma c^*_{t+1}} - E_t e^{-\gamma c^*_{t+1} - er_{1,t+1} - \tau} = 0.5\psi(\hat{z}^*_{1t} - \hat{z}^*_{1,t-1})e^{-r^*_t} \tag{B.26}$$

$$E_t e^{-\gamma c^*_{t+1} + er_{2,t+1} - er_{1,t+1}} - E_t e^{-\gamma c^*_{t+1} - er_{1,t+1} - \tau} = 0.5\psi(\hat{z}^*_{2t} - \hat{z}^*_{2,t-1})e^{-r^*_t} \tag{B.27}$$

$$E_t e^{-\gamma c^*_{t+1} + er_{3,t+1} - er_{1,t+1} - \tau_L} - E_t e^{-\gamma c^*_{t+1} - er_{1,t+1} - \tau} = 0.5\psi(\hat{z}^*_{3t} - \hat{z}^*_{3,t-1})e^{-r^*_t} \tag{B.28}$$

Assuming again that consumption and returns are log-linear, taking expectations and then linearizing $e^x$ as $1+x$, we have

$$E_t er_{1,t+1} + \tau - 0.5\sigma_1^2 - \gamma cov(er_{1,t+1}, c^*_{t+1}) = \frac{0.5\psi}{R}(\hat{z}^*_{1t} - \hat{z}^*_{1,t-1}) \tag{B.29}$$

$$E_t er_{2,t+1} + \tau + 0.5\sigma_2^2 - \sigma_{12} - \gamma cov(er_{2,t+1}, c^*_{t+1}) = \frac{0.5\psi}{R}(\hat{z}^*_{2t} - \hat{z}^*_{2,t-1}) \tag{B.30}$$

$$E_t er_{3,t+1} + \tau - \tau_L + 0.5\sigma_3^2 - \sigma_{13} - \gamma cov(er_{3,t+1}, c^*_{t+1}) = \frac{0.5\psi}{R}(\hat{z}^*_{3t} - \hat{z}^*_{3,t-1}) \tag{B.31}$$

Log-linearizing (B.22), we have

$$c^*_{t+1} = r^*_t - er_{1,t+1} + z^*_{1t} er_{1,t+1} + z^*_{2t} er_{2,t+1} + z^*_{3t} er_{3,t+1} \tag{B.32}$$

The first-order conditions then become

$$E_t er_{1,t+1} + \tau + 0.5\sigma_1^2 - (1-\gamma)\sigma_1^2 - \gamma z^*_{1t}\sigma_1^2 - \gamma z^*_{2t}\sigma_{12} - \gamma z^*_{3t}\sigma_{13} = \frac{0.5\psi}{R}(\hat{z}^*_{1t} - \hat{z}^*_{1,t-1})$$

$$E_t er_{2,t+1} + \tau + 0.5\sigma_2^2 - (1-\gamma)\sigma_{12} - \gamma z^*_{1t}\sigma_{12} - \gamma z^*_{2t}\sigma_2^2 - \gamma z^*_{3t}\sigma_{23} = \frac{0.5\psi}{R}(\hat{z}^*_{2t} - \hat{z}^*_{2,t-1})$$

$$E_t er_{3,t+1} + \tau - \tau_L + 0.5\sigma_3^2 - (1-\gamma)\sigma_{13} - \gamma z^*_{1t}\sigma_{13} - \gamma z^*_{2t}\sigma_{23} - \gamma z^*_{3t}\sigma_3^2 = \frac{0.5\psi}{R}(\hat{z}^*_{3t} - \hat{z}^*_{3,t-1})$$

We can write these first-order conditions compactly as

$$E_t \mathbf{er}_{t+1} + \begin{pmatrix} \tau \\ \tau \\ \tau - \tau_L \end{pmatrix} + 0.5 diag(\Sigma) - (1-\gamma)\Sigma_1 - \gamma\Sigma\mathbf{z}^*_t = \frac{0.5\psi}{R}(\hat{\mathbf{z}}^*_t - \hat{\mathbf{z}}^*_{t-1}) \tag{B.33}$$

where $\mathbf{z}^*_t = (z^*_{1t}, z^*_{2t}, z^*_{3t})'$ is the vector of portfolio shares of Foreign agents and $\hat{\mathbf{z}}^*_t$ subtracts $z^*_{4t}$ from each element of $\mathbf{z}^*_t$. $\Sigma_1$ is the first column of $\Sigma$.

