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VOLATILITY AND LINKS BETWEEN NATIONAL STOCK MARKETS

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ABSTRACT

The empirical objective of this study is to account for the time-variation the covariances between markets. Using data on sixteen national stock rkets, we estimate a multivariate factor model in which the volatility of turns is induced by changing volatility in the orthogonal factors. Excess turns are assumed to depend both on innovations in observable economic riables and on unobservable factors. The risk premium on an asset is a near combination of the risk premia associated with factors.

The main empirical finding is that only a small proportion of the time-riation in the covariances between national stock markets can be accounted r by observable economic variables. Changes in correlations markets are iven primarily by movements in unobservable variables.

We also estimate the risk premia for each country, and are able to entify substantial movements in the required return on equity. Our results so suggest that, although inter-correlations between markets have risen since e 1987 stock market crash this is not necessarily evidence of a trend crease.

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1. Introduction

Attempts to explain the "excess volatility" of stock markets have focused in recent research on the issue of modelling time-varying volatility and the implied stochastic process for expected returns. A large family of statistical models for the variation over time in conditional variances has grown up (for example, ARCH, GARCH, EGARCH and many others). But these models do not enable us to disentangle the **source** of changes in volatility. As Nelson and Kim (1988) have commented, "if one knew the factors that may cause conditional variances to vary, more efficient estimates could be obtained than with the ARCH specification by explicitly considering these factors". In this paper we try to identify those factors - both observable and unobservable - that are responsible for changes over time in stock market volatility.

Another feature of the existing literature is that, by and large, it is concerned with explaining volatility only in the US stock market. When data on many stock markets are examined an explicit multivariate model of the time-varying variance-covariance matrix of returns is required. The links between national stock markets have been attracting increasing attention (Roll(1989) surveys the recent literature). There are certain periods - the 1987 stock market crash is a conspicuous example - when markets move in unison, and others when the correlation between them is low (see Figure 1). The empirical objective of this

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study is to account for the time-variation in the covariances between markets. We use data on sixteen national stock markets to estimate a multivariate factor model in which the volatility of returns is induced by changing volatility in the orthogonal factors. This allows us to obtain a parsimonious representation of the conditional variance-covariance matrix of excess returns as a function of the variances of a small number of factors. Excess returns are assumed to depend both on innovations in observable economic variables and on unobservable factors. We allow the conditional variances of the underlying factors to vary over time and parameterize this in terms of ARCH processes (Engle (1982)). We then use arbitrage pricing theory to make the risk premium on an asset a linear combination of the risk premia associated with the factors. We estimate jointly the model that generates factors from observable economic variables and the equation for equilibrium excess returns.

A significant advantage in assuming that the conditional variances of the factors vary over time is that statistical identification of the factor model is less problematical than is the case in the usual static setting. In conventional factor analytic tests of the APT, the individual risk premia are only identifiable up to an orthogonal transformation (for example, Dhrymes, Friend and Gultekin (1984)). In our model, the time-variation in the conditional variances enables us to identify the individual risk premia.

Our approach enables us to decompose the time-variation in the covariance between markets into components that can be attributed to observable and unobservable components respectively. It may also be used to examine whether there is any evidence for the hypothesis that markets have become more integrated over time. We estimate a time series for the implied expected return on equity for each of the sixteen markets. This can be decomposed into observable and unobservable components. We are also able to construct a time-series of the "price of risk" - the ratio of expected return to conditional variance - and to test whether country-specific risk is priced in each market.

This research bears on the issue of whether time-varying equity risk premia can account for the observed volatility of share prices. If innovations in expected returns are persistent then shocks to the risk premium on equity may account for a substantial proportion of the volatility of returns. Because our model implies that there are several components of volatility, the degree of persistence in expected returns can be very different from that of observed volatility. We also investigate whether shocks to expected returns can be explained in terms of news about observable economic variables or whether unobservable factors are responsible for innovations in the risk premium. The distinction between these two sources of innovations in expected returns seems a fruitful way to examine the relative merits of an efficient markets and a noise trader view of volatility.

The paper is organized as follows. In section 2, we discuss the theoretical model and the estimation procedure. The results obtained by estimating the model using monthly data over the period 1970-1988 are presented in section 3. In section 4 we examine the robustness of the results, and our conclusions are stated in section 5.

2. A Factor Model of Multivariate Asset Returns with Time-varying Volatility

The model that we propose has four components. The first is a factor pricing model for excess returns. The second is a method for constructing factors that explain the behaviour of excess returns. The third is a model for the variation over time in the conditional variance-covariance matrix of unanticipated returns. The fourth is a dynamic model for the asset risk premia in terms of the changing volatility of the factors.

2.1 The Model

We shall assume that the returns on each of N assets are generated by a factor pricing model in which

$$r_t = \mu_t + \eta_t \qquad \textbf{(1a)}$$

$$\eta_t = Bf_t + v_t \equiv B_1 f_{1t} + B_2 f_{2t} + v_t \qquad \textbf{(1b)}$$

where $\mathbf{r}_t$ is the Nx1 vector of returns, which we measure as excess returns above the safe rate of interest, μ_t is the Nx1 vector of time-varying risk premia on the N assets, and η_t is the Nx1 vector of unexpected (at time t-1) components of excess returns. The stochastic component of returns is decomposed in equation (1b) into the sum of the influence of some common factors, $\mathbf{f}_t$, and idiosyncratic noise, $\mathbf{v}_t$. The common factors are in turn decomposed into k_1 observable factors, $\mathbf{f}_{1t}$, related to the unanticipated innovations in measurable economic variables, and a set of unobservable factors, $\mathbf{f}_{2t}$. The latter might represent fundamental influences on returns that are not captured by innovations in published statistics or changes in investor sentiment of the kind associated with noise trader models. $\mathbf{B}_1$ and $\mathbf{B}_2$ are, respectively, Nxk_1 and Nxk_2, full column rank matrices of factor loadings. $\mathbf{B} = (\mathbf{B}_1; \mathbf{B}_2)$, and $\mathbf{f} = (\mathbf{f}_1; \mathbf{f}_2)$. We assume that $E_{t-1}(\mathbf{f}_t) = \mathbf{0}$, $E_{t-1}(\mathbf{v}_t) = \mathbf{0}$, $E_{t-1}(\mathbf{f}_t\mathbf{v}_t') = \mathbf{0}$, $E_{t-1}(\mathbf{f}_t\mathbf{f}_t') = \mathbf{\Lambda}_t$ and $E_{t-1}(\mathbf{v}_t\mathbf{v}_t') = \mathbf{\Omega}_t$, where $\mathbf{\Lambda}_t$ and $\mathbf{\Omega}_t$ are $(k_1+k_2)x(k_1+k_2)$ and NxN, respectively, time-varying diagonal positive definite matrices because the factors are assumed to be (conditionally) orthogonal. E_{t-1} denotes the expectations operator conditional on the information set at time t-1.

It follows from these assumptions that the NxN conditional variance-covariance matrix of excess returns is

$$V_{t-1}(r_t) = B\Lambda_t B' + \Omega_t = B_1\Lambda_{1t}B_1' + B_2\Lambda_{2t}B_2' + \Omega_t \tag{2}$$

In the absence of any structure on the behaviour of excess returns, a model of time-varying volatility would consist of time series processes for each of the $N(N+1)/2$ distinct elements of the variance-covariance matrix of excess returns. One of the attractions of the factor model is that the dimension of this problem can be reduced to (k_1+k_2+N) independent time-varying processes. In our empirical application, $k_1 = 4$, $k_2 = 2$, $N = 16$. The number of processes that must be estimated is reduced, therefore, from 136 to only 22.

If we assume that investors can diversify away idiosyncratic risk, then the arbitrage pricing theory (Ross (1976)) implies that the risk premia of the N assets satisfy the cross-equation restriction

$$\mu_t = B\pi_t \tag{3}$$

where π_t is a (k_1+k_2) vector the i^{th} element of which can be interpreted as the risk premium of a portfolio with unit loading on the i^{th} factor and zero loadings on the other factors. The model implies, therefore, that the risk premium of an asset is a linear combination of the risk premia associated with the factors.

But (3) is not a model of the time-variation in asset risk premia. Given that the factors are orthogonal it seems natural to assume that the risk premia associated with them are proportional to their conditional own variance, that is

$$\pi_t = \Lambda_t \tau = \begin{bmatrix} \Lambda_{1t}\tau_1 \\ \Lambda_{2t}\tau_2 \end{bmatrix} \tag{4}$$

In this way time-variation in asset risk premia is induced by the pattern over time of the volatility of the factors. It is the orthogonality of the factors that enables us to express the risk premia as (4) and also to obtain a decomposition of the variance of asset returns into components attributable to observable factors, unobservable factors, and idiosyncratic noise (see equation (2)).

One way to derive (4) from more fundamental considerations is to interpret the factor model in terms of the consumption capital asset pricing model. If we assume that innovations in the marginal rate of intertemporal substitution in consumption, S_t, can be expressed as a linear function of the factors then the asset risk premia, which are proportional to the covariance between r_t and S_t, are a linear combination of the conditional variances of the orthogonal factors (see Engle, Ng and Rothschild (1989)[1].

Combining equations (1), (3) and (4) yields the basic model for excess returns

$$r_t = B\Lambda_t\tau + Bf_t + v_t = B_1\Lambda_{1t}\tau_1 + B_2\Lambda_{2t}\tau_2 + B_1f_{1t} + B_2f_{2t} + v_t \tag{5}$$

Equation (5) encompasses the models of Engle, Ng and Rothschild (1989), who examined US bond and stock returns, and Diebold and Nerlove (1989), who looked at exchange rates. The former assumed a linear factor model with only unobservable factors ($k_1=0$) and constant idiosyncratic variances ($\Omega_t=\Omega$). These assumptions mean that the variance-covariance matrix of returns can be expressed as a FACTOR ARCH model (Engle (1987)) which is a particular parameterization of the covariance matrix of a multivariate ARCH model in terms of less than full rank matrices (see Baba et.al. (1987)). It is important to distinguish between our factor model with ARCH processes for the factors and the specification that has become known as the FACTOR ARCH model. In our model the variance-covariance matrix of asset returns is an explicit function of the volatility of observable economic variables and the separate contributions of common unobservable factors and idiosyncratic noise. This means that in our model excess returns are a linear combination of conditionally heteroscedastic processes, whereas in the FACTOR ARCH model it is assumed that linear combinations of the excess returns follow ARCH processes. Diebold and Nerlove (1989) assumed only (one) unobservable factor and a zero risk premium ($k_1=0$, $k_2=1$, $\tau_2=0$). In the case of stock returns it seems essential to relate the risk premium to the model of changes in volatility. Equation (5) also encompasses the model of Burmeister and McElroy (1988) who considered observable and unobservable factors but with constant variances[2].

If we are to distinguish between observable and unobservable factors then it is obviously important that we include a relatively comprehensive set of economic variables in the set of observable factors. However, the inclusion of each additional observable factor requires us to estimate an additional $N+M+2$ parameters, where N is the number of assets and M the number of economic variables. It is necessary, therefore, that the number of factors be kept to a manageable level. For that reason we use a factor analytic approach to extract a small number of factors from a larger number of observable economic variables. Since the factors represent unanticipated shocks to asset returns we first estimate a vector autoregressive process for the observable economic variables, and then extract common factors from the estimated innovations of these processes.

