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ABSTRACT

We examine the recent literature that studies the spatial distribution of economic activity across both space and time. We discuss the methodological advances enabling the incorporation of dynamic forces of economic activity—such as endogenous innovation, forward-looking location choices, capital and asset accumulation, idea diffusion, and stochastic fundamentals—into frameworks with many heterogeneous locations and a rich economic geography. These frameworks remain tractable for quantitative evaluations. We also discuss the wide range of empirical questions explored in recent work through the lens of these frameworks, including the global and local economic impacts of climate change, the dynamic effects of trade and migration policy, labor market adjustments to import competition, the spatial consequences of structural change, the dynamic effects of place-based policies, and the long-run spatial effects of large-scale infrastructure projects.

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Spatial Dynamics*

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Abstract

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Keywords: Spatial dynamics, innovation, migration, trade, endogenous growth
JEL Codes: F10, F16, F22, O11, O18, O33, R11, R12, R23

1 Introduction

Economic geographers have long explored how both static and dynamic agglomeration economies shape the distribution of economic activity over time and space.[1] According to the early theories of Marshall (1890) and Jacobs (1969), local density facilitates the exchange and transmission of ideas. Living and working in close proximity allows individuals to learn from each other. These localized knowledge spillovers not only benefit productivity in a static sense, they also promote innovation and economic growth in a dynamic sense. For example, Glaeser et al. (1992) find that diversity

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[1] For an overview of this literature, see Desmet and Rossi-Hansberg (2010).

within cities fosters growth, highlighting the role of dynamic knowledge spillovers across different industries.

Despite these insights, formally incorporating both a time and a space dimension into economic geography models has been challenging. The New Economic Geography (NEG) literature of the 1990s primarily focused on two-sector static models, with the goal of showing how transport costs and agglomeration economies shape the spatial distribution of economic activity. In the late 1990s and early 2000s, some of these NEG models started to incorporate endogenous innovation and dynamics (Martin and Ottaviano, 1999, 2001; Baldwin et al., 2001; Baldwin and Martin, 2004). In these models, agglomeration economies and innovation tend to reinforce each other. Denser locations have larger markets, increasing the incentives for local firms to innovate. As higher productivity translates into higher income, the local market further grows in size. As such, these models make the essential point that density is not just related to static agglomeration economies, it also leads to more innovation and thus higher growth over time. However, these NEG models were often highly stylized: they typically only included a few locations, and were therefore unable to fully capture the complex geography of an economy. While valuable as conceptual models to gain insights into the circular relationship between agglomeration and growth, they were limited in their capacity of connecting with empirical data.

Incorporating a more realistic high-resolution geography into dynamic frameworks poses three significant challenges. First, clearing markets can be complex in spatial economies with many locations and heterogeneous economic fundamentals, as it requires solving for a large number of prices in general equilibrium. Second, incorporating a realistic economic geography requires access to detailed spatial data in order to conduct counterfactual analysis. Third, allowing for many locations in a dynamic setting where individuals can move across space increases the dimensionality of the problem. Agents' forward-looking migration and investment decisions depend not only on the fundamentals of their own locations, but also on the evolution of the state of the economy across all locations.

Since the 2000s, the development of static spatial frameworks with a rich economic geography has made significant progress in addressing the first two challenges.[2] These static general equilibrium models incorporate many locations, heterogeneous fundamentals across locations, agents moving across space, a gravity trade structure, and agglomeration and congestion forces, among other elements.[3] To handle market clearing, these spatial models have relied on tools from the international trade literature that facilitate solving for the general equilibrium (Eaton and Kortum, 2002). An alternative methodology is to clear markets sequentially, as proposed by Rossi-Hansberg (2005). This literature has also leveraged the recent surge in spatial data, allowing researchers to take their models to the data and conduct counterfactual analysis.

However, static spatial frameworks fail to account for the forces that shape the distribution of

[2]For an in-depth discussion of these static models, see the chapters in this volume by Allen and Arkolakis (2025) and Redding (2025).

[3]See also the review in Redding and Rossi-Hansberg (2017).

economic activity across time and space. These forces include endogenous innovation, dynamic agglomeration externalities, forward-looking location and investment choices, the accumulation of physical and human capital, and the evolution of wealth across space. While many of these forces have been extensively studied in the macroeconomic literature, integrating them into spatial economies with many locations results in a large state space—the dimensionality issue highlighted in the third challenge above. The additional complexity of agents having the choice of moving across space further complicates the problem, as it requires jointly solving for agents' location decisions along with other economic choices. These are difficulties that standard dynamic macroeconomic models do not face.

This chapter reviews recent advances in the dynamic spatial literature that have developed frameworks to effectively address these challenges. In reviewing these methodological achievements, we structure the discussion around the dynamic spatial framework with endogenous innovation and growth by Desmet et al. (2018) and the framework with forward-looking migration decisions by Caliendo et al. (2019). We describe the environment's rich geography and provide details about agents' dynamic innovation and migration decisions. Additionally, we show how these models can be used for quantitative evaluations, and discuss recent generalizations and extensions of these models.

Beyond describing these methodological advances, the chapter discusses how dynamic spatial frameworks have proven useful in elucidating a wide range of phenomena that are inherently dynamic and spatial. One such phenomenon is how spatial dynamics affect aggregate growth. For instance, spatial heterogeneity in economic damages from global warming may lead to underestimating the aggregate impact of climate change because some of the most affected areas are densely populated. Therefore, even if we are only interested in the aggregate global impact of rising temperatures, it is still essential to incorporate spatial heterogeneity into climate assessment models. Likewise, the effects of important drivers of aggregate growth, such as human capital accumulation, depend on where this accumulation takes place, as individuals may accumulate human capital at different rates across locations. More generally, it is intuitive that aggregate growth is a consequence of spatial growth. Specifically, understanding aggregate growth requires incorporating the processes of factor accumulation and productivity growth at the local level, along with their general equilibrium interactions across space.

A second dynamic phenomenon relevant to spatial economies is transitional dynamics. For instance, an important question in recent work is how long it takes for labor markets to adjust to import competition shocks, tariff shifts, or local productivity changes. Considering space is essential because the uneven distribution of economic activity, combined with trade and mobility frictions, means that different locations have varying exposure to these trade and productivity changes. Accordingly, the transition of workers displaced from industries facing negative shocks depends crucially on the geography of those industries and how easily workers can relocate across space. Understanding the transitional dynamics of spatial economies is also important for analyzing the role of initial conditions versus shocks to economic fundamentals in shaping the spatial distribution of labor, capital accumulation, and other economic outcomes over time. This includes phenomena such as income convergence or divergence across locations, the speed of convergence of the economy to its steady state (or its balanced growth path), and, more generally, how initial conditions influence the economy's long-term outcomes.

A third important aspect of spatial economies relates to the distributional effects of shocks. In static models, local shocks affect the spatial distribution of economic activity. In dynamic models, this reallocation may magnify the impact of such shocks. For instance, locations that gain population experience an expansion of the local market, incentivizing endogenous innovation and growth. In addition, in the presence of migration frictions, the option value of future decisions depends on the choices agents make today. For example, the optimal mobility decision for an agent today might be to remain in a given location, even if current payoffs are higher elsewhere, if the agent anticipates that future payoffs will be greater in that location due to factors such as infrastructure projects or improved accessibility. The option value of future decisions shapes differential forward-looking choices across space, which, in turn, impacts the distributional consequences of location-specific shocks.

The rest of this chapter is organized as follows. Section 2 reviews different classes of dynamic spatial models. Section 3 describes applications of these models. Section 4 summarizes the state of the literature, discusses open questions, and concludes.

2 Dynamic Spatial Models

This section discusses two classes of dynamic spatial models. A first class incorporates endogenous growth and innovation. To keep these models computationally tractable, it introduces a structure that simplifies forward-looking migration and innovation decisions to a sequence of static problems. A second class incorporates forward-looking decisions, where agents consider future payoffs in their choices. Unlike the first class, these models do not include endogenous innovation decisions but focus on dynamic decisions related to migration, capital and assets accumulation, and infrastructure investment. Their tractability comes from computing transitional dynamics without identifying location fundamentals and leveraging insights from conditional choice probability theory.

2.1 Dynamic Spatial Models with Endogenous Innovation

A first class of models focuses on high-resolution dynamic spatial models with endogenous innovation. Many important economic problems are spatial and dynamic in nature, and analyzing them often requires a high-resolution approach that allows for endogenous growth. As an example, consider evaluating the economic consequences of sea-level rise. As rising oceans cause the decline of certain high-productivity coastal clusters, people and economic activity will relocate, and over time new clusters of economic activity will emerge. Having a model with high spatial resolution is crucial, not only because coastal flooding is highly localized, but also because localized agglomeration economies shape the evolution of the world's economic geography. Allowing for endogenous innovation and growth is essential if we want to capture the emergence of new economic clusters, driven by the relationship between local density and innovation.

In this subsection, we describe the main building blocks of such a model. As already mentioned, developing tractable dynamic spatial frameworks is challenging. A firm's decision of how much to invest in innovation in a particular location depends not just on what might happen in that location, but in all other locations as well. When the number of locations is very high and the model features both migration and innovation decisions, this creates a dimensionality issue that makes these forward-looking decisions computationally infeasible to solve.

In what follows, we discuss in detail how to simplify forward-looking innovation and migration decisions. We then outline the main elements of a dynamic spatial framework that incorporates, innovation, migration and trade. The discussion builds on the work of Desmet and Rossi-Hansberg (2009, 2014) and Desmet et al. (2018).

2.1.1 Innovation Decisions

Assume the output per unit of land of a firm located in r at time t is:

$$q_t(r) = z_t(r)\, \phi_t(r)^{\gamma_1}\, L_t(r)^{\mu}, \tag{1}$$

where $z_t(r)$ is a TFP shifter drawn from a probability distribution, $\phi_t(r)$ is an innovation which requires the employment of $\nu \phi_t(r)^{\xi}$ units of innovation labor, and $L_t(r)$ is the amount of production labor.

Following Desmet et al. (2018), we make two assumptions about the productivity draws $z_t(r)$. First, productivity draws are spatially correlated. Specifically, as the distance between locations goes to zero, the productivity draws in a particular period become perfectly correlated. Second, any purposeful investment in innovation by a firm in location r at time $t-1$, measured by $\phi_{t-1}(r)$, diffuses to any potential entrants at that location by time t. A simple way of modeling this is to assume that any improvement in technology in period $t-1$ in location r shifts up the distribution from which any potential entrant in location r draws its productivity $z_t(r)$ in period t. Specifically, suppose $z_t(r)$ is drawn from a Fréchet distribution with c.d.f.

$$Pr\left[z_{it}(r) \leq z\right] = e^{-(Z_{it}(r)/z)^{\theta}}, \tag{2}$$

where $\theta > 0$ and Z_{it} scales up linearly with $\phi_{t-1}(r)^{\gamma_1}$. In that case, the average productivity draw in location r in period t also scales up linearly with $\phi_{t-1}(r)^{\gamma_1}$. As a result, any potential entrant at location r has access to the same technology.

The spatial correlation in productivity draws implies that in any small interval firms face similar profit maximization problems. As the size of the interval goes to zero, firms face perfect local competition. Because of perfect competition, in each period firms bid up the price of land until they make zero profits net of the cost of innovating. To the extent that innovating allows firms to bid up the price of land, firms will innovate. As discussed in Desmet and Rossi-Hansberg (2012), this leads to innovation by firms in the presence of perfect competition.

In principle, because of local competition for land and local diffusion of technology, a firm's dynamic profit maximization problem simplifies to a sequence of static problems. However, it is still possible that a firm's innovation decision today affects its scale tomorrow, and hence its innovation decision tomorrow. If so, this would make the innovation decision dynamic. However, if productivity draws are spatially correlated, any decision by a firm today would not affect its productivity and scale tomorrow. Therefore, this setup guarantees that a firm maximizes profits period by period. Desmet and Rossi-Hansberg (2015) give further details of this argument.

Formally, a firm in location r chooses how much labor to employ and how much to innovate to maximize its static profits per unit of land:

$$p_t(r,r)\, z_t(r)\, \phi_t(r)^{\gamma_1} L_t(r)^{\mu} - w_t(r)\left[L_t(r) + \nu \phi_t(r)^{\xi}\right] - R_t(r), \tag{3}$$

where $p_t(r,r)$ is the price of the variety produced by the firm and sold in r. The maximum per unit land rent that a firm is willing to pay at time t, the bid rent, is given by

$$R_t(r) = p_t(r,r)\, z_t(r)\, \phi_t(r)^{\gamma_1} L_t(r)^{\mu} - w_t(r)\left[L_t(r) + \nu \phi_t(r)^{\xi}\right], \tag{4}$$

which ensures all firms make zero profits.