Taking the average of (B.20) and (B.33), we have

$$E_t \mathbf{er}_{t+1} + \frac{1}{2}\begin{pmatrix} 0 \\ \tau - \tau_L \\ \tau - \tau_L \end{pmatrix} + \frac{1}{2} diag(\Sigma) - \frac{1}{2}(1-\gamma)\Sigma_1 - \gamma\Sigma\mathbf{z}^A_t = \frac{\psi}{2R}(\hat{\mathbf{z}}^A_t - \hat{\mathbf{z}}^A_{t-1}) \tag{B.34}$$

where $\mathbf{z}^A_t = 0.5(\mathbf{z}_t + \mathbf{z}^*_t)$ and $\hat{\mathbf{z}}^A_t = 0.5(\hat{\mathbf{z}}_t + \hat{\mathbf{z}}^*_t)$.

## B.2 Market Equilibrium

Next impose asset market equilibrium:

$$z_{1t} + Q_t z^*_{1t} = Q_t b^S \tag{B.35}$$
$$z_{2t} + Q_t z^*_{2t} = Q_t P^{L,*}_t b_t \tag{B.36}$$
$$z_{3t} + Q_t z^*_{3t} = P^L_t b_t \tag{B.37}$$
$$z_{4t} + Q_t z^*_{4t} = b^S \tag{B.38}$$

Here $b^S$ is the constant supply of the short-term bond, while $b_t$ is the quamtity of long-term bonds. Both are equal in the two countries. Adding up these market clearing conditions, we have

$$(1 + Q_t)(1 - b^S) = b_t \left( Q_t P^{L,*}_t + P^L_t \right) \tag{B.39}$$

The steady state value of $b_t$ must then be $\bar{b} = (1 - b^S)/\bar{P}^L$, where $\bar{P}^L = \kappa/(R - 1 + \delta)$ is the steady state long term bond price. It follows that $\bar{b}\bar{P}^L = 1 - b^S$. We refer to $\bar{b}\bar{P}^L$ as $b^L$, the value (in terms of purchasing power) of long term bonds in both countries. Therefore $b^S + b^L = 1$. Furthermore, linearizing (B.39) gives

$$b_t = -p^{L,A}_t \tag{B.40}$$

where $p^{L,A}_t = 0.5(p^L_t + p^{L,*}_t)$ is the average log bond price.

In log-linear form the first three market clearing conditions are then

$$\mathbf{z}^A_t = 0.5 \begin{pmatrix} b^S \\ b^L \\ b^L \end{pmatrix} + 0.5 \begin{pmatrix} b^S \\ b^L \\ 0 \end{pmatrix} q_t - 0.5 \bar{\mathbf{z}}^* q_t + 0.25 b^L \begin{pmatrix} 0 \\ -p^{L,D}_t \\ p^{L,D}_t \end{pmatrix} \tag{B.41}$$

where $\bar{\mathbf{z}}^*$ is the steady state of $\mathbf{z}^*_t$ and $p^{L,D}_t = p^L_t - p^{L,*}_t$ is the relative log long term bond price.

Since the steady state portfolio shares $\bar{\mathbf{z}}^*$ enter in (B.41), we need to say something about them. We will relate then to portfolio home bias. Let $\bar{z}_i$ and $\bar{z}^*_i$ be the steady state portfolio shares of Home and Foreign agents. By symmetry

$$\bar{z}_1 + \bar{z}_4 = \bar{z}^*_1 + \bar{z}^*_4 = b^S \tag{B.42}$$
$$\bar{z}_2 + \bar{z}_3 = \bar{z}^*_2 + \bar{z}^*_3 = b^L \tag{B.43}$$

