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CYCLICAL PRODUCTIVITY WITH UNOBSERVED INPUT VARIATION

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ABSTRACT

In this paper, we derive and estimate relationships governing variable utilization of capital and labor for a firm solving a dynamic cost-minimization problem. Our method allows for (i) imperfect competition, (ii) increasing returns to scale, (iii) unobserved changes in utilization, (iv) unobserved changes in technology, (v) unobserved fluctuations in the factor prices of capital and labor, (vi) unobserved fluctuations in the shadow price of output, and (vii) the non-existence of a value-added production function. We can estimate the parameters of interest without imposing specific functional forms or using restrictions from assuming the existence of a representative consumer. We find that variable capital and labor utilization explain 40-60 percent of the cyclicality of the Solow residual in U.S. manufacturing, so true technology shocks have a lower correlation with output than the RBC literature assumes. Controlling for variable utilization also eliminates the evidence for increasing returns to scale. We show that our model-based proxies for variable utilization are valid even when extending the workweek of capital potentially has two costs: a shift premium paid to workers, as well as a higher rate of depreciation. Thus, these proxies can be used under very general conditions in a wide range of empirical work.

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Why is productivity procyclical? That is, why do measures of labor productivity and total factor productivity rise in booms? The answer to this question sheds light on the relative merits of different models of business cycles. The literature suggests three main explanations. First, measured fluctuations in productivity might reflect *exogenous* changes in production technology. Second, productivity (appropriately measured) may be procyclical because of increasing returns to scale: in this case the economy *endogenously* becomes more efficient by moving to higher levels of activity. Third, if inputs are systematically mismeasured, *measured* productivity may be procyclical even if true productivity does not change. The gap between actual and measured productivity most likely comes from unobserved changes in capital utilization or in the intensity of work effort.

It is important to know the extent to which the cyclicality of productivity truly reflects variable factor utilization, for several reasons. First, if variable utilization is present and economically significant, it can serve as an important propagation mechanism for both technology and non-technology shocks. Second, if factor utilization varies over the business cycle, it can help explain some of the lead-lag relationships observed in the data (for example, the fact that productivity is a leading indicator).[1] Third, it allows one to explain the cyclicality of measured productivity without invoking increasing returns to scale or unobserved fluctuations in technology. This is important, since models based on these mechanisms often paint a very different picture of the causes of business cycles: for example, recent models incorporating large increasing returns stress the possibility of "indeterminacy," in which Keynesian "animal spirits" drive business cycles.[2] Fourth, unobserved factor utilization interacts with other issues: for example, models with imperfect price flexibility have dramatically different implications if firms can vary the service flows of their factors of production.[3] Our work is related to a burgeoning empirical literature on the puzzle of cyclical productivity.[4] To our knowledge, however, we are the first to nest variable capital utilization, variable labor effort, imperfect competition, and increasing returns to scale in an empirical framework derived explicitly from a dynamic optimizing model.

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[1] Burnside and Eichenbaum (1996) make the first point; Bils and Cho (1994) make the second.
[2] For example, Farmer and Guo (1994).
[3] See Ball and Romer (1990) and Kimball (1995).
[4] Recent papers include Abbott, Griliches and Hausman (1988), Aizcorbe and Kozicki (1995), Basu (1996), Bils and Cho (1994), Burnside and Eichenbaum (1996), Finn (1995), Gordon (1990), Hall (1988, 1990) Rotemberg and Summers (1990), Sbordone (1994), and Shapiro (1993).

Given the alternative explanations for procyclical productivity and the need to distinguish among them, it is clearly important to estimate the basic parameters governing variable utilization of capital and labor in the U.S. economy. The resulting estimates can be used to calibrate general-equilibrium models incorporating variable factor utilization, such as Bils and Cho (1994) or Burnside and Eichenbaum (1996), augmented to include increasing returns and imperfect competition if necessary.

But in order to estimate these parameters, we require an estimation method that is robust to a large variety of phenomena. These of course include technology shocks, increasing returns and variable factor utilization. But it is also important to allow for the possibility of imperfect competition — which must exist if the increasing returns are internal to the firm — as well as for implicit contracts between firms and their factors of production. Imperfect competition implies that marginal revenue is unobserved; implicit contracts imply that we cannot observe high-frequency fluctuations in wage and rental rates. Finally, we wish to do all this in a setup that avoids identification through functional form assumptions — throughout the paper, we lean towards the use of general functional forms — and also eschews identification achieved by assuming the existence of a representative consumer or an efficient equilibrium.

Our requirements may seem stringent, but we show that under quite general conditions one can in fact estimate the key parameters governing variable factor utilization in a setup that meets all our criteria. The key features of our approach are the use of an optimizing model, imposing conditions from cost-minimization, and using gross-output data to estimate the parameters of interest. When we make the standard assumption that the penalty for utilizing capital more intensively is that it depreciates faster, our method allows us to estimate the parameters of interest even though we assume that we observe neither the true quantities of capital and labor input nor their shadow prices. We show, however, that if the cost of higher utilization involves a shift premium paid to workers who work late at night as well as a higher rate of depreciation, the parameters are generally not all identified. But even under these conditions we can obtain some important results, notably estimates of the properties of technology shocks and the degree of returns to scale. That is, we can control for variable service flow even when we cannot say whether it represents a change in labor utilization or a change in capital utilization.[5]

[5] This result should be useful in a wide range of applied work that attempts to estimate properties of production technology or firm behavior using productivity data. Applications range from trade (e.g. Levinsohn, 1993) to

Our basic insight comes from the premise that a cost-minimizing firm should equate the marginal benefits of all factors of production to their marginal costs. This is true for all inputs, observed or unobserved. Thus, certain increases in observed inputs should also proxy for unobserved changes in utilization.[6] By including both the usual inputs and the proxies in an otherwise-standard estimating equation for the parameters of the production function, we can recover the true production function parameters (especially the degree of returns to scale) as well as the parameters of the technology governing changes in utilization. Residuals from this equation give us a time series for technology shocks, which are of interest in their own right.

We find that controlling for variable utilization eliminates the evidence for large increasing returns of the sort found by Hall (1990). Our results indicate that both variable capital utilization and variable labor utilization are important. We estimate two parameters that are crucial for calibrating structural general-equilibrium models: the elasticity of unobserved changes in labor effort with respect to observed changes in hours per worker, and the elasticity of the rate of depreciation with respect to capital utilization. We discuss the interpretation of the estimates, especially the issue of whether the special role of hours per worker reflects variable utilization or increasing returns to the length of the workday. We conclude that the latter cannot account for the size of the coefficient we estimate. Since we can control for cyclicality of the Solow residual coming from both non-constant returns to scale and variable utilization, our regression residuals should be interpreted as true technology shocks. We find that controlling for variable utilization significantly reduces the cyclicality of technology shocks relative to naive Solow residual accounting. However, although we find that changes in utilization are important in accounting for fluctuations in output, we also find that variations in the capital stock resulting from utilization-induced changes in the depreciation rate are quite small. Thus, most applied researchers will probably not find it necessary to correct the capital stock for this type of measurement error.

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industrial organization (e.g. Olley and Pakes, forthcoming). In ongoing work with John Fernald we use this result to study the properties of aggregate technology shocks, and find that they are negatively correlated with contemporaneous employment — a result similar to that of Gali (1996).

[6] Thus our paper is closely related to Sbordone (1994). The major difference is that Sbordone exploits the intertemporal implications of this insight, while we concentrate on intratemporal conditions that we believe are more robust (for example, to alternative hypotheses about expectations formation).

We derive these results by assuming that the cost of utilizing capital more intensively is that it depreciates faster, which has become a standard assumption in the literature. We then extend the literature on capital utilization to include a second cost of using capital more intensively: a shift premium paid to workers for working at unusual times, in addition to the cost of faster depreciation in use.[7] We show that conventional data cannot distinguish between variable labor utilization and variable capital utilization when the shift premium — paid to labor — is part of the cost of changing the workweek of capital. Definitively apportioning the costs of higher utilization between the shift premium and variable depreciation requires better data on capital utilization than those currently available.[8]

Previous papers have generally met one or two of our criteria, but not all of them. For example, Bils and Cho (1994) and Burnside and Eichenbaum (1996) assume constant returns and perfect competition; Basu (1996) and Bils and Cho (1994) assume that observed factor prices are allocative; and Burnside and Eichenbaum (1996) calibrate the key parameter of their model via a functional-form assumption that allows them to obtain an elasticity from a steady-state condition.[9] Bils and Cho (1994) also restrict capital and labor utilization to move together one for one, and calibrate their model based on Schor's (1987) data set on how actions per hour (a measure of labor effort) vary over the business cycle in the United Kingdom. Since the extent of variable labor utilization depends partly on labor market institutions (which determine adjustment costs), Schor's estimates may not apply to the U.S. economy. Finally, Burnside, Eichenbaum and Rebelo (1995) and Basu (1996) estimate the degree to which factor utilization varies over the business cycle, but their methods cannot uncover the structural parameters that one needs in order to model firms' decisions to vary utilization in response to different shocks.

Our more general model helps us understand and unify insights from earlier papers, including some that are not explicitly based on optimization. For example, we clarify a debate in the literature over the right variable to use as a proxy for variable capital utilization. Abbott, Griliches and Hausman (1988) use hours per worker as a proxy for all forms of factor utilization.[10] On the other hand, Burnside and

[7] Shapiro (1996a) estimates that the shift premium is empirically quite significant; we discuss his estimates below.
[8] Shapiro (1996b) uses unpublished data from the Survey of Plant Capacity, which we plan to use in later work. However, these data are only available starting in 1974. A longer time series would be invaluable.
[9] We discuss the last point at length in Section IV.
[10] Bils and Cho (1994) assume that capital utilization must vary in proportion to labor input, which also makes hours per worker a good proxy for both labor effort and variable capital utilization.

