NBER WORKING PAPERS SERIES

LABOR HOARDING AND THE BUSINESS CYCLE

Craig Burnside
Martin Eichenbaum
Sergio Rebelo

Working Paper No. 3556

NATIONAL BUREAU OF ECONOMIC RESEARCH 1050 Massachusetts Avenue Cambridge, MA 02138 December 1990

We would like to thank Lawrence Christiano, Robert Gordon, Dale Mortensen, and Mark Watson for many helpful conversations and suggestions. This paper is part of NBER's research program in Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

LABOR HOARDING AND THE BUSINESS CYCLE

ABSTRACT

Existing Real Business Cycle (RBC) models assume that the key impulses to business cycles are stochastic technology shocks. RBC analysts typically measure these technology shocks by the Solow residual. This paper assesses the sensitivity of inference based on Solow residual accounting to labor hoarding behavior. Our main results can be summarized as follows. First, the quantitative implications of RBC models are very sensitive to the possibility of labor hoarding. Allowing for such behavior reduces our estimate of the variance of technology shocks by 50%. Depending on the sample period investigated, this reduces the ability of technology shocks to account for aggregate output fluctuations by 30% to 60%. Second, our labor hoarding model is capable of quantitatively accounting for the observed correlation between government consumption and the Solow residual. Third, unlike standard RBC models, our labor hoarding model is consistent with three important qualitative features of the joint behavior of average productivity and hours worked: (i) average productivity and hours worked do not display any marked contemporaneous correlation, (ii) average productivity is positively correlated with future hours worked, and (iii) average productivity is negatively correlated with lagged hours worked.

Craig Burnside
Department of Economics
Queen's University
Kingston Canada
K&L 3NL

Sergio Rebelo Kellog School of Management Northwestern University Evanston, IL 60208 Martin Eichenbaum Department of Economics Northwestern University Evanston, IL 60208

1. INTRODUCTION

The phenomenon of procyclical productivity has traditionally been regarded as inconsistent with neoclassical theories of the business cycle. This empirical regularity seems to contradict the notion that changes in aggregate output correspond to movements along a fixed neoclassical production function, i.e. one which exhibits diminishing marginal productivity of labor. Existing Real Business Cycle (RBC) models resolve this puzzle by simply abandoning the assumption of a time invariant production function. According to this class of theories, the key impulses to business cycles are stochastic technology shocks which shift the aggregate production possibilities frontier. As a result, higher levels of output represent movements along higher marginal product of labor schedules. In this way the model can generate procyclical labor productivity.

This explanation relies heavily on the frequency and magnitude of aggregate technology shocks. To provide evidence in favor of the importance of such shocks, authors like Prescott (1986) have examined the univariate time series properties of the Solow residual under the null hypotheses that they represent exogenous technology shocks. However, various authors, ranging from Lucas (1989) to Summers (1986), have conjectured that many of the movements in the Solow residual do not represent exogenous shocks to technology, but instead are artifacts of labor hoarding type behavior. While labor hoarding may arise for a variety of different reasons, the existence of such behavior always implies that reported aggregate hours worked will not accurately reflect actual labor input. This introduces non—classical measurement error in those measures of labor input typically used to calculate the Solow residual. Moreover, many labor hoarding models (see for example

¹For example, Sargent (1987, page 468) cites the procyclical behavior of productivity as one of the key empirical patterns commonly thought to cast doubt on classical models of the business cycle. For a review of different approaches to modeling the cyclical behavior of average productivity, see Rotemberg and Summers (1988).

²See Fay and Medoff (1985) for a discussion of different types of labor hoarding and of measures of their quantitative importance based on survey data.

Hall (1989) and Rotemberg and Summers (1988)) suggest that this measurement error will be procyclical, say because firms use their work force more intensively in periods of high aggregate demand.³ To the extent that this is true, empirical work which identifies technology shocks with the Solow residual will systematically overstate their importance to the business cycle and their contribution to the procyclical behavior of labor productivity.

There is by now a substantial amount of evidence that casts doubts on the view that Solow residuals represent only exogenous technological shocks. Hall (1988) has shown that the growth rates of the Solow residual and military expenditures are correlated. Also, Evans (1990) has demonstrated that the Solow residual is correlated with various measures of the money supply. Hall (1988) has argued that such findings are inconsistent with the way in which Solow residuals are interpreted by RBC analysts.

This paper has two primary goals. First, we attempt to assess the sensitivity of inference based on Solow residual accounting to labor hoarding behavior. Second, we investigate whether our model is capable of accounting for certain aspects of the data which seem anomalous from the perspective of existing RBC theory. In order to simultaneously accomplish these objectives we introduce labor hoarding in a way that represents a minimal perturbation of standard RBC models. The advantage of this approach is that we can isolate the quantitative impact of labor hoarding per se in a standard real business cycle model. The disadvantage is that we are restricted to one particular type of labor hoarding. Still, we feel that our labor hoarding model succeeds in capturing the essence of existing critiques of Solow residual based measures of technology shocks.

Our model builds upon the standard indivisible labor model associated with Rogerson (1988) and Hansen (1985). We modify this model by assuming that firms must make their employment decisions before observing the current state of demand and technology. Once these shocks are observed firms can adjust labor input by varying the

³In contrast, Horning (1990) and Prasaad (1990) present models in which labor hoarding can be either pro or counter cyclical.

level of effort that they require from their workers. However, these adjustments are costly to firms because workers care about effective hours of work, defined as the product of work effort times hours worked. This extension of the Hansen-Rogerson model is intended to capture the idea that it is not feasible for firms to vary the size of their work force in response to every bit of new information regarding the state of demand and technology. In addition, our extension can be thought of as capturing, in a rough manner, the type of measurement error induced by the fact that, in many industries, reported hours worked do not vary in a one to one manner with actual hours worked.⁴

In the competitive equilibrium of our economy, workers' labor effort increases in response to a positive innovation in government purchases or technology, i.e. effort will be procyclical. In this sense our model captures the notion that labor hoarding implies a procyclical measurement error in hours worked. Consequently our model predicts that naive Solow residual accounting systematically overstates the importance of technology shocks as impulses to business cycles. Since the equilibrium law of motion for effort is a function of variables like government purchases the model is also, in principle, capable of rationalizing the fact that the Solow residual is correlated with different measures of fiscal policy.

An important shortcoming of our model is that we do not allow for variations in the degree to which capital is utilized. However it is clear that allowing for such effects would only strengthen our conclusions. While poorly measured, capital utilization rates are clearly procyclical. Consequently, the measurement error involved in using the stock of capital for the purpose of calculating Solow residuals would also be procyclical. The same sorts of impulses which cause labor effort to increase presumbaly also induce increases in capital utilization rates. To the extent that this is true, our results understate the sensitivity of

⁴To take an extreme example, the actual number of hours which professors and their secretaries work is highly variable. This fact is not reflected in the official reported number of hours worked.

⁵Gordon (1990) provides some evidence that the biases in standard Solow residual accounting exercises that arise from time varying effort and time varying capital utilization rates work in the same direction.

RBC models to more general types of "hoarding" behavior.

To investigate the quantitative implications of our theory, we estimate the model using the variant of Hansen's (1982) Generalized Method of Moments procedure discussed in Christiano and Eichenbaum (1990). Given point estimates for the structural parameters, we are able to disentangle movements in hours worked from movements in effort. This in turn allows us to distinguish between movements in the Solow residual and exogenous technology shocks. Our main results can be summarized as follows:

- (1) We find that the quantitative implications of RBC models are very sensitive to the possibility of labor hoarding. Allowing for such behavior reduces our estimate of the variance of technology shocks by roughly 50%. Depending on the sample period investigated, this reduces the ability of technology shocks to account for aggregate output fluctuations by 30% to 60%.
- (2) We find that our labor hoarding model, which embodies perfect competition and complete markets, is capable of quantitatively accounting for the observed correlation between government consumption and the Solow residual. This provides evidence against Hall's (1988, 1989) conjecture that imperfect competition and increasing returns are essential ingredients of an empirically plausible explanation of these correlations.
- (3) We find that our model is consistent with three important qualitative features of the joint behavior of average productivity and hours worked. First, average productivity and hours worked do not display any marked contemporaneous correlation. Second, average productivity leads the cycle in the sense that it is positively correlated with future hours worked. Third, average productivity is negatively correlated with lagged hours.⁶
- (4) While our model certainly cannot account for all features of the aggregate data, we conclude that incorporating labor hoarding into the analysis, substantially enhances its

⁶Gordon (1979) presents evidence on this general phenomenon which he labels as the

[&]quot;End-of-Expansion-Productivity-Slowdown". McCallum (1989) also documents a similar pattern for the dynamic correlation between average productivity and output.

overall empirical performance.

The remainder of this paper is organized as follows. In section 2 we describe our basic model. Section 3 describes our econometric methodology. In section 4 we present our empirical results. Finally, section 5 contains some concluding remarks.

2. A Model of Time Varying Effort and The Business Cycle

In this section we present a variant of Hansen's (1985) indivisible labor model. This setup allows us to assess the sensitivity of existing RBC models to labor hoarding based critiques of Solow residual accounting. As in Christiano and Eichenbaum (1990) we modify the basic Hansen (1985) model to allow for aggregate demand shocks in the form of stochastic movements in government consumption. Absent uncertainty regarding the level of technology and government purchases, our model and the version of Hansen's model considered in Christiano and Eichenbaum (1990) are observationally equivalent.

2.1 The Model

Our model economy is populated by a finite number of infinitely lived individuals. In order to go to work each individual must incur a fixed cost, ξ , denominated in terms of hours of foregone leisure. Once at work, an individual stays there for a fixed shift length of f hours. The momentary utility at time t of such a person is given by

$$(2.1) \quad \ln(C_{t}^{p}) + \theta \ln(T - \xi - W_{t}^{f}).$$

Here, T is a scalar denoting the individual's time endowment, θ is a positive scalar, C_t^p denotes time t privately purchased consumption and W_t denotes the level of time t effort. According to this specification, what individuals care about is total effective work, $W_t f$.

The time t utility of a person who does not go to work is simply given by

(2.2)
$$ln(C_t^p) + \theta ln(T)$$
.

Output, \boldsymbol{Y}_{t} , is produced via the Cobb Douglas production function

$$(2.3) \quad \mathbf{Y}_{t} = \mathbf{Z}_{t} \mathbf{K}_{t}^{1-\alpha} (\mathbf{N}_{t} \mathbf{W}_{t} \mathbf{f})^{\alpha}.$$

where 0 < α < 1, N_t denotes the total number of bodies going to work at time t, and K_t denotes the beginning of period t capital stock. The variable Z_t represents the time t level of technology which evolves according to

$$(2.4) \quad Z_t = \gamma^{\alpha t} A_t$$

where γ is a positive scalar which determines the unconditional growth rate of technology and A_t is the stationary component of Z_t . We suppose that A_t evolves according to

$$(2.5) \quad \ln(\mathbf{A_t}) = (1 - \rho_{\mathbf{a}}) \ln(\mathbf{A}) + \rho_{\mathbf{a}} \ln(\mathbf{A_{t-1}}) + \epsilon_{\mathbf{t}}.$$

The unconditional mean of $\ln(A_t)$ equals $\ln(A)$, $|\rho_a| < 1$ and ϵ_t is the innovation in $\ln(A_t)$ with standard deviation of σ_ϵ .