2.1.2 Migration Decisions

Assume the period-utility of an agent who resides in location r in period t and who previously lived in a sequence of locations $\bar{r}_- = (r_0, ..., r_{t-1})$ is given by:

$$u_t(\bar{r}_-, r) = a_t(r)\, C_t(r) \varepsilon_t(r) \prod_{s=1}^{t} m(r_{s-1}, r_s)^{-1}, \tag{5}$$

where $a_t(r)$ denotes the level of amenities in location r, $C_t(r)$ refers to the consumption of goods, $\varepsilon_t(r)$ is an individual idiosyncratic preference shock for location r, and $m(r_{s-1}, r_s)^{-1}$ is a permanent flow utility discount of moving from r_{s-1} to r_s in period s. If each period agents consume all their income, the utility function above can be written as $u_t(\bar{r}_-, r) = a_t(r)\, y_t(r) \varepsilon_t(r) \prod_{s=1}^{t} m(r_{s-1}, r_s)^{-1}$, where $y_t(r)$ denotes the real income of an agent in location r.

After observing their idiosyncratic preference shocks, agents decide every period where to live. Bilateral moving costs between s and r are modeled as a permanent flow-utility discount $m(s,r)$. In particular, bilateral moving costs take the form of the product of an origin-specific leaving cost $m_1(s)$ and a destination-specific entry cost $m_2(r)$, so $m(s,r) = m_1(s)\, m_2(r)$. There is no cost to staying in the same place, so $m(r,r) = 1$. This implies that the cost of leaving a place is the inverse of the cost of entering that same place: $m_1(r) = 1/m_2(r)$. As a result, a location that is costly to enter (say, the United States) is easy to leave. Vice versa, a location that is costly to leave (say, the Democratic Republic of Congo) is easy to enter. Although migration costs are bilateral, the cost of entering a location is independent of the country of origin. It is as difficult to enter the U.S. for a Ghanaian as it is for a German. However, because it is more difficult to leave Ghana than it is to leave Germany, the migration cost from Ghana to the U.S. is higher than the cost of moving from Germany to the U.S.

These assumptions make migration decisions fully reversible. A migrant who leaves location r incurs a permanent flow utility benefit (cost) $m_1(r)$ which is the inverse of the permanent flow utility cost (benefit) $m_2(r) = 1/m_1(r)$ that she experienced upon entering location r. Take the example of a Mexican migrant who moves to the U.S. in period t, and either returns to Mexico or moves to a third country in period $t+5$. Upon initially entering the U.S. in period t and for all future periods, she pays a permanent flow utility cost. However, upon leaving the U.S. in period $t+5$ and for all periods thereafter, she gets a permanent flow utility exit benefit that fully compensates the flow utility entry cost. As a result, she only pays the utility cost of entering the U.S. during the period that she actually resides in the U.S. As such, the forward-looking migration decision becomes fully reversible.

Formally, the value function of an agent living in r_0 in period 0, after observing a distribution of taste shocks in all locations, $\bar{\varepsilon}_1 \equiv \varepsilon_1(\cdot)$, can be written as

$$\begin{aligned} V(r_0, \bar{\varepsilon}_1) &= \max_{r_1} \left[\frac{a_1(r_1)}{m(r_0, r_1)} y_1(r_1) \varepsilon_1(r_1) + \beta E \left(\frac{V(r_1, \bar{\varepsilon}_2^i)}{m(r_0, r_1)} \right) \right] \\ &= \frac{\max_{r_1} \left[\frac{a_1(r_1)}{m_2(r_1)} y_1(r_1) \varepsilon_1(r_1) + \beta E \left(\frac{V(r_1, \bar{\varepsilon}_2)}{m_2(r_1)} \right) \right]}{m_1(r_0)} \end{aligned}$$

where the second equality relies on $m(r_0, r_1) = m_1(r_0) m_2(r_1)$. Since an agent residing in r_1 in period 1 needs to decide where to move to in period 2, then

$$V(r_1, \bar{\varepsilon}_2) = \max_{r_2} \left[\frac{a_2(r_2)}{m_1(r_1) m_2(r_2)} y_1(r_2) \varepsilon_1(r_2) + \beta E \left(\frac{V(r_2, \bar{\varepsilon}_2)}{m_1(r_1) m_2(r_2)} \right) \right].$$

Because $m_1(r_1) m_2(r_1) = 1$, the equation above can be rewritten as

$$V(r_0, \bar{\varepsilon}_1) = \frac{\max_{r_1} \left[\frac{a_1(r_1)}{m_2(r_1)} y_1(r_1) \varepsilon_1(r_1) \right] + \beta E \left(\max_{r_2} \left[\frac{a_2(r_2)}{m_2(r_2)} y_2(r_2) \varepsilon_2(r_2) + \frac{V(r_2, \bar{\varepsilon}_2)}{m_2(r_2)} \right] \right)}{m_1(r_0)}.$$

Hence, the decision on where to reside in period 1 does not depend on the decision of where to reside in period 2. Because today's decision does not depend on tomorrow's decision, this simplifies the dynamic migration problem to a sequence of static problems.

2.1.3 Other Model Features

In what follows we sketch some of the other main features of the dynamic spatial model of Desmet et al. (2018). In doing so, we will emphasize the dynamic elements of the model.

Endowments and preferences. The economy is modeled on a two-dimensional surface, with points r in space defined by longitude and latitude. Having two dimensions gives the model a realistic geography, and hence facilitates taking the model to the data. In a simple setup, the economy's endowments are labor and land. As above, there could be different types of labor, such as innovation and production labor. Land is key to allow for standard economic geography concerns, such as agglomeration economies and congestion forces. Endowments could be taken as constant, or they could vary with time if we want to bring in fertility decisions that affect the population size or climate change concerns that affect the quantity of land available for productive purposes.

Preferences are given by (5). Agents derive utility from consumption, amenities, and idiosyncratic location-specific taste shocks, with migration costs acting as a utility discount. In spatial models, trade is central. For there to be a motive for locations to trade with each other, we must allow for either comparative advantage between sectors or product differentiation within sectors. In its most general expression, $C_t(r)$ can be thought as an aggregator of different varieties ω of different goods i. In addition to real income, amenities $a_t(r)$ are a key driver of location choices. In its easiest formulation, we can think of amenities as representing exogenous and time-invariant features of a location. Examples include coastal proximity or average hours of sunshine. Of course, amenities could also be endogenous to economic forces. For instance, amenities might be subject to congestion:

$$a_t(r) = \overline{a}_t(r) L_t(r)^{-\lambda}$$

where $\overline{a}_t(r)$ denotes the location's fundamental amenity and $L_t(r)^{-\lambda}$ is a local dispersion force coming from congestion due to population density. Idiosyncratic location-specific taste shocks tend to be important to modulate the migratory response to income or other shocks. For example, taste shocks can be assumed to be distributed according to a Fréchet distribution with shape parameter $1/\Omega$. A higher value of Ω then implies greater taste heterogeneity, and hence a lower elasticity of migration to changes in income. Taste heterogeneity implies that not everyone wants to live in the same place, and hence acts as dispersion force.

Technology and innovation. A firm's production function is given by (1). We already discussed how dynamic forward-looking innovation decisions can be simplified to a sequence of static decisions. Here, we bring in some of the spatial forces that affect economic growth, such as agglomeration economies and spatial diffusion. The productivity shifter $z_t(r)$ is drawn from a Fréchet distribution with c.d.f. given by (2), where $Z_t(r)$ is proportional to the average idiosyncratic productivity of varieties in location r. Suppose the term $Z_t(r)$ depends on a location's density of production and innovation workers, $\overline{L}_t(r)$, and on its fundamental productivity, $\tau_t(r)$:

$$Z_t(r) = \tau_t(r)\overline{L}_t(r)^{\alpha}.$$

The greater the parameter α, the stronger the agglomeration force. Suppose that fundamental productivity, in turn, evolves over time, depending on past innovation and productivity, in the own location and in other locations:

$$\tau_t(r) = \phi_{t-1}(r)^{\gamma_1}\left[\int \eta\tau_{t-1}(s)\,ds\right]^{\frac{1-\gamma_2}{\theta}}\tau_{t-1}(r)^{\frac{\gamma_2}{\theta}}. \tag{6}$$

As mentioned before, the average productivity draw scales up with past innovation in the own location, ensuring that any potential entrant has access to the same technology. Spatial diffusion acts as a dispersion force, and is essential to avoid excessive spatial concentration over time.

Trade. Let $\varsigma(s,r) \geq 1$ denote the iceberg cost of transporting a good from r to s. Using the approach of Eaton and Kortum (2002), one can write an expression of the spending of location r on goods from location s, relative to the total spending of location r. These expressions are consistent with standard gravity equations of trade.

Equilibrium. When discussing the equilibrium of any dynamic spatial model, two questions arise. First, economic geography models often feature multiple equilibria because of the presence of externalities, making it important to address not just existence but also uniqueness. Second, as in any endogenous growth model, we are interested in establishing conditions that ensure the economy converges to a balanced growth path.

On the issue of existence and uniqueness of the equilibrium, we start by describing the different agglomeration and congestion forces. In this model, there are two static agglomeration forces. One comes from the scale effect of innovation. Locations that produce more face a lower cost of innovation per unit of production. Another comes from the impact of local density on the average productivity draw. Counteracting this are several congestion forces, one coming from land and another from the existence of idiosyncratic location-specific taste shocks. In addition, amenities may also suffer from congestion. Following arguments in Zabreyko et al. (1975) and Allen and Arkolakis (2014), one can establish conditions that guarantee the existence and uniqueness of the equilibrium:

$$\alpha + \frac{\gamma_1}{\xi} \leq \lambda + 1 - \mu + \Omega. \tag{7}$$

This condition is intuitive. It states that the static agglomeration economies associated with the local production externalities (α) and the degree of returns to innovation (γ_1/ξ) should not dominate the three congestion forces coming from amenities (λ), land ($1-\mu$), and idiosyncratic locational preferences (Ω).

The spatial economy converges to a balanced-growth path if the following condition holds:

$$\alpha + \frac{\gamma_1}{\xi} + \frac{\gamma_1}{\xi\left[1-\gamma_2\right]} \leq \lambda + 1 - \mu + \Omega. \tag{8}$$

As can be seen, this condition is slightly more restrictive than the uniqueness condition (7). Specifically, the left-hand side includes an additional dynamic agglomeration term coming from local innovation which is mitigated by diffusion, $\gamma_1/(\xi\left[1-\gamma_2\right])$. This dynamic agglomeration force captures the fact that increased local innovation attracts more people, which in turn leads to more innovation in the future. Diffusion ensures that employment does not concentrate excessively in one location. If there were no diffusion so that $1-\gamma_2 = 0$, there would be no balanced-growth path with a non-degenerate distribution of employment. Similar to (7), the balanced-growth path condition states dispersion forces have to be large enough compared to agglomeration forces.

2.2 Dynamic Spatial Frameworks with Forward-Looking Agents

In a second class of dynamic spatial models, the effects of changes to economic fundamentals depend on how forward-looking agents anticipate the trajectory of these fundamentals. This anticipation shapes their economic decisions and, consequently, the actual economic outcomes. As a result, when making forward-looking decisions—such as mobility decisions—agents evaluate not only the current payoffs across locations but also the future payoffs of their location choices. Therefore, agents' forward-looking decisions are intrinsically dynamic; the option value of future decisions depends on the choices they make today. For example, the optimal mobility decision for an agent today might be to stay in a given location, even if current payoffs are higher elsewhere, if the agent anticipates that future payoffs will be greater in locations that are more easily accessible from the current one. The joint decisions of forward-looking agents imply that a general equilibrium analysis must solve for a large number of prices, all determined by the decisions of agents across many locations. Recent work has integrated these forward-looking decisions into frameworks with detailed economic geography. Progress in this area has been twofold. First, on the modeling side, the literature has introduced forward-looking decisions by agents in a tractable manner. Second, advances in solution methods have made it possible to align these frameworks with data, enabling general equilibrium counterfactual analysis.

To introduce forward-looking decisions into a dynamic spatial framework, Caliendo et al. (2019) leverage the assumption of a continuum of atomistic and heterogeneous agents who make forward-looking decisions taking the aggregate state of the economy as given. The tractability of the dynamic spatial framework also leverages the idea of renewal actions from conditional choice probability theory.[4] Counterfactual analysis through the lens of these frameworks relies on the insight that

[4]For example, with forward-looking migration decisions, the renewal action is the location choice, which implies that the distribution of future states is identical for individuals residing in the same location. See Arcidiacono (2011) and Kalouptsidi et al. (2021) for examples of a class of dynamic discrete choice frameworks with a renewal structure, applied to various dynamic problems in the literature on industrial organization.

observable allocations (e.g., trade, migration) contain information on economic fundamentals, which, as we describe in the next subsections, allows for conducting counterfactual analysis without the need to identify a potentially large set of fundamentals or to assume that the actual economy is in a steady state.

