NBER WORKING PAPER SERIES

INFLATION FORECASTS AND MONETARY POLICY

Ben S. Bernanke
Michael Woodford

Working Paper 6157
http://www.nber.org/papers/w6157

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 1997

This work has been supported by grants from the National Science Foundation to the National Bureau of Economic Research and Princeton University. This paper is part of NBER's research programs in Economic Fluctuations and Growth and Monetary Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

Inflation Forecasts and Monetary Policy
Ben S. Bernanke and Michael Woodford
NBER Working Paper No. 6157
September 1997
JEL Nos. E5, E52
Economic Fluctuations and Growth
and Monetary Economics

ABSTRACT

Proposals for "inflation targeting" as a strategy for monetary policy leave open the important operational question of how to determine whether current policies are consistent with the long-run inflation target. An interesting possibility is that the central bank might target current private-sector forecasts of inflation, either those made explicitly by professional forecasters or those implicit in asset prices. We address the issue of existence and uniqueness of rational expectations equilibria when the central bank uses private-sector forecasts as a guide to policy actions. In a dynamic model which incorporates both sluggish price adjustment and shocks to aggregate demand and aggregate supply, we show that strict targeting of inflation forecasts is typically inconsistent with the existence of rational expectations equilibrium, and that policies approximating strict inflation-forecast targeting are likely to have undesirable properties. We also show that economies with more general forecast-based policy rules are particularly susceptible to indeterminacy of rational expectations equilibria. We conclude that, although private-sector forecasts may contain information useful to the central bank, ultimately the monetary authorities must rely on an explicit structural model of the economy to guide their policy decisions.

Ben S. Bernanke
Woodrow Wilson School
Princeton University
Princeton, NJ 08544
and NBER
bernanke@wws.princeton.edu

Michael Woodford
Department of Economics
Princeton University
Princeton, NJ 08544
and NBER
woodford@phoenix.princeton.edu

Introduction

In recent years a number of major central banks have adopted, or at least actively considered, some form of "inflation targeting" as a framework for monetary policy.[1] In an inflation-targeting regime, the central bank (usually in conjunction with the government) establishes explicit goals for the inflation rate at medium-term and long-term horizons. Although pursuit of an inflation target does not preclude other objectives, such as short-run output or exchange-rate stabilization, it is understood that these objectives are subsidiary to achieving the target level of inflation.

As a strategy for conducting monetary policy, inflation targeting has both advantages and disadvantages. One potential advantage is increased "transparency" of monetary policy, i.e., better communication of policy-makers' objectives and intentions to the public and the financial markets; see Bernanke and Mishkin (forthcoming). A second desirable feature is that, by setting targets for its goal variable rather than for an intermediate indicator (such as money growth or the exchange rate), the inflation-targeting central bank may avoid the "velocity instability" problem, which arises when there are unexpected changes in the relationship between the intermediate target and the ultimate objective.

The main disadvantages of the inflation-targeting approach follow from the empirical observation that inflation responds to changes in monetary policy only with a substantial lag, from one to two years or more. The lack of quick feedback from the economy to policy implies two related problems: First, the information the central bank requires in order to implement inflation targeting may be much greater than that needed to target an intermediate variable, whose response to policy changes can be observed with less delay. Second, as it is difficult for the inflation-targeting central bank to tell whether it is "on track",

it is equally difficult for the public and the financial markets to make that judgment, which has potentially adverse consequences for the central bank's accountability and credibility.

Is there some way to overcome the problems associated with the long lag between changes in policy and changes in the inflation rate? An interesting possibility is for the central bank to target current *forecasts* of medium-term inflation, rather than inflation itself. The current forecast of inflation, unlike actual future inflation, is (at least in principle) a contemporaneously observable variable; thus, in a regime which targets inflation *forecasts*, both the central bank and the public would be able to monitor policy continuously. Further, the rationally-formed forecast of inflation incorporates, by definition, all information currently available about future inflation, so that there can never be any conflict between the objectives of targeting the inflation forecast and targeting inflation itself. For both of these reasons, it has been argued that the forecast of inflation is the "ideal" intermediate variable for an inflation-targeting regime (Svensson, forthcoming; Svensson, 1997).

In practice, how could the central bank go about targeting the forecast of inflation? At least three types of approaches have been proposed. First, the central bank could try to "target" the predictions of private-sector forecasters, for example, by raising interest rates when the consensus private-sector forecast has inflation above the central bank's announced target and lowering rates when the inflation forecast is below the target. Hall and Mankiw (1994) propose a strategy of this type.[2] Second, the central bank might attempt to target the forecast of inflation implicit in various asset prices; for example, as we discuss in Section 4, there have been proposals to adjust monetary policy automatically in response to movements in commodity prices, to changes in long-term bond yields or in interest-rate spreads, to CPI

futures prices, and to changes in the spread between nominal and indexed government bonds, among others. Finally, the central bank might try to target its own internal forecasts of inflation (Svensson, forthcoming), in the sense of adjusting its instrument to eliminate any discrepancy between its staff's forecast of inflation and the target. Examples approximating each of these proposals may be found in central bank practice: E.g., in its quarterly *Inflation Report*, the Bank of England reports extensively on both private-sector and its own internal forecasts of inflation; and, though the Bank does not follow a mechanical rule, there is a presumption that if a preponderance of forecasts are above the Bank's inflation target, a tightening of policy will be recommended (see Bowen, 1995, and King, 1996, for discussions of the Bank of England's strategy). A similar approach has been followed by the Reserve Bank of New Zealand (Mayes and Riches, 1996). Virtually all central banks pay close attention to financial-market indicators of inflation, such as long-term bond yields and yield spreads.

The objective of this paper is to study the behavior of the economy when the central bank attempts to "target the forecast" of inflation, setting its instrument to eliminate deviations of some explicit or implicit inflation forecast from a pre-specified target. For concreteness, for most of the paper we consider the case in which (1) the inflation forecast being targeted is the consensus private-sector forecast, and (2) the private sector has some information about the economy that the central bank does not have.[3] However, as we discuss in the final section of the paper, thorough analysis of this case allows us to draw conclusions about other types of forecast targeting, and about the cases in which the central bank has equal or superior information to the private sector.

Unfortunately, we find that targeting the forecast of inflation, in the sense of allowing monetary policy to respond strongly to

deviations between the inflation forecast and the target, is not likely to be a useful tactic for monetary policy, for two broad sets of reasons: First, somewhat paradoxically, to the extent that targeting the forecast is successful, the signal-to-noise ratio in the inflation forecast is likely to become (endogenously) small. In the limit, as perfect stabilization of the inflation forecast is approached, there is no incentive for the private sector to gather information, and the inflation forecast becomes uninformative. We show further that policies approximating stabilization of the inflation forecast are also likely to have undesirable properties. These findings confirm and extend the analysis of Woodford (1994a); see also West (1994).

Second, we find that attempts to target the inflation forecast lead, for broad classes of policies, to indeterminacy of the rational expectations equilibrium. An implication is that even successful attempts to target the inflation forecast may be associated with arbitrary volatility in inflation itself, as well as in output and other goal variables. Thus, direct targeting of private-sector inflation forecasts is *not* a panacea for the problems raised by the long lag between monetary policy actions and the response of inflation.

On a somewhat more positive note, our analysis shows that, despite the problems with strict forecast targeting, a more subtle approach in which forecasts are simply used as one of several sources of information can be helpful. In particular, the central bank may well be able to infer useful information from private-sector forecasts of macroeconomic variables *other than inflation*, such as output or interest rates.[4] However, again caution must be urged, as we show that the problem of indeterminacy of equilibrium can apply to monetary policy rules based on forecasts of output and interest rates, just as it can to policies based on forecasts of inflation. The most general conclusion of our paper is that central banks should be careful not to tie monetary policy too

closely to any variable that is too sensitive to the expectations of the public.

To avoid misunderstanding, we should emphasize that our results have little to say about the desirability or feasibility of inflation targeting *per se*, as opposed to inflation-*forecast* targeting; indeed, this policy strategy has many attractive aspects. Our claim is only that, for successful implementation of inflation targeting, there appears to be no substitute for explicit structural modeling of the economy and extensive information gathering by the central bank. Private-sector forecasts, and forecasts inferred from financial markets, should be part of the information gathered by the bank, but they should be combined with other information in the making of policy.

The rest of the paper proceeds as follows. Section 1 analyzes the effects of inflation-forecast targeting in a simple, reduced-form model due to Woodford (1994a). We confirm Woodford's earlier result that, when the central bank attempts to target the inflation forecast precisely at the target, no rational expectations equilibrium exists. More generally, we find that attempts by the central bank to keep the inflation forecast close to the target, while technically feasible, may lead to excessive volatility in actual inflation outcomes.

The model of Section 1 is static and thus has no role for private-sector expectations about future policies. To remedy this shortcoming, in Section 2 we analyze a dynamic macroeconomic model which incorporates price stickiness and disturbances to both aggregate demand and aggregate supply. We find that the results from the static model generalize to the dynamic case. In Section 3 we show further that, in the dynamic model, forecast-based policy rules in many cases lead typically to non-uniqueness of rational expectations equilibrium; in particular, under such rules, the economy may be subject to "sunspot" equilibria and related pathologies.

As noted above, our analysis focuses on the case in which the central bank attempts to target private-sector inflation forecasts, and in which the private sector has some information about the economy that the central bank does not have. Section 4 discusses the application of our results to alternative forms of forecast targeting, and to alternative assumptions about information. The basic message of this paper--that there is no alternative to structural modelling of the economy for implementing forward-looking monetary policies--survives these generalizations. Section 5 is a brief conclusion. An Appendix provides some technical details.

1. Macroeconomic equilibrium when monetary policy depends on private-sector forecasts: A simple model

In this section we extend the example originally due to Woodford (1994a), who used it to illustrate the potential incompatibility of inflation forecast targeting and the revelation of private-sector information in rational expectations equilibrium. Suppose that next period's inflation is given by

$$\pi_{t+1} = s_t + u_t + \varepsilon_{t+1} \tag{1}$$

where π_{t+1} is realized inflation in period t+1, s_t is a state variable indicating underlying inflation pressures, u_t is the instrument of the central bank (which, as the subscript indicates, must be chosen one period prior to the realization of the inflation rate), and ε_{t+1} is an unforecastable disturbance that also affects the realized inflation rate. The random variables s_t and ε_{t+1} are assumed to be independent of the policy action u_t; the difference between them is that s_t is realized

prior to the choice of policy action and ε_{t+1} is realized subsequent to the policy action. Without loss of generality we may also assume that s_t and ε_{t+1} are mutually independent. We denote the variances of s and ε by σ_s^2 and σ_ε^2, respectively, and we normalize their means to zero. Finally, below we drop the time subscripts when there is no potential ambiguity.

For now, assume that the objective of the central bank is to minimize uncertainty about inflation, as measured by the conditional variance. (Implicitly, we assume that the targeted inflation level is zero; this assumption is easily modified.) If the central bank observes the underlying state s, then its task is simple; it need only set its instrument $u = -s$. In this case, $\text{var}(\pi) = \sigma_\varepsilon^2$, which is evidently the lower bound that can be achieved.

However, suppose instead that the underlying state s is observed by private forecasters but *not* by the central bank. The authority may then seek to infer the true state from the private-sector forecasts. If the central bank takes this approach, will it be able to achieve the same minimal variance of inflation attainable when it has direct information of the state? Clearly the answer is no; if the central bank could infer the state s from the private-sector forecasts, then it would set $u = -s$, so that $\pi = \varepsilon$. But then the rational private-sector forecasts of inflation would have to be independent of s, so that (contrary to hypothesis) the central bank would not be able to infer the realization of s from the forecasts.