The aggregate resource constraint is given by

$$(2.6) \quad \mathrm{C}^{\mathrm{p}}_{t} + \mathrm{K}_{t+1} - (1 - \delta) \mathrm{K}_{t} + \mathrm{X}_{t} \leq \mathrm{Y}_{t}.$$

The assumption that the parameter α enters into the law of motion for Z_t is made only to simplify algebraic computations, and involves no restrictions.

The parameter δ represents the depreciation rate on capital and satisfies the condition $0 < \delta < 1$. The random variable X_t denotes time t government consumption, which evolves according to

$$(2.7) \quad X_t = \gamma^t G_t,$$

where G_t is the stationary stochastic component of X_t . We suppose that G_t has the law of motion

(2.8)
$$\ln(G_t) = (1-\rho_g)\ln(G) + \rho_g\ln(G_{t-1}) + \mu_t$$

Here $\ln(G)$ is the mean of $\ln(G_t)$, $|\rho_g|<1$ and μ_t is the innovation in $\ln(G_t)$ with standard deviation $\sigma_{_{II}}{}^8$

In the presence of complete markets the decentralized competitive equilibrium corresponds to the solution of a social planning problem. Proceeding as in Rogerson (1988) it is easy to show that, since agents' criteria functions are separable across consumption and leisure, the social planner will equate the consumption of employed and unemployed individuals. Under these circumstances, the Pareto optimal competitive equilibrium corresponds to the solution of the following planning problem:

Maximize

(2.9)
$$E_0 \sum_{t=0}^{\infty} \beta^t \{ \ln(C_t^p) + \theta N_t \ln(T - \xi - W_t f) + \theta (1 - N_t) \ln(T) \},$$

subject to (2.3) - (2.8), and K_0 , by choice of contingency plans for $\{C_t^p, K_{t+1}, N_t, W_t; t \geq 0\}$

⁸See Aiyagari, Christiano and Eichenbaum (1990) and Baxter and King (1990) for discussions of the effects of government purchases in the stochastic one sector growth model.

0}. In (2.9), we have normalized the number of agents in the economy to one. Also E_0 is the time 0 conditional expectations operator, and β is the subjective discount rate, $0 < \beta < 1$.

To complete the specification of the model we must specify the planner's time t information set, $\Omega_{\mathbf{t}}^*$. If $\mathbf{Z}_{\mathbf{t}}$ and $\mathbf{X}_{\mathbf{t}}$ are seen before $\mathbf{N}_{\mathbf{t}}$ and $\mathbf{W}_{\mathbf{t}}$ are chosen, then the model is observationally equivalent to the standard indivisible labor model, modified to incorporate government consumption into the analysis (see Christiano and Eichenbaum [1990]).

One simple way to capture labor hoarding type behavior is to change the information structure of the model. In particular, suppose that N_t must be chosen before, rather than after, X_t and A_t are known. Let Ω_t denote agents' common information set at the beginning of time t. We assume that Ω_t includes the lagged values of all variables in the model. Let Ω_t^* consist of Ω_t plus (A_t, X_t) . Then the planner's contingency plans for N_t will be a function of the elements of Ω_t , while the contingency plans for W_t , K_{t+1} and C_t will be functions of the elements of Ω_t^* .

This perturbation of the standard model intends to capture the notion that it is costly for firms to vary the size of their work force. It is simply not feasible for a firm to change employment in response to every bit of new information regarding the state of demand or of technology. The previous formalization of the planner's problem incorporates the notion that firms must make employment decisions conditional on their views about the future state of demand and technology. Once employment decisions are made firms adjust to observed shocks along other dimensions. In our model this adjustment occurs through variations in the labor effort that workers are asked to supply. Workers' compensation will naturally depend on the effort supplied. But to compute the laws of motion for the quantity variables we do not have to be precise about the exact

⁹If $\xi=0$ it is efficient for all individuals to go to work in every period $(N_t=1)$, given that they can adjust their labor effort, W_t , in response to shocks . For this reason we assume that $\xi>0$.

compensation scheme adopted by firms.

It is convenient to represent the planning problem in terms of variables that converge to a nonstochastic steady state. To this end we define

$$(2.10) \quad \overline{C}_t^p = C_t^p/\gamma^t, \quad \overline{Y}_t = \overline{Y}_t/\gamma^t, \quad \overline{K}_t = \overline{K}_t/\gamma^t, \quad \overline{X}_t = \overline{X}_t/\gamma^t.$$

Using this transformation, constraint (2.6) can be written as

$$(2.11) \quad \gamma K_{t+1} = A_t K_t^{1-\alpha} (N_t W_t i)^{\alpha} - C_t^p + (1-\delta) K_t - X_t$$

Also, using (2.10), the planner's criterion function can be written as

$$(2.12) \qquad \kappa + E_0 \sum_{t=0}^{\infty} \beta^t \{ \ln(\overline{C}_t^p) + \theta N_t \ln(T - \xi - W_t f) + \theta (1 - N_t) \ln(T) \}$$

where $\kappa = \sum\limits_{t=0}^{\infty} \beta^t \log(\gamma^t)$ is a constant which can be ignored in the maximization problem. It follows that the original planning problem is equivalent to the problem of maximizing (2.12) subject to (2.5), (2.8), (2.11), and K_0 by choice of a contingency plan for N_t which is a function of the elements of Ω_t , and contingency plans for W_t , K_{t+1} and C_t^p which are functions of the elements of Ω_t^* .

In general it is not possible to obtain an analytical solution for the problem just discussed unless $\delta=1$. Here we use King, Plosser and Rebelo's (1988) log linear modification of the procedure used by Kydland and Prescott (1982) to obtain an approximate solution. To display the form of this solution, let $(w_t, n_t, k_t, a_t, c_t^p, g_t)$ denote the deviations of the logs of $(W_t, N_t, K_t, A_t, C_t^p, G_t)$ from their nonstochastic steady state

values, e.g. $a_t = \ln(A_t/A)$. It follows from results in King, Plosser and Rebelo (1988) that the log linear laws of motion for w_t , n_t , k_t and c_t^p will be of the form

$$(2.13a) w_t = \pi_1 k_t + \pi_2 n_t + \pi_3 a_t + \pi_4 g_t$$

(2.13b)
$$n_t = \pi_5 k_t + \pi_6 a_{t-1} + \pi_7 g_{t-1}$$

(2.13c)
$$\mathbf{k}_{t+1} = \pi_8 \mathbf{k}_t + \pi_9 \mathbf{n}_t + \pi_{10} \mathbf{a}_t + \pi_{11} \mathbf{g}_t$$

(2.13d)
$$c_t^p = \pi_{12}k_t + \pi_{13}n_t + \pi_{14}a_t + \pi_{15}g_t.$$

Here the coefficients π_i , i = 1,15 are scalar functions of the model's underlying parameters.

2.2 Solow Residual Accounting and Time Varying Effort

It is useful to briefly consider the implications of our model for the standard RBC practice of interpreting Solow residuals as exogenous technology shocks. Most RBC studies (see for example Prescott [1986]) assume that output is produced via the Cobb-Douglas production function:

$$(2.14) \quad \mathbf{Y}_{\mathbf{t}} = \mathbf{Z}_{\mathbf{t}} \mathbf{K}_{\mathbf{t}}^{1-\alpha} (\mathbf{H}_{\mathbf{t}})^{\alpha}.$$

Here $\mathbf{H_t}$ denotes total hours worked. Under the maintained assumptions of the standard indivisible labor model, $\mathbf{H_t}$ equals total bodies at work times the fixed shift length, \mathbf{f} :

(2.15)
$$H_t = N_t f$$
.

Implicit in this formulation is the assumption that effort is constant over time, say equal to one.

The standard method used to calculate the Solow residual in RBC studies is to solve

(2.14) for Z_t given some value for α . Suppose that we maintain the assumption that Z_t is a trend stationary process with unconditional growth rate, $\gamma^{\alpha t}$, i.e.

$$(2.16) \quad Z_{t} = \gamma^{\alpha t} S_{t}.$$

Here S_t represents the time t realization of a stationary stochastic process, and is interpreted by existing RBC studies as the time t technology shock. 10

This interpretation implies certain testable restrictions, a subset of which have been investigated in the literature. Hall (1989) in particular argues (Proposition 1) that under the maintained assumptions of these models, $S_{\rm t}$ ought to be orthogonal to all variables known to neither cause productivity shifts nor to be caused by productivity shifts. This leads to the restriction,

(2.17)
$$\operatorname{ES}_{\mathbf{t}} \mathbf{q}_{\mathbf{t}-\tau} = 0$$
, for all τ ,

for variables q_t satisfying the conditions of Hall's proposition. Hall (1988) tests and rejects a version of (2.17) for $\tau=0$, using different candidate variables for q_t , including per capita military expenditures. Evans (1990) also tests and rejects variants of (2.17) using different measures of the money supply.

There exist a variety of potential explanations of why restriction (2.17) appears to be at variance with the data. Hall (1988, 1989) argues that the most probable explanation is the prevalence of imperfect competition and increasing returns to scale. In the remainder of this section we show why our model can also account for the failure of (2.17). In section 4 we present quantitative evidence on this issue and respond to Hall's argument that time varying effort is implausible as an explanation of the correlations in question.

 $^{^{10}}$ If Z_t is modeled as a difference stationary process, as in Christiano and Eichenbaum (1990), then $\Delta \ln Z_t$ is taken to measure the stationary stochastic component of technology.

Let s_t denote the deviation of the log of S_t from its nonstochastic steady state value. According to our model s_t , a_t and w_t are related via the relationship,

$$(2.18) \quad \mathbf{s}_{t} = \mathbf{a}_{t} + \alpha \mathbf{w}_{t}.$$

It follows that objects which are correlated with w_t will also be correlated with s_t , even though they are not correlated with a_t . Since our model predicts that w_t depends on g_t , restriction (2.17) will not be satisfied. Put differently, according to our model, s_t constitutes an error ridden signal regarding the level of the technology shock. However, the measurement error is not of the classical type, since $Ea_tw_{t-\tau}$ is not equal to zero. As Hall (1989) points out, classical measurement error in s_t cannot rationalize the observations in question. For that, the measurement error must be systematically related to either K_t or N_t , which is the case according to our model.

Not surprisingly, given our estimates of the model's structural parameters, both π_3 and π_4 are positive. This implies that, other things equal, it is optimal to work harder when faced with a positive innovation in either government consumption or technology, i.e. effort will be procyclical. By assumption neither the stock of capital nor aggregate hours worked can change at period t in response to an innovation in government consumption. Consequently, the Solow residual will rise as a result of the increase in government consumption. This is true despite the absence of any technology shock whatsoever. Similarly, the observed Solow residual will rise by more than a technology shock, i.e. the innovation to s_t will be larger than the corresponding innovation in a_t .