An important feature of the model is that because we are only interested in those innovations to economic variables that also drive stock returns, we allow the choice of $\mathbf{f}_{1t}$ to be determined endogenously by estimating jointly the excess returns equation and the equation that determines the common factors. This system is described by equation (5) and the following equations for the mapping from observable economic time series to factors.

$$x_t = \sum_{j=1}^{p} A_j x_{t-j} + \epsilon_t \tag{6a}$$

$$\epsilon_t = C_1 f_{1t} + w_t \tag{6b}$$

where $\mathbf{x}_t$ is the (Mx1) vector of economic variables, $\mathbf{A}_j$ is the matrix of coefficients on the jth lag in the VAR, $\mathbf{C}_1$ is a (Mxk_1) full column rank matrix of factor loadings for the economic variables, and $\mathbf{w}_t$ is a (Mx1) vector of idiosyncratic error terms.[3] We assume that $E_{t-1}(\mathbf{w}_t) = 0$, $E_{t-1}(\mathbf{f}_t\mathbf{w}_t') = 0$, $E_{t-1}(\mathbf{w}_t\mathbf{v}_t') = 0$ and $E_{t-1}(\mathbf{w}_t\mathbf{w}_t') = \Gamma_t$, a positive semidefinite diagonal matrix.[4] Given these assumptions the conditional variance of the economic variables is

$$V_{t-1}(x_t) = C_1 \Lambda_{1t} C_1' + \Gamma_t \tag{7}$$

The estimation procedure consists of estimating the excess returns equations (5) jointly with the process for the economic variables (6) by maximum likelihood methods. Before turning to a description of the actual estimation procedure, we consider first the important issue of identification.

2.2 Identification

Since the units of measurement of the factors are irrelevant, then in the case of constant variance ($\mathbf{\Lambda}_t = \mathbf{\Lambda}$, $\forall$ t) it is usual to impose the condition that the variance of each of the factors is unity, that is $\mathbf{\Lambda} = \mathbf{I}$. In this case, however, $\mathbf{Bf}_t$, $\mathbf{C}_1\mathbf{f}_t$, and $\mathbf{B}\boldsymbol{\tau}$ are indistinguishable from $\mathbf{B}^0\mathbf{f}^0_t$, $\mathbf{C}^0{}_1\mathbf{f}^0_t$ and $\mathbf{B}^0\boldsymbol{\tau}^0$, where, for arbitrary orthogonal $\mathbf{Q}$, $\mathbf{B}^0=\mathbf{BQ}'$, $\mathbf{C}^0{}_1=\mathbf{C}_1\mathbf{Q}'$, $\mathbf{f}^0_t=\mathbf{Qf}_t$, and $\boldsymbol{\tau}^0=\mathbf{Q}\boldsymbol{\tau}$. One approach in standard factor analysis models is to use Dunn's (1973) sufficiency conditions for identification.[5] In the context of the present model these amount to imposing a zero restriction on the upper triangle of $\mathbf{B}_2$ and $\mathbf{C}_1$. In practice, most attempts to estimate the APT have not imposed these restrictions. Instead, the identification problem is merely acknowledged. (for example, Roll and Ross (1980) and Dhrymes, Friend and Gultekin (1984)).

The advantage of modelling time-varying volatility is that identification is less problematic as in general the orthogonal transformation will not preserve the form of Λ_t. In fact it can be shown that if Λ_t is diagonal (but not scalar) and $E(\Lambda_t) = \mathbf{I}$, then these conditions are sufficient to ensure that $\mathbf{B}$ and τ' are always identifiable up to column sign changes (Sentana (1990)).

In conventional factor analytic estimation of the APT, it is common for researchers to stress the identification problem and not report the individual risk premia, because "......they are identified up to an orthogonal transformation and no economic significance can be attributed to them" (p. 867 in Gultekin, Gultekin and Penati (1989); see also Dhrymes, Friend and Gultekin (1984) for a detailed discussion). Since our model is, in general, identified, we do not face this problem, so, in this respect, our approach has a non-trivial advantage over the conventional approach.

2.3 Estimation

If we assume that the factors and the idiosyncratic noise have a conditional normal distribution, we may write the log-likelihood of the sample (ignoring initial conditions) as:

$$L = K - \frac{1}{2}\sum_{t=1}^{T} \ln|\Sigma_t| - \frac{1}{2}\sum_{t=1}^{T} \zeta_t' \Sigma_t^{-1} \zeta_t \tag{8}$$

where

$$\zeta_t = \begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \tag{9}$$

$$\eta_t = r_t - B_1 \Lambda_{1t} \tau_1 - B_2 \Lambda_{2t} \tau_2 \tag{10}$$

$$\epsilon_t = E^{-\frac{1}{2}} \left(x_t - \sum_{j=1}^{p} A_j x_{t-j} \right) \tag{11}$$

$$E = diag\left(C_1 C_1' + \Gamma \right) \tag{12}$$

$$\Sigma_t = \begin{bmatrix} B_1 & B_2 \\ C_1 & 0 \end{bmatrix} \begin{bmatrix} \Lambda_{1t} & 0 \\ 0 & \Lambda_{2t} \end{bmatrix} \begin{bmatrix} B_1' & C_1' \\ B_2' & 0 \end{bmatrix} + \begin{bmatrix} \Omega_t & 0 \\ 0 & \Gamma_t \end{bmatrix} \tag{13}$$

The model has a natural state-space representation . Taking the common factors as the state, the measurement and transition equations are given, respectively, by:

$$\begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ C_1 & 0 \end{bmatrix} \begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} + \begin{bmatrix} v_t \\ w_t \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} = \qquad\qquad \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \end{bmatrix} \tag{15}$$

where

$$V_{t-1} \begin{bmatrix} v_t \\ w_t \end{bmatrix} = \begin{bmatrix} \Omega_t & 0 \\ 0 & \Gamma_t \end{bmatrix} \tag{16a}$$

$$V_{t-1} \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \end{bmatrix} = \begin{bmatrix} \Lambda_{1t} & 0 \\ 0 & \Lambda_{2t} \end{bmatrix} \tag{16b}$$

In this context the Kalman filter is ideally suited to decompose the observed series into their unobservable components. The prediction equations for the state vector f_t, and its covariance matrix, Λ_t are (Diebold and Nerlove (1989), Sentana (1990))

$$\hat{f}_{t|t-1} = 0 \tag{17a}$$

$$\hat{\Lambda}_{t|t-1} = \Lambda_t \tag{17b}$$

The updating equations are

$$\hat{f}_t = \Lambda_t B^{+\prime} \Sigma_t^{-1} \xi_t \tag{18a}$$

$$\hat{\Lambda}_t = \Lambda_t - \Lambda_t B^{+\prime} \Sigma_t^{-1} B^{+} \Lambda_t \tag{18b}$$

where

$$B^{+} = \begin{bmatrix} B_1 & B_2 \\ C_1 & 0 \end{bmatrix} \tag{19}$$

We shall assume that the variances of f_t, v_t, and w_t, follow univariate ARCH processes (Engle, 1982). In order to economize on the number of parameters, we assume a 12th order ARCH process with linearly declining weights. The basic intuition of the ARCH model is that "high" volatility today is likely to be associated with "high" volatility tomorrow. In that spirit, it is not unreasonable to expect a squared innovation from the distant past to have a smaller effect on the variance as compared to a squared innovation from the recent past.

Unfortunately, the true values of the factors do not belong to the information set - only the estimated values are available (see Harvey and Ruiz, 1990, Sentana 1990). For computational simplicity, however, we shall ignore the uncertainty in those estimates and assume that the diagonal elements of Λ_t can be represented by an equation of the form

$$\lambda_{it} = (1 - \psi_{i0}) + \psi_{i0} \sum_{j=1}^{12} \left(\frac{13-j}{78}\right) \hat{f}^{2}_{i,t-j} \qquad 0 \leq \psi_{i0} \leq 1 \quad \textbf{(20a)}$$

The diagonal elements of the Ω_t are represented as

$$\omega_{it} = \phi_{i0} + \phi_{i1} \sum_{j=1}^{12} \left(\frac{13-j}{78}\right) \hat{v}^{2}_{i,t-j} \quad \phi_{i0} > 0,\ 0 \leq \phi_{i1} \leq 1 \ \textbf{(20b)}$$

with an analogous equation for the diagonal elements of Γ_t. The simulation results reported by Sentana (1990) suggest that, for our purpose, this simplification makes little difference to the parameter estimates.

The restrictions on the coefficients of the ARCH process are necessary to ensure stationarity and positivity of variances, while, for λ_{it}, we also need to ensure that the variance is unity.

The computation of our estimates is considerably simplified also if we first obtain consistent estimates of $\mathbf{A}_j$ (the coefficients of $\mathbf{x}_{t-j}$ in the vector autoregression) independently by using ordinary least squares. At the risk of some loss in asymptotic efficiency this is the procedure that we followed. Nevertheless, the number of parameters to estimate by full information maximum likelihood is $k_1(N+M+2)+k_2(N+2)+3N+2M$. In order to obtain initial values for the maximisation of the global loglikelihood function in (8) we use a two-step procedure. Given the estimates of $\mathbf{A}_j$ obtained by OLS, we take the residuals ϵ_t, and, having standardised them, we use a principal components analysis to obtain estimates of the factor loadings $\mathbf{C}_1$. Then we use the estimates obtained by this method as starting values for maximum likelihood estimation of the parameters in the sub-model given by equation (6). In this way we obtain Kalman filter based factors $\mathbf{f}_{1t}$ and variances $\mathbf{\Lambda}_{1t}$. We next regress the actual excess returns on our estimated factors, $\mathbf{f}_{1t}$ and apply a principal components procedure on the residuals in order to extract estimates of the matrix of factor loadings, $\mathbf{B}_2$. These estimates are then used as starting values for a maximum likelihood procedure to estimate the sub-model consisting of the excess returns equation (5)) taking $\mathbf{f}_{1t}$ and $\mathbf{A}_{1t}$ as fixed regressors.

This procedure gives us consistent estimates of all of the parameters of the model, and these are then used as initial values in the maximization of the global likelihood function given by equation (8).[6]

3. Empirical Application

We estimate the multivariate factor asset pricing model described above on monthly data for returns on sixteen national stock markets from 1970:1 to 1988:10. The sixteen countries are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom and the United States. Data on stock returns - the percentage change in the share price index plus the dividend yield - were obtained from the Morgan Stanley Capital International world indices. All stock returns are measured in US dollars, and excess returns were computed as the return on each market during the month minus the one-month US Treasury Bill yield at the beginning of the month.