More formally, suppose that the loss function of each forecaster is given by

$$(2) \quad L^f = E_t\{(\pi^f - \pi_{t+1})^2\}$$

where π^f is the individual's (publicly announced) forecast of inflation. Minimization of this loss requires the announcement

(3) $\pi^f = E_t\{\pi_{t+1}|s_t\}$

(For the moment we assume that forecasters have the same information and thus make the same forecast.) Now suppose that the central bank observes the (common) forecast and chooses

(4) $u = \phi\pi^f$

Equations (1), (3), and (4) jointly imply a unique rational expectations equilibrium, so long as $\phi \neq 1$, in which

(5) $\pi^f = \psi s$, with

(6) $\psi = \frac{1}{1-\phi}$

Now as long as $\psi \neq 0$, the value of s can be recovered from the forecast (by using (5)). So it might seem that the central bank can achieve the minimum variance for inflation by using a rule of the form (4) for which

(7) $\phi = -\frac{1}{\psi}$

However, it can be readily seen that there exists no joint solution for (6) and (7): From (7), minimization of the variance of inflation requires $\phi\psi = -1$, which in turn implies, from (6), that forecasters will rationally choose $\psi = 0$. But then (7) has no solution.

Some might argue that this example is of little practical import, since there exist well-behaved rational expectations equilibria in which the central bank achieves a level of inflation variability *arbitrarily close* to the theoretical minimum. Note that equations (1), (3), and (4) imply a unique solution for any finite choice of the policy reaction coefficient ϕ, so long as $\phi \neq 1$. In the associated equilibria, the variance of inflation is given by

$$(8) \quad \text{var}(\pi) = \frac{\sigma_s^2}{(\phi - 1)^2} + \sigma_\varepsilon^2$$

Thus it is possible to make the variance of inflation arbitrarily close to its minimum value σ_ε^2 by choosing a sufficiently large positive or negative value of ϕ (although the lower bound is attained only in the limit as ϕ is made unboundedly large, with either sign).

But the equilibria of this simple model associated with very large (in absolute value) ϕ are unappealing as a basis for a policy recommendation, even though they are technically well-behaved. One problem is that the model assumes that forecasters will make the effort to observe the true state s precisely, even though in equilibrium the forecaster's loss will barely depend on whether she knows s or not. To be specific, suppose that observation of the realized value of s costs the forecaster an amount $c > 0$, where this cost is measured in the same units as the loss in (2). Since each forecaster can achieve a loss of

$E\{\pi^2\}$ simply by choosing $\pi^f = 0$, that is, by forecasting inflation to be at its unconditional mean, it will be worthwhile for the forecaster to gather information about the realization of s if and only if

$$(9) \quad E\{(\pi^f(s)-\pi)^2\}+c \leq E\{\pi^2\}$$

where $\pi^f(s)$ is the optimal forecast conditional on knowledge of the state s. Equation (9) represents an additional constraint on the problem of the central bank. Using (1), (3), and (4), one can show that (9) requires $\text{var}(\pi) \geq \sigma_\varepsilon^2 + c$, or equivalently

$$(10) \quad (\phi-1)^2 \leq \frac{\sigma_s^2}{c}$$

Equation (10) shows that there is a limit to how large ϕ can be set, and therefore to the degree to which the variability of inflation can be reduced, without eliminating the incentive of the forecasters to gather information. On the other hand, it should be noted, as long as $c < \sigma_s^2$, the constraint (10) does permit values of ϕ that imply a variance of inflation lower than the central bank could achieve without using the information in the private-sector forecasts (i.e., lower than $\sigma_s^2 + \sigma_\varepsilon^2$).[5] So there is a sense in which the central bank can use outside forecasts to improve its policy, as we noted in the introduction. But this is not, strictly speaking, an inflation-forecast targeting policy, since in equilibrium forecasts different from the inflation target can occur without implying that the central bank's policy is too "tight" or too "loose".

Another practical concern about recommending a "large-ϕ" policy to the central bank (i.e., one in which the central bank's actions are highly sensitive to private-sector forecasts) is that--even if there is sufficient incentive for forecasters to gather information--some forecasters may be "incompetent" at using their information to produce optimal forecasts. To illustrate, suppose there is a random component in the average forecast made by the private sector, conditional on the underlying state. E.g., we might replace (3) with

$$\pi^f = E\{\pi|s\} + \upsilon \tag{11}$$

where υ is a random variable that we assume (for simplicity) to be independent of s and ε and to have mean zero and variance σ_υ^2. There are various interpretations that may be given to the noise term υ in (11): As already suggested, one possibility is "incompetence"--i.e., systematic errors in the computation or communication of the forecast--or, perhaps, "herd behavior", even though over many trials the forecasters make the correct inference on average. An alternative possibility is strategic randomization by the forecasters, which may occur if in fact their loss function is not given by (2). For example, Laster et al. (1996) show that if forecasters obtain benefit not only from the accuracy of their forecasts in an absolute sense, but also from being observed to have made the relatively most accurate forecast, then in equilibrium they will distribute their forecasts rather than announce their conditional expectations, even if all forecasters have identical information.[6] If we suppose that there are three or more forecasters and the forecasts are announced simultaneously, then this game can have a mixed-strategy equilibrium in which the average forecast is random, even conditional on the true state s.[7] In such an equilibrium (11)

will hold, where π^f refers now to the average private-sector forecast.

If the central bank follows a rule of the form (4), and the average forecast is given by (11) rather than (3), then equation (5) is replaced by $\pi^f = \psi s + v$, where ψ is determined by (6). The analogue to equation (8), which describes the variance of inflation attainable by the central bank, is

$$(12) \quad \text{var}(\pi) = \frac{\sigma_s^2}{(\phi - 1)^2} + \phi^2 \sigma_v^2 + \sigma_\varepsilon^2$$

Equation (12) shows that the variance of inflation is now bounded above its minimum value under perfect information (equal to σ_ε^2), regardless of the value of the central bank's reaction coefficient ϕ. Furthermore, the variance of inflation is no longer minimized by choosing ϕ as large as possible (either positive or negative); indeed, "targeting the forecast" by choosing ϕ large may lead to a very high variance of inflation, because of the presence of the second term on the right side of (12). Analysis of (12) shows that the global minimum for the variance of inflation is attained for a finite value $\phi^* < 0$, though there is also a local minimum at a value $\phi^{**} > 1$.[8]

Like the problem of inducing forecasters to bear the costs of gathering information, the problem of either potential "incompetence" or strategic behavior does not imply that the central bank cannot benefit from using private-sector forecasts in making its policy: After all, the variance of inflation is still lower for $\phi = \phi^*$ than for $\phi = 0$. However, again it is also true that it is dangerous to literally try to

"target the forecast"; although monetary policy should react to deviations of private forecasts from the inflation target, a policy of completely eliminating deviations of private forecasts from the official inflation target is generally not optimal, and can lead to an extremely high variance of inflation.

"Targeting the forecast" is a misleading description of the proper use of private-sector forecasts in another sense as well: The strategy of targeting the forecast seems to imply that the private-sector forecast of the goal variable (e.g., inflation) is a sufficient statistic for the information that the central bank can usefully obtain from outside forecasters. In fact, in general, the central bank may find it useful to observe private-sector forecasts of variables other than the goal variable, even in the extreme case when its objective function depends only on the stabilization of the single goal variable. To illustrate this point in the context of our simple reduced-form model, suppose that, prior to the central bank's choice of policy, it observes private-sector forecasts of both inflation and also the central bank's policy variable u. (Think of u as the short-term interest rate, for example; the choice of u as the particular additional variable to be forecast is inessential, as we show later.) With this assumption, we can now consider the consequences of monetary policy rules that depend on the average private-sector forecast u^f of the central bank's policy action, as well as on the forecast π^f of subsequent inflation.

We will want to assume that the private-sector forecasters care about the accuracy of both their inflation forecasts and their forecasts of central bank actions; for example, their clients may be interested in the likely path of short-term interest rates. Generalizing (2), let us suppose that forecasters minimize a loss function of the form

(13) $L^f = E\{(\pi^f - \pi)^2\} + \alpha E\{(u^f - u)^2\}$

where $\alpha > 0$ is the relative weight forecasters put on accurate prediction of the central bank's policy variable. Because the loss function is quadratic, the forecasters optimally choose to announce the conditional expectations of π and u as their forecasts:

(14) $\pi^f = E\{\pi|s\}$, $u^f = E\{u|s\}$

Now suppose that the monetary policy rule is of the form

(15) $u = \phi_\pi \pi^f + \phi_u u^f$.

The combination of (1), (14), and (15) implies a unique rational expectations equilibrium, as long as $\phi_\pi + \phi_u \neq 1$, in which

(16) $\pi^f = \psi s$, $\psi \equiv \dfrac{1-\phi_u}{1-\phi_\pi - \phi_u}$

$u^f = \theta s$, $\theta \equiv \dfrac{\phi_\pi}{1-\phi_\pi - \phi_u}$

$\pi = \psi s + \varepsilon$

$u = \theta s$

It may be observed that, if in addition we impose $\phi_u \neq 1$, the equilibrium described by (16) is equivalent to the one resulting from the simpler policy rule (4), with $\phi = \dfrac{\phi_\pi}{1-\phi_u}$. Thus at first glance it

might appear that imposing $\phi_u = 0$ in (15) is innocuous, and that the inflation forecast of the private sector is all that the central bank needs to know. However, this conclusion is incorrect. Indeed, as we show next, allowing for $\phi_u \neq 0$ can mitigate the problems associated with rules of the form (4) with very large values of ϕ.

First, we observed above that no rule of the form (4) can completely eliminate the effects of the state variable s on inflation, and thereby reduce the variance of inflation to its full-information lower bound. More precisely, we showed that there is no equilibrium in which 1) inflation is independent of the state variable in equilibrium and 2) the central bank is able to infer the value of the state variable from private-sector forecasts. However, this problem disappears when the central bank uses a rule of the form (15), i.e., it responds to forecasts both of inflation and of an additional variable (in this example, the central bank's policy variable). In particular, if the central bank sets $\phi_u = 1$, $\phi_\pi \neq 0$, then in equilibrium $\psi = 0$ and $\theta = -1$, with the consequences that π is independent of s and $\mathrm{var}(\pi) = \sigma_\epsilon^2$, its theoretical minimum. It is interesting to note that, when the central bank uses this rule, the private-sector inflation forecast never deviates from the target rate of inflation in equilibrium. (If it did, the central bank would choose a value of u different from its forecasted value, but in a rational expectations equilibrium such a deviation could not predictably occur.) But the central bank does not achieve this result by the heavy-handed means of moving the policy instrument violently in response to deviations of the forecast from the target; instead, it induces the private forecasters to reveal their information through their forecast of the policy instrument.

Second, a rule of the form (14) can eliminate, or at least ameliorate, the force of the incentive constraint that arises when

forecasters have a cost of collecting information. In particular, if the forecasters' loss function is given by (13), plus the cost $c > 0$ which is incurred if the true state is observed, then the incentive constraint (9) becomes

$$(17)\quad E\{(\pi^f(s)-\pi)^2+\alpha(u^f(s)-u)^2\}+c \le E\{\pi^2+\alpha u^2\}$$

That is, the improvement in forecast accuracy from observing the true state must exceed the cost of gathering the information. Note that, if $c \le \alpha\sigma_s^2$, then the constraint (17) is satisfied in the equilibrium resulting from the (unconstrained) optimal policy rule, $\phi_u = 1$, $\phi_\pi \neq 0$. Thus if information-gathering costs, though positive, are not too large, it may still be possible to reduce the variance of inflation around its target to the theoretical minimum.