It follows that our model formalizes conjectures in Summers (1986) and Lucas (1989) that naive Solow residual accounting systematically overestimates the level of technology in booms, systematically underestimates the level of technology in recessions and systematically overestimates the variance of the true technology shock. From this perspective, existing RBC models systematically overstate the role of technology shocks in

explaining the time series behavior of average productivity. Whether these sources of bias are quantitatively important is an empirical issue. In the next section we discuss our econometric methodology for empirically examining these issues.

3. Econometric Methodology

In this section we accomplish three tasks. First, we describe our strategy for estimating the structural parameters of the model as well as various second moments of the data. Second, we discuss our methodology for assessing the model's empirical performance. Finally, we describe the data used in our empirical analysis.

3.1 Estimation

The parameters of interest can be divided into two groups. The first group, which we denote by Ψ_1 , consists of the structural parameters of the model:

$$(3.1) \quad \Psi_{1} = \{\delta, \; \theta, \; \alpha, \; \rho_{\mathbf{a}}, \; \sigma_{\epsilon}, \; \rho_{\mathbf{g}}, \; \sigma_{\mu}, \; \gamma_{\mathbf{g}}, \; \mathsf{G}, \; \gamma, \; \mathsf{Y}, \; \sigma_{\mathbf{v}^{\mathsf{h}}}^{2} \}.$$

Here $\ln(Y)$ and $\ln(G)$ denote the unconditional means of linearly detrended $\ln(Y_t)$ and $\ln(G_t)$, respectively. The parameter γ_g denotes the unconditional growth rate of government purchases, G_t . In describing our theoretical model γ_g was assumed to equal the unconditional growth rate of technology, γ . In order to assess the plausibility of this assumption, we did not impose it restriction in our empirical work.

Since our estimation strategy allows for classical measurement error in hours worked, the vector Ψ_1 includes the parameter $\sigma_{\rm vh}^2$. This parameter equals the variance of the measurement error, whose other properties are discussed below. The parameters, T, β and ξ were not estimated. Instead we fixed T at 1369 hours per quarter. The parameter β

was set so as to imply a 3% annual subjective discount rate, i.e. $\beta = (1.03)^{-25}$. We experimented with a variety of values of ξ and found that our results were very insensitive to choices of ξ between 20 and 120. The results reported in section 4 correspond to a value of ξ equal to 60.

The second set of parameters, Ψ_2 , correspond to various second moments of the data which are used to diagnose the empirical performance of our model.

$$(3.2) \quad \Psi_2 = \{ \sigma_{\rm C}^{\rm p}/\sigma_{\rm v}, \, \sigma_{\rm dk}/\sigma_{\rm v}, \, \sigma_{\rm g}/\sigma_{\rm v}, \, \sigma_{\rm h}, \, \sigma_{\rm h}/\sigma_{\rm APL}, \, \rho_{-\tau}[{\rm APL,h}] \}$$

for $\tau=0, \pm 1, \pm 2, \pm 3, \pm 4$. Here APL denotes the average productivity of labor, $\sigma_{\rm x}$ denotes the standard deviation of the variable x, x = {c_p, y, APL,dk, h} and $\rho_{-\tau}[{\rm APL,h}]$ denotes the correlation between average productivity at time t- τ and hours worked at time t.

Since the data display marked time trends, some stationary inducing transformation must be adopted to ensure that the moments in (3.2) exist. In this paper, for diagnostic purposes, we detrend both model output and actual data with the Hodrick-Prescott (1980) filter. Consequently, the population moments in Ψ_2 pertain to Hodrick-Prescott (HP) filtered versions of the data.

We choose to work with this filter for two reasons. First, many authors in the RBC literature report results based on the HP filter. 11 By using this filter we are able to minimize the differences between their procedures and ours. This in turn helps us to isolate the effects of time varying effort. Second, the HP filter is, in fact, a stationary inducing transformation for trend stationary processes (See King and Rebelo (1988)).

The Unconditional Moments Underlying Our Estimator of Ψ_1

¹¹See for example Kydland and Prescott (1982), Hansen (1985), Prescott (1986), Kydland and Prescott (1988), Backus, Kehoe and Kydland (1989), Christiano and Eichenbaum (1990), and Benhabib, Rogerson, and Wright (1990).

In order to estimate Ψ_1 we use a variant of Hansen's (1982) GMM procedure. As in Christiano and Eichenbaum (1990) we consider an exactly identified version of GMM in which the number of unconditional moment restrictions used to estimate the structural parameters of the model coincides with the dimension of Ψ_1 . The moment restrictions which we use to estimate Ψ_1 result in parameter values that have two appealing features. First, these parameter values are very similar to those used in most RBC studies. This allows us to isolate the effects of labor hoarding per se in those models. Second, at these parameter values, the model succeeds in reproducing the first sample moments of the data. We recognize that there is no a priori reason to use a small subset of the model's implications for the purpose of estimating Ψ_1 . However ignoring these other moment implications certainly affects the asymptotic efficiency of our estimator but not its consistency.

According to our model, $\delta=1+\mathrm{DK}_t/\mathrm{K}_t-\mathrm{K}_{t+1}/\mathrm{K}_t$, where DK_t represents gross investment. Let δ^* denote the unconditional mean of the time series $[1+\mathrm{DK}_t/\mathrm{K}_t-\mathrm{K}_{t+1}/\mathrm{k}_t]$, i.e.

$$(3.3) \quad \mathrm{E}[\delta^{*} - (1 - \mathrm{D}\mathrm{K_{t}}/\mathrm{K_{t}} - \mathrm{K_{t+1}}/\mathrm{K_{t}})] = 0.$$

We identify δ with a consistent estimate of the parameter δ^* .

The social planner's first order condition for capital accumulation requires that the time t expected value of the marginal rate of substitution of goods in consumption equals the time t expected value of the marginal return to physical investment in capital. It follows that

$$(3.4) \quad \mathrm{E}\{\beta^{-1} - [(1-\alpha)\mathrm{Y}_{\mathbf{t}+1}/\mathrm{K}_{\mathbf{t}+1} + (1-\delta)]\mathrm{C}^{\mathrm{p}}_{\mathbf{t}}/\mathrm{C}^{\mathrm{p}}_{\mathbf{t}+1}\} = 0.$$

This is the moment condition that underlies our estimate of α .

The first order condition for effort requires that, for all t, the marginal productivity of an extra unit of effort equals the marginal disutility of effort of those engaged in working. It follows that

$$(3.5)' \quad \mathbb{E}\{\theta(\mathbf{T} - \xi - \mathbf{W_t}\mathbf{f})^{-1} - \alpha[\mathbf{Y_t}/(\mathbf{C_t^pW_tN_t}\mathbf{f})]\} = 0.$$

Since $H_t = N_t f$ this implies the condition

$$(3.5) \quad \mathrm{E}\{\theta(\mathrm{T}-\xi-\mathrm{W_tf})^{-1}-\alpha\mathrm{Y_t}/(\mathrm{C_t^pW_tH_t})\}=0.$$

This is the moment condition used to estimate the parameter θ .

A key problem with implementing (3.5) is that it involves the unobserved value of time t effort, W_t . However by exploiting the structure of our model we can express W_t in terms of the elements of Ψ_1 and the variables which we do observe. Recall that $S_t = Z_t/\gamma^{\alpha t} = (Y_t/\gamma^t)/[(K_t/\gamma^t)^{1-\alpha}H_t^{\alpha}]$. Since $s_t = \ln[S_t/S]$, we can construct a time series of observations on s_t . Equations (2.13a) and (2.18) can be solved to express w_t and a_t as functions of Ψ_1 and of observations on K_t , H_t , G_t , and Y_t .

The scale parameter f also appears in these solutions because n_t is calculated using the fact that $N_t = H_t/f$. To choose a value for f we exploit the planner's first order condition for N_t . This condition equates the utility cost of employing an additional worker to the expected marginal productivity of employment:

$$(3.6) \quad \mathbf{E}_{\mathbf{t}}\{\theta \mathbf{l} \mathbf{n} (\mathbf{T} - \boldsymbol{\xi} - \mathbf{W}_{\mathbf{t}} \mathbf{f}) - \theta \mathbf{l} \mathbf{n} (\mathbf{T}) - \alpha \mathbf{Y}_{\mathbf{t}} / [\mathbf{C}_{\mathbf{t}} \mathbf{N}_{\mathbf{t}}]\} = 0.$$

Relations (3.5) and (3.6) imply that in the nonstochastic steady state:

(3.7)
$$ln[T/(T - \xi - Wf)] = Wf/(T - \xi - Wf).$$

Conditional on our assumed values of T and ξ , (3.7) can be solved for Wf. The parameter f was chosen to equate the average value of N_t f to the sample average value of our empirical measure of per capita hours worked, H_t . Given this value of f and the value of Wf which emerges from (3.7), we deduce W, the nonstochastic steady state value of W_t . Finally, by exploiting the fact that $w_t = \ln(W_t/W)$, we can construct a time series on W_t , which can be used in implementing (3.5).

The law of motion for A_t summarized by (2.5) and our definition of $a_t = \ln(A_t/A)$ imply the unconditional moment restrictions

(3.8a)
$$E[a_t - \rho_a a_{t-1}] a_{t-1} = 0$$
,

(3.8b)
$$E\{[a_t - \rho_a a_{t-1}]^2 - \sigma_{\epsilon}^2\} = 0.$$

Absent measurement error in hours worked relations (3.8) could be used as unconditional moment restrictions for estimating ρ_a and σ_{ϵ} . Below we discuss how (3.8) must be modified to take into account our assumptions regarding measurement error.

The laws of motion for X_t and G_t summarized by (2.7)–(2.8) as well as our definition that $g_t = \ln(G_t/G)$ implies that

$$(3.9a) \hspace{1cm} \mathrm{E}[\ln(\mathrm{X}_{\mathbf{t}}) - \ln(\mathrm{G}) - \ln(\gamma_{\mathbf{g}})\mathbf{t}] = 0$$

$$(3.9b) \qquad \qquad \mathrm{E}[\ln(\mathrm{X}_{\mathbf{t}}) - \ln(\mathrm{G}) - \ln(\gamma_{\mathbf{g}})t]t/\mathbf{T} = 0$$

(3.9c)
$$E[g_t - \rho_g g_{t-1}]g_{t-1} = 0$$

(3.9d)
$$E\{[g_t - \rho_g g_{t-1}]^2 - \sigma_\mu^2\} = 0.$$

where T denotes the number of observations in our sample. ¹² Relation (3.9) summarizes the unconditional moment restrictions that underlie our estimates of $\rho_{\rm g}$, $\sigma_{\mu^{\prime}}$, G and $\gamma_{\rm g}$.

¹²The second equation in (3.9) is scaled by the constant T so that the asymptotic distribution results discussed in Eichenbaum and Hansen (1990) apply.