3.1 Vector autoregressions for the economic variables

Data on the ten macroeconomic variables that were available monthly, and might reasonably be expected to affect stock returns, were analysed. The variables are (i) short interest rates measured by the yield on US Treasury Bills, (ii) long interest rates (index of yields on long-term bonds weighted by GNP), (iii) the dollar-deutschmark exchange rate, (iv) the dollar-yen exchange rate, (v) industrial production for the G3 group of countries (an index weighted by GNP), (vi) inflation in G3 (consumer price index weighted by GNP), (vii) US trade deficit (% of GNP), (viii) real money supply in G3 (weighted by GNP), (ix) real oil price in US dollars, and (x) an index of real commodity prices. Details of the definitions and sources of these variables may be found in the data appendix.

Because of delays in the publication of economic statistics the values of the variables for industrial production, the US trade deficit, real money supply, and inflation in period t were assumed to be the published values for month t-1. We report below the results of varying this assumption.

The first step is to estimate innovations in the economic variables by fitting vector autoregressions described by equation (8a). We experimented with different lag lengths in order to obtain a final specification with no detectable serial correlation in the residuals.[7] The fitted equations also included seasonal dummies. Our preferred specification, and the values of the Lagrange multiplier-based test against 12th order serial correlation (see Harvey, 1981), are shown in Table 1. These VARs were estimated over the sample period 1970:8 to 1988:10 - a total of 219 observations. For eight of the ten variables considered, a specification with thirteen lags for the dependent variable and three lags for the other variables was sufficient to ensure that the null hypothesis of no serial correlation could not be rejected at the 5% level. But for two of the variables - inflation and real money supply - somewhat different lag lengths were necessary in order to ensure that the residuals were white-noise (see Table 1).

3.2 Estimation of the joint factor and multivariate returns model

Standardized residuals from the VARs reported in Table 1 above are used as data in order to estimate the joint factor and multivariate returns model described by equations (7) and (8b) by maximum likelihood methods. The general model permits both observable and unobservable factors. We used four factors, that is $\mathbf{f}_{1t}$ is a 4x1 vector. When estimating (8b) in order to generate initial values for the global maximum likelihood procedure, we tested for the appropriateness of four factors. On the one hand, a likelihood ratio test for the existence of a fifth factor yielded a value of 16.96 (with eleven degrees of freedom) for the difference in loglikelihoods, which is statistically insignificant at conventional levels. On the other hand, the omission of the fourth factor could be rejected with a LR value of 25.7, which is significant at the 1% level (see section 4 for a discussion of the effect of including additional observable factors).[8] In addition to the four observable factors, we allowed for two unobservable factors. Attempts to allow for a third unobservable factor led to the idiosyncratic variance for the Netherlands market being driven to zero. Since this seemed an implausible outcome we used only two unobservable factors.

The matrix, $\mathbf{C}_1$, of factor loadings for the economic variables that is obtained from estimation of the complete system is shown in Table 2. The first factor has relatively large coefficients in the equations for the innovations in interest rates, both short and long rates. It might be interpreted as an "interest rate factor". For the second factor the loadings are high in the equations for the dollar-yen and dollar-DM exchange rates. This might be thought of as a "dollar exchange rate factor". The third factor also has substantial loadings on the two exchange rate variables, but in this case the loading on the dollar-yen cross-rate has the opposite sign to that on the dollar-DM rate. It measures exchange rate shocks among currencies other than the dollar. The fourth factor reflects principally innovations in G3 real money supply and the inflation rate.

Estimates of the matrix of country factor loadings **B** and the vector $\boldsymbol{\tau}$ of risk premia associated with each of the six factors are shown in Table 3. The coefficients of the first observable factor, $\mathbf{f}_{11}$, are, with one exception, negative. If $\mathbf{f}_{11}$ is an interest rate factor, the implication is that unanticipated increases in interest rates reduce excess returns. The estimate of the risk premium associated with the interest rate factor is positive, which implies that an **increase** in the variance of innovations to interest rates is associated with a **reduction** in the required rate of return. An interpretation in terms of the consumption capital asset pricing model would suggest that this occurs because the rate of growth in consumption is positively correlated with innovations in interest rates.

As far as the dollar exchange rate factor is concerned, there is no unambiguous theoretical prediction about the sign of the coefficients. An appreciation of the dollar stimulates the exports of companies in other countries, which raises share prices in those markets. But it also may increase inflation in non-US economies and higher unanticipated inflation lowers share prices (see, for example, Fama and Schwert (1977)). In addition, for a given change in stock prices in local currency, a dollar appreciation implies a lower dollar return in non-US stock markets. Our estimates suggest that an unanticipated appreciation of the dollar leads to a fall (measured in dollars) in all non-US stock markets and a rise in the US market.

Estimates for the third observable factor imply that an unanticipated appreciation of the yen and deutschmarklocal currency leads to increases in the stock market in Japan and Germany, respectively. As one might expect, the size of the coefficient is bigger for countries that belong to the "DM block", that is for Germany, Austria, Switzerland, Netherlands and Belgium. The positive estimates of the risk premia associated with both the second and third observable factors mean that an increase in the volatility of the dollar raises the equity risk premium in the US and lowers reduces the required rate of return in non-US stock markets. These effects are, however, small.

The coefficients on the real money supply factor imply that unanticipated increases in the money supply or unexpected falls in the inflation rate raise share prices in twelve of the sixteen national markets. The negative estimate of the associated risk premium means that an increase in the volatility of money innovations increases the equity risk premium.

In order to interpret the unobservable factors Figures 2a and 2b plot the estimated two factors over the sample period. The first unobservable factor is extremely volatile. For example, it exhibits a sharp spike in October 1987 which coincides with the worldwide stock market crash. Other sharp downward movements can be seen in September 1979, March 1980, and September 1981, and upward surges in January 1975 and January 1987. This factor is - by construction - uncorrelated with the innovations in the observable economic variables that we have analysed. The difficulty in explaining the 1987 stock market crash in terms of changes to observable fundamental variables is illustrated clearly in Figure 1a. The plot suggests that there are other episodes in the last twenty years when markets have moved together without any obvious explanation in terms of observable economic variables. The first unobservable factor has a substantial loading on most countries, the conspicuous exception being Austria which fell least in the 1987 crash. An increase in the volatility of this factor leads to a higher estimated equity risk premium. The second unobservable factor also exhibits high volatility. But the estimated risk premium associated with this factor is very small.

The estimation method involves fitting a linearly declining ARCH(12) process for the variances of each of the factors. The estimates of the coefficients of these processes are shown in Table 4. Although some factors exhibit a fairly high degree of persistence (for example, the interest rate and dollar exchange rate factors have coefficients of persistence exceeding 0.8), others are much less persistent, for example the first unobservable factor. A components model of volatility of this kind can help to reconcile the observation that univariate models of the volatility of stock returns sometimes conclude that persistence is low - because the dominant component exhibits very little persistence - with a model that relates stock returns to a time-varying required return that reflects innovations in volatility.

3.3 The Contribution of Observables to the Covariance Matrix of Returns

The model enables us to estimate the extent to which the covariance of world stock markets can be explained in terms of innovations in observable economic variables. The conditional variance-covariance matrix of excess retuns is given by equation (2). Figure 3a shows the estimated conditional covariance, and the contribution to this of the variance of the observable factors, for the US and UK markets. The picture is striking. Although the covariance between the two markets has ranged between 0.14 to values in excess of 0.3 during the sample period, the contribution of the observable factors has remained firmly in the range $\mp$ 0.01. The relative contribution of observable factors to changes in the covariance between the US and UK appears to have been miniscule.

Figure 3b shows the estimated conditional covariance between the US and Japan. The relative contribution of observables in this case is somewhat higher. The decline in the covariance in 1978-79 is clearly linked to changes in the volatility of observables. Nevertheless, although the covariance has ranged between about 0.03 to about 0.17 over the sample period, the contribution of observables has been between approximately zero and -0.03. Not only do observables account for a small proportion of the covariance between markets, they also fail to explain a substantial fraction of the variance of the markets. Figures 4a, 4b and 4c show the decomposition of the variance of the US, UK and Japanese markets, respectively, into three main components attributable to observable factors, unobservable factors, and idiosyncratic noise. For the US the contribution of the observables to the variance is negligible. It is clear that the contribution of the unobservables essentially mirrors the actual change in the variance. The contribution of the idiosyncratic component is constant for the US because the persistence coefficient in the ARCH process was estimated to be zero. In the UK also the contribution of the observables is small. The idiosyncratic component. however, is important. There was a sharp rise in volatility between December 1974 and February 1975 during which period there was a major fall in the market followed by a rapid recovery. This emphasises the need to allow for time-varying idiosyncratic variances

The behaviour of volatility in Japan differs from that in the Anglo-Saxon world. The most interesting difference is that, as shown in Figure 4c, in Japan observable variables are **more** important than unobservables in accounting for changes in conditional volatility. This is because our estimates suggest that the Japanese market is both more sensitive to unanticipated changes in interest rates and the dollar exchange rate than either the US or UK, and less affected by the first unobservable factor. Our finding that Japan is relatively insensitive to the first unobservable factor is consistent with the observation - using monthly returns - that the Tokyo market was less affected by either the October 1987 crash (which is in our sample period) or the mini-crash of October 1989 (which is outside the sample period) than other major markets.

Finally, Figure 4d shows the (unweighted) average of the variances for all sixteen markets. It is clear that the observable component fluctuates little (except during 1979), and that the unobservable component accounts for most of the movement in conditional volatility.

3.4 Estimates of the Risk Premia

The model generates estimates of the equity risk premium for each market for each year of the sample period. The estimated ex ante required rate of excess return, μ_t, for the US is plotted in Figure 5a. It shows considerable variability, ranging from a value of 3 1/2% per month in 1975 to -1% per month in 1981. This estimate of the risk premium is considerably more variable than that generated by estimating univariate processes for volatility such as an ARCH or GARCH model (see Attanasio and Wadhwani (1989)).

The source of the difference is the components structure to our model of returns and hence conditional volatility. From equations (3) and (4) the risk premium may be written as the sum of $k_1 + k_2$ components

$$\mu_t = B_1 \Lambda_{1t} \tau_1 + B_2 \Lambda_{2t} \tau_2 \tag{21}$$

The first term is the contribution to the risk premium of the observable economic variables and the second term is that of the unobservables. Figure 5b shows that changes in the volatility of observables account for only a relatively small proportion of the variance of the equity risk premium in the US. Figures 5c and 5d show the (equally-weighted) average risk premium for all sixteen countries, and its decomposition into observable and unobservable components.

The results are very similar to those for the US. Note that the required rate of return is estimated to be negative in 1981. On the basis of our estimates increases in the variance of f_{11}, f_{13} and f_{22} lower the risk premium, whereas decreases in the variance of the remaining factors (f_{12}, f_{14} and f_{21}) have the same effect. Inspection of the conditional volatility of each of the factors (see Figure 6) reveals that the variances of f_{11} and f_{13} were well above the sample mean in 1981 (a period of exceptional volatiltiy in interest rates), whereas the variance of f_{21} reached almost its lowest value. This is nevertheless surprising because it is not obvious that an increase in the volatility of interest rates would lower the risk premium.

