CAPITAL GAINS AND THE DISTRIBUTION OF INCOME IN THE UNITED STATES*

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Abstract

This paper constructs a new data series on aggregate capital gains and their distribution, and documents that since 1980 capital gains have been the main driver of wealth accumulation. Over this period, capital gains averaged 8% of national income and comprised a third of total capital income. Capital gains are not included in the national income and product accounts, where the definition of national income reflects the goal of measuring current production. To explain the accumulation of household wealth and distribution of capital income, both of which are affected by changes in asset prices, this paper uses the Haig-Simons income concept, which includes capital gains. Accounting for capital gains increases the measured capital share of income by 5 p.p., increases the comprehensive savings rate (inclusive of capital gains) by 6 p.p., and leads to a greater measured increase in income inequality.

Keywords: Capital gains, inequality, capital share, savings rate
JEL Classification: E01, D63, E22, E21

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1 Introduction

This paper documents a new fact: aggregate capital gains have increased substantially in the United States over the past forty years. It defines a new measure of aggregate capital gains, Gross National Capital Gains (GNKG), which quantifies the yearly increase in national wealth driven by changes in asset prices, and not by savings or investment. Capital gains are not included in the national income and product accounts, where national income is defined in order to measure current production and output. Since this paper is concerned not with production but by wealth accumulation and its distribution, we use the Haig-Simons income concept,[1] a broad concept of income that combines market and capital gains income.

The capital gains we document provide a wider and improved window to understanding three macroeconomic trends involving the measurement and distribution of income: (i) the decline in the national accounts savings rate[2] (ii) the secular increase in the capital share of income (iii) the level and trend of income inequality. We find that including GNKGs in a comprehensive savings rate shows that savings has *increased* post-1980 by 5 percentage points, reversing the conclusion that comes from traditional national accounts data. In addition, accounting for GNKGs increases the Haig-Simons capital share of income by 5 p.p., amplifying the increasing capital share (and declining labor share) documented using standard national accounts data.[3] This paper then studies how GNKGs are distributed, combining aggregate and micro-level data to create distributional tables of Haig-Simons income. We show that capital gains are extremely concentrated, and Haig Simons income significantly increases the measured share of income of the upper percentiles of the distribution, as compared to income reported on tax returns or in survey data.

To understand and rationalize the emergence of GNKGs, we explore a model in which changes in wealth are not generated solely by changes in savings or investment, but also through changes in asset prices. We build a quantitative model of the US economy that includes unmeasured investment, imperfect competition, and the trading of pure profits. Our model shows that the three primary drivers of capital gains have been an increase in market power, an increase in intangible investment, and a decline in interest rates.

Figure 1 tells the aggregate story of capital gains. The blue 'X' series is the aggregate wealth-to-income ratio in the US, where wealth is defined as the market value of all stocks, bonds, and real estate held by individuals.[4] The red '+' series is the capital-to-income ratio of the US, computed by accumulating investment through the perpetual inventory method.[5] Beginning in 1980, the two series

[1] See Haig (1921) and Simons (1938).

[2] Net private savings.

[3] See Karabarbounis and Neiman (2014) and Elsby, Hobijn and Şahin (2013).

[4] The data is from the Financial Accounts, the aggregate balance sheet of households compiled by the Federal Reserve.

[5] The data is from the BEA.

diverge. Wealth increases substantially, due to an increase in the price of stocks, bonds, and real estate. In contrast, capital remains flat, due to low rates of savings and investment. The difference between the two series is analogous to how we will measure GNKGs: whenever wealth increases without a corresponding increase in saving or investment, there is an aggregate capital gain.

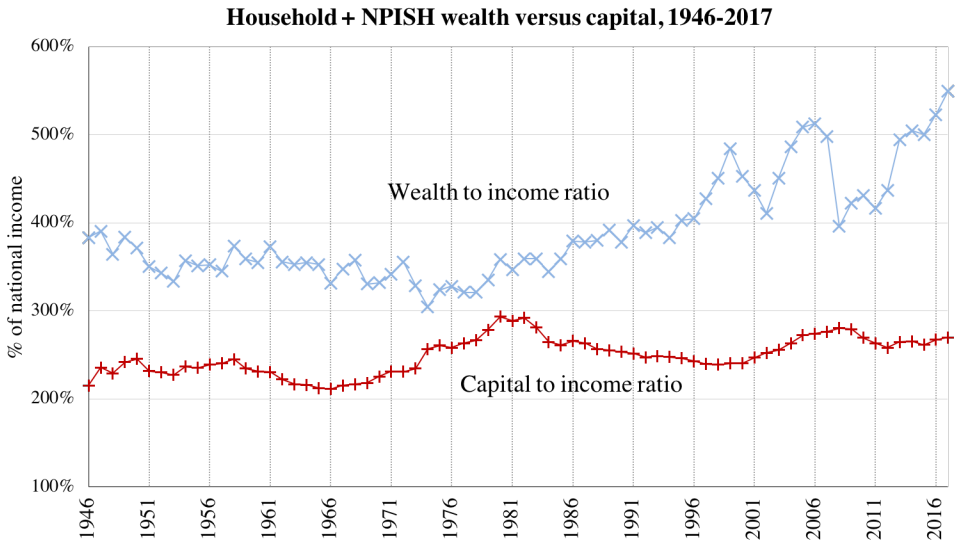

Figure 1: Trends in wealth and capital, 1946-2017. Wealth data is from the Financial Accounts of the Federal Reserve, and consists of the market value of stocks, bonds, housing, pensions, and business assets held by households and nonprofits (NPISH). Capital is the replacement value of the capital stock, computed by the Bureau of Economic Analysis (BEA). Income is net national income, from the BEA.

The economic literature contains two separate concepts of income, with each definition corresponding to a different purpose in economic theory and practice. The first school, 'income as production', defines income as equal to current output. The original national accounts were primarily created to measure production, either of consumption goods or stocks of capital. The national income concept was thus defined so as to equal production, with no place for capital gains.[6] As stated by Simon Kuznets (1947), "capital gains and losses are not increments to or drafts upon the heap of goods produced by the economic system for consumption or for stock destined for future use, and they should be excluded from measures of real income and output." There are other important drawbacks to including capital gains as income. Asset prices are highly volatile, and incorporating them would make income and savings volatile as well.

The second concept of income is 'income as well being', or Haig-Simons income. Haig-Simons income (see Haig (1921), Simons (1938), and Hicks (1946))

[6]See Landefeld (2000).

was well described by Hicks: "a person's income is what he can consume during the week and still expect to be as well off at the end of the week as he was at the beginning." In practical terms, this is measured as consumption plus change in wealth, or equivalently market income plus capital gains.[7] The contribution of changes in asset prices to welfare falls straight out of consumer theory:[8] to a first approximation, individuals are indifferent between receiving income as a dividend or as a capital gain. Importantly, as we will show below, capital gains have grown significantly as a share of national income. While there are still pros and cons to including capital gains in measures of income and savings, this change in economic reality suggests there are important things to be learned from looking at this alternative measure. To overcome the issue of volatility, we will focus our analysis on long run changes in capital gains, taking moving averages while eschewing discussion of individual years.

To measure GNKGs, we combine data on wealth from the Financial Accounts of the Federal Reserve with income and savings data from the Bureau of Economic Analysis (BEA) to create stock-flow consistent categories of assets and savings. As the calculation of capital gains necessitates valuations at market rather than book value, we make several key adjustments to the Financial Account data to move series from book to market value.

Our new data series for GNKGs shows that capital gains have grown to become a substantial and sustained component of capital income. GNKGs were small in magnitude from 1946-1980, with a mean of approximately zero. In contrast, from 1980-2017 GNKGs averaged 7% of net national income. On a yearly basis, GNKGs can be highly variable: as high as 50% of national income in the heyday of the dot-com boom, to as low as -115% of national income during the financial crisis of 2008. Over the past four decades, however, the gains have outpaced losses, making them a sustained source of income for US asset holders.

The first implication of the post-1980 upsurge of GNKGs is that standard stories about the rise of wealth in the US are missing a crucial element: the increase in asset prices. Typical models of increasing wealth focus on the declining growth rate of the economy combined with an increase in the savings rate. However, over the past forty years the wealth was not primarily accumulated by classical notions of savings and investment: it was accumulated by capital gains. And just as individuals can save out of dividends or wages, so they can save out of capital gains. We compute a new comprehensive savings rate, incorporating personal savings, corporate savings, and capital gains, and find that the traditional story of a decline in savings post-1980 is reversed. Savings *increased* post-1980; comprehensive savings averaged 11% from 1946-1982, then increasing to 16.2% from 1983-2017. It is precisely through this comprehensive

[7] Haig (1921) wrote that income is "the money value of the net accretion to one's economic power between two points of time", and Simons (1938) wrote that income is "the algebraic sum of (1) the market value of the rights exercised in consumption and (2) the change in the value of the store of property rights between the beginning and end of the period in question."

[8] See Shell, Sidrauski and Stiglitz (1969), for example, or section 2 below.

savings that the rise in wealth was accumulated.

The second direct implication of GNKGs is their effect on the capital share. GNGKs accrue to the owners of financial assets, i.e. to capital. We compute a new series for a comprehensive capital share, which includes capital gains as well as traditional capital income. GNKGs have a large effect on measured capital income, increasing the post-1980 capital share by 6 percentage points.

Finally, we study the impact of capital gains on the distribution of income. Most previous studies examine the contribution of capital gains *reported on income tax returns* to inequality. Importantly, the increase in aggregate capital gains that we document is not present in tax data on realized capital gains from the IRS. Capital gains reported on tax returns average around 3% of national income before 1980, but only increase modestly to 4% of national income from 1980 to the present. Aggregate capital gains are poorly measured in tax data for three reasons: (i) a growing share of realized capital gains are not subject to tax (ii) individuals can delay realizing capital gains, sometimes indefinitely (iii) capital gains reported to tax authorities are conceptually different than GNKGs, as taxable gains include nominal gains as well as real gains. Existing studies of income inequality that include only realized capital gains on tax returns have missed the surge of post-1980 capital gains.

To study the distributional effects of capital gains income beyond what is reported on income tax returns, we extend the Distributional National Accounts (DINAs), of Piketty, Saez and Zucman (2016) (henceforth PSZ). The DINAs contain data on the distribution of national income, which by definition does not include GNKGs.[9] To distribute aggregate capital gains, we use the same method used by PSZ to distribute capital income. For a given asset class, capital gains are distributed in proportion to an individual's holdings.[10]

We find that accounting for GNKGs significantly increases measured income inequality. The reason behind this is straightforward: since wealth and capital income are more concentrated than labor income, an increase in capital income will tend to increase top income shares. Our comprehensive income series show a large increase in top 10% and 1% shares from 1970 to the present. The top 10%'s share of income increases by 18 p.p. over the time period, compared with a 13 p.p. increase without GNKGs. The top 1%'s share increases by 14 p.p., while the share without capital gains increases by 8 p.p..

The measurement of GNKGs also contributes to the ongoing debate about whether top income shares since 2000 are being driven by capitalist rentiers (the view of Piketty) or whether they are being driven by the working rich (the view of Smith et al. (2017)).[11] Taking into account capital gains produces a story

[9] Although PSZ do not study the distribution of capital income directly, they do incorporate information from taxable capital gains in order to distribute national income.

[10] This method relies upon the assumption that, for a given asset class, individuals across the income distribution have the same expected return on assets. To the extent this is not true, and that the rich have a higher return on assets, our analysis will understate the concentration of capital gain income.

[11] Rognlie (2016) also discusses the impact of capital gains on the capital versus the labor

that contains elements of both of these views. The capital share of top income inclusive of GNKGs started increasing in the mid-1990s. By 2015, the top 10% received 55% of income from capital, while the top 1% received almost 70%.

1.1 Previous literature

Several previous papers study the magnitude of aggregate capital gains. Eisner (1989) compiles a 'Total Incomes System of Accounts', which includes revaluations in the price of tangible capital as income. Eisner (1980) estimates aggregate capital gains from the stock market. Bhatia (1970) computes capital gains on corporate stock, real estate, and livestock, and McElroy (1971) compiles capital gains for corporate equities. Piketty and Zucman (2014) and Roth (2016) provide a breakdown of savings versus capital gains for the 1970-2010 period. Rognlie (2016) emphasizes the important of housing capital gains for explaining the capital share.

This paper distinguishes itself from these prior studies in three aspects. The first is methodological. Our estimation of capital gains is part of a consistent framework that ensures no double counting of income, and embodies a stock-flow consistent relationship between wealth, national income, and savings. Second, our analysis accounts for a number of asset classes that previous work does not, such as fixed income and pension assets. In this aspect we can use a recent data source, the Integrated National Accounts, which was first published in 2006. Third, our data series extends much longer than previous work, from 1946-2017. This is important for two reasons. First, capital gains are quite volatile, and thus in order to correctly interpret their magnitude it is necessary to have many years of data. Second, with a long time series we are able to detect a change in trend for pre versus post 1980.

Two categories of papers have previously studied the distribution of capital gain income. First are studies of the distribution of *taxable realized* capital gains. In an early contribution, Liebenberg and Fitzwilliams (1961) examined the distribution of realized gains for 1958. Piketty and Saez (2003) study the distribution of income reported on tax returns, and in some specifications include capital gains. Piketty and Saez (2003) compute two capital gain series: one in which individuals are ranked using non capital gain income, but capital gains are included in the income shares, and a second series in which capital gain income is included in both ranking and income shares. Feenberg and Poterba (2000) also include capital gains in their study of top income inequality.

The second category of papers studies the distribution of capital gain income by imputing returns based on asset holdings. In an early work, Goldsmith et al. (1954) imputes retained earnings of different income groups. Bhatia (1974) examines income inequality inclusive of capital gains for 1955-1964. He estimates aggregate *nominal* capital gains for three categories of assets, corporate stock, non farm real estate, and farm assets. Then, he allocates capital gains to indi-

share.

viduals based on estimates of wealth derived from individual tax returns and the SCF. McElroy (1971) also studies the distribution of capital gains by imputing income based upon asset holdings. In a paper which is closest in scope to this current study, Armour, Burkhauser and Larrimore (2013) measure the distribution of capital income, inclusive of capital gains, using asset holding data from the Survey of Consumer Finances (SCF). For every asset class, they impute capital income using the average return of that asset class, allowing the return to differ by year.

Our paper distinguishes itself from Armour, Burkhauser and Larrimore (2013) in two aspects. The first is methodological. Our estimation of capital gains is part of a consistent framework that embodies a stock-flow consistent relationship between aggregate measures of wealth, national income, and savings. This method allows us to study the distribution of 100% of GNKGs. The second difference is the scope of our study. Our data stretches from 1946 to 2017, while Armour, Burkhauser and Larrimore (2013) have a more restricted sample of 1989-2007. We study capital gains on a wider variety of assets, including pension, retirement accounts, and fixed income assets. The greater scope and longer time period lead us to draw different conclusions from this earlier study. Our longer time series allows us to identify an important trend break in capital gains, beginning in the early 1980s. In addition, the longer data series shows that accounting for capital gains *increases* the trend in top-income inequality post-1980. This reverses the conclusion of Armour, Burkhauser and Larrimore (2013), which argued that capital gains would dampen the level and trend of income inequality. We will show that this conclusion is being driven by the endpoints of the sample. The year 1989 was a year of large capital gains, which tended to increase income inequality, while gains in 2007 were relatively modest. These two facts combine to flatten the profile of income inequality.

The paper will proceed as follows. Section 2 will introduce a simplified model of aggregate capital gains. Section 3 explains the measurement issues and data sources, and presents a time series of GNKGs. Section 4 defines aggregate Haig-Simons income and savings, and presents a data series for both of these variables. Section 5 studies the distribution of Haig-Simons income. Section 6 returns to the model to explain the empirical results.

2 A theory of capital gains

Before measuring aggregate capital gains in the data, we formally define them and explore how they relate to the income concept in the national accounts. We will show there is a close theoretical connection between capital gains, consumption, and welfare.

An agent born at time τ, with a lifespan of M years, optimizes lifetime utility $U(c_\tau, c_{\tau+1}, ..., c_{\tau+M-1})$, with $U(\cdot)$ concave and increasing. Her flow budget constraint is given by equation 1. At each time t, the agent chooses between consumption c_t and purchasing two types of assets. The first is a capital asset,

K_{t+1}, which is purchased at price q_t, depreciates at rate δ, and yields a rental rate ρ_t. The second asset is a financial asset, S_{t+1}, purchased at price X_t, which pays a dividend of d_t. The agent works, and receives labor income of $w_t l_t$. Initial asset holdings are zero, and the agent leaves no bequests.

$$c_t + q_t K_{t+1} + X_t S_{t+1} = q_t(1-\delta)K_t + S_t(X_t + d_t) + \rho_t K_t + w_t l_t \quad (1)$$

There are two distinct types of income implicit in this budget constraint.

Definition 1. *National income is equal to income from wages plus rental income from capital plus dividends from securities, minus depreciation.*

$$Y_t^n = w_t l_t + \rho_t K_t - \delta q_{t-1} K_t + d_t S_t. \quad (2)$$

National income is income received from production. If we sum equation 2 over all US residents, definition 1 is in line with the BEA definition of national income, which, absent measurement errors, equals the national production of output.[12]

Definition 2. *Capital gains for an asset class are equal to the change in the price of the un-depreciated portion of the asset. Then* $KG_t^S \equiv S_t(X_t - X_{t-1})$*, and* $KG_t^K \equiv (1-\delta)K_t(q_t - q_{t-1})$*,* $KG_t = KG_t^S + KG_t^K$*.*

Capital gains are not included in the BEA definition of national income. However, they enter in the budget constraint in the same way as other types of capital income. With no transaction costs of selling assets and perfect information, the agent is indifferent between a one dollar increase in the share price of their asset or a dollar in additional dividends.[13]

We now formally define Haig-Simons income, which includes both national income and capital gains.

Definition 3. *Haig-Simons income is equal to national income plus capital gains:*

$$HS_t \equiv Y_t^n + KG_t. \quad (3)$$

There is a close theoretical connection between Haig-Simons income and consumption:

Proposition 1. *If initial wealth is zero and there are no bequests, the average consumption of an agent over her lifetime is equal to her average Haig-Simons income:* $\sum_{t=\tau}^{\tau+M-1} c_t/M = \sum_{t=\tau}^{\tau+M-1} HS_t/M$*.*

Proof. See appendix G □

[12] See the BEA Handbook, Fox and McCully (2009).

[13] We formally show this in Appendix Proposition 7.

Proposition 1 follows directly from the budget constraints. You can only spend on consumption what you make in income. It makes clear that in a standard optimization setting, the income which is mostly tied to consumption is Haig-Simons income, not national income. In fact, proposition 1 might be termed the 'ex-post exact permanent income hypothesis':[14] ex-post consumption must exactly equal ex-post income, where income in this case is inclusive of capital gains.

The dynamics of wealth accumulation are also closely tied to Haig-Simons income, which is inclusive of capital gains. We formally define wealth as follows.

Definition 4. *End of period financial wealth is equal to the market value of capital and securities:* $W_t \equiv q_t K_{t+1} + X_t S_{t+1}$.

Remark 1. *The change in wealth between two periods is equal to Haig-Simons income minus consumption:* $\Delta W_t = W_t - W_{t-1} = HS_t - c_t$.

As remark 1 shows, to understand the dynamics of wealth accumulation, it is necessary to take into account capital gains. A savings rate that excludes capital gains excludes a major determinant of wealth.

3 Measuring capital gains

In this section, we move from theory to measurement and estimate aggregate capital gains in the United States. We then compare our estimates with aggregate capital gains reported on tax returns.

We extend equation 1 to allow for multiple types of financial assets a^j, indexed by j— there is still a single type of capital asset held directly by households, as there will be in the data.

We also extend our theory to incorporate retained earnings. In equation 1 we made the simplifying assumption that all capital income is paid out to shareholders as dividends. We now remove this assumption, allowing some income to be held internal to the firm as *retained earnings*, RE_t. In the spirit of Miller and Modigliani (1961), we make the assumption that a dollar of retained earnings contributes a dollar to the market value of a firm: $p_t^j = p_t^{j,ER} + RE_t^j$, where $p_t^{j,ER}$ is the price of asset j ex-retained earning. Retained earnings are already measured as income in the BEA's definition of national income. In order to avoid double counting the income of retained earnings,[15] in our definition of capital gains we will attempt to net-out the effect of retained earnings on share prices.

With multiple types of assets and retained earnings, the budget constraint is given as

[14] See Friedman (1957).

[15] That is, once as corporate income, and once as a capital gain in the share price of the firm with the retained earnings.

$$c_t + q_t K_{t+1} + \sum_{j \in J} (p_t^{j,ER} + RE_t^j) a_{t+1}^j = \tag{4}$$
$$(1 - \delta + \rho_t/q_t) q_t K_t + \sum_{j \in J} \left(p_t^{j,ER} + RE_t^j + d_t^j \right) a_t^j + w_t l_t,$$

We modify our definition of national income to include multiple types of assets and retained earnings, $Y_t^n = w_t l_t + \rho_t K_t - \delta q_{t-1} K_t + \sum_{j \in J} a_t^j (d_t^j + RE_t^j)$, and similarly modify our definition of capital gains:

$$KG_t^j \equiv a_t^j ((p_t^j - RE_t^j) - p_{t-1}^j). \tag{5}$$

We can now proceed to our measurement equations.

Definition 5. *Gross National Capital Gains (GNKGs) equal the increase in the total value of all assets directly owned by US residents due to changes in the price of the assets, minus retained earnings from the financial assets:*

$$GNKG_t = \sum_{i \in USA} \left\{ \sum_{j \in J} KG_t^{i,j} + (1 - \delta) K_t^i (q_t - q_{t-1}) \right\} = \tag{6}$$
$$\sum_{j \in J} KG_t^j + (1 - \delta) KG_t^K. \tag{7}$$

To measure aggregate capital gains in a world of perfect data, we would have data on the individual asset holdings of all US residents along with data on the market prices of each of these assets. Often this data is not available, however there is a way to calculate capital gains indirectly, using data which is available. Rearranging the terms for capital gains from equation 5,

$$KG_t^j \equiv a_t^j ((p_t^j - RE_t^j) - p_{t-1}^j) = \tag{8}$$
$$\left[p_t^j a_{t+1}^j - p_{t-1}^j a_t^j \right] - (a_{t+1}^j - a_t^j) p_t^j - a_t^j RE_t^j =$$
$$W_t^j - W_{t-1}^j - FL_t^j - a_t^j RE_t^j.$$

Measuring capital gains boils down to measuring changes in aggregate household wealth, minus the "flows" F_t^j for the asset class, which are the net purchases during the time period.

3.1 Gross National Capital Gains

We measure equation 8 using data from the Financial Accounts (formerly the Flow of Funds), which is compiled by the Federal Reserve, as well as data from the National Income and Product Accounts (NIPAs), compiled by the BEA. The level of aggregation we will use in this paper is the 'national' level, comprising

all US residents. Our measures of income and wealth will thus include totals that are earned or held abroad, although it may fail to capture income and wealth in tax havens.[16]

The Financial Accounts compiles a national balance sheet for US residents and non-profit institutions. The Financial Accounts often cannot distinguish between household holdings and non-profit holdings, thus for all of our results we will show combined results for the two sectors. Non-profit institutions held 8% of combined wealth in 2017. For simplicity, we will refer to the combined household and non-profit sector results as simply 'household' results. For more than thirty different asset classes, the Financial Accounts provides information on the market value of the assets held, i.e. wealth $W_t^j = p_t^j a_{t+1}^j$.