Our model implies that the log of output is a trend stationary stochastic process with unconditional growth rate γ . It follows that

(3.10a)
$$E[ln(Y_+) - ln(Y) - tln(\gamma)] = 0$$

(3.10b)
$$E[\ln(Y_{t}) - \ln(Y) - \tan(\gamma)]t/T = 0.$$

These are the unconditional moment restrictions underlying our estimates of Y and γ .

Classical Measurement Error In Hours Worked

A variety of authors have implemented RBC models allowing for classical measurement error in average hours worked. Prescott (1986) has argued that (a) the magnitude of this error is large, and (b) failure to account for it leads to large positive biases in standard estimates of the variance of the innovation to technology shocks. ¹³

In order to minimize the differences between our procedure and those adopted in the existing literature we incorporate Prescott's (1986) model of measurement error into our analysis. This is accomplished by exploiting the two different measures of hours worked that we have at our disposal. The first is Hansen (1984)'s measure which was constructed using the results of the household survey conducted by the Bureau of the Census. The second is based on the establishment survey conducted by the Bureau of Labor Statistics. For convenience we refer to these two measures as household hours, H_t^h , and establishment hours worked, H_t^e , respectively.

Suppose that H_t^* denotes true hours worked at time t. Proceeding as in Prescott (1986) we assume that the measurement errors in $\ln(H_t^h)$ and $\ln(H_t^e)$ are i.i.d. and

¹³Other authors have implicitly used Prescott's (1986) model of measurement since they use his measurement error corrected estimates of the variance of the innovation to the technology shock. See for example, Hansen (1988), Kydland and Prescott (1989), and Chari, Christiano, and Kehoe (1990).

orthogonal to each other as well as to $log(H_t^*)$:

(3.11)
$$\ln(\mathbf{H}_{t}^{e}) = \ln(\mathbf{H}_{t}^{*}) + \mathbf{v}_{t}^{e}$$
$$\ln(\mathbf{H}_{t}^{h}) = \ln(\mathbf{H}_{t}^{*}) + \mathbf{v}_{t}^{h}.$$

Equation (3.11) implies that

$$(3.12) \quad \mathrm{E}\{\sigma_{v^h}^2 - .5[\Delta \ln(\mathrm{H}_t^h)]^2 + .5\Delta \ln(\mathrm{H}_t^e)\Delta \ln(\mathrm{H}_t^h)\} = 0.$$

This unconditional moment restriction allows us to estimate the variance of Π_t^h , $\sigma_{v^h}^2$.

Not surprisingly, the presence of the measurement error associated with hours worked affects unconditional moment restrictions (3.8a) and (3.8b) which involve the parameters ρ_a and σ_e . In Appendix A we show that these must be replaced by

$$(3.13) \qquad \qquad \mathrm{E}\{[\mathbf{a_t} \ -\rho_{\mathbf{a}}\mathbf{a_{t-1}}]\mathbf{a_{t-1}} + \rho_{\mathbf{a}}\phi^2\sigma_{\mathbf{v}^{\mathbf{h}}}^2\} = 0,$$

(3.14)
$$E\{[a_t - \rho_a a_{t-1}]^2 - \sigma_\epsilon^2 - \phi^2 (1 + \rho_a^2) \sigma_{vh}^2\} = 0,$$

where $\phi = \omega(1+\pi_2)/(1+\alpha\pi_3)$. The only other moment condition which involves H_t is the one which defines our estimator of α , equation (3.5). In Appendix A we show that this restriction remains valid, up to a first order approximation.

To summarize then, we have displayed 12 unconditional moment restrictions, (3.3)–(3.5), (3.9), (3.10), (3.12)–(3.14), which involve the twelve dimensional parameter vector Ψ_1 and the data. These can be summarized as:

(3.15)
$$E[M_{1t}(\Psi_1^0)] = 0$$
 for all $t \ge 0$,

where Ψ_1^0 is the true value of Ψ and $M_{1t}(\cdot)$ is the 12 x 1 vector valued function whose elements are the left hand sides of (3.3)-(3.5), (3.9)-(3.10), (3.12)-(3.14) before expectations are taken.

The Unconditional Moments Underlying Our Estimator of Ψ_{γ}

Data which are transformed via the HP filter have zero mean by construction. It follows that we can estimate the parameters of Ψ_2 by exploiting the unconditional moment restrictions

$$\begin{split} \text{(3.16)} \qquad & \quad \mathbb{E}\{\mathbf{y}_{\mathbf{t}}^{2}(\sigma_{\mathbf{x}}/\sigma_{\mathbf{y}})^{2}-\mathbf{x}_{\mathbf{t}}^{2}\,\} = 0 \quad \mathbf{x}_{\mathbf{t}} = c_{\mathbf{t}}^{p},\, \mathrm{d}\mathbf{k}_{\mathbf{t}},\, \mathbf{g}_{\mathbf{t}}, \quad \sigma_{\mathbf{x}} = \sigma_{\mathbf{c}}^{p},\, \sigma_{\mathbf{d}\mathbf{k}},\, \sigma_{\mathbf{g}} \\ & \quad \quad \mathbb{E}[\mathbf{h}_{\mathbf{t}}^{2}-\sigma_{\mathbf{h}}^{2}] = 0 \\ & \quad \quad \quad \mathbb{E}\{\mathrm{APL}_{\mathbf{t}}^{2}(\sigma_{\mathbf{h}}/\sigma_{\mathrm{APL}})^{2}-\mathbf{h}_{\mathbf{t}}^{2}\,\} = 0 \\ & \quad \quad \quad \mathbb{E}\{\sigma_{\mathbf{h}}^{2}/[\sigma_{\mathbf{h}}/\sigma_{\mathrm{APL}}]\rho_{-\tau}(\mathrm{APL},\mathbf{h}) - \mathrm{APL}_{\mathbf{t}-\tau}\mathbf{h}_{\mathbf{t}}\} = 0, \quad \tau = 0,\, \pm 1,\, \pm 2,\, \pm 3,\, \pm 4. \end{split}$$

Equation (3.16) consists of 14 unconditional moment restrictions involving the 12 elements of Ψ_2 and the HP filtered data. These can be summarized as

$$(3.17) \quad {\rm E}[{\rm M}_{2t}(\Psi_2^0)] = 0 \quad {\rm for \ all} \ t \geq 0,$$

where Ψ_2^0 is the true value of Ψ_2 and $H_{2t}(\cdot)$ is the 14 x 1 vector valued function whose elements are the left hand sides of (3.16) before expectations are taken.

Using the notation $\Psi=[\Psi_1 \ \Psi_2]$ and $M_t=[M_{1t},\ M_{2t}]$ we can write the unconditional moment restrictions (3.15) and (3.17) as

(3.18)
$$E[M_t(\Psi^0)] = 0 \quad \forall \ t \ge 0,$$

where Ψ^0 is the true value of Ψ . Let $\mathbf{g}_{\mathbf{T}}$ denote the 26 x 1 vector valued function

$$(\mathbf{3.19}) \quad \mathbf{g}_{\mathbf{T}}(\boldsymbol{\Psi}) = (1/\mathbf{T}) \sum_{t=0}^{\mathbf{T}} \mathbf{M}_{t}(\boldsymbol{\Psi}).$$

According to our model, $\mathbf{g_T}(\cdot)$ is a stationary and ergodic stochastic process. Since $\mathbf{g_T}(\cdot)$ is of the same dimension as Ψ it follows from Hansen (1982) that the estimator Ψ_T defined by the condition $\mathbf{g_T}(\Psi_T) = 0$ is a consistent estimator of Ψ^0 . Hansen (1982) also shows that a consistent estimator of the variance—covariance matrix of Ψ_T is given by

(3.20)
$$Var(\Psi_T) = [D_T'(S_T)^{-1}D_T]^{-1}/T.$$

Here S_T is a consistent estimate of the spectral density matrix of $H_t(\Psi^0)$ at frequency zero and $D_T = \partial g_T(\Psi_T)/\partial \Psi$.14

3.2 Testing

In order to assess the model's empirical performance we examine its ability to account for various second moments of the data, say the $q \times 1$ vector of moments Λ . Given a set of values for Ψ_1 our model implies particular values for Λ . We represent this relationship via the function Π that maps \mathbb{R}^{12} into \mathbb{R}^q :

¹⁴Let $S_o = \sum_{k=-\infty}^{\infty} E[H_{t+k}, \Psi^o]M_t(\Psi^o)']$ denote the true spectral density matrix of $M_t(\Psi^o)$ at frequency zero. Proceeding as in Hansen (1982) we can estimate S_o by replacing population moments in the previous expression by their sample counterparts evaluated at Ψ_T . In order to guarantee that our estimate of S_o is positive definite we used the damped autocovariance estimator discussed in Eichenbaum and Hansen (1990). The reported results were calculated by truncating after 4 lags.

$$(3.21) \quad \Pi(\Psi_1) = \Lambda.$$

To compute $\Pi(\cdot)$ we used the log-linear approximation discussed in King, Plosser and Rebelo (1988).

Let B be the q x 26 matrix composed of zeros and ones with the property

(3.22)
$$B\Psi = \Lambda$$

and let

(3.23)
$$F(\Psi) = \Pi(\Psi_1) - B\Psi$$
.

Under the null hypothesis that the model is true

(3.24)
$$F(\Psi^{O}) = 0$$
.

Christiano and Eichenbaum (1990) show that

$$(3.25) \quad \operatorname{Var}[F(\Psi_{\overline{T}})] = [F'(\Psi_{\overline{T}})][\operatorname{Var}(\Psi_{\overline{T}})]F'(\Psi_{\overline{T}})]^{'}.$$

Also the test statistic

(3.26)
$$\mathbf{J} = \mathbf{F}(\Psi_{\mathbf{T}})' \mathbf{Var}[\mathbf{F}(\Psi_{\mathbf{T}})]^{-1} \mathbf{F}(\Psi_{\mathbf{T}})$$

is asymptotically distributed as a chi-square random variable with q degrees of freedom. This fact can be used to test null hypotheses of the form (3.24).

3.3 Data

In this section we describe the data used in our empirical analysis. Private consumption, c_t^p , was measured as quarterly real expenditures on nondurable goods plus services, plus the imputed service flow from the stock of durable goods. The first two measures were obtained from the Survey of Current Business. The third measure was obtained from the data base documented in Brayton and Mauskopf (1985). Government consumption, gt, was measured by real government purchases of goods and services minus real government (federal, state and local) investment. A measure of government investment was provided to us by John Musgrave of the Bureau of Economic Analysis. This measure is a revised and updated version of the measure discussed in Musgrave (1980). Gross investment, dk, was measured as private sector fixed investment plus real expenditures on durable goods plus government fixed investment. The capital stock series, k_t , was chosen to match the investment series. Accordingly, k_t was measured as the stock of consumer durables, producer structures and equipment, plus government and private residential capital plus government nonresidential capital. Gross output, yt, was measured as c_t^p plus g_t plus dk_t plus time t inventory investment. Our basic measure of hours worked, described in Section 3.1, is the one constructed by Hansen (1984), which we refer to as household hours. The second measure of hours worked, establishment hours, which we use to help estimate the measurement error in household hours, is also described in Section 3.1. All data were converted to per capita terms using an efficiency weighted measure of the population.15

Empirical Results

¹⁵For further details on our data set, see Christiano (1987a).