In what follows, we explore dynamic spatial frameworks with forward-looking agents. These models differ from those in the previous subsection in two key ways. First, agents' forward-looking decisions take into account their expectations of future changes to economic fundamentals and their impact on dynamic decisions, such as location choices, and the accumulation of capital and assets. In contrast, the models discussed in the previous subsection relied on assumptions that allowed forward-looking agents to abstract from such anticipatory effects. Second, while some frameworks in this section incorporate economic growth, they abstract from tractably incorporating endogenous innovation in agents' forward-looking decisions.

2.2.1 Dynamic Spatial Model with Forward-Looking Migration Decisions

One main focus of the spatial dynamics literature is the study of the adjustment process of an economy across space and over time following changes in economic fundamentals at the local or aggregate levels—such as infrastructure projects, natural disasters, place-based policies, among others. An important factor shaping the distribution of economic activity across space is agents' mobility decisions.[5] When making these mobility decisions, agents must consider not only current payoffs across locations but also their lifetime utility, which depends on the future payoffs associated with different location choices.

In this context, we discuss the dynamic spatial framework with forward-looking migration decisions in Caliendo et al. (2019). To simplify the description of the framework and methodology, we present a single-sector version of the model. Time is discrete and denoted by t. The world consists of N locations, indexed by n or i. At each moment in time, each location i is populated by a mass $L_{i,t}$ of heterogeneous, forward-looking agents who observe economic conditions and optimally decide where to locate in each period by maximizing the present discounted value of their utility. In each location i, agents supply one unit of labor inelastically and consume local goods where they live. Agents' consumption capacity $c_{i,t}$ is determined by their labor income $w_{i,t}$ and the local price index $P_{i,t}$. Forward-looking location choices are also affected by bilateral mobility frictions when moving from location i to location n, denoted by $\tau_{in,t}$, and by additive idiosyncratic taste shocks that vary across locations, denoted by $\epsilon_{n,t}$.

[5]Recent work has provided empirical evidence on how the differential exposure of locations to trade shocks (e.g., tariff changes, import competition, etc.) shapes distributional effects across space (e.g., Dix-Carneiro and Kovak, 2017; Autor et al., 2013).

2.2.1.1 Dynamic Migration Decisions

Forward-looking migration decisions are modeled as a dynamic-discrete choice problem within this economic environment. In particular, denote by $v_{i,t}(\epsilon_t, \tilde{L}_t, \Theta_t)$ the location value for an agent in location i at time t. It depends on the vector of idiosyncratic shocks across locations $\epsilon_t \equiv \{\epsilon_{1,t}, \epsilon_{2,t}...\epsilon_{N,t}\}$, on the endogenous state of economy $\tilde{L}_t$, and on the exogenous state of the economy Θ_t. The endogenous state space in Caliendo et al. (2019) is defined by the distribution of agents across space, given by $L_t \equiv \{L_{1,t}, L_{2,t}...L_{N,t}\}$. Later, we discuss other frameworks where the endogenous state of the economy includes additional variables, such as the stock of capital, assets, or productivity. The exogenous state of the economy, Θ_t, is the set of economic fundamentals, including productivities, migration costs, and trade costs, which we specify later.

The timing for the agent's problem and decisions is as follows: in a given period t, the agent earns the market wage, observes the economic conditions in all locations along with the realizations of their own idiosyncratic shocks, and decides at the end of that period where to supply labor in $t+1$. Therefore, the location value for an agent in location i at time t is given by

$$v_{i,t}(\epsilon_t, \tilde{L}_t, \Theta_t) = U(c_{i,t}) + \max_{\{n\}_{n=1}^N} \left\{ \beta \mathbb{E}_t \left[v_{n,t+1}(\epsilon_{t+1}, \tilde{L}_{t+1}, \Theta_{t+1}) \right] - \tau_{in,t} + \nu \epsilon_{n,t} \right\}, \tag{9}$$

where $U(c_{i,t})$ is the flow utility of the agent at location i, β is the discount factor, and the idiosyncratic shocks $\epsilon_{n,t}$ are $i.i.d.$ realizations from a Gumbel (Type I Extreme Value) distribution with dispersion parameter ν. Let $V_{i,t} = \mathbb{E}_t[v_{i,t}]$ be the expectation at time t over the future realizations of the idiosyncratic shocks that shape the continuation value of each location, taking as given the aggregate state of the economy. Using the properties of the Gumbel distribution, both sides of equation (9) can be integrated over $\epsilon_{n,t}$ to obtain the value of location i for a representative agent at time t:

$$V_{i,t}(\tilde{L}_t, \Theta_t) = U(c_{i,t}) + \nu \log \left[\sum_{n=1}^N exp \left[\beta V_{n,t+1}(\tilde{L}_{t+1}, \Theta_{t+1}) - \tau_{in,t} \right]^{1/\nu} \right]. \tag{10}$$

The location value (10) depends on two terms. The first term on the right-hand side is the utility flow in the agent's current location. Importantly, the second term in the location value reflects both the continuation value and the option value of moving to another location. The option value of migration differentiates frameworks with forward-looking decisions from static ones and shapes agents' mobility decisions as well as the welfare effects of changes in economic fundamentals.

Forward-looking agents move to the location with the highest value, subject to mobility costs and idiosyncratic preference shocks. In particular, the fraction of workers that moves from location i to location n, denoted by $\mu_{in,t}$, can be expressed in closed form using the properties of the Gumbel distribution:

$$\mu_{in,t} = \frac{\exp\left[\beta V_{n,t+1}(\tilde{L}_{t+1}, \Theta_{t+1}) - \tau_{in,t}\right]^{1/\nu}}{\sum_{h=1}^{N} \exp\left[\beta V_{h,t+1}(\tilde{L}_{t+1}, \Theta_{t+1}) - \tau_{ih,t}\right]^{1/\nu}}. \tag{11}$$

This equilibrium condition is a forward-looking equation that determines the gross migration flows of agents across space at each moment in time. It shows that forward-looking agents decide where to locate by assessing the relative net future value of each location, taking into account the option value of migrating to different locations in the future. The gross migration flows determine the evolution of the distribution of agents across space. Specifically, the supply of workers at location i at time $t+1$ is given by the individuals who migrate to location i from all locations n (including those who stay in i) at time t. Therefore, the stock of workers at each location evolves according to

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}. \tag{12}$$

Hence, conditional on the real wages $w_{i,t}/P_{i,t}$, which determine consumption across locations, the system of equations given by (10), (11), and (12) solves for the location values $V_{i,t}$, the forward-looking location choices $\mu_{in,t}$, and, consequently, the evolution of the distribution of agents across space $L_{i,t}$.

Before turning to the demand side of the framework, a few remarks are in order. First, the assumption of idiosyncratic taste shocks from a Gumbel value distribution plays an important role in dynamic discrete choice models. The properties of the Type-I extreme value distribution add tractability by allowing for a closed-form solution of the location values and gross flows across locations. At the same time, it captures the empirical regularity that gross flows are an order of magnitude larger than net flows (see Artuc et al., 2010). Additionally, the presence of idiosyncratic preference shocks, along with bilateral mobility frictions, generates a gradual adjustment of the economy, as only a fraction of agents (i.e., those with sufficiently favorable draws) optimally relocate following a change in fundamentals. This sluggish adjustment of the economy in response to changes in economic fundamentals aligns with the empirical evidence on the persistent effects of trade shocks across space, as documented in recent literature.[6]

Relatedly, one might think that the assumption of *i.i.d.* shocks could lead to excessive mobility across locations. However, this dynamic discrete choice framework delivers outcomes for a representative agent in each location, not for individuals moving across locations. In other words, as equation (11) clearly shows, an outcome of the framework is the fraction of agents reallocating

[6]For instance, Dix-Carneiro and Kovak (2017) show that the differential spatial effects on wages after a trade liberalization episode in Brazil were not eliminated but increased after twenty years. Menezes-Filho and Muendler (2011) find that displaced workers from import competition in Brazil often took years to find new employment. Autor et al. (2014) provide evidence that increased import competition from China adversely affected blue-collar incomes for over a decade, while Hummels et al. (2014) find that offshoring negatively impacted blue-collar wages in Danish firms for nearly four years.

across locations. These gross flows are disciplined by the migration elasticity $1/\nu$, but remain agnostic about the transition rates of individuals.[7] Finally, the assumption that mobility frictions and idiosyncratic taste shocks are additive is not strictly needed for analytical tractability. Caliendo et al. (2019) show that assuming multiplicative migration costs with idiosyncratic shocks that are Fréchet distributed leads to similar analytical expressions to those in (10) and (11).

Trade and production structure. The production structure of the model is characterized by a general gravity trade structure with economic geography components. In particular, the demand side in Caliendo et al. (2019) features a continuum of heterogeneous firms in each labor market producing intermediate goods à la Eaton and Kortum (2002). Firms are competitive, have constant-returns-to-scale technology, and demand labor, local factors, and materials from all sectors according to the input-output structure of the economy as in the static spatial framework of Caliendo et al. (2017). As noted earlier, to simplify the exposition, we abstract from different industries here. The productivity of an intermediate good is composed of two terms: a time-varying location component, $A_{i,t}$, common to all varieties in a location, and a variety-specific component, z_i. The output for a producer of an intermediate variety with efficiency z_i is then given by

$$q_{i,t} = z_i \left[A_{i,t} \left[h_i \right]^{\xi_i} \left[l_{i,t} \right]^{1-\xi_i} \right]^{\gamma_i} \left[M_{i,t} \right]^{1-\gamma_i},$$

where $l_{i,t}, h_i$, and $M_{i,t}$ are labor, capital structures (a fixed factor), and material inputs, respectively. The parameters γ_i and ξ_i represent the shares of value added in gross output and the share of capital structures in value added, respectively. Input shares are allowed to be location-specific, which introduces another source of heterogeneity to the framework. Denoting by $r_{i,t}$ the rental price of structures in location i, the input bundle cost in that location is given by $x_{i,t} = B_i \left((r_{i,t})^{\xi_i} (w_{i,t})^{1-\xi_i} \right)^{\gamma_i} (P_{i,t})^{1-\gamma_i}$, where $P_{i,t}$ denote the price of materials (and final goods), defined below, and where B_i is a location-specific constant.

Shipping goods across locations, from n to i, is subject to iceberg trade costs, $\kappa_{in,t}$. Following Eaton and Kortum (2002), productivities z are distributed according to a Fréchet distribution with a dispersion parameter θ. Firms in location i purchase varieties from the lowest-cost supplier, and therefore, the fraction of goods in location i sourced from location n, denoted by $\lambda_{in,t}$, is given by

$$\lambda_{in,t} = \frac{\left[\kappa_{in,t} x_{n,t} \right]^{-\theta} \left[A_{n,t} \right]^{\gamma_n \theta}}{\sum_{h=1}^{N} \left[\kappa_{ih,t} x_{h,t} \right]^{-\theta} \left[A_{h,t} \right]^{\gamma_h \theta}}. \tag{13}$$

The price index in location i, can be expressed as

$$P_{i,t} = \Gamma \left[\sum_{n=1}^{N} \left[\kappa_{in,t} x_{n,t} \right]^{-\theta} \left[A_{n,t} \right]^{\gamma_n \theta} \right]^{-1/\theta}, \tag{14}$$

[7] See Artuc et al. (2010), Artuc and McLaren (2015), Caliendo et al. (2019), and Traiberman (2019) on different approaches to estimate migration elasticities in dynamic discrete choice frameworks.

where Γ is a constant. Total expenditure in location i, which we denote by $X_{i,t}$, is

$$X_{i,t} = (1-\gamma_i) \sum_{n=1}^{N} \lambda_{ni,t} X_{n,t} + I_{i,t},$$

where the total expenditure on goods in location i is the sum of the expenditure on intermediate goods and agents' income given by $I_{i,t} = w_{i,t} L_{i,t} + r_{i,t} H_i$.[8] Finally the market clearing conditions for labor and capital structures are

$$L_{i,t} = \frac{\gamma_i (1-\xi_i)}{w_{i,t}} \sum_{n=1}^{N} \lambda_{ni,t} X_{n,t}, \text{ and } H_i = \frac{\gamma_i \xi_i}{r_{i,t}} \sum_{n=1}^{N} \lambda_{ni,t} X_{n,t}. \tag{15}$$

Before describing the equilibrium of the economy, it is worth emphasizing that the demand side of the framework can accommodate a variety of building blocks in terms of production functions (e.g., CES, nested-CES, etc.), production factors (e.g., skills), the number and types of sectors (tradable and non-tradable), market structure, local fixed factors, and other sources of congestion effects, as well as both endogenous and exogenous amenities, among other features. Redding and Rossi-Hansberg (2017) provide a description of alternative economic environments or building blocks in the recent static spatial frameworks in the literature.