Eichenbaum (1996) claim that one needs to include a measure of the marginal revenue product of capital. Our model shows that Abbott et al. (1988) are essentially correct if the sole cost of higher capital utilization is the higher labor cost associated with the shift premium. Burnside and Eichenbaum are right if the sole cost of higher capital utilization is a higher rate of depreciation.[11]

The rest of the paper comprises six sections. In the first, we present our model and derive our estimating equations. In keeping with the rest of the literature, we initially assume that the sole cost of variable capital utilization is higher depreciation. The second section discusses the data we use, and the extent to which it does and does not conform to the theoretical concepts we have in mind. The third section presents our results. In section four we discuss why our results on variable depreciation are at odds with some of the other literature on this topic. In the next section, we discuss how to generalize the one functional-form assumption we use in our first derivation. In section five, we extend our model by including a shift premium. We present results suggesting that the shift premium is an important determinant of capital utilization. The final section summarizes our conclusions and indicates directions for future research.

I. The Model

We model a firm that faces adjustment costs in both investment and hiring, so that both the amount of capital (number of machines and buildings), K, and employment (number of workers), L, are quasi-fixed. However, the number of hours per week for each worker, H, can vary freely, with no adjustment cost. Both capital and labor also have freely variable utilization rates. The cost of higher capital utilization, U, is quicker depreciation through extra wear and tear on the capital. The cost of higher labor utilization, E, is a higher disutility on the part of workers that must be compensated with a higher wage. The benefit of higher utilization is its multiplication of effective inputs. (We first consider the problem

[11] However, due to their calibration method, Burnside and Eichenbaum (1996) do not in fact estimate the elasticity of depreciation with respect to utilization. We discuss their calibration method in Section IV. Burnside and Eichenbaum also assume that there is no adjustment cost for changing the capital stock. We show that when one departs from this assumption, one needs to include an indicator of Tobin's q in addition to an indicator of the marginal revenue product of capital.

without a shift premium, which considerably complicates the derivation. The full derivation, including the shift premium, is presented in Appendix A.)

Consider the following problem for the representative firm of an industry:

$$\underset{E,U,H,I,A,M,N}{\text{Min}} \int_0^{\infty} e^{-\int_0^t r\,d\tau}\left[WLG(H,E)+P_M M+P_N N+WL\Psi(A/L)+P_I K\,J(I/K)\right]dt \quad \text{(1a)}$$

subject to

$$\overline{Y} = F(UK,EHL,M,N;Z) = Z\Gamma\left((UK)^{c_K}(EHL)^{c_L}M^{c_M}N^{c_N}\right) \quad \text{(1b)}$$

$$\dot{K} = I - \delta(U)K \quad \text{(1c)}$$

$$\dot{L} = A \quad \text{(1d)}$$

Variables are defined as follows. Y is output; H is hours per worker; L is employment (number of workers); E is work intensity (hourly effort expended); U is capital utilization; M is materials usage; N is energy usage; I is gross investment; and A is hiring net of separations. $WG(H,E)$ is total compensation per worker as specified by an implicit contract; $WL\Psi(A/L)$ is the total cost of changing the number of employees; $P_I K J(I/K)$ is the total cost of investment; P_M is the price of materials; and P_N is the price of energy. F is a generalized Cobb-Douglas production function; Z is the gross-output-augmenting level of technology; $\delta(U)$ is the variable rate of depreciation; and c_K, c_L, c_M, and c_N are the shares of the inputs in total cost. We omit time subscripts for clarity.

Note that we have set up a cost-minimization problem, and we shall use only the intra-temporal conditions associated with minimizing costs. Profit-maximization is thus irrelevant to our derivations. This allows us to ignore the firm's behavior in product markets, which may be very complex. Firms may sell output with sticky prices or engage in strategic interactions in a repeated-game setting, but the existence of such behavior does not affect our results.

One may ask why we expend considerable effort to make our procedure robust to imperfect competition and increasing returns when various authors have suggested that returns to scale are approximately constant, which implies that markups of price over marginal cost must be small.[12] The reason is that even small markups, well within the standard-error bands of current estimates, are

[12] See, for example, Burnside et al. (1995), Basu (1996), Burnside (1996), and Basu and Fernald (1997).

consistent with large cyclical fluctuations in the markup.[13] These fluctuations imply that the marginal revenue product of inputs can change substantially in ways that are not captured by observed changes in the output price. Hence, one needs an empirical procedure robust to such fluctuations.

Using a perfect-foresight model amounts to making a certainty-equivalence approximation but even departures from certainty equivalence should not disturb the key results, which rely only on intra-temporal optimization conditions.

We assume the production function has constant cost shares, but can have increasing returns. These increasing returns may even involve fixed costs: in that case, we need to bear in mind that the degree of returns to scale is only locally constant. Writing $\Omega = (UK)^{c_K}(EHL)^{c_L} M^{c_M} N^{c_N}$ for the input aggregate, the (local) degree of returns to scale is

$$\gamma(\Omega) = \frac{\Omega\Gamma'(\Omega)}{\Gamma(\Omega)}. \tag{2}$$

In addition to the assumption that Ψ and J are convex, and the appropriate technical assumptions on G in the spirit of convexity and normality,[14] it is helpful to make some normalizations in relation to the normal or "steady state" levels of the variables. Using an asterisk to denote these normal levels, let

$$\delta(U^*) = \delta^*$$
$$J(\delta^*) = \delta^*$$
$$J'(\delta^*) = 1$$
$$\Psi(0) = 0.$$

Also, we assume that the marginal employment adjustment cost is zero at a constant level of employment:

$$\Psi'(0) = 0.$$

We solve the representative firm's problem using the standard current-value Hamiltonian, letting λ, q, and θ be the multipliers on constraints (1b), (1c) and (1d) respectively. Using numerical subscripts for derivatives of the production function F with respect to its first, second and third arguments, and literal

[13] See Bils (1987) and Rotemberg and Woodford (1991) for evidence indicating that there are substantial cyclical fluctuations in the markup.

[14] The conditions on G are easiest to state in terms of the function Φ defined by $\ln G(H,E) = \Phi(\ln H, \ln E)$. Convex Φ guarantees a global optimum; assuming $\Phi_{11} > \Phi_{12}$ and $\Phi_{22} > \Phi_{12}$ ensures that optimal H and E move together.

subscripts for derivatives of the labor cost function G, the firm's seven intra-temporal first-order conditions for cost-minimization are:

$$\lambda EL\, F_2(UK, EHL, M, N; Z) = WL\, G_H(H,E) \tag{3}$$

$$\lambda HL\, F_2(UK, EHL, M, N; Z) = WL\, G_E(H,E) \tag{4}$$

$$\lambda K\, F_1(UK, EHL, M, N; Z) = qK\, \delta'(U) \tag{5}$$

$$\lambda\, F_3(UK, EHL, M, N; Z) = P_M \tag{6}$$

$$\lambda\, F_4(UK, EHL, M, N; Z) = P_N \tag{7}$$

$$\theta = W\Psi'(A/L) \tag{8}$$

$$q = P_I\, J'(I/K). \tag{9}$$

The Euler equations for employment and capital are:

$$\dot{\theta} = r\theta - \lambda EHF_2 + WG(H,E) + W\left[\Psi(A/L) - (A/L)\Psi'(A/L)\right] \tag{10}$$

$$\dot{q} = \left[r + \delta(U)\right]q - \lambda UF_1 + P_I\left[J(I/K) - (I/K)J'(I/K)\right]. \tag{11}$$

As the Lagrange multiplier associated with the level of output, λ can be interpreted as marginal cost. Since the firm internally values output at marginal cost, λF_1 is the marginal value product of effective capital input, λF_2 is the marginal value product of effective labor input, λF_3 is the marginal value product of materials input, and λF_4 is the marginal value product of energy input.[15] The marginal physical products themselves are:

$$F_1 = \gamma c_K \frac{Y}{UK} \tag{12a}$$

$$F_2 = \gamma c_L \frac{Y}{EHL} \tag{12b}$$

$$F_3 = \gamma c_M \frac{Y}{M} \tag{12c}$$

$$F_4 = \gamma c_N \frac{Y}{N} \tag{12d}$$

These equations are actually valid for any homothetic production function, with c_K, c_L, c_M, and c_N representing the cost shares and γ the degree of returns to scale. Assuming a generalized Cobb-Douglas production function ensures that the cost shares are constant.

[15] Typically, of course, marginal cost equals marginal revenue, so these are also the marginal revenue products.

Labor Utilization

Equations (3) and (4) can be combined into an equation implicitly relating E and H:

$$\frac{HG_H(H,E)}{G(H,E)} = \frac{EG_E(H,E)}{G(H,E)}. \tag{13}$$

In words, the elasticity of labor costs with respect to H and E must be equal. Why? Because on the benefit side, the elasticities of effective labor input with respect to H and E are equal. Given the assumptions on G, (13) implies a unique, upward-sloping E-H expansion path, so that we can write

$$E = E(H), \qquad E'(H) > 0. \tag{14}$$

Thus, our theory implies that the unobservable intensity of labor utilization E can be expressed as a monotonically increasing function of the observed number of hours per worker H.

By the same logic as embodied in (13), we can argue that in the long run, the elasticities of labor costs with respect to H and E must be equal to the long-run elasticity of labor costs with respect to L, which is 1, since the elasticities of effective labor input with respect to all three of L, H and E are equal to each other. Only the adjustment cost prevents this from being true in the short run. To look at the long run formally, the steady state of constant[16] employment toward which the model adjusts has $A = 0$ and $\theta = \dot{\theta} = 0$, so most of the terms in (10) disappear.[17] Combining the steady-state version of (10) with the steady-state version of (3) shows that the steady-state elasticity of labor cost with respect to hours per worker is 1; combining the steady-state versions of (10) and (4) shows that the elasticity of labor cost with respect to effort must also be 1.