So both Home and Foreign investors invest a fraction $b^S$ in short term bonds and a fraction $b^L$ in long term bonds. Within short-term bonds and within long-term bonds, the extent

of home bias is determined by $\tau$ and $\tau_L$, which we can use to set home bias at any value. Denoting home bias as $h$ for both short-term and long-term bonds, we have

$$h = 1 - \frac{\bar{z}_1/b^S}{0.5} = 1 - \frac{\bar{z}_4^*/b^S}{0.5} \tag{B.44}$$

$$h = 1 - \frac{\bar{z}_2/b^L}{0.5} = 1 - \frac{\bar{z}_3^*/b^L}{0.5} \tag{B.45}$$

Therefore

$$\bar{z}_1 = \bar{z}_4^* = 0.5(1-h)b^S \tag{B.46}$$

$$\bar{z}_2 = \bar{z}_3^* = 0.5(1-h)b^L \tag{B.47}$$

These equations, together with (B.42) and (B.43) map the home bias parameter $h$ into all steady state portfolio shares in both countries. We have

$$\bar{z}_4 = \bar{z}_1^* = 0.5(1+h)b^S \tag{B.48}$$

$$\bar{z}_3 = \bar{z}_2^* = 0.5(1+h)b^L \tag{B.49}$$

Define

$$\mathbf{v} = 0.25(1-h)\begin{pmatrix} b^S \\ b^L \\ -b^L \end{pmatrix} \tag{B.50}$$

Then (B.41) becomes

$$\mathbf{z}_t^A = 0.5\begin{pmatrix} b^S \\ b^L \\ b^L \end{pmatrix} + \mathbf{v}q_t + 0.25b^L\begin{pmatrix} 0 \\ -p_t^{L,D} \\ p_t^{L,D} \end{pmatrix} \tag{B.51}$$

Combining these market equilibrium conditions with (B.34), and focusing on the deviation from the steady state, we have

$$E_t\mathbf{er}_{t+1} - \gamma\Sigma\mathbf{v}q_t - 0.25\gamma b^L p_t^{L,D}\Sigma\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \tag{B.52}$$

$$\frac{0.5\psi}{R}\mathbf{v}(q_t - q_{t-1}) + \frac{\psi b^S}{8R}(1-h)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(q_t - q_{t-1}) + \frac{\psi b_L}{8R}(p_t^{L,D} - p_{t-1}^{L,D})\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

## B.3 Solution

In order to solve the model, we will write (B.52) as a second-order difference equation in the variables $(q_t, p_t^{L,D}, p_t^{L,A})'$. We first need to write $\mathbf{er}_{t+1}$ in terms of these variables. Log-linearizing the long term bond returns, we have

$$r_{t+1}^{L} = \lambda p_{t+1}^{L} - p_t^{L} \tag{B.53}$$
$$r_{t+1}^{L,*} = \lambda p_{t+1}^{L,*} - p_t^{L,*} \tag{B.54}$$

where $\lambda = (1-\delta)/R$. We then have

$$er_{1,t+1} = q_{t+1} - q_t + r_t^{D} \tag{B.55}$$
$$er_{2,t+1} = q_{t+1} - q_t + \lambda p_{t+1}^{L,*} - p_t^{L,*} - r_t \tag{B.56}$$
$$er_{3,t+1} = \lambda p_{t+1}^{L} - p_t^{L} - r_t \tag{B.57}$$

We can also write

$$er_{2,t+1} = q_{t+1} - q_t - 0.5\lambda p_{t+1}^{L,D} + \lambda p_{t+1}^{L,A} + 0.5 p_t^{L,D} - p_t^{L,A} + 0.5 r_t^{D} - r_t^{A} \tag{B.58}$$
$$er_{3,t+1} = 0.5\lambda p_{t+1}^{L,D} + \lambda p_{t+1}^{L,A} - 0.5 p_t^{L,D} - p_t^{L,A} + 0.5 r_t^{D} - r_t^{A} \tag{B.59}$$

where $r_t^{D} = r_t^{*} - r_t$.