Finally, a rule of the form (15) that eliminates (or nearly so) the influence of the state variable s on inflation (with ϕ_u close to 1), will in general be much more robust to random noise in private-sector forecasts--even though policy now responds to two distinct forecasts, each of which may be noisy. To illustrate, suppose that (14) is replaced by

$$(18)\quad \pi^f = E\{\pi|s\}+\upsilon, \quad u^f = E\{u|s\}+\omega$$

where υ and ω are mean-zero random variables, independent of s, ε, and each other, and with variances σ_υ^2 and σ_ω^2, respectively. If the policy rule is again of the form (15), in equilibrium the private forecasts will be given by $\pi^f = \psi s+\upsilon$, $u^f = \theta s+\omega$, where ψ and θ are as defined in (16). The resulting inflation rate is

(19) $\pi = \psi s + \phi_x v + \phi_u \omega + \varepsilon$

and the variance of inflation is

(20) $\text{var}(\pi) = \left(\frac{1-\phi_u}{1-\phi_x-\phi_u}\right)^2 \sigma_s^2 + \phi_x^2 \sigma_v^2 + \phi_u^2 \sigma_\omega^2 + \sigma_\varepsilon^2$

In the benchmark case $\phi_u = 1$, $\phi_x \neq 0$, which eliminates the effects of the state variable on inflation (but which is not necessarily the optimum in this class of policies), (20) implies

(21) $\text{var}(\pi) = \phi_x^2 \sigma_v^2 + \sigma_\omega^2 + \sigma_\varepsilon^2$

Forecast noise does cause additional inflation volatility, but overall this policy dominates policies that do not condition on the forecast of the policy instrument (i.e., for which $\phi_u = 0$) as long as the idiosyncratic noise in the forecast of the policy instrument is not too large (specifically, we need $\sigma_\omega^2 < \frac{\sigma_s^2}{(\phi^*-1)^2} + \phi^{*2}\sigma_v^2$, where $\phi^* < 0$ is the optimal policy reaction coefficient described above). More generally, one can write the first-order conditions defining the optimal reaction coefficients ϕ_x and ϕ_u for the case of noisy forecasts, verifying that $\phi_u = 0$ cannot be a solution; hence, in general it can never be optimal to ignore the information in the forecast of the policy instrument.

The desirable properties of the rule (15) with $\phi_u = 1$, $\phi_x \neq 0$ show that use of private-sector forecasts, including forecasts of inflation,

can improve the performance of monetary policy. We reiterate, however, that the ideal rules are not usefully described as "targeting the inflation forecast": First, the best policy rule does not usually involve responding to forecasts only of inflation (even though stabilizing inflation may be the central bank's only objective); and second, the optimal policy rule will typically not involve a high degree of sensitivity of the policy instrument to deviations of private-sector inflation forecasts from the inflation target.

2. Monetary Policy and Private-Sector Forecasts in a Dynamic Model

The model of Section 1 is static and rather stylized. We now present a more detailed, and explicitly dynamic model which allows for both monetary and non-monetary sources of inflation. In this more realistic model the inflation rate depends not solely upon a monetary policy action taken at a single point in time but also upon the *rule* (or reaction function) which is expected to dictate monetary policy in the future. This modification introduces important complications, in particular, the possible indeterminacy of rational expectations equilibrium under certain types of rules. As we show in Section 3, this potential indeterminacy is a further problem with simple proposals to "target the forecast".

In Section 1 we were not specific about the central bank's policy instrument. In line with the actual practice of most of the world's central banks, we now assume that the policy instrument is the short-term nominal interest rate, R_t, and ask how private-sector forecasts might be used in setting this particular instrument.

Apart from the monetary policy rule itself, to be discussed below, our dynamic model consists of two structural equations, an "expectational IS equation" (Kerr and King, 1996; Woodford, 1996; McCallum and Nelson, 1997) and an aggregate supply or price

determination equation. The expectational IS equation, which relates spending decisions to the interest rate, is given by

$$(22) \quad y_t = E_t y_{t+1} - \sigma[R_t - E_t \pi_{t+1} - \rho_t]$$

where y_t is the log of real output in period t, π_{t+1} is the rate of inflation between periods t and t+1, and ρ_t is an exogenous disturbance. Equation (22) is derivable as a log-linear approximation for optimal consumption on the part of the representative household, into which has been substituted the equilibrium condition that consumption demand equals the economy's output (there is no investment, government spending, or net foreign demand in the model). In this interpretation, the parameter $\sigma > 0$ is the intertemporal elasticity of substitution of consumption, and the exogenous disturbance term $\{\rho_t\}$ represents random (percentage) variations in the intertemporal marginal rate of substitution, arising for example from variations in the rate of time preference and evaluated at a plan involving constant consumption over time.[9]

The aggregate supply relationship is assumed to be

$$(23) \quad \pi_t = \beta E_{t-1} \pi_{t+1} + \kappa E_{t-1}(y_t - \theta_t)$$

where θ_t is an exogenous stochastic process interpretable as the log of the "natural rate" level of output in period t. Equation (23) can be obtained as a log-linear approximation to a first-order condition for optimal price-setting in a discrete-time version of the model of staggered price changes introduced by Calvo (1983). Alternatively, (23) can be derived from a model with a convex cost of changing prices, as in

Cochrane (1995). The parameter β, $0<\beta<1$, may be interpreted as the discount factor of the price-setters while $\kappa>0$ is a measure of the speed of price adjustment.

The model underlying eq. (23) is one in which only some suppliers are allowed to choose a new price for their output in any given period, these "lucky" suppliers being randomly selected in an independent drawing each period. It is also assumed that any price change chosen at date t takes effect only one period later, in t+1; hence, (23) differs from the form of the aggregate supply function obtained in several recent papers (e.g., Roberts (1995), Yun (1996), King and Watson (1996), Woodford (1996)) which assume that price changes take effect within the same period. In particular, whereas the cited papers obtain a first-order condition of the form $\pi_t = E_t X_{t+1}$, we here obtain instead a relationship of the form $\pi_t = E_{t-1} X_{t+1}$. Thus, in our specification, the period-t inflation rate depends only on period-(t-1) information, because all prices in effect in period t were chosen in t-1 or earlier. We choose this specification to capture the notion that inflation is "inertial", in particular, that it is affected by monetary policy actions only with a lag.[10]

For concreteness, assume the shocks to the IS equation and the aggregate supply equation are first-order autoregressive:

$$\rho_t = \lambda \rho_{t-1} + \upsilon_t \tag{24}$$

$$\theta_t = \delta \theta_{t-1} + \eta_t$$

where λ and δ have absolute values less than one and the innovations series $\{\upsilon_t\}, \{\eta_t\}$ are serially uncorrelated, mean-zero disturbances, also mutually uncorrelated at all leads and lags. Equations (22)-(24),

together with the monetary policy rule, constitute a complete model of the inflation process. In this model, both aggregate demand and aggregate supply shocks, if not offset by monetary policy, may lead to changes in the inflation rate.

We turn now to the analysis of monetary policy in this model. Let us suppose that the central bank's objective is not only to stabilize the inflation rate but also to minimize deviations of output y_t from the natural rate level of output, θ_t. Stabilizing output is not only consistent with the pursuit of stable inflation in this model but actually implies it, since in any stationary equilibrium (23) implies

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j E_{t-1}(y_{t+j} - \theta_{t+j}). \tag{25}$$

In any case, as a practical matter, not even the most hawkish inflation targeters among central banks have not demonstrated that they are willing to ignore output fluctuations entirely.

It is easily seen how interest rates must vary if the objectives of inflation stabilization and output at the natural rate are to be fully achieved. If we assume without loss of generality that the inflation target is zero, substitution of $\pi_t = 0$ and $y_t = \theta_t$ (for all t) into (22) yields

$$R_t = \rho_t + \frac{1}{\sigma}(E_t\theta_{t+1} - \theta_t) = \rho_t - \frac{(1-\delta)\theta_t}{\sigma} \tag{26}$$

where we have used (24) to substitute for $E_t\theta_{t+1}$. Equation (26) shows that, if the central bank directly observes the current realizations of

the two shocks, ρ_t and θ_t, it will be able to set the nominal interest rate R_t to stabilize both inflation and output perfectly.

What if the central bank does not directly observe the two shocks, but only the history of output and inflation? To be precise (and to make the most generous assumption about the timing of information receipt), suppose that at the time that it must choose R_t the central bank can observe y_{t-j} and π_{t-j} for all $j \geq 0$.[11] Is it possible in this case for the central bank to implement the first-best outcome? The answer is plainly no: Equation (26) requires that R_t respond to the innovations υ_t and η_t, or--more specifically--to the linear combination $\upsilon_t - \frac{(1-\delta)\eta_t}{\sigma}$. It is therefore necessary that the central bank be able to infer this quantity. But y_t is the only variable in the central bank's information set that is not determined at date t-1 or earlier; and since $y_t = \theta_t$ in the first-best case, observing output in period t would permit the central bank (under the hypothetical first-best policy) to infer only η_t, the innovation to θ_t. Thus if the central bank observes only current and past values of output and inflation, it cannot implement the first-best.[12]

What if, besides observing the histories of output and inflation, the central bank can also observe a private-sector inflation forecast? To give private forecasters an information advantage over the central bank, let us assume that (perhaps at some cost), the forecasters can make direct observations in period t of the shocks ρ_t and θ_t, as well as the histories of output and inflation (equivalently, the histories of the shocks), upon which they base their forecast of inflation between periods t and t+1.

Let π_t^f denote the forecast of π_{t+1} announced by the private forecasters in period t. We continue to assume that forecasters seek to minimize the variance of their forecast error and hence announce $\pi_t^f = E\{\pi_{t+1}|I_t\}$, where the information set I_t consists of $\{\rho_{t-j}, \theta_{t-j}\}, \forall j \geq 0$. Now, supposing that the central bank observes the private-sector forecasts, let us consider what can be achieved by a monetary policy rule of the form

$$R_t = \phi_\pi \pi_t^f + \phi_y y_t \tag{27}$$

We restrict attention, for now, to rational expectations equilibria of the form

$$y_t = a_1 \rho_t + a_2 \theta_t \tag{28}$$
$$\pi_{t+1} = b_1 \rho_t + b_2 \theta_t$$

Note that the first-best outcome, $y_t = \theta_t$, $\pi_{t+1} = 0$, can be expressed in the form of (28), so that equilibria yielding full stabilization can be considered.

Using equations (22) and (23), it is straightforward to show that, given a policy rule of the form (27), (28) can describe a rational expectations equilibrium if and only if the coefficients $\{a_i, b_i\}$ satisfy

$$b_1 = \beta\lambda b_1 + \kappa\lambda a_1 \tag{29}$$
$$b_2 = \beta\delta b_2 + \kappa\delta a_2 - \kappa\delta$$

(30) $(1+\sigma\phi_y)a_1 + \sigma(\phi_\pi - 1)b_1 = \lambda a_1 + \sigma$

$(1+\sigma\phi_y)a_2 + \sigma(\phi_\pi - 1)b_2 = \delta a_2$

Since $\beta\lambda < 1$ and $\beta\delta < 1$, it is possible to solve conditions (29) for (a_1, a_2) given (b_1, b_2), or vice versa. Thus equilibria of the class characterized by (28) may be indexed by the coefficients (a_1, a_2) describing the behavior of output, or alternatively by the coefficients (b_1, b_2) describing the behavior of inflation. Let us consider now whether, using information from private-sector forecasts, the central bank will be able to achieve stabilization of inflation. For this purpose, it is convenient to index equilibria by (b_1, b_2); the goal of inflation stabilization then requires $b_1 = b_2 = 0$. Note that this also implies that $a_1 = 0$, $a_2 = 1$, so that inflation stabilization is associated with output always equal to the natural rate. (This is because of our restriction of attention to the special class of candidate equilibria of form (28).)

Using equations (29)' to eliminate a_1 and a_2 from equations (30), we obtain

(31) $[(1-\lambda+\sigma\phi_y)\frac{1-\beta\lambda}{\kappa\lambda} + \sigma(\phi_\pi - 1)]b_1 = \sigma$

$[(1-\delta+\sigma\phi_y)\frac{1-\beta\delta}{\kappa\delta} + \sigma(\phi_\pi - 1)]b_2 = -(1-\delta+\sigma\phi_y)$

Since there are two free policy parameters to be chosen in order to determine the values of b_1 and b_2, it is clear that "almost" any pattern of inflation responses to the underlying shocks is consistent

with some policy of the form (27). In this respect, we have verified the usefulness of allowing policy to make use of the information contained in the inflation forecast: In general, π_t^f will not depend upon ρ_t and θ_t with the same relative weights as does y_t, and so the central bank will be able to extract information from the inflation forecast that can be used to reduce the central bank's expected loss.