The coverage of assets we include in our calculation of household wealth is similar to the concept of household net worth from the Financial Accounts, and is also in line with the concept of household wealth used in Saez and Zucman (2016). Wealth is the market value of all assets owned by US households, net of their debts. Assets include all financial and non-financial assets over which ownership rights can be enforced. It includes all pension wealth, with the exception of Social Security benefits and unfunded defined benefit pensions.

The Financial Accounts also has data on "flows", net purchases of financial assets, by asset type, i.e. $FL_t^j = (a_{t+1}^j - a_t^j)p_t^i$. We make one modification to the 'flows' in order to harmonize the data with the NIPA data. In theory, across all asset types the sum of financial flows for households should equal 'personal saving' from the NIPAs,[17] however there is a statistical discrepancy between the two measures.[18] We will use the personal savings from the NIPAs as our baseline measure of net flows, and distribute the statistical discrepancy between the different asset classes in the Financial Accounts flows in proportion to their relative magnitudes.

We can therefore measure GNKGs as the aggregate increase in the market value of household wealth beyond what is saved:

$$GNKG_t = W_t - W_{t-1} - s_t^{personal} - RE_t = W_t - W_{t-1} - s_t^{private}. \quad (9)$$

The final equality follows from the definition of private savings. GNKGs are thus measured as a residual: they are what remain after subtracting savings from changes in wealth.

There are five main categories of assets: housing, equities, fixed-income, business, and pensions and life insurance. Liabilities consist of mortgage and non-mortgage debt. The calculation of GNKGs requires that assets are at market value, and thus we make several modifications to the Financial Accounts data. First, we convert bond wealth data from par value to market value.[19]

[16] See Zucman (2013).

[17] Total flows equal personal savings (NIPA variable A071RC1) minus capital transfers paid by households and nonprofits (NIPA W981RC1).

[18] Financial accounts variable FA157005005.

[19] For the exact method, see appendix A. The par value of a bond is the amount it pays at maturity, and is often the initial selling price of the bond.

This is important, because declining interest rates since 1980 have tended to increase bond prices and generate capital gains.[20] Without this modification, bonds show capital losses. Second, we convert non-corporate business valuations from book value to market value. This is important, because most other data sources show an increase in the market value of closely held businesses relative to book value since the mid-1990s, perhaps due to "sweat equity" in partnerships and sole-proprietorships (see Bhandari and McGrattan (2018)). Finally, we subtract durable good wealth and deferred life insurance payments.

Data on private savings comes from the NIPAs, and consists of personal savings plus corporate retained earnings[21], minus capital transfers.[22] Figure 2 shows trends in the private savings rate. From 1946 to 1980 the savings rate was relatively stable, however since 1980 NIPA savings have been trending downward, a period during which wealth has been rising (see figure 1).

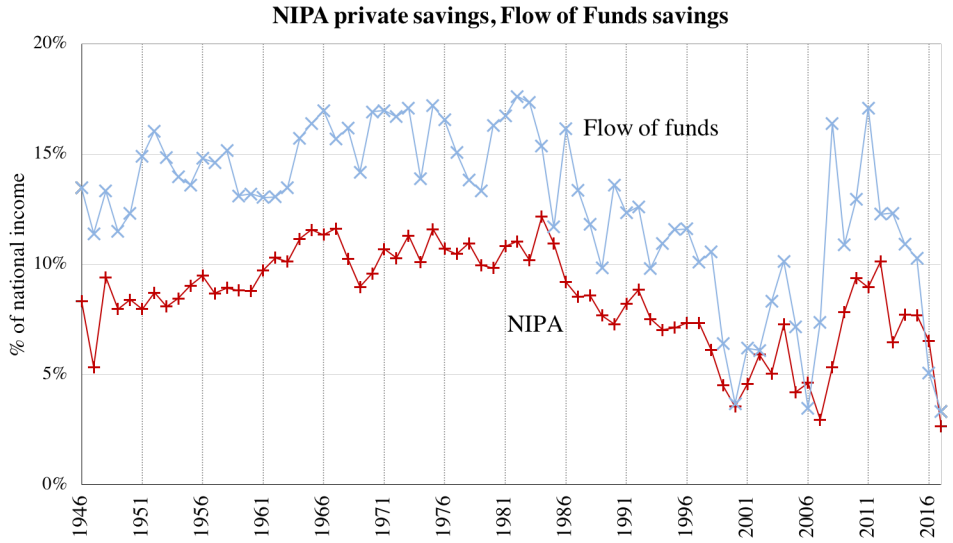

Figure 2: Trends in savings, 1946-2017. Data on NIPA savings is from the BEA, and consists of personal savings plus corporate retained earnings, minus capital transfers. Data on Flow of fund savings is from the Financial Accounts, and consists of capital expenditures plus net acquisition of financial assets plus retained earnings, less net increase in liabilities.

We calculate GNKGs using equation 9, with all nominal amounts converted to average 2010 dollars.[23] For the results in the section, we show five year moving averages of GNKGs.

[20] In fact, a large proportion of totals yield of bonds since 1980 have been due to capital gains, not yield to maturity. See Dobbs et al. (2016).

[21] NIPA varaible A127RC1.

[22] Including net transfers paid by corporations, W976RC1.

[23] We denote W_t as the end of period market value of wealth, thus we must convert these to mid year prices.

Figure 3 shows our main result for aggregate capital gains. The most striking result is a large increase in the mean and variance of capital gains starting around the year 1980. Looking at broad trends in the time series, figure 3 can roughly be divided into three eras. In the first era, from 1946-1968, there are moderate capital gains of 2% of national income per year. In the second era, from 1969-1982, there are moderate capital *losses* of 3% per year. In the final era, from 1983-2017, there are large capital gains averaging 8% per year.

Until the early 1980s, capital gains were small in magnitude, averaging less than 1% of national income per year. That is not to say there weren't individual years with moderate capital gains, however on the balance years of capital losses netted out the gains. Beginning in the early 1980s, capital gains increased in magnitude. During the 1990s internet boom capital gains boomed as well, and during the financial crisis of 2008 there were massive capital losses. Since 1980, however, capital losses have outpaced the gains. A stark representation of this is present in figure 1. Until 1980 the path of wealth followed the path of capital, but starting in 1980 wealth diverged and has not come back.

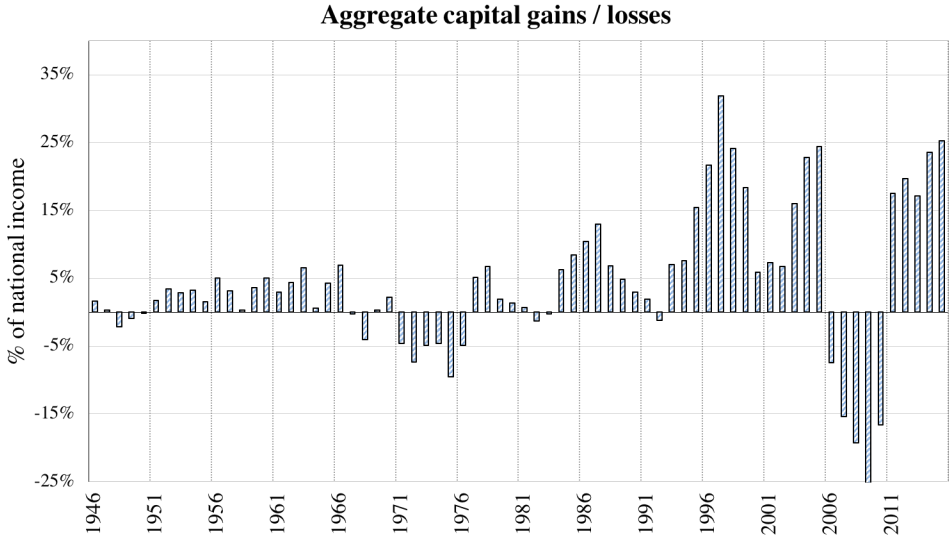

Figure 3: Aggregate capital gains, 1946-2015. GNKGs calculated as the real increase in the market value of wealth, minus net private savings. See equation 9. Data on wealth is from the Financial Accounts, data on savings is from the BEA.

3.2 Capital gains reported on tax returns

The long-run increase in measured capital gains using aggregate data (depicted in figure 3) *is not present* in individual level income-tax data on realized capital gains reported to the IRS. Realized capital gains reported on tax returns are only a fraction of GNKGs, and they show only a moderate change in trend post

1980. The absence of capital gains from tax returns has likely concealed their macroeconomic importance, as well as their contribution to income inequality.

There are two strands of literature that measure the distribution of capital gains on tax returns. The first strand studies the distribution of capital gain income that is a part of adjusted gross income (AGI) (see, for example, Feenberg and Poterba (2000) or CBO (1992)). Measuring capital gains using this method is straightforward but theoretically problematic, because in the past the tax code allowed for a significant portion of realized capital gains to be 'excluded' from AGI. When a capital gain is excluded from AGI, the individual does not pay tax on the gain. From 1942-1978, 50% of capital gains were excluded, and from 1979-1986 60% were excluded. For example, in 1978, an individual with a realized capital gain of $100 will report the $100 on Schedule D, line 6. However, only $50 will be reported as AGI on form 1040. In recent years excluded gains are less of a problem: from 1987 to the present, 100% of capital gains are in AGI.

Figure 4, red '+' series, displays realized capital gains in AGI on individual tax returns. All series in the figure are mid-point moving averages, to better serve as comparison to the GNKG series. This series does display an upward trend, but a significant portion of the trend is due to changes in tax laws that changed the amount of capital gains excluded from AGI in 1987.

The second strand of the literature studies the distribution of capital gains reported on Schedule D of individual tax returns (this includes Piketty and Saez (2003)). Schedule D includes capital gains that are included in AGI, as well as those that are excluded but are still reported on the tax return. Figure 4, blue 'X' series, shows capital gains reported on Schedule D of individual income taxes. Schedule D capital gains averaged about 3% of national income before 1980, and increase modestly to 4% of national income from 1980 to the present.

In order to compare these tax based measures to our aggregate measure, we make one adjustment to the schedule D series. There are several categories of capital gains that realized by individuals but are excluded even from schedule D. This includes, for example, a significant portion of capital gains on the sale of primary residences. Since 1997, up to $500,000 of capital gains on the sale of a primary residence are exempt from tax. Individuals that don't owe any tax on the sale of their home do not need to report the sale to the IRS. We estimate these further excluded capital gains using the following method:

1. The size of the 'tax expenditure' for each category (e.g., exemption for the sale of primary residences) of excluded gains is taken from the Joint Committee on Taxation's yearly estimate (see on Taxation (2008)). This yields the estimated tax revenue that capital gain category would have yielded in the absence of the exemption.

2. We use the average tax rates, in combination with the tax expenditure, to back out the size of the capital gains not reported on Schedule D.

Figure 4, teal circles, shows the sum of Schedule D capital gains plus these

Schedule D exclusions. This modification is important for the post 1997 period, when housing capital gains increase substantially. 'Schedule D + Exclusion' capital gains average 3% of national income for the 1954-1979 period, increasing to 4.6% of national income from 1980-2012.

Figure 5 compares GNKGs with the 'Schedule D + exclusions' series. Aside from the differences in trend noted above, there are also stark differences in the magnitude of capital gains reported on tax returns and the level of GNKGs computed from aggregate data. For example, in 2012, total GNKGs calculated using aggregate data were $2.5 trillion, while on individual tax returns ('Schedule D +') only $871 billion in capital gains were reported.

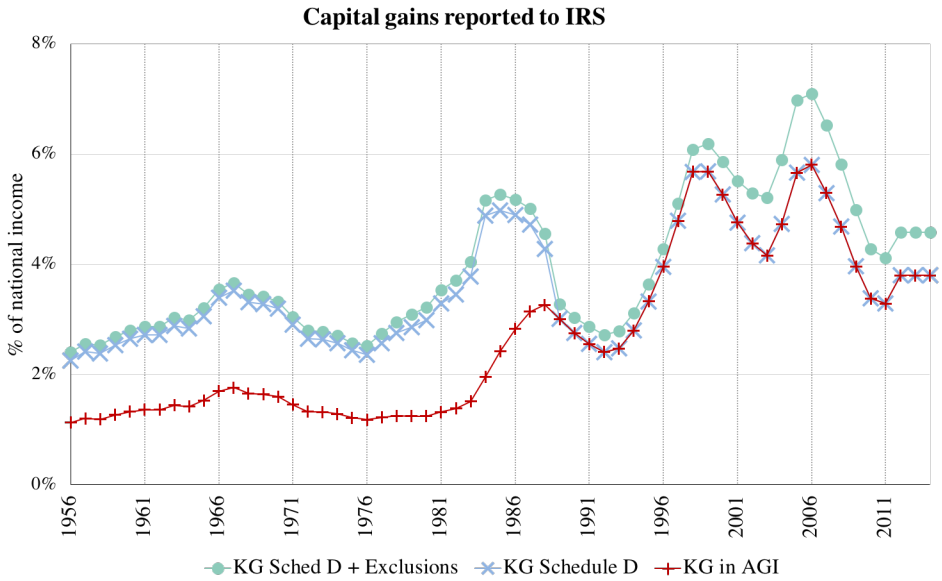

Figure 4: Capital gains included in adjusted gross income (AGI) of form 1040 of individual tax returns, capital gains reported on Schedule D of form 1040, and capital gains reported on Schedule D plus estimates of excludable capital gains. All series are five year mid-point moving averages.

There are three reasons why the patterns for GNKGs are not mirrored in the tax data. First, tax return capital gains are conceptually different than aggregate capital gains, as they include nominal gains and retained earnings. Individuals pay taxes on nominal capital gains, while purely nominal gains are excluded from the definition of GNKGs. In addition, GNKGs are calculated net of retained earnings, while taxable capital gains will include gains from any increase in the market value of equities that is due to retained corporate earnings. Thus in eras of high inflation and high retained earnings there will be high taxable capital gains, but not necessarily high GNKGs. Figure 5, red '+' series, shows aggregate nominal capital gains, defined as simply the yearly change in the market value of household wealth minus personal savings, without adjustment for retained earnings or inflation. Due to the presence of inflation nominal capital gains are large in value, trend upwards until 1980, and have no trend from 1980

to the present.

Second, a growing share of realized capital gains are not subject to the individual income tax, and thus do not show up on tax returns. Pension and IRA capital gains are not reported on tax returns. In addition, a growing proportion of total wealth are held by non-profits, and thus are not subject to tax. Figure 5, purple triangle series, estimates the flow of nominal capital gains that are subject to tax. While before the 1960s most capital gains were subject to tax, since then a gap has appeared between taxable and non-taxable capital gains. Overall nominal taxable capital gains do not display a trend over the time period.

Third, individuals can delay realizing capital gains, sometimes indefinitely. Capital gains are only taxed when they are realized, and thus the time path of realized capital gains does not necessarily match the path of accrued capital gains. Even upon death capital gains are not taxed. Instead, the tax basis of the deceased's assets is stepped up to the market value at the time of death. When heirs eventually sell the inherited asset, they only pay capital gains tax on the difference between the value when inherited and the sale price. Of the capital gains that were realized in 2012, the majority were for long term transactions, those with a holding period of more than one year. And of the long-term transactions, over 50% had a holding period of more than five years.

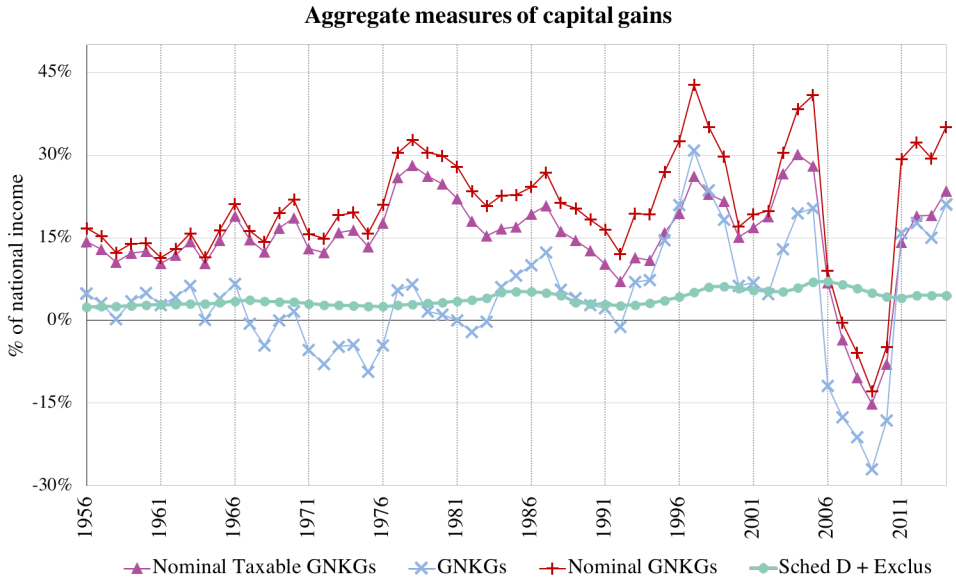

Figure 5: GNKGs, nominal KGs , nominal taxable KGs, and Schedule D based measures of capital gains. GNKGs calculated as the real increase in the market value of wealth, minus net private savings. See equation 9. Nominal capital gains are the nominal increase in household wealth. Taxable capital gains equal nominal capital gains minus gains that are not subject to capital gains tax. 'Schedule D + exclusion' capital gains equal the total amount of capital gains reported on Schedule D of individual tax returns plus estimated exclusions. All series a five year mid-point moving averages.

Prior research consistently shows that capital gains reported on tax returns captures only a small fraction of total capital gains. Bourne et al. (2018) link federal estate tax returns from decedents in 2007 to panel data on income tax returns prior from 2002-2006. Although this was a period of very high returns in the stock and housing markets, the majority of wealth individuals reported *nominal* returns on capital to the IRS of less than 2%. Steuerle (1985) and Steuerle (1982) also provide evidence that realized capital gains bear little relation to actual returns.

The above analysis explains why taxable capital gains are not a good measure of GNKGs, and lend support to our method of studying the distribution of capital gains in section 5.

3.3 Capital gains by asset class

GNKGs can also be computed by asset class. The Financial Accounts breaks down wealth and saving into stock-flow consistent groups, and we combine them into five main categories of assets. Using equation 9, we calculate capital gains by asset class. For housing, we subtract mortgage capital gains from gross housing, and for fixed income, we subtract capital gains on debt. Figures A.1 displays GNKGs by asset class. By far the largest component of GNKGs are capital gains on equities and housing, while pensions are a growing source of capital gains post 1980.

4 Measuring Haig-Simons income

In this section, we compute our estimates of Haig-Simons income, Haig-Simons savings, and the Haig-Simons capital share.

We define our aggregate measure of Haig-Simons income using equation 3.

Definition 6. *National Haig-Simons Income (NHSI) is the sum of National Income and Gross National Capital Gains:* $NHSI_t = Y_t^n + GNKG_t$.

The first component of this is 'national income'. In our theoretical model, national income to equal the sum of labor income, dividends, and retained earnings. We call this 'national income' because it aligns well with how the BEA measures aggregate national income in the data.

National income is a concept very closely tied to production. We briefly describe this measurement process, in the context of the national accounting system. Gross national product (GNP), Y_t is the amount of output produced by US citizens. Gross national income (GNI) is the amount of income from production received by US citizens, and is measured as the sum of payments to labor, $w_t L_t$, net operating surplus, $Y_t - w_t L_t - \delta K_t$, and consumption of fixed capital, δK_t. As their definitions make clear, GNP is equal to GNI, although they are computed using different data sources so there is sometimes a discrepancy. Net operating surplus consists of the sum of two types of capital income: dividends

d_t, and retained earnings, RE_t. Net national income, which we will refer to as national income, equals gross national income minus depreciation.

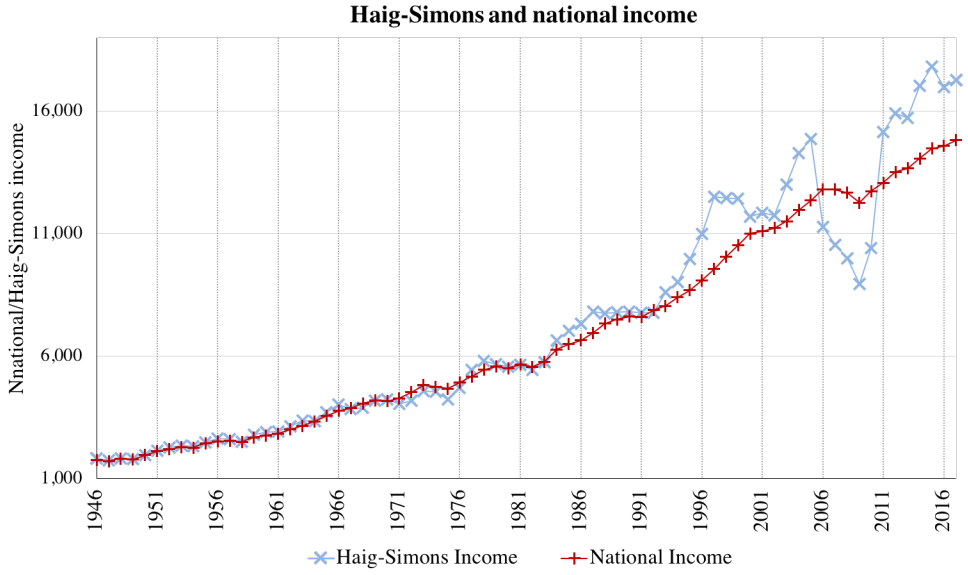

Figure 6: Haig-Simons and national income. Haig-Simons income equals national income plus gross national capital gains (GNKGs). Data on national income is from the BEA. For the construction of GNKGs, see section 3.

We use NIPA data on national income[24] along with GNKGs calculated in section 3 to measure NHSI. Figure 6 presents the time series from 1946-2017, in constant 2010 dollars, and compares the series to national income. Haig-Simons income tracks national income until the early 1990s, when it begins to diverge. From 1990-2017 Haig-Simons income is mainly above national income, with the exception of the years of the financial crisis around 2008.

4.1 Haig-Simons savings

Definition 7. *Haig-Simons Savings (HSS) is the sum of net private savings and GNKGs:* $s_t^{HS} = s_t^{private} + GNKG_t$. [25]

We calculate HSS using data on private savings from the NIPA. Figure 7 presents the time series of Haig-Simons savings from 1946 to the present, and shows as a comparison group net private savings from the NIPAs. The pattern for HSS is at odds with the traditional story of a post-1980 decline in savings. The HSS rate does not decline post 1980s, as NIPA savings does, but *increases* in magnitude. When individuals accrue capital gains in the stock and housing markets, they hold on to them, serving as an engine of wealth accumulation.

[24] Series A032RC1.

[25] In previous literature, capital gains are sometimes referred to as "passive savings". See also the "comprehensive savings" of Eisner (1980).