It has been argued that shocks to volatility cannot account for the observed variation in stock prices because these shocks are not sufficiently persistent to explain large movements in the prices of discounted cash flows over an infinite horizon (for example, Poterba and Summers (1986))[9]. This claim, however, is at least partly the result of modelling volatility as a univariate process. With the components structure to volatility our measure of the risk premium does display considerable persistence. In order to demonstrate this Table 5 shows the first-order autoregression coefficient of the risk premium for each country in the sample. It ranges between 0.92 and 0.97. The dynamic model for expected returns implied by our model is much more complicated than an AR(1) process - it is a mixture of six processes each of which has a high order ARMA structure. Hence the AR(1) process shown in Table 5 is no more than a simple summary statistic of the estimated degree of persistence which may be compared to other estimates in the literature.

The estimated series for the risk premium has some ability to predict excess returns. The second column of Table 5 shows the value of R^2 in a regression of actual excess returns on the estimated series for the risk premium. These values are similar to those obtained by studies that find predictability of stock returns using variables such as lagged dividend yields (for example, Fama and French (1988)). The final column of Table 5 shows the correlation coefficient between expected returns and conditional variance in each market. In a one-component model of volatility expected returns and conditional variance would be proportional. Table 5 shows that the components structure to volatility implied by our model allows expected returns to follow a different time-series process from that for volatility.

3.5 Time-Variation in the Price of Risk

For each factor in our model there is a separate risk premium. But we may compute an overall "price of risk" that can be compared with the price that would be implied by the capital asset pricing model. For each market the price of risk - the trade-off between mean and variance - is denoted by ρ_t and defined by the relationship

$$\mu_t - \rho_t \sigma_t^2 \tag{22}$$

The model implies estimates of both expected returns and conditional variances each period, and, therefore, of the time-varying price of risk. Estimates of ρ_t for the US, UK, and Japan are shown in Figures 7a through 7c. There is substantial variation in the price of risk. In the US, for example, the price of risk exceeded unity in the early part of 1975 and the end of 1987, and actually turned negative in 1981.[10] There is similar variation in the estimates of ρ_t for the UK and Japan, although in Japan the time path of the price of risk is quite different from that in the US (for example, the price of risk was not especially high in Japan in the aftermath of the October 1987 crash). The equally-weighted average price of risk across all countries also exhibits substantial variation over time (Figure 7d).

Previous attempts to estimate the price of risk directly have often encountered difficulty in obtaining positive and statistically significant estimates of the price of risk because they typically constrain ρ_t to be constant (see, for example, Merton (1980) and French et.al. (1987)). In the context of the model presented here, there is no necessary *a priori* restriction on the sign of ρ_t and the fact that our estimates suggest substantial variation over time may help to explain the difficulties associated with previous estimates of (22).

3.6 Changing Links Between National Stock Markets

It is often asserted that the links between national stock markets have increased with improved electronic communications and the abolition of exchange controls in a number of countries. The Brady Commission (1988), however, pointed out that there had been no trend increase in the actual correlations (computed by using a monthly window of the last twelve observations) between markets. Figure 8 plots the average ex ante estimated correlation coefficient between the markets implied by our parameter estimates. when we regress this mean correlation coefficient on a constant, a dummy variable for the 1980s and a dummy variable for the period after the 1987 stock market crash we obtain

$$\text{CORR} = 0.381 + 0.018\ D_{1980s} + 0.105\ D_{crash} \qquad (25)$$

Hence the case for a trend increase in correlations between markets depends upon the weight that is attached to the observations surrounding the 1987 stock market crash. Those authors who argue that markets have become increasingly integrated from data in the period 1986-88 (for example, von Furstenberg and Nam Jeon (1989)) may be confusing a transitory with a permanent increase in correlations.

A stylised fact that has been noted before is that periods when markets are increasingly correlated are also times when markets are volatile (for example, King and Wadhwani (1990) and Roll (1989)). Indeed, King and Wadhwani argued that this might be because a rise in volatility caused by factors that are not closely related to "news", might lead agents to pay greater attention to other markets in an attempt to determine the change in the "taste for equity". In our model, periods when the volatility of the unobservable factors rises are also those when, ceteris paribus, markets appear to exhibit greater inter-correlation. To see this, consider a two-asset two-factor model

$$r_{1t} = \mu_{1t} + \beta_{11} f_{1t} + \beta_{12} f_{2t} + \nu_{1t} \tag{23a}$$

$$r_{2t} = \mu_{2t} + \beta_{21} f_{1t} + \beta_{22} f_{2t} + \nu_{2t} \tag{23b}$$

where $\beta_{ij} > 0 \; \forall$ i,j. The correlation coefficient between the two markets is given by

$$\rho_{12t} = \frac{\beta_{11}\beta_{21}\lambda_{1t} - \beta_{12}\beta_{22}\lambda_{2t}}{(\beta_{11}^{2}\lambda_{1t} + \beta_{12}^{2}\lambda_{2t} + \omega_{1t})^{\frac{1}{2}} (\beta_{21}^{2}\lambda_{1t} + \beta_{22}^{2}\lambda_{2t} + \omega_{2t})^{\frac{1}{2}}} \tag{24}$$

It follows that the correlation coefficient is an increasing function of λ_{1t} and a decreasing function of λ_{2t} and the idiosyncratic variances. Hence an increase in the volatility of those factors that affect all stock markets with the same sign - the unobservable factors in our sample - will be associated with an increase in the correlation between markets. This is consistent with the observed rise in both volatility and inter-correlation around the time of the 1987 crash. Rises in the volatility of factors that move markets in different directions - exchange rates, for example - may be associated with falls in correlation coefficients. The positive link between volatility and correlation that has been noted by previous authors appears to reflect the fact that the unobservable factors have historically been more important in explaining stock returns than the observable factors. Our analysis provides some confirmation over a longer sample period for the stylised fact that correlation is related to volatility. It does not, however, enable us to assess whether there is a causal relation between volatility and correlation.

4. Robustness of the Results

In this section, we investigate the robustness of our decomposition of excess returns into observable and unobservable components.

4.1 The Number of Observable Factors

In order to economise on the number of parameters, we used only four observable factors. The inclusion of one additional observable factor would raise the number of parameters to be estimated from 216 to 244. To allow as many factors as variables would entail the estimation of 374 parameters. In order to illustrate the effect of including additional economic variables, we regressed excess returns on **all** ten series of innovations from the VARs, computed the R^2, and compared them with the values obtained when we included only the four observable factors. The results are shown in Table 6. As measured by the average R^2, four factors enable us to capture over 75% of the explanatory power that can be obtained by using all ten innovations separately. This suggests that the use of four factors has not led us to underestimate significantly the contribution of observable factors.[11]

It is possible that the observable factors are measured with error. For example, the information set used in our VARs may be too restricted. Under the efficient markets hypothesis other relevant information is incorporated in stock prices. Hence we re-estimated the VARs including in the information set three lags of the return in the world index as a proxy for variables that had been omitted. This produced new estimates of the ten innovations in the observable economic variables. We then regressed excess returns on these innovations. There was very little change in the explanatory power of the economic variables - the average R^2 actually fell from 0.136 to 0.128. Because of delays in the publication of economic statistics, we assumed a one month lag in the production of certain monthly series (see section 3.1). We experimented, however, with the current values of variables. Again this makes little difference - the mean R^2 rose marginally from 0.136 to 0.144.

4.2 Distributional Assumptions

One problem with the estimation method that we have used is that its validity is predicated on the conditional normality of the factors. Table 7 shows Jarque-Bera tests of this assumption which suggest that it is violated for some of the factors. Other assumptions that we have made in order to keep the estimation of the model manageable (for example, the parameterisation of the ARCH process) also raise the question of the robustness of the estimates. We have, therefore, also estimated a simpler version of the model which is not dependent on the assumption of a normal distribution for the factors. We initially extracted the first four principal components of our "news" variables (i.e. f_{1t}). This procedure should yield consistent estimates of C_1. We then regressed the excess returns series on the four principal components (f_{it}), and we then extracted the first two principal components of the resulting residuals. Provided that $V(B\Lambda_t\tau)$ is "small", this simpler principal components (PC) procedure should provide a good alternative to the estimates that we obtained from the maximum likelihood procedure.[12]

In order to compare our results using the PC method with the MLE method, we use

$$tr\,(V(r_t)) = \underbrace{tr\,(B_1B_1')}_{\text{observable components}} + \underbrace{tr(B_2B_2')}_{\text{unobservable components}} + \underbrace{tr\,(\Omega)}_{\text{idiosyncratic components}}$$

to obtain a decomposition of the variance of returns into the appropriate components. Our results, alongside those obtained using the MLE procedure, are shown in Table 8.

The proportion of the variance of returns that can be explained in terms of the observable factors is slightly higher when we use maximum likelihood estimation. This is not surprising since the MLE method chooses the factors to be common to both the economic variables and the stock returns. In contrast, the principal components method ignores the behaviour of stock returns when choosing the factors to extract. The relative contribution of the unobservable factors does differ more markedly across the two methods with the principal components analysis suggesting it accounts for up to half of the variance, while the maximum likelihood procedure suggests that is accounts for little over a third. This is not very surprising as the unobservable factors that are obtained when using the PC method are chosen to maximize the explained proportion of the unconditional variance of $\mathbf{B}_2\mathbf{f}_{2t} + \mathbf{v}_{2t}$.

4.3 The Pricing of Idiosyncratic Risk

The factor model for excess returns - equation (5) - implies that the risk premium of an asset is equal to the sum of the risk premia associated with the common factors. The possibility of diversification means that idiosyncratic risk is not priced. This implication is testable. We included the estimated idiosyncratic volatility, w_{it}, in the equation for excess returns. The null hypothesis is that the coefficients on w_{it} are zero. The results are shown in Table 9. In 14 out of 16 countries idiosyncratic risk is not priced. The exceptions are Austria and the UK.[13] Given the existence of a variety of legal and other barriers to international portfolio investment over much of the sample period, it is gratifying that the results are consistent with our basic assumption that only risk associated with common factors is priced in the market.

5. Conclusions

The main empirical finding is that only a small proportion of the time-variation in the covariances between national stock markets can be accounted for by observable economic variables. Changes in correlations between markets are driven primarily by movements in unobservable variables. These may be interpreted as unobservable fundamental variables that influence the price of risk, or as variables that measure the demand for equities by "noise" traders. There is also substantial variation over time in the "price of risk" - the trade-off between the mean and variance of returns. For example, over our sample period it is estimated to have fluctuated between -0.5 and 1.2 in the US.

We also estimated the risk premia for each country, and were able to identify substantial movements in the required return on equity. For example, our estimates suggest that the risk premium demanded by investors in the US has varied between about 3 1/2 % per month to about zero. The evidence also suggests that country-specific risk is not priced. This is consistent with the basic factor model.

Future work is required in two main directions. The first is to explore alternative processes for the time-series behaviour of the factors. In particular, it would be interesting to examine a non-normal distribution for the first unobservable factor to see whether this would account for the well-known skewness and excess kurtosis of returns. The second is to integrate more closely the estimation of the innovations in observable variables - the VARs - with the estimation of the model for excess returns.