Nevertheless, it is the case that--as in the simple model of the previous section--complete stabilization of inflation is not possible by means of policies of the form (27). In particular, inspection of equations (31) reveals that these equations cannot be solved for (ϕ_π, ϕ_y) when we impose $b_1 = b_2 = 0$. As before, the problem is essentially that perfect stabilization of inflation is inconsistent with private-sector inflation forecasts being informative about the underlying shocks. Specifically, in any equilibrium with $b_1 = 0$, inflation forecasts cease to reveal information about the value of ρ_t. (There is no problem in choosing a policy that induces $b_2 = 0$; this simply requires $\phi_y = -\frac{1-\delta}{\sigma}$, $\phi_\pi \neq 1$.)

As in Section 1, although perfect stabilization is impossible, there exist rational expectations equilibria in this model in which the central bank comes arbitrarily close to full stabilization of both inflation and output, i.e., equations (31) can be solved for policy parameters that achieve $b_2 = 0$ and an arbitrarily small value for b_1. This near-optimal policy involves $\phi_y = -\frac{1-\delta}{\sigma}$ and ϕ_π large in absolute value. Such a policy not only makes use of the private-sector inflation forecast but comes arbitrarily close to a policy of "targeting the forecast".

However, as in the example explored in Section 1, this policy has undesirable features that we believe should rule out its application in practice. First, if policy is highly sensitive to the forecast, the resulting low variability of expected inflation could eliminate the incentive for forecasters to gather information about the underlying state variables, as we saw earlier. Second, if forecasters are subject to "trembling hands", so that--due for example to systematic errors or strategic behavior--there is additive noise in the average forecast, then extreme sensitivity of policy to the forecast will induce excessive volatility in both inflation and output.

However, these conclusions do not rule out the productive use of private-sector forecasts in policy-making. One can again show that, if forecasters make predictions of variables other than inflation, the central bank may be able to use the information implied by these additional forecasts--even if the additional variables being forecasted do not enter the central bank's loss function. For example, suppose that there are available private-sector forecasts of the short-term nominal interest rate, as well as of inflation. To be precise, suppose that in period t the following sequence of events occurs: First, the shocks ρ_t and θ_t are realized and revealed to the forecasters (as well as to the private agents whose decision problems are affected by these shocks). Next, the forecasters announce their forecasts of π_{t+1} and R_t, denoted π_t^f and R_t^f respectively. Finally, the central bank and private decision-makers jointly act, determining π_{t+1}, y_t, and R_t. Specifically, we assume that the policy-makers are able to condition the value of their instrument R_t on y_t (but not π_{t+1}) and that private decision-makers observe R_t when choosing y_t and π_{t+1}.

The central bank's policy rule is of the form

(32) $R_t = \phi_\pi \pi_t^f + \phi_R R_t^f + \phi_y y_t$

and we again consider rational expectations equilibria of the form (28). It is easily verified that if $\phi_R \neq 1$, and the parameters of the policy rule are such that an equilibrium of the form (28) exists, then the equilibrium is identical to one arising from a policy rule of the form (27), with coefficients $(\tilde{\phi}_\pi, \tilde{\phi}_y)$ given by $\tilde{\phi}_\pi = \frac{\phi_\pi}{1-\phi_R}$, $\tilde{\phi}_y = \frac{\phi_y}{1-\phi_R}$.

However, if $\phi_R = 1$, more interesting possibilities arise: In particular, complete inflation and output stabilization (an equilibrium with $b_1 = b_2 = 0$) is possible if $\phi_R = 1$, $\phi_y = 0$, and $\phi_\pi \neq 0$. Furthermore, this way of stabilizing inflation is superior to the use of a rule of the form (27), with $|\phi_\pi|$ large, in that complete stabilization may not eliminate the incentive for forecasters to gather information about the state ρ_t, and the effects of forecast noise on output and inflation will be bounded. Thus the points developed in the context of the simple model of Section 1 carry over to this more complete model.

We conclude that the monetary authorities should use private-sector forecasts in their decision-making, if the private forecasters are likely to have information that the central bankers do not have. However, a regime of strict targeting of private-sector inflation forecasts is unlikely to be desirable. Instead, the policy-makers should draw information from a variety of private forecasts.

3. Determinacy of Equilibrium Under Forecast Targeting

We saw in Sections 1 and 2 that, although private-sector forecasts may provide useful information to policy-makers, targeting the inflation

forecast directly is not likely to be an effective device for stabilizing the inflation rate; the problem is that, if inflation equals the target in equilibrium, then the information of the private forecasters is not revealed. In this section we show that, in the context of a dynamic model like that of Section 2, there is an additional potential problem with naive forecast targeting. This problem is that such a policy rule need not imply a determinate rational expectations equilibrium and so may permit fluctuations arising purely from self-fulfilling expectations. Not only may the response of the economy to exogenous structural disturbances be indeterminate, but there may also exist "sunspot equilibria", in which the endogenous variables respond to random variables unrelated to the structure of the model simply because they are expected to.

Indeterminacy of equilibrium is of course a general problem for policy rules in settings in which the current equilibrium depends on expectations about the indefinite future.[13] However, rules that link policy actions to forecasts, thereby making the current equilibrium especially sensitive to expectations about the future, are particularly vulnerable in this regard, as we shall see. Thus, in analyzing forecast-based policy rules, it is insufficient to demonstrate (as in Section 2) that the rule is *consistent* with a particular desirable equilibrium; we must also check whether the desirable equilibrium is the *unique* outcome associated with that rule. We show in this section that forecast-based policy rules often do not imply unique rational-expectations equilibria, which provides another caveat to the use of such rules.

Let us consider again the dynamic model of Section 2 and the stationary rational expectations equilibria consistent with a policy rule of the form (27). However, we no longer restrict attention to equilibria of the form (28)--though we know that, for almost all choices

of (ϕ_π, ϕ_y), there is *an* equilibrium of that form. Substitution of the policy rule (27) into the "expectational IS curve" (22) yields

$$(33) \quad E_t y_{t+1} = \sigma(\phi_\pi - 1)\pi_{t+1} + (1 + \sigma\phi_y)y_t - \sigma\rho_t$$

where we have used the fact that $\pi_t^f = E_t\pi_{t+1} = \pi_{t+1}$, because all prices are fixed at least one period in advance.[14] Next, advancing the period by one in the aggregate supply relationship (23) and using (33) to substitute for $E_t y_{t+1}$, we obtain

$$(34) \quad E_t\pi_{t+2} = \beta^{-1}[1 - \kappa\sigma(\phi_\pi - 1)]\pi_{t+1} - \beta^{-1}\kappa(1 + \sigma\phi_y)y_t + \beta^{-1}\kappa(\sigma\rho_t + E_t\theta_{t+1})$$

The system (33)-(34) can be written compactly in vector form as

$$(35) \quad E_t x_{t+1} = \mathbf{M}x_t + z_t$$

where $x_t = |\pi_{t+1}, y_t|'$ is a vector of endogenous variables and z_t is a vector of exogenous disturbances. Note that both elements of x_t are determined at date t (despite the fact that "π_{t+1}" is written with a t+1 subscript, following conventional notation for the rate of inflation), and that each is free to respond to disturbances, news of future disturbances, or sunspot states realized at date t.

Suppose that the disturbance processes $\{\rho_t, \theta_t\}$ are bounded random variables. Then, from Blanchard and Kahn (1980), we know that the system (35) has a unique bounded solution if and only if both eigenvalues of $\mathbf{M}$ lie outside the unit circle. When this condition holds, (35) implies that

$$x_t = -\sum_{j=0}^{k-1} \mathbf{M}^{-j-1} E_t z_{t+j} + \mathbf{M}^{-k} E_t x_{t+k}$$

for all $k \geq 1$. Because in this case $\mathbf{M}^{-1}$ has both eigenvalues inside the unit circle, in any bounded solution for $\{x_t\}$ it must be that

$$\lim_{k \to \infty} \mathbf{M}^{-k} E_t x_{t+k} = 0$$

Hence the unique bounded solution to (35) is given by

$$x_t = -\sum_{j=0}^{\infty} \mathbf{M}^{-j-1} E_t z_{t+j} \tag{36}$$

In the case in which the structural disturbances follows the processes described in (24), it is possible to write $E_t z_{t+j}$ for arbitrary $j \geq 0$ as a linear function of the disturbance vector $|\rho_t, \theta_t|'$. It follows from (36) that x_t is a function of the period-t disturbance vector as well. Then, in this case, the unique bounded solution is of the form (28), and is the stationary equilibrium characterized in Section 2.

However if $\mathbf{M}$ has an eigenvalue *inside* the unit circle, there is no longer a unique rational expectations equilibrium, even if we restrict our attention to bounded stationary solutions.[15] For example, suppose that $\mathbf{M}$ has one eigenvalue satisfying $|\lambda_s| < 1$ while the other satisfies $|\lambda_u| > 1$. (This turns out to be the case of greatest relevance, since it corresponds to a policy of attempting to "target the inflation

forecast" by choosing a large value for ϕ_x; see below.) Then, letting (e_s, e_u) be the corresponding right eigenvectors and (v_s', v_u') the corresponding left eigenvectors, normalized so that $v_s' e_s = v_u' e_u = 1$, we know that any process of the form

(37a) $$x_t = s_t e_u + w_t e_s$$

is a bounded solution to (35), where (1) the scalar process $\{s_t\}$ is defined by

(37b) $$s_t = -\sum_{j=0}^{\infty} \lambda_u^{-j-1} v_u' E_t z_{t+j} \quad ;$$

and (2) the scalar process $\{w_t\}$ evolves according to

(37c) $$w_{t+1} = \lambda_s w_t + \varepsilon_{t+1},$$

where $\{\varepsilon_{t+1}\}$ is any bounded random variable such that $E_t \varepsilon_{t+1} = 0$.

The conditions (37a-c) admit of a wide variety of solutions, depending on the choice of the unforecastable random variable $\{\varepsilon_{t+1}\}$. One solution arises from setting $w_t = 0$ for all t, so that $x_t = s_t e_u$; it is easily verified that this solution is of the form (28), so that this is the solution characterized in Section 2. However, if instead we let $\{\varepsilon_{t+1}\}$ be white noise independent of the model's structural disturbances, then (37) describes a "sunspot" equilibrium. For yet

another disturbing possibility, suppose that $\{\varepsilon_{t+1}\}$ depends on the innovations to the model's structural disturbances, e.g.,

$$\varepsilon_{t+1} = \alpha_v v_{t+1} + \alpha_\eta \eta_{t+1} \tag{38}$$

In this case it can be shown that, although output and inflation respond only to the structural disturbances of the model, in equilibrium the responses of both endogenous variables to the structural shocks are *indeterminate*.

Evidently, the robustness of rules that relate the policy instrument to private-sector forecasts depends critically on the magnitudes of the eigenvalues of $\mathbf{M}$. It turns out that whether the unique- or multiple-equilibrium case obtains depends on the parameters of the policy rule, (27), itself. One can show that $\mathbf{M}$ has both eigenvalues outside the unit circle if and only if *either*

$$\phi_y > -(1-\beta)\sigma \quad \text{and} \quad -(1-\beta)\phi_y < \kappa(\phi_\pi - 1) < (1+\beta)(\phi_y + 2\sigma) \tag{39}$$

or

$$\phi_y < -(1-\beta)\sigma \quad \text{and} \quad -(1-\beta)\phi_y > \kappa(\phi_\pi - 1) > (1+\beta)(\phi_y + 2\sigma) \tag{40}$$

Clearly, there exist policy rules that satisfy either (39) or (40) and so lead to a determinate equilibrium of the form (28). However, the requirement that policy satisfy one of these conditions represents a further restriction on what can be achieved through policy rules of the form (27), in which the nominal interest rate is allowed to depend on output and the inflation forecast.