In this section we report empirical results both for the model of section 2 and for the standard indivisible labor model. In the version of the latter model that we consider the planner's criterion function is of the form $E_0 \Sigma_{t=0}^{\infty}[\ln(C_t^p) - \tilde{\theta}H_t]$. Here $\tilde{\theta}$ is some positive scalar. It is straightforward to show that all of the unconditional moment restrictions used to estimate the parameters of the labor hoarding model continue to hold for the standard indivisible labor model, with the exception of (3.5). In this model the planner's first order condition for choosing H_t implies that $\tilde{\theta} = \alpha Y_t/[C_t^pH_t] = 0$. Let $\tilde{\theta}$ denote the unconditional expected value of the time series on the right hand side of the previous expression, i.e.

(4.1)
$$E\{\tilde{\theta}^* - \alpha Y_t / [C_t^p H_t]\} = 0.$$

We identify $\tilde{\theta}$ with a consistent estimate of the parameter $\tilde{\theta}^*$. The only difference between our estimator of the parameters of this model and the time varying effort model, is that (4.1) replaces (3.5) and $W_t \equiv W$, i.e. work effort is constant over time. Subject to these changes, the discussion of our econometric methodology in section 3 applies to the standard indivisible labor model.

The first columns of Tables 1 and 2 report our estimates of Ψ_1 for the two models, obtained using data over the whole sample period. There is reason to believe that, conditional on the variables being trend stationary stochastic processes, there is a break in the data. It is well known that the growth rate of average productivity slowed down substantially in the late 1960's. This can be seen visually from Figure 1 where we graph the log of the Solow residuals. To document the likelihood of a "break" in the process governing the Solow residual, we performed a series of iterative Chow tests. The probability values for the test statistics are graphed as a function of calendar time in

¹⁵These were calculated using the point estimate of α reported in column one of Table 2 but not correcting for classical measurement error in hours worked.

Figure 2. From this figure we see that the null hypothesis of no break is rejected at very high significance levels at all dates during the interval of time spanning 1966 to mid 1974. The break point underlying the results reported in this section coincides with 1969:4. As it turns out our results are quite insensitive to the precise break point chosen.

In Tables 1 and 2 we report the parameter estimates of the two models for the two subsamples. Comparing these to the estimates obtained using the whole sample period we see four important differences. First, our estimates of the unconditional growth rate of the Solow residual, in the first and second sample periods, .0069 and .0015, respectively, are quite different. Second, our estimates of ρ_a are very sensitive to the sample period used to carry out the estimation. For example, using the standard indivisible labor model, over the whole sample, our point estimate of this parameter is .986, whereas it equals .86 and .88 in the first and second sample periods, respectively. The fact the estimate of ρ_a is substantially larger when the whole sample is used is exactly what we would expect to find if there actually were a break in the Solow residual process (see Perron (1988)). Third, the estimated value of σ_{ϵ} is also very sensitive to allowing for a break in the sample as it equals .0060 and .0101 in the first and second sample periods, respectively in the standard indivisible labor model (the corresponding point estimate of σ_ϵ for the whole sample is .0089). Fourth, the estimates of $\gamma_{\rm g}$, $\rho_{\rm g}$ and σ_{μ} are affected in the same qualitative ways as the analog parameters governing the evolution of the Solow residual. However the quantitative differences are even larger.

To understand the quantitative properties of our model it is useful to consider a subset of the impulse response functions of the system, evaluated at the parameter values reported in the first column of Table 1. Figure 3 presents the response of the system to a 1% innovation in government consumption. By assumption employment cannot immediately respond to this shock. However effort rises by over 15% in the first period and then reverts to its steady state level. Panel (a) shows the implied movement in the Solow residual. Since effort has gone up in the first period but total hours of work have not

changed, the Solow residual increases (by about .25%). This is true even though there has been no technology shock whatsoever. Naive Solow residual accounting falsely interprets the increase in average productivity as a shift in technology rather than an exogenous increase in government consumption. Consistent with this observation panel (d) shows that labor productivity, measured as Y_t/N_t , also rises in the first period (by .1%). Like the mechanisms in Lucas (1970) or Hansen and Sargent (1988), time varying effort provides an alternative to technology shocks as the sole explanation for the procyclical behavior of average productivity.

Figure 4 depicts the models's response to a 1% innovation in technology. Given agents' willingness to intertemporally substitute effective leisure over time, they respond to the shock in the first period by increasing effort by about .5%. As a result, the Solow residual rises by 1.3%. Again naive Solow residual accounting exaggerates the true magnitude of the technology shock.

Consistent with the previous discussion, the first columns of Tables 1 and 2 indicate that the main difference between the estimates of Ψ_1 emerging from the two models concerns σ_ϵ . While allowing for time varying effort reduces somewhat our estimate of $\rho_{\bf a}$, the main effect is a large reduction in σ_ϵ . Based on the whole sample period the variance of the innovation to technology shocks drops by roughly 35 per cent. In the first and second samples this variance drops by 48 and 56 percent, respectively. Breaking the sample magnifies the sensitivity of estimates of σ_ϵ to time varying effort. A different way to assess this sensitivity is to consider the unconditional variance of the stationary component of the technology shock, $\sigma_{\bf a}^2$, which equals $[\sigma_\epsilon^2/(1-\rho_{\bf a}^2)]$. Allowing for time varying effort reduces the volatility of technology shocks by over 58 per cent in whole sample period, 49% in the first sample period and 57% over the second sample period.

In summary, incorporating time varying effort in the standard model substantially reduces point estimates of both σ_{ϵ}^2 and σ_a^2 . For this reason our results provide substantial support for the view that a large percentage of the movements in the observed Solow

residual are artifacts of labor hoarding type behavior.

A key question that remains to be answered is how these changes translate into the models' implications for observable variables. To answer this question we begin by analyzing the impact of these changes on the percentage of the variability of output which the model can account for, defined as $\lambda = \sigma_{ym}^2(\Psi_T)/\sigma_y^2$. Here the numerator denotes the variance of HP filtered model output, evaluated at Ψ_T , and the denominator denotes the variance of HP filtered US output. Kydland and Prescott (1989), have emphasized the importance of this statistic. Their claim that technology shocks account for most of the fluctuations in postwar US output corresponds to the claim that λ is a large number, with the current estimate being between .75 and 1.0, depending on exactly which RBC model is used (see for example Hansen (1988)).

Table 3 presents the values of λ implied by the two models over the different sample periods that we consider. For the whole sample period introducing time varying effort causes λ to decline by over 28%, from .81 to .58. The sensitivity of λ to labor hoarding is even more dramatic once we allow for a break in the sample. Labor hoarding reduces λ by 58% in the first sample period and by 63% in the second period. Evidently the finding that technology shocks account for most of the variability in aggregate output is very sensitive to the presence of labor hoarding.

The previous evidence is of interest only to the extent that introducing labor hoarding does not lead to a substantial deterioration in the model's empirical performance along other dimensions of the data. To address this issue we begin by considering the models' implications for a different set of moments which have been emphasized in the RBC literature: the volatility of hours worked, $\sigma_{\rm n}$, the relative volatility of consumption, investment, and government purchases to output, $\sigma_{\rm c}/\sigma_{\rm y}$, $\sigma_{\rm dk}/\sigma_{\rm y}$, and $\sigma_{\rm g}/\sigma_{\rm y}$, respectively, as well as the volatility of hours worked relative to average productivity, $\sigma_{\rm n}/\sigma_{\rm APL}$. Table 4A reports the models' predictions for these moments as well our estimates of the corresponding data moments. Table 4B reports the analogous results obtained using the

two subsample periods. For each moment in the two tables there are three numbers entered in the columns labelled "Time Varying Effort Model" and "Constant Effort Model". The top number equals the probability limit of the moment implied by the relevant model. These were calculated using the relevant estimate of Ψ_1 . The middle number is the estimated standard error of the first number, and reflects sampling uncertainty in our estimates of Ψ_1 . For each moment we tested the null hypothesis that the model moment equals the data population moment. This was done using the J statistic discussed in section 3. The bottom number equals the probability value of the relevant J statistic. 17

According to the results summarized in Table 4A it is very difficult to distinguish between the two models on the basis of their implications for the moments in question. Indeed, according to the J statistics, there is very little evidence against the individual hypotheses that the value of $\sigma_{\rm n}$, $\sigma_{\rm c}/\sigma_{\rm v}$, $\sigma_{\rm dk}/\sigma_{\rm v}$, or $\sigma_{\rm g}/\sigma_{\rm v}$ that emerges from either model is different from the corresponding data population moments. However, the performance of both models deteriorates significantly when we allow for a break in the sample. This deterioration is quite pronounced with respect to the relative volatility of consumption and investment. As Table 4B indicates, using either sample period, we can reject at any conventional significance level, the hypotheses that these model moments equal the corresponding data population moments. Interestingly these rejections do not arise so much because the data moment estimates change substantially. Rather they arise because the models' implications for the two moments appear to be quite sensitive to allowing for a break in the sample. For example over the whole sample period, both models imply that consumption is roughly half as volatile as output. However, when estimated on the separate sample periods, both models predict that consumption is only a fourth as volatile as output.

The intuition behind this last result is straightforward. According to the permanent

¹⁷Since we are considering only one moment at a time, the J statistic is asymptotically distributed as a chi—square random variable with one degree of freedom.

income hypothesis, an innovation to labor income causes households to revise their consumption plan by the annuity value of that innovation. If income were an AR(1) process about a deterministic trend, then the annuity value of the innovation would be a strictly increasing function of the size of the AR(1) coefficient. Using a model very similar to our version of the standard indivisible labor model, Christiano (1987b) shows that the income effect of an innovation to the technology shock is also increasing in ρ_a , the parameter which governs the serial correlation of the technology shock. Since the point estimate of ρ_a falls in both subsamples, we would expect that, holding interest rates constant, the response of consumption to an innovation in the technology shock should also fall. Given that Christiano (1987b) also shows that the impact of technology shocks on the interest rate in standard RBC models is quite small, it is not surprising the model predicts lower values for σ_c/σ_y in the subsample periods. Since output equals consumption plus investment plus government consumption, and the latter does not respond to technology shocks, it follows that, other things equal, with consumption less volatile, investment will be more volatile.