Note that we are able to derive the labor utilization proxy without either solving a full general equilibrium model or assuming perfect competition in the product market — the two approaches taken by Burnside, Eichenbaum and Rebelo (1993) and Burnside and Eichenbaum (1996). By contrast, our key assumptions are cost minimization, competitive behavior in *factor* markets, and compensation per hour that depends on both effort and hours worked in such a way that it makes sense to increase effort per hour when workers are putting in longer hours.[18,19] These are much more plausible assumptions than perfect

[16] If there is a trend, the entire model can be expressed in terms of detrended quantities from the very beginning.
[17] Remember that $\Psi(0) = \Psi'(0) = 0$.
[18] While we work with implicit wage contracts, so that work in one period may be paid in later periods, Bils (1987) presents evidence indicating that firms find increases in hours per worker quite costly even within the same year.

competition in both output and factor markets, or the large number of assumptions needed to solve a full general-equilibrium model.

Capital Utilization

Substituting from (12a) into (5) and rearranging, we find that the level of capital utilization depends on the degree to which the current marginal value product of capital exceeds future marginal products:

$$U\delta'(U) = \lambda\gamma c_K \frac{Y}{qK} \tag{15}$$

Since fluctuations in marginal cost λ, the degree of returns γ and the marginal value of capital q are difficult to observe directly, we would like to express these factors in terms of other variables that are more readily observed.

The problem with trying to measure q directly is not just the difference between the marginal and average value of capital but also the noisiness of the asset prices one would use to gauge the average value of capital. Instead of trying to measure q directly, we use equation (9) to express q as the price of investment goods times a function of I/K. Note that Tobin's q is actually q/P_I in our notation. Equation (9) can be inverted to say that I/K is a function of Tobin's q.

The first order conditions for materials usage (6) and energy usage (7) are the keys to expressing the product $\lambda\gamma$ in terms of observables. Combining these with the two expressions for the marginal products of these two inputs, equations (12c) and (12d), we find

$$\lambda\gamma = \frac{P_M M}{c_M Y} = \frac{P_N N}{c_N Y} \tag{16}$$

Bils's results assume that the statutory overtime premium is at least partially allocative. Trejo (1991) finds evidence that the overtime premium is partially offset by implicit contracting arrangements that alter the base wage. Trejo's evidence supports our basic framework, since it suggests that labor compensation is in fact governed by implicit contracts, and it is difficult to imagine any sensible implicit contracts that would *not* satisfy our assumptions in footnote 11.

[19] For the existence of the *E(H)* function, it is only necessary that more intensive use of employees optimally involves an increase in hours, not an increase in effort alone. *E'(H) > 0*, which depends on effort increasing along with hours is not needed for the derivation. This assumption just provides an expected sign for the parameter ζ which we define below.

Thus, we have two alternative expressions for $\lambda\gamma$, one based on the ratio of the revenue share to the cost share of materials, the other based on the ratio of the revenue share to the cost share of energy. This gives us two measures of the marginal value product of capital:

$$\lambda\gamma c_K = \frac{c_K}{c_M}\frac{P_M M}{K}. \tag{17a}$$

and

$$\lambda\gamma c_K = \frac{c_K}{c_N}\frac{P_N N}{K} \tag{17b}$$

These measures of the marginal value product of capital are important because there is no reliable direct measure of short-run fluctuations in the rental rate of capital. In the absence of such measures, fluctuations in c_K/c_M or c_K/c_N also cannot be observed directly. We focus on equation (17a) to identify movements in c_K/c_M from changes in the quantities of inputs and the structure of the production function. (Identifying movements in c_K/c_M is the *only* place in which the structure of the production function matters for our method.) Although in principle one could use a fully general production function with c_K/c_M determined as a function of all four input quantities, estimating that many parameters would put a large burden on our data set (in particular, it would put too large a burden on our instruments).[20] Therefore, we impose more structure on the production function. For simplicity, we begin with the generalized Cobb-Douglas assumption, which makes c_K/c_M constant. In Section V we conduct a sensitivity analysis, examining other forms of the production function.

We use equation (17a) rather than (17b) to identify the marginal value product of capital because of our concern that the mode of production may not adjust very quickly and finely to changes in the price of energy because energy has a small cost share in most industries.[21] Equation (17a) does not depend on the first-order conditions for optimal energy usage since it depends only on the price of materials. Thus (17a)

[20] We emphasize, however, that given sufficiently rich data and powerful instruments, we could easily work with the fully-general case.

[21] We view the existence of noisy and highly visible energy conservation measures for a substantial period of time after the OPEC oil price increase of 1973— often prodded along by the government — as evidence of such slow adjustment.

is robust to adjustment costs of changing energy usage in a way that equation (17b), which depends on the energy price, is not.[22]

Substituting the expression for the marginal revenue product of capital (equation 17a) and the expression for q (equation 9) into (15) leads to our desired expression for capital utilization in terms of observables:

$$U\delta'(U) = \frac{c_K}{c_M}\frac{P_M M}{P_I K}\frac{1}{J'(I/K)}. \tag{18}$$

Estimating the Determinants of Unobserved Factor Utilization

We find it useful to define a number of elasticities in terms of steady-state values of different variables. In the definitions that follow, a starred variable is the steady-state value of that variable.

To find the percentage change in effective labor input, first define

$$\zeta \equiv \frac{H^* E'(H^*)}{E(H^*)}.$$

Then, using lowercase letters for natural logarithms and d for changes, a log-linearization for effective labor input implies that

$$d\ln(EHL) = dl + dh + de = dl + (1+\zeta)dh.$$

To find the percentage change in effective capital input, first define

$$\Delta \equiv \frac{U^* \delta''(U^*)}{\delta'(U^*)}$$

and

$$j \equiv \frac{(I/K)^* J''((I/K)^*)}{J'((I/K)^*)} = \frac{\delta^* J''(\delta^*)}{J'(\delta^*)}$$

Then with a constant c_K/c_M, (18) implies

[22] We do not model energy adjustment costs explicitly because the details, which could be quite intricate, are irrelevant to the validity of the equations we use in our estimation, just as we need not know the precise nature of the adjustment costs facing the firm as long as we know that it minimizes costs. Note, for example, that the shape of the adjustment cost function for the number of employees does not affect our estimation. The investment adjustment cost function does matter but only because changes in capital utilization affect the rate of capital accumulation by destroying capital.

$$du = \frac{1}{1+\Delta}\left(dp_M + dm - dp_I - dk\right) - \frac{j}{1+\Delta}(di - dk). \tag{19}$$

Adding *dk* gives a measure of the percentage change in effective capital input.

Finally, define a conventional measure of overall input growth based only on the readily-observable H, K, L, M and N: $X = K^{c_K}(HL)^{c_L} M^{c_M} N^{c_N}$. Then

$$dx = c_K\, dk + c_L\,(dl + dh) + c_M\, dm + c_N\, dn .$$

Putting everything together, the production function implies the log-linearization

$$\begin{aligned} dy &= \gamma^*\, d\omega + dz \\ &= \gamma^*\, c_K\left[dk + \frac{1}{1+\Delta}\left(dp_M + dm - dp_I - dk\right) - \frac{j}{1+\Delta}(di - dk)\right] \\ &\quad + \gamma^*\, c_L\left[dl + (1+\zeta)dh\right] \\ &\quad + \gamma^*\, c_M\, dm + \gamma^*\, c_N\, dn + dz \\ &= \gamma^*\, dx + \gamma^*\zeta c_L dh + \frac{\gamma^*}{1+\Delta} c_K\left(dp_M + dm - dp_I - dk\right) - \frac{\gamma^* j}{1+\Delta} c_K (di - dk) + dz \end{aligned} \tag{20}$$

This suggests a regression of dy on dx, $c_L dh$, $c_K\left(dp_M + dm - dp_I - dk\right)$ and $c_K(di - dk)$ with the error interpreted as the *true* technology shock dz. By actually running a regression of $dy - dx$ on the same four variables, including dx, the results are equivalent, and we can see how much of the cyclicality of the measured (cost-based) Solow residual[23] can be explained by these four variables and how much should be attributed to the technology shock dz.

II. The Data

Performing the tests proposed here requires that we examine the usage of materials relative to other inputs, so we need a data set that has data on gross output and materials input, as well as the usual capital and labor input. We use unpublished data provided by Dale Jorgenson and Barbara Fraumeni on industry-level inputs and outputs. The data consist of a panel of U.S. manufacturing industries, at

[23] For the distinction between the usual revenue-based Solow residual and the cost-based residual, see Hall (1990).

approximately the 2-digit S.I.C. level, for the years 1949-85.[24] For a complete description of the data set, see Jorgenson, Gollop, and Fraumeni (1987).

These sectoral accounts seek to provide accounts that are, to the extent possible, consistent with the economic theory of production. Output is measured as gross output, and inputs are separated into capital, labor, materials, and energy.

Jorgenson's input data are available both with and without an adjustment for input quality. Without the quality adjustment, the Jorgenson measures of labor and capital input are essentially the standard ones — hours worked and the capital stock. From the perspective of a firm, however, the relevant measure of its input of, say, labor is not merely labor hours. The firm also cares about the relative productivity of different workers. In creating a series for labor input, Jorgenson, Gollop, and Fraumeni assume that wages are proportional to marginal products. This allows them, in essence, to calculate quality-adjusted labor input by weighting the hours worked by different types of workers (distinguished by various demographic and occupational characteristics) by relative wage rates. Hence, labor input can increase either because the number of hours worked increases, or because the "quality" of those hours increases. Similarly, Jorgenson, Gollop and Fraumeni adjust inputs of capital and intermediate goods for changes in quality. We use quality-adjusted data for labor but not for capital. The capital adjustment is based on implicit rental rates that are inconsistent with our model of imperfect competition — for example, it assumes that economic profits are zero. The labor quality adjustment is not subject to the same problems.[25]

The main discrepancy between our theory and the available data comes in the measurement of capital input. We have assumed that the depreciation rate for capital goods varies over time as the degree of utilization of capital changes. Our capital stock data, however, are based on a perpetual inventory

[24] The only major difference between the industry groupings in the Jorgenson data and the standard 2-digit S.I.C. classification is that Motor Vehicles (S.I.C. 371) are separated from other transportation equipment (S.I.C. 372-79). Thus, there are 21 manufacturing industries in the data set rather than the usual 20.