In particular, it is not possible to approximate closely the first-best outcome $b_1 = b_2 = 0$ by using a policy rule of the form (27), without leaving the range of parameter values for which there exists a determinate equilibrium.[16] Thus the concern to maintain the determinacy of equilibrium limits the degree to which a rule of the form (27) can be used to stabilize inflation, even supposing there is no problem of inducing forecasters to collect information and no noise in the forecasts. Furthermore, there is *no* rule of the form (27) that can guarantee that, in a rational expectations equilibrium, fluctuations in the inflation rate will be small. In particular, vigorous "targeting" of the inflation forecast ($|\phi_\pi|$ large) can never ensure that inflation will be stable: To see why, note that if $|\phi_y|$ is also made large, so as to preserve determinacy of equilibrium, then inflation variability in the unique equilibrium with bounded fluctuations is not small, because of the excessive response of monetary policy to the fluctuations in output caused by supply shocks. But if $|\phi_y|$ is chosen to be small, so that b_2 is small in the unique solution of the form (28), then this equilibrium is no longer the only one; and the set of possible rational expectations equilibria in this case includes equilibria with arbitrarily large fluctuations in inflation.

In the last section we saw that the problems with using a rule of the form (27), in which only the inflation forecast was considered, could be ameliorated by a rule of the form (32), which allows policy to respond to forecasts of the policy instrument itself as well as to forecasts of inflation. Unfortunately, problems of indeterminacy afflict rules of the form (32) as well. To analyze policy rules of this form we may restrict attention to the case $\phi_R = 1$, since only this case

is not covered by the discussion above. A rule of the form (32) with $\phi_R = 1$ implies that in any rational expectations equilibrium

$$\phi_\pi \pi_{t+1} = -\phi_y y_t \tag{41}$$

Let us assume $\phi_\pi \neq 0$.[17] Then (41) implies that $\pi_{t+1} = \gamma y_t$, where $\gamma \equiv -\phi_y / \phi_\pi$. Combining this with (23) yields

$$\gamma y_t = (\beta\gamma + \kappa) E_t y_{t+1} - \kappa E_t \theta_{t+1} \tag{42}$$

Equation (42) has a unique bounded solution if and only if $\gamma \neq 0$ and

$$-(1+\beta) < \frac{\kappa}{\gamma} < 1 - \beta \tag{43}$$

This solution is given by

$$y_t = -\frac{\kappa}{\gamma} \sum_{j=1}^{\infty} (\beta + \frac{\kappa}{\gamma})^{j-1} E_t \theta_{t+j} = -\frac{\delta(\frac{\kappa}{\gamma})}{1 - \delta(\beta + \frac{\kappa}{\gamma})} \theta_t \tag{44a}$$

Substitution of (44a) into (41) yields the unique solution for inflation:

$$\pi_{t+1} = -\frac{\delta\kappa}{1 - \delta(\beta + \frac{\kappa}{\gamma})} \theta_t \tag{44b}$$

On the other hand, if $\gamma = 0$ or if either inequality in (43) fails to hold, equilibrium is indeterminate under a rule of the form (32). In particular, any process of the form

$$(45) \quad y_{t+1} = (\frac{\gamma}{\beta\gamma + \kappa})(y_t + \frac{\delta\kappa}{\gamma}\theta_t) + \varepsilon_{t+1}$$

where $\{\varepsilon_{t+1}\}$ is any bounded random variable such that $E_t\varepsilon_{t+1} = 0$, represents a bounded solution to (42). The corresponding solution for inflation is then given by $\pi_{t+1} = \gamma y_t$. As discussed above, these solutions admit of arbitrary responses of inflation and output to fundamental shocks or responses to "sunspot" events. In particular, solutions of the form (45) include equilibria with arbitrarily large fluctuations in output (both in absolute terms and relative to the "natural rate") and, except when $\gamma = 0$, arbitrarily large fluctuations in inflation as well.

Condition (43) shows that determinacy of equilibrium requires, in the case of a rule of the form (32) with $\phi_R = 1$, that $|\gamma|$ be sufficiently large.[18] However, (44b) indicates that the response of inflation to supply shocks can be made small only by making $|\gamma|$ sufficiently small. (Recall that perfect stabilization was achieved, at the end of the last section, by a policy corresponding to $\gamma = 0$.) Thus the requirement of determinacy not only excludes complete stabilization of inflation and output (around the natural rate) through this type of policy, but it also does not allow the first-best equilibrium to be approximated. Indeed, condition (43) substituted into conditions (44a,b) implies that, in any locally unique equilibrium,

$\text{var}(\pi) > (\frac{\kappa\delta}{1+\delta})^2 \text{var}(\theta)$ and $\text{var}(y-\theta) > (\frac{1-\beta\delta}{1+\delta})^2 \text{var}(\theta)$. Any policy implying an equilibrium involving more stabilization of inflation or output than this results in indeterminacy. In particular, the policy $\phi_R = 1$, $\phi_y = 0$, $\phi_\pi \neq 0$ discussed at the end of Section 2 results in indeterminacy.[19]

We have examined the properties of relatively simple policy rules, in the form of (27) or (32), that link the monetary authority's policy instrument to forecasts of inflation and other variables; and we have shown that, if we impose the additional restriction that these rules do not open the economy to possible indeterminacy of equilibrium, these rules cannot be used to perfectly stabilize the economy. Now, as a theoretical matter, it is not true that the requirement of determinacy rules out perfect stabilization by policy rules of any form: To display a counterexample, let R_t^{1f} denote the forecast at the beginning of period t of R_{t+1}, R_t^{2f} the forecast at the beginning of date t of R_{t+2}, and y_t^f the forecast at the beginning of date t of y_{t+1}. Then a policy rule that results in a determinate rational expectations equilibrium with complete stabilization of both inflation and output (around its natural rate) is given by[20]

$$(46) \quad R_t = R_{t-1}^{1f} + \lambda^{-1}(R_t^{1f} - R_{t-1}^{2f}) + \phi_\pi \pi_t + \frac{(\delta\lambda^{-1} - 1)(1-\delta)}{\sigma}(y_t - y_{t-1}^f)$$

where ϕ_π is any nonzero quantity. (The Appendix analyzes the implications of adopting (46) and shows further that it is a specific example of a much wider class of rules with these properties.)

Nevertheless, because of the complexity of (46), and its sensitivity to some elements of the model's structure, we do not wish to offer this particular rule as a practical policy proposal.

The main conclusion of this section is that the requirement of determinacy puts important restrictions on the central bank's choice of policy rules, particularly policy rules that relate the central bank's instrument to explicit or implicit private-sector forecasts. We reiterate that this result by no means precludes the use of private-sector forecasts by the central bank as sources of information not easily attainable by other means. But the information content of private-sector forecasts should be evaluated in the context of an explicit structural model of the economy; and this is especially crucial in the case of policy rules that respond in a highly sensitive manner to changes in private-sector forecasts, whether of inflation or of other macroeconomic variables. Such policies must be adopted with care. Finally, because conclusions regarding the determinacy of rational expectations equilibrium are often sensitive to the dynamic specification of one's model, it would be prudent to analyze the predicted performance of a contemplated rule under alternative specifications.

With these caveats as background, it is also interesting to note that the rule given in (46), which does achieve perfect stabilization within the requirement of determinacy, ties policy to forecasts of interest rates and output but not of inflation. Thus the fact that a central bank's main objective is to hit an inflation target does *not* imply that forecasts of inflation are more useful than forecasts of other macroeconomic variables. Indeed, inflation forecasts are problematic as a source of information for the central bank, for the reason stressed by Woodford (1994a) and in Sections 1 and 2: If complete stabilization of inflation *is* possible, private-sector

inflation forecasts cease to contain any information about exogenous shocks to the economy. On the other hand, even with complete stabilization, output forecasts will reveal the conditional expectation of future values of the aggregate supply shock θ (and hence, if $\delta \neq 0$, the current θ); and interest rate forecasts (or, perhaps, the term structure) will reveal the conditional expectation of future values of the IS disturbance ρ (and hence, if $\lambda \neq 0$, current ρ). Thus, as a general rule, it makes more sense for the central bank to make use of the information revealed by these other forecasts, rather than forecasts of the goal variable.

4. Discussion of Related Proposals

Although we have focused for concreteness on the effects of targeting private-sector forecasts, there are several proposals for managing monetary policy that raise closely related issues. In this section we briefly consider some of these proposals.

4.1. Asset Prices as Indicators of Inflation Expectations

It is sometimes argued that monetary policy should respond to the changes in inflation expectations that may be inferred from various asset prices, as opposed to the explicit forecasts made by individual forecasters. Well-known examples include proposals to adjust monetary policy in response to movements in a commodity price index (e.g., Reynolds, 1982), to movements in long-term bond yields or in interest-rate spreads (e.g., Goodfriend, 1993), to CPI futures prices (e.g., Dowd, 1994; Sumner, 1995), or in response to the yield spread between indexed and non-indexed government bonds (e.g., Hetzel, 1990, 1992).[21] Reasons for making use of asset prices as indicators, instead of (or at least in addition to) explicit forecasts might include the belief that

financial markets aggregate a greater amount of information than is possessed by any small number of market participants alone; a belief that people may reveal their beliefs more truthfully in the way that they risk their own money than in their public statements; or a concern that forecasters could be subject to political manipulation.

Proposals of this kind, however, are potentially subject to all of the drawbacks of "forecast targeting" discussed above. To illustrate, consider a rule of the form

$$R_t = \phi_\pi (R_t - R_t^r) + \phi_y y_t \tag{47}$$

where R_t^r denotes the real interest rate paid on a one-period indexed bond. Suppose furthermore that in a rational expectations equilibrium, the yields on the two types of bonds are linked by

$$R_t^r = R_t - E_t \pi_{t+1} \tag{48}$$

Then equilibrium is determined by relations (33)-(34) and (47)-(48). But, using (48) to eliminate R_t^r, one obtains the same system of three equations as in the case of policy rule (27), when $\pi_t^f = E_t \pi_{t+1}$ is used to eliminate $E_t \pi_{t+1}$. Thus all conclusions with regard to rules of the form (27) are obtained in this case as well. Because our criticism of "forecast targeting" rules of that kind did not depend upon any assumed inadequacy of the forecasters' information or difficulty in eliciting honest reports of their conditional expectations, the use of asset prices to infer "market expectations" does not solve any of the problems previously discussed.

It is true that our discussion of strategic randomization by forecasters as a source of forecast "noise" would not apply to the "market expectations" implied by asset prices. However, asset-price measures of inflation expectations are likely to be contaminated by other sources of noise. For example, Campbell and Shiller (1996) discuss the possible use the spread between indexed and non-indexed bond yields as a measure of inflation expectations, and point out that such a measure could easily be contaminated by changes in expectations regarding the future tax treatment of the two kinds of bonds, or by changes in the inflation risk premium. The presence of these extraneous sources of variation in the yield spread makes a policy rule of the form (47) with a large $|\phi_\pi|$ as unappealing as a policy rule of the form (27) with a large $|\phi_\pi|$.

Further problems with using the inflation expectations implicit in asset prices arise when the connection between asset prices and expected inflation is not reasonably straightforward (as it is in the indexed bond case) but instead may be sensitive to the policy regime. Consider for example Goodfriend's (1993) proposal that the funds rate be raised whenever long rates rise (an increase in long rates being taken to indicate an increase in expected inflation). To simplify the analysis, suppose that the central bank observes the nominal yield R_t^l on a consol. In accordance with the expectations theory of the term structure, suppose that this yield is determined in equilibrium by the relation

$$(49) \quad R_t^l = (1-\beta)\sum_{T=t}^{\infty}\beta^{T-t}E_t R_T + \xi$$

where the term premium ξ_t is assumed to be an exogenous stochastic process. Then it is easy to describe policy regimes under which equilibrium fluctuations in the long rate R_t^l correspond largely to variations in expected inflation. Suppose for simplicity that $\lambda = \delta = 0$ in (24). Conditions (22)-(24) then imply that

$$E_t R_T = E_t \pi_{T+1} + (\sigma\kappa)^{-1}(E_t \Delta\pi_{T+1} - \beta E_t \Delta\pi_{T+2})$$

Substitution of this into (49) yields

$$R_t^l = (1-\beta)R_t + (1-\beta)\sum_{T=t+1}^{\infty} \beta^{T-t} E_t \pi_{t+1} + (1-\beta)\beta(\sigma\kappa)^{-1} E_t \Delta\pi_{t+2} + \xi_t$$

If β is near one (because periods are short), then the return on the consol is essentially a weighted average of the expected rate of inflation over various future periods (plus the term premium, if any).