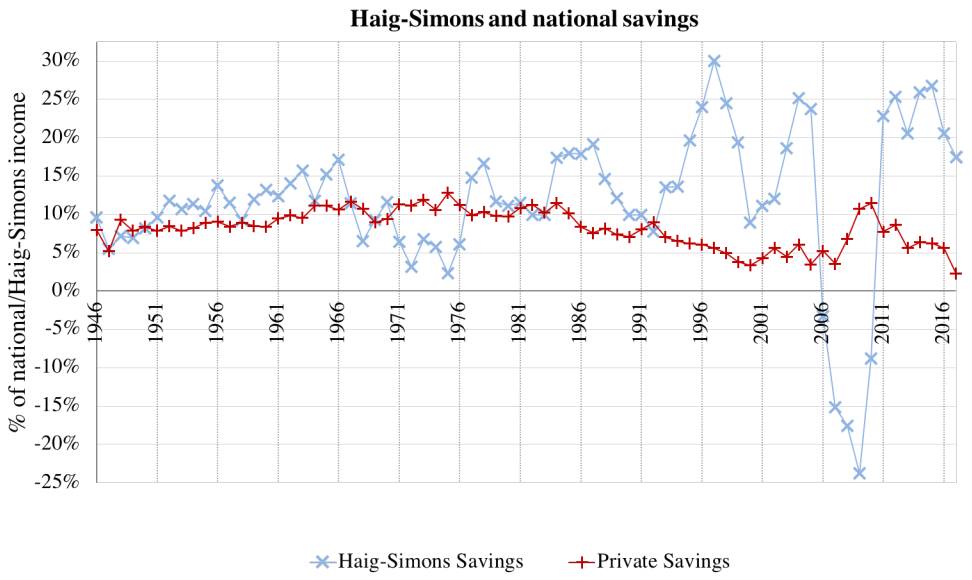

Figure 7: Haig-Simons saving and net private saving. Haig-Simons saving is the sum of net private savings and GNKGs. Data on net private savings is from the BEA. For the construction of GNKGs, see section 3.

Figure 8 compares the magnitudes of the two vehicles of wealth accumulation, savings and capital gains, throughout the three eras. In the first two eras, savings drove the increase in wealth. However, in the third era, wealth was accumulated on the back of GNKGs.

Our finding of a post-1980 rise in GNKGs dovetails nicely with the strand of literature that tries to understand the post-1980 decline of the personal savings rate in the United States. Juster et al. (2006), using panel data from the PSID, finds that the decline in personal saving is largely due to capital gains from corporate equities. This is consistent with other studies, such as Bostic, Gabriel and Painter (2009), that find moderate effects of a rise in wealth on consumption.

4.2 Haig-Simons capital share

GNKGs accrue to the owners of financial assets, i.e. to capital. If a firm's market value increases, this is income to a firm's owners and not to its workers. The rise of GNKGs since the 1980 shown in figure 3 thus has immediate implications for the level and trend of the capital share of income. A growing literature (see, for example, Karabarbounis and Neiman (2014) and Elsby, Hobijn and Şahin (2013)) documents a declining labor share of income in the US, and a corresponding rise in the capital share. This literature measures capital income using NIPA income, and does not account for capital gains.

Definition 8. *The Haig-Simons capital share of income equals NIPA capital income plus GNKGs, divided by Haig-Simons income.*

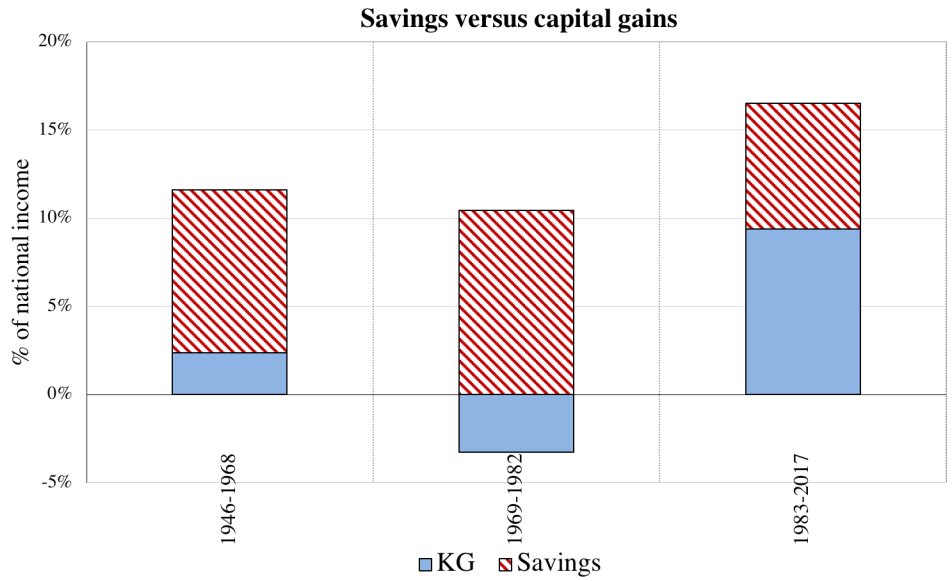

Figure 8: Capital gains: three eras. Savings is net private savings is from the BEA. For the construction of GNKGs, see section 3.

Figure 9 shows two measure of the capital share for the US. The first is a traditional measure, without capital gains, derived from national account data on capital income. Capital income is the sum of corporate profits, income from owner and tenant occupied housing, and the capital component of non-corporate income.[26] We divide capital income by factor-price national income to yield the capital share.[27] This measure, in line with the literature, shows an increasing trend, from 21% in 1980 to 26% in 2017.

The second measure of the capital share incorporates capital gains. We add GNKGs to NIPA capital income, and take as the denominator factor-price Haig-Simons income.[28] This measure shows an even larger increase post-1980, from 22% in 1980 to 38% in 2017. The large GNKGs post-1980 ensure that in the absence of a deep recession the capital share of Haig-Simons is above the NIPA capital share. Figure 10 compares the two measures of the capital share for the post 1983 period. Capital gains in the stock and housing markets push up the Haig-Simons capital share to 28% of national income, a quarter of which originates from GNKGs.

[26] We assume that 30% of mixed income is labor. Our analysis in this section is robust to other assumptions about income shares.

[27] Factor price income equals national income, minus production taxes, plus subsidies, minus net government profits.

[28] Equal to factor-price national income plus GNKGs.

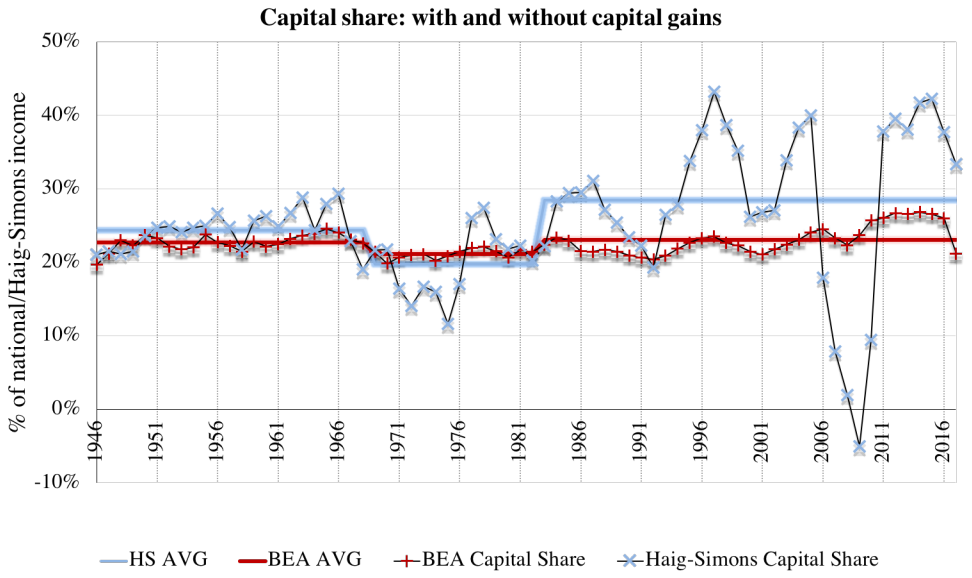

Figure 9: Capital share, with and without capital gains. BEA capital share is the sum of corporate profits, income from owner and tenant occupied housing, and the capital component of non-corporate income, divided by national income. Data is from the BEA. Haig-Simons capital share is BEA capital income plus GNKGs, divided by Haig-Simons income. For the construction of GNKGs, see section 3. For the construction of Haig-Simons income, see section 4.

5 The distribution of Haig-Simons income

We now turn to the question of the distribution of capital gain income. Section 3 documents substantial capital gains for the post-1980 period, capital income which has the potential to influence the measurement of income inequality.

While there is disagreement about whether capital gains should be included in income for the purpose of measuring aggregate output, theoretically there are good reasons for including capital gains when measuring income inequality. When restricted to annual measures of income, the Haig-Simons concept is widely agreed to be the ideal measure of income (see JCT (2012)); it is the embodiment of the Hicksian notion that income is what you can spend while keeping capital intact.[29] Section 2 shows the close theoretical connection between Haig-Simons income and individual utility.

While Haig-Simons may possess theoretical merits, it has several practical drawbacks. Aggregate capital gains are extremely volatile, an embodiment of

[29]When not restricted to annual measures, in theory the ideal income concept is the lifetime, or permanent, income (see, for example, Auerbach, Gokhale and Kotlikoff (1991) and Fullerton and Rogers (1993)). Measuring lifetime income inequality is quite difficult, however, due to the lack of long time series on individual income (exceptions include Guvenen et al. (2017) and Gustman and Steinmeier (2001)). Due to these limitations economists and tax policy have generally taken an annual approach to measuring income.

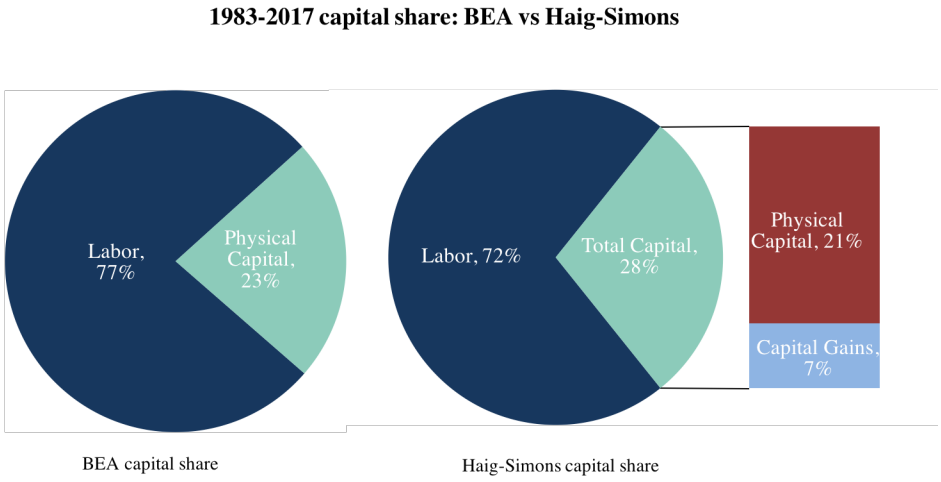

Figure 10: Post-1983 capital share comparison. BEA capital share is the sum of corporate profits, income from owner and tenant occupied housing, and the capital component of non-corporate income, divided by national income. Data is from the BEA. Haig-Simons capital share is BEA capital income plus GNKGs, divided by Haig-Simons income. For the construction of GNKGs, see section 3. For the construction of Haig-Simons income, see section 4.

the stock and housing markets which drive them. This volatility poses a challenge for measuring and interpreting trends in Haig-Simons income inequality. In years when the stock and housing markets boom, top-income shares increase, as capital gains are very concentrated. In turn, during stock market crashes, top-income shares drop. Volatility of measured inequality in and of itself is not a problem, as long it accurately reflects the volatility of individual wellbeing. It might be argued that in years in which the stock markets declines, the top of the distribution do in fact suffer welfare losses in proportion to the market. During the financial crisis of 2008, the wealth of the richest individuals in the US was almost cut in half.[30]

In another sense, however, single year movements in asset market prices are not a good measure of individual well-being. Most individuals have an investment horizon that is significantly longer than one year. The 2016 Survey of Consumer Finance (SCF) asks individuals for the reasons why they save and invest. The 5 top choices for savings all point towards a longer term investment horizon: for retirement (33% of individuals), precautionary savings for emergencies (24%), in order to make a bequest for children (7%), for children's education (6%), "for the future" (5%). The SCF also asks individuals directly what their saving and investment horizon is: 69% have a horizon greater than one year,

[30]For example, Warren Buffet's fortune fell from $62 billion to $37 billion, and likewise Bill Gates's net worth dropped from $58 billion to $40.

while 42% have a horizon more than 5 years. For the purposes of achieving these long term goals, it is the returns over the holding period that matter, not returns in individual years. For this reason, we will focus our analysis on longer run changes in capital gains, by using a five year moving average of capital gains.

5.1 Distributing GNKGs

The starting point of our analysis is data from the Distributional National Accounts (DINAs), a micro data source with information on the distribution of national income and wealth from 1946-2016. The DINAs encompass data on the distribution of national income,[31] but not GNKGs. To compute the distribution of Haig-Simons income, we need to estimate the distribution of GNKGs.[32]

The advantage of the DINAs over previous studies is they capture the total distribution of aggregate national income, not only the income reported on tax-returns or reported to surveys. A large percentage of national income doesn't show up on individual tax returns, including implicit rents on housing, the retained earnings of corporations, and employer fringe benefits. Figure 11 shows the relationship between the micro-data of the DINAs and the macroeconomic aggregates from the national accounts. Total income in the DINAs sums to national income from the NIPAs, and total wealth sums to aggregate wealth from the financial accounts.

The advantage of Haig-Simons income over the DINA's pre-tax income concept is it captures capital gains not included in the NIPA concept of national income. The red portion of figure 12 reproduces a figure from Piketty, Saez and Zucman (2016), and shows that only a third of capital income is reported on personal tax returns. The blue area of 12 shows the DINAs are still missing a key component of capital income, GNKGs.

In an ideal world, GNKGs could be measured through individual level data on specific asset holdings.[33] Since this data is not available for the United States, we distribute capital gains using the same method Piketty, Saez and Zucman (2016) use to study the distribution of (non capital gain) capital income. The method works as follows. First, for each asset class, we compute the macroeconomic yield of GNKGs by dividing the flow of aggregate capital gains by the total value of the corresponding asset. For example, for equities we will divide total capital gains on stocks for a given year by the total value of the stock market (see equation 10). We then multiply individual wealth holdings by the macroeconomic yield to compute individual capital gain income (see equation 11). This procedure ensures that individual capital gains sum to aggregate GNKGs.

$$Yield_t^j = GNKG_t^j / W_t^j \tag{10}$$

[31] For a overview of the DINA data, see appendix C.

[32] We will use the original DINA results as the main comparison data for our Haig-Simons series.

[33] In addition, data would be needed on the retained earnings of the underlying securities for equity holdings.

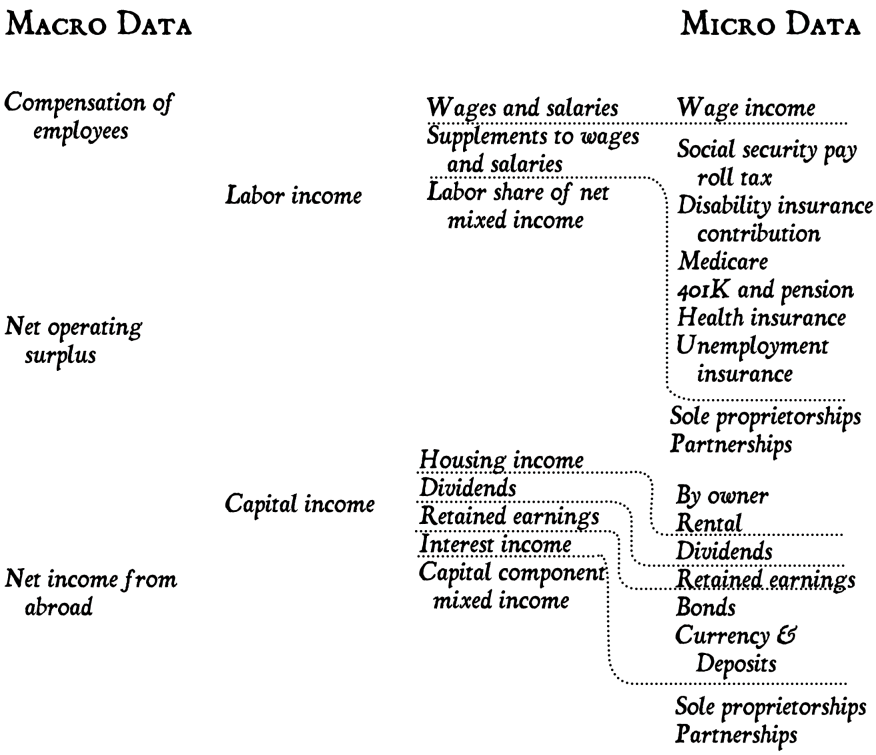

Figure 11: DINAs: National Income from macro to micro data.

$$\widehat{GNKG}_t^{i,j} = Yield_t^j \cdot W_t^{i,j} \tag{11}$$

Our method of distributing capital gains relies upon the crucial assumption that for a given asset class, individuals across the income distribution have the same expected total return on assets. To the extent that this is false, and richer individuals have higher returns, we will tend to understate the amount of capital gains inequality. To the extent that richer individuals have lower returns, we will tend to overstate the amount of capital gains inequality.

5.2 Top income shares

Figure 13 shows two series for the top 10% share of income. The first, the red '+' series, is the DINA baseline. It shows, first, a decline in the top 10% income share from 1946-1970 from 37% to 34%, and then a subsequently rise until a present share of 46%. The decline and subsequent rise in income shares is fairly smooth, and there is fairly little pro-cyclicality in top income shares.

The blue 'X' series shows the distribution of Haig-Simons income. For our baseline series, we rank individuals on Haig-Simons income, and compute shares of Haig-Simons income. There is a larger increase in the top 10% share

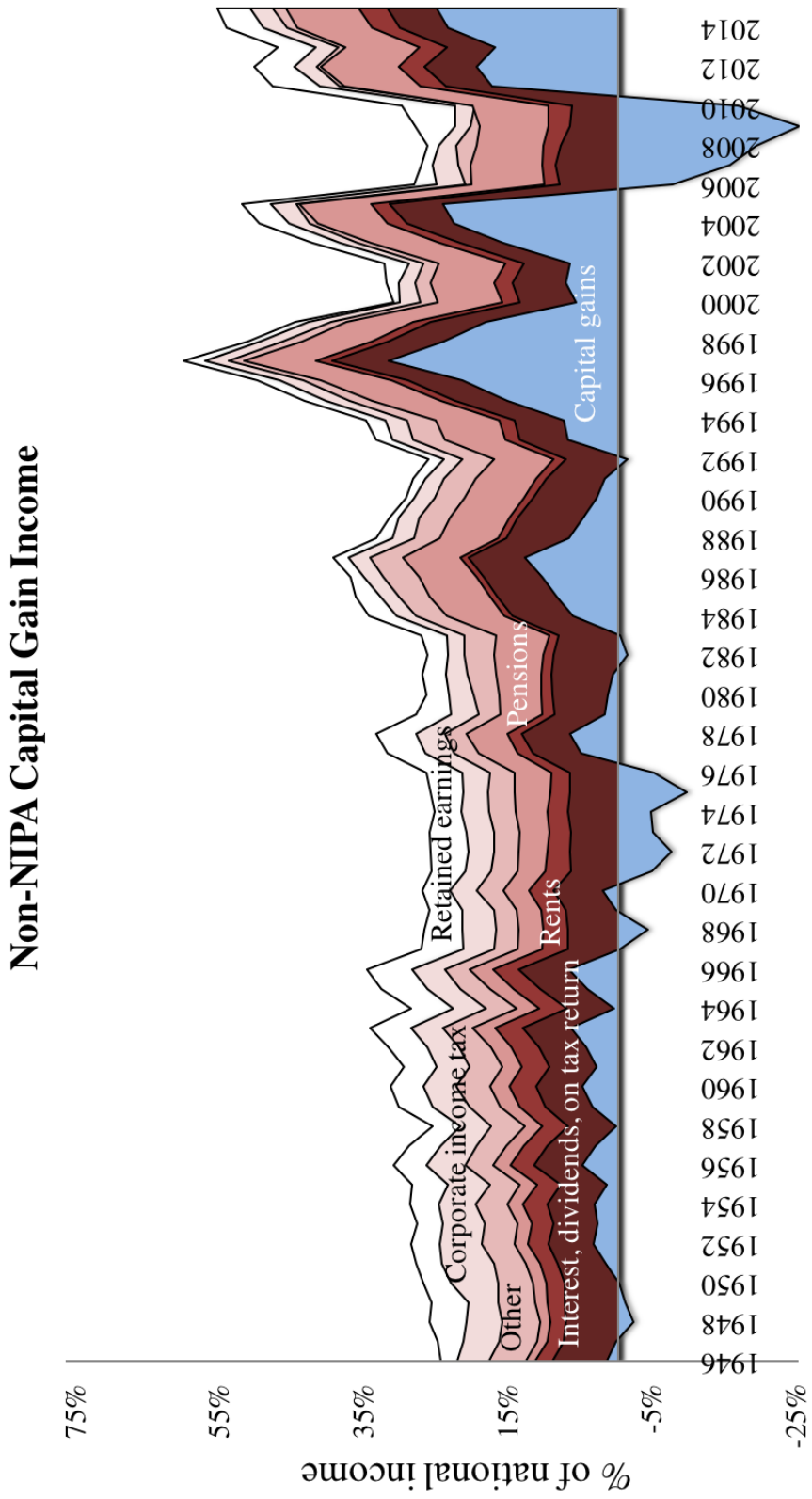

Figure 12: Capital income: taxable income, national income, Haig-Simons income. Red and white parts of the figure adopted from Piketty, Saez and Zucman (2016). The blue area is GNKGs. For the construction of GNKGs, see section 3.

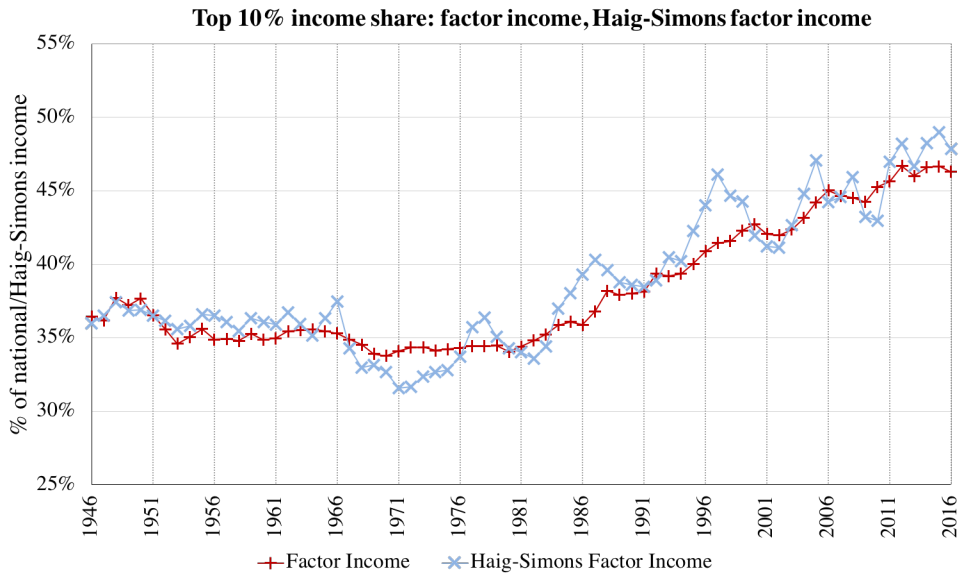

Figure 13: The top 10% share of income. Factor income series is from Piketty, Saez and Zucman (2016), and is the percentage of factor national income received by individuals in the top 10% of the income distribution. Haig-Simons factor income series is the percentage of Haig-Simons income received by individuals in the top 10% of the income distribution.

post-1970, from 31% of income to 48%. In addition, Haig-Simons top income shares are more pro-cyclical than national income. This is unsurprising, since as figure 3 shows, Haig-Simons income inherits some of the pro-cyclicality of stock and housing market prices. In periods of recession, the top 10% share drops precipitously. The overall picture is, however, that Haig-Simons income is even more unequally distributed than National Income, and there has been a larger increase over the time period.