[25] For example, the validity of Jorgenson's labor quality adjustment is unaffected by markup pricing, because markups affect the overall level of wages but not relative wages.

method that assumes constant exponential depreciation. Our results indicate that this source of measurement error is not significant.[26] We discuss this issue at length in Section IV.

To estimate the required payments to capital, we follow Hall and Jorgenson (1967), Hall (1990), and Caballero and Lyons (1992), and compute a series for the user cost of capital r. The required payment for any type of capital is then $rP_K K$, where $P_K K$ is the current-dollar value of the stock of this type of capital. In each sector, we use data on the current value of the 51 types of capital, plus land and inventories, distinguished by the BEA in constructing the national product accounts. Hence, for each of these 53 assets, the user cost of capital is

$$r_s = (\rho + \delta_s)\frac{(1 - ITC_s - \tau d_s)}{(1-\tau)}, \qquad s = 1 \text{ to } 53.$$

ρ is the required rate of return on capital, and δ_s is the depreciation rate for this asset. ITC_s is the asset-specific investment tax credit, τ is the corporate tax rate, and d_s is the asset-specific present value of depreciation allowances. We follow Hall (1990) and Caballero-Lyons (1992) in assuming that the required return ρ equals the dividend yield on the S&P 500. Jorgenson and Yun (1990) provide data on ITC_s and d_s that is specific to each type of capital good. Given required payments to capital, computing the cost shares is straightforward. To calculate the steady-state cost shares, we take time averages of the observed shares over our sample period.

Since input use is likely to be correlated with technology shocks, we seek demand-side instruments for input use. To solve the "transmission problem" of endogeneity between productivity shocks and input growth, we use versions of the instruments advocated by Ramey (1989) and Hall (1988): the growth rate of the price of oil deflated by the GDP deflator; the growth rate of real government defense spending; and the political party of the President. We use the current value and one lag of each instrument. We also use annual versions of the monetary policy instruments suggested by Romer and Romer (1989); we use two lags of a dummy variable that is one in a year when there is a Fed contraction and zero otherwise.

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[26] Nevertheless, we could correct even for this small deviation by using a recursive procedure: first estimate variable depreciation based on the existing capital stock figures, construct a new time series for the capital stock based on those estimates, and iterate to convergence.

III. Results

We estimate several versions of equation (20) using our data set of two-digit manufacturing industries, restricting coefficients to be equal across industries. All of the right-hand-side variables are instrumented with the aggregate demand instruments discussed in Section II. All the equations include industry-specific constants and a post-1974 dummy variable. Perron (1989) and others have argued that allowing a trend break in 1974 is a useful way to model trends over the postwar period.

Table 1 presents our basic results. The table reports two sets of estimates. The first line reports results using standard BLS data. The second line uses Jorgenson's correction for the composition of labor.

Table 1 shows that once we correct for cyclical utilization, we find no evidence of increasing returns: controlling for variable utilization, returns to scale are precisely estimated as being almost exactly 1. This result confirms the findings of Basu (1996) and Eichenbaum et al. (1995), but using proxies predicted by a dynamic, optimizing model of firm behavior.[27] As noted by Rotemberg and Woodford (1995), finding constant returns to scale, along with the low rates of economic profits observed in the data, implies that the average ratio of price to marginal cost must also be close to one. As we noted in the introduction, these results are problematic for the increasing-returns models of business cycles (and growth) that have become popular in the literature.[28]

We do, however, find strong evidence of variable utilization: the estimates all have the right sign, the sizes are generally plausible, and all the reduced-form coefficients are significant. The evidence for variable labor utilization is particularly strong: the estimate of ζ is more than 1.1, indicating that a one percent increase in hours per worker is accompanied by a greater than one percent increase in effort per hour. The coefficient on the marginal value product of capital is about 0.5, indicating values of

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[27] Our method avoids the problems noted by Basu (1995, 1996) with using materials or energy as proxies for variable factor services. Of course, another problem with these ad-hoc proxies is that they do not allow one to estimate the structural parameters that are needed to calibrate stochastic general-equilibrium models.

[28] See Rotemberg and Woodford (1995) for an excellent survey of such models. However, if one assumes that measured profit rates understate true profits (perhaps because some profits are extracted by labor rather than capital), one can probably justify a markup ratio of 1.1 or so. Since this is a markup of 10 percent on gross output, it is still large enough to imply significantly countercyclical markups on value added in the empirical model of Rotemberg and Woodford (1991).

approximately 1 for Δ. This is about twice the size assumed by Burnside and Eichenbaum (1996). We discuss below why their different methodology leads them to assume a smaller Δ. Our estimates are not statistically different from their calibrated value of $\Delta = 0.56$; indeed our estimates are not significantly different from zero at conventional significance levels. We believe, however, that to give a range of reasonable parameters for calibrated models it is the 95 percent confidence interval that is most useful, since it gives a sense of the likely range of values for this important parameter, given the data. Rather than assuming a precise value obtained through a restrictive assumption, it is important to establish that the data are not very informative about Δ, which warns modelers to consider a wide range of values.

The 95 percent confidence interval for Δ is about [-0.2, 2], although on economic grounds we should truncate the negative values. The point estimates imply that depreciation increases with utilization in a convex fashion as we would expect: a one percent increase in utilization increases the *marginal* rate of depreciation by about one percent. We also find significant investment adjustment costs: we estimate values of j of about 0.4.[29] Quality-adjusting the data does not significantly affect any of the coefficients.

We now discuss the implications of our results for variable labor and capital utilization.

Labor Utilization

A strong role for variable labor utilization is certainly consistent with other evidence. Schor (1987) uses meticulously-measured data on time-and-motion studies in a representative sample of British manufacturing firms, and concludes that actions per hour (a good indication of effort) have an elasticity of about 0.5 with respect to the workweek of labor.[30] Hence, we are confident that hours per worker is a good proxy for labor effort. Our estimate is higher than Schor's, however. Thus, it is worth asking whether the true ζ may be smaller than it appears from our estimates.

We have argued that hours per worker proxies for labor effort. The main alternative hypothesis is that the positive ζ comes from a technological fact: increasing returns to hours per worker, perhaps due to

[29] A j of 0.42 implies that the capital stock of a firm adjusts at a rate of about 25.5 percent per year toward its long-run value; see Kimball and Weil (forthcoming).
[30] See Bils and Cho (1994) for a good summary of the macroeconomic implications of Schor's paper.

a daily "setup cost."[31] (This increasing returns to hours per week is conceptually separate from increasing returns to scale in production, since hours per week is not a scale variable but rather is confined to a well-defined range.) We can use evidence from labor economics to estimate how large ζ should be for this reason alone. Suppose we specify the increasing returns to hours per worker by assuming that there is a fixed setup cost associated with working — for example, the time required to talk with one's manager, catch up on the office gossip, and settle down to work. Suppose this fixed cost is $\bar{h}$, and the total labor input of each worker is h. For reasons we discuss below, we believe that $\bar{h}$ is about one-eighth of the total workday, so increasing hours per worker increases total labor input $8/7 = 1.14$ times more than hiring new workers. Thus, even if hours per worker were not a proxy for labor hoarding, we should find ζ approximately equal to 0.14, rather than zero.

Why assume that $\bar{h}$ is about one-eighth of the workday? Hamermesh and Rees (1988) find that the premium for full-time as opposed to half-time work is 16 percent. If wages are linearly proportional to daily marginal worker products, an $\bar{h}$ of this size implies that a full-time worker produces $\left(\frac{7}{8}\right)/\left(\frac{3}{4}\right) = 1.167$ as much per paid hour as a half-time worker, yielding a wage premium for full-time workers of 16.7 percent relative to the half-time wage.[32,33]

Even allowing for increasing returns to hours per worker of 1.17, and taking Schor's estimate that the elasticity of effort with respect to hours is about 0.5, we should expect an estimated ζ of about 0.75 (calculated as 1.5*1.17 - 1). Our estimate of 1.1-1.3, however, is somewhat higher. This might be simply because labor market institutions (and hence the compensation function, G, or adjustment costs) are different in the U.S. and the U.K. But we show that there might also be another reason. We investigate this issue further in our discussion of capital utilization.

[31] See Hansen and Sargent (1988) or Bils and Cho (1994) for models where output depends on the composition of the labor input as well as on total hours worked. G. Hall (1996) compares the propagation mechanisms in the Hansen-Sargent model with those in the variable-effort model of Burnside, Eichenbaum and Rebelo (1993).
[32] This full-time premium should not be confused with the shift premium that we discuss below.
[33] Selection issues probably lead us to overestimate the full-time premium, since high-quality workers are also likely to be the ones who are offered full-time work. Thus, an $\bar{h}$ of one hour based on Hamermesh and Rees's estimates is probably somewhat high.

Capital Utilization

One interesting prediction of our model is that capital utilization should be negatively related to marginal q, and hence to investment. The intuition is that investment is a way of building up the capital stock, while utilization accelerates depreciation, diminishing the capital stock. One is not apt to do both simultaneously, unless the current marginal value product of capital greatly exceeds future marginal value products. We view this prediction as a stringent test of our model. Productivity is a highly procyclical series, as is investment. But our model requires that the partial correlation between them be negative, given the other variables in the regression. It is thus gratifying that we do in fact find a strong negative relationship between the two, and therefore conclude that capital adjustment costs, j, are both economically and statistically significant.

Taking variable depreciation to be the cost of utilizing capital more heavily, we also find evidence of changes in capital utilization in response to variations in the marginal value product of capital. As we noted, however, our estimate of Δ, the elasticity of marginal depreciation with respect to utilization, is about twice the 0.56 assumed by Burnside and Eichenbaum (1996). Given this parameter value, Burnside and Eichenbaum conclude that changes in capital utilization are the major cause of the measured procyclicality of productivity, and an important propagation mechanism for exogenous shocks. The larger Δ we find implies less variation in utilization than Burnside and Eichenbaum suggest.[34]

At this point, we note an alternative possible cost of increasing capital utilization: part of that cost may come from extra payments to workers necessary to compensate them for working on later shifts. In this case, as we show in Section V, hours per worker becomes a proxy for both capital and labor utilization. So our larger estimates of ζ may come from the dual role of dh, and indicate that variable utilization of both capital and labor are important. We find this explanation persuasive because, as we noted before, our measures of ζ are somewhat higher than we would expect from Schor's (1987) estimates.