Nonetheless, in this setting a policy rule of the kind that Goodfriend appears to advocate would not help to guarantee stable prices. Consider a policy rule of the form

(50) $R_t = \phi R_t^l$

for some $\phi > 0$. The complete system of equilibrium conditions is then given by (22)-(23) and (49)-(50). Note that the subsystem (49)-(50) determines the processes $\{R_t, R_t^l\}$, with no reference to the evolution of prices or output, while the subsystem (22)-(23) then determines the

processes $\{\pi_t, y_t\}$, given the equilibrium process for nominal interest rates.

Subsystem (49)-(50) may or may not uniquely determine rational expectations equilibrium processes for short and long nominal rates. If

$$(51) \quad 1 < \phi < \frac{1+\beta}{1-\beta}$$

then the solution for $\{R_t, R_t^l\}$ is not unique, even if one restricts attention to bounded solutions (and assumes a bounded stochastic process for the term premium).[22] In particular, any process for the long rate of the form

$$R_{t+1}^l = \psi R_t^l + (\xi_{t+1} - \beta^{-1}\xi_t) + \upsilon_t$$

where $\psi \equiv [1-(1-\beta)\phi]/\beta$, and $\{\upsilon_t\}$ is any sequence of mean-zero random variables unforecastable one period in advance, solves (49)-(50).[23] The corresponding solution for the short rate is then given by (50). Thus equilibrium variability in interest rates may be arbitrarily large, and may include response to "sunspot" variables. The problem, essentially, is that expectations that interest rates will be high become self-fulfilling, because the expectation of high rates in the future causes long rates to rise, leading the central bank (that interprets this as an "inflation scare") to raise short rates. This variation in interest rates due to self-fulfilling expectations results, of course, in instability of prices and output as well.

But even when (49)-(50) uniquely determine the equilibrium path of nominal interest rates (because (51) does not hold), the resulting system of equilibrium conditions is equivalent to that which obtains if the monetary policy specifies an *exogenous* path for the short-term

nominal interest rate, corresponding to the process $\{R_t\}$ that solves (49)-(50). But a monetary policy of that sort results in indeterminacy of the paths of inflation and output, for the same reason as in our analysis above of a rule of the form (27) with $\phi_\pi = \phi_y = 0$. Thus a policy rule of the form (50) results in indeterminacy *regardless* of the value of ϕ.

4.2. Internal Central Bank Forecasts

A related question concerns the proper use of internal central bank forecasts of inflation in setting monetary policy. Certainly central banks do prepare forecasts of inflation (and of other goal variables), and these forecasts play an important role in discussions of how to adjust policy to current conditions. Svensson (forthcoming) suggests that a central bank that seeks to achieve a target rate of inflation ought to treat its own internal inflation forecast as its "intermediate target".[24] Svensson's discussion seems to imply at times that the inflation forecast, once prepared, provides all the information that policymakers need to have to set policy. For example, he writes (pp. 14-15)[25]: "Although the construction of the forecast is difficult and resource-demanding, the monetary policy conclusions from a given inflation forecast are straightforward: If the forecast is above (below) the target, monetary policy should be adjusted in a contractionary (expansionary) direction. If the forecast is on target, monetary policy is appropriate."

Svensson asserts (p. 3) that targeting of "the central bank's own structural forecast" does not give rise to the problems of circularity associated with targeting the consensus private-sector forecast. Some care must be taken here, however: The mere fact that the forecast is prepared internally does not, in itself, have any effect on the validity

of the analysis in the previous sections. Suppose, for example, that a central bank assigns to a part of its staff the duty of preparing the inflation forecast, after collecting all relevant data as to the current state of the economy, and then asks this part of the staff to report *a single number*--their unconditional inflation forecast--to the policymaking arm of the central bank. The policymakers then act in the way indicated in the above quotation. Suppose furthermore that the staff of the forecasting department are rewarded simply on the basis of the accuracy of the forecasts that they deliver. Then the situation is exactly the same as if the central bank solicits the forecasts of outside forecasters, and our previous analysis applies exactly.

Thus it is certainly not true that the policymakers need no other information from the staff about current conditions other than their inflation forecast. If, for example, the central bank regards the model of Sections 2-3 as an accurate representation of the economy, the policymakers should ask the staff for the information needed to implement rule (46), or some other rule that achieves the desired responses to ρ and θ shocks as a unique rational expectations equilibrium.[26] Of course, if that information is provided and the policy rule (46) is implemented, then in the resulting rational expectations equilibrium the forecasting department should always forecast inflation to equal the target rate. But that fact does not imply that advising the policymakers to implement (26), and providing the information about current conditions needed to do so, is equivalent to advising them to target the inflation forecast and providing information only about the current value of that forecast.

In fact, a careful reading of Svensson's discussion indicates that he does not advocate a regime as simple as our hypothetical. His qualification that the central bank must target a "structural" forecast is a crucial one. This appears to mean not only that the central bank's

internal forecast is prepared with the use of a structural model, but that the model and data on the current state of the economy are used to determine the policy action that, according to the model, should result in a forecast of inflation equal to the target. We have no quarrel with Svensson's recommendation, if it is understood in this way.

But it seems odd to us (and possibly dangerous) to summarize such a recommendation with the phrase "inflation forecast targeting". Such a summary invites the interpretation suggested in our hypothetical (as does the language cited earlier). Further, in places Svensson seems to suggest that it would be sufficient for the staff to present the policymakers with a report of the form, "Our model predicts that inflation will be 1% above target two years from now if the funds rate remains at its current level, but it will be on target if the funds rate is raised immediately by 50 basis points". In other words, it might seem that a forecast that is *conditional* upon the policy action chosen is all that the forecasting department needs to produce. But this result is only possible, in Svensson's analysis, because he assumes a model of inflation determination in which the dynamic linkages between monetary policy and inflation are extremely rudimentary (as in our analysis in Section 1).

To illustrate this point, note that Svensson's structural model implies that inflation is determined by an equation of the form

$$\pi_{t+2} = s_t + x_t - \gamma R_t + \varepsilon_{t+2} \tag{52}$$

where s_t is an exogenous state variable known at date t, x_t is an endogenous state variable that is predetermined at date t, ε_{t+2} is an exogenous random disturbance that is unforecastable as of date t, and we have used the same dating conventions as in previous sections. Given

(52), it is clear that the policy that minimizes the expected squared deviation of inflation from target is given by

$$(53) \quad R_t = \gamma^{-1}(s_t + x_t)$$

Thus, revelation to the policymakers of the conditional forecast implied by (52), namely

$$\pi_t^f(R_t) = s_t + x_t - \gamma R_t$$

suffices to allow them to determine the optimal action (53).

But (52) is an unrealistically oversimplified model of inflation determination. In particular, expectations of future policy play no role, so that it is possible for the model to yield a forecast conditional solely upon the current policy action, without any specification of a policy rule that the central bank is committed to follow in the future. In general, expectations of future policy *will* be relevant to the determination of current inflation (as in our model of Sections 2-3), so that a conditional forecast of that kind cannot be "structural". In the dynamic model analyzed in Sections 2-3 of this paper, there is no obvious way to communicate the information needed for the policymakers to implement the optimal policy (26) in terms of a conditional inflation forecast.[27]

4.3. The Role of Inflation Forecasts in Monitoring Central Bank Performance

Svensson (1996) argues that another advantage of "inflation forecast targeting" as a strategy for monetary policy is that it facilitates monitoring by the public of the central bank's commitment to

its putative inflation target. As we noted in the introduction, the general problem is that there can be a long delay after the announcement of an inflation target until the time when it can determined whether the target is being met (and even then, inflation may deviate from the target rate for reasons outside the control of the central bank). However, Svensson suggests, if people can observe immediately whether or not inflation is *forecasted* to equal the target rate, then central bank misbehavior should become apparent soon after the wrong actions have been taken. Recognizing this, the central bank should be less willing to deviate from its promised policy.

Taking this line of reasoning further, one might seek to ensure central bank accountability by mandating public testimony as to the bank's success in keeping inflation forecasts on target (for example, as part of the Humphrey-Hawkins report to Congress that is made twice a year by the Chairman of the Federal Reserve Board in the U.S.), or even to impose penalties under a "central banker's contract" for deviations of inflation forecasts from the target level. Insofar as accountability is judged to be a serious problem, attention to the behavior of inflation forecasts does seem desirable as a central element in the monitoring of central bank behavior; for it is indeed true under quite general circumstances that one property of an optimal policy is that (rationally formed) inflation forecasts ought not deviate from the target inflation rate. And this particular property of optimal policy is one that is easy to explain, and perhaps easier than others to verify as well.

However, again, these observations do not imply that it is desirable for the central bank to target the particular inflation forecast that is used by the public or the government in evaluating the central bank's performance. For the same reasons discussed earlier, the current level of the inflation forecast should not be the only

aspect of current conditions that is closely tracked by the central bank, nor is it desirable that monetary policy be adjusted sharply in response to small deviations of such a forecast from target. However, on the other hand, a narrow focus on the inflation forecast by the central bank is not required in order for the performance of monetary policy to be monitored in the manner suggested: Suppose that the central bank has sufficient information to pursue an optimal policy; say, (26) is implemented. Then if the public forms a rational forecast, based upon correct knowledge of the state of the economy and of the central bank's action, and also upon a correct model of inflation determination (as assumed by Svensson), then the public will find that its forecast of inflation always equals the target. In other words, the central bank can choose a policy that has the *effect* of keeping private inflation forecasts equal to the target without its having the *form* of a "forecast targeting" rule, and it is desirable that it do so.

If the private sector's information, or model of the economy, does not agree with that of the central bank, however, matters are more complex. Awareness that its performance will be evaluated in terms of private-sector forecasts that it regards as incorrect will surely interfere with the central bank's pursuit of inflation stabilization (according to its own beliefs). But this does not mean that the central bank must practice naive "forecast targeting". It might, for example, implement rule (26), but using the private sector's estimates of the states ρ_t and θ_t (even though it believes that it has better information), and using the private sector's beliefs about the parameters δ and σ (even though it believes these values are incorrect). Implementation of such a rule (and achievement of a reputation for doing so) would result in private sector inflation forecasts that never deviated from target, except perhaps for the

"noise" component of those forecasts. If the central bank's information is more accurate than that of the private sector, then requiring the central bank to adopt this aim would involve a sacrifice of some degree of inflation stabilization that might otherwise have been achievable; and such a method for monitoring central bank performance would not be desirable if the informational difference were believed to be too severe. But it would not result in the difficulties stressed earlier, as long as the central bank responds to its estimate of the factors that determine the private forecasts rather than to its observation of the private forecasts themselves.

A version of such a proposal that might also reduce the extent to which the central bank is required to stabilize incorrect private forecasts rather than actual inflation would be to simply require the central bank to give public testimony about the motivation for its policy stance, that might well include discussion of its *own* inflation forecast. Private-sector forecasts that disagree with that of the central bank might well be matters that would require comment on the part of the central bank, but one could accept an explanation on the part of the central bank of how its own forecasts are made as sufficient demonstration of a good-faith effort to achieve the inflation target. In such a case, the fact that publicly available inflation forecasts play an important role in ensuring central bank accountability would not in any way require that the central bank naively target those forecasts.

5. Conclusion

Our conclusion, in brief, is that there are no short cuts for monetary authorities attempting to target inflation or other macroeconomic variables. To achieve their policy objectives, central banks must both develop structural models of the macroeconomy and gather relevant information from a variety of sources.[28] Monitoring forecasts

of both the bank's target variable and other variables is likely to be a useful part of the information-gathering process; but "targeting" macroeconomic forecasts, in the sense described in this paper, is likely to be risky or even counter-productive.