Figure 14 shows a similar story for the top 1% share of income as for the top 10%. For national income, there is an increase from 11% in 1970 to 19% in 2016. For Haig-Simons the increase is larger, from 8% to 20%. In addition, the top 1% share of Haig-Simons income is much more pro-cyclical, dropping precipitously during the dot-com crash and the great recession.

5.3 Capital share of top income groups

Top income shares can be decomposed into a labor income share and a capital income share, just as total national income and Haig-Simons income was analyzed in section 4.2. For NIPA income, labor income consists of compensation of employees, and the labor component of mixed income. Capital income is the sum of corporate profits, income from owner and tenant occupied housing, and the capital component of non-corporate income. For Haig-Simons income, we add capital gains to the numerator and the denominator of the capital share.

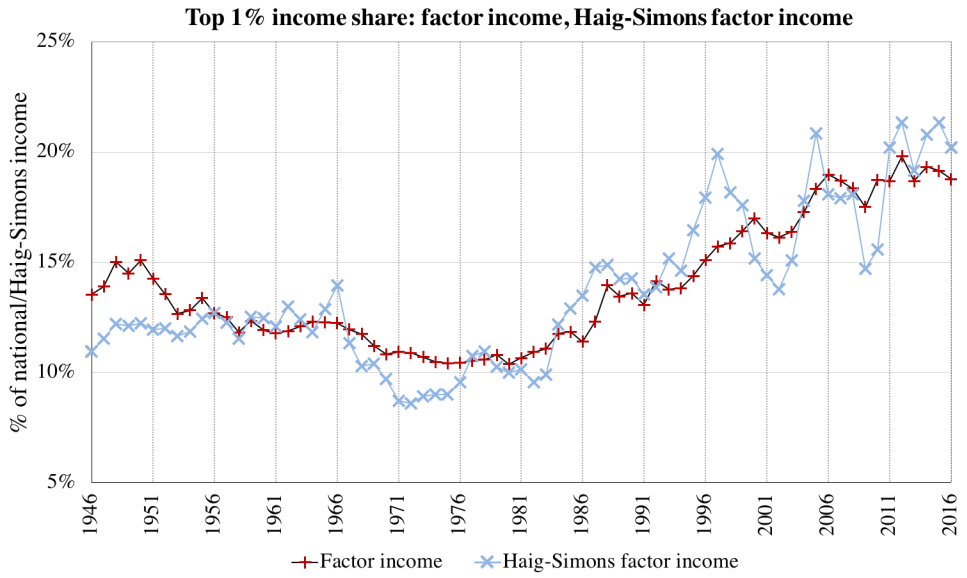

Figure 14: The top 1% share of income. Factor income series is from Piketty, Saez and Zucman (2016), and is the percentage of factor national income received by individuals in the top 1% of the income distribution. Haig-Simons factor income series is the percentage of Haig-Simons income received by individuals in the top 1% of the income distribution.

Figures 15 and 16 shows the capital share of the top 10% and top 1%, respectively, of the income distribution. The Haig-Simons capital share is depicted by blue 'X' series, while the national income capital share is the red '+' series. The red series show, in line with Piketty, Saez and Zucman (2016), that until 2001 the rise of top income shares was mostly a labor-income phenomena. After 2001 capital shares increased, and henceforth drove the large increase in income inequality.

The blue series shows the capital share for top income groups. Rather than a gradual decline in the capital share seen in the DINA series, there is a sharp decline in the late 1960s and early 1970s. In the 1980s and 1990s the capital share recovers. During the dot-com bust and great recession, the capital share dropped precipitously, as asset market prices crashed during these recessions.

6 Capital gains in a neoclassical model

The data analysis of sections 3 shows a large and sustained increase in capital gains. We now show that a standard neoclassical model, in which capital is the only asset, has trouble generating the magnitude of capital gains in the data. We introduce a few parsimonious modifications to the neoclassical model that allows the generation of capital gains of a magnitude commensurate with the empirical facts.

The key to modeling large and sustained capital gains is the existence of an

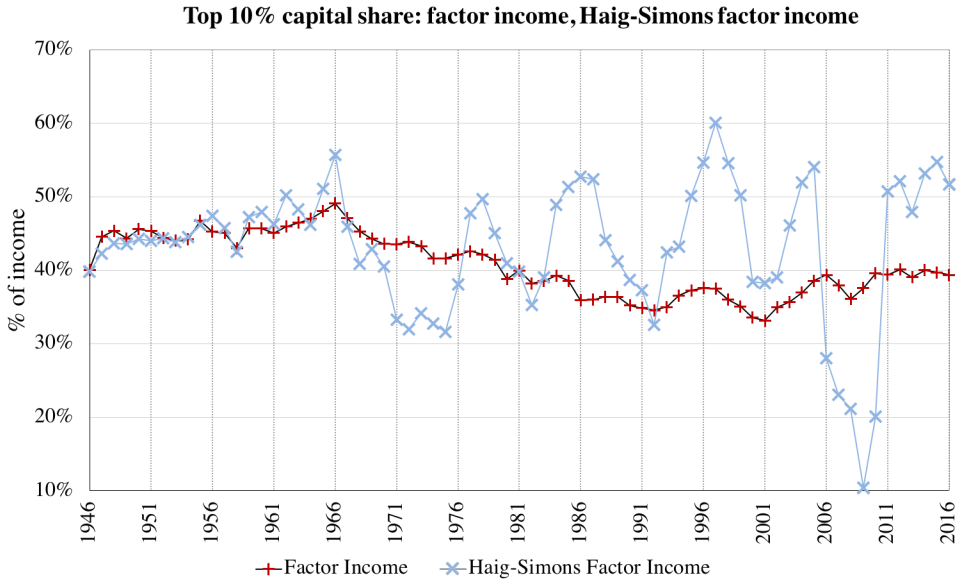

Figure 15: Capital share, top 10%. Factor capital income series is from Piketty, Saez and Zucman (2016), and equals the total factor capital income received by individuals in the top 10% of the income distribution divided by total factor income. Haig-Simons factor capital income series equals the total Haig-Simons capital income received by individuals in the top 10% of the income distribution divided by total Haig-Simons income.

asset which is *nonreproducible*. A reproducible asset has an anchor on its price, limiting capital gains.

Proposition 2. *Let c be the replacement cost of an asset, inclusive of any installation costs, such that c units of output can be converted to 1 unit of the asset. Let p be the price of the asset. If $p = c$, then $KG \leq \Delta c$. If $p < c$, then $KG \leq \Delta c + (c - p)$.*

Proposition 2 might be termed the 'iron law of capital gains'. If an asset's price is equal to its replacement cost, the cost will limit price appreciation. If an asset's price is below its replacement cost, the difference between price and cost limits the extent of capital gains. A non-reproducible asset will have an infinite replacement cost, leaving a large latitude for capital gains. A Leonardo da Vinci painting is non-reproducible; it's price has no anchor. Land is an intermediate case. While the price of land lies below the cost of land reclamation or development there is room for price appreciation. If and when the price of land rises to the point of equality, technological factors will limit capital gains.

In a standard neoclassical model, the 'iron law' precludes the existence of large capital gains. If capital is the only asset, capital gains can be generated by two channels: (1) a change in investment-specific technological progress, which we term the 'GHK' channel (see Greenwood, Hercowitz and Krusell (1997)) (2) a change in the price of installed capital relative to uninstalled capital, which we

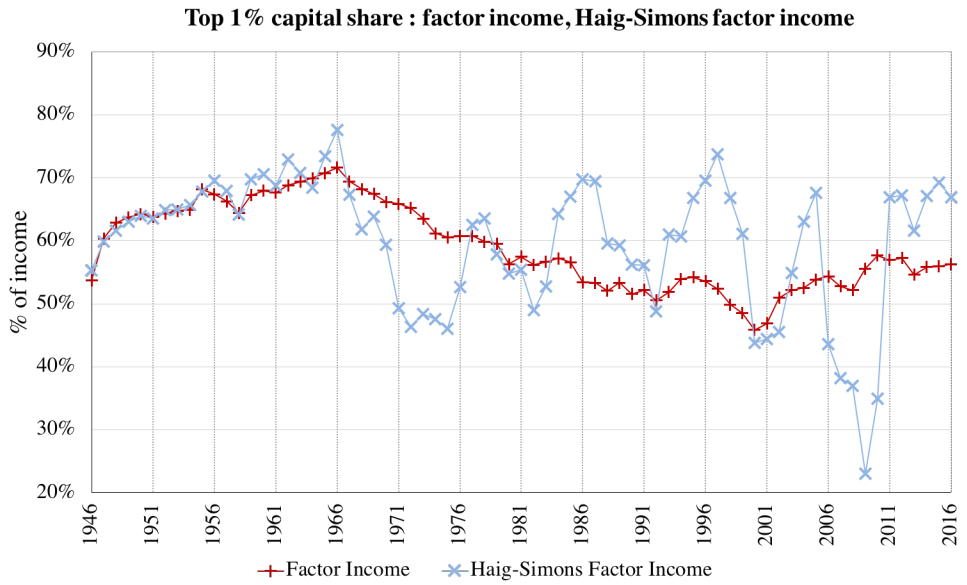

Figure 16: Capital share, top 1%. Factor capital income series is from Piketty, Saez and Zucman (2016), and equals the total factor capital income received by individuals in the top 1% of the income distribution divided by total factor income. Haig-Simons factor capital income series equals the total Haig-Simons capital income received by individuals in the top 1% of the income distribution divided by total Haig-Simons income.

call the 'Hayashi' channel (see Hayashi (1982)).[34] We can thus write the price of capital as the product of two terms, 'GHK q' and 'Hayashi q': $q_t = q_t^{GHK} \cdot q_t^h$. We discuss each in turn.

Figure 17 presents data on the change in relative price of uninstalled capital goods from the NIPAs, 'GHK q'. The relative price is calculated as a fraction, in which the numerator is the implicit price deflator for investment goods, and the denominator is the price deflator for GDP. Figure 17 shows that the relative price of capital has steadily declined from 1980-2017, an average of roughly 1% per year. Given that the capital-to-output ratio for this period has been around 200%, this implies capital *losses* of 2% of GDP per year.

To estimate Hayashi q, we follow Hall (2001) and assume a quadratic adjustment cost function $c(\cdot)$:

$$c\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) = \frac{\nu}{2}\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)^2. \tag{12}$$

Capital installation occurs up to the point where the marginal adjustment cost equals the difference between the price of installed capital, q_t^h, and the price of uninstalled capital:

[34] The story may change when looking at investment at the micro-level, or in a model with lumpy investment. See Khan and Thomas (2008).

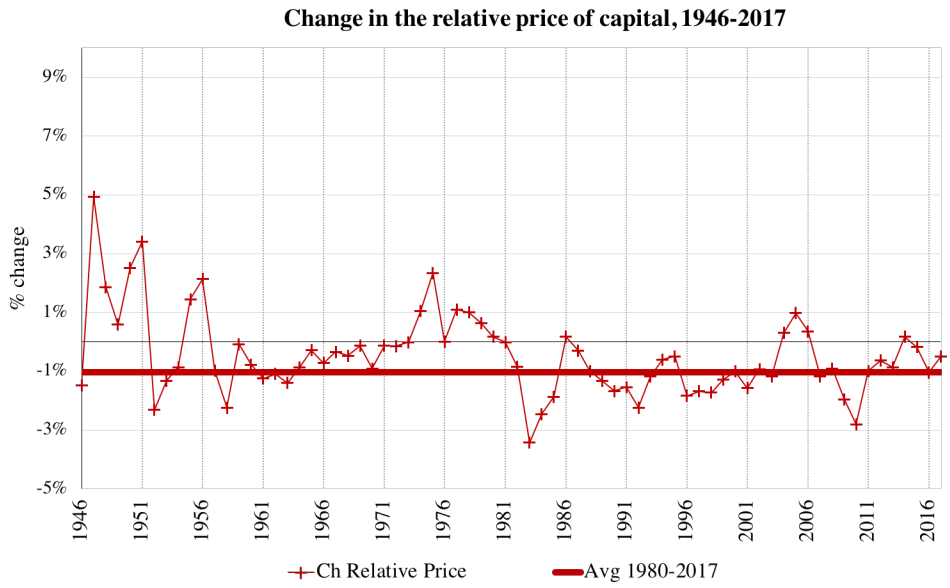

Figure 17: The change in relative price of capital, computed as a ratio. The numerator is the implicit price deflator for investment goods from the BEA, and the denominator is the implicit price deflator for GDP.

$$\nu\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) + 1 = q_t^h. \tag{13}$$

We calculate q_t^h from equation 13 using data on the real quantity of capital from the BEA, under different assumptions about the adjustment parameter, ν, also from Hall (2001). Figure 18 shows the results. Independently of the adjustment cost parameter, all series show capital losses from 1980 to the present.

To explain the capital gains seen in the data, we must therefore move away from a world in which reproducible capital is the only asset. We make a single deviation from the neoclassical model: the introduction of a nonreproducible asset class, termed a security S_t.[35]

But what will be the yield on the asset? In a world of perfect competition and constant returns to scale, factors are paid their marginal product and total output equals the sum of factor income. We therefore introduce an exogenous wedge between factor prices and their marginal products: the difference between output and these reduced factor payments will be the security's dividend.[36]

The starting point of the theory is an open economy neoclassical model:

1. Production is constant returns to scale in capital and labor:

$$Y_t = f(K_t, A_t L_t), \tag{14}$$

[35] In section 7, we present a full version of the simplified model presented in this section.
[36] In section 7, the wedge will be fully endogenized.

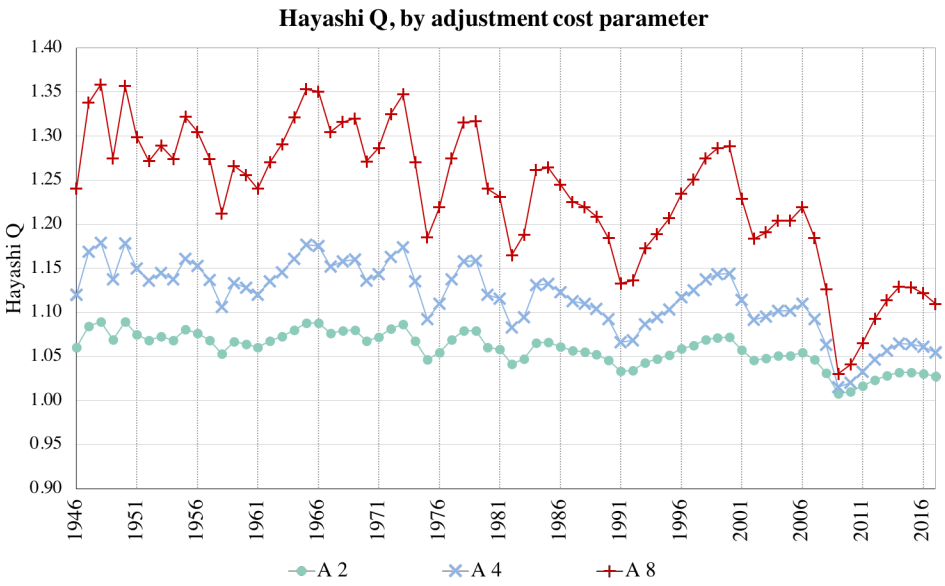

Figure 18: Hayashi q under different adjustment cost parameters. Computed using equation 13. The real value of the capital stock is from the BEA.

with $f(\cdot)$ concave and increasing. Denote the elasticity of output with respect to capital as $\epsilon_t^{Y,K} \equiv \frac{\partial \log(Y_t)}{\partial \log(K_t)}$, and the elasticity of capital utilized with respect to the wedge as $\epsilon_t^{K,\mu} \equiv \frac{\partial \log(K_t)}{\partial \log(\mu_t)}$. Productivity follows a random walk process with drift,

$$ln(A_{t+1}) = g + ln(A_t) + z_t^A, \tag{15}$$

where z_t^A is a shock to future productivity.

2. Agents have rational expectations, markets are complete, and there is no arbitrage.

3. 1 unit of capital K_t is produced by q_t units of output, and thus the price of capital is q_t. The series for q_t is exogenously specified.[37]

4. Labor supply is exogenous.

5. There is an open economy with world interest rate r_t, with all assets receiving the same return.[38] The interest rate follows the stochastic process

$$r_{t+1} = r_t + z_t^r. \tag{16}$$

To these we add the following elements:

[37] For simplicity, we abstract from capital adjustment costs and thus 'Hayashi q'.

[38] In section 7, the interest rate will be endogenized, and there will be heterogeneous interest rates across assets.

6. Factors of production capital and labor are paid their marginal products divided by a wedge μ_t:

$$\begin{aligned} w_t &= f_L/\mu_t \\ \rho_t = r_t + \delta &= f_k/\mu_t. \end{aligned} \tag{17}$$

The wedge follows the random walk process

$$ln(\mu_{t+1}) = ln(\mu_t) + z_t^{\mu}, \tag{18}$$

where z_t^{μ} is a shock to the wedge.

7. The wedge between factor prices and their marginal products means that factor income does not equal output. This difference between output and factor income we term the dividend. We assume the dividend is paid out to the owners of a financial asset, termed a *security* S_t. The aggregate dividend $D_t = Y_t - w_t L_t - \rho_t K_t = \frac{\mu_t - 1}{\mu_t} Y_t$ is divided equally amongst shares outstanding in the security, thus

$$d_t = \frac{D_t}{S_t} = \frac{\mu_t - 1}{\mu_t} Y_t / S_t. \tag{19}$$

We assume there is a single share outstanding, and thus $S_t = 1 \; \forall t$. Denote the elasticity of the aggregate dividend share of output, $\frac{\mu_t - 1}{\mu_t}$, with respect to the wedge, as $\epsilon_t^{DS,\mu} \equiv \frac{\partial \log(DS_t)}{\partial \log(\mu_t)}$.

Given assumption 2 and 5 (i.e., an interest rate of r_t, complete markets, and a no-arbitrage condition), the price of the security S_t is the present discounted value of the dividends it receives:

$$X_t = E_t \left[\sum_{s=t+1}^{\infty} d_s \frac{1}{\prod_{n=t+1}^{s} (1 + r_n)} \right]. \tag{20}$$

The model can be summarized in three equations (recapitulated below) in three endogenous variables: capital, output, and the price of securities.

$$r_t + \delta = f_k(K_t)/\mu_t \tag{21}$$

$$Y_t = f(K_t, A_t L_t) \tag{22}$$

$$X_t = E_t \left[\sum_{s=t+1}^{\infty} d_s(Y_t) \frac{1}{\prod_{n=t+1}^{s} (1 + r_n)} \right]. \tag{23}$$

Given the exogenous processes of interest rates, the rate of productivity growth, the price of capital q, and the wedge μ, equation 21 determines the capital stock.

Given the path for the capital stock, the path for output is determined by the production function (equation 22), since labor is exogenous. The path for output determines dividends per share (equation 19), given our assumption there is a single share outstanding. The level of dividends determines the price of the security (equation 23). The price of security determines the capital gains of the model, through definition 2.[39]

We now characterize different paths that can generate capital gains.

Proposition 3.

1. $\frac{\partial KG_t^S}{\partial z_t^A} > 0$. *An increase in future productivity will increase capital gains for securities.*

2. $\frac{\partial KG_t^S}{\partial z_t^r} < 0$. *A decline in interest rates will lead to an increase in capital gains for securities.*

3. *If* $\mu_t < \frac{-1}{\epsilon_t^{Y,K}\epsilon_t^{K,\mu}} + 1$, $\frac{\partial KG_t^S}{\partial z_t^\mu} > 0$. *If an increase in the wedge increases dividends, then an increase in the wedge will lead to an increase in capital gains for securities.*

Proof. See appendix G.

□

We leave the proof to the appendix, and discuss the intuition here. Figure 19, panel (a), shows the effect of an increase in future productivity. Future productivity has no effect on the current value of the capital stock, which is tied down by q_t. When productivity does increase, this will lead to a higher capital stock, and an increase in output. This will increase future dividends, which are reflected in the present price of securities. There will thus be a jump in the price of securities, generating a capital gain.

Figure 19, panel (b), shows the effect of an increase in the wedge μ. An increase in the wedge will lead to an increase in the share of output that goes towards security owners, which would tend to increase dividends. On the other hand, a higher wedge will also lead to a decline in the capital stock, which will tend to lower the path of output, capital, and dividends. As long as wedges as small enough, the increase in output share towards dividends is the more powerful force, however, leading to a net increase in future dividends and thus an increase in the price of securities, generating a capital gain.

We conclude our discussion with a final important source of capital gains: unmeasured intangible investment. Previous research has identified intangible investment as an important source of stock market gains.[40] In section 3, we measured capital gains by subtracting measured savings from changes in the

[39] Given the above discussion on 'GHQ' and 'Hayashi' q, we will limit our discussion of capital gains arising from changes in the price of capital below.

[40] See, for example, McGrattan and Prescott (2010) and Hall (2001).

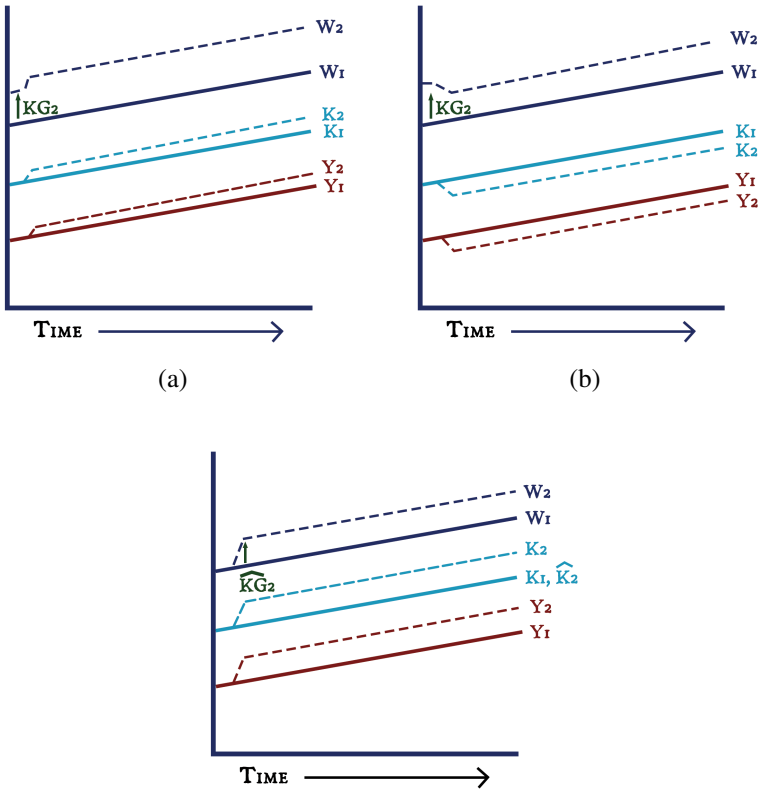

Figure 19: Pathways to capital gains. W is financial wealth, K the capital stock, and Y output. (a) effect of change in future productivity and/or decline in interest rates (b) effect of an increase in μ_t (c) effect of an increase in I_t^u.

market value of wealth. But if there is unmeasured investment, there will be unmeasured savings, and our estimated capital gains will be biased.