[34] A larger Δ implies that variations in utilization are more costly, so utilization should vary less in response to any given shock. This does not mean, however, that changes in utilization, if they occur, are unimportant.

We have now described three reasons why we might find a significant *dh* in our estimation: It may indicate variable labor effort, variable capital utilization, or increasing returns to hours per worker. Clearly we would like to know the relative importance of these three factors. But it is important to recognize that all three explanations imply that *dh* belongs in the regression. For some purposes — for example, estimating the degree of retuns to scale, or computing the size and properties of true technology shocks — the reason for the positive coefficient on *dh* is unimportant. So while we try to sort out these differing interpretations of ζ, it is important to bear in mind that for some purposes knowing the correct interpretation is immaterial.

The Corrected Solow Residual

Finally, we see how well our approach does at explaining the cyclicality of the Solow residual. In Table 3, we regress the industry cost-based Solow residuals and our corrected residuals on a business-cycle indicator, the growth rate of aggregate manufacturing output. We use two measures of aggregate output: the growth of manufacturing value added and the growth of manufacturing gross output. According to the first measure, we explain about 40 percent of the cyclicality of the Solow residual; by the second measure we explain 60 percent. Thus, we explain a significant fraction of the cyclicality of the Solow residual, though according to our decomposition technology shocks still account for about half of the cyclicality of productivity.

We go on to investigate the economic effects of technology shocks. We are interested in the effect of technology shocks on the average industry. In Table 4 we examine the average correlation of each measure of technology with output growth for the corresponding industry. As with aggregate output, we find substantial differences between our residuals and Solow residuals: the sectoral Solow residual has a correlation with sectoral output that is almost twice the correlation of our residual. If we accept that our residuals are better measures of technology shocks, then this finding is problematic for the standard real-business-cycle model. That model already produces too large a correlation between output and the Solow residual, which it takes to be the true measure of technology shocks. If the correlation between true technology shocks and output is even smaller, then the problem is more severe.

IV. Depreciation in Use

Given an estimate of the variations in capital utilization we can calculate the effects of this variable capital utilization on the capital stock. Based on the assumption that additional depreciation is the only cost of higher capital utilization, the first-order condition (5) implies that the value of the additional capital destroyed is exactly equal to the value of the additional output. To make this principle operational, start with (11), which implies at the steady state that,

$$q^* = \frac{\lambda^* U^* F_1}{r + \delta(U^*)}.$$

Substituting this value for q^* into the steady-state version of (5) implies that $U^* \delta'(U^*) = r + \delta(U^*)$. To a linear approximation, the capital destroyed by additional wear and tear is

$$\delta'(U^*)[U - U^*] \tag{21a}$$

or, to a log-linear approximation,

$$U^* \delta'(U^*)[\ln(U) - \ln(U^*)] = [r + \delta(U^*)][u - u^*]. \tag{21b}$$

Equation (19), in turn, implies that

$$u - u^* = \frac{1}{1+\Delta}\left[\ln\left(\frac{P_M M}{P_I K}\right) - \ln\left(\frac{P_M^* M^*}{P_I^* K^*}\right)\right] - \frac{j}{1+\Delta}\left[\ln\left(\frac{I}{K}\right) - \ln\left(\frac{I^*}{K^*}\right)\right]. \tag{22}$$

Using the values for j and Δ from the second line of Table 1, we construct a time series for u by summing our implied series for du from the beginning of our sample to each time period t. We then study the time-series properties of u-u^*, which we define as the demeaned and detrended version of the constructed series u. We find that this series is well approximated as an AR(1) process with autoregressive coefficient $\phi = 0.77$ (standard error = 0.11, s.e.e. = 0.0023, $R^2 = 0.55$). Thus, the deviations of utilization from its steady-state value are quite persistent. Such persistence tends to create greater capital mismeasurement through long-lasting departures of the depreciation rate from its steady-state value.

Even though utilization changes are persistent, however, we find that variable depreciation has only a small effect on the capital stock. Using the estimated value of ϕ and given values for the steady-state real interest rate, r, and depreciation rate, $\delta(U^*)$, we can readily calculate the variance of the measurement error in the capital stock coming from capital destruction.[35] We find that the standard error in the measurement of the log capital stock relative to its true value is 0.0025 (or 0.25 percent). We can also calculate the standard deviation of the growth of the true capital stock relative to its measured value, dk. This is the statistic that is relevant for gauging the error in TFP calculations, or in estimates like ours that use the growth rate of the capital stock rather than its level. We find that this standard deviation is 0.00032 (or 0.032 percent) per year.

Thus, variable depreciation does not seem a significant source of error in the capital stock figures reported by the BEA. By comparison, the standard deviation of the measured annual growth rate of the privately-owned capital stock is about 0.0068 (or 0.68 percent), so the extra volatility coming from variable depreciation is less than 5 percent of the normal variation in capital. As a source of measurement error in the capital growth rate, this strikes us as second-order relative to other issues that have been identified — for example, changes in capital quality, variations in capital composition, or even deviations in the steady-state rate of depreciation from the strict exponential form assumed in the perpetual inventory method.[36]

Comparison with Burnside and Eichenbaum

These results can be compared with those of Burnside and Eichenbaum (1996), who claim that allowing for variable capital utilization and variable depreciation requires one to make larger corrections to the figures for the capital stock and for true capital input. We show, however, that the larger effects they find arise from a restrictive functional form assumption that forces Δ to be smaller and therefore forces the implied movements in capital utilization to be correspondingly larger. Indeed, Burnside and

[35] We take $r = 0.025$ and $\delta(U^*) = 0.13$. For the latter, see Jorgenson and Sullivan (1981).

[36] For example, Jorgenson et al. (1987) note that capital equipment with a higher depreciation rate must, *ceteris paribus*, yield a higher service flow per unit time. Applying this composition correction, they find that the usually-reported growth rate of private capital services has a measurement error from this source with an annual standard deviation of about 0.02 — a mismeasurement of capital input about two orders of magnitude more important than neglecting variable depreciation.

Eichenbaum's functional form assumptions are so restrictive that they can deduce $\Delta = U^* \delta''(U^*)/\delta'(U^*)$ from those assumptions knowing only the values for r and $\delta(U^*)$!

Burnside and Eichenbaum assume that $\delta(U) = \delta_1 U^{1+\Delta}$, where δ_1 and Δ are constants. Since our equations (5) and (11) hold in their model, too,[37]

$$1+\Delta = \frac{U^* \delta'(U^*)}{\delta(U^*)} = \frac{r + \delta(U^*)}{\delta(U^*)}. \tag{23}$$

But then, given this functional form,

$$\frac{U^* \delta''(U^*)}{\delta'(U^*)} = \Delta = \frac{r}{\delta(U^*)}.$$

This is a small number, since the real interest rate is only a fraction of the depreciation rate for most types of capital. Burnside and Eichenbaum get $\Delta = 0.56$ by assuming a fairly high value for the real interest rate and a low value for $\delta(U^*)$. Using our parameters, their approach would yield $\Delta = 0.19$. Our point estimate for Δ is about 1, although statistically we are unable to reject the value of 0.56 that Burnside and Eichenbaum assume.

We see no good reason to assume their particular functional form *a priori*. Our nonparametric approach is equivalent to using the flexible functional form $\delta(U) = \delta_0 + \delta_1 U^{1+\Delta}$. From the perspective of this flexible functional form, one can see that Burnside and Eichenbaum make the implicit assumption that there is no "rust and dust." That is, their functional form precludes any fixed component to depreciation that does not depend on the level of capital utilization. In effect, they assume that "wear and tear" is the whole story. Thus their approach rules out the heretofore standard case of "rust and dust" only, $\delta(U) = \delta_0$. By contrast, our approach allows as special cases both Burnside and Eichenbaum's assumption of only "wear and tear" and the heretofore standard case of only "rust and dust," along with all the possibilities in between. With the flexible functional form, since $r + \delta(U) = U\delta'(U) = (1+\Delta)\delta_1 U^{1+\Delta}$, the steady-state share of depreciation that is wear and tear is

$$\frac{\delta_1 U^{1+\Delta}}{\delta_0 + \delta_1 U^{1+\Delta}} = \frac{r + \delta(U^*)}{(1+\Delta)\delta(U^*)}. \tag{24}$$

[37] This equation corresponds to Burnside and Eichenbaum's unnumbered formula after their equation (29). In their notation, $\Delta = \phi - 1$.

Thus, our overall estimate (giving the average over all the industries we study) implies that about 60 percent of depreciation is wear and tear, while the other 40 percent is rust and dust.[38]

Despite our methodological differences, this summary statistic shows that at one level our result is not substantially different from that of Burnside and Eichenbaum. They take one polar view: that depreciation is only a function of utilization. The traditional view is at the opposite pole: it holds that depreciation is only a function of time. Our point estimates suggest that Burnside and Eichenbaum's position is closer to being right than is the traditional view, but the data actually favor a position about halfway in between the two extremes.

Nevertheless, the difference between our positions still has important practical consequences. First, as we have seen, the two methods imply different degrees of measurement error of the official capital stock. Second, our method naturally implies that utilization and capital services are less variable than Burnside and Eichenbaum's calibration would imply.[39] Third and perhaps most important, our method makes clear that Δ is a parameter that needs to be estimated, and in fact is not pinned down very precisely by the data because it has to be estimated as the reciprocal of a fairly small number. Thus, even the small standard error of the reduced-form parameter necessarily implies that there is large uncertainty about the structural parameter Δ. Consequently, economic modelers should conduct sensitivity analysis of their results using a wide range of values for this parameter.