DATA APPENDIX

Details of the data series used are as follows (where appropriate the name of the series if followed by its Datastream code):

Stock prices and dividend yields:	from Morgan Stanley Capital International Perspectives.
Safe interest rate:	yield on 1 month US T Bills, beginning of period.
Short interest rate:	yield on 3 Month US T Bills, end of period (USOCTBL%).
Long Interest Rate: period	US Yield on long-term government bonds, end of (USOCLNG%) W. Germany: Yield on long-term government bonds, end of Period (BD0CLNG%) Japan: Yield on Central Government Bonds end of Period (JP0CLNG%)
Exchange rates:	US$-Yen, end of period (JP0CEXCH). US$-DM, end of period (BD0CEXCH).
Index of Industrial Production	US (USOCIPRDJ) West Germany (BD0CIPRDI) Japan (JP0CIPRDH)
Consumer Prices:	US - all items (USOCPCONF) W. Germany - total (BD0CPCONF) Japan - Tokyo, all items (JP0CCPTKF)
Trade Account:	US Foreign Trade Balance, US $million (USOCVBALA)
Money Supply: (BDTU0800A)	US: M3, US $billion, current prices (USOCM3MNA) W. Germany: M3 DM million, current prices Japan: M1 + Quasi Money, yen billion current prices (JP0CM1QSA)
Oil Price:	Saudi Arabian Light Oil Spot Price, US$ per barrel end of period (SAUDISPT) (up to Dec 72 from IMF Financial Statistics)

Commodity Prices:	The Economist World Commodity Price Index (last week of each month) (up to Dec 72 monthly average)
GNP Weights:	US GNP $billion at annual rates, current prices (USOCGNPDB) W. Germany GNP DM billion at annual rates, current prices (BDOCGNPDB) Japan GNP Yen billion at annual rates, current prices) (JPOCGNPDB)

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FOOTNOTES

1. An additional advantage of using the consumption capital asset pricing model is that it implies an exact relationship between the risk premium and the variances of the factors, irrespective of the number of assets. In the standard APT model such a relationship holds only as the number of asssets tends to infinity.

2. If we define

$$f_t^* = \Lambda_t^{-\frac{1}{2}} f_t; \quad B_t^* = B\Lambda_t^{\frac{1}{2}}; \quad \tau_t^* = \Lambda_t^{\frac{1}{2}} \tau$$

then our model can also be interpreted as a time-varying factor-betas model with homoscedastic factors in which the betas of different assets on a factor change proportionally (see Engle, Ng and Rothschild (1989)).

3. In order to make our analysis invariant to charges in scale, we divide the residuals from the vector autoregression by their own standard deviation - so that all the transformed values of ϵ_t have, by construction, unit variance.

4. The matrix Γ_t is assumed to be positive semidefinite in order to allow for the possibility that the variance of the idiosyncratic noise is zero.

5. See Jennrich (1978) for an alternative set of identifiability conditions.

6. Maximization of the log-likelihood function was carried out on the LSE VAX using the NAG library E04JBF routine. The block triangularity of the global factor loading matrix $\mathbf{B}^+$ and the special form of the covariance matrices is exploited by means of the Woodbury formula (Householder, 1964) so that the inversion of $\boldsymbol{\Sigma}_t$ (a 26 x 26matrix) only involves the inversion of the (4 x 4) matrix $\boldsymbol{\Lambda}^{-1}_{1t} + \mathbf{C}_1' \boldsymbol{\Gamma}_t^{-1} \mathbf{C}_1$ and the (2 x 2) matrix $\boldsymbol{\Lambda}_{2t}^{-1} + \mathbf{B}_2' \boldsymbol{\Omega}_t^{-1} \mathbf{B}_2$ (for the case of $\boldsymbol{\Gamma}_t, \boldsymbol{\Omega}_t$ singular see Sentana, 1989).

7. Given that we have only just over 200 monthly observations the dimensionality of the VAR has to be somewhat restricted.

8. The appropriateness of four observable factors is also confirmed by the Akaike information criterion.

9. Although their method of proxying volatility by using lagged squared returns may lead to an underestimate of the degree of persistence (see Pagan and Ullah (1988) for a theoretical discussion and Attanasio and Wadhwani (1989) for Monte Carlo evidence).

10. This estimate of a negative risk premium in 1981 appears to be related to the fact that all returns are measured in US dollars. If returns were measured in a different currency, say DM, the structure of the model would be unchanged but the interpretation of the estimated coefficients would be different. The mean correlation coefficient computed when returns are measured in US dollars is in fact very similar to that computed using DM returns except for the calendar year 1981.

11. Our result that the contribution of observable factors to an explanation of returns is small is consistent with the fiindings of Cutler, Poterba and Summers (1989)).

12. Note that $V(r_t) = V(B\Lambda_t\tau) + B B' + \Omega$, where $\Omega = E(\Omega_t)$. If $V(B\Lambda_t\tau)$ can be neglected, the unconditional variance of r_t has a factor analytic form, and can, therefore, be consistently estimated by a principal components procedure. The assumption that $V(B\Lambda_t\tau) \approx 0$ will be valid if for each factor either $\tau_i \approx 0$ or if λ_{it} is constant.

13. Since the value of w_{it} is constant for the Netherlands and the US, the estimated coefficients of the constant term in the regression are reported in these two cases.

TABLE 1

VECTOR AUTOREGRESSIONS FOR ECONOMIC VARIABLES

VARIABLE	LAG LENGTH Dependent Variable	Other Variables	F-test for Serial Correlation (degrees of freedom)
1. SHORT INTEREST RATE	13	3	1.27 (12,155)
2.LONG INTEREST RATE	13	3	0.49 (12,155)
3.DOLLAR/ YEN EXCH RATE	13	3	1.24 (12,155)
4. DOLLAR/ DM EXCH RATE	13	3	1.14 (12,155)
5.INDUSTRIAL PRODUCTION	13	3	0.94 (12,155)
6.INFLATION	13	1, 12	2.18 (12,164)
7.US TRADE ACCOUNT	13	3	0.73 (12,155)
8.REAL MONEY SUPPLY	18	3	1.13 (12,151)
9.OIL PRICE	13	3	0.55 (12,155)
10.COMMODITY PRICES	13	3	1.05 (12,152)

NOTES:

$F_{0.05}(12,155) = 1.82$

$F_{0.01}(12,155) = 2.30$

TABLE 2

MATRIX OF FACTOR LOADINGS

VARIABLE	FIRST	SECOND	THIRD	FOURTH
innovat-ions in	f_{11}	f_{12}	f_{13}	f_{14}
1.SHORT INTEREST RATE	0.980 (0.120)	-0.028 (0.055)	0.075 (0.068)	-0.058 (0.056)
2.LONG INTEREST RATE	0.818 (0.109)	0.045 (0.060)	-0.005 (0.082)	-0.105 (0.061)
3.DOLLAR/ YEN EXCH. RATE	-0.283 (0.079)	-0.883 (0.164)	0.341 (0.106)	-0.026 (0.134)
4.DOLLAR/ DM EXCH. RATE	-0.297 (0.083)	-0.797 (0.169)	-0.529 (0.138)	0.020 (0.152)
5.INDUST-RIAL PROD-UCTION	0.173 (0.087)	-0.061 (0.064)	0.032 (0.092)	-0.196 (0.067)
6.INFLAT-ION	0.052 (0.088)	-0.156 (0.070)	-0.023 (0.104)	0.208 (0.065)
7.US TRADE ACCOUNT	-0.198 (0.094)	-0.008 (0.063)	0.060 (0.066)	0.036 (0.067)
8.REAL MONEY SUPPLY	-0.011 (0.101)	0.007 (0.100)	-0.044 (0.323)	-0.995 (0.088)
9.OIL PRICE	0.032 (0.069)	0.075 (0.062)	-0.104 (0.067)	-0.107 (0.069)
10.COMMOD-ITY PRICES	-0.077 (0.086)	-0.305 (0.086)	-0.152 (0.089)	0.055 (0.075)

TABLE 3

ESTIMATES OF B AND τ

Country	Coeff (Std.Err) f_{11} 'Int Rate'	Coeff (Std.Err) f_{12} 'Dollar'	Coeff (Std.Err) f_{13} 'DM-Yen'	Coeff (Std.Err) f_{14} 'Money Supply'	Coeff (Std.Err) f_{21}	Coeff (Std.Err) f_{22}
AUSTRALIA	-0.106 (0.056)	-0.099 (0.053)	-0.019 (0.052)	-0.069 (0.034)	-0.522 (0.059)	0.180 (0.113)
AUSTRIA	-0.095 (0.026)	-0.228 (0.021)	-0.169 (0.022)	-0.002 (0.017)	-0.046 (0.028)	0.213 (0.068)
BELGIUM	-0.106 (0.046)	-0.191 (0.035)	-0.144 (0.036)	-0.002 (0.023)	-0.194 (0.055)	0.499 (0.127)
CANADA	-0.085 (0.047)	0.012 (0.036)	-0.063 (0.038)	0.002 (0.028)	-0.468 (0.042)	0.115 (0.081)
DENMARK	-0.036 (0.038)	-0.173 (0.031)	-0.070 (0.034)	-0.086 (0.025)	-0.167 (0.041)	0.241 (0.079)
FRANCE	-0.154 (0.052)	-0.262 (0.044)	-0.124 (0.045)	0.007 (0.030)	-0.282 (0.061)	0.478 (0.128)
GERMANY	-0.135 (0.044)	-0.192 (0.033)	-0.126 (0.033)	-0.007 (0.022)	-0.087 (0.053)	0.580 (0.149)
ITALY	0.005 (0.055)	-0.199 (0.050)	-0.003 (0.053)	-0.033 (0.040)	-0.189 (0.063)	0.403 (0.119)
JAPAN	-0.095 (0.036)	-0.265 (0.035)	0.070 (0.038)	-0.057 (0.024)	-0.158 (0.046)	0.308 (0.094)
NETHER-LANDS	-0.079 (0.041)	-0.136 (0.034)	-0.139 (0.034)	-0.016 (0.021)	-0.284 (0.049)	0.430 (0.114)
NORWAY	-0.092 (0.060)	-0.113 (0.055)	-0.137 (0.056)	-0.011 (0.046)	-0.391 (0.066)	0.382 (0.123)
SPAIN	-0.081 (0.042)	-0.162 (0.041)	-0.007 (0.043)	-0.059 (0.026)	-0.154 (0.050)	0.270 (0.091)
SWEDEN	-0.058 (0.043)	-0.165 (0.038)	-0.039 (0.038)	-0.082 (0.026)	-0.192 (0.047)	0.304 (0.089)
SWITZ-ERLAND	-0.122 (0.044)	-0.181 (0.033)	-0.167 (0.033)	-0.040 (0.022)	-0.222 (0.051)	0.476 (0.122)
UK	-0.061 (0.049)	-0.107 (0.044)	-0.095 (0.046)	-0.028 (0.039)	-0.423 (0.060)	0.343 (0.113)
USA	-0.022 (0.034)	0.025 (0.030)	-0.045 (0.031)	-0.029 (0.023)	-0.363 (0.036)	0.162 (0.072)
τ	1.138 (0.339)	0.048 (0.075)	0.854 (0.515)	-5.825 (3.738)	-1.116 (0.782)	-0.033 (0.206)