Intangible investment is spending by firms on activities that will increase future output, but that accounting agencies and statistical authorities recognize as an expense rather than an investment. For example, in the past firm expenditures on software was not recognized as investment, however recent revisions to the NIPAs has rectified this, reclassifying software expenditures as an investment. A growing body of research shows there are still large categories of spending that increase the future value of a firm, but are not currently classified as investment.[41] If this spending were properly classified, the measured profits and retained earnings of firms would increase.

To explore the effect of unmeasured investment on capital gains, we must extend our framework slightly to include measurement. In what follows, variables with a hat signify observed quantities. We make the following assumptions:

[41] See, for example, Corrado, Hulten and Sichel (2005) and Corrado, Hulten and Sichel (2009).

1. Capital gains are measured as in equation 9: $\widehat{KG_t} = W_t - W_{t-1} - \widehat{s_t^{private}}$.

2. True investment I_t is only observed with error. The relationship between observed and unobserved investment is described by a measurement error equation, $\widehat{I_t} = I_t - I_t^u$, where I_t^u is unobserved investment.

3. The replacement value of the capital stock is not observed, but must be estimated through the perpetual inventory method, $\widehat{K_{t+1}} = (1-\delta)\widehat{K_t} + \widehat{I_t}$. The estimated value of the capital stock inherits the measurement error from investment. The true level of the capital stock, K_t, is unobserved. Defining unobserved capital as K_t^u, we have $K_t = \widehat{K_t} + K_t^u$.

4. The statistical authority of the model computes gross domestic product (GDP) as expenditures on consumption and investment goods: $\widehat{Y_t} = C_t + \widehat{I_t}$. Gross national income is measured as the sum of payments to labor, $w_t L_t$, net operating surplus, $\widehat{Y_t} - w_t L_t - \delta\widehat{K_t}$, and consumption of fixed capital, $\delta\widehat{K_t}$.

5. The statistical authority measures savings as measured income minus consumption, and thus the measurement error in investment is also inherited in national savings and net national savings (savings net of depreciation):

$$\widehat{s_t} = \widehat{Y_t} - C_t = s_t - I_t^u \tag{24}$$

$$\widehat{s_t^{net}} = \widehat{Y_t} - C_t - \delta\widehat{K_t} = s_t^{net} - I_t^u + \delta K_t^u. \tag{25}$$

6. Financial market professionals observe investment without error, and a dollar of unmeasured intangible investment increases the market value of wealth by a dollar. Total wealth is therefore measured without error: $\widehat{W_t} = W_t$.

Since investment, and thus savings, is measured with error, capital gains will be measured with error as well.

Proposition 4. *If there is unmeasured investment, estimated capital gains will reflect both actual capital gains and this unmeasured investment:* $\widehat{KG_t} = KG_t + I_t^u$.

Figure shows 19, panel (c), shows the effect of an increase in unmeasured investment. Although there is an increase in capital, measured capital does not change. But there is an increase in measured wealth, which is interpreted as a capital gain.

Finally, we note there can be non-zero capital gains in the "steady state" of a balanced growth environment.

Proposition 5. *Let the economy be on a balanced growth path, with output growth rate* g*. If* $\bar{\mu} > 0$*, then* $KG_t^S > 0$*.*

Proof. If markups are greater than one ($\bar{\mu} > 1$), then there is a positive security price ($X_t > 0$). On a constant growth path, the value of final goods firms grows at the same rate of dividends (see equation 20), namely the rate of output growth g. The growth in security will generate capital gains: $KG_t^S = S_t(X_t - X_{t-1}) = g \cdot S_t X_{t-1}$. For example, if the value of securities is 200% of GDP and the economy is growing at 2% per year, there are capital gains of 4% of GDP per year. □

Proposition 6. *Let the economy be on a balanced growth path, with output growth rate g. Denote by $\mathscr{U} = I_t^u / I_t$ the fraction of investment which is unmeasured on the balanced growth path. If $\mathscr{U} > 0$, then $\widehat{KG}_t > 0$*

7 A model of capital gains and inequality

In section 6, we explained capital gains through an additional asset class, a security S_t, which receives exogenously specified dividends. We now endogenize the additional asset class, and move from a partial equilibrium concept to the general equilibrium.

The few parsimonious changes we make to the neoclassical model open the door to a host of novel results. Every shock in the model now has the ability to affect not only output, but also capital gains and the present value of wealth. We will use our model to theoretically and quantitatively study the accumulation of wealth through traditional savings as well as Haig-Simons savings inclusive of capital gains. Finally, we will examine the impact of capital gains on the distribution of income. Our results show that capital gains, the observed phenomena in the data, can be the product of a number of disparate channels, from mismeasurement to an increase in market power.

The model's starting point is the setup presented in section 6, which contained several simplifications. We extend the model to the minimal set of features that allows a quantitatively study capital gains and inequality in a general equilibrium framework. In particular, we make the following modifications:

1. There is imperfect competition, and the rights to the pure profits of firms are sold as securities.[42]
2. There is long-run productivity risk and convex investment adjustment costs.
3. The interest rate is determined through the loanable funds market.
4. There are two types of agents: capitalists and workers. Capitalists will be optimizing agents, and save more than workers. Workers will consume and save via a 'rule of thumb' decision process.

[42] These are the 'wedges' of section 6.

Modification 1 is necessary in order to generate sustained capital gains. As discussed in section 6, in order to generate capital gains a non-reproducible asset is needed. In section 6, this asset was a security that received dividends from an exogenous wedge. We now endogenize this wedge. We introduce a simple form of imperfect competition, through which firms will make pure profits through a markup of price over marginal cost.

The level of capital gains in the economy is determined by the price of securities, which in turn is determined by the discount rate on future profits. This discount rate is the sum of the risk-free interest rate and the risk premium on securities. In order to quantitatively match the equity risk premium in the data, we introduce a minimal set of features: Epstein-Zinn preferences, convex adjustment costs, and long-run productivity risk as in Bansal and Yaron (2004).[43]

In order to study inequality, we introduce a simple form of heterogeneity, with two types of agents. This modeling choice will allow us to match a key moment in the data, the joint distribution of income and wealth for the top 10% versus the bottom 90%. The 'capitalist' type of agent will represent the upper percentiles of the income distribution. They will have higher labor productivity, save more (in line with the data), and consequently receive a greater fraction of their income through capital income. 'Workers' have lower productivity and a lower savings rate, but still have positive wealth. In order to include this heterogeneity of savings rates in a tractable way, we assume the 'worker' follows a rule of thumb type of consumption plan.

7.1 Profits, rents, and the stock market

The key to introducing capital gains is the existence of a non-reproducible asset. In our model, these assets will be firms that have a monopoly on their industry. There is no free entry in the short run, and thus firms are non-reproducible, allowing their prices to rise if profits increase.

There is a unit mass of monopolistically competitive final goods firms that produce output by differentiating an intermediate good. The individual final goods are combined in a composite final good which is the CES aggregate of these differentiated final goods, which are indexed by i:

$$Y_t = \left[\int_0^1 y_t^f(i)^{\frac{\Lambda_t - 1}{\Lambda_t}} di \right]^{\frac{\Lambda_t}{\Lambda_t - 1}}.$$

Final goods firms produce output using intermediate goods according to a linear production function, $y_t^f = y_t^m$. We derive in the appendix that a firm's optimal price is a time-varying markup μ_t over marginal cost: $\mu_t = \frac{\Lambda_t}{\Lambda_t - 1}$.

The key determinant of market power is the CES elasticity Λ_t, which determines the level of markups. We assume that $\frac{\Lambda_t}{\Lambda_t - 1}$, i.e. markups μ_t, follows an AR(1) process given by

[43] In this, we follow Croce (2014).

$$ln(\mu_t) = (1 - \rho_\mu)ln(\bar{\mu}) + \rho_\mu ln(\mu_{t-1}) + \epsilon_t^\mu \tag{26}$$

In this expression, $\bar{\mu}$ is the long run level of markups in the economy and ϵ_t^μ is a temporary shock to the level of markups.

There are strong barriers to entry for final goods firms, which ensures entering firms do not compete away profits. Since entry is fixed, final goods firms are non-reproducible in the short run.[44]

The barriers to entry allow the existence of non-zero profits in the economy. Final goods firms make aggregate profits Π_t equal to $\Pi_t = \frac{\mu_t - 1}{\mu_t} Y_t$. The profit share of income in the economy is given by $PS_t = \frac{\mu_t - 1}{\mu_t}$.

Pure profits flow to two distinct group: workers, and shareholders of the final good firms. We assume workers have some level of bargaining power which yields them a share bp_t of aggregate profits. Worker bargaining power follows the following stochastic process:

$$ln(bp_t) = (1 - \rho_{bp})ln(\bar{bp}) + \rho_{bp} ln(bp_{t-1}) + \epsilon_t^{bp} \tag{27}$$

The long-run level of bargaining power is given by $\bar{bp}$, and ϵ_t^{bp} are temporary shocks to bargaining power.

The shareholders of final goods firms receive the residual profits as dividends. Aggregate dividends distributed to shareholders at time t are given by $d_t^f = (1 - bp_t)\Pi_t$. All firms in our economy make identical profits, and thus each firm receives an equal share of aggregate dividends.

The rights to the pure profits of final goods firms are traded on security markets. At the end of period $t - 1$, securities S_t^f are traded for each firm, which give the rights to all future dividends d_t^f of these firms for as long as they survive. Individuals do not buy shares of individual final goods firms, which are infinitesimally small. They instead buy positive fractions of the continuum of firms. Since the continuum spans from 0 to 1, every period there is a single share of securities S_t^f outstanding.

The value of securities S_t^f is given by the present discounted value of the dividends the shares receive:

$$X_{t-1}^f = E_{t-1}\left[\sum_{s=t}^{\infty} m_s d_s^f \prod_{n=t+1}^{s} (1 - \Delta_n)\right]. \tag{28}$$

There are two discount factors in this equation. The first, m_t, is the stochastic discount factor that is determined in equilibrium by the optimal asset choice of households.

The second discount factor, Δ_t, is the probability that a final goods firm goes out of business during year t. There is exogenous firm entry in our model as in

[44] As will be seen, the barriers to entry are not permanent, and all firms will eventually go out of business and be replaced by new entrants.

Melitz (2003). Firm exit ensures that despite the significant barriers to entry in our model, all firms eventually go out of business. The bankruptcy rate is a key determinate of asset prices, and including firm exit will allow us to better match the level of financial wealth in the economy. The information on whether a firm goes out of business is revealed at the end of period $t-1$, before asset decisions are made for the next period.

Entry in our model is exogenous. Each period a mass Δ_t of new firms enters, replacing exiting firms. New "IPO" securities are issued at the end of time $t-1$ that give the rights to these firms' profits.

7.2 Intermediate goods firms

Final goods are produced using intermediate goods. A representative intermediate goods firm uses labor L_t and capital K_t to produce output Y_t^m according to the production function

$$Y_t^m = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{29}$$

where σ is the production elasticity of substitution and A_t the level of labor augmenting productivity. Capital K_t includes both tangible and intangible capital that contributes to the production of goods and services.

In order to help match the equity premium, we introduce investment adjustment costs. Investment increases the firm's future stock of capital according to

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \tag{30}$$

where δ is the rate of depreciation and adjustment costs $\Phi(\cdot)$ are a positive concave function. Following Jermann (1998), we use an adjustment cost function given by $\Phi(I_t/K_t) = \frac{a_1}{1-\xi}\left(\frac{I_t}{K_t}\right)^{1-\xi} + a_2$.[45]

7.3 Long Run Risk

Capital gains depend on the value of securities S_t^f, which in turn depends upon the rate at which pure profits are discounted, and thus the equity premium. In order to match the equity premium in the data, we follow the macro-finance literature and include long-run productivity risk in our model, as in Bansal and Yaron (2004) and Croce (2014). There are two sources of uncertainty in productivity growth: an i.i.d short-run shock that is standard in RBC models (ϵ_a), and a long-run component (ϵ_x) that leads to small but persistent movement in long-run growth. Let A_t denote the level of labor augmenting productivity, and lowercase

[45] For a full discussion of the adjustment cost function, see appendix F.5.

letters denote log-units. The growth rate of productivity is given by:

$$\Delta a_{t+1} = \zeta + x_t + \sigma_a \epsilon_{a,t+1} \tag{31}$$

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t} \tag{32}$$

$$\begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} \sim iid\ N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right) \tag{33}$$

Here x_t is the "long run risk" of productivity growth, and $\epsilon_{a,t+1}$ is the short run risk.

7.4 Heterogeneity

To study the effect of capital gains on inequality, we include a simple form of heterogeneity. There are two types of agents in the economy: capitalists, and workers. The capitalist is a "Ramsey" type of agent, that maximizes lifetime utility. The worker is a rule of thumb type of agent, or "Solow" agent, that saves an exogenous fraction of utility. A unit mass of agents are capitalists, and a mass Υ are workers. We will index capitalist variables with a 'c' subscript, and worker variables with a 'w' subscript.

Capitalists maximize utility subject to a series of budget constraints. Preferences are of the Epstein and Zin (1989) variety, which will allow us to more easily match the equity premium. Utility is given by

$$V_t = \left[(1 - \beta D_t) \left(c_{c,t}^{\nu} (A_{t-1}(1 - L_{c,t}))^{1-\nu} \right)^{\frac{1-\gamma}{\theta}} + \beta D_t \left(E_t V_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{34}$$

where the time discount factor is β, ν is a weight determining the average share of total hours worked, γ is the risk aversion parameter, and θ is a parameter defined as $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. In this expression, ψ is the elasticity of intertemporal substitution.[46] The preference shock D_t follows the stochastic process

$$ln(D_t) = \rho_D ln(D_{t-1}) + \epsilon_t^D \tag{35}$$

D_t is a lever we will use in our quantitative exercises to simulate a decrease in the natural rate of interest.

Individuals maximize this utility subject to a series of budget constraints,

$$\begin{aligned} c_{c,t} + q_t K_{c,t+1} + X_t^f S_{c,t+1}^f = w_t L_{c,t} + \Gamma_t cb_t + \rho_t K_{c,t} + d_t^f S_{c,t}^f \\ + \Delta_t X_t^f + (1 - \Delta_t) X_t^f S_{c,t}^f + (1 - \delta) K_{c,t}. \end{aligned} \tag{36}$$

[46] The main advantage of using Epstein-Zin utility is that there is no longer a link between the intertemporal elasticity of substitution and the coefficient of risk aversion, which makes it easier to match the equity-risk premium. If $\gamma = 1/\psi$, the utility collapses to the CRRA variety. As in Croce (2014), leisure utility is scaled by productivity.

On the left hand side of the budget constraint, individuals use their income to purchase either consumption, capital goods K_{t+1}, or shares of final goods firms S^f_{t+1}. On the right hand side of the budget equation, agents receive income from a variety of sources: labor income $w_t L_t$, the return on capital $\rho_t K_t$, dividends from final goods firms $d^f_t S^f_t$, IPO issued securities of final goods firms $\Delta_{t+1} X^f_t$, remaining share value of final goods firms $(1-\Delta_t) X^f_t S^f_t$, and remaining capital, $(1-\delta)K_t$. The Γ_t term is the fraction of the rents from collective bargaining that capitalists receive. We assume the rents from collective bargaining are split amongst capitalists and workers according to the fraction of labor income received.

The worker class lives according to a rule of thumb, and saves an exogenous fraction s^w_t of income $Y_{w,t}$, where income is defined as

$$Y_{w,t} = w_t \Omega_t L_{w,t} + (\rho - \delta) K_{w,t} + d^f_t S^f_{w,t} + S^f_{w,t}(X^f_t - X^f_{t-1}) + (1-\Gamma_t) c b_t. \quad (37)$$

Workers receive income from wages, bargaining rents, returns from capital, capital gains, and dividends from final goods firms. We assume the labor supply of workers is the same as that of capitalists, and thus $L_{w,t} = L_{c,t}\ \forall t$. Workers have lower productivity than capitalists. The relative productivity of workers is denoted Ω_t.

Workers use their savings to purchase final goods securities and intermediate goods securities. We assume they spend an exogenous fraction $\mathscr{P}$ of savings on capital goods, and a corresponding fraction $(1-\mathscr{P})$ on final goods securities. The law of motion for assets is given by

$$K_{w,t+1} = (1-\delta) K_{w,t} + s^w_t \mathscr{P} Y_{w,t} \frac{1}{q_t} \quad (38)$$

$$S^f_{w,t+1} = (1-\Delta) S^f_{w,t} + s^w_t (1-\mathscr{P}) Y_{w,t} \frac{1}{X^f_t}. \quad (39)$$

7.5 Aggregation and equilibrium

Aggregate capital, labor, and consumption are the sum of worker and capitalist totals:

$$c_t = c_{c,t} + \Upsilon c_{w,t} \quad (40)$$

$$K_t = K_{c,t} + \Upsilon K_{w,t} \quad (41)$$

$$S^f_t = S^f_{c,t} + \Upsilon S^f_{w,t} \quad (42)$$

$$L_t = L_{c,t} + \Upsilon \Omega_t L_{w,t}. \quad (43)$$

In our equilibrium, we make the assumption that the marginal buyer of securities is the 'capitalist'. This assumption assures that the security (via equation 28) is priced using the capitalist's stochastic discount factor.

An equilibrium is a set of quantities

$\{c_{c,t}, c_{w,t}, L_{c,t}, L_{w,t}, K_{c,t}, K_{w,t}, d_t^f, cb_t, S_{c,t}^f, S_{w,t}^f\}_{t=0}^{\infty}$, a set of prices $\{w_t, X_t^f, q_t, m_t\}_{t=0}^{\infty}$, and a set of exogenous processes such that (i) capitalists maximize utility subject to their budget constraint (ii) workers follow their law of motion (iii) firms maximize profits, and (iv) markets clear. For the full description of the equilibrium, see appendix F.

The assumption that the capitalist is the marginal purchaser of securities means that in a steady state or balanced growth path, the capitalist is on her consumption Euler equation, and thus the steady state interest rate will be a function of her time preference.[47] This in turn implies that the steady state interest rate and capital to labor ratio is only a function of capitalist variables.

We can summarize the model as follows. In a steady state, aggregate variables are largely the same as they would be in a purely representative agent model, with the exception of the aggregate labor supply. The distribution of assets between the two classes depends upon three factors: (i) the relative productivity of workers Ω (ii) the overall savings rate of workers, s^w (iii) the portfolio choice of workers, $\mathscr{P}$.

7.6 Capital gains and distribution

The existence of capital gains opens up a new channel to influence the distribution of income, through the Haig-Simons capital share. We will use our model to describe how different shocks flow through to the distribution of income.

The statistical authority of the model world computes the labor share using the same method as the BLS. Labor receives income from both wages and from their bargaining power.[48] The capital share is measured in the data as the complement of the labor share.

$$\widehat{LS_t} = \frac{w_t L_t + cb_t}{\widehat{Y}_t}, \quad \widehat{KS_t} = \frac{\widehat{Y}_t - w_t L_t - cb_t}{\widehat{Y}_t}. \tag{44}$$

Note that 'hats' on the variables, which denote measurement. The "true" labor and capital shares, purged of measurement error from investment, are defined in the same way, except without the hats.

The Haig-Simons labor share is defined as labor income divided by Haig-Simons income. The SA's estimate of KS_t^{HS} takes the form

$$\widehat{KS_t^{HS}} = \frac{\widehat{Y}_t - w_t L_t - cb_t + \widehat{KG_t}}{\widehat{HS_t}}. \tag{45}$$

For each of these income shares, we can compute corresponding distributional income shares: the share of income received by the different classes, capi-

[47] See appendix section F.11 for details.

[48] Although wages and the rents from labor bargaining power may not be observed separately, we assume total payments to workers, $w_t L_t + bp_t \Pi_t$, are observed.

Table 1: Parameters taken from the data and related literature

Panel A: Data	*Symbol*	*Value*	*Source*
Productivity growth (/yr)	ζ	2.02%	Fernald (2012)
Panel B: Related literature			
Long run risk persistence	ρ	.98	Croce (2014)
Long run risk std. dev.	σ_x	.0010	Croce (2014)
Short run risk std. dev.	σ_a	.01	Croce (2014)
Depreciation rate (/yr)	δ	6%	Jorgensen (1996)
Adjustment costs	ξ	.12	Croce (2014)

talists and workers. For example, the labor share of the worker class is computed as

$$\widehat{LS_t^w} = \frac{w_t L_{t,w} \Omega_t \Upsilon + (1 - \Gamma_t) cb_t}{\widehat{Y}_t}. \tag{46}$$

In our simulations, we will examine the effect of the different capital gains channels on the distributional national accounts of our model.

7.7 Calibration

We calibrate our model to the US economy. There are three main sets of parameters. The first are those that we take from the existing literature, displayed in table 1. We take productivity growth ζ from Fernald (2012), and use Croce (2014)'s process for long-run risk. The depreciation rate is from Jorgenson (1996). For our baseline results, we assume workers have no bargaining power and there is no unmeasured investment.

The second set of parameters are those we choose to match the joint distribution of income and wealth inequality. We interpret the 'capitalist' class to be the top 10% of the income distribution, and thus we set Υ to match this. We aim to match the joint distribution of income and wealth of the top 10% of income compared to the bottom 90%. From the distributional accounts, the bottom 90% receives 61% of labor income, and we set $\Omega = .17$ to match this.[49] The top 10% have 51% of the wealth, and we set the savings rate of the workers to match this $s^w = .066$. As discussed in section 7.5, given the relative productivity of the working class, the relative wealth holdings of workers is determined by their savings rate.[50] Finally, we need a parameter to determine the relative holdings of final goods and intermediate goods firms for the worker class, $\mathscr{P}$. In this choice we are guided by the desire to reflect the empirical fact that wealthier individuals tend to hold riskier assets. We will see in our results that final goods firms are riskier than intermediate goods firms. In our model we will thus interpret

[49] Labor share of workers in our model is $\frac{\Upsilon\Omega}{\Upsilon\Omega+1} = .61$. Solving yields $\Omega = .17$.

[50] See also appendix equation A.134.

Table 2: Calibrated parameter results

Parameters chosen to match targets	*Symbol*	*Value*
Capital production elasticity	α	0.35
Production elasticity	σ	0.85
Firm exit rate	Δ	0.0058
Rate of time preference	β	0.9963
Risk Aversion	γ	6.32
Hours supplied	ν	0.21
AR(1) persistence	ρ_μ	0.97
AR(1) error variance	σ_μ	.005

these final goods as equities, and we set $\mathscr{P}$ to match the relative asset holdings of equities between the two classes in the data.

We choose the remaining parameters to match six key data moments from the US economy for 1970. We choose the year 1970 to precede the large increase in capital gains in the early 1980s, as well as the large fluctuations in the real interest rate during the great inflation and Volcker deflation. The six moments are the real interest rate, the labor share, the investment-to-output ratio, the share of hours worked, the equity premium, and the wealth-to-income ratio. The parameters chosen this way are the rate of time preference β, the capital production coefficient α, the production elasticity of substitution σ, the firm exit rate Δ, the labor supply coefficient ν, and the risk aversion parameter γ.

To calibrate the model, we minimize an objective function which is the weighted sum of the squared differences between the data moments and our model moments. The results are displayed in table 2, and appendix table A.1 compares the resulting model moments with the data moments. Overall, our calibration procedure produces a close fit between model and data moments.

7.8 Simulating capital gains

Figure 20 shows the impact of a one-standard deviation temporary shock to markups ϵ_t^μ, an increase of about .5 percentage points. The temporary increase in markups boosts the value of final goods firms X_t^f by around .6%, which leads to a capital gain of .2% of national income. While Haig-Simons income increases, national income decreases due to the distortion of markups. There is a small increase in the capital share of income of .35 p.p., and a large increase in the Haig-Simons capital share of 2 p.p.. Finally, there is a decline in traditional measure of savings, but an increase in comprehensive savings inclusive of capital gains. Because this is a temporary shock, markups eventually return to their long run level. As markups decline so do profits, and thus there is a decline in the value of final goods firms and corresponding capital losses.