V. Low Elasticities of Substitution Between Capital Services, Materials and Energy

Up to this point we have maintained the assumption of a generalized Cobb-Douglas production function. That assumption is actually used only to derive an expression for the movements in capital utilization. There, we used the Cobb-Douglas assumption to ensure constancy of c_K/c_M. In principle, the approach we have been using can be extended to a production function of arbitrary shape by using the relationship

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[38] Since the nonparametric approach itself focuses only on the local properties of the depreciation function, it does not by itself identify the fraction of wear and tear versus rust and dust.

[39] Craig Burnside noted in a personal communication that using our estimate of Δ in the Burnside-Eichenbaum (1996) model would lower the standard deviations for both utilization and capital services by about one-third.

$$c_K / c_M = \psi(UK, EHL, M, N).$$

for some function ψ in conjunction with the other equations of the model (including the equation giving labor effort as $E(H)$) to solve for movements in U. Assuming only homotheticity for the production function (which implies homogeneity of degree zero for the function ψ) or a departure from homotheticity of known form, it is still theoretically possible to identify all of the parameters of the model. Even without the assumption of homotheticity, it is still theoretically possible to identify all parameters other than γ.[40]

However, in practice, allowing for an arbitrary form of the production function would require us to estimate a regression with seven right-hand-side variables, with the parameters identified from the projections of these variables onto a relatively small set of instruments. This would put too great a burden on our limited set of instrumental variables. Therefore, in our examination of the data at hand, we will be modest in the departures from Cobb-Douglas we implement empirically.

Jorgenson and Griliches (1967) suggest the assumption of a zero elasticity of substitution between energy and capital services, leading energy and capital services to be used in fixed proportion. Burnside, Eichenbaum and Rebelo (1995) have recently revived this proposal and used it to estimate the degree of returns to scale, controlling for variable utilization. In our notation, this fixed proportion is $UK = \text{constant} * N$, implying $du = dn - dk$.

On substitution into the production relation, the counterpart to (20) becomes

$$dy = \gamma\left[c_L(dl + dh) + c_M dm + (c_K + c_N)dn\right] + \gamma\zeta c_L dh + dt.$$

This leads to a regression with only two independent variables, $\left[c_L(dl + dh) + c_M dm + (c_K + c_N)dn\right]$, and $c_L dh$, rather than four.

An alternative, parallel assumption would be a zero elasticity of substitution between *materials* and capital services; this is close to the suggestion of Basu (1996). This implies fixed proportions between materials and capital services, $UK = \text{constant} * M$, implying $du = dm - dk$ and

40 Basu (1996) suggests additional assumptions that might identify γ even if the production function is non-homothetic.

$$dy = \gamma\left[c_L(dl + dh) + (c_K + c_M)dm + c_N dn\right] + \gamma\zeta c_L dh + dt.$$

Again, this leads to a regression with only two independent variables, one that is a modified version of dx and the other $c_L dh$.[41]

The results of the regressions with a Leontief subproduction function between capital services and energy and with a Leontief subproduction function between capital services and materials are reported in Table 5. In both cases, the reduction in the number of independent variables gives us confidence that the instruments are up to the task. Thus, these regressions provide a check of the robustness of certain results. Corresponding to the benefit of simplicity is the cost that it becomes impossible to identify the parameters Δ and j, since the lack of substitution makes these parameters controlling prices irrelevant. What is evident, though, is that the finding of substantial variation in unobserved labor effort is robust to this departure from Cobb-Douglas. The degree of returns to scale is also identified, although in the Jorgenson-Griliches case — using energy as a proxy for U — it is now statistically *smaller* than 1. Economically speaking, however, we continue to find constant returns to scale.

VI. Extensions

A. The Shift Premium as a Cost of Variable Capital Utilization

We now return to the issue of interpreting our estimate of ζ. As we have noted, the estimate has a number of interpretations. In particular, we argued that a portion of ζ may reflect variable *capital* utilization if there are shift premia. The reason is that shift premia create a link between capital hours and labor compensation. If there is such a link, we need to change our specification of the compensation function. Let the true compensation function be $G(H, E)V(U)$. Then, as we show in Appendix A, in addition to proxying for labor utilization, dh proxies for part of capital utilization.

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[41] Basu (1996) actually assumes that materials have a zero elasticity of substitution with *true value added* rather than with capital services alone. This leads to the equation $dy = \gamma\left[(1 - c_N)dn + c_M dm\right]$, which Basu uses to estimate the degree of returns to scale.

The intuition for this result comes from the role played by hours per worker in our estimation. If hours per worker is high, then the *marginal* hourly cost of labor is high relative to its *average* hourly cost. Increasing capital utilization by opening a second shift means that one pays only the *average* hourly cost of labor for workers on that shift, and is able to utilize the capital stock at an hour when it was not being used. So it makes sense that capital utilization should increase at a time when the marginal cost of labor is high relative to its average cost — i.e. at a time when dh is large.

Recall from our previous discussion of labor utilization that our estimate of ζ is higher than we expected. Given Schor's data from time-and-motion studies, we expected the true ζ to be on the order of 0.5. Even allowing for a large setup cost of working, which implied a full-time wage premium of 17 percent, we expected the estimated ζ to rise to only 0.7. The estimated ζ, however, is about 1.3. Given the theoretical results in Appendix A, this finding is less surprising. In recent empirical work, Shapiro (1996a) concludes that there is in fact a shift premium of at least 25 percent of the base wage. With a shift premium the estimated ζ should be higher.

Our theory predicts a link between capital utilization and labor hours when there is a shift premium. There is also some direct evidence of a reduced-form link between these two variables. Bils and Cho (1994) find a positive relation between the workweek of looms and hours per worker in the textile industry. Using electricity consumption as a proxy for capital utilization, they also find mixed evidence of a relationship between electricity consumption and hours per worker for 19 of the industries we study. Shapiro (1996b) confirms that there is generally a positive correlation between the workweeks of capital and labor in manufacturing using the Survey of Plant Capacity to observe the workweek of capital over the limited time period for which these data are available. Bils and Cho (1994) give a structural interpretation to this evidence; by contrast, our model implies that firms simply find it optimal to increase the workweeks of capital and labor at the same time.

However, with existing data we cannot identify the components of ζ that reflect the separate effects of capital and labor utilization. We show in Appendix A that, in the presence of a shift premium:

$$dy = \gamma^* dx + \gamma^* \zeta c_L dh + \frac{\gamma^* \xi \eta}{(1-\xi)(1+\Delta)+\xi v} c_K dh$$
$$+ \frac{\gamma^*(1-\xi)}{(1-\xi)(1+\Delta)+\xi v} c_K \left[(dp_M + dm - dp_I - dk) \right] \tag{A.14}$$
$$- \frac{\gamma^* j(1-\xi)}{(1-\xi)(1+\Delta)+\xi v} c_K (di - dk) + dz,$$

where ξ is the share of the cost of higher capital utilization due to the shift premium, $1-\xi$ is the share of the cost of higher capital utilization due to variable depreciation, η is the rate of increase of the ratio of marginal to average labor costs with respect to observed hours, and v is the rate of increase of the elasticity of labor costs with respect to capital utilization. Equation (A.14), thought of as a regression, indicates that both variable labor utilization and variable capital utilization tend to yield a positive coefficient on *dh*, which is what we asserted above. Unfortunately, a regression like (A.14) does not allow us to identify ξ without more knowledge about some of the other parameters. We can, however, test the hypothesis that $\xi = 1$ by testing for the significance of the third and fourth terms in equation (A.14). Using our data set, we find that these terms are jointly significant (at a level greater than 1 percent), which indicates that part of the penalty for variable utilization is higher depreciation of capital. This result is consistent with other evidence. Based on equations (A.7) and (A.10) and the estimates of Shapiro (1996a), we believe a reasonable range for ξ is between 0.25 and 0.50. Note, however, that even knowing ξ *a priori* would not allow us to identify all of the structural parameters: the elasticities Δ and v would remain unidentified (with a linear restriction across the two).

On the positive side, equation (A.14) establishes an important result: Even in this complex model, with capital utilization costs that are very general, the same three variables that we identified in Section I continue to be sufficient proxies for variable utilization of both capital and labor. Thus, our estimate of returns to scale in Table 1 is unbiased, as are our estimates of the properties of technology shocks in Tables 3 and 4.[42] This is a useful result for applied researchers who wish to investigate technological change using Solow's method, since it establishes that under very general conditions one can obtain a

[42] To our knowledge, this is the first estimate of returns to scale *per se* that controls rigorously for variable factor utilization. The estimates of Basu (1996) and Burnside, Eichenbaum and Rebelo (1995) are better regarded as estimates of the slope of the marginal cost curve. See Basu (1995).

series of true technology shocks by regressing output growth on input growth and our three utilization proxies. For example, such a series provides an interesting way of testing Gali's (1996) hypothesis that technology improvements *reduce* employment on impact, perhaps because prices are not perfectly flexible in the short run.

These results also modify our conclusions regarding the amount of depreciation due to wear and tear. If we decide that a substantial portion of the cost of capital utilization comes from the shift premium, then we must revise downward our estimate of the share of depreciation due to variable utilization. In theory, the adjustment is simple: Just multiply the cost of variable utilization given in (21b) by the share of the marginal cost of extra capital utilization due to wear and tear, as opposed to the shift premium.