Equilibrium. The endogenous state of the economy at any moment in time is given by the distribution of agents across locations $L_{i,t}$ for all i. The fundamentals of the economy are deterministic and given by the vector of fundamental productivities across locations, $A_t = \{A_{i,t}\}_{i=1}^{N}$, the matrices of bilateral mobility frictions and bilateral trade costs, $\tau_t = \{\tau_{ni,t}\}_{n=1,i=1}^{N,N}$, $\kappa_t = \{\kappa_{ni,t}\}_{n=1,i=1}^{N,N}$, and the stock of local fixed factors, $H = \{H_i\}_{i=1}^{N}$. The set of economics fundamentals is then summarized by $\Theta_t = \{A_{i,t}, \kappa_{ni,t}, \tau_{ni,t}, H_i\}_{i=1,n=1}^{N,N,}$. Given $(L_0, \{\Theta_t\}_{t=1}^{\infty})$, a sequential competitive equilibrium is a sequence of equilibrium outcomes $\{L_{i,t}, \mu_{in,t}, V_{i,t}, w_{i,t}, r_{i,t}, P_{i,t}\}_{i=1,n=1,t=1}^{N,N,\infty}$ that solves equilibrium conditions (10), (11), (12), (13), (14), and (15) at each t.

Computing the sequential equilibrium in this dynamic spatial framework presents three challenges. First, the equilibrium definition states that solving for the equilibrium requires conditioning on Θ_t, namely, the level of the economy's fundamentals (such as location productivities, endowments of local structures, mobility costs, and trade costs) at each point in time. As the number of locations increases, the number of fundamentals to be identified grows geometrically, which might constrain the number of locations in the framework if there is not enough variation in the data to identify these fundamentals. Second, solving the agent's dynamic problem requires considering that the economy might not be in a steady state in the initial period. In particular, the data might reflect non-zero net flows in the initial period, which would contradict the assumption that the economy is

[8] For simplicity, we also abstract from trade imbalances here. The framework in Caliendo et al. (2019) features a system of transfers of the revenues from the fixed factor that generates endogenous imbalances across locations, following the formulation in Caliendo et al. (2017).

in a steady state at that time. Third, solving for the transitional dynamics requires computing the steady state across many locations with heterogeneous economic fundamentals. To overcome these challenges, Caliendo et al. (2019) develop a solution method referred to as dynamic-hat algebra, which we explain next.

2.2.1.2 Dynamic-Hat Algebra with Constant Fundamentals: Ex-ante Counterfactuals

We discuss the key aspects of dynamic-hat algebra that address the challenges of solving for the sequential equilibrium in the dynamic spatial framework described earlier. Our discussion centers on two types of counterfactual questions: ex-ante and ex-post.

Ex-ante counterfactuals study the effects of unmaterialized shocks to economic fundamentals, such as future changes in migration or trade policy, alternative infrastructure projects to address congestion, and predicted impacts of environmental changes. Since these outcomes are not yet observed, one has to take a stand on the evolution of fundamentals other than those being studied. A common assumption in the literature is that all these other economic fundamentals remain constant. For example, a researcher studying a future productivity change in a given location aims to answer the question: how would the economy evolve in that location with a certain productivity path while keeping other fundamentals constant? In what follows, we refer to ex-ante counterfactuals as counterfactuals with constant fundamentals.

Suppose a researcher wants to study the spatial effects of future productivity increases in a specific location while assuming other fundamentals remain unchanged. This can be achieved by computing the sequential equilibrium of the dynamic spatial model, assuming constant fundamentals except for the productivity path in the studied location. One approach is to assume that the economy is in a steady state and analyze how changes in productivity impact the economy both in the long run and during the transition, similar to an impulse response. However, this approach is problematic because the economy may not be in a steady state in the initial period. Hence, one might want to study how an economy transitioning to a steady state is affected by a productivity change. Consequently, transitional dynamics reflect both the productivity shock and the initial conditions influencing the path to the steady state. This can potentially bias counterfactual outcomes, as one might attribute all effects to the productivity shock, when in reality, only part of the observed outcomes is due to the productivity change.

Alternatively, the researcher could compute transitional dynamics twice: once with constant fundamentals and once with changes only in the productivity of the studied location. The difference between these two equilibrium outcomes would capture the effects of the productivity shocks. While this approach addresses the issue that the economy might not initially be in a steady state, computing the sequential competitive equilibrium requires identifying all exogenous fundamentals that are assumed to remain constant (such as trade, migration costs, productivities across all locations) in addition to the productivity changes. As the number of locations increases, the number of fundamentals grows exponentially, which can result in an intractable number of fundamentals to identify for empirically realistic numbers of locations.

Caliendo et al. (2019) demonstrate that computing the sequential equilibrium in time differences can address these challenges. Dynamic-hat algebra first solves the transition dynamics of a baseline economy with constant fundamentals, and then for a counterfactual economy that includes the productivity change in the location of interest. The difference between the two reflects the effects of the productivity changes.

Solving for the baseline economy. To describe the solution for the baseline economy with constant fundamentals, consider one of the equilibrium conditions of the agent's dynamic problem, namely the fraction of agents that relocate from i to n at $t = 0$,

$$\mu_{in,0} = \frac{\exp\left[\beta V_{n,1}(\tilde{L}_1, \Theta) - \tau_{in}\right]^{1/\nu}}{\sum_{h=1}^{N} \exp\left[\beta V_{h,1}(\tilde{L}_1, \Theta) - \tau_{ih}\right]^{1/\nu}}, \tag{16}$$

where $\Theta = \{A, \kappa, \tau, H\}$ represents the set of time-invariant fundamentals. This equilibrium condition shows that various combinations of migration costs and location values can rationalize migration flows at $t = 0$. A key insight of dynamic-hat algebra is that time differences in migration flows reflect variations in location values, while constant fundamentals differentiate out. To illustrate this, by applying the same equilibrium condition at $t = -1$, migration flows at $t = 0$ can be rewritten in terms of time differences as follows:

$$\dot{\mu}_{in,0} = \frac{\exp\left[V_{n,1}(\tilde{L}_1, \Theta) - V_{n,0}(\tilde{L}_0, \Theta)\right]^{\beta/\nu}}{\sum_{h=1}^{N} \mu_{ih,-1} \exp\left[V_{h,1}(\tilde{L}_1, \Theta) - V_{h,0}(\tilde{L}_0, \Theta)\right]^{\beta/\nu}}. \tag{17}$$

Thus, expression (17) shows that by conditioning on observed initial migration flows and expressing gross flows as time differences, solving for the location values in these differences determines the entire migration flow path for all t without identifying migration costs. The key insight is that all relevant information about migration costs is captured in the initial period's observed flows, and since these fundamentals remain constant, time differences differentiate them out.

Similarly, all equilibrium conditions can be expressed in time differences. Let $\dot{x}$ denote the time difference of the variable x, specifically $\dot{x}_{t+1} = x_{t+1}/x_t$. To simplify the notation further, let $\dot{u}_{i,t+1} = \exp\left[V_{i,t+1}(\tilde{L}_{t+1}, \Theta) - V_{i,t}(\tilde{L}_t, \Theta)\right]$. The equilibrium conditions in time differences are then given by

$$\dot{u}_{i,t+1} = \left[\dot{w}_{i,t+1}/\dot{P}_{i,t+1}\right] \left[\sum_{n=1}^{N} \mu_{in,t} \left[\dot{u}_{n,t+2}\right]^{\beta/\nu}\right]^{1/\nu}, \tag{18}$$

$$\dot{\mu}_{in,t+1} = \frac{\left[\dot{u}_{n,t+2}\right]^{\beta/\nu}}{\sum_{h=1}^{N} \mu_{ih,t} \left[\dot{u}_{h,t+2}\right]^{\beta/\nu}}, \tag{19}$$

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}, \tag{20}$$

where $\dot{w}_{i,t+1}/\dot{P}_{i,t+1}$ solves for the static trade equilibrium in time differences at $t+1$, given by

$$\dot{\lambda}_{in,t+1} = \frac{\left[\dot{x}_{n,t+1}\right]^{-\theta}}{\sum_{h=1}^{N} \lambda_{ih,t} \left[\dot{x}_{h,t+1}\right]^{-\theta}}, \tag{21}$$

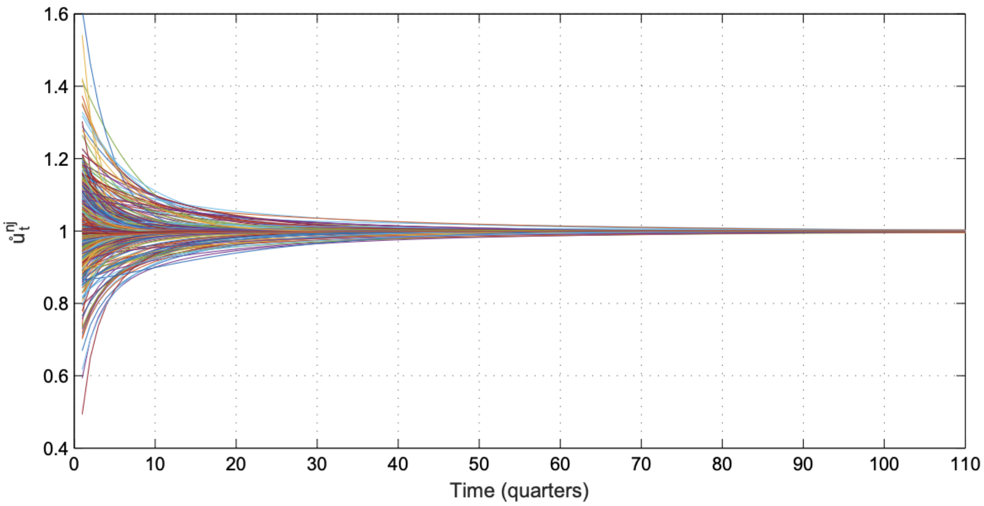
$$\dot{P}_{i,t+1} = \left[\sum_{n=1}^{N} \lambda_{in,t} \left[\dot{x}_{n,t+1}\right]^{-\theta}\right]^{-1/\theta}, \tag{22}$$

$$\dot{L}_{i,t+1} = \frac{\gamma_i \left(1 - \xi_i\right)}{\dot{w}_{i,t+1} w_{i,t} L_{i,t}} \sum_{n=1}^{N} \lambda_{ni,t+1} X_{n,t+1}, \tag{23}$$

where $X_{n,t+1} = (1 - \gamma_n) \sum_{n=1}^{N} \lambda_{ni,t+1} X_{n,t+1} + I_{n,t+1}$, and income is $I_{i,t+1} = \dot{w}_{i,t+1} \dot{L}_{i,t+1} w_{i,t} L_{i,t} + r_{i,t} H_i$.

The system of equilibrium conditions (18) to (23) shows that constant fundamentals differentiate out since $\dot{\Theta} = \{1, 1, 1, 1\}$. Solving in time differences requires conditioning on initial allocations: migration flows μ_{-1}, trade shares λ_0, value added $w_0 L_0$, and capital payments $r_0 H_0$ from the first period, which can be obtained from data. These observed allocations contain all relevant information on fundamentals (trade costs, migration costs, productivities, etc.). Therefore, solving the system in time differences does not necessitate identifying the set of fundamentals.

Another aspect of dynamic-hat algebra is its steady-state equilibrium, characterized by the solution $\dot{u}_{i,steady\ state} = 1$ for all i. This solves for both the agent's dynamic problem and trade equilibrium in the steady state, regardless of the level of the fundamentals. This property is advantageous because it shows that the steady-state equilibrium can be determined for any number of locations without computing location-specific steady states, which would require knowledge of all fundamentals and could become intractable as the number of locations increases. Additionally, since the system in time differences conditions on initial observed allocations, the model matches all cross-sectional moments for the initial period without assuming that the economy is in steady state. If the initial allocations indicate non-zero net flows in each location, the baseline economy will not be in steady state at that time, meaning $\dot{u}_{i,steady\ state} = 1$ for all i won't solve the system. This property is crucial for accurately computing transitional dynamics and studying the effects of initial conditions on equilibrium outcomes, which is relevant for many spatial topics. Figure 1 illustrates these properties by showing the evolution of changes in location values across over a thousand U.S. labor markets in the baseline economy with constant fundamentals from Caliendo et al. (2019). The figure shows that initial conditions lead to location-specific transitional dynamics, even with constant fundamentals, and that the location values in time differences converge to $\dot{u}_{i,steady\ state} = 1$ for all i.



Note: The figure presents the evolution of the value functions in time differences across U.S. labor markets in a baseline economy with constant fundamentals. A labor market is defined as the combination of a U.S. state and a sector, denoted by n and j, respectively. Source: Caliendo et al. (2019).