Next a couple of comments on the matrix $\Sigma$. Let $\sigma_q^2 = var_t(q_{t+1})$, $\sigma_L^2 = var_t(\lambda p_{t+1}^{L})$, $\sigma_{LL} = cov(\lambda p_{t+1}^{L}, \lambda p_{t+1}^{L,*})$ and $\sigma_{qL} = cov_t(q_{t+1}, \lambda p_{t+1}^{L})$. Then we have

$$\Sigma = \begin{pmatrix} \sigma_q^2 & \sigma_q^2 - \sigma_{qL} & \sigma_{qL} \\ \sigma_q^2 - \sigma_{qL} & \sigma_q^2 + \sigma_L^2 - 2\sigma_{qL} & \sigma_{qL} + \sigma_{LL} \\ \sigma_{qL} & \sigma_{qL} + \sigma_{LL} & \sigma_L^2 \end{pmatrix} \tag{B.60}$$

Let $\sigma_{ij}$ be element $(i,j)$ of the matrix $\Sigma$. Denote $\sigma_i^2 = \sigma_{ii}$. In the data we compute $\sigma_1^2$, $\sigma_3^2$, $\sigma_{13}$ and $\sigma_{23}$. Then

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1^2 - \sigma_{13} & \sigma_{13} \\ \sigma_1^2 - \sigma_{13} & \sigma_1^2 + \sigma_3^2 - 2\sigma_{13} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix} \tag{B.61}$$

Consider the system (B.52). First take the third equation, plus the second equation, minus the first equation. This gives

$$E_t(\lambda p_{t+1}^{L,A} - p_t^{L,A} - r_t^{A}) = 0 \tag{B.62}$$

Assuming that $r_t^A$ follows an AR process with AR coefficient $\rho$, the solution is

$$p_t^{L,A} = -\frac{1}{1-\lambda\rho} r_t^A \tag{B.63}$$

Next consider the first equation of (B.52), together with the third minus second plus first equation. This gives

$$\begin{aligned} &E_t q_{t+1} - q_t + r_t^D + a_1 q_t + 0.25\gamma(\sigma_1^2 - 2\sigma_{13}) b^L p_t^{L,D} = \\ &\quad \frac{\psi}{4R} b^S (1-h)(q_t - q_{t-1}) \end{aligned} \tag{B.64}$$

$$\begin{aligned} &\lambda E_t p_{t+1}^{L,D} - p_t^{L,D} + r_t^D + 2a_2 q_t + 0.5\gamma(\sigma_{23} - \sigma_3^2) b^L p_t^{L,D} = \\ &\quad \frac{\psi}{4R}(1-h)(b^S - b^L)(q_t - q_{t-1}) + \frac{\psi}{4R} b^L (p_t^{L,D} - p_{t-1}^{L,D}) \end{aligned} \tag{B.65}$$

where

$$a_1 = -0.25\gamma(1-h)\left(\sigma_1^2 b^S + (\sigma_1^2 - 2\sigma_{13}) b^L\right) \tag{B.66}$$

$$a_2 = -0.25\gamma(1-h)\left(\sigma_{13} b^S + (\sigma_{23} - \sigma_3^2) b^L\right) \tag{B.67}$$

This system can also be written as

$$A_1 E_t \begin{pmatrix} q_{t+1} \\ p_{t+1}^{L,D} \end{pmatrix} + A_2 \begin{pmatrix} q_t \\ p_t^{L,D} \end{pmatrix} + A_3 \begin{pmatrix} q_{t-1} \\ p_{t-1}^{L,D} \end{pmatrix} + A_4 r_t^D = 0 \tag{B.68}$$

The matrices are defined as follows. We have

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix} \tag{B.69}$$

$$A_2 = \begin{pmatrix} -1 + a_1 - \frac{\psi}{4R}(1-h) b^S & 0.25\gamma(\sigma_1^2 - 2\sigma_{13}) b^L \\ 2a_2 - \frac{\psi}{4R}(1-h)(b^S - b^L) & -1 + 0.5\gamma(\sigma_{23} - \sigma_3^2) b^L - \frac{\psi}{4R} b^L \end{pmatrix} \tag{B.70}$$

$$A_3 = \frac{\psi}{4R} \begin{pmatrix} b^S(1-h) & 0 \\ (1-h)(b^S - b^L) & b^L \end{pmatrix} \tag{B.71}$$

$$A_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{B.72}$$

The system is driven by exogenous AR processes for $r_t^D$:

$$r_t^D = \rho r_{t-1}^D + \varepsilon_t \tag{B.73}$$

Let $\sigma^2$ be the variance of $\varepsilon_t$.