B. Constraining the Workweek of Capital to Equal the Workweek of Labor

So far, we have considered the situation where the workweek of capital and the workweek of labor can be varied independently, but one might think this is not always so. In general, this problem is one of integer constraints on the number of shifts. Such constraints may make it difficult to vary independently the workweek of capital and the workweek of an individual worker. It would be quite difficult to analyze all possible effects of such integer constraints, but it is instructive to look at least at the extreme case in which there is always a fixed number of shifts. In this case, any variation in the workweek of labor clearly corresponds to a change in the workweek of capital, so the workweek of capital and the workweek of a worker must covary perfectly.[43]

If the number of shifts is fixed, then without loss of generality we can assume that there is only one shift. Thus, we analyze the case of a fixed number of shifts in Appendix B by reworking our basic model of Section I under the constraint $U = H$. With $U = H$, $G(H,E)$ and $G(H,E)V(H)$ are both functions of H and E and so are observationally equivalent. Thus, all the results of this section still apply even if there is a shift premium. We define a function

[43] We are grateful to Matthew Shapiro for raising this point. Bils and Cho (1994) work under the maintained hypothesis that there is only one shift.

$$\psi(H,E)=\frac{HG_H}{EG_E}=\frac{HG_H/G}{EG_E/G},$$

where $\psi_H(H,E)>0$ and $\psi_E(H,E)<0$. Then our estimating equation becomes

$$\begin{aligned} dy = \gamma(dx + c_K dk) &- \frac{\gamma c_L}{E\psi_E(H,E)} \frac{c_M}{c_L} \frac{P_I K}{P_M M} J'(I/K) H\delta'(H) \\ &\times\left[j(di-dk)-(dp_M+dm-dp_I-dk)+(1+\Delta)dh\right] \\ &-\frac{\gamma c_L}{E\psi_E(H,E)} H\psi_H(H,E)dh+dz. \end{aligned} \tag{B.9}$$

The steady-state relationships are not of material help in interpreting the coefficients of the various logarithmic changes, but a log-linear approximation would take these coefficients as constant. Except that dx is augmented by $c_K dh$, the variables involved are the same as before: $(dx+c_K dh)$, dh, $(dp_M+dm-dp_I-dk)$ and $(di-dk)$. Other than the ratio of the coefficient of $(di-dk)$ to the coefficient of $(dp_M+dm-dp_I-dk)$ being $-j$, the magnitudes of the coefficients are difficult to interpret. The predicted signs, however, are telling. Since $\psi_E(H,E)<0$ and $\psi_H(H,E)>0$, the predicted sign of dh is positive as before, but the predicted sign of $(dp_M+dm-dp_I-dk)$ is now negative, while the predicted sign of $(di-dk)$ is now positive! Thus, at least in principle, the signs of these terms allow one to distinguish between our primary model with a continuously variable number of shifts, allowing H and U to be determined independently in this model with a fixed number of shifts.

Intuitively, with H and U chosen independently, the unobserved capital utilization (and its contribution to production) should be positively related to the ratio of the marginal product of capital to the marginal value of capital. With H and U codetermined, and both of them directly observed as H, a high MP_K/q means that H and U should be high relative to effort E, and so this ratio is a *negative* indicator of unobserved effort (and the contribution of that unobserved effort to production). Since effort is the only unobserved variable, that is the entire story of the signs of $(dp_M+dm-dp_I-dk)$ and $(di-dk)$ in the regression. Increases in the observed workweek $U=H$ raise the cost of the workweek relative to the cost of effort for any given level of effort, and so lead to an increase in unobserved effort E (and its contribution to production).

We estimate equation (B.9), reporting the results in Table 6. The evidence on the signs of the two variables is not quite conclusive, because we cannot reject the hypothesis that at least one is zero. But the point estimates certainly favor our model with continuously variable shifts over the model with a fixed number of shifts.

VII. Conclusion

We use a dynamic, optimizing model to solve for variable utilization of capital and labor in terms of observables. Estimating a model that nests cyclical utilization, increasing returns to scale, and variable technology as alternative explanations for cyclical productivity, we find strong evidence for cyclical utilization but none for increasing returns. This is an important result, since it indicates that much of the recent work on business-cycles models with large increasing returns may be fundamentally misguided. Furthermore, given the absence of large pure profits in the U.S. economy, finding constant returns to scale indicates that the ratio of price to marginal cost must also be close to one.

Variable utilization also reduces the cyclicality of true technology shocks, which we estimate is 40 to 60 percent lower than the cyclicality of the usual Solow residual. This result implies that standard RBC models, which generally have problems amplifying and propagating shocks,[44] may fit the data even worse than previously thought. This is not a foregone conclusion, however, since cyclical utilization reduces the cyclicality of the impulse coming from technology shocks, but adds a new propagation mechanism. Burnside and Eichenbaum (1996) argue that the second effect more than offsets the first. Their conclusion, of course, is subject to our caveats about their calibration methods. Thus, it is important to redo their simulation exercise using our estimated elasticities.

We doubt that variable utilization will rescue RBC models, however, because our work provides a direct way to study the effects of technology change on output and employment. One of the most generally-applicable results in our paper is that we derive model-based proxies for utilization that are valid under very general circumstances. Using these proxies, we find in preliminary empirical work

[44] See Cogley and Nason (1995), and Rotemberg and Woodford (1996).

going beyond the scope of this paper that aggregate technology change, corrected both for utilization and for the aggregation biases emphasized by Basu and Fernald (1997), is actually *negatively* correlated with contemporaneous employment changes. This result presents a challenge to standard Real Business Cycle theory.

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Table 1. Estimating Returns to Scale with Variable Utilization

$$dy_i - dx_i = c_i + d\,d_{74} + \alpha_1\,dx_i + \alpha_2 c_{L_i} dh_i + \alpha_3 c_{K_i}\left(dp_{M_i} + dm_i - dp_{I_i} - dk_i\right) - \alpha_4 c_{K_i}\left(di_i - dk_i\right) + dz_i$$
$$= c_i + d\,d_{74} + \left(\gamma^* - 1\right)dx_i + \gamma^* \zeta\, c_{L_i} dh_i + \frac{\gamma^*}{1+\Delta} c_{K_i}\left(dp_{M_i} + dm_i - dp_{I_i} - dk_i\right) - \frac{\gamma^* j}{1+\Delta} c_{K_i}\left(di_i - dk_i\right) + dz_i$$

	α_1	α_2	α_3	α_4	γ^*	ζ	Δ	j
Non-Quality-Adjusted	-0.01 (0.02)	1.26 (0.25)	0.46 (0.14)	0.21 (0.06)	0.99 (0.02)	1.28 (0.26)	1.13 (0.68)	0.47 (0.20)
Quality-Adjusted.	-0.02 (0.02)	1.10 (0.27)	0.50 (0.15)	0.21 (0.06)	0.98 (0.02)	1.12 (0.28)	0.96 (0.62)	0.42 (0.18)

Table 2. Using both Materials and Energy to Measure the Marginal Product of Capital

$$dy_i - dx_i = c_i + d\,d_{74} + \alpha_1\,dx_i + \alpha_2 c_{L_i} dh_i + \alpha_3 c_{K_i}\left(\tfrac{1}{2}\left(dp_{M_i} + dm_i\right) + \tfrac{1}{2}\left(dp_{N_i} + dn_i\right) - dp_{I_i} - dk_i\right)$$
$$-\ \alpha_4 c_{K_i}\left(di_i - dk_i\right) + dz_i$$
$$= c_i + d\,d_{74} + \left(\gamma^* - 1\right)dx_i + \gamma^*\zeta\, c_{L_i} dh_i + \frac{\gamma^*}{1+\Delta} c_{K_i}\left(\tfrac{1}{2}\left(dp_{M_i} + dm_i\right) + \tfrac{1}{2}\left(dp_{N_i} + dn_i\right) - dp_{I_i} - dk_i\right)$$
$$-\ \frac{\gamma^* j}{1+\Delta} c_{K_i}\left(di_i - dk_i\right) + dz_i$$

	α_1	α_2	α_3	α_4	γ^*	ζ	Δ	j
Quality-Adjusted.	0.02 (0.02)	1.49 (0.30)	-0.38 (0.14)	0.27 (0.06)	1.02 (0.02)	1.43 (0.30)	-0.66 (0.92)	-0.69 (0.30)

Table 3. Cyclicality of Residuals

	Solow Residuals		Estimated Residuals	
RHS Var.	Non-Quality-Adjusted	Quality-Adjusted	Non-Quality-Adjusted	Quality-Adjusted
dv	0.060 (0.011)	0.058 (0.011)	0.037 (0.009)	0.035 (0.009)
dy	0.034 (0.012)	0.034 (0.012)	0.015 (0.010)	0.014 (0.010)

Table 4. Correlations with Sectoral Gross Output

(Averages across all Industries)

Solow Residuals		Estimated Residuals	
Non-Quality-Adjusted	Quality-Adjusted	Non-Quality-Adjusted	Quality-Adjusted
0.36	0.36	0.20	0.23

Table 5. Departures from Cobb-Douglas

$$dy_i = c_i + d\, d74 + \alpha_1\left[c_{L_i}\left(dl_i + dh_i\right) + c_{M_i} dm_i + \left(c_{K_i} + c_{N_i}\right) dn_i\right] + \alpha_2 c_{L_i} dh_i + dt_i$$
$$= c_i + d\, d74 + \gamma\left[c_{L_i}\left(dl_i + dh_i\right) + c_{M_i} dm_i + \left(c_{K_i} + c_{N_i}\right) dn_i\right] + \gamma\zeta c_{L_i} dh_i + dt_i.$$

$$dy_i = c_i + d\, d74 + \alpha_1\left[c_{L_i}\left(dl_i + dh_i\right) + \left(c_{K_i} + c_{M_i}\right) dm_i + c_{N_i} dn_i\right] + \alpha_2 c_{L_i} dh_i + dt_i$$
$$= c_i + d\, d74 + \gamma\left[c_{L_i}\left(dl_i + dh_i\right) + \left(c_{K_i} + c_{M_i}\right) dm_i + c_{N_i} dn_i\right] + \gamma\zeta c_{L_i} dh_i + dt_i.$$

	α_1	α_2	γ^*	ζ
Energy as a Proxy for *U*	0.95 (0.01)	0.67 (0.25)	0.95 (0.01)	0.71 (0.27)
Materials as a Proxy for *U*	1.00 (0.01)	1.07 (0.29)	1.00 (0.01)	1.06 (0.30)

Table 6. Constraining the Model to a Fixed Number of Shifts

$$dy_i = c_i + d\, d_{74} + \alpha_1\left(dx_i + c_{K_i} dh_i\right) + \alpha_2 dh_i + \alpha_3\left(dp_{M_i} + dm_i - dp_{I_i} - dk_i\right) + \alpha_4 c_{K_i}\left(di_i - dk_i\right) + dz_i$$