TABLE 4

PARAMETERS OF ARCH PROCESS

Estimated equation:

$$\lambda_{it} = (1 - \psi_{i0}) + \psi_{i0} \sum_{j=1}^{12} \frac{13-j}{78} f_{i,t-j}^{\;2}$$

VARIABLE	ψ_{i0}
f_{11}	0.822
f_{12}	0.825
f_{13}	0.357
f_{14}	0.225
f_{21}	0.249
f_{22}	0.825

TABLE 5
PROPERTIES OF ESTIMATED RISK PREMIA

OUNTRY	PERSISTENCE COEFFICIENT	R^2	CORRELATION BETWEEN EX ANTE MEAN AND VARIANCE
USTRALIA	0.940	0.0297	0.656
USTRIA	0.967	0.0485	0.112
ELGIUM	0.961	0.0250	-0.095
ANADA	0.932	0.009	0.204
ENMARK	0.939	0.0175	0.258
RANCE	0.958	0.0183	-0.028
ERMANY	0.967	0.0193	0.051
ALY	0.921	2.13×10^{-5}	0.117
APAN	0.949	0.006	0.382
ETHER ANDS	0.951	0.0198	0.284
ORWAY	0.944	0.001	0.130
PAIN	0.955	0.007	0.225
WEDEN	0.945	0.0123	-0.004
WITZER AND	0.963	0.0230	0.182
K	0.934	0.0345	0.584
SA	0.925	0.0200	0.898

(i) Column (1) reports the first-order autocorrelation for μ_t.
(ii) Column (2) reports the R^2 from the regression of excesss returns on the risk premium
(iii) Column (3) reports the correlation coefficient between the estimated expected return and the conditional variance

TABLE 6

PROPORTION OF VARIANCE OF EXCESS RETURNS EXPLAINED BY ECONOMIC FACTORS

COUNTRY	4 FACTORS	10 INNOVATIONS
AUSTRALIA	0.0145	0.0525
AUSTRIA	0.1961	0.2083
BELGIUM	0.1658	0.1781
CANADA	0.0674	0.0918
DENMARK	0.1194	0.1497
FRANCE	0.1481	0.1701
GERMANY	0.1523	0.2035
ITALY	0.0554	0.0896
JAPAN	0.1939	0.2204
NETHERLANDS	0.1212	0.1412
NORWAY	0.0650	0.1567
SPAIN	0.0595	0.0927
SWEDEN	0.0784	0.0961
SWITZERLAND	0.1867	0.2161
UK	0.0388	0.0604
US	0.0373	0.0525
AVERAGE	0.1062	0.1362

TABLE 7

TESTS FOR NORMALITY OF FACTORS

FACTOR	χ^2 test (2 degrees of freedom)
f_{11}	8.23
f_{12}	24.05
f_{13}	7.97
f_{14}	0.02
f_{21}	24.05
f_{22}	8.29

Notes:

(i) The Jarque and Bera (1980) test was carried out on each factor normalised by its time-varying variance.

(ii) $\chi^2_{0.05}(2) = 5.99$
$\chi^2_{0.01}(2) = 9.21$

TABLE 8

VARIANCE DECOMPOSITION
(per cent)

ESTIMATION METHOD	OBSERVABLE FACTORS	UNOBSERVA-BLE FACTORS	IDIOSYNCRA-TIC FACTORS
PRINCIPAL COMPONENTS	9.22	46.09	44.69
MAXIMUM LIKELIHOOD	10.95	35.60	53.45

TABLE 9

ESTIMATES OF THE PRICE OF IDIOSYNCRATIC RISK

	Price	Asymptotic t-ratio
Australia	0.628	1.143
Austria	0.508	2.490
Belgium	0.540	1.322
Canada	1.247	1.357
Denmark	0.217	0.642
France	1.449	1.647
Germany	-0.151	-0.140
Italy	-0.192	-0.206
Japan	0.028	0.032
Netherlands	0.286	0.093
Norway	3.272	0.664
Spain	-0.110	-0.166
Sweden	0.298	0.797
Switzerland	0.529	0.263
UK	0.433	2.162
US	-4.837	-0.949

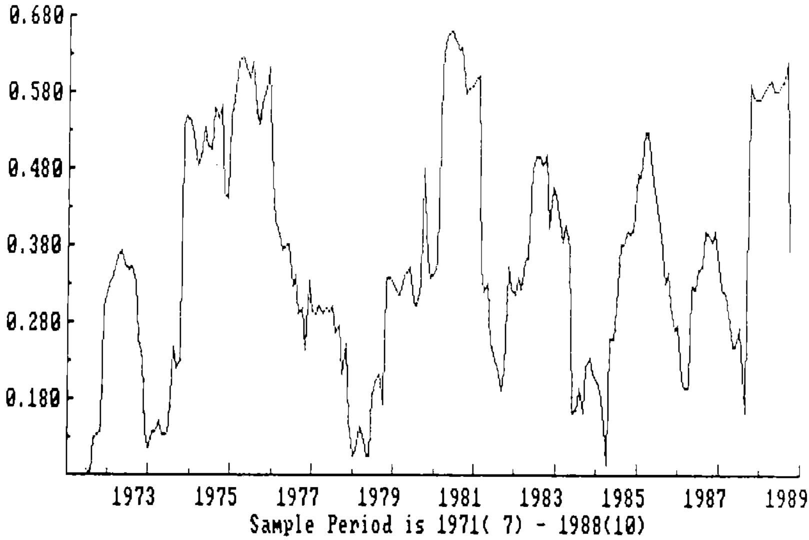

Figure 1

Equally-weighted average of individual cross-country correlation coefficients

(over preceding 12 months)