Figure 1: Equilibrium Location Values in Time Differences

To compute transitional dynamics with constant fundamentals, Caliendo et al. (2019) develop an iterative algorithm that can be sketched as follows. Conditional on the allocations in the initial period (μ_{-1}, λ_0, $w_0 L_0$, $r_0 H_0$) and the values of the trade and mobility elasticities, the process begins with an initial guess for the path of changes in values, such as $\dot{u}_{i,t} = 1$ for all i, t. This guess is then used to calculate the evolution of gross flows $\mu_{in,t+1}$ via (19). The initial allocation of agents $L_{i,0}$ and the path of gross flows determine the evolution of the spatial distribution of agents $L_{i,t+1}$, from which the time changes $\dot{L}_{i,t+1}$ are determined. Given $\dot{L}_{i,t+1}$, static trade equilibria at each time t can be computed using the Alvarez and Lucas (2007) and Caliendo and Parro (2015) algorithms. This results in a path of changes in real wages $\dot{w}_{i,t+1}/\dot{P}_{i,t+1}$, which is then used to generate a new path of changes in values $\dot{u}_{i,t+1}$ for all t. If this new path of values differs from the initial guess, the algorithm updates the guess and repeats the process.

Solving for the counterfactual economy. Suppose a researcher aims to study a counterfactual productivity path in a given location i. Let $\dot{x}^{'}$ be the time difference in the variable $x^{'}$ in the counterfactual economy, i.e., $\dot{x}^{'}_{t+1} = x^{'}_{t+1}/x^{'}_{t}$. The path of fundamentals is now given by $\Theta^{'}_t = \left\{A^{'}_{i,t}, A_{\neq i}, \kappa, \tau, H\right\}$ for all t, namely it contains the time-varying productivity path in the location of interest and all other constant fundamentals. In time differences, $\dot{\Theta}^{'}_t = \left\{\dot{A}^{'}_{i,t}, 1, 1, 1, 1\right\}$.

The equilibrium conditions for the counterfactual economy mirror those from (18) to (23) used for the baseline economy. The key difference is that $\dot{A}^{'}_{i,t}$ must be included in the system, affecting trade shares (21) and the price index (22) in locations sourcing goods from i.

Since the counterfactual productivity path is treated as given, the same approach used to compute transition dynamics in the baseline applies here. This includes conditioning on initial allocations and following the iterative algorithm previously discussed. The only difference is that the counterfactual economy requires assumptions about agents' expectations regarding the productivity path. In particular, Caliendo et al. (2019) assume perfect foresight. In the context of our example, at $t = 0$ agents do not expect a change in productivity in location i, and only subsequentally learn about the new productivity path. Under this timing assumption, the equilibrium gross flows (19) at t =0 do not hold, because agents are surprised by the productivity changes. The gross flows in the first period reflect the surprise in agents' expectations, namely the difference in location values at $t = 1$ under the changes in fundamentals, $V'_{n,1}(\tilde{L}'_1, \dot{\Theta}'_1)$, and the location values had the agents' expected fundamentals remained constant, $V_{n,1}(\tilde{L}_1, \Theta)$. Accordingly, it can be shown that migration flows at $t = 0$ in the counterfactual economy are given by $\mu'_{in,1} = \frac{\vartheta_{in,0}\dot{u}'_{n,2}}{\sum_{h=1}^{N}\vartheta_{ih,0}\dot{u}'_{h,2}}$, where $\vartheta_{in,0} = \mu_{in,0}\exp\left[V'_{n,1}(\tilde{L}'_1, \dot{\Theta}'_1) - V_{n,1}(\tilde{L}_1, \Theta)\right]^{\beta/\nu}$.

2.2.1.3 Dynamic-Hat Algebra with Time-Varying Fundamentals: Ex-post Counterfactuals

Ex-post counterfactuals study shocks that have already occurred, such as the impact of China's productivity growth on the U.S. regions or the economic effects of past EU enlargements. For example, suppose a researcher intends to study the effects of historical productivity growth in a specific location. To do so, the researcher analyzes observed outcomes, such as changes in employment, and seeks to understand how these would differ if productivity growth in that location had not occurred. Unlike ex-ante counterfactuals, researchers cannot assume that other fundamentals remain constant since they have already unfolded. The central question here is: how would the economy have evolved if all fundamentals had evolved as they did in the data, except for the productivity change under consideration? In what follows, we refer to ex-post counterfactuals as counterfactuals with time-varying fundamentals.

To study the effects of a past shock to economic fundamentals, building on the previous example, a researcher observes economic outcomes (e.g., employment changes) in a specific location and aims to uncover a counterfactual scenario that reflects outcomes resulting solely from productivity shifts in that location. The observed changes may arise from changes in multiple economic fundamentals, including trade and migration costs, as well as productivity changes across various locations. To isolate the impact of the productivity changes, the researcher must disentangle its effects from the influences of other fundamentals on the outcomes.

The challenge in conducting counterfactual analysis in this scenario lies in the need to account for all time-varying fundamentals, which requires identifying their evolution. Moreover, computing the sequential equilibrium described in the previous subsection does not effectively differentiate these fundamentals. Caliendo et al. (2019) extend the dynamic-hat algebra method for counterfactual analysis involving time-varying fundamentals. In the context of our example, where the researcher studies the effects of a realized productivity change in a location, the dynamic-hat algebra compares a baseline economy with a counterfactual economy. Unlike the counterfactuals with constant fundamentals discussed earlier, the baseline economy now includes all time-varying fundamentals (trade costs, migration costs, productivity), while the counterfactual economy excludes the productivity changes in the location of interest. The difference between these economies reveals the impact of the productivity change the researcher aims to study.

Conducting counterfactuals with time-varying fundamentals relies on two key insights. First, the baseline economy uses data under the perfect foresight assumption, as the observed time series contains all information about the actual path of time-varying fundamentals. Second, when solving the counterfactual economy relative to the baseline, all time-varying fundamentals—except for the specific productivity change under study—will differentiate out. This second insight is akin to a structural difference-in-differences approach. The baseline economy reflects the actual path of all fundamentals, while the counterfactual economy is identical except that it excludes productivity changes in the location of interest. Thus, the relative change between the two economies isolates the effect of the productivity path in the location being analyzed, differentiating out all other time-varying fundamentals.

To illustrate these insights, and following the analogous illustration described with constant fundamentals, denote by $\hat{x}_{t+1}$ the relative change between the counterfactual economy and the actual (baseline) economy, both expressed in time differences, namely $\hat{x}_{t+1} = \dot{x}'_{t+1}/\dot{x}_{t+1}$. Using this notation, the change in gross flows between the counterfactual economy and the baseline economy at any time t is given by

$$\hat{\mu}_{in,t} = \frac{[\hat{\tau}_{in,t}]^{-1/\nu} [\hat{u}_{n,t+1}]^{\beta/\nu}}{\sum_{h=1}^{N} \mu'_{ih,t-1} \dot{\mu}_{ih,t} [\hat{\tau}_{ih,t}]^{-1/\nu} [\hat{u}_{h,t+1}]^{\beta/\nu}}. \tag{24}$$

In the counterfactual gross flows (24), the term $\dot{\mu}_{ih,t}$ represents the change in migration flows in the baseline (observed) economy. Furthermore, since in the example the researcher is studying a productivity change in a specific location, and migration costs evolve identically in both the baseline and counterfactual economies, the researcher can set $\hat{\tau}_{in,t} = 1$ in condition (24). All relevant information about the actual path of fundamentals in the counterfactual migration flows $\mu'_{in,t}$ is contained in the evolution of migration flows in the actual economy, represented by $\dot{\mu}_{ih,t}$. It follows that the set of fundamentals in hat-changes at t is given by $\hat{\Theta}_t = \left\{\hat{A}_{i,t}, 1, 1, 1, 1\right\}$ as the path of time-varying fundamentals in the baseline and counterfactual economies is the same, except for the productivity path in location i.

The remaining equilibrium conditions (18) to (23) can similarly be expressed in hat-changes, specifically comparing the counterfactual to the baseline economy. Thus, dynamic-hat algebra with time-varying fundamentals solves the counterfactual economy relative to the baseline, conditioning on the actual time series that reflects the baseline economy with time-varying fundamentals, rather than initial allocations as in the previous subsection. After the last data point, the remaining trajectory of the baseline economy can be simulated with constant fundamentals, as described in the previous subsection. The other properties of dynamic-hat algebra discussed earlier still hold with time-varying fundamentals. In hat-changes, the equilibrium reaches a steady state characterized by $\hat{u}_{i,steady\ state} = 1$ for all i. The model in hat-changes conditions on data without assuming that the economy is initially in a steady state, and the iterative algorithm to compute transitional dynamics follows the same steps as before, but iterates over $\hat{u}_{i,t}$ instead of $\dot{u}_{i,t}$ for all i, t. Caliendo et al. (2019) discuss the types of ex-ante and ex-post questions that can be answered using the dynamic hat algebra methodology.[9] We now turn to briefly describe the application of dynamic hat algebra in Caliendo et al. (2019) to study the impact of the China shock on U.S. local labor markets.

2.2.1.4 The Impact of Import Competition on U.S. Local Labor Markets

Caliendo et al. (2019) apply their framework and the dynamic-hat algebra methodology to study the effects of increased import competition due to China's trade expansion (i.e., the China shock) on U.S. local labor markets. In particular, to apply dynamic-hat algebra, the authors follow the steps discussed in the previous subsection and compute the counterfactual economy with constant Chinese productivity, relative to the baseline economy, which reflects the actual path of fundamentals contained in the data from 2000 to 2007. The authors rely on the identification restriction suggested by Autor et al. (2013) to measure China's trade shock and use their model to compute the changes in sectoral productivities in China between 2000 and 2007 that generate the same change in U.S. imports from China as predicted by the Autor et al. (2013) regression. Among other results, the authors find that increased Chinese competition reduces the aggregate manufacturing employment share by 0.36 percentage points in the long run, equivalent to a reduction of about 0.55 million manufacturing jobs, or about 16 percent of the observed decline in manufacturing employment from 2000 to 2007. The authors also find that the United States experiences aggregate welfare gains, but due to trade and migration frictions, the welfare and employment effects vary across U.S. labor markets. The authors also show that the differential employment effects across local labor markets implied by their model align with those in Autor et al. (2013).

We now turn to discussing recent work that has extended this framework with forward-looking location decisions by incorporating other important economic mechanisms, such as capital accumulation, asset accumulation, human capital accumulation, idea diffusion, and stochastic fundamentals.

2.2.2 Forward-Looking Capital Accumulation

The dynamic spatial framework described in the previous subsection assumes that capital structures are a fixed factor in each location. A natural question to ask is how capital accumulation across locations shapes mobility decisions and the evolution of economic activity across space. Addressing this question requires extending the previous dynamic spatial framework to incorporate forward-looking

[9] See also the online appendix in Caliendo et al. (2019). Additionally, a MATLAB code and a readme file, including an example algorithm to compute counterfactuals with constant fundamentals in an economy with four sectors, 50 U.S. states, 37 countries, and the rest of the world, can be downloaded from the authors' webpages.

capital accumulation decisions. Capital accumulation has a long tradition in the macroeconomic literature and has more recently been introduced in dynamic models of international trade (e.g., Eaton et al., 2016; Alvarez, 2017; Ravikumar et al., 2019; Anderson et al., 2019). The crucial distinctions in dynamic spatial models are the large state space and the mobility of agents across locations. These two features are not innocuous for the tractability of the framework. Intuitively, if agents who accumulate capital also move across space, locations will feature agents with different mobility histories, leading to different capital accumulation decisions and, consequently, different mobility decisions (beyond their heterogeneity in idiosyncratic taste shocks). Therefore, characterizing mobility decisions would require tracking individuals' mobility histories, which quickly results in a prohibitively large state space for empirically realistic numbers of locations.

Kleinman et al. (2023) build on Angeletos (2007) and Moll (2014) to introduce forward-looking capital accumulation into the dynamic spatial framework described in Section 2.2.1. Each location features landlords who are geographically immobile but make forward-looking capital accumulation decisions by having access to an investment technology in local capital, such as buildings and structures. Importantly, the tractability of the framework stems from the immobility of landlords, or in other words, the separability of capital accumulation decisions from mobility decisions.