One can write the system as a first-order difference equation of the form $AE_t x_{t+1} + Bx_t = 0$, where $x_t = (q_t, p_t^{L,D}, q_{t-1}, p_{t-1}^{L,D}, r_t^D)'$. This allows us to solve for the control variables $(q_t, p_t^{L,D})$ as a function of the state variables $(q_{t-1}, p_{t-1}^{L,D}, r_t^D)'$. Define

$$v_t = \begin{pmatrix} q_t \\ p_t^{L,D} \end{pmatrix} \tag{B.74}$$

Then the solution takes the form

$$v_t = M_1 v_{t-1} + M_2 r_t^D \tag{B.75}$$

We can also integrate this and write

$$v_t = \sum_{k=0}^{\infty} M_1^k M_2 r_{t-k}^D \tag{B.76}$$

with $M_1^0$ being the identity matrix.

## B.4 Model Moments

Consider the regression of excess returns on $r_t^D$. First consider the excess return

$$er_{4,t+1} = -\lambda p_{t+1}^{L,D} + p_t^{L,D} + q_{t+1} - q_t \tag{B.77}$$

This is equal to $er_{2,r+1} - er_{3,t+1}$, which is the excess return of the Foreign long term bond over the Home long term bond. The coefficient of a regression of $er_{4,t+1}$ on $r_t^D$ is equal to

$$\beta_1 = \frac{cov(er_{4,t+1}, r_t^D)}{var(r_t^D)} \tag{B.78}$$

Define the vectors $e_1 = (1, -\lambda)$ and $e_2 = (-1, 1)$. Then

$$er_{4,t+1} = e_1 M_2 r_{t+1}^D + \sum_{k=0}^{\infty} \left(e_1 M_1^{k+1} + e_2 M_1^k\right) M_2 r_{t-k}^D \tag{B.79}$$

We then have

$$\beta_1 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 \tag{B.80}$$

Next consider $er_{1,t+1}$, the excess return of the Foreign short term bond over the Home short term bond. Defining $e_1 = (1, 0)$ and $e_2 = (-1, 0)$, the regression coefficient of $er_{1,t+1}$ on $r_t^D$ is

$$\beta_2 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 + 1 \tag{B.81}$$

Finally consider the difference between the Foreign and the Home local excess returns of long term over short term bonds. This is equal to $-\lambda p_{t+1}^{b,D} + p_t^{b,D} - r_t^D$. Defining $e_1 = (0, -\lambda)$ and $e_2 = (0, 1)$, this coefficient of a regression on $r_t^D$ is

$$\beta_3 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 - 1 \tag{B.82}$$

We can also consider predictability reversal in this model for the FX excess return. Defining again $e_1 = (1, 0)$ and $e_2 = (-1, 0)$, the regression coefficient of $er_{1,t+k}$ on $r_t^D$ is

$$\beta_k = \rho^k e_1 M_2 + \sum_{i=0}^{k-2} (e_1 M_1^{i+1} + e_2 M_1^i) M_2 \rho^{k-i-1} + (e_1 M_1 + e_2) M_1^{k-1} (I - \rho M_1)^{-1} M_2 + \rho^{k-1} \tag{B.83}$$

# C Empirical Analysis of Equity Portfolio Shares

## C.1 Data Description

The data is monthly and we consider the 44 countries included in the MSCI indices.[1] For portfolio positions, we use U.S. investors' international equity claims as computed by Bertaut and Tryon (2007) (www.federalreserve.gov/PUBS/ifdp/2007/910/default.htm) and Bertaut and Judson (2014) (www.federalreserve.gov/pubs/ifdp/2014/1113/default.htm). We combine these two sources to get estimated equity positions from January 1994 to January 2017 for 44 countries. In this section we use $z_{i,t}$ to denote the share of country $i$ in U.S. equity portfolios with the respect to the 44 countries considered:

$$z_{i,t} = \frac{a_{i,t}}{\sum_{j=1}^{44} a_{j,t}} \tag{C.1}$$

where $a_{i,t}$ is the U.S. claim on country $i$.