All in all, it is difficult to distinguish between the different models on the basis of the previous moments emphasized in existing RBC studies. Fortunately, there exist other dimensions of the data along which the models have quite different implications. One of these dimensions is suggested by Hall's (1988) observation that the growth rate of the Solow residual is positively correlated with his measure of government consumption. Existing RBC models imply that this correlation coefficient ought to equal zero. To understand the quantitative implications of our model for this correlation we proceeded as in Hall (1988) and estimated the regression coefficient bg, of the growth rate of the Solow residual on the growth rate of our measure of government consumption. Using the whole sample period the estimated value of bg equals .187 with standard error equal to .07. The probability limit of bg implied by our model equals .104 with standard error of .024. Testing the hypothesis that the two regression coefficients are the same in population, we

obtain a value for our J statistic which has a probability value of .29. Consequently, one cannot reject, at conventional significance levels, the view that our model fully succeeds in accounting for the correlation in question. However, there is somewhat more evidence against the null hypothesis once we allow for a break in the sample. The probability value of the J statistic is .9999 in the first part of the sample but it is only .008 in the second part.18

Hall (1988) interprets his positive estimate for bg as evidence in favor of the notion that imperfect competition and increasing returns to scale are important determinants of the time series properties of average productivity. While he does not construct and test a model incorporating these features, he does review and reject alternative explanations of regression results. To argue that unobserved variation in labor effort is not a plausible explanation he first calculates the growth rate of effective labor input required to explain all of the observed movements in total factor productivity. From this measure he subtracts the growth rate of actual hours of work to generate a time series on the growth rate in work effort. Based on these calculations Hall argues that the implied movements in work effort are implausibly large. However, this calculation is not germane to our analysis because it presumes that there are no technology shocks whatsoever. In our context the relevant issue is what must the time series properties of effort be to explain the regression coefficient in question, not whether time varying effort can explain all movements in total factor productivity. Our analysis indicates that time varying effort seems quite plausible as an explanation for the phenomenon in question. Hall's conjectures notwithstanding, the contemporaneous correlation between government consumption and the Solow residual can be accounted for within a model embodying the twin assumptions of perfect competition and a constant returns to scale production technology. In any event it is clear that the time

¹⁸In the first sample the point estimate of bg is .0798 with standard error .0795. The probability limit of bg that emerges from our model equals .0797 with a standard error of .0259. For the second sample the point estimate of bg is .280 with a standard error of .099, while the probability limit of bg implied by the model is .0225 with standard error .004.

varying effort model does substantially better than standard RBC models on this dimension of the data.

We now investigate the models' implications for the dynamic correlations between hours worked and average productivity. The first column of Table 5 reports our estimates of the correlation between H_{t-i} and APL_t , i=-4,-3,...,3,4, over the whole sample period. The second and third columns report the corresponding model predictions about what the econometrician would see in the data if hours worked were not corrupted by measurement error. To be more precise, these probability limits refer to the correlations between $\ln(APL_t^*)$, and $\ln(H_{t-i}^*)$, i=-4,-3,...,3,4. Here APL_t^* equals $\ln(Y_t) - \ln(H_t^*)$ and H_t^* denotes the true unobserved level of hours worked at time t. The last two columns of Table 5 report the models' predictions for the probability limits of the dynamic correlations between observed average productivity and hours worked.

We wish to emphasize four interesting features of the estimated correlations in the data. First, contemporaneous average productivity and hours display a weak negative correlation (-.19). One cannot reject the null hypothesis that this correlation actually equals zero. This is consistent with findings by numerous authors that different measures of the returns to working do not display a pronounced correlation with hours worked (see Christiano and Eichenbaum (1990)). Second, average productivity is positively correlated with hours worked at all leads. However, with one exception, average productivity is negatively correlated with hours worked at all lags. In the exceptional case, this correlation is insignificantly different from zero. Third, the maximal correlation occurs between average productivity at time t and hours worked at time t+2. In this sense average productivity leads the cycle. Fourth, the maximal negative correlation occurs between average productivity at time t and hours worked at time t-4. The is consistent with Gordon's (1979) finding that average productivity tends to fall at the end of expansions.

Consider the models' predictions for the dynamic correlations between true average productivity and hours worked. Columns 2 and 3 of Table 5 reveal that the predictions of

the two models for these correlations differ in three important ways. First, while the contemporaneous correlation between average productivity and hours worked is large and positive in the indivisible labor model, it is insignificantly different from zero in our model. Second, the maximal correlation between true hours worked and average productivity in the standard indivisible labor model occurs contemporaneously. In our model the maximum value of this correlation occurs at lead 1. Third, according to the standard indivisible labor model true average productivity displays a strong positive correlation with lagged hours worked, and these correlations are systematically larger than those between average productivity and future hours worked. In contrast our model predicts that average productivity ought to display a larger correlation with future hours worked than with lagged hours worked. These observations suggest that the basic structure of our model seems more consistent with the data than the constant effort model. However these results cannot be used to test the two models. For that we must consider the models' implications for the dynamic correlations between observed average productivity and hours worked.

These are reported in the fourth and fifth columns of Table 5, each element of which contains three numbers. The top number corresponds to the probability limit of the model for the moment in question. The middle number corresponds to the estimated standard error of the probability limit. The bottom number equals the probability value of the J statistic associated with testing the null hypothesis that the model moment and the data moment are the same in population.

Both models do quite well in accounting for the correlation between average productivity and hours worked at time t+i, i=+1,0,-1. However the models do less well at longer leads and lags. Table 6 reports estimates of the dynamic correlations and the model predictions allowing for a break in the sample. Comparing Tables 5 and 6 we can see that model predictions are not substantially affected by the sample split. However, the estimated correlations in the data appear to be quite sensitive. It is still the case that in both samples average productivity leads the cycle. But, relative to the whole data set, this

feature is muted in the first sample and more salient in the second sample.

In order to assess the models' ability to simultaneously account for these correlations, we tested the joint hypothesis that the values of $\rho(APL_t, H_{t+i})$, i = -L,...+L in the model and the data are the same in population. The results of test for L = 1 and L = 2 are reported in Table 7. For L = 1 (2) our J statistic is asymptotically distributed as a chi-square statistic with 3 (5) degrees of freedom. Consider first the results obtained using the whole sample. With L = 1 the probability values of the J statistics for the time varying effort and standard indivisible labor models are .675 and .017, respectively. With L = 2, the J statistics for the time varying effort and constant effort models have probability values of .001 and .037. So with L=1 there is less evidence against our model. But with L=2 there is less evidence against the standard indivisible labor model. However, this inference depends sensitively on allowing for a break in the sample.

In the first sample, with L=1, the J statistics for the time varying and constant effort models have probability values of .526 and .381. For the second sample, the corresponding J statistics have probability values of .278 and .003. So this test yields very little evidence against the time varying effort model in either subsample. On the other hand this test yields strong evidence against the standard indivisible labor model in the second sample. In the first sample, when L=2, the J statistics for the time varying effort and indivisible labor models have probability values equal to .172 and .085. For the second sample, the corresponding J statistics have probability values of .001 and .000. Evidently, once we allow for a split in the sample the time varying effort model out—performs the constant effort model both when L=1 and when L=2, irrespective of which subsample is used. Indeed, the only evidence against the time varying effort model occurs for the L=2 test in the second subsample. In contrast, there is a great deal of evidence against the constant effort model in the second sample irrespective of whether L equals one or two.

Overall we conclude that neither of the two models that we consider explains all aspects of the data. Still, our evidence indicates that our model is at least as successful as

the indivisible labor model in accounting for those aspects of the data investigated in this paper. Indeed we have argued that along a variety of dimensions the time varying effort model out—performs the constant effort model. From this perspective, we conclude that (a) existing estimates of the importance of technology shocks are very sensitive to allowing for labor hoarding type behavior, and (b) there is reason to believe that labor hoarding type behavior is plausible in the sense that incorporating it into the analysis moves the model into closer conformity with the aggregate data.

5. CONCLUSION

This paper has investigated the sensitivity of Solow residual based measures of technology shocks. Our evidence provides support for the view that a significant proportion of movements in the Solow residual reflect labor hoarding behavior. Our findings cast doubt on existing claims in the RBC literature that technology shocks account for a large proportion of the variance in post—war US aggregate output.

Our model embodies a number of strong assumptions which ought to be relaxed in future research. First, we supposed that workers' shift length is constant. This means that firms can only respond to unanticipated demand and technology shocks by inducing labor to work more intensively. It would be desirable to extend the model to allow firms to vary the shift length itself, say by allowing for overtime. In principle, such an extension would allow us to study the movements in labor, both along the extensive and intensive margins. Second, we assumed that employment is chosen on a quarterly basis. While this corresponds to the standard practice of identifying agents' decision intervals with the data frequency, the quantitative implications of our theory may be sensitive to this assumption. We suspect that allowing for temporal aggregation effects will reduce the impact of labor hoarding on our estimates of the variance of technology shocks. On the other hand, we assumed that there are no costs to changing employment over different periods.

Introducing adjustment costs would probably magnify the effects of labor hoarding over time and increase the impact of such behavior on estimates of the variance of technology shocks.

References

- Aiyagari, S. R., Christiano, L.J., and Eichenbaum, M., "The Output, Employment, and Interest Rate Effects of Government Consumption," 1990, NBER Working Paper No. 3330.
- Backus, D. K., Kehoe, P. J. and Kydland, F. E., "International Trade and Business Cycles," 1989, Federal Reserve Bank of Minneapolis Working Paper 425.
- Baxter, M. and King, R., "Fiscal Policy in General Equilibrium," 1990, manuscript, University of Rochester.
- Benhabib, J., Rogerson, R. and Wright, R., "Homework in Macroeconomics: Aggregate Fluctuations," 1990, NBER Working Paper No. 3344.
- Brayton, F. and Mauskopf, E., "The MPS Model of the United States Economy," 1985, Board of Governors of the Federal Reserve System, Division of Research and Statistics, Washington D.C.
- Chari, V.V., Christiano, L. J. and Kehoe, P. J., "Optimal Fiscal Policy," manuscript, 1990, Federal Reserve Bank of Minneapolis.
- Christiano, L. J., "Technical Appendix to 'Why Does Inventory Investment Fluctuate So Much?'", 1987a, Working Paper 380, Federal Reserve Bank of Minneapolis.
- Christiano, L. J., "Why is Consumption Less Volatile Than Income," Federal Reserve Bank of Minneapolis, Quarterly Review, 1987b, 1, 2-20.
- Christiano, L. J. and Eichenbaum, M., "Current Real Business Cycle Models and Aggregate Labor Markets, manuscript, 1990, Northwestern University.
- Eichenbaum, M., and Hansen, L.P., "Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data," Journal of Business and Economic Statistics, 1990, 8, 53-71.
- Evans, C. L., "Productivity Shocks and Real Business Cycles, 1990, manuscript, University of Southern Carolina.
- Fay, Jon and Medoff, James, "Labor Input over the Business Cycle," American Economic Review, vol. 75, September, 1985, 638-55.
- Gordon, R. J., "The End-of-Expansion Phenomenon in Short-Run Productivity Behavior," *Brookings Papers on Economic Activity*, 1979, 2, 447-61.
- Gordon, R.J., "Are Procyclical Productivity Fluctuations a Figment of Measurement Error?", 1990, manuscript, Northwestern University.
- Hall, R. E., "The Relation between Price and Marginal Cost in U.S. Industry," Journal of Political Economy, October 1988, 96, 921-47.
- Hall, R. E., "Invariance Properties of the Solow Residual," 1989, NBER Working Paper