In particular, immobile landlords in each location choose their consumption and investment to maximize their intertemporal utility, subject to their budget constraint. The expected present discounted value of their flow utility is given by

$$v_{i,t}^{k} = \sum_{s=0}^{\infty} \beta^{t+s} \frac{[c_{i,t+s}^{k}]^{1-1/\psi}}{1-1/\psi}.$$

Here, the superscript k denotes landlords, $c_{i,t}^{k}$ represents the consumption basket, β is the discount rate, and ψ is the intertemporal elasticity of substitution. The investment technology in each location is in units of local goods, and capital structures depreciate at a constant rate δ. Landlords maximize their intertemporal utility subject to the following budget constraint,

$$r_{i,t}k_{i,t} = p_{i,t}\left[c_{i,t}^{k} + k_{i,t+1} + (1-\delta)k_{i,t}\right].$$

This budget constraint states that the revenues from renting capital to firms must equal the total value of the landlords' consumption plus the total value of net investment. Solving this maximization problem yields the landlords' optimal consumption-investment decisions:

$$c_{i,t} = \varsigma_{i,t} R_{i,t} k_{i,t}, \tag{25}$$

$$k_{i,t+1} = (1-\varsigma_{i,t}) R_{i,t} k_{i,t}, \tag{26}$$

where $R_{i,t} = 1 - \delta + r_{i,t}/p_{i,t}$, and $\varsigma_{i,t}^{-1} = 1 + \beta^{\psi}\left[R_{i,t+1}^{\frac{\psi-1}{\psi}} \varsigma_{i,t+1}^{-\frac{1}{\psi}}\right]^{\psi}$.

As shown in this optimal policy, landlords have a linear saving rate $[1-\varsigma_{i,t}]$, as in Angeletos

(2007), which is endogenous and forward-looking. For log-utility, the saving rate is a constant fraction of capital rents, as in Moll (2014), namely $k_{i,t+1} = \beta R_{i,t} k_{i,t}$. In steady state, the real rental rate is constant across locations, namely $(r_i/p_i)^{\text{steady state}} = [1 - \beta [1 - \delta]] / \beta$ for all i. However, since landlords across locations have different incentives to accumulate capital during the transition, the capital-labor ratio remains heterogeneous across space in steady state.

The rest of the framework has a structure similar to that in Caliendo et al. (2019). The sequential equilibrium in Kleinman et al. (2023) is analogous to the one described in Section 2.2.1, with the addition of the equilibrium condition in equation (26). Once this condition is expressed in relative time differences, which requires conditioning on real rental rates across locations, the dynamic-hat algebra can be applied as previously discussed. Linearizing the model yields a closed-form solution for the economy's transition path, in terms of an impact matrix that captures the initial impact of shocks and a transition matrix that governs the updating of the state variables. The paper characterizes various properties of the model, such as the speed of convergence, and provides a sufficient condition for the existence and uniqueness of the steady-state equilibrium.

Kleinman et al. (2023) apply their framework to examine the determinants of income convergence across U.S. states from 1965 to 2015, as well as the persistent and heterogeneous impact of local shocks. They find that the declining rate of income convergence across U.S. states is primarily driven by initial conditions that shape the subsequent transitional dynamics of labor distribution and capital accumulation, rather than by changes in economic fundamentals such as state-level productivity, amenities, trade costs, and migration frictions that occurred from 1965 to 2015. Using spectral analysis, the authors show that the speed of convergence to steady state is influenced by the interplay between capital accumulation and migration. Specifically, convergence toward steady state is slow when the gaps in capital and labor from the steady state are positively correlated across locations.

Following the approach of Allen et al. (2024), Kleinman et al. (2023) provide a sufficient condition for the existence of a unique steady-state equilibrium in the model with forward-looking migration decisions and capital accumulation. They also show that this condition holds in the empirical version of their model. The proof focuses on the properties of a matrix denoted as A, which contains the power coefficients—such as input shares, trade and migration elasticities, and discount factors—of the steady-state equilibrium conditions. Specifically, a sufficient condition for the existence of a unique steady-state equilibrium is that the spectral radius of the coefficient matrix A of the model parameters is less than or equal to one. Additionally, the authors show that the linearized system has a unique stable transition path.

Allen and Donaldson (2022) explore the full non-linear properties of a model that incorporates forward-looking migration, agglomeration, and congestion forces. They provide sufficient conditions for achieving both unique dynamic and steady-state equilibria. Furthermore, they characterize the parameter restrictions—particularly the relative magnitudes of agglomeration and dispersion forces—that ensure uniqueness.

2.2.3 Idea Diffusion through Trade and Migration

In the dynamic spatial frameworks described in the previous subsections, transition dynamics are shaped by forward-looking migration and capital accumulation decisions, as well as by an exogenous path of productivities across locations. To study how forward-looking migration, capital accumulation, and trade impact productivity and growth across locations, Cai et al. (2023) develop a framework of spatial growth that incorporates idea diffusion through trade and migration.

The authors first provide causal evidence of the impact of trade and migration on local knowledge. Using microdata from China, migrants from more productive provinces are shown to contribute more to the local stock of knowledge (proxied by patent stocks and flows) than migrants from less productive locations. In addition, local knowledge grows more in provinces that are more open to imports. These findings on migration and knowledge align with recent empirical evidence on the impact of immigrants on innovation and growth in the United States (Burchardi et al., 2020) and on the role of migration in disseminating knowledge across Brazilian regions (Pellegrina and Sotelo, 2024).

Guided by these empirical insights, the authors develop a dynamic spatial model in which spatial growth is driven by the endogenous evolution of total factor productivity, resulting from the diffusion of global and local ideas. Ideas spread across locations through trade and migration. Migrating workers learn local ideas and diffuse knowledge when interacting with producers at their destinations. Global ideas, embedded in imported intermediate goods, diffuse more to areas with higher exposure to international trade.

To keep the idea diffusion process tractable, it is modeled as a stochastic process. Specifically, ideas arrive stochastically to each producer, and their productivity is a combination of a random original component, a random insight drawn from the ideas of sellers to that location, and a random insight drawn from workers in that location. Formally, the productivity of a new idea, denoted by q, is given by $q = z q_\ell^{\rho_\ell} q_m^{\rho_m}$, where z is the original component, q_ℓ are insights from workers, and q_m are insights from imported goods.

Under the assumptions that the distribution of original ideas is Pareto, the number of new ideas arriving between t and $t+1$ follows a Poisson distribution with mean $\Lambda_t = \alpha_t \bar{z}^{-\theta}$, the distribution of original ideas has a sufficiently thin tail, and trade and migration are two independent sources of insights, the local stock of knowledge in location n, denoted by $A_{n,t+1}$, evolves over time as

$$A_{n,t+1} - A_{n,t} = \alpha_t \int_0^\infty \int_0^\infty \left(q_\ell^{\rho_\ell} q_m^{\rho_m} \right)^\theta dG_{n,t}^\ell(q_\ell) G_{n,t}^m(q_m). \tag{27}$$

Expression (27) states that the knowledge stock in location n grows at a faster rate if ideas arrive faster or if the location benefits from better sources of insights from workers and sellers. Under the assumptions stated above, the source distribution of insights from migrants is given by $G_{n,t}^\ell(q_\ell) = \sum_{i=1}^N s_{in,t} F_{i,t}(q_\ell)$, where $s_{in,t} = \frac{\mu_{in,t} L_{i,t}}{\sum_{h=1}^N \mu_{hn,t} L_{h,t}}$ is the share of workers in location n who arrived from i at the end of period t, and $F_{i,t}(q_\ell)$ is the probability that the best productivity for a variety is no greater than q at time t. The authors refer to $F_{i,t}(q_\ell)$ as the frontier of knowledge. The source of insights from trade is analogous to Buera and Oberfield (2020).

Under these mechanisms for idea diffusion, the law of motion of the stock of knowledge across locations $A_{n,t}$ can be written as a difference structural equation given by

$$A_{n,t+1} - A_{n,t} = \alpha_t \Gamma_{\rho_\ell,\rho_m} \underbrace{\left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t}\right)^{\rho_\ell}\right]}_{people} \underbrace{\left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}\right]}_{goods}. \tag{28}$$

Here, the evolution of the stock of the stock of knowledge in each location depends on the arrival rate of new ideas, α_t, the exposure to ideas from migrants, $s_{in,t}$, and the exposure to trade, $\lambda_{ni,t}$. The diminishing returns to technological improvement from insights implies that in the balanced growth path the economy grows at the exogenous growth rate of α_t, a feature shared by semi-endogenous growth models in Buera and Oberfield (2020), Jones (1995), Kortum (1997), and Atkeson and Burstein (2019), and consistent with the evidence in Peters (2022) discussed later in this chapter. The idea diffusion process across locations shares similarities with the endogenous growth frameworks discussed in Section 2.1 (see also Berkes et al., 2025). The framework in Cai et al. (2023) considers agents' forward-looking decisions and endogenous idea diffusion through trade and migration but abstracts from endogeneous innovation decisions.[10]

The authors then embed this idea diffusion process into a spatial framework with forward-looking migration and capital accumulation decisions, as in Caliendo et al. (2019) and Kleinman et al. (2023). Crucially, the productivity across locations that affects trade shares (13) and price indices (14) in the framework described in Section 2.2.1 now evolves endogenously according to the structural equation (28).

Cai et al. (2023) show how to apply dynamic-hat algebra without requiring that the economy be on a balanced growth path initially. This aspect is crucial to examine the role of initial conditions in shaping the transition dynamics of the economy. The authors also estimate the idea diffusion parameters ρ_ℓ and ρ_m. Applying their framework to study how idea diffusion through trade and migration shaped China's growth in the 1990s and 2000s, they find that initial conditions significantly influenced China's growth trajectory. The economic fundamentals of 1990 set the stage for high growth in subsequent decades. In the 1990s, idea diffusion and capital accumulation contributed equally to growth. By the 2000s, idea diffusion became more important as trade openness and worker mobility facilitated knowledge diffusion. Reforms in trade costs and migration after 1990 had only a minor impact on growth compared to these initial conditions. Furthermore, the paper shows that idea diffusion through trade and migration is crucial for understanding the observed heterogeneity in spatial growth during China's economic transition in the 1990s and 2000s.

[10] In the appendix, Buera and Oberfield (2020) show how to accommodate innovation in their model with idea diffusion and trade.

2.2.4 Endogenous Wealth Accumulation

In the dynamic spatial frameworks discussed in the previous subsections, agents make forward-looking location choices but consume their income, meaning they do not solve a consumption-savings decision or accumulate wealth. However, location decisions may depend on individuals' wealth and consumption-saving decisions. For example, using administrative data for France, Bilal and Rossi-Hansberg (2021) document that, conditional on location, income, occupation, and age, low-wealth individuals (likely to be financially constrained) on average move to lower-ranked locations (those offering lower wages but cheaper rents) following negative income shocks, while high-wealth individuals do not adjust their location much. Low-wealth individuals who downgrade their location do not adjust their holdings of financial assets, consistent with being close to the credit constraint. In contrast, unconstrained individuals smooth consumption by adjusting their financial assets.

To rationalize these empirical findings, Bilal and Rossi-Hansberg (2021) develop a dynamic infinite-horizon model incorporating consumption-saving decisions, endogenous wealth accumulation, and location choices. Agents optimize their intertemporal consumption path by choosing their holdings of financial assets and what the authors coin as the "location asset". In particular, the model interprets the individual's problem as a standard portfolio choice problem in which the risk-free bond is subject to a borrowing constraint, and the return to the location asset is endogenous and depends on the individual's skill. In equilibrium, consistent with the empirical evidence, financially unconstrained individuals use their financial assets to smooth consumption after a negative temporary income shock. Once they reach the borrowing constraint, they start borrowing using the location asset; namely, they downgrade their location in order to minimize fluctuations in their level of consumption.

Giannone et al. (2023) also find evidence of individual location decisions varying with wealth determinants, using microdata for Canada. Their key empirical result shows that migration propensities decline monotonically with the ability to borrow.

They develop a quantitative life-cycle spatial equilibrium model incorporating wealth accumulation and forward-looking migration decisions. In this model, overlapping generations of risk-averse households, facing idiosyncratic income shocks, accumulate wealth through liquid and illiquid assets (housing), subject to a borrowing constraint. Each period, households make forward-looking decisions about location, tenure status (own or rent), consumption, and savings. Moving is subject to monetary and utility costs, assumed to be homogeneous across locations. Wages, house prices, and rental prices are determined endogenously. Following our discussion in Section 2.2.2, adding endogenous wealth accumulation shaped by housing ownership complicates tractability, as agents who accumulate wealth also move across space. To address this, the authors assume that homeowners trade houses every period, even when staying in the same house. This avoids the need to track individuals carrying assets across space, which would otherwise make the state space intractably large.

The model's main mechanism closely relates to that in Bilal and Rossi-Hansberg (2021). Agents facing a negative shock weigh the value of staying in their current location against the cost of moving. The value of staying depends on their ability to smooth the shock using savings or accessing financial markets, while moving offers better opportunities but comes at a cost. In equilibrium, low-wealth households with limited borrowing ability find moving more valuable. The ability of financially constrained individuals to smooth consumption by relocating to cheaper locations is consistent with the empirical evidence.