	α_1	α_2	α_3	α_4
Quality-Adjusted	1.02 (0.02)	1.07 (0.28)	0.11 (0.15)	0.38 (0.08)

Appendix A: Adding a Shift Premium to the Model

Consider what happens when we modify the firm's problem by changing the labor cost function from $WLG(H,E)$ to $WLG(H,E)V(U)$, where $V(U)$ is an increasing, convex function representing the increase in labor costs incurred by making workers work at off-hours, as is necessary in order to lengthen the work-week of capital. Here, we are implicitly assuming that the number of shifts is a continuous variable, so that the workweek of capital U can be varied independently of the workweek of an individual worker. (In Appendix B we look at the polar opposite case in which the number of shifts is fixed so that the workweek of capital can only vary in proportion to the workweek of an individual worker.) Many equations remain unaltered by this change. Equations (3) and (4) do change slightly to

$$\lambda EL\, F_2(UK, EHL, M, N; Z) = WL\, G_H(H,E)V(U) \qquad \text{(A.1)}$$

and

$$\lambda HL\, F_2(UK, EHL, M, N; Z) = WL\, G_E(H,E)V(U). \qquad \text{(A.2).}$$

However, these two modified equations (A.1) and (A.2) can still be combined to yield an unaltered (13) and therefore an unaltered $E(H)$ function. The only first-order condition that changes markedly is (5), which becomes

$$\lambda K\, F_1(UK, EHL, M, N; Z) = qK\,\delta'(U) + WLG(H,E)V'(U). \qquad \text{(A.3).}$$

The benefit of increasing the workweek of capital U is the same as before, but now the cost has two components: additional wear and tear on the capital, and the shift premium for the labor at the tail end of the workweek of capital. Equation (A.1) can be combined with the still-valid (12b) to get

$$WLH\, G_H(H,E)V(U) = \lambda\gamma c_L Y. \qquad \text{(A.4).}$$

Together with (12a), this allows us to rewrite (A.3) as

$$\lambda\gamma c_K \frac{Y}{U} = qK\delta'(U) + \lambda\gamma c_L Y \frac{G(H,E)}{HG_H(H,E)} \frac{V'(U)}{V(U)}. \qquad \text{(A.5).}$$

Define

$$g(H) = \frac{HG_H(H, E(H))}{G(H, E(H))} \qquad \text{(A.6)}$$

and

$$\mathrm{v}(U)=\frac{UV'(U)}{V(U)}. \tag{A.7}$$

$\mathrm{v}(U)$ is thus the ratio of the marginal shift premium to the average shift premium. Based on Shapiro's (1996a) estimate, $.125 < \mathrm{v} < 0.25$.Then we can divide through (A.5) by $\lambda\gamma c_K Y/U$ to get

$$1=\frac{qK}{\lambda\gamma c_K Y}U\delta'(U)+\frac{c_L}{c_K}\frac{\mathrm{v}(U)}{g(H)}. \tag{A.8}$$

The labor cost elasticity with respect to hours given by the function $g(H)$ is positive and increasing by the assumptions we have made on $G(H,E)$. The labor cost elasticity with respect to capital utilization given by the function $\mathrm{v}(U)$ is positive as long as there is a positive shift premium. We also assume that the shift premium increases rapidly enough with U to make the elasticity increasing in U. If there is no shift premium, $\mathrm{v}(U)=0$, and the equation reduces to (15). Following the same strategy in expressing the first term in (A.8) as we did for (15), we get

$$1=\frac{c_M}{c_K}\frac{P_I K}{P_M M}J'(I/K)U\delta'(U)+\frac{c_L}{c_K}\frac{\mathrm{v}(U)}{g(H)}. \tag{A.9}$$

Now we can log-linearize (A.9) around the steady state values. First, define

$$\xi=\frac{c_L}{c_K}\frac{\mathrm{v}(U^*)}{g(H^*)}=\frac{c_L}{c_K}\frac{\mathrm{v}(U^*)}{1}, \tag{A.10}$$

$$\eta=\frac{H^*g'(H^*)}{g(H^*)}, \tag{A.11}$$

and

$$\nu=\frac{U^*\mathrm{v}'(U^*)}{\mathrm{v}(U^*)}. \tag{A.12}$$

From (A.9), we see that ξ, as defined in (A.12), is the share of the cost of higher capital utilization due to the shift premium, while $1-\xi$ is the share of the cost of higher capital utilization due to wear and tear. η indicates the rate at which the elasticity of labor costs with respect to hours increases. ν indicates the rate at which the elasticity of labor costs with respect to capital utilization increases. Using this notation, the log-linearization is

$$du = \frac{\xi}{(1-\xi)(1+\Delta)+\xi v}\eta\, dh + \frac{1-\xi}{(1-\xi)(1+\Delta)+\xi v}\left[(dp_M + dm - dp_I - dk) - j(di - dk)\right]. \tag{A.13}$$

The counterpart of (20) is therefore

$$dy = \gamma^* dx + \gamma^* \zeta c_L dh + \frac{\gamma^* \xi \eta}{(1-\xi)(1+\Delta)+\xi v} c_K\, dh + \frac{\gamma^*(1-\xi)}{(1-\xi)(1+\Delta)+\xi v} c_K\left[(dp_M + dm - dp_I - dk)\right] - \frac{\gamma^* j(1-\xi)}{(1-\xi)(1+\Delta)+\xi v} c_K (di - dk) + dz. \tag{A.14}$$

Appendix B: Constraining the Workweek of Capital to Equal the Workweek of a Worker

Here we analyze the case where the number of shifts is fixed, so that a change in the workweek of labor also represents a change in the workweek of capital. By choosing units appropriately we can, without loss of generality, represent this case by the constraint $U = H$. The easiest way to add this constraint is to substitute H for U wherever it appears. Otherwise the model is unchanged. With $U = H$, $G(H,E)$ and $G(H,E)V(H)$ are both functions of H and E and so are observationally equivalent. Therefore, for simplicity we will write $WLG(H,E)$ for labor costs.

In either case, we must strengthen the "normality" assumption on $G(H,E)$ or $G(H,E)V(H)$ to allow for the different "relative prices" of $\ln(H)$ and $\ln(E)$ on the benefit side that can result from the fact that changes in $\ln(H)$ now interact with both capital and labor's shares in production. (Before, with H and U chosen independently, we knew that the "relative price" of $\ln(H)$ and $\ln(E)$ on the benefit side was 1.)

With H substituted in for U, the derivative of the Hamiltonian with respect to H functioning in a dual role as hours per worker and capital utilization yields the first-order condition

$$\lambda[KF_1 + ELF_2] = qK\delta'(H) + WLG_H. \tag{B.1}$$

The marginal benefit of a longer-workweek (the left-hand side) involves extra capital input and extra labor input. The marginal cost (the right-hand side) involves wear and tear on capital plus the utility cost of a longer workweek for labor (indeed, of a longer workweek with some of the time at worse times of day or worse days of the week). The first-order condition for optimal effort is little changed:

$$\lambda HLF_2 = WLG_E. \tag{B.2}$$

Dividing (B.1) through by (E/H) times (B.2) yields

$$\frac{KF_1}{ELF_2} + 1 = \frac{qK\delta'(H)}{\lambda ELF_2} + \frac{HG_H}{EG_E} \tag{B.3}$$

Define

$$\psi(H,E) = \frac{HG_H}{EG_E} = \frac{HG_H/G}{EG_E/G}.$$

Then the strengthened "normality" condition implies that $\psi_H(H,E)>0$ and $\psi_E(H,E)<0$. That is, a longer workweek raises the elasticity of labor cost with respect to the workweek more than the elasticity of labor cost with respect to effort, while more effort raises the elasticity of labor cost with respect to effort more than the elasticity of labor cost with respect to the workweek. Using the fact that

$$HK\frac{F_1}{c_K}=EHL\frac{F_2}{c_L}=M\frac{F_3}{c_M}=N\frac{F_4}{c_N}, \tag{B.5}$$

equation (B.3) can be rewritten as

$$1+\frac{c_K}{c_L}=\frac{qKH\delta'(H)}{\lambda\gamma c_L Y}+\psi(H,E). \tag{B.6}$$

Finally, one can use the first order conditions (6) and (7) to get

$$\begin{aligned}1+\frac{c_K}{c_L}&=\frac{c_M}{c_L}\frac{P_I K}{P_M M}J'(I/K)\delta'(H)+\psi(H,E)\\&=\frac{c_N}{c_L}\frac{P_I K}{P_N N}J'(I/K)\delta'(H)+\psi(H,E)\end{aligned} \tag{B.7}$$

Taking a total derivative with respect to the changes in the logarithms of the variables and solving for the change in effort de, we find:

$$\begin{aligned}de=&\frac{-1}{E\,\psi_E(H,E)}\frac{c_M}{c_L}\frac{P_I K}{P_M M}J'(I/K)H\delta'(H)\\&\times\left[j(di-dk)-(dp_M+dm-dp_I-dk)+(1+\delta)dh\right]+H\,\psi_H(H,E)dh\end{aligned} \tag{B.8}$$

The corresponding productivity equation is

$$\begin{aligned}dy&=\gamma\left[c_K(dk+dh)+c_L(dl+dh+de)+c_M dm+c_N dn\right]+dz\\&=\gamma\left[(dx+c_K dk)+c_L de\right]+dz\\&=\gamma\left(dx+c_K dk\right)-\frac{\gamma c_L}{E\psi_E(H,E)}\frac{c_M}{c_L}\frac{P_I K}{P_M M}J'(I/K)H\delta'(H)\\&\qquad\times\left[j(di-dk)-(dp_M+dm-dp_I-dk)+(1+\Delta)dh\right]\\&\qquad-\frac{\gamma c_L}{E\psi_E(H,E)}H\,\psi_H(H,E)dh+dz\end{aligned} \tag{B.9}$$