Appendix figure A.2 shows the IRF of a temporary shock to labor bargaining power. An increase in bargaining power of 1 p.p. leads to a .3% decline in the

Figure 20: IRF of one standard deviation shock to markups em.

price of final goods firms, and a capital loss of .2 percent of GDP. The fall in security prices leads to a fall in the Haig-Simons capital share, as well as the Haig-Simons savings rate.

Appendix figure A.3 shows the impulse response of a temporary increase in unmeasured investment. A change in measurement does not affect any 'true values', and thus true national income, capital share, and capital gains do not change. However, the measured quantities do change. Lower measured investment leads to lower measured income and savings. Under our assumptions about market valuation, investors value the contributions of intangibles, and thus the market value of assets is unchanged despite the lower savings rate. This generates a measured capital gain.

7.9 Decomposing capital gains

We combine our model with estimates from the data to understand the relative contributions of four different channels towards generating capital gains: changes in productivity growth, changes in the discount factor, changes in monopoly profits, and changes in unmeasured intangible investment. The analysis of section 2 shows that a rise in asset prices has increased the wealth to income ratio from 2.66 in 1970 to 3.61 in 2015. We use our model to estimate the contribution three of the four factors towards generating this increase in wealth to income. The fourth factor, unmeasured intangible, we will estimate by capitalizing investment flows through the perpetual inventory method.

We perform the decomposition through a comparative statics analysis. Start-

Table 3: Decomposition of capital gains: 1970-2015

Forcing variable	Δ in W/Y	% of total Δ
Total changes	1.01	100%
Productivity growth	.13	13%
Discount rate	.254	25%
Intangible investment	.083	8%
Monopoly profits	.53	52%

ing from the initial 1970 steady state, we plug into the model one-by-one changes in productivity growth, changes in the discount rate, changes in markups, and calculate their effect on the wealth to income ratio.

All of our data series are taken as a five year moving average around the two steady states, 1970 and 2015. Data on markups is taken from Eggertsson, Robbins and Wold (2018), who estimate an increase from 11% in 1970 to 23% in 2015. Data on productivity growth is taken from Fernald (2012), who find a decrease in the rate of productivity growth rate (the parameter ζ), from 2.02% per year in 1970 to a level of .65% in the present. We estimate the increase in the discount factor D such that the combined effect of the changes in μ, ζ, and D lead to a decrease in interest rates over the time period of 2%, which is in line with the data.[51]

Table 3 decomposes the contribution of each of these factors to the decline in interest rates. We change each parameter from its steady-state value in 1970 to its steady-state value in 2015, holding all other parameters constant. We then examine the effect of this change on the wealth-to-income ratio. For example, changing markups from their level of 11% in 1970 to their level in 2015 of 23% results in an increase in the wealth to income ratio of $.53$. The table shows a decomposition of the relative importance of all the other factors. The increase in markups plays the largest role in the increase of wealth, contributing to 53% of the increase.

The final component of the decomposition is the contribution of unmeasured investment. To compute this component, we will use estimates of aggregate business spending that the BEA does not consider investment, but may plausibly contribute to future production, and thus the valuation of firms. We compute stocks of unmeasured intangible spending using depreciation rates from Corrado et al. (2012) and capital price indices from the BEA.

The spending categories we capitalize are taken from the literature on intangible investment. Corrado, Hulten and Sichel (2005) classify intangible capital into three categories: (i) computerized information, which includes software and cloud computing (ii) innovative property, which includes investment in R&D and

[51]This is a conservative estimate of the decline. The literature estimates a range of a 1 to 3.5 percentage point decline. Holston, Laubach and Williams (2017) find a fall $\approx$ 3.5 percentage points, from 3.91 in 1970 to .43% in 2015. Del Negro et al. (2017) estimate a decline of 1-1.5 percentage points, from 2-2.5% to 1-1.5%.

artistic originals, and (iii) economic competencies, which consists of branding, worker training, and advertising. Most of categories (i) and (ii) are already categorized as investment by the BEA, and thus contribute to the capital stock. Our estimate of unmeasured intangible investment will involve capitalizing category (iii), which is by far the largest category of intangible capital.

Data on business expenditure was provided from Corrado and Hulten (2010), and consists of yearly estimates of spending on five types of category (iii) series: finance and insurance new product development, industrial design, branding, worker training, and organizational capital. Stock calculation for each series follows the BEA (see Herman (2001)):

1. Nominal investment for year i in asset class j is converted to real investment using a price index.

2. The contribution of real investment in year i of asset j in year t is given by $N_{tij} = I_{ij} * (1 - \delta_j/2)(1 - \delta_j)^{t-i}$.

3. Current cost estimates are estimated of the stocks of type j by multiplying the stocks by the price index that was used to deflate investment. The total stock is estimated as the sum across all types of assets j: $C_t = \sum_{j \in J} C_{ij}$.

Appendix figure A.4 shows the time series of investment flows, and shows that tangible investment has decreased since 1977 as a percentage of national income, while intangible investment has increased. We split intangible investment into two categories: national accounts intangibles, i.e. those that the BEA currently measures as part of investment, and non-national account investment flows. Of the two series, non-national account has increased the fastest, comprising 9% of national income by 2015.

Figure 21 shows the stock estimates, shows that non-national account intangible stocks have only moderately increased over the time period. This is somewhat puzzling, given that non-national account intangibles is the largest category of investment. The reason behind the low replacement value of the stock is that estimates of depreciation are very high for non-national account intangibles. For example, the depreciation rate on advertisement and brand spending is 55%, while the depreciation rate on worker training is 40%. Figure A.5 shows average depreciation rates for the different categories. The high depreciation rate is why we find a small contribution of intangible investment for generating capital gains in the US.

8 Conclusion

Our analysis shows that prior to 1980, increases in household wealth were largely driven by the forces of accumulation, through savings and investment. After 1980, the role of savings diminished, and the increase in wealth was largely generated through the appreciation of asset prices. We quantify the increase in

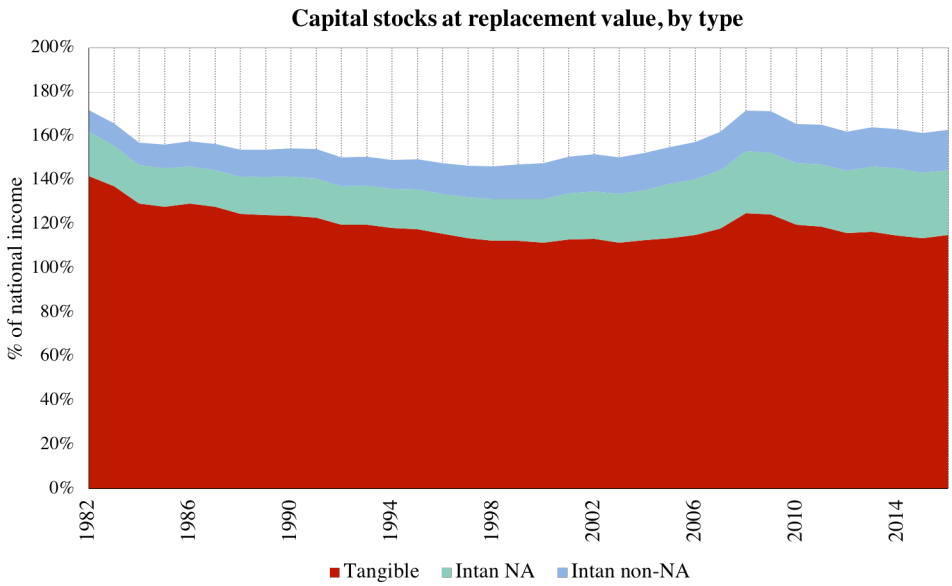

Figure 21: Estimates of intangible capital stocks, compiled using the perpetual inventory method. Data on investment flows provided by Carol Corrado. Depreciation rates are from Corrado et al. (2012), and capital price indexes are from the BEA.

asset prices through an aggregate measure of capital gains, Gross National Capital Gains. Our theoretical analysis shows a close connection between capital gains, consumption, and welfare, which motivates us to explore the implications of this source of income on measures of aggregate savings and the distribution of income. We find that measures of savings inclusive of capital gains increased post-1980, compared to the traditional finding that savings has decreased. We also find that including capital gains as income increases the measured capital share of income, and increases the share of income received by the top percentiles of the income distribution.

The analysis of section 7.9 shows capital gains in the post-1980 were primarily driven by changes in market power and interest rates. This raises the question of whether the large capital gains seen in the data are a temporary force, or will be a sustained source of capital income in the future. Our theoretical model shows that there can be capital gains on a balanced growth path with positive productivity growth. For example, if the ratio of the market value of securities to GDP is 200%, and growth is 2%, there can be capital gains of 4% of GDP in a steady state.

In order to draw welfare and policy conclusions, it is necessary to study in more detail the reasons underlying the increase in capital gains. Capital gains driven by unmeasured intangible investment will contribute positively to output welfare. Capital gains driven by an increase in monopoly power may be either "malignant" or "benign", depending on whether this change is due to benign technological change or lax antitrust enforcement. Changes driven by bargain-

ing power have important distributional consequences, but potentially limited aggregate effects.

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Online Appendix for *Capital gains and Inequality*

Jacob A. Robbins

A Modifications to Financial Accounts

While most assets are listed in the Financial Accounts at market value, there are two exceptions to this. First, government and corporate bonds are listed at book value instead of market. This is potentially important, because declining interest rates have generated capital gains on bonds. In fact, since 1980 capital gains have been one of the largest drivers of bond returns (see Dobbs et al. (2016)). Second. the value of non-corporate business assets, consisting of sole-proprietorships and partnerships, is listed at book value.[52] As shown in Antoniewicz et al. (1996) and Henriques and Hsu (2014), non-corporate business valuations are significantly higher in the Survey of Consumer Finance, which surveys individuals about the *market* value of their businesses.

We estimate the market value of bonds by using indexes of bond market prices to par values. We use a separate index for corporate and government bonds. For the 1946-1996 period, we use a price index from historical New York Stock Exchange data. For government bonds, 1997-2017, we use a price index from the Dallas Federal Reserve. For corporate bonds, 1997-2017, we use a price index from Bank of America Merrill Lynch.

We use the Survey of Consumer Finance valuations for non-corporate business for 1989-2017.

B Comparison with literature

There are a number of important differences between the savings and capital gains concepts used in this paper, and in the work of Piketty and Zucman (2014). In terms of the aggregate savings concept, Piketty and Zucman (2014) use personal savings from the Financial Accounts, add corporate savings from the NIPA, and then subtract the statistical discrepancy from the NIPA between savings and investment. Since the Financial Accounts savings has tended to be above NIPA personal savings, and the statistical discrepancy has tended to be negative, this decreases their measured capital gains. Piketty and Zucman (2014) make two more modifications in order to bring down "implausibly high" capital gains. First, they replace corporate savings from the flow of funds with NIPA corporate savings; since NIPA savings is significantly higher than corpo-

[52] With the exception of financial assets held by the businesses, which are listed at market value.

rate savings, this will tend to decrease capital gains. Second, they use housing investment derived from data from Robert Shiller; since this tends to be higher than NIPA residential investment, this will once again tend to bring down capital gains for housing.

Aside from differences in aggregate saving, there are differences in savings rates by asset class. This differences are due to this paper's attempt, as much as possible, to align the saving concepts with the wealth concept as to maintain a consistent stock-flow relation. The differences, by asset class, are as follows:

- *Fixed income:* This paper's definition of fixed income includes trade receivables, the acquisition of non-produced non-financial assets, insurance receivables due from property-casualty insurance companies, non-life insurance reserves at life insurance companies, and Federal Government Retiree Health Care Funds. It does not include net nonprofit investment in equipment and IP.
- *Equities:* This paper's equity saving includes money market shares.
- *Pension:* This paper does not include investment in "miscellaneous assets".
- *Non-corporate business:* This paper includes nonprofit investment in equipment and IP.

C Description of the DINAs

The DINAs are a micro-dataset of the distribution of total national income and household wealth. There is yearly data from 1962-2014, but estimates for the earlier period can be attained through an imputation procedure using IRS tabulations.

The main advantage of the DINAs over other micro data sources, such as the Survey of Consumer Finance, is that there is conformity between the micro data and aggregate macro data from the national accounts. Total income in the DINA micro data sum to national income from the NIPAs, and total wealth sums to aggregate wealth from the Financial Accounts.

Conformity between micro and macro totals is achieved through taking aggregate income or wealth from the national accounts, and *distributing* it to the micro level. We give the example for the case of compensation of employees. Micro-data on compensation of employees is gathered from tax data, as well as survey data on health insurance and pension benefits. However, the sum of compensation of employees from micro data, $comp_t^{micro} \equiv \sum_{i \in USA} comp_t^i$, does not equal the sum from macro data, $comp_t^{macro}$. PSZ distribute the macro compensation to the micro-level in proportion to the individual level holdings: $\widehat{comp_t^i} = comp_t^i \cdot \frac{comp_t^{macro}}{comp_t^{micro}}$. This ensures that the sum of the new micro-variable, $\widehat{comp_t^i}$, equals the macro total.

The different income categories in the DINAs correspond to the categories from the national accounts. Factor labor income consists of the compensation of employees, which includes wages, benefits, and pension contributions. It also includes the labor component of net mixed income. Factor capital income includes implicit rent from housing, dividends,

D Additional tables

Table A.1: 1970 calibration results

Targets	*Model*	*Data*	*Source*
Real interest rate	3.00%	3.00%	Federal Reserve
Wealth-to-output ratio	2.66	2.66	Financial Accounts
Investment-to-output ratio	15.34%	16.15%	NIPA
Labor share	71.87%	71.49%	Elsby (2013)
Equity premium	3.45%	4.71%	Croce (2014)
Labor supply	0.18%	0.18%	Croce (2014)

E Additional figures

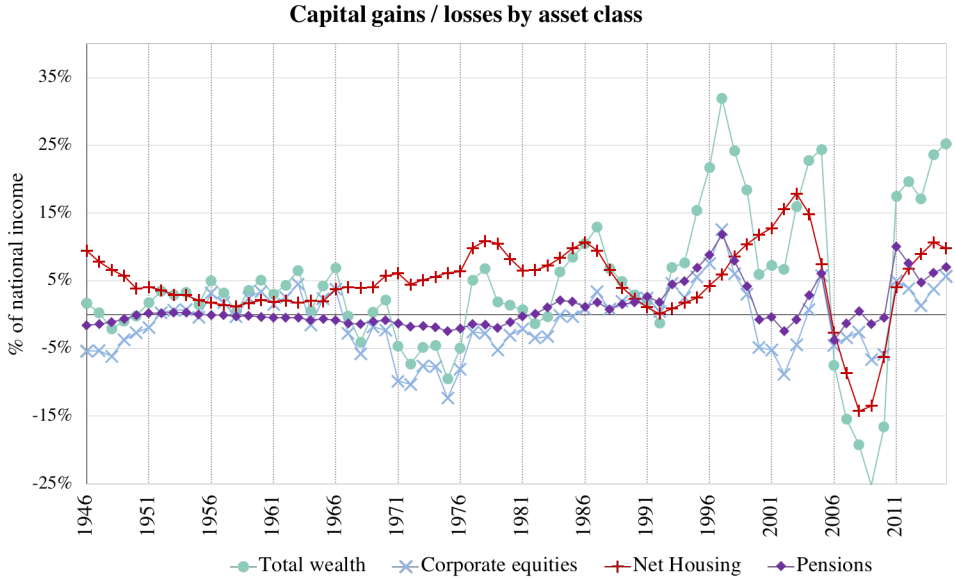

Figure A.1: Capital gains by asset class.

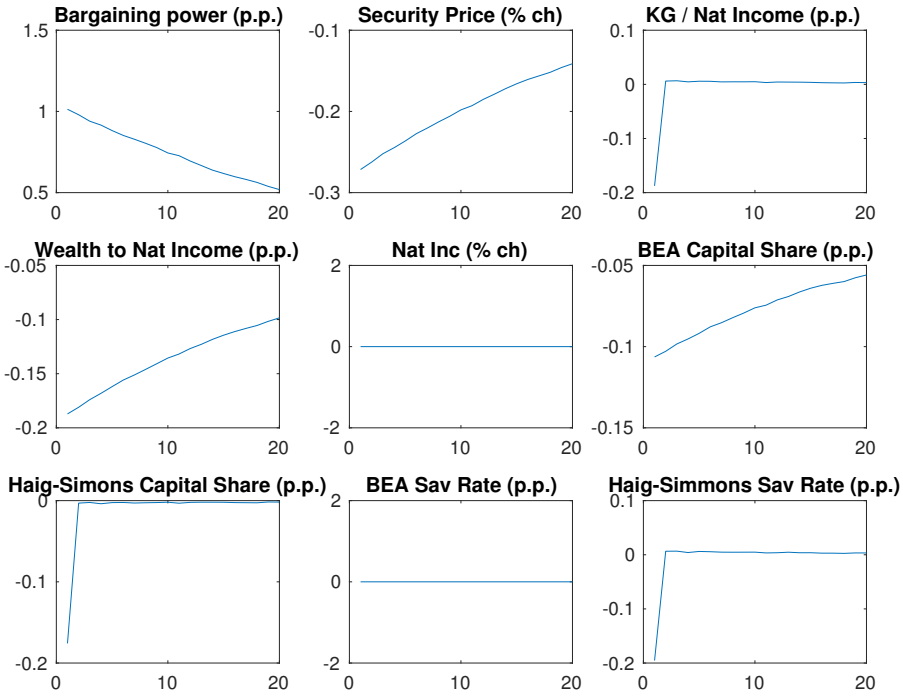

Figure A.2: IRF of shock to labor bargaining power eb

F Full equations of model

F.1 Final goods firms

There is a unit mass of monopolistically competitive final goods firms that differentiate an intermediate good and resell it to consumers. The final good composite is the CES aggregate of these differentiated final goods, which are indexed by i:

$$Y_t = \left[\int_0^1 y_t^f(i)^{\frac{\Lambda_t - 1}{\Lambda_t}} di\right]^{\frac{\Lambda_t}{\Lambda_t - 1}}.$$

Final goods firms set prices in each period, and face a demand curve that takes the following form: $y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\Lambda_t}$, where Λ_t is a time-varying measure of a firm's market power. An increase in Λ_t decreases a firm's market power and lowers equilibrium markups. The nominal price index is defined as

$$P_t = \left(\int_0^1 p_t(i)^{1-\Lambda_t} di\right)^{\frac{1}{1-\Lambda_t}}.$$

Each final goods producer uses y_t^m of intermediate goods to produce output, according to a linear technology function $y_t^f = y_t^m$. A final goods firm chooses real prices $\frac{p_t(i)}{P_t}$ and $y_t^f(i)$ to maximize real profits, subject to the production and

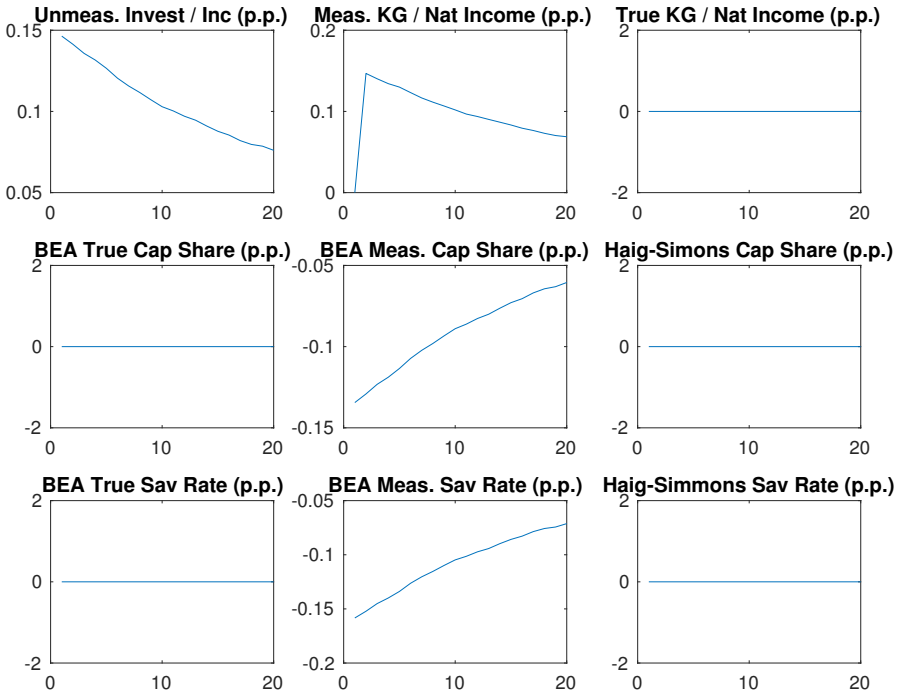

Figure A.3: IRF of shock to intangible capital share eu

demand constraints:

$$\max \ \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}}{P_t} y_t^f(i)$$
$$\text{subject to } \ y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\Lambda_t},$$

where $\frac{p_t^{int}}{P_t}$ is the price of the intermediate good taken as given by the firm.

The optimality condition for the real price of the firm's good is a time-varying markup over the price of the intermediate good:

$$\frac{p_t(i)}{P_t} = \frac{\Lambda_t}{\Lambda_t - 1} \frac{p_t^{int}}{P_t} = \mu_t \frac{p_t^{int}}{P_t}, \tag{A.1}$$

where μ_t is the optimal markup of the firm.

Since the price of the intermediate good is the same, all final goods firms make the same pricing decisions, and thus $p_t(i) = P_t$, yielding $\frac{p_t^{int}}{P_t} = \frac{1}{\mu_t}$.

Final goods firms have market power which allows them to set prices above marginal costs. Market power is determined by the CES elasticity Λ_t, which determines markups. We assume that $\frac{\Lambda_t}{\Lambda_t - 1}$, i.e. markups μ_t, follows an AR(1) process given by

$$ln(\mu_t) = (1 - \rho_\mu) ln(\bar{\mu}) + \rho_\mu ln(\mu_{t-1}) + \epsilon_t^\mu + z_t^\mu \tag{A.2}$$
$$z_t^\mu = z_{t-1}^\mu + \iota_t^\mu. \tag{A.3}$$

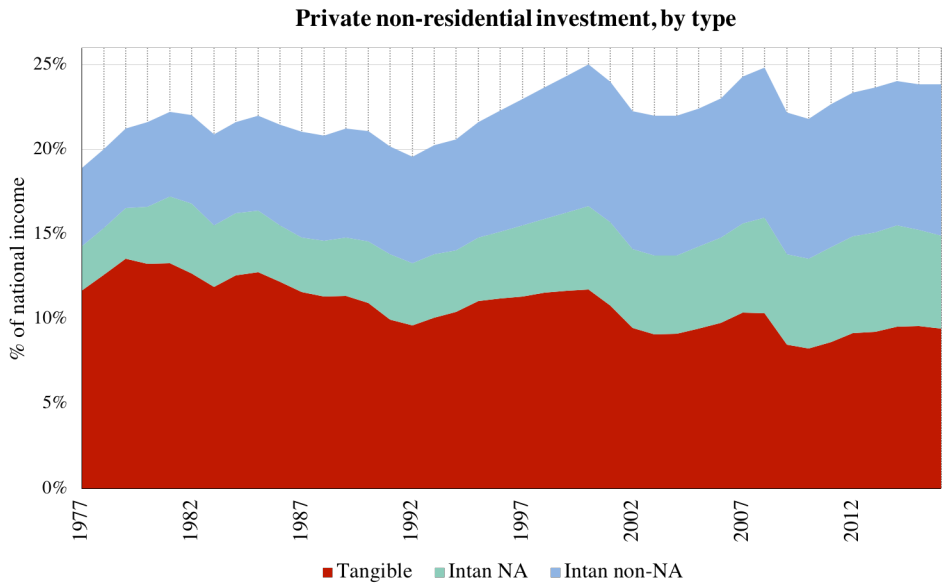

Figure A.4: Estimates of intangible investment. Data on investment flows provided by Carol Corrado.