For each country, we collect the MSCI dividend-adjusted return index in USD, the MSCI earning-price ratio and the MSCI dividend-yield ratio from Datastream. Data is available for the whole sample but for the United Arab Emirates where it starts in May 2005. We take the logarithm of these variables. The return differential $er_{i,t}$ is defined as

[1] The countries are Australia, Austria, Belgium, Brazil, Canada, Chile, China, Colombia, Czech Republic, Denmark, Egypt, Finland, France, Germany, Greece, Hong-Kong, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Republic of Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, Poland, Portugal, Russian Federation, Singapore, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Arab Emirates, and United Kingdom.

the log return of country $i$, $r_{i,t}$, minus the weighted average of log returns in the other 43 countries:

$$er_{i,t} = r_{i,t} - \frac{\sum_{j \neq i} a_{j,t} r_{j,t}}{\sum_{j \neq i} a_{j,t}} \tag{C.2}$$

We do the same to compute the differential in the log earning-price and in the log dividend-yield.

## C.2 Portfolio Regressions

To estimate an expected returnd differential, we regress $er_{i,t+1}$ of the differentials in the log dividend-price and in the log earning-price at time $t$, as well as on $er_{i,t}$. We then compute the predicted value $\widehat{er}_{i,t+1}$ and use it in the portfolio regressions.

To estimated portfolio shares $z_{i,t}$, we run a pooled regression with or without country or period fixed effects. The portfolio regression we consider is

$$z_{i,t} = \alpha_0 + \alpha_1 z_{i,t-1} + \alpha_2 \widehat{er}_{i,t+1} + \varepsilon_{i,t} \tag{C.3}$$

To estimate standard errors, we use time and/or country clustering.

Table 1 shows the results for three different specifications. Column 1 assumes the same constant across time and across countries, while columns 2 and 3 add a country or a time fixed effect. Overall, the results are consistent across specifications. The estimate of $\alpha_1$ is very close to 1, which shows a very high persistence of $z_{i,t}$. The estimate of $\alpha_2$ is around 0.06.

The results in Table 1 are consistent with the specification of gradual portfolio adjustment, e.g. equation (8) in the paper. From equation (8), we have $\alpha_1 = \psi/(\psi + \gamma\sigma^2)$ and $\alpha_2 = 1/(\psi + \gamma\sigma^2)$. In our benchmark calibration, we would get $\alpha_1 = 0.998$ $\alpha_2 = 0.0665$. These numbers are very similar to the results of Table 1. For $\alpha_1$, the estimates are slightly higher in columns 1 and 3 and slightly lower in column 2. The point estimates of $\alpha_2$ are slightly lower, but the number of 0.0665 cannot be rejected.

Table 1: Portfolio Allocation with Predicted Future Return Differentials

| Dependent variable: $z_{i,t}$ | | | |
|---|---|---|---|
| $z_{i,t-1}$ | 0.9997*** | 0.9871*** | 0.9999*** |
| | (0.0003) | (0.0042) | (0.0004) |
| $\widehat{er}_{i,t+1}$ | 0.0576*** | 0.0635*** | 0.0650*** |
| | (0.0109) | (0.0073) | (0.0112) |
| Constant | -0.0000 | 0.0003** | -0.0000 |
| | (0.0000) | (0.0001) | (0.0003) |
| Month FE | No | No | Yes |
| Country FE | No | Yes | No |
| Month Cluster | Yes | Yes | No |
| Country Cluster | Yes | No | Yes |
| Observations | 11389 | 11389 | 11389 |
| $R^2$ | 0.998 | 0.998 | 0.998 |

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

# References

[1] Bertaut, Carol C. and Ruth Judson (2014), "Estimating U.S. Cross-Border Securities Positions: New Data and New Methods," Board of Governors of the Federal Reserve System (U.S.), International Finance Discussion Paper 1113.

[2] Bertaut, Carol C. and Ralph W. Tryon (2007), "Monthly Estimates of U.S. Cross-Border Securities Positions, " Board of Governors of the Federal Reserve System (U.S.), International Finance Discussion Paper 910.