- No. 3034.
- Hansen, G. D., "Fluctuations in Total Hours Worked: A Study Using Efficiency Units," 1984, manuscript, University of Minnesota.
- Hansen, G. D., "Indivisible Labor and the Business Cycle," Journal of Monetary Economics, November 1985, 16, 309-28.
- Hansen, G. D., "Technical Progress and Aggregate Fluctuations," 1988, manuscript, University of California, Los Angeles.
- Hansen, G. D. and Sargent, T. J., "Straight Time and Over Time in Equilibrium," Journal of Monetary Economics, March/May 1988, 21, 281-308.
- Hansen, L. P., "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 1982, 50, 1029 54.
- Hodrick, R. J. and Prescott, E. C., "Post—War Business Cycles: An Empirical Investigation," manuscript, Carnegie—Mellon University, 1980.
- Horning, B. C., "Labor Hoarding and the Business Cycle," 1990, manuscript, University of Minnesota.
- King, R. G. and Rebelo, S. T., "Low Frequency Filtering and Real Business Cycle Cycles," manuscript, 1988, University of Rochester.
- King, R. G., Plosser, C. I. and Rebelo, S. T., "Production, Growth and Business Cycles," Journal of Monetary Economics, March/May 1988, 21, 195-232.
- Kydland, F. E. and Prescott, E. C., "Time to Build and Aggregate Fluctuations," Econometrica, November 1982, 50, 1345-70.
- Kydland, F. E. and Prescott, E. C., "The Work Week of Capital and Its Cyclical Implications," Journal of Monetary Economics, March/May 1988, 21, 343-60.
- Kydland, F. E. and Prescott, E. C., "Hours and Employment Variation in Business Cycle Theory, 1989, Institute for Empirical Economics, Discussion Paper No. 17.
- Lucas, R. E. Jr., "Capacity, Overtime and Empirical Production Functions," American Economic Review, Papers and Proceedings, 1970, 6, 1345-1371.
- Lucas, R. E. Jr., "The Effects of Monetary Shocks When Prices Are Set in Advance," 1989, manuscript, University of Chicago.
- McCallum, B. T., "Real Business Cycle Models," in Modern Business Cycle Theory, ed. by Robert J. Barro, 1989, 16-50, Harvard University Press.
- Musgrave, J., "Government Owned Fixed Capital in the United States," Survey of Current Business, 1980, March, 33-43.
- Perron, P., "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis," Econometrica, 1988, 55, 277-302.
- Prassad, A., "A Model of Labor Hoarding," 1990, manuscript, University of Chicago.

- Prescott, E. C., "Theory Ahead of Business Cycle Measurement," Federal Reserve Bank of Minneapolis, Quarterly Review, Fall 1986, 10, 9-22.
- Rotemberg, J. J. and Summers, L. H., "Labor Hoarding, Inflexible Prices and Procyclical Productivity," 1988, NBER Working Paper No. 2591, forthcoming, Quarterly Journal of Economics.
- Rogerson, R., "Indivisible Labor, Lotteries and Equilibrium," Journal of Monetary Economics, January 1988, 21, 3-17.
- Summers, L. H., "Some Skeptical Observations on Real Business Cycle Theory," Federal Reserve Bank of Minneapolis, Quarterly Review, Fall 1986, 10, 23 - 27.

APPENDIX

In this appendix we show how the unconditional moment restrictions implied by the labor hoarding model must be modified to take account of measurement error in hours

Equation (3.11) shows that, for either measure of hours worked, we are assuming that

$$\ln(\mathbf{H}_{t}) = \ln(\mathbf{H}_{t}^{*}) + \mathbf{v}_{t}$$

where H_t is observed hours worked, H^{*}_t is true hours worked, and v_t is some i.i.d.

measurement error. Accordingly we denote all true measures of variables with asterisks.

Recall that with labor hoarding we have the two equations

(2.13a)
$$\begin{aligned} \mathbf{w}_{t}^{*} &= \pi_{1}\mathbf{k}_{t} + \pi_{2}\mathbf{n}_{t}^{*} + \pi_{3}\mathbf{a}_{t}^{*} + \pi_{4}\mathbf{g}_{t} \\ \mathbf{g}_{t}^{*} &= \mathbf{a}_{t}^{*} + \alpha\mathbf{w}_{t}^{*}. \end{aligned}$$

It can be shown that s, is given by

(A.1)
$$\mathbf{s}_{\mathbf{t}}^* = \mathbf{y}_{\mathbf{t}} - (1-\alpha)\mathbf{k}_{\mathbf{t}} + \alpha\mathbf{h}_{\mathbf{t}}^*.$$

Our model implies that $n_{t}^{*} = h_{t}^{*}$, so that (2.13a), (2.18) and (A.1) can be solved for a_{t}^{*} as a function of y_t , k_t , h_t^* and g_t . This solution is given by

(A.2)
$$\mathbf{a}_{\mathbf{t}}^* = [\mathbf{y}_{\mathbf{t}} - (1 - \alpha + \alpha \pi_1) \mathbf{k}_{\mathbf{t}} - \alpha (1 + \pi_2) \mathbf{h}_{\mathbf{t}}^* - \alpha \pi_4 \mathbf{g}_{\mathbf{t}}] / (1 + \alpha \pi_3).$$

Unfortunately, h, is not observable. Therefore, in practice we can only generate the series $\mathbf{a}_{\mathbf{t}} = [\mathbf{y}_{\mathbf{t}} - (1 - \alpha + \alpha \pi_1) \mathbf{k}_{\mathbf{t}} - \alpha (1 + \pi_2) \mathbf{h}_{\mathbf{t}} - \alpha \pi_4 \mathbf{g}_{\mathbf{t}}] / (1 + \alpha \pi_3).$

Since $h_t = h_t^* + v_t$ follows from our definition of the measurement error, a_t and a_t^* are related according to:

(A.3)
$$a_t = a_t^* - [\alpha(1+\pi_2)/(1+\alpha\pi_3)] v_t$$

Letting $\phi = \alpha(1+\pi_2)/(1+\alpha\pi_3)$ we obtain $\mathbf{a}_t = \mathbf{a}_t^* - \phi \mathbf{v}_t$.

The moment conditions (3.8a) and (3.8b) hold for the true technology shock a.

(3.8a)
$$E[a_{t}^{*} - \rho_{a} a_{t-1}^{*}] a_{t-1}^{*} = 0$$

(3.8b)
$$\mathbb{E}\{[\mathbf{a}_{t}^{*} - \rho_{a} \mathbf{a}_{t-1}^{*}]^{2} - \sigma_{\epsilon}^{2}\} = 0$$

Now,
$$\mathbf{E}[\mathbf{a_t} - \rho_{\mathbf{a}} \mathbf{a_{t-1}}] \mathbf{a_{t-1}} = \mathbf{E}[\mathbf{a_t}^* - \phi \mathbf{v_t} - \rho_{\mathbf{a}} (\mathbf{a_{t-1}}^* - \phi \mathbf{v_{t-1}})] (\mathbf{a_{t-1}}^* - \phi \mathbf{v_{t-1}})$$

$$= \mathbf{E}[(\mathbf{a_t}^* - \rho_{\mathbf{a}} \mathbf{a_{t-1}}) - \phi (\mathbf{v_t} - \rho_{\mathbf{a}} \mathbf{v_{t-1}})] (\mathbf{a_{t-1}}^* - \phi \mathbf{v_{t-1}})$$

$$= -\mathbf{E}[\phi^2 \rho_{\mathbf{a}} \mathbf{v_{t-1}}^2]$$

$$= -\phi^2 \rho_{\mathbf{a}} \sigma_{\mathbf{v}}^2.$$

Also,
$$\mathrm{E}\{[\mathbf{a_t} - \rho_{\mathbf{a}} \mathbf{a_{t-1}}]^2 - \sigma_{\epsilon}^2\} = \mathrm{E}\{[(\mathbf{a_t}^* - \phi \mathbf{v_t}) - \rho_{\mathbf{a}} (\mathbf{a_{t-1}}^* - \phi \mathbf{v_{t-1}})]^2 - \sigma_{\epsilon}^2\}$$

$$\begin{split} &= \mathrm{E}\{[(\mathbf{a}_{\mathsf{t}}^* - \rho_{\mathsf{a}} \mathbf{a}_{\mathsf{t}-1}^*) - \phi(\mathbf{v}_{\mathsf{t}} - \rho_{\mathsf{a}} \mathbf{v}_{\mathsf{t}-1})]^2 - \sigma_{\epsilon}^2\} \\ &= \mathrm{E}\{(\mathbf{a}_{\mathsf{t}}^* - \rho_{\mathsf{a}} \mathbf{a}_{\mathsf{t}-1}^*)^2 - 2\phi(\mathbf{a}_{\mathsf{t}}^* - \rho_{\mathsf{a}} \mathbf{a}_{\mathsf{t}-1}^*)(\mathbf{v}_{\mathsf{t}} - \rho_{\mathsf{a}} \mathbf{v}_{\mathsf{t}-1}) + \phi^2(\mathbf{v}_{\mathsf{t}} - \rho_{\mathsf{a}} \mathbf{v}_{\mathsf{t}-1})^2 - \sigma_{\epsilon}^2\} \\ &= \sigma_{\epsilon}^2 + \phi^2(1 + \rho_{\mathsf{a}}^2)\sigma_{\mathsf{v}}^2 - \sigma_{\epsilon}^2 \\ &= \phi^2(1 + \rho_{\mathsf{a}}^2)\sigma_{\mathsf{v}}^2. \end{split}$$

These results justify the forms of the moment restrictions given in (3.13) and (3.14). Equation (3.5) holds for the true variables.