In this expression, $\bar{\mu}$ is the long run level of markups in the economy and ϵ_t^{μ} is a temporary shock to the level of markups. z_t^{μ} follows a random walk process. Final goods firms make aggregate profits equal to

$$\Pi_t = \frac{\mu_t - 1}{\mu_t} Y_t. \tag{A.4}$$

The profit share of income in the economy is given by $PS_t = \frac{\mu_t - 1}{\mu_t}$. Pure profits flow to two distinct group: workers, and shareholders of the final good firms. We assume workers have some level of bargaining power which yields them a share bp_t of aggregate profits. Worker bargaining power follows the following stochastic process:

$$ln(bp_t) = (1 - \rho_{bp}) ln(\bar{bp}) + \rho_{bp} ln(bp_{t-1}) + \epsilon_t^{bp} + z_t^{bp} \tag{A.5}$$

$$z_t^{bp} = z_{t-1}^{bp} + \iota_t^{bp}. \tag{A.6}$$

The long-run level of bargaining power is given by $\bar{bp}$, ϵ_t^{bp} are temporary shocks to bargaining power, and z_t^{bp} follows a random walk. Total bargaining rents distributed to workers are termed the 'collective bargin', denoted cb_t, and are given by

$$cb_t = bp_t \Pi_t. \tag{A.7}$$

The shareholders of final goods firms receive the residual profits as dividends. Aggregate dividends distributed to shareholders at time t are thus given by

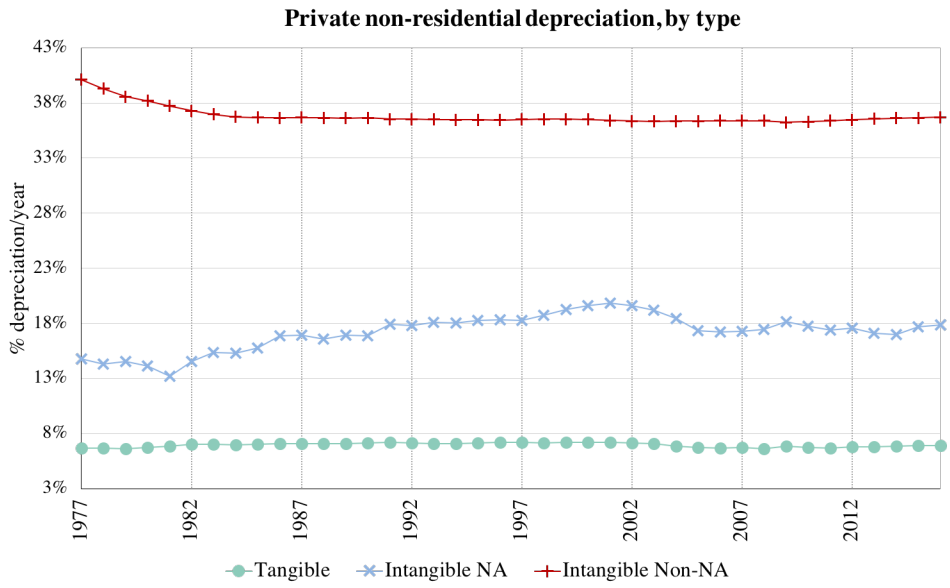

Figure A.5: Depreciation by asset type.

$$d_t^f = (1 - bp_t)\Pi_t. \tag{A.8}$$

Since all firms make identical profits, each firm receives an equal fraction of aggregate dividends.

F.2 Long Run Risk

Let A_t denote the level of productivity, and lowercase letter denote log-units. The growth rate or productivity is given by:

$$\Delta a_{t+1} = \zeta + x_t + \sigma_a \epsilon_{a,t+1} \tag{A.9}$$

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t} \tag{A.10}$$

$$\begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} \sim iid\ N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right) \tag{A.11}$$

Here x_t is the "long run risk" of productivity growth, and $\epsilon_{a,t+1}$ is the short run risk.

F.3 Two agent model

There are two types of agents in the economy: capitalists, and workers. The capitalist is a "Ramsey" type of agent, that maximizes lifetime utility. The worker is a "Solow" type of agent that saves an exogenous fraction of utility. A unit mass of agents are capitalists, and a mass Υ are workers. We will index capitalist variables with a 'c' subscript, and worker variables with a 'w' subscript.

F.3.1 Capitalists

The model mainly follows Caldara et al. (2012) and Croce (2014). Capitalists have Epstein-Zin utility given by

$$V_t = \left[(1-\beta D_t)\left(c_{c,t}^{\nu}(A_{t-1}(1-L_{c,t}))^{1-\nu}\right)^{\frac{1-\gamma}{\theta}} + \beta D_t \left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{A.12}$$

where the time discount factor is β, the labor supply coefficient is ν, γ is the risk aversion parameter, and θ is defined as

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}. \tag{A.13}$$

In this last expression, ψ is the elasticity of intertemporal substitution. The term D_t is an additional wedge between utility in period $t+1$ and period t, beyond the time discount rate of β. The preference shock D_t follows the stochastic process

$$ln(D_t) = \rho_D ln(D_{t-1}) + \epsilon_t^D + z_t^D \tag{A.14}$$

$$z_t^D = z_{t-1}^D + \iota_t^D. \tag{A.15}$$

Individuals maximize utility subject a series of budget constraints,

$$\begin{aligned} c_{c,t} + X_t^i S_{c.t+1}^i + X_t^f S_{c,t+1}^f = w_t L_{c,t} + \Gamma_t cb_t + d_t^i S_{c,t}^i + d_t^f S_{c,t}^f \\ +\Delta_{t+1} X_t^f + (1-\Delta_t) X_t^f S_{c,t}^f + X_t^i S_{c,t}^i. \end{aligned} \tag{A.16}$$

On the left hand side of the budget constraint, individuals use their income to purchase either consumption, shares of intermediate good firms ($X_t^i S_{c,t+1}^i$), or shares of final goods firms ($X_t^f S_{c,t+1}^f$). On the right hand side of the budget equation, agents receive income from a variety of sources:

1. Labor income $w_t L_{c,t}$
2. Bargaining power payments of $\Gamma_t cb_t$
3. Dividends from intermediate goods firms $d_t^i S_{c,t}^i$
4. Dividends from final goods firms $d_t^f S_{c,t}^f$
5. IPO issued securities of final goods firms $\Delta_{t+1} X_t^f$
6. Remaining share value of final goods firms. Agents come into the period holding $S_{c,t}^f$ shares of the security. Because of firm exit, a fraction Δ_t of shares lose their value, and thus the value remaining is $(1-\Delta_t) X_t^f S_{c,t}^f$.
7. Remaining share value of intermediate goods firms, $X_t^i S_{c,t}^i$

Due to the nature of the recursive utility, we can write the optimal solution as a recursive function

$$\begin{aligned} V_t(S^i_{c,t}, S^f_{c,t}) = max_{c_{c,t}, L_{c,t}, S^i_{c,t+1}, S^f_{c,t+1}} \Big[(1-\beta D_t) \left(c^{\nu}_{c,t} (A_{t-1}(1-L_{c,t}))^{1-\nu} \right)^{\frac{1-\gamma}{\theta}} + \\ \beta D_t \left(E_t V_{t+1}(S^i_{c,t+1}, S^f_{c,t+1})^{1-\gamma} \right)^{\frac{1}{\theta}} \Big]^{\frac{\theta}{1-\gamma}} \\ s.t.\ c_{c,t} + X^i_t S^i_{c,t+1} + X^f_t S^f_{c,t+1} = w_t L_{c,t} + \Gamma_t cb_t + d^i_t S^i_{c,t} + d^f_t S^f_{c,t} \\ + \Delta_{t+1} X^f_t + (1-\Delta_t) X^f_t S^f_{c,t} + X^i_t S^i_{c,t}. \end{aligned} \tag{A.17}$$

From this equation, we can derive the first order conditions for optimization. Setting up the Lagrangean and differentiating with respect to $c_{c,t}$, we have

$$\frac{\partial \mathcal{L}}{\partial c_{c,t}} : (1-\beta) V_t^{1-\frac{1-\gamma}{\theta}} \left(c^{\nu}_{c,t} (A_{t-1}(1-L_{c,t}))^{1-\nu} \right)^{\frac{1-\gamma}{\theta}} \nu \frac{1}{c_{c,t}} = \lambda_t. \tag{A.18}$$

Taking first order conditions with respect to $L_{c,t}$, we have

$$\frac{\partial \mathcal{L}}{\partial L_{c,t}} : (1-\beta) V_t^{1-\frac{1-\gamma}{\theta}} \left(c^{\nu}_{c,t} (A_{t-1}(1-L_{c,t}))^{1-\nu} \right)^{\frac{1-\gamma}{\theta}} (1-\nu) \frac{1}{(1-L_{c,t})} = w_t \lambda_t. \tag{A.19}$$

Combining the first order conditions with respect to labor and with respect to consumption, we have

$$\frac{(1-\nu)}{\nu} \frac{c_{c,t}}{(1-L_{c,t})} = w_t, \tag{A.20}$$

and taking the first order condition with respect to $S^f_{c,t+1}$, we have

$$\frac{\partial \mathcal{L}}{\partial S^f_{c,t+1}} : X^f_t \lambda_t = \beta E_t [\lambda_{t+1} ((1-\Delta_{t+1}) X^f_{t+1} + d^f_{t+1})]. \tag{A.21}$$

Now, taking the first order condition with respect to $c_{c,t+1}$, we have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_{c,t+1}} : V_t^{1-\frac{1-\gamma}{\theta}} \beta D_t \left(E_t V_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}-1} \times \\ E_t \left[V_{t+1}^{-\gamma} (1-\beta) V_{t+1}^{1-\frac{1-\gamma}{\theta}} \left(c^{\nu}_{c,t+1} (A_t(1-L_{c,t+1}))^{1-\nu} \right)^{\frac{1-\gamma}{\theta}} \nu \frac{1}{c_{c,t+1}} \right], \end{aligned} \tag{A.22}$$

where in the last step we make a substitution by forwarding $\frac{\partial}{\partial c_{c,t}}$ one period. Canceling redundant terms, we get

$$m_{t+1} = \frac{\partial V_t / \partial c_{c,t+1}}{\partial V_t / \partial c_{c,t}} = \beta D \left(\frac{c_{c,t+1}}{c_{c,t}} \right)^{\frac{\nu(1-\gamma)}{\theta}-1} \left(\frac{A_t(1-L_{c,t+1})}{A_{t-1}(1-L_{c,t})} \right)^{\frac{(1-\nu)(1-\gamma)}{\theta}} \left(\frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} \tag{A.23}$$

F.4 Workers

The worker class saves an exogenous fraction s_t^w of income $Y_{w,t}$, where income is defined as

$$Y_{w,t} = w_t \Omega_t L_{w,t} + r_t^i S_{w,t}^i X_t^i + d_t^f S_{w,t}^f + (1 - \Gamma_t) cb_t. \tag{A.24}$$

Workers receive income from wages, bargaining rents, returns from intermediate good firms, and dividends from final goods firms. We assume the labor supply of workers is the same as that of capitalists, and thus $L_{w,t} = L_{c,t}\ \forall t$. Workers have lower productivity than capitalists. The relative productivity of workers is denoted Ω_t.

Workers use their savings to purchase final goods securities and intermediate goods securities. We assume they spend an exogenous fraction $\mathscr{P}$ of savings on intermediate goods securities, and a corresponding fraction $(1 - \mathscr{P})$ on final goods securities. The law of motion for securities is then given by

$$S_{w,t+1}^i = S_{w,t}^i + s_t^w \mathscr{P} Y_{w,t} \frac{1}{X_t^i} \tag{A.25}$$

$$S_{w,t+1}^f = (1 - \Delta) S_{w,t}^f + s_t^w (1 - \mathscr{P}) Y_{w,t} \frac{1}{X_t^f}. \tag{A.26}$$

.

Worker consumption is given by

$$c_{w,t} = (1 - s_t^w) Y_{w,t}. \tag{A.27}$$

F.5 Intermediate Goods Firm's Problem

Representative intermediate goods firms use labor L_t and capital K_t to produce intermediate goods Y_t^m according to the production function

$$Y_t^m = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{A.28}$$

where σ is the production elasticity of substitution. $\sigma = 1$ corresponds to Cobb-Douglas. The firm finances part of its investment I_t through retained earnings RE_t and issues shares to cover the remaining part,

$$I_t = X_t^i (S_{t+1}^i - S_t^i) + RE_t. \tag{A.29}$$

It distributes the excess of its profits over retained earnings to its shareholders as a dividend,

$$d_t^i S_t^i = \frac{1}{\mu_t} Y_t^m - w_t L_t - RE_t. \tag{A.30}$$

Since $Y_t = Y_t^m$, we have

$$d_t^i S_t^i = \frac{1}{\mu_t} Y_t - w_t L_t - R E_t. \tag{A.31}$$

Investment increases the firm's future stock of capital according to

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \tag{A.32}$$

where δ is the rate of depreciation and adjustment costs $\Phi(\cdot)$ are a positive concave function. The adjustment costs function is given by

$$\Phi(I_t/K_t) = \frac{a_1}{1-\xi}\left(\frac{I_t}{K_t}\right)^{1-\xi} + a_2. \tag{A.33}$$

Following Jermann (1998), we will choose the adjustment costs parameters so that the steady state ratio of investment to capital is not affected. In a model with productivity growth, the investment to capital ratio is given by $\frac{I}{K} = (\delta + e^{\zeta} - 1)$. From equation (A.32), in the steady state with productivity growth we have that $K(\delta + e^{\zeta} - 1) = \Phi(I/K)K$, thus we must have in the steady state $\Phi(I/K) = (\delta + e^{\zeta} - 1)$. Note that this also ensures that if a firm replaces depreciation and accounts for growth, adjustment costs are zero.

In addition, we also need q to be 1 in the steady state, and thus $\Phi'(I/K) = 1$. These conditions imply the following two conditions:

$$\frac{a_1}{1-\xi}\left(\delta + e^{\zeta} - 1\right)^{1-\xi} + a_2 = (\delta + e^{\zeta} - 1) \tag{A.34}$$

$$a_1(\delta + e^{\zeta} - 1)^{-\xi} = 1. \tag{A.35}$$

Thus we have that $a_1 = (\delta + e^{\zeta} - 1)^{\xi}$, and $a_2 = (1 - \frac{1}{(1-\xi)})(\delta + e^{\zeta} - 1)$.

F.5.1 Computation of the intermediate good firm's value

Intermediate goods firms maximize the expected value of cash flow to the shareholders, discounted by the stochastic discount factor of capitalists. Defining cash flow, $CF_t^i = \frac{1}{\mu_t} Y_t - w_t L_t - I_t$, the value of the intermediate good firm is given by

$$V_t^i = E_t\left[\sum_{s=t}^{\infty} \beta^{s-t} D^{s-t} \frac{\lambda_s}{\lambda_t} CF_s^i\right]. \tag{A.36}$$

Firms maximize (A.36) subject to (A.32). The first order conditions are given by:

$$\frac{\partial}{\partial I_t} : \frac{1}{\Phi'(I_t/K_t)} = q_t, \tag{A.37}$$

where q_t is the the Lagrange multiplier of the maximization problem. Continuing, we have

$$\frac{\partial}{\partial K_{t+1}} : -q_t + E_t \left[\beta D \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{1}{\mu_{t+1}} f'(K_{t+1}) + q_{t+1} \left(-\Phi' \left(\frac{I_{t+1}}{K_{t+1}} \right) \left(\frac{I_{t+1}}{K_{t+1}} \right) + \Phi \left(\frac{I_{t+1}}{K_{t+1}} \right) + (1-\delta) \right) \right] \right].$$

Then, using the fact that $\Phi'(I_{t+1}/K_{t+1}) = \frac{1}{q_{t+1}}$ and

$$F'(K_{t+1}) = \left(\alpha (K_{t+1})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_{t+1}L_{t+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha K_{t+1}^{\frac{-1}{\sigma}}, \tag{A.38}$$

we have that

$$q_t = E_t \left[\beta D \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{1}{\mu_{t+1}} \left(\alpha (K_{t+1})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_{t+1}L_{t+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha K_{t+1}^{\frac{-1}{\sigma}} - \left(\frac{I_{t+1}}{K_{t+1}} \right) + q_{t+1} \left(\Phi \left(\frac{I_{t+1}}{K_{t+1}} \right) + (1-\delta) \right) \right] \right]. \tag{A.39}$$

Finally, we have

$$\frac{\partial}{\partial L_t} : \frac{1}{\mu_t} \left(\alpha (K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} (1-\alpha) L_t^{\frac{-1}{\sigma}} = w_t. \tag{A.40}$$

F.6 Asset pricing implications

As usual, the return on the risk free rate is given by

$$R_t = \frac{1}{E_t[m_{t+1}]} \tag{A.41}$$

The return for investing in intermediate goods firms is given by

$$R^i_{t+1} = \frac{d^i_{t+1} S^i_{t+1} + X^i_{t+1} S^i_{t+1}}{X^i_t S^i_{t+1}} = \frac{d^i_{t+1} + X^i_{t+1}}{X^i_t}. \tag{A.42}$$

Now, using the fact that $q_t K_{t+1} = X^i_t S^i_{t+1}$, we have

$$\begin{aligned} R^i_{t+1} &= \frac{\frac{1}{\mu_t} Y^m_{t+1} - w_{t+1} L_{t+1} - R E_{t+1} + X^i_{t+1} S^i_{t+1}}{q_t K_{t+1}} \\ &= \frac{\frac{1}{\mu_t} Y_{t+1} - w_{t+1} L_{t+1} - I_{t+1} + X^i_{t+1}(S^i_{t+2} - S^i_{t+1}) + X^i_{t+1} S^i_{t+1}}{q_t K_{t+1}} \\ &= \frac{\frac{1}{\mu_t} Y_{t+1} - w_{t+1} L_{t+1} - I_{t+1} + q_{t+1} K_{t+2}}{q_t K_{t+1}}. \end{aligned} \tag{A.43}$$

As usual, we have the asset pricing equation

$$1 = E_t[m_{t+1} R^i_{t+1}]. \tag{A.44}$$

The return for investing in final goods firms is given by

$$R^f_{t+1} = \frac{d^f_{t+1} S^f_{t+1} + (1 - \Delta_{t+1}) S^f_{t+1} X^f_{t+1}}{S^f_{t+1} X^f_t} = \frac{\frac{\mu_{t+1}-1}{\mu_{t+1}} Y_{t+1} + (1 - \Delta_{t+1}) X^f_{t+1}}{X^f_t} \tag{A.45}$$

F.7 Aggregation

Aggregate securities, labor, and consumption are the sum of worker and capitalist totals:

$$c_t = c_{c,t} + \Upsilon c_{w,t} \tag{A.46}$$
$$S^i_t = S^i_{c,t} + \Upsilon S^i_{w,t} \tag{A.47}$$
$$S^f_t = S^f_{c,t} + \Upsilon S^f_{w,t} \tag{A.48}$$
$$L_t = L_{c,t} + \Upsilon \Omega_t L_{w,t}. \tag{A.49}$$

F.8 Equilibrium

An equilibrium is a set of quantities:
$\{c_{c,t}, c_{w,t}, c_t, K_t, L_{c,t}, L_{w,t}, L_t, I_t, Y_t, Y_{w,t}, Y^m_t, d^i_t, d^f_t, cb_t, S^i_{c,t}, S^i_{w,t}, S^i_t, S^f_{c,t}, S^f_{w,t}, S^f_t\}^\infty_{t=0}$,
a set of prices $\{w_t, X^i_t, X^f_t, q_t, m_t\}^\infty_{t=0}$, and a set of exogenous processes
$\{\mu_t, A_t, x_t, \Delta_t, bp_t, D_t, s^w_t\}^\infty_{t=0}$ that jointly satisfy:

1. Capitalist consumption maximizes (A.12) subject to (A.16)
2. The capitalist stochastic discount factor is given by (A.23)
3. Intermediate firms maximize (A.36) subject to (A.32)
4. Intermediate good production is given by (A.28), and final good production is given by $Y_t = Y^m_t$
5. Aggregate profits of final goods firms are given by (A.4), and aggregate dividends are given by (A.8)
6. The collective bargain is given by (A.7)
7. The price of securities satisfies (A.21)
8. The wage is given by (A.40)

9. The stochastic processes for μ_t, A_t, x_t, bp_t, and D_t, are given by (A.2), (A.9), (A.10), (A.5), and (A.14)

10. The paths for Δ_t and s_t^w are exogenously specified

11. Worker income is given by (A.24), worker consumption by (A.27), and worker asset choice by (A.25)

12. Aggregate consumption, securities, and labor are given by (A.46)

While this model is not stationary, we can make it so by applying a standard transformation: divide all quantities (except labor) by A_{t-1}, as well as wages and the price of securities X_t^i and X_t^f.