In related work, Dvorkin (2023) develops a tractable spatial equilibrium model with location and wealth choices. The author uses dynamic-hat algebra to solve a model with simultaneous location and wealth decisions, employing log-linear value functions that make migration elasticities independent of wealth. Greany (2023) also proposes a model with similar components and mechanisms to study how uneven regional growth affects wealth inequality. The framework is in partial equilibrium, taking the interest rate as exogenous. To maintain tractability, the model is formulated in continuous time and implements the finite differences algorithm in Achdou et al. (2021). However, as the number of locations becomes large, the efficiency of this algorithm decreases, making computation prohibitively slow. To regain tractability, Greany (2023) assumes migration only occurs at a finite number of ages known ex-ante (shock ages). This separates the timing of migration from household consumption-savings decisions.

2.2.5 Human Capital Accumulation and Spatial Growth

When studying spatial growth, an important question is how individuals accumulate human capital across space and how the spatial distribution of human capital shapes aggregate growth. This raises the question of where individuals accumulate human capital at a faster rate. The challenge in answering this question is that human capital, unlike physical assets, is embodied in each individual. As a result, it cannot be accumulated by someone else, like the rentier in Section 2.2.2. Therefore, incorporating human capital accumulation into dynamic spatial frameworks requires solving the problem of individuals who migrate and accumulate human capital, with the rate of the latter depending on the destination choice of the former.

Crews (2023) develops a spatial endogenous growth model in continuous time with a discrete number of cities. In this framework, heterogeneous agents make forward-looking migration decisions and human capital investments over the life cycle. At the core of the local law of motion of human capital there is a local externality: individuals learn from others and take the level of human capital in each city as given when evaluating their return on investing in human capital. Additionally, individuals learn more in larger cities and in more skilled cities. The evolution of human capital in Crews (2023) nests the specification of the standard Ben-Porath (1967) model.

Agent's location and human capital decisions are expressed as a Hamilton-Jacobi-Bellman equation, and the state of the economy evolves according to a Kolmogorov forward equation, shaping the general equilibrium of the economy. The paper defines a balanced growth path as a scenario where the growth rate and city size distribution remain constant over time. Under this characterization, the spatial distribution of human capital impacts aggregate growth. To solve for the balanced growth path, the paper adapts a fixed-point algorithm from Achdou et al. (2021). A calibrated version of the model using data from U.S. metropolitan areas shows that it can rationalize the faster wage growth of workers in big cities, as well as life-cycle wage profiles. Through the lens of the model, policies that further concentrate skilled workers in large cities are growth-enhancing, as individuals accumulate human capital more rapidly there.

Overall, Crews (2023) takes a step forward in incorporating human capital accumulation into dynamic spatial models by presenting a continuous-time framework that allows for tracking the distribution of the state space. Future research should explore whether this formulation is also tractable enough to solve for the transition path.

2.2.6 Stochastic Fundamentals

In the dynamic spatial frameworks discussed in previous subsections, the spatial effects of changes in economic fundamentals are shaped by forward-looking agents who perfectly anticipate the future trajectory of these fundamentals. However, the macroeconomic literature has extensively studied how uncertainty and beliefs about future economic fundamentals influence agents' forward-looking decisions and economic outcomes. In this subsection, we discuss recent work that has incorporated stochastic fundamentals into dynamic spatial frameworks.

In the absence of perfect foresight, it is the agents' beliefs about future fundamentals—rather than the actual trajectory—that drive their forward-looking decisions and, consequently, shape economic outcomes. Fan et al. (2023) integrate evolving beliefs and uncertainty about future fundamentals across locations into a dynamic spatial framework with forward-looking location choices and a general equilibrium gravity trade structure.

In their model, the world is composed of many locations, and location-specific productivities follow a stochastic process. Importantly, agents' beliefs about these stochastic processes can be uncertain, evolve over time, or be heterogeneous across agents. Forward-looking agents might not know the true stochastic process governing productivities and instead base their decisions on their beliefs about future productivity. Beyond nesting the case of rational expectations, beliefs in their framework can capture different cases by specifying how they vary with new information. For example, agents' learning can be Bayesian or myopic; regardless of the case, agents could be naive learners, or they might be sophisticated and understand that they will revise their beliefs as new data arrive.

To solve this stochastic spatial framework, Fan et al. (2023) implement a local approximation around a deterministic path with perfect foresight, rather than around a steady state. This approach captures important aspects for conducting counterfactual analysis in spatial economies, such as not relying on the existence of a steady state or assuming that the economy is in (or close to) a steady state. Additionally, an approximation path accommodates other time-varying fundamentals that are not the primary focus of the counterfactual analysis.

The authors build on this solution method to address two main challenges. The first challenge is how to introduce uncertainty around a path of fundamentals. The second is how to recover counterfactual allocations when observed outcomes result from the forward-looking decisions of agents with imperfect beliefs about future fundamentals.

The main challenge in introducing uncertainty into dynamic spatial frameworks is solving the model with many locations (a large state space) and multiple sources of aggregate uncertainty. A natural starting point for introducing uncertainty is to rely on standard macroeconomic methods (e.g., Sims, 2003; Schmitt-Grohé and Uribe, 2004), which relate policy rules to the state of the economy and approximate them around the steady state. Recent research has used this approach to incorporate heterogeneous agents in models with multiple locations (Bhandari et al., 2023; Bilal, 2023). Relatedly, Egger et al. (2024) solve a dynamic spatial framework with rational expectations as a mean field game in discrete time, over a discrete state space, preserving the full nonlinear structure of the problem while imposing AR(1) stochastic processes across locations.

Alternatively, Fan et al. (2023) adopt a sequence-space representation of the economy (i.e., in the space of sequences of endogenous outcomes). A key advantage of this approach is its ability to handle non-stationary productivity and history-dependent beliefs without expanding the state space. Additionally, the derivatives of the decision rules (first, second, and higher-order) are closed-form expressions of equilibrium outcomes along the approximation path. This feature eliminates the need for numerically computing these derivatives, as is typically required in standard perturbation methods. It also allows simulating the model's solution a large number of times, computing second-order terms, and solving the model with second-order accuracy for an arbitrary number of locations and location-specific stochastic processes.

In addition, in Section 2.2.1.3 we discussed how to recover counterfactual outcomes using dynamic-hat algebra when the data is generated by forward-looking agents who perfectly anticipate the future trajectory of fundamentals. A second challenge addressed in Fan et al. (2023) is to show how to recover counterfactual outcomes when the data are generated by beliefs about future fundamentals that did not materialize.

To do so, they propose a recursive algorithm to recover the expected path of endogenous outcomes that rationalizes observed outcomes. This algorithm relies on two key insights. First, it builds on dynamic-hat algebra, as the deviations between expected and realized fundamentals allow for the recovery of deviations between realized outcomes and expected future outcomes, accounting for other time-varying fundamentals. Second, it leverages the fact that the expected path of endogenous outcomes solves the forward-looking agents' maximization problem, thereby rationalizing the actual migration decisions. In doing so, the authors discipline beliefs about fundamentals so that the expected paths align with external data that provide insights into agents' beliefs (e.g., employment forecasts by statistical agencies).[11] Having recovered the expected path of endogenous

[11] See also Porcher (2022), who imposes structure migration costs to identify agents' beliefs from migration decisions. Also, related to studies by Dickstein and Morales (2018), Bombardini et al. (2023), and Porcher et al. (2024), recover agents' beliefs based on the agents' decisions in partial equilibrium trade or spatial frameworks.

outcomes that rationalizes the observed outcomes (the data-generating process), deviations from these paths yield the counterfactual outcomes. The authors also show how to extend this approach to settings with heterogeneous beliefs by imposing some structure on higher-order beliefs.

Fan et al. (2023) present two applications of their stochastic spatial framework. In the first application, they find that forward-looking agents gradually learned about China's productivity growth in the 2000s. Notably, they show that if researchers had assumed data were generated under perfect foresight, the decline in U.S. manufacturing employment would have been larger, and U.S. welfare gains smaller. The differences in these findings are due to large variations in inferred migration costs and the imperfect beliefs about the China shock. The second application is an evaluation of the impacts of rising temperatures over 2014-2100 on welfare and the spatial allocation of economic activity in the United States. Compared to perfectly anticipated climate change, their results reveal that uncertainty leads to substantially larger welfare losses in the United States, and a moderately faster reallocation of individuals from southern U.S. locations. They also find that the presence of climate skeptics slows down spatial reallocation, resulting in an increase in the welfare of individuals in the North.

Although dynamic stochastic general equilibrium models have a long tradition in macroeconomics, incorporating stochastic fundamentals into dynamic spatial frameworks is an emerging and promising research area with significant potential for future research.

2.3 Relative Strengths of Different Approaches

The dynamic spatial framework with endogenous innovation discussed in Section 2.1 and the framework with forward-looking agents discussed in Section 2.2 both achieve a level of realism that was lacking in early spatial models, particularly in their ability to accommodate the spatial heterogeneity and complex geography observed in the data. While both approaches can be used for quantitative evaluations across a wide range of settings, we conclude this section by highlighting some relative strengths of each approach in specific contexts.

Using a spatial endogenous growth model that simplifies the forward-looking migration and innovation decisions to a sequence of static problems is useful when the spatial resolution to be analyzed is very high, and endogenous growth and innovation are central to the problem at hand. A good example are spatial integrated assessment models (S-IAM) that analyze the economic impact of climate change at a high spatial resolution (for an overview of such models, see Desmet and Rossi-Hansberg, 2024). The reason for needing a very large number of locations is two-fold: first, there is substantial spatial heterogeneity, even at the relatively local level, in terms of vulnerability to global warming, so we need high resolution models; second, climate change is a global phenomenon, so it is necessary to model the interaction between the economy, emissions and temperature at a global scale. Combining the global scale with a high spatial resolution yields models with a large number of locations. As a reference point, the world is made up of 64,800 $1^\circ \times 1^\circ$ cells, of which around 19,000 are on land. We already mentioned why endogenous innovation is a key adaptation mechanism to climate change. As certain locations lose and others gain from global warming, endogenous innovation will play a key role in the emergence of new economic clusters.

Another question when evaluating the relative merits of different modeling strategies relates to the data requirements. A good example are bilateral migration costs. Suppose we have N locations. To estimate a full set of bilateral migration costs one would need to estimate N^2 parameter values. Unfortunately, there are no good bilateral migration flow data at a high resolution for the entire globe. In contrast, using the assumptions in Desmet et al. (2018) that simplify the dynamic migration decision into a sequence of static decisions, it is enough to have data on changes in population at the local level. Recall that in this setup the bilateral migration costs between r and s are the product of the cost of leaving r, $m_1(r)$, and the cost of entering s, $m_2(s)$. In addition, $m_1(r) = m_2(r)^{-1}$. Hence, researchers only need to estimate N parameters. This is why having data on the population change between two time periods for each location is enough.

For other questions, it might be crucial to consider agents' forward-looking decisions. That is, agents account for future payoffs in their dynamic decisions. For example, mobility decisions are often not solely based on arbitraging current wage differentials but also factor in the option value of remaining in a location. This means that welfare can sometimes move in the opposite direction of wages (see, e.g., Artuc et al., 2010). This perspective aligns with several applications that will be presented in Section 3, such as the study of expected changes in import competition in Caliendo et al. (2019), the impact of anticipated changes to migration policy, as discussed in Caliendo et al. (2021), the importance of accounting for the dynamic effects of environmental change when deciding where to allocate infrastructure today (e.g., Balboni, 2025), and the role of agents' beliefs in understanding the spatial effects of the China shock in the United States (e.g., Fan et al., 2023), which was discussed in the previous subsection.

As discussed earlier, using dynamic-hat algebra requires conditioning on data, such as gross migration flows, which do not exist at a very high spatial resolution (e.g., $1° \times 1°$ cells). This might constrain its application to questions where such a level of granularity is relevant. The availability of more micro-level data in the future will allow for the construction of observable allocations, such as migration flows, trade, and production data, at an even higher spatial resolution. On the other hand, as discussed in Section 2.2, conditioning on observable allocations with dynamic-hat algebra allows for conducting counterfactual analysis while remaining agnostic about the structure of a potentially large set of economic fundamentals across locations. It also allows for the computation of transitional dynamics in models with forward-looking decisions, without assuming that the economy is in a steady state, which is important for studying many empirical questions, as discussed in the preceding paragraph.

3 Applications of Dynamic Spatial Models

Many empirical phenomena have both a time and a spatial dimension. In this section, we discuss many applications, including climate change, trade and migration policy, local labor market effects of import competition, structural change and infrastructure investment.

3.1 Climate Change

To convincingly evaluate the economic impact of climate change, we need high-resolution dynamic models that cover the entire globe. Having high spatial resolution is important because global warming is a spatial phenomenon. It matters whether you reside in the equatorial regions of sub-Saharan Africa, where it is already very warm, or in northern Scandinavia, where temperatures are cold. This spatial heterogeneity in vulnerability to climate change is also present at smaller geographic scales. Miami or Houston are much more vulnerable to coastal flooding than locations even a few miles more inland. Similarly, temperature differences can be quite substantial over relatively short distances, such as when comparing San Francisco to close by cities away from the Pacific Ocean. Taking a global perspective is essential as well, because global temperatures depend on the amount of carbon in the atmosphere, regardless of where the emissions originated.