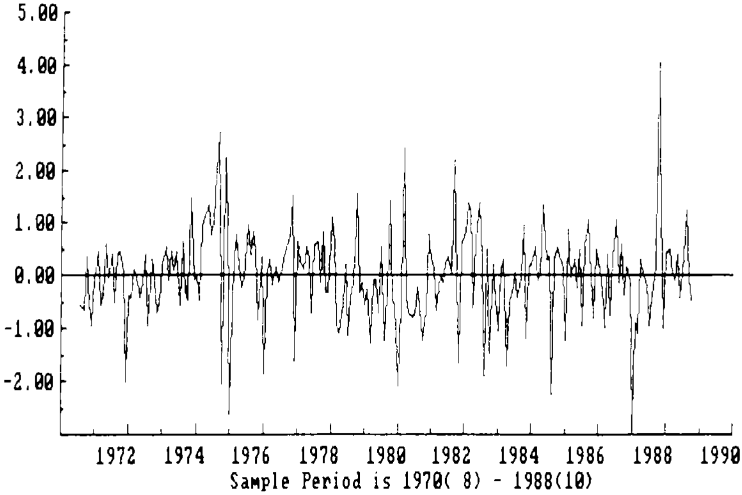

Figure 2a

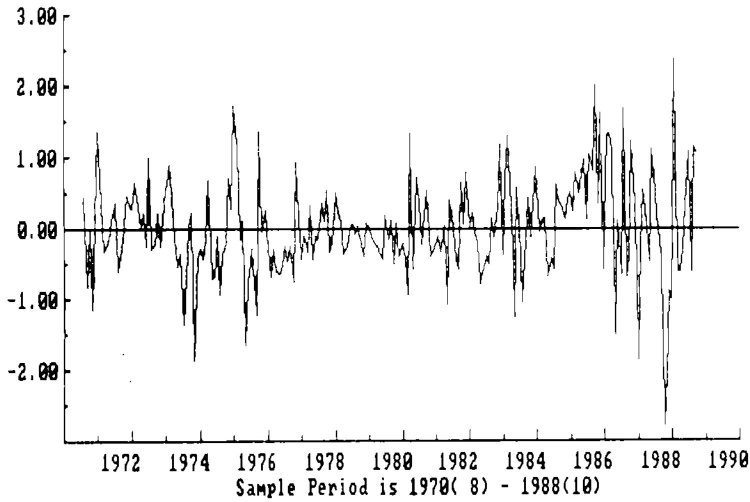

Figure 2b

Unobservable factors

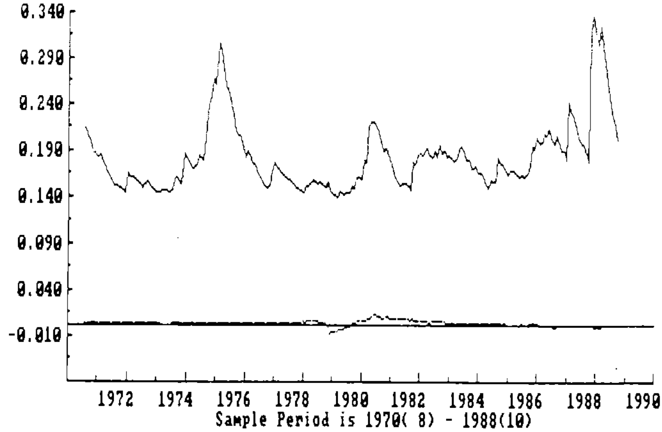

Figure 3a

Conditional covariance between
the US and UK stock markets

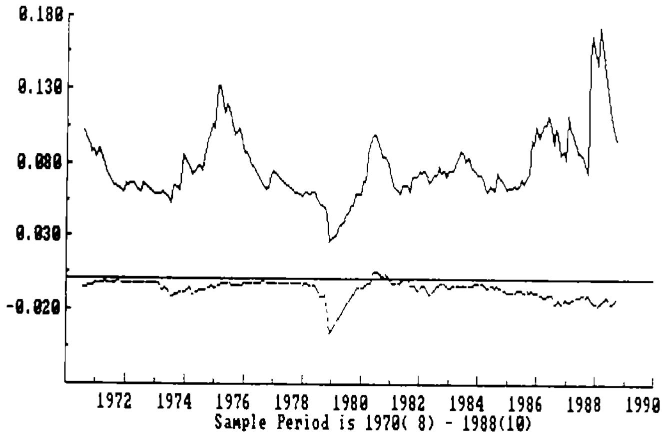

Figure 3b

Conditional covariance between
the US and Japanese stock markets

Key: —— Conditional covariance
--- Contribution by observables

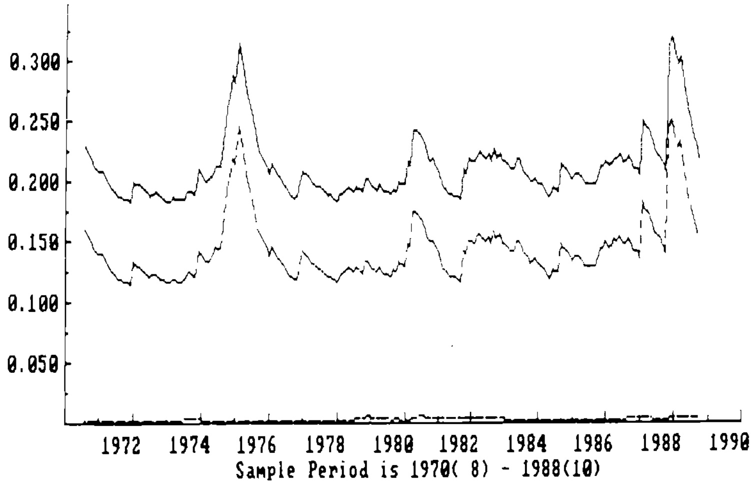

Figure 4a

Variance of the US stock market

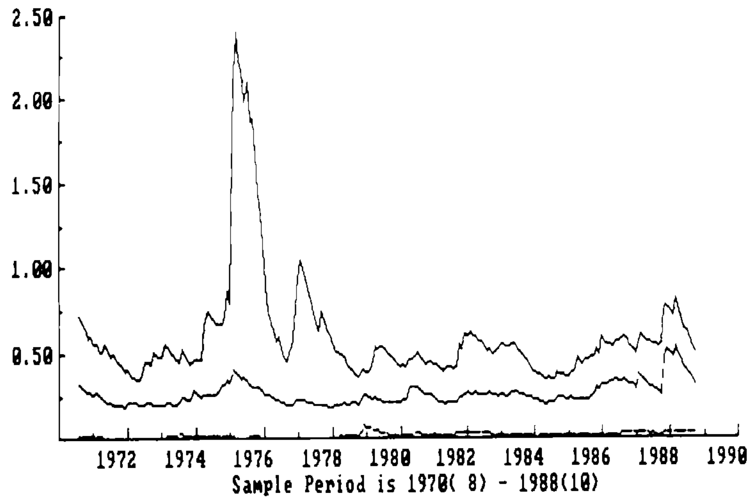

Figure 4b

Variance of the UK stock market

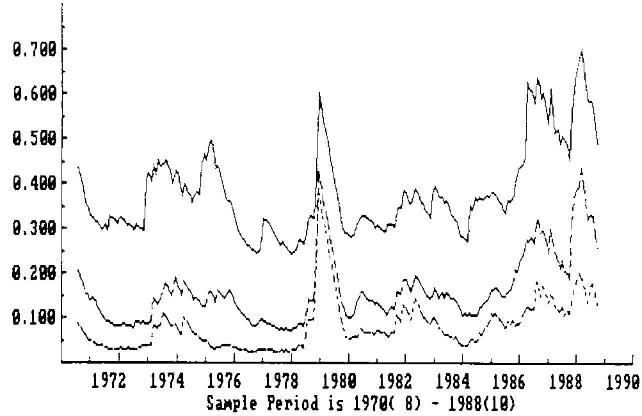

Figure 4c

<u>Variance of the Japanese stock market</u>

Key: —— Conditional variance
--- Contribution by observables
- - observables and unobservables

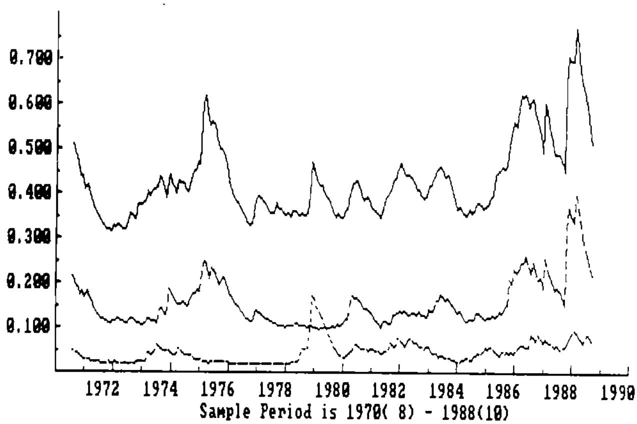

Figure 4d

<u>Equally-weighted average of</u>
<u>the variance of all markets</u>

Key: —— Conditional variance
--- Contribution by observables
- - Contribution by unobservables