F.9 Full Equations of Model

Now, collecting the equations of the model, we have

$$V_t = \left[(1-\beta D_t)\left(c_{c,t}^{\nu}(A_{t-1}(1-L_{c,t}))^{1-\nu}\right)^{\frac{1-\gamma}{\theta}} + \beta D_t\left(E_t(V_{t+1})^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \tag{A.50}$$

$$m_{t+1} = \beta D_t\left(\frac{c_{c,t+1}}{c_{c,t}}\right)^{\frac{\nu(1-\gamma)}{\theta}-1}\left(\frac{A_t(1-L_{c,t+1})}{A_{t-1}(1-L_{c,t})}\right)^{\frac{(1-\nu)(1-\gamma)}{\theta}}\left(\frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}}\right)^{1-\frac{1}{\theta}} \tag{A.51}$$

$$\frac{(1-\nu)}{\nu}\frac{c_{c,t}}{(1-L_{c,t})} = w_t \tag{A.52}$$

$$\frac{1}{\Phi'(I_t/K_t)} = q_t, \tag{A.53}$$

$$q_t = E_t\left[m_{t+1}\left[\frac{1}{\mu_{t+1}}\left(\alpha(K_{t+1})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_{t+1}L_{t+1})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}\alpha K_{t+1}^{\frac{-1}{\sigma}}\right.\right. \tag{A.54}$$
$$\left.\left. - \left(\frac{I_{t+1}}{K_{t+1}}\right) + q_{t+1}\left(\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) + (1-\delta)\right)\right]\right]$$

$$\frac{1}{\mu_t}\left(\alpha(K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_tL_t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}(1-\alpha)A_t^{\frac{\sigma-1}{\sigma}}L_t^{\frac{-1}{\sigma}} = w_t \tag{A.55}$$

$$Y_t^m = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_tL_t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{A.56}$$

$$Y_t = Y_t^m \tag{A.57}$$

$$Y_t = c_t + I_t \tag{A.58}$$

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \tag{A.59}$$

$$\Delta a_{t+1} = \zeta + x_t + \sigma_a \epsilon_{a,t+1} \tag{A.60}$$

$$x_t = \rho_t x_{t-1} + \sigma_x \epsilon_{x,t} \tag{A.61}$$

$$ln(\mu_t) = (1-\rho_\mu) ln(\bar{\mu}) + \rho_\mu ln(\mu_{t-1}) + \epsilon_t^\mu. \tag{A.62}$$

$$R_t = \frac{1}{E_t[m_{t+1}]} \tag{A.63}$$

$$R_{t+1}^i = \frac{\frac{1}{\mu_t} Y_{t+1} - w_{t+1} L_{t+1} - I_{t+1} + q_{t+1} K_{t+2}}{q_t K_{t+1}}. \tag{A.64}$$

$$d_t^f = (1 - bp_t) \frac{\mu_t - 1}{\mu_t} Y_t \tag{A.65}$$

$$X_t^f = E_t[m_{t+1}((1-\Delta_{t+1})X_{t+1}^f + d_{t+1}^f)]. \tag{A.66}$$

$$R_{t+1}^f = \frac{\frac{\mu_{t+1}-1}{\mu_{t+1}} Y_{t+1} + (1-\Delta_{t+1}) X_{t+1}^f}{X_t^f} \tag{A.67}$$

$$q_t K_{t+1} = X_t^i S_{t+1}^i \tag{A.68}$$

$$c_{w,t} = (1 - s_t^w) Y_{w,t} \tag{A.69}$$

$$c_t = c_{c,t} + \Upsilon c_{w,t} \tag{A.70}$$

$$L_t = L_{c,t} + \Upsilon \Omega_t L_{w,t} \tag{A.71}$$

$$L_w = L_c \tag{A.72}$$

$$Y_{w,t} = w_t \Omega_t L_{w,t} + r_t^i S_{w,t}^i X_t^i + d_t^f S_{w,t}^f + (1-\Gamma_t) cb_t. \tag{A.73}$$

$$S_{w,t+1}^f = (1-\Delta) S_{w,t}^f + s_t^w (1 - \mathscr{P}) Y_{w,t} \frac{1}{X_t^f} \tag{A.74}$$

$$S_t^f = S_{c,t}^f + \Upsilon S_{w,t}^f \tag{A.75}$$

$$S_t^f = 1 \tag{A.76}$$

$$S_t^i = S_{c,t}^i + \Upsilon S_{w,t}^i \tag{A.77}$$

$$S_{w,t+1}^i = S_{w,t}^i + s_t^w \mathscr{P} Y_{w,t} \frac{1}{X_t^i} \tag{A.78}$$

$$S_t^i = 1 \tag{A.79}$$

$$ln(bp_t) = (1-\rho_{bp})ln(\bar{bp}) + \rho_{bp} ln(bp_{t-1}) + \epsilon_t^{bp} + z_t^{bp} \tag{A.80}$$

$$z_t^{bp} = z_{t-1}^{bp} + \iota_t^{bp} \tag{A.81}$$

$$ln(D_t) = \rho_D ln(D_{t-1}) + \epsilon_t^D + z_t^D \tag{A.82}$$

$$z_t^D = z_{t-1}^D + \iota_t^D \tag{A.83}$$

$$z_t^\mu = z_{t-1}^\mu + \iota_t^\mu \tag{A.84}$$

The variables are $V, Y, Y^m, c, L, m, w, q, K, I, a, x, \mu, R, R^i, R^f, d_t^f, X^f$ $X^i, c_w, c_c, L_c, L_w, Y_w, S_w^f, S_c^f, S^f, S_w^i, S_c^i, S^f, bp_t, z^{bp}, D_t, z^D, z^\mu$. Thus there are 35 equations and 35 variables.

F.10 Making the model stationary

We now make a standard transformation by dividing the following variables by A_{t-1}: $V, Y, Y^m, c, w, K, I, X^f, X^i, d^f, d^i, c_w, c_c, Y_w$. We then have the following set of equations:

$$V_t = \left[(1-\beta D_t)\left(c_{c,t}^\nu((1-L_{c,t}))^{1-\nu}\right)^{\frac{1-\gamma}{\theta}} + e^{\left(\frac{1-\gamma}{\theta}\right)\Delta a_t}\beta D_t \left(E_t(V_{t+1})^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \tag{A.85}$$

$$m_{t+1} = \beta D_t \left(\frac{c_{c,t+1}}{c_{c,t}}\right)^{\frac{\nu(1-\gamma)}{\theta}-1} \left(\frac{(1-L_{c,t+1})}{(1-L_{c,t})}\right)^{\frac{(1-\nu)(1-\gamma)}{\theta}} \left(\frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}}\right)^{1-\frac{1}{\theta}} e^{\left(\frac{(1-\gamma)}{\theta}-1\right)\Delta a_t} \tag{A.86}$$

$$\frac{(1-\nu)}{\nu}\frac{c_{c,t}}{(1-L_{c,t})} = w_t \tag{A.87}$$

$$\frac{1}{\Phi'(I_t/K_t)} = q_t, \tag{A.88}$$

$$q_t = E_t\left[m_{t+1}\left[\frac{1}{\mu_{t+1}}\left(\alpha(K_{t+1})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\Delta a_{t+1}}(L_{t+1})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}\alpha K_{t+1}^{\frac{-1}{\sigma}}\right.\right. \tag{A.89}$$
$$\left.\left. - \left(\frac{I_{t+1}}{K_{t+1}}\right) + q_{t+1}\left(\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) + (1-\delta)\right)\right]\right]$$

$$\frac{1}{\mu_t}\left(\alpha(K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\Delta a_t}(L_t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}(1-\alpha)e^{\frac{\sigma-1}{\sigma}\Delta a_t}L_t^{\frac{-1}{\sigma}} = w_t \tag{A.90}$$

$$Y_t^m = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\Delta a_t}(L_t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{A.91}$$

$$Y_t = Y_t^m \tag{A.92}$$

$$Y_t = c_t + I_t \tag{A.93}$$

$$K_{t+1}e^{\Delta a_t} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \tag{A.94}$$

$$\Delta a_{t+1} = \zeta + x_t + \sigma_a\epsilon_{a,t+1} \tag{A.95}$$
$$x_t = \rho_t x_{t-1} + \sigma_x\epsilon_{x,t} \tag{A.96}$$

$$ln(\mu_t) = (1-\rho_\mu)ln(\bar{\mu}) + \rho_\mu ln(\mu_{t-1}) + \epsilon_t^\mu. \tag{A.97}$$

$$R_t = \frac{1}{E_t[m_{t+1}]} \tag{A.98}$$

$$R_{t+1}^i = \frac{\frac{1}{\mu_t}Y_{t+1} - w_{t+1}L_{t+1} - I_{t+1} + q_{t+1}K_{t+2}e^{\Delta a_{t+1}}}{q_t K_{t+1}}. \tag{A.99}$$

$$d_t^f = (1-bp_t)\frac{\mu_t - 1}{\mu_t}Y_t \tag{A.100}$$

$$X_t^f = E_t[m_{t+1}((1-\Delta_{t+1})X_{t+1}^f e^{\Delta a_t} + d_{t+1}^f e^{\Delta a_t})]. \tag{A.101}$$

$$R_{t+1}^f = \frac{d_{t+1}^f e^{\Delta a_t} + (1-\Delta_{t+1}) X_{t+1}^f e^{\Delta a_t}}{X_t^f} \tag{A.102}$$

$$q_t K_{t+1} e^{\Delta a_t} = X_t^i S_{t+1}^i \tag{A.103}$$

$$c_{w,t} = (1 - s_t^w) Y_{w,t} \tag{A.104}$$

$$c_t = c_{c,t} + \Upsilon c_{w,t} \tag{A.105}$$

$$L_t = L_{c,t} + \Upsilon \Omega_t L_{w,t} \tag{A.106}$$

$$L_w = L_c \tag{A.107}$$

$$Y_{w,t} = w_t \Omega_t L_{w,t} + r_t^i S_{w,t}^i X_t^i + d_t^f S_{w,t}^f + (1-\Gamma_t) c b_t. \tag{A.108}$$

$$S_{w,t+1}^f = (1-\Delta) S_{w,t}^f + s_t^w (1-\mathscr{P}) Y_{w,t} \frac{1}{X_t^f} \tag{A.109}$$

$$S_t^f = S_{c,t}^f + \Upsilon S_{w,t}^f \tag{A.110}$$

$$S_t^f = 1 \tag{A.111}$$

$$S_t^i = S_{c,t}^i + \Upsilon S_{w,t}^i \tag{A.112}$$

$$S_{w,t+1}^i = S_{w,t}^i + s_t^w \mathscr{P} Y_{w,t} \frac{1}{X_t^i} \tag{A.113}$$

$$S_t^i = 1 \tag{A.114}$$

$$ln(bp_t) = (1-\rho_{bp}) ln(\bar{bp}) + \rho_{bp} ln(bp_{t-1}) + \epsilon_t^{bp} + z_t^{bp} \tag{A.115}$$

$$z_t^{bp} = z_{t-1}^{bp} + \iota_t^{bp} \tag{A.116}$$

$$ln(D_t) = \rho_D ln(D_{t-1}) + \epsilon_t^D + z_t^D \tag{A.117}$$

$$z_t^D = z_{t-1}^D + \iota_t^D \tag{A.118}$$

$$z_t^\mu = z_{t-1}^\mu + \iota_t^\mu \tag{A.119}$$

F.11 Steady State

In the steady state, all transformed variables are constant.

We begin by finding steady state investment. From equation (A.94), and using the assumed properties of the $\Phi(\cdot)$ function, in particular that in the steady state $\Phi(\cdot) = \delta + e^{\zeta} + 1$ we have that $\frac{I}{K} = \delta + e^{\zeta} - 1$. Next, using the fact that

$$\bar{m} = \beta D e^{\left(\frac{(1-\gamma)}{\theta} - 1\right)\zeta}, \tag{A.120}$$

from equation (A.89) we have that

$$1 = \bar{m}\left[\frac{1}{\bar{\mu}}\left(\alpha\left(\frac{\bar{K}}{\bar{L}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta}\right)^{\frac{1}{\sigma-1}}\alpha\left(\frac{\bar{K}}{\bar{L}}\right)^{\frac{-1}{\sigma}} + 1 - \delta\right]. \tag{A.121}$$

Rearranging, we have

$$\left(\frac{\bar{K}}{\bar{L}}\right) = \left[\frac{(1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta}}{\left[\left(\frac{1}{\bar{m}} - 1 + \delta\right)\frac{\mu}{\alpha}\right]^{\sigma-1} - \alpha}\right]^{\frac{\sigma}{\sigma-1}}. \tag{A.122}$$

Using this equation, steady state output-to-labor can be derived as well:

$$\frac{\bar{Y}}{\bar{L}} = \left(\alpha\left(\frac{\bar{K}}{\bar{L}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta}\right)^{\frac{\sigma}{\sigma-1}}. \tag{A.123}$$

Similarly, the wage can be written as

$$w = \frac{1}{\mu}M\bar{P}L = \frac{1}{\mu}(1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta}\left(\alpha\left(\frac{\bar{K}}{\bar{L}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta}\right)^{\frac{1}{\sigma-1}}. \tag{A.124}$$

Having derived the overall capital-to-labor ratio, we now derive the worker's capital to labor ratio. From the laws of motion for workers' capital and final goods securities, steady state asset allocations are given by

$$S_w^f X^f = s^w(1-\mathscr{P})Y_w\frac{1}{\Delta} \tag{A.125}$$

$$K_w = \mathscr{P}s^w Y_w\frac{1}{e^{\zeta} - 1 + \delta}. \tag{A.126}$$

Combining the two equations, we can write optimal final goods securities in terms of capital:

$$S_w^f = \frac{K_w}{X^f}\frac{(1-\mathscr{P})}{\mathscr{P}}\frac{e^{\zeta} - 1 + \delta}{\Delta}. \tag{A.127}$$

The value of final goods firms X^f can be computed in a steady state using the formula for the sum of an infinite geometric series:

$$X^f = \frac{m \cdot d^f \cdot e^\zeta}{1-(1-\Delta)e^\zeta m}, \tag{A.128}$$

and thus the ratio of dividends to the value of final goods securities can be written

$$\frac{\bar{d^f}}{\bar{X^f}} = \frac{1-(1-\Delta)e^\zeta m}{me^\zeta}. \tag{A.129}$$

We are thus able to write

$$d^f S_w^f = K_w \cdot \frac{1-(1-\Delta)e^\zeta \cdot m}{m \cdot e^\zeta} \frac{(1-\mathscr{P})}{\mathscr{P}} \frac{e^\zeta - 1 + \delta}{\Delta} \equiv \mathscr{C} K_w. \tag{A.130}$$

We assume that bargaining rents are distributed proportionally to labor earnings. We define the total labor supplied by workers as $\tilde{L_w} = \Upsilon\Omega L_w$ and the total labor supplied by capitalists as $\tilde{L_c} = L_c$. The percentage of bargaining rents received by workers is given by

$$(1-\Gamma) = \frac{\tilde{L_w}}{\tilde{L_c} + \tilde{L_w}} = \frac{\Upsilon\Omega}{\Upsilon\Omega + 1} \equiv (1-\mathscr{F}) \tag{A.131}$$

Then the total collective bargain received by workers is

$$(1-\Gamma)cb = (1-\mathscr{F})bp \cdot PS \cdot Y = (1-\mathscr{F})bp \cdot PS \frac{Y}{L}\frac{L}{\tilde{L_w}}\tilde{L_w} \tag{A.132}$$

$$= bp \cdot PS \frac{Y}{L}\tilde{L_w} \tag{A.133}$$

Turning now to the worker's law of motion for capital, in the steady state we have

$$K_w e^\zeta = (1-\delta)K_w + \mathscr{P}s^w\left(w\tilde{L_w} + rK_w + d^f S_w^f + (1-\Gamma)cb\right) \tag{A.134}$$

$$K_w(e^\zeta - 1 + \delta - \mathscr{P}s^w r) = \mathscr{P}s^w\left(w\tilde{L_w} + \mathscr{C}K_w + bp \cdot PS\frac{Y}{L}\tilde{L_w}\right)$$

$$\frac{K_w}{\tilde{L_w}} = \frac{\mathscr{P}s^w(w + bp \cdot PS\frac{Y}{L})}{e^\zeta - 1 + \delta - \mathscr{P}s^w(r+\mathscr{C})}$$

Consumption of a worker is given by

$$c_w = (1-s^w)\left(w\tilde{L_w} + \mathscr{C}K_w + bp \cdot PS\frac{Y}{L}\tilde{L_w}\right) \tag{A.135}$$

Scaling this by total labor, we have

$$\frac{c_w}{L} = (1-s^w)\left(w(1-\mathscr{F}) + \mathscr{C}(1-\mathscr{F})\frac{K_w}{L_w} + bp \cdot PS\frac{Y}{L}(1-\mathscr{F}) + r\frac{K_w}{L_w}(1-\mathscr{F})\right) \tag{A.136}$$

Continuing, from (A.91) and (A.93) we have that

$$\frac{\bar{c_c}}{\bar{L}} = \frac{\bar{Y}}{\bar{L}} - (e^{\zeta} + \delta - 1)\frac{\bar{K}}{\bar{L}} - \frac{c_w}{L}, \tag{A.137}$$

Now, combining the definition for the wage with the first order condition for labor, and rearranging, we have

$$w = \frac{(1-\nu)}{\nu}\frac{\bar{c_c}}{(1-\bar{L_c})}, \tag{A.138}$$

Rearranging, we have

$$\frac{\bar{c_c}}{\bar{L}} = \frac{\nu}{1-\nu}\frac{(1-\mathscr{G}L)}{\bar{L}}w, \tag{A.139}$$

where $\mathscr{G} = \frac{L_c}{L}$. Now combining (A.137) and (A.139), we have

$$\frac{\bar{Y}}{\bar{L}} - (e^{\zeta} + \delta - 1)\frac{\bar{K}}{\bar{L}} - \frac{c_w}{L} = \frac{\nu}{1-\nu}\frac{(1-\mathscr{G}L)}{\bar{L}}w. \tag{A.140}$$

Solving for $\bar{L}$, we have

$$\bar{L} = \frac{\frac{\nu}{1-\nu}w}{\frac{\bar{Y}}{\bar{L}} - (e^{\zeta} + \delta - 1)\frac{\bar{K}}{\bar{L}} - \frac{c_w}{L} + \mathscr{G}\frac{\nu}{1-\nu}w}. \tag{A.141}$$

Turning now to the value function, we have

$$\bar{V} = \left(\frac{(1-\beta)\left(\bar{c_c}^{\nu}((1-\bar{L_c}))^{1-\nu}\right)^{\frac{1-\gamma}{\theta}}}{1 - e^{\left(\frac{1-\gamma}{\theta}\right)\zeta}\beta}\right)^{\frac{\theta}{1-\gamma}}. \tag{A.142}$$

G Proofs

Proof of **Proposition 1**.

Proof. Subtracting the market value of securities and capital in period $t-1$ from both sides of the budget constraint, equation 1 can be rewritten

$$c_t + q_t K_{t+1} - q_{t-1}K_t + X_t S_{t+1} - X_{t-1}S_t = \tag{A.143}$$

$$(1-\delta)K_t(q_t - q_{t-1}) - \delta q_{t-1}K_t + \rho K_t + S_t(X_t - X_{t-1}) + S_t d_t + w_t l_t \tag{A.144}$$

$$\Rightarrow c_t + q_t K_{t+1} - q_{t-1}K_t + X_t S_{t+1} - X_{t-1}S_t = Y_t^n + KG_t. \tag{A.145}$$

Summing the left side of the budget constraint over the agent's lifespan, we have

$$\sum_{t=\tau}^{\tau+M-1} \{c_t + q_t K_{t+1} - q_{t-1} K_t + X_t S_{t+1} - X_{t-1} S_t\} \tag{A.146}$$

$$= \left(\sum_{t=\tau}^{\tau+M-1} c_t \right) + q_{\tau+M-1} K_{\tau+M} - q_{\tau-1} K_\tau + X_{\tau+M-1} S_{\tau+M} - X_{\tau-1} S_\tau \tag{A.147}$$

Since by assumption of zero initial assets and zero bequests $0 = K_\tau = K_{\tau+M} = S_\tau = S_{\tau+M}$, the sum of the left side of the BC is simply the sum of consumption, and the sum of the right side is the sum of Haig-Simons income.

□

Proposition 7. *Let $X = \{X_\tau, X_{\tau+1}, ...\}$ be an arbitrary sequence of security prices, and $d = \{d_\tau, d_{\tau+1}, ..\}$ a sequence of dividends. Let the solution to the agent's optimal consumption problem, given a sequence of security prices and dividends, be denoted $c^*(X, d) = (c^*_\tau, c^*_{\tau+1}, ...)$. Then there exists a sequence of prices $\widetilde{X} = \{\widetilde{X_\tau}, ...\}$ such that $c^*(X, d) = c^*(\widetilde{X}, \mathbf{0})$, thus the individual is indifferent between receiving returns as dividends or through price increases.*

Proof. Define $Z_t = X_{t-1} S_t$, and $R_t = (X_t + d_t)/X_{t-1}$. Then we can rewrite the budget constraint purely in terms of returns R_t:

$$c_t + q_t K_{t+1} + Z_{t+1} = q_t(1-\delta)K_t + \rho_t K_t + Z_t R_t + w_t l_t. \tag{A.148}$$

We can replicate the returns from this budget constraint with price changes rather than dividends. Define $\widetilde{X_\tau} = X_\tau$, and $\widetilde{X_{t+1}} = R_{t+1}\widetilde{X_t}$ $\forall t \geq \tau + 1$. From equation A.148, the budget sets are the same, and thus optimal consumption is the same.

□

Proof of **Proposition 3**.

Proof. From definition 2, $\frac{\partial KG^S_t}{\partial X_t} > 0$. To show the effect of shocks to the economy on capital gains, it is thus sufficient to show the effect of shocks on the security prices.

1. An increase in future productivity will increase future dividends, raising the present value of the security price. From equation 14 and assumption 7, $\frac{\partial d_s}{\partial z^A_t} \geq 0$ $\forall s \geq t$, and > 0 for $s = t+1$. From equation 20, an increase in dividends will raise the security price.

2. A positive shock to interest rates will have two effects on the price of securities: it will increase the discounting of dividends in the security price, and it will lower the level of future dividends through a decline in the capital stock. Both forces will tend to decrease the price of securities. An increase in the discount rate will lower security prices: from 20, $\frac{\partial X_t}{\partial r_s} \leq 0$ $\forall s > t$. An increase in interest rate will lead to a decrease in capital. From equation 17, $\frac{\partial K_s}{\partial r_s} \leq 0 \, \forall s$, since $f_k < 0$. A lower capital stock will lead to a decline in output, from equation 14, and thus a decline in dividends, from assumption 7. A decline in dividends will lead to a decline in the security price, from equation 20.

3. There are two opposing forces affecting aggregate dividends D_t when the wedge changes. First, there is an increase in the share of output paid out as a dividend. However, with a higher wedge firms will use less capital, leading to lower output and dividends. The elasticity of aggregate dividends with respect to a change in the wedge can be written $\epsilon_t^{D,\mu} = \epsilon_t^{DS,\mu} + \epsilon_t^{Y,K}\epsilon_t^{K,\mu}$ (the elasticities are described in assumption 1 and 7). There will only be an increase in dividends if the increased output share that goes to dividends outweighs the decline in output from an increase in the wedge. By assumption the wedge is small enough such that $\epsilon_t^{DS,\mu} = \frac{1}{\mu_t - 1} > -\epsilon_t^{Y,K}\epsilon_t^{K,\mu}$, and thus it follows that $\epsilon_t^{DS,\mu} + \epsilon_t^{Y,K}\epsilon_t^{K,\mu} > 0$.

□

Proof of **Proposition 6**.

Proof. For the purpose of this proof, assume that markups are 1 (and thus the value of final goods firms is zero). If markups are greater than 1, proposition 5 already shows there will be positive capital gains on the balanced growth path. Under the assumption that markups are 1, changes in wealth are equivalent to changes in capital (see definition 4), and thus measured capital gains can be written $\widehat{KG}_t = K_{t+1} - K_t - \widehat{s_t^{net}}$. In a balanced growth path, changes in capital are equal to net investment, which is equal to net savings: $K_{t+1} - K_t = I_t - \delta K_t = s_t^{net}$. We can then write measured capital gains as $\widehat{KG}_t = s_t^{net} - \widehat{s_t^{net}}$.

We thus see that measurement error in savings can lead to measured capital gains. We now derive an expression for balanced growth measured savings. From equation 24 we have $\widehat{s_t^{net}} = s_t^{net} - I_t^u + \delta K_t^u$. If $I_t^u > \delta K_t^u$, there will be measured capital gains.

The SA measures capital as $\widehat{K_{t+1}} = \sum_{i=0}^{t}(1-\delta)^{t-i}\widehat{I}_i = (1 - \mathscr{U})K_t$. On a balanced growth path, $s_t = I_t = (e^{\zeta} - 1 + \delta)K_t$, and $I_u^t = \mathscr{U}(e^{\zeta} - 1 + \delta)K_t$, and thus we have $\widehat{s}_t = (1 - \mathscr{U})(e^{\zeta} - 1 + \delta)K_t$. Finally, $\widehat{s_t^{net}} = (1 - \mathscr{U})(e^{\zeta} - 1 + \delta)K_t - (1 - \mathscr{U})\delta K_t = (1 - \mathscr{U})(e^{\zeta} - 1)K_t$. We can thus write

$$\widehat{KG} = \mathscr{U}(e^{\zeta} - 1)K_t. \tag{A.149}$$

□