Appendix: Policy rules implying complete stabilization as the unique REE

For the model used in Sections 2 and 3, there do exist rules that achieve complete stabilization ($\pi_t = 0$ and $y_t = \theta_t$ at all times) as a unique REE. Such rules must make use of information other than the current and lagged values of output and inflation alone, for reasons explained in the text. Under the assumption that the private sector but not the central bank observes current shocks, private forecasts may well provide the necessary additional information---but they must be forecasts of variables other than inflation!

An example of such a rule is given by equation (46) in the text. In this appendix we show constructively how this rule was obtained and verify that it implies complete stabilization as the unique REE of the model. Similar methods can be used to generate many alternative rules with the same properties.

Rule (46) rule belongs to a general class of rules of the form

(A.1) $$R_t = R_{t-1}^{1f} + \phi_\pi \pi_t + (X_t - X_{t-1}^f)$$

where X_t denotes some variable observable by the central bank at time t (perhaps involving private-sector forecasts itself), X_{t-1}^f is the forecast of this variable as of time t-1[29], and $\phi_\pi \neq 0$. Rules of this form are interesting in that, if we assume that forecasters seek to minimize the expected mean squared forecast error, in equilibrium their forecasts will satisfy $R_t^{1f} = E_t R_{t+1}$ and $X_t = E_t X_{t+1}$. If at date t (when π_{t+1} is determined) agents expect (A.1) to hold at date t+1, it follows that in any rational expectations equilibrium we must have

(A.2) $$\pi_{t+1} = 0$$

This result follows directly from advancing (A.1) by one period and taking the conditional expectation of each term with respect to period t information. Further, as we will demonstrate next, a rule of this kind can also keep output at the natural rate in each period, if the variable X_t in (A.1) is chosen appropriately.

To see how X_t should be chosen, note that in an equilibrium with complete stabilization of output we must have $R_t = \rho_t - (\frac{1-\delta}{\sigma})\theta_t$ in each period (eq. (26)), which implies that

$$R_t - E_{t-1}R_t = \upsilon_t - (\frac{1-\delta}{\sigma})\eta_t \tag{A.3}$$

Under the proposed form of the policy rule (A.1), (A.3) holds if and only if

$$X_t - E_{t-1}X_t = \upsilon_t - (\frac{1-\delta}{\sigma})\eta_t \tag{A.4}$$

So we need to choose the variable X_t such that (A.4) is satisfied, in an equilibrium in which $\pi_t = 0$, $y_t = \theta_t$, and $R_t = \rho_t - (\frac{1-\delta}{\sigma})\theta_t$ holds in all periods.

As in the text, suppose that the central bank can observe only current and lagged values of inflation, output, and the nominal interest rate. Then, restricting ourselves to linear deterministic expressions, the central bank must pick a variable of the form

$$\begin{aligned} X_t = &\sum_{j=0}^{\infty}(\psi_{\pi j}\pi_{t-j} + \psi_{yj}y_{t-j} + \psi_{Rj}R_{t-j}) \\ &+ \sum_{k=1}^{\infty}(\omega_{\pi k}E_t\pi_{t+k} + \omega_{yk}E_t y_{t+k} + \omega_{Rk}E_t R_{t+k}) \end{aligned} \tag{A.5}$$

For what patterns of coefficients will a variable defined as in (A.5) satisfy (A.4)? First, the ψ terms in (A.5) for $j \geq 1$ are plainly irrelevant, since their values do not affect the quantity $X_t - E_{t-1}X_t$; so set them to zero WLOG. Second, terms of the form $\psi_{\pi j}$ (for all j) and of the form $\omega_{\pi k}$ (for all k) are likewise irrelevant, since for any rule of the proposed form, in the REE $\pi_t = 0$ in all periods; so set them to zero. Finally, the ψ_{R0} term is redundant, since it can always be

eliminated by a rescaling of the coefficients of the policy rule.[30] So set ψ_{R0} to zero as well.

We may thus restrict attention to variables of the form

$$X_t = \psi_y y_t + \sum_{k=1}^{\infty} (\omega_{yk} E_t y_{t+k} + \omega_{Rk} E_t R_{t+k}) \tag{A.6}$$

In the case of an equilibrium of the desired kind, this would result in equilibrium fluctuations in X_t of the form

$$\begin{aligned} X_t &= \psi_y \theta_t + \sum_{k=1}^{\infty} [\omega_{yk} \delta^k \theta_t + \omega_{Rk} (\lambda^k \rho_t - \frac{1-\delta}{\sigma} \delta^k \theta_t)] \\ &= [\psi_y + \sum_{k=1}^{\infty} \delta^k (\omega_{yk} - \frac{1-\delta}{\sigma} \omega_{Rk})] \theta_t + (\sum_{k=1}^{\infty} \lambda^k \omega_{Rk}) \rho_t \end{aligned} \tag{A.7}$$

which implies that

$$X_t - E_{t-1} X_t = [\psi_y + \sum_{k=1}^{\infty} \delta^k (\omega_{yk} - \frac{1-\delta}{\sigma} \omega_{Rk})] \eta_t + (\sum_{k=1}^{\infty} \lambda^k \omega_{Rk}) \upsilon_t \tag{A.8}$$

Thus (A.4) is satisfied if and only if

$$\sum_{k=1}^{\infty} \lambda^k \omega_{Rk} = 1 \qquad \text{and} \tag{A.9}$$

$$-(\frac{\sigma}{1-\delta}) \psi_y + \sum_{k=1}^{\infty} \delta^k (\omega_{Rk} - \frac{\sigma}{1-\delta} \omega_{yk}) = 1 \tag{A.10}$$

Obviously, there are many ways to satisfy (A.9) and (A.10). An especially simple way is to let $\omega_{Rk} = 0$ for all $k > 1$ and $\omega_{yk} = 0$ for all $k \geq 1$; this implies $\omega_{R1} = \lambda^{-1}$ and $\psi_y = (\frac{1-\delta}{\sigma})(\delta\lambda^{-1} - 1)$, which corresponds to the policy rule (46) in the text. We have thus verified that rule (46), and many rules of the same class, allows an REE with complete stabilization.

But is the equilibrium with complete stabilization the only REE consistent with policy rule (46)? One can show that it is. First, we have already noted that any REE consistent with a policy rule of the form (A.1) must satisfy (A.2.). If we consider only periods t such that the policy rule has been expected to be in force, at least since period t-1, then we must have $\pi_t = 0$, i.e., inflation is stabilized.[31] Substitution of this result into (23) and using (24) yields:

(A.11) $$E_{t-1}y_t = \delta\theta_{t-1};$$

and substitution of both of these results into (22) gives us

(A.12) $$R_t = \rho_t + \frac{\delta\theta_t - y_t}{\sigma}$$

Now we have seen that all rules of the class (A.1) satisfy

(A.13) $$R_t - E_{t-1}R_t = X_t - E_{t-1}X_t,$$

while (A.12) implies that

(A.14) $$R_t - E_{t-1}R_t = (\rho_t - \lambda\rho_{t-1}) + \frac{\delta}{\sigma}(\theta_t - \delta\theta_{t-1}) - \frac{1}{\sigma}(y_t - \delta\theta_{t-1})$$

where we have used (A.11) and (24) to simplify the right-hand side. Equating the right-hand sides of (A.13) and (A.14) and using the definition of X from above, we find that

(A.15)

$$\lambda^{-1}(E_tR_{t+1} - E_{t-1}R_{t+1}) = \frac{(1-\delta\lambda^{-1})(1-\delta)-1}{\sigma}(y_t - \delta\theta_{t-1}) + (\rho_t - \lambda\rho_{t-1}) + \frac{\delta}{\sigma}(\theta_t - \delta\theta_{t-1})$$

where (A.11) has again been used to eliminate $E_{t-1}y_t$.

Taking the conditional expectation of both sides of (A.12) with respect to t-1 information, and using (A.11) and (24) to simplify, we obtain

(A.16) $$E_{t-1}R_t = \lambda\rho_{t-1} - \frac{\delta(1-\delta)}{\sigma}\theta_{t-1}$$

Similarly, taking the conditional expectation based on information from one period earlier, we obtain

(A.17) $$E_{t-2}R_t = \lambda^2\rho_{t-2} - \frac{\delta^2(1-\delta)}{\sigma}\theta_{t-2}$$

Finally, substituting expressions (A.16) and (A.17) for the two conditional expectations on the left-hand side of (A.15), we get

(A.18) $$\frac{\delta[1+\lambda^{-1}(1-\delta)]}{\sigma}(y_t - \theta_t) = 0$$

So long as $\delta \neq 0$ and $\delta \neq 1+\lambda$, (A.18) implies that $y_t = \theta_t$. Thus we have uniquely determined the equilibrium paths of both output and inflation and shown that, in this unique equilibrium, output and inflation are perfectly stabilized at all times ($y_t = \theta_t$, $\pi_t = 0$, for all t).[32] (A.12) implies a unique equilibrium path for interest rates as well, given by (26).

ENDNOTES

Acknowledgments: We would like to thank V.V. Chari, Larry Christiano, Marvin Goodfriend, Ken Kuttner, Ed Nelson, Ken Rogoff, Julio Rotemberg, Lars Svensson, Alberto Trejos, and an anonymous referee for helpful comments. Both authors acknowledge research support from the National Science Foundation.

---

[1] Central banks recently making highly-publicized switches to an inflation-targeting approach include New Zealand, Canada, the United Kingdom, Sweden, and Australia; many other countries have moved toward making price stability the primary objective of the central bank, often in conjunction with institutional reforms to increase central bank independence (Goodhart and Vinals, 1994; Debelle and Fischer, 1994; Leiderman and Svensson, 1996; Haldane, 1996; Bernanke and Mishkin, forthcoming). A few countries, notably Germany and Switzerland, have long used medium-term inflation objectives as an important component of their monetary policy-making (Bernanke and Mishkin, 1992; Bernanke and Mihov, forthcoming). In the United States, the Fed has so far resisted the adoption of explicit inflation targets, although the perception is that the relative importance of price stability among the Fed's objectives has increased. Formal legislation introduced in the U.S. Senate by Senator Mack would amend the Humphrey-Hawkins bill and require the Fed to pursue an inflation target.

[2] Actually, Hall and Mankiw suggest that the central bank target forecasts of nominal GDP growth rather than inflation; however, many of the criticisms made here are independent of the particular variable whose forecast is to be targeted.

[3] Romer and Romer (1996) have presented evidence that Fed forecasts are superior to those of the private sector. Even if this finding is correct, however, it does not rule out the possibility that the private sector has information that the Fed would like to infer (the reverse may also be true, of course). It is well known, for example, that the Fed attempts to use private-sector information implicit in asset prices.

[4] More generally, our point is that the most useful information variable will *not* be the target variable. Thus inflation forecasts might be useful to a central bank that seeks to stabilize nominal GDP rather than inflation.

[5] In this case, one can show that there are two constrained-optimal solutions to the central bank's problem, corresponding to the two values of ϕ--one negative and one greater than 2--that satisfy (10) with equality. Under either of these policies, $\text{var}(\pi) = \sigma_\varepsilon^2 + c < \sigma_\varepsilon^2 + \sigma_\varepsilon^2$.

[6] For a related analysis of strategy regarding incentives for scattering of forecasts, see Lamont (1996).

[7] Note that the strategic dispersion of forecasts requires that the forecasters be uncertain about what the realized value of inflation will be, which in this model is guaranteed by the presence of the shock ε.