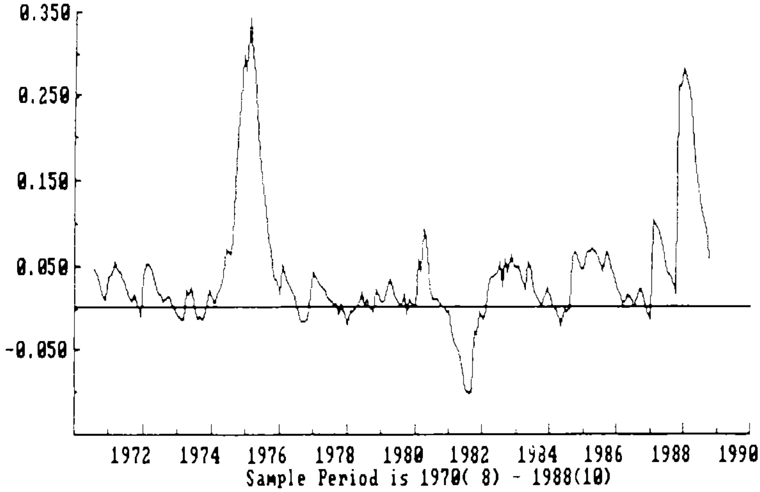

Figure 5a

Required rate of return for the US

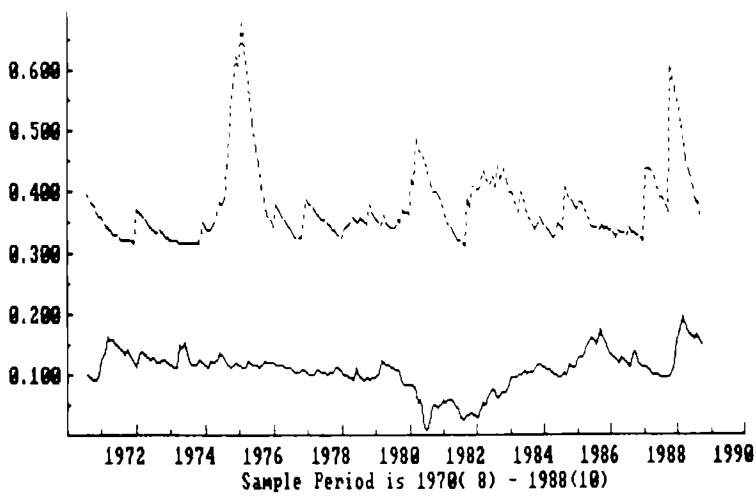

Figure 5b

Decomposition of the required rate of return for the US

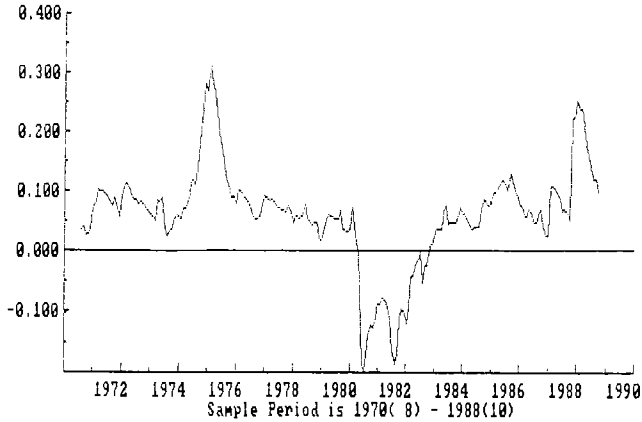

Figure 5c

Equally-weighted average of the required rates of return

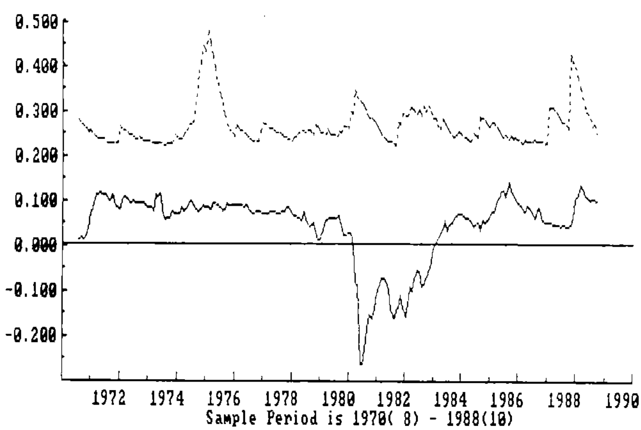

Figure 5d

Decomposition of the equally-weighted average of required rates of return

Key: —— Contribution by observables

--- Contribution by unobservables

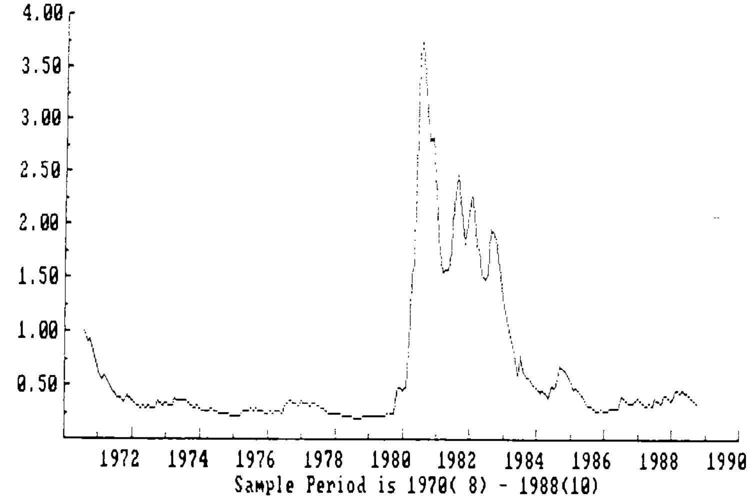

Figure 6a

Variance of f_{11t}

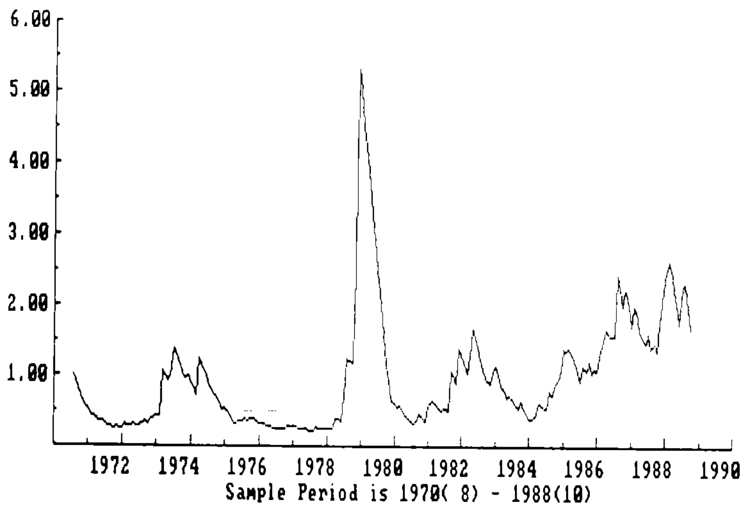

Figure 6b

Variance of f_{12t}

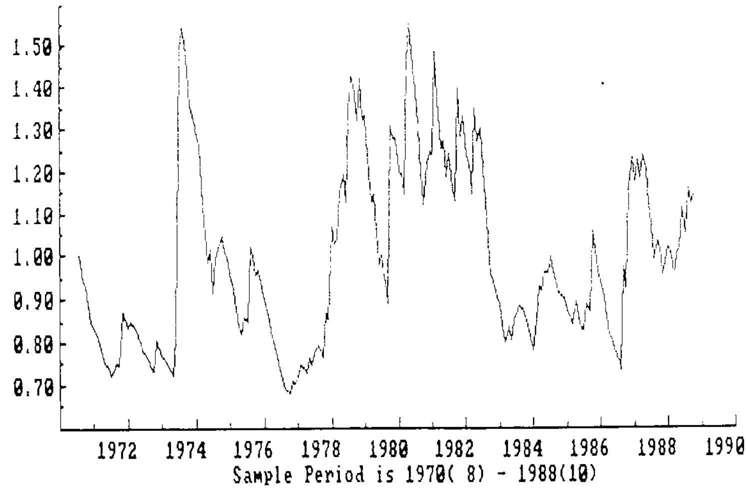

Figure 6c

<u>Variance of</u> f_{13t}

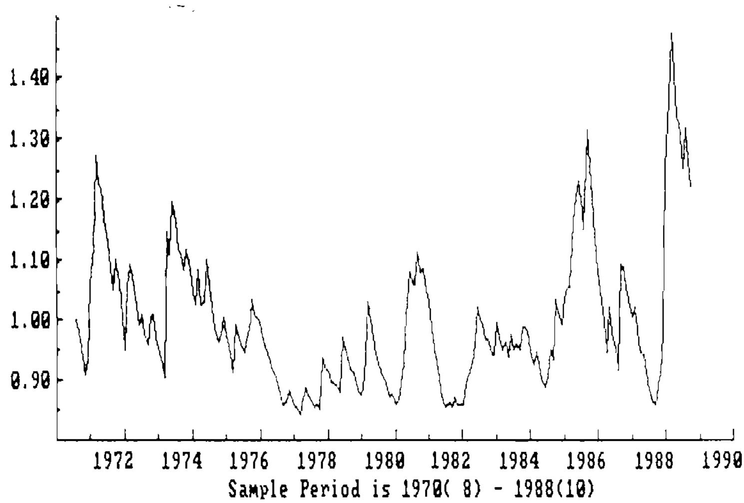

Figure 6d

<u>Variance of</u> f_{14t}

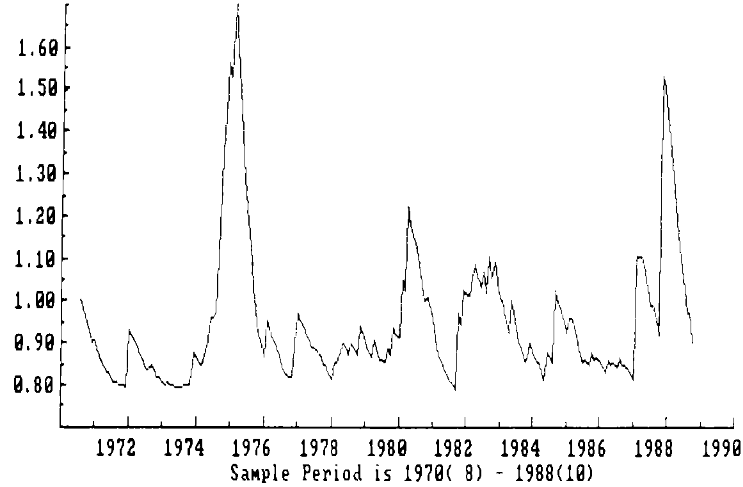

Figure 6e

<u>Variance of</u> f_{21t}

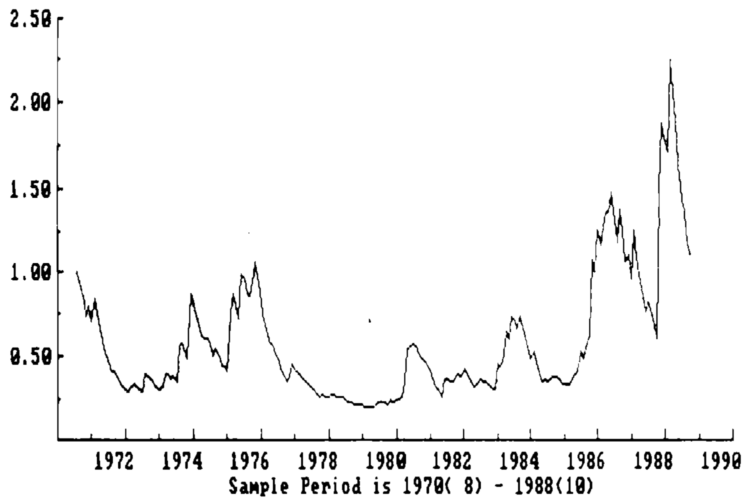

Figure 6f

<u>Variance of</u> f_{22t}

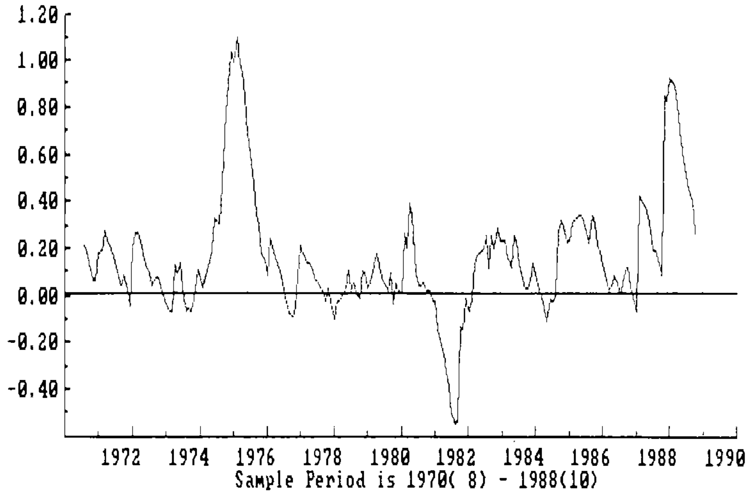

Figure 7a

Price of risk for the US market

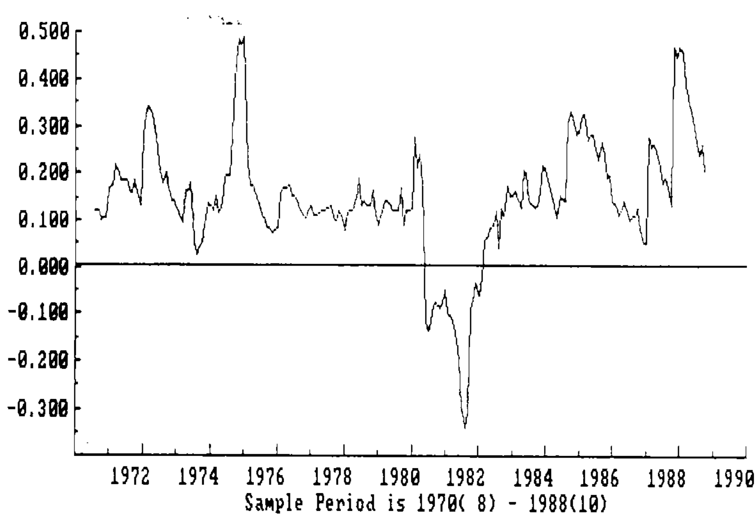

Figure 7b

Price of risk for the UK market

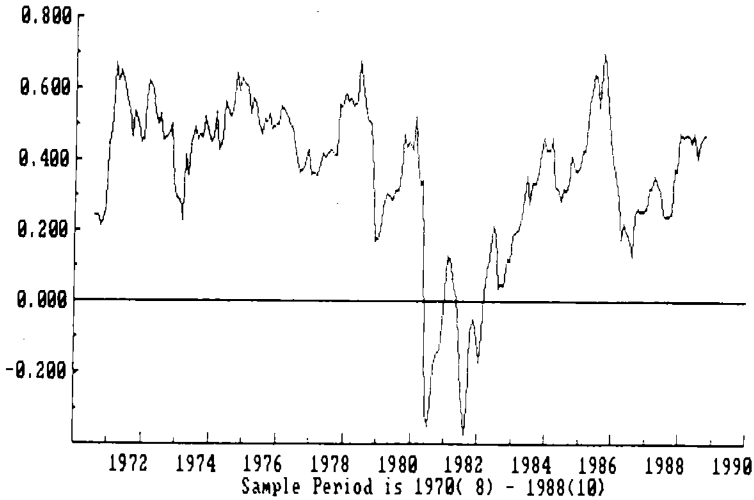

Figure 7c

Price of risk for the Japanese market

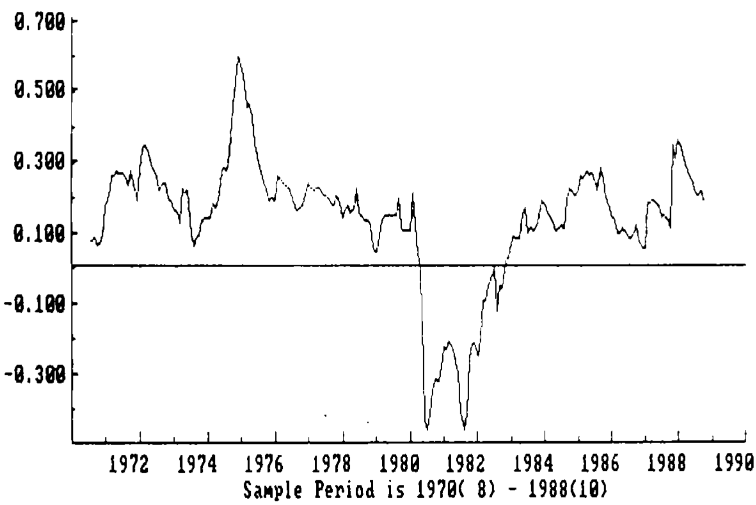

Figure 7d

Equally-weighted average of the price of risk for all markets

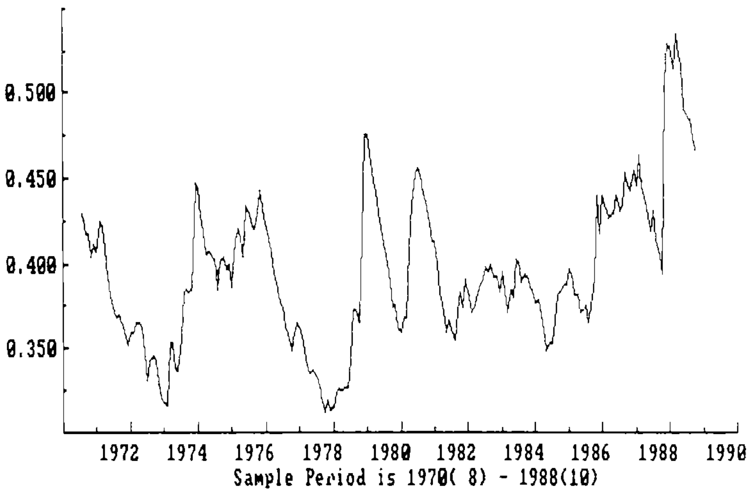

Figure 8

<u>Equally-weighted average of</u>

<u>ex-ante correlations between markets</u>