(3.5)
$$E\{\theta(T-\psi-W_{+}^{*}f)^{-1} - \alpha[Y_{+}/(C_{+}^{P}W_{+}^{*}H_{+}^{*})]\} = 0$$

If (2.13a) is used to solve for the level of work effort but measures of hours worked, and therefore values of the technology shock, are corrupted with error, we will obtain a time series:

$$\begin{split} \mathbf{w_t} &= \pi_1 \mathbf{k_t} + \pi_2 \mathbf{h_t} + \pi_3 \mathbf{a_t} + \pi_4 \mathbf{g_t} \\ &= \pi_1 \mathbf{k_t} + \pi_2 \mathbf{h_t} + \pi_3 \mathbf{a_t} + \pi_4 \mathbf{g_t} + (\pi_2 - \phi \pi_3) \mathbf{v_t} \\ &= \mathbf{w_t}^* + (\pi_2 - \phi \pi_3) \mathbf{v_t}. \end{split}$$

This implies that

$$\mathbf{w}_{t} = \mathbf{w}_{t}^{*} \exp[(\pi_{2} - \phi \pi_{3}) \mathbf{v}_{t}].$$

We will construct the left hand side of (3.5) when W_t is substituted for W_t^* , and H_t is substituted for H.

$$\begin{split} \theta(\mathbf{T} - \psi - \mathbf{W_t} \mathbf{f})^{-1} - \alpha[\mathbf{Y_t}/(\mathbf{C_t^p W_t H_t})] &= \\ \theta\{\mathbf{T} - \psi - \mathbf{W_t^*} \exp[(\pi_2 - \phi \pi_3) \mathbf{v_t}] \mathbf{f}\}^{-1} - \alpha\{\mathbf{Y_t}/[\mathbf{C_t^p W_t^* H_t^*} \exp((1 + \pi_2 - \phi \pi_3) \mathbf{v_t})]\}. \end{split}$$

Taking a first-order Taylor series expansion of the right hand side around $v_{\star} = 0$ we obtain

$$\begin{split} \theta(\mathbf{T} - \phi - \mathbf{W_t} \mathbf{f})^{-1} - \alpha[\mathbf{Y_t} / (\mathbf{C_t^p W_t H_t})] & \cong \\ \theta[(\mathbf{T} - \psi - \mathbf{W_t^*} \mathbf{f})^{-1} - 0.5 (\mathbf{T} - \psi - \mathbf{W_t^*} \mathbf{f})^{-2} \ \mathbf{W_t^*} \mathbf{f}(\pi_2 - \phi \pi_3) \mathbf{v_t}] - \\ \alpha\{[\mathbf{Y_t} / (\mathbf{C_t^p W_t^* H_t^*})] - 0.5 [\mathbf{Y_t} / (\mathbf{C_t^p W_t^* H_t^*})] (1 + \pi_2 - \phi \pi_3) \mathbf{v_t}\}. \end{split}$$

Taking expectations of both sides of the equation we find that
$$\mathbb{E}\{\ell(\mathbf{T}-\psi-\mathbf{W}_t\mathbf{f})^{-1}-\alpha[\mathbf{Y}_t/(\mathbf{C}_t^p\mathbf{W}_t\mathbf{H}_t)]\}\cong\mathbb{E}\{\ell(\mathbf{T}-\psi-\mathbf{W}_t^*\mathbf{f})^{-1}-\alpha[\mathbf{Y}_t/(\mathbf{C}_t^p\mathbf{W}_t^*\mathbf{H}_t^*)]\}.$$

As a result, we do not modify equation (3.5) to account for measurement error. Similarly, it can be shown that the same modifications must be made for the standard indivisible labor model with α substituted for ϕ wherever it appears.

Table 1 Model Parameters (Standard Errors) Time Varying Effort Model

Parameter	Whole Sample	Sample Period 1	Sample Period 2
δ	.0209	.0196	.0221
	(.0003)	(.0001)	(.0002)
α	.6553	.6593	.650 4
	(.0057)	(.0062)	(.0095)
θ	3.6779	3.7810	3.5826
	(.0334)	(.0324)	(.0537)
$ ho_{ m a}$.9772	.8691	.8815
	(.0289)	(.0430)	(.0611)
σ_{ϵ}	.0072	.00 42	.0067
	(.0012)	(.0006)	(.0006)
ln(Y)	8.5722	8.4914	8.8733
	(.0181)	(.0138)	(.0107)
$\ln(\gamma_{\mathbf{y}})$.0041	.0069	.0015
	(.0003)	(.0004)	(.0003)
ln(G)	6.9488	6.8090	7.1618
	(.0278)	(.0167)	(.0111)
$\ln(\gamma_{\rm g})$.0021	.0073	00 13
	(.0004)	(.0007)	(.0 003)
$ ho_{ m g}$.9791	.9380	.6618
	(.0212)	(.0467)	(.0769)
σ_{μ}	.0145	.0143	.0115
	(.0011)	(.0013)	(.0012)
$\sigma_{ u}$.0087	.0084	.0091
	(.0016)	(.0020)	(.0 022)

Table 2
Model Parameters (Standard Errors)
Constant Effort Model

Parameter	Whole Sample	Sample Period 1	Sample Period 2
δ	.0209	.0196	.0221
	(.000 3)	(.0001)	(.0002)
α	.6553	.6593	.6504
	(.0057)	(.0062)	(.0537)
θ	3.6977	3.8014	3.5959
	(.0401)	(.0364)	(.0610)
$ ho_{f a}$.9857	.8624	.8842
	(.0259)	(.0706)	(.0647)
σ_{ϵ}	.0089	.0060	.0101
	(.0013)	(.0022)	(.0015)
ln(Y)	8.5722	8.4914	8.8733
	(.0181)	(.0138)	(.0107)
$\ln(\gamma)$.0041	.0069	.0015
	(.0003)	(.0004)	(.0003)
ln(G)	6.9488	6.8090	7.1618
	(.0278)	(.0167)	(.0111)
$\ln(\gamma_{ m g})$.0021	.0073	0013
	(.0004)	(.0007)	(.0003)
$^{ ho}g$.9791	.9380	.6618
	(.0212)	(.0467)	(.0769)
σ_{μ}	.01 45	.0143	.0115
	(.0011)	(.0013)	(.0012)
$\sigma_{ u}$.0087	.0084	.0091
	(.0016)	(.0020)	(.0022)

Table 3
Variability of Output Accounted by Models

	Time Varying Effort Model		Constant Effort Model		US Data	
HP Filter	$\sigma_{ exttt{y}}$	λ	$\sigma_{\mathbf{y}}$	λ	$\sigma_{\mathbf{y}}$	
Whole Sample	.015	.58	.017	.81	.019	
	(.001)	(.14)	(.006)	(.56)	(.002)	
Sample Period 1	.011	.71	.017	1.69	.01 3	
	(.001)	(.20)	(.007)	(1.51)	(.002)	
Sample Period 2	.017	.52	.028	1.42	.024	
	(.001)	(.12)	(.005)	(.65)	(.003)	

Table 4A Selected Second Moments Whole Sample

2nd Moment	US Data	Time Varying Effort Model	Constant Effort Model	
$\sigma_{\rm c}/\sigma_{\rm y}$.44 (.03)	.48 (.19) [.80]	.53 (.24) [.69]	
$^{\sigma}\mathrm{d}\mathbf{k}^{/\sigma}\mathrm{y}$	2.22 (.07)	2.77 (.45) [.23]	2.65 (.59) [.47]	
$\sigma_{\rm g}/{ m dy}$	1.15 (.20)	1.29 (.15) [.50]	1.090 (.35) [.89]	
$^{\sigma}$ n/ $^{\sigma}$ APL	1.22 (.12)	1.017 (.41) [.39]	1.053 (.46) [.72]	
$\sigma_{ m n}$.017 (.002)	.013 (.003) [.76]	.013 (.005) [.94]	

Table 4B Selected Second Moments: Subsamples

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					•		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Parameter	US Data	Varying		US Data	Varying	Constant Effort
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\sigma_{\rm c}/\sigma_{ m y}$		(.03)	(.05)		(.05)	(.05)
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$^{\sigma}_{ m d}{}_{ m k}/^{\sigma}{}_{ m y}$		(.11)	(.16)		(.17)	(.18)
$ \begin{bmatrix} .38 \end{bmatrix} \begin{bmatrix} .27 \end{bmatrix} \qquad \begin{bmatrix} .40 \end{bmatrix} \begin{bmatrix} .14 \end{bmatrix} $ $ \sigma_{\mathbf{n}} \qquad \qquad .017 \qquad .012 \qquad .017 \qquad .02 \qquad .016 \qquad .025 \\ (.002) \qquad (.002) \qquad (.005) \qquad (.002) \qquad (.001) \qquad (.005) $	$\sigma_{ m g}/{ m dy}$		(.29)	(.43)		(.10)	(.09)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$^{\sigma}$ n $/^{\sigma}$ APL		(.05)	(.70)		(.05)	(.64)
	$\sigma_{ m n}$		(.002)	(.005)		(.001)	(.005)

Table 5

Data and Model Predictions For Dynamic Correlations Between Average Productivity and Hours Worked

$\rho(APL_t, H_{t+i})$	US Data	Time	Constant	Time	Constant
· · · · · · · · · · · · · · · · · · ·		Varying Effort	Effort	Varying Effort	Effort
	Measurem	ent Error F	ree	Error F	Ridden
4	.27 (.12)	.04 (.02)	19 (.07)	01 (.01) [.02]	08 (.07) [.01]
3	.38 (.12)	.20 (.04)	07 (.03)	.05 (.01) [.001]	01 (.04) [.002]
2	.43 (.11)	.39 (.08)	.09 (.06)	.14 (.03) [.01]	.08 (.03) [.003]
1	.27 (.10)	.64 (.12)	.31 (.13)	.25 (.05) [.92]	.21 (.10) [.70]
0	19 (.13)	.18 (.15)	.60 (.24)	08 (.18) [.58]	09 (.37) [.79]
-1	.08 (.08)	.16 (.11)	.52 (.21)	.14 (.08) [.60]	.33 (.17) [.19]
-2	01 (.08)	.14 (.08)	.44 (.17)	.12 (.06) [.16]	.28 (.14) [.08]
-3	20 (.10)	.11 (.05)	.37 (.14)	.11 (.04) [.003]	.24 (.12) [.004]
-4	31 (.10)	.09 (.03)	.29 (.11)	.09 (.03) [0.0]	.19 (.09) [0.0]

 ${\bf Table~6}$ ${\bf Correlations~(n_t,~APL_{t-i})}$ Data and Model Predictions Incorporating Measurement Error

i	US Data	Time Varying Effort Model	Constant Effort Modell	US Data	Time Varying Effort Model	Constant Effort Model
•	Sample Period 1			Sample Pe	riod 2	
4	024 (.14)	005 (.02) [.89]	14 (.12) [.48]	.41 (.16)	02 (.02) [.01]	21 (.07) [0.0]
3	.14 (.13)	.04 (.008) [.47]	10 (.10) [.11]	.48 (.17)	.05 (.02) [.008]	13 (.07) [0.0]
2	.23 (.12)	.11 (.02) [.31]	03 (.07) [.04]	.53 (.16)	.16 (.02) [.01]	$\begin{bmatrix}02 \\ (.08) \\ [0.0] \end{bmatrix}$
1	.03 (.10)	.21 (.06) [.15]	.07 (.05) [.73]	.38 (.14)	.30 (.05) [.54]	.14 (.09) [.09]
0	47 (.10)	23 (.23) [.33]	25 (.35) [.56]	05 (.18)	.09 (.20) [.51]	.06 (. 2 0) [.62]
-1	05 (.12)	.02 (.05) [.60]	.22 (.11) (.14]	.14 (.10)	.07 (.04) (.45)	.37 (.09) [.07]
-2	04 (.11)	.04 (.04) [.50]	.22 (.11) [.07]	.007 (.11)	.07 (.04) [.50]	.34 (.08) [.004]
-3	11 (.15)	.04 (.03) [.31]	.21 (.10) [.04]	23 (.13)	.07 (.03) (.02)	.31 (.07) [0.0]
-4	31 (.17)	.05 (.02) [.03]	.19 (.08) [.003]	30 (.13)	.06 (.02) [.005]	.27 (.06) [0.0]