[8] The proof proceeds as follows: Differentiation of (12) indicates that two local minima exist, corresponding to the roots of the equation $k\phi = (\phi - 1)^{-3}$, where $k \equiv \sigma_u^2/\sigma_\varepsilon^2$. The two roots are as described in the text. Furthermore, defining $\xi \equiv 1/\phi$, this equation may equivalently be written $k(1-\xi)^3 = \xi^4$. Letting $\xi^* = 1/\phi^*$, $\xi^{**} = 1/\phi^{**}$, one observes that $0 < 1 - \xi^{**} < 1 < 1 - \xi^*$, implying that $|\xi^*| > |\xi^{**}|$ and hence $|\phi^*| < |\phi^{**}|$. Finally, since at either of the local minima $|\phi - 1|^{-3} = k|\phi|$, one must have $|\phi^* - 1| > |\phi^{**} - 1|$. Thus each of the first two terms on the right side of (12) takes a smaller value at $\phi = \phi^*$ than at $\phi = \phi^{**}$, so that ϕ^* is the global minimum.

[9] Other interpretations of the random disturbance term ρ_t are possible. For example, in a model with government spending, exogenous shifts in the share of resources consumed by the government would give rise (by changing the relationship between private consumption and total output) to a disturbance term of the same form, in the log-linear approximation.

[10] For further discussion of aggregate supply specifications with time lags in price-setting of the kind assumed here, see Rotemberg and Woodford (forthcoming).

[11] Strictly speaking, we have in mind that the central bank can condition the nominal interest rate on the current value of output, so that the interest rate and output are simultaneously determined. It is not necessary to think of y_t as being determined strictly before R_t is chosen.

[12] Nor is it possible, we may add, to approximate this outcome arbitrarily closely. An informal argument is as follows: In any

equilibrium, under the information assumptions of the last paragraph, the central bank can infer only the linear combination of v_t and η_t that is revealed by the observation of y_t. For any equilibrium that is near-optimal, innovations in y_t will be almost perfectly correlated with η_t and so not a close approximation to the variable that the central bank needs to know. Thus the assumption that prices are determined at least one period in advance implies that observing the history of output and inflation alone does not provide the central bank enough information to achieve complete stabilization.

[13] For general discussion of the issue of determinacy of equilibrium as a criterion for choosing among policy rules, see Guesnerie and Woodford (1992, section 8) and Woodford (1994b).

[14] Although we wish now to allow for the possibility of sunspot equilibria, we assume that any sunspot state affecting pricing decisions made in period t (and taking effect in t+1) is observed by the forecasters before they announce the forecast π_t^f.

[15] We consider only bounded solutions because these solutions to our log-linear approximations to the exact equilibrium conditions constitute approximate solutions to the exact (nonlinear) conditions. Even in the case that there is a unique bounded solution to the log-linear equilbrium conditions, there may exist a multiplicity of solutions to the exact conditions; but one cannot determine this from the form of the log-linearized conditions alone. Further, even when the exact conditions have multiple solutions, the bounded solution represents an approximate characterization of a rational expectations equilibrium that is at least locally unique, in an appropriate topology (Woodford, 1986). In the "indeterminate" case analyzed here, by contrast, the REE is not even locally unique; thus it is less plausible that the economy can be relied to settle into the particular stationary equilibrium of the form (28), rather than into one of the large multiplicity of nearby equilibria.

[16] Proof: Suppose instead that there exists a sequence of policies $\{\phi_\pi^n, \phi_y^n\}$, and an associated sequence of equilibria $\{b_1^n, b_2^n\}$ satisfying (31) for each n, with the property that $b_1^n \to 0$ and $b_2^n \to 0$ as $n \to \infty$. Then the quantity in square brackets in the first equation of (31) must become unboundedly large as n grows, while the quantity $(1-\delta+\sigma\phi_y^n)$

must not grow at the same rate, and indeed must eventually become an arbitrarily small fraction of the quantity in the square brackets. This can occur only if $|\phi_\pi^n| \to \infty$ while $|\phi_y^n|/|\phi_\pi^n| \to 0$. But neither condition (39) nor condition (40) allows $|\phi_\pi^n|$ to grow without bound unless $|\phi_y^n|$ grows at the same rate, so that $|\phi_\pi^n|/|\phi_y^n|$ remains bounded and $|\phi_y^n|/|\phi_\pi^n|$ remains bounded away from zero. Thus one obtains a contradiction.

[17] It is easy to show that a rule of form (32) with $\phi_\pi \neq 0$ does not stabilize inflation, unless there is no stochastic disturbance to the aggregate supply equation (23).

[18] In the case in which all prices are fixed one period in advance, which corresponds to the limiting case of the model of Section 2 with κ infinitely large, no finite $|\gamma|$ is large enough, i.e., any rule of the form discussed in the text results in indeterminacy.

[19] Actually, in this case, because $\gamma = 0$, complete stabilization of inflation is guaranteed in equilibrium; however, equilibrium output is indeterminate and the fluctuations of output around the "natural rate" can be unboundedly large.

[20] We assume here that $\lambda \neq 0$, $\delta \neq 0$, and $\delta \neq 1+\lambda$. In the special case that $\delta = 1+\lambda$ exactly, a more complicated variant of this rule will still work.

[21] Proposals of this kind were given particular attention within the Federal Reserve System following the endorsement of an approach of this general type by Vice Chairman Manuel H. Johnson (1988).

[22] This can be shown using the same methods as in Section 3. The analysis is simplest if (49) is replaced by the quasi-differenced form $R_t^l = (1-\beta)R_t + \beta E_t R_{t+1}^l + \zeta_t$, where $\zeta_t \equiv \xi_t - \beta E_t \xi_{t+1}$. Note that bounded processes $\{R_t, R_t^l\}$ satisfy this equation if and only if they satisfy (49).

[23] Condition (51) is just the condition under which $|\psi| < 1$, so that this defines a bounded process for the long rate, if $\{\xi_t, v_t\}$ are bounded processes.

[24] The idea that an inflation forecast should be treated as an "intermediate target" is found in various discussions of recent monetary

policy strategy in the U.K., such as King (1994) and Bowen (1995). Svensson goes beyond these discussions in stressing that it is an internal forecast that should be targeted.

[25] We quote here from the working paper version of Svensson's paper, dated June 1996.

[26] Of course, the advice of the staff will also be required to explain the advantages of rule (26), which depends upon the form and parameter values of the structural model (22) -- (24).

[27] Svensson (1997) provides an example of an economy with forward-looking elements in which next period's expected inflation is a sufficient statistic for policy. This example is rather special, though, in that timing assumptions are made so that the expectation of inflation one period ahead contains information useful to the policymakers, but is independent of current policy actions. Further, Svensson does not show that the rational expectations equilibrium he considers is unique under the forecast-based instrument rule that he proposes.

[28] It was not our intention in writing this paper to improve employment prospects for economists, though our conclusions have that implication.

[29] If X_t is itself the forecast at t of some variable Y_{t+1}, then X^f_{t-1} may alternatively be the forecast at t-1 of Y_{t+1}; the essential property is that, in a rational expectations equilibrium, $X^f_{t-1} = E_{t-1}X_t$.

[30] This statement requires $\psi_{R0} \neq 1$, which we must assume since the proposed policy rule does not determine the nominal interest rate if $\psi_{R0} = 1$.

[31] In the remainder of this derivation we maintain the assumption that the policy rule has been expected to be in force since at least the previous period. Obviously, if period t is the first period of a new policy regime, π_t depends on expectations of future policy held at t-1 and need not equal zero.

[32] Even if $\delta = 1 + \lambda$, it can be shown that it is possible to obtain complete stabilization in the unique rational expectations equilibrium, using a slightly more complicated policy rule. When $\delta = 0$, no rule achieves the fully optimal outcome, for in this case the $\{\theta_t\}$ shocks do not perturb the aggregate supply relation (23), since $E_{t-1}\theta_t = 0$. It follows that there is no way that policy can arrange for output to vary with the $\{\theta_t\}$ shocks unless these shocks can be directly observed by the central bank (rather than inferred from forecasts).

LITERATURE CITED

Bernanke, Ben, and Ilian Mihov, "What Does the Bundesbank Target?", European Economic Review, forthcoming.

Bernanke, Ben, and Frederic Mishkin, "Central Bank Behavior and the Strategy of Monetary Policy: Evidence from Six Industrialized Countries", in Olivier Blanchard and Stanley Fischer, eds., NBER *Macroeconomics Annual*, Cambridge MA: MIT Press, 1992.

_____, "Inflation Targeting: A New Strategy for Monetary Policy?, *Journal of Economic Perspectives*, forthcoming.

Bernanke, Ben and Michael Woodford, "Inflation Forecasts and Monetary Policy", NBER working paper no. xxx, 1997.

Blanchard, Olivier, and Charles M. Kahn, "The Solution of Linear Difference Models Under Rational Expectations", *Econometrica* 48 (1980), 1305-11.

Blinder, Alan S., "Central Banking in Theory and Practice -- Lecture II: Credibility, Discretion and Independence," Marshall Lecture, University of Cambridge, May 5, 1995.

Bowen, Alexander, "British Experience with Inflation Targeting", in Leonardo Leiderman and Lars E.O. Svensson, eds., *Inflation Targets*, London: CEPR, 1995.

Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12 (1983), 383-398.

Campbell, John and Robert Shiller, "A Scorecard for Indexed Government Debt", in Ben Bernanke and Julio Rotemberg, eds., NBER *Macroeconomics Annual*, Cambridge MA: MIT Press, 1996.

Cochrane, John H., "Identifying the Output Effects of Monetary Policy," NBER working paper no. 5154, June 1995.

Kerr, William, and Robert G. King, "Limits on Interest Rate Rules in the IS Model," *Economic Quarterly*, Federal Reserve Bank of Richmond (Spring 1996), 47-75.

King, Mervyn, "Monetary Policy in the U.K.", *Fiscal Studies* 15 (1994), 109-128.

King, Robert G., and Mark W. Watson, "Money, Prices, Interest Rates and the Business Cycle," *Review of Economics and Statistics* 78 (1996), 35-53.

Lamont, Owen, "Macroeconomic Forecasts and Microeconomic Forecasters", unpublished, University of Chicago, May 1996.

Laster, David, Paul Bennett, and In Sun Geoum, "Rational Bias in Macroeconomic Forecasts," Research paper no. 9617, Federal Reserve Bank of New York, July 1996.

Leiderman, Leonardo, and Lars E. O. Svensson, eds., *Inflation Targets*, London: Centre for Economic Policy Research, 1995.

Mayes, David G. and Brendon Riches, "The Effectiveness of Monetary Policy in New Zealand", *Reserve Bank Bulletin* (New Zealand) 59 (1996), 5-20.

McCallum, Bennett T., and Edward Nelson, "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis", NBER working paper no. 5875, January 1997.

Reynolds, Alan, "The Trouble with Monetarism", *Policy Review* 21 (Summer 1982), 19-42.

Roberts, John M., "New Keynesian Economics and the Phillips Curve," *Journal of Money, Credit and Banking* 27 (1995), 975-984.

Romer, Christina, and David Romer, "Federal Reserve Private Information and the Behavior of Interest Rates", NBER working paper no. 5692, July 1996.

Rotemberg, Julio and Michael Woodford, "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy", Ben Bernanke and Julio Rotemberg, eds., *NBER Macroeconomics Annual*, forthcoming.

Sumner, Scott, "The Impact of Futures Price Targeting on the Precision and Credibility of Monetary Policy", *Journal of Money, Credit, and Banking* 27 (1995), 89-106.

Svensson, Lars E. O., "Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets", *European Economic Review*, forthcoming.

_____, "Inflation Targeting: Some Extensions", unpublished, IIES, Stockholm, February 1997.

West, Kenneth D., "Comment on Hall and Mankiw", in N.G. Mankiw, ed., *Monetary Policy*, Chicago: Univ. of Chicago Press for NBER, 1994.

Woodford, Michael, "Nonstandard Indicators for Monetary Policy: Can Their Usefulness be Judged from Forecasting Regressions?" in N.G. Mankiw, ed., *Monetary Policy*, Chicago: Univ. of Chicago Press for NBER, 1994 (a).

Woodford, Michael, "Determinacy of Equilibrium under Alternative Policy Regimes: Introduction," *Economic Theory* 4 (1994), 323-6 (b).

Woodford, Michael, "Control of the Public Debt: A Requirement for Price Stability?" NBER working paper no. 5684, July 1996.

Yun, Tack, "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37 (1996), 345-370.