As for dynamics, allowing for endogenous innovation that can reshape the economic geography of the world is important in the context of climate change. On the one hand, global warming is a gradual and protracted phenomenon that will play out over a long period of time, measured in decades, and possibly even centuries. Hence, focusing on long-run growth and development with a spatial angle is bound to be important. On the other hand, as climate change hurts the productivity of equatorial regions and coastal cities, there will be reallocation of economic activity and population across space. As certain economic clusters decline, others will emerge. As this occurs, new local markets will provide incentives for firms to innovate, and over time these new clusters will gain in productivity and will thrive. Allowing for these types of dynamics playing out over space seems essential to any realistic evaluation of the economic impact of climate change.

Bringing climate into dynamic spatial models. Before discussing some applications of dynamic spatial models to climate change, we briefly describe how to bring climate change into these models. Starting in the 1990s, Nordhaus pioneered the use of integrated assessment models (IAM). These combine standard economic models with the main insights of climate science with the goal of evaluating the economic impact of global warming (Nordhaus, 1993, 2008, 2010). However, these models lack spatial heterogeneity. As a result, they cannot account for the spatial variation in the impact of climate change and they are unable to evaluate how migration and trade might act as adaptation mechanisms.

In the past decade, advances in high-resolution dynamic spatial models have facilitated the emergence of spatial integrated assessment models (S-IAM). These build on earlier integrated assessment models, with the difference that they incorporate both high spatial resolution and endogenous innovation. This is accomplished by combining a model similar to the one described in Section 2.1 with the basic interaction between the economy and climate found in the Nordhaus models. On the one hand, temperature affects the economy in two ways. First, productivity is made dependent on temperature—a relationship referred to as the damage function in IAMs. Specifically, the average productivity draw $Z_t(r)$ described in equations (2) and (29) now depends on temperature:

$$Z_t(r) = \tau_t(r)\, g(T_t(r)) \left(\frac{L_t(r)}{H_t(r)}\right)^{\alpha} \tag{29}$$

where $g(T_t(r))$ is the damage function—a productivity discount that applies to productivity. Conte et al. (2021) model $g(T_t(r))$ as a bell-shaped function. At the optimal temperature, it takes a value of 1, and this value then declines when the temperature is either too hot or too cold. Alternatively, this damage function could be estimated nonparametrically as in Cruz and Rossi-Hansberg (2024). In a multi-sector model, we would expect the damage function to differ across sectors: agriculture is much more vulnerable to temperature than services (Cruz, 2023). Second, in addition to productivity being a function of temperature, it is natural to think of amenities as depending on temperature too (Rappaport, 2007; Albouy et al., 2016). The effect of global warming on the amenity value of a location is introduced in Cruz and Rossi-Hansberg (2024).

On the other hand, the economy also affects temperature. Production requires energy that generates emissions. As this increases the stock of carbon in the atmosphere, temperatures increase. To incorporate emissions, energy $E_t(r)$ is introduced in the production function (1) as an additional factor of production:

$$q_t(r) = L_{\phi t}(r)^{\gamma}\, z_t(r)\, L_t(r)^{\mu}\, E_t(r)^{\sigma}\, H_t(r)^{1-\gamma-\mu-\sigma}. \tag{30}$$

Energy is assumed to come from fossil fuels, but one could introduce clean energy as well. The relation between energy use, emissions and the atmospheric stock of carbon can be written as:

$$K_t = \varepsilon_1 K_{t-1} + \varepsilon_2 E_t, \tag{31}$$

where K is the atmospheric stock of carbon, ε_2 determines how the use of fossil fuels E translates into carbon emissions, and ε_1 determines the rate of decay of the atmospheric stock of carbon due to the carbon cycle. The expression above is a simplified version of more sophisticated versions of carbon cycles, where emissions never fully decay (Hassler and Krusell, 2012; IPCC, 2013). The increase in the atmospheric stock of carbon then affects the average global temperature T:

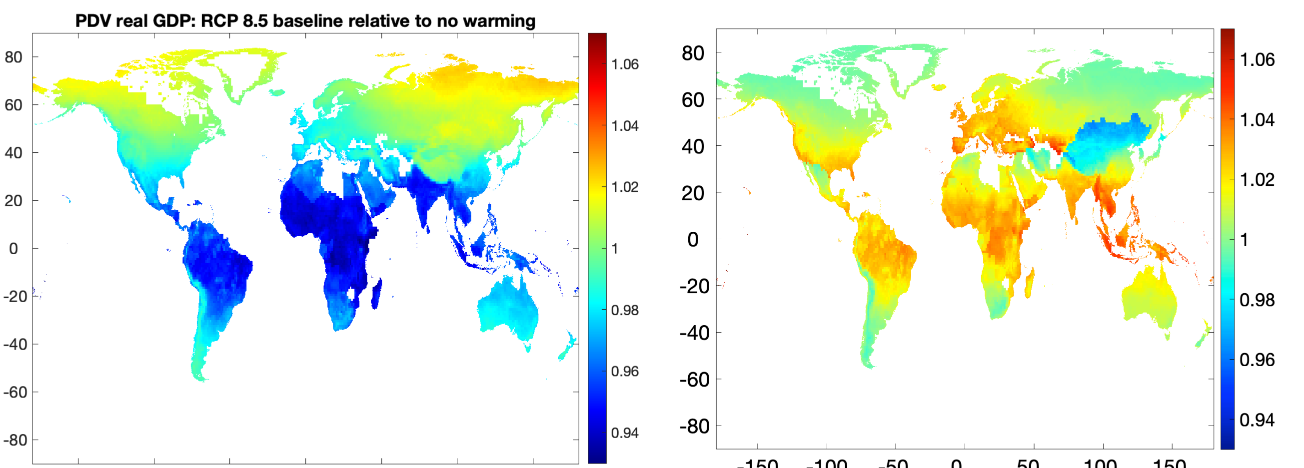
$$T_t = T_{t-1} + \nu(K_t - K_{t-1}), \tag{32}$$

where ν reflects the sensitivity of temperature to the carbon stock. Any change to the average global temperature, T, then influences local temperature, $T(r)$, though not necesarily one-for-one.

Spatial heterogeneity in the impact of climate change. To illustrate the importance of spatial heterogeneity when assessing the impact of global warming, we rely on the S-IAM of Cruz and Rossi-Hansberg (2024). This model closely follows the model description above, with both productivity and amenities depending on temperature through damage functions. It uses a spatial resolution of $1° \times 1°$ for the entire globe, and most of the analysis is done for Representative Concentration Pathway (RCP) 8.5, corresponding to a rather pessimistic scenario of fossil-fuel intensive growth with average global temperature increasing by 3.7°C by the end of the 21^{st} century.

Based on this S-IAM, Panel (a) of Figure 2 displays the present discounted value (PDV) of real GDP in a world with global warming relative to a world without global warming. What stands out is that the welfare effects are heterogeneous across space: whereas real GDP losses are more than 5 percent in some regions of sub-Saharan Africa, the most northern latitudes gain more than 5 percent in terms of real GDP. Given that the regions that are hardest hit tend to also be some of the poorest, these heterogeneous welfare effects imply an important increase in spatial inequality in the world. In many IAMs the average global effects of climate change are relatively small. The important point here is that even if the average cost is not too large, the local impact is very heterogeneous and will lead to an important increase in spatial inequality.

(a) Baseline real GDP　　　(b) Real GDP: averaging vs baseline



Note: Panel (a) displays the PDV of welfare in a world with global warming relative to a world without global warming; Panel (b) displays the PDV of real GDP when averaging damages at the level of World Bank regions relative to the baseline where damages are measured at the $1° \times 11°$ grid-cell level.

Figure 2: Effect of Global Warming on the PDV of Welfare and Real GDP

As a further illustration of how spatial heterogeneity matters, Desmet and Rossi-Hansberg (2024) analyze the error induced by averaging out damages at the level of the world's main regions. Specifically, using the S-IAM in Cruz and Rossi-Hansberg (2024), they take the damages to amenities and productivities at the $1^\circ \times 1^\circ$ level, and average them at the level of each of the seven World Bank (WB) regions.[12] For example, focusing on sub-Saharan Africa, for any given year they take the average damage to productivity at the level of the entire region, and apply that same damage to each one of the grid-cells within the regions. This implies that equatorial regions, where the temperature is already very warm, suffer the exact same productivity damage as regions in southern Africa, where the temperature is more temperate. Apart from averaging damages, the rest of the analysis is done at the $1^\circ \times 1^\circ$ resolution.

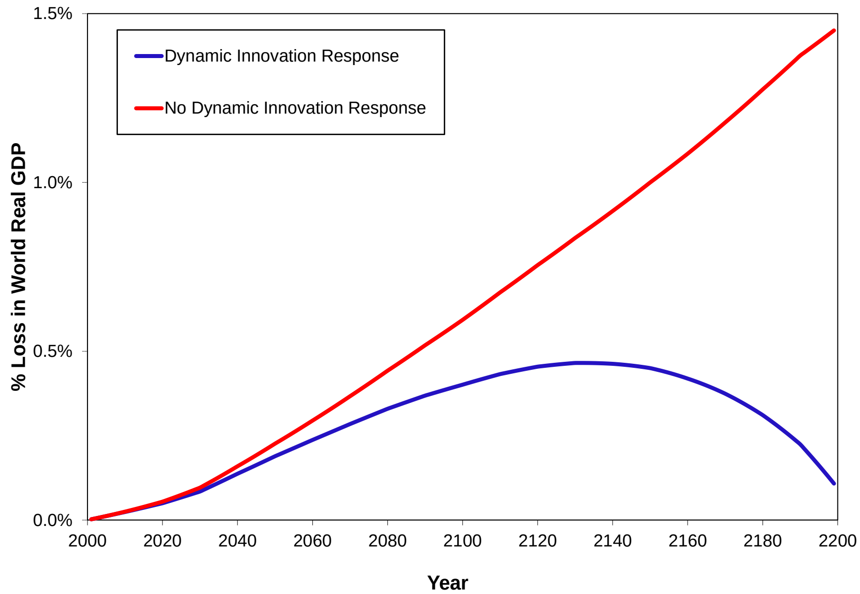
Panel (b) of Figure 2 displays the PDV of real GDP when averaging damages at the level of WB regions relative to the baseline without averaging. The comparison is based on a world with global warming. As can be seen, averaging out damages leads to underestimating the loss in real GDP in southern Europe, the southern United States, and the equatorial regions of America, Africa and Asia. Intuitively, by applying the average damage of North America, the southern U.S. suffers less, and by applying the average damage of sub-Saharan Africa, the Democratic Republic of Congo experiences a smaller negative impact. Overall, the errors induced by averaging out damages go up to 6 percent of the PDV of real GDP. Ignoring the full spatial heterogeneity in damages also leads to underestimating the global impact of climate change. The peak negative impact of climate change drops from 5.4 percent of GDP in the baseline to around 3.8 percent when averaging at the level of WB regions.

Dynamics and adaptation. To illustrate the importance of endogenous innovation and innovation, the S-IAM by Desmet et al. (2021) on the economic impact of coastal flooding is useful. As some coastal cities decline because of rising sea levels, the productivity embedded in these clusters of economic activity is lost. However, as people relocate, new clusters emerge. These new markets provide incentives for firms to innovate, making these new agglomerations more productive. Figure 3 shows how the flooding-induced loss in world real GDP between 2000 and 2200 depends on endogenous innovation. In addition to adaptation through migration and trade, the baseline also allows for an endogenous innovation response as economic activity relocates. As can be seen, the real GDP loss from sea-level rise is around 0.1 percent in 2200. In contrast, when switching off the endogenous innovation response, while still allowing costly migration and trade, this loss rises to around 1.5 percent.

Other dynamic spatial models of climate change. Spatial frameworks of climate change that incorporate forward-looking migration decisions have been applied using a coarser spatial resolution than the S-IAM described above.

Cruz (2023) introduces climate change into a dynamic spatial framework with forward-looking migration decisions. The model's environment comprises multiple countries, multiple regions within each country, and multiple sectors. The production structure of the model features intermediate

[12] The seven World Bank regions that we consider are East Asia and the Pacific, Europe and Central Asia, Latin America and the Caribbean, Middle East and North Africa, North America, South Asia, and sub-Saharan Africa.



Source: Based on Figure 3A in Desmet et al. (2021).

Figure 3: World Losses in Real GDP due to Sea-Level Rise: Dynamic Reponse

goods and input-output linkages, as in Caliendo and Parro (2015). Workers have non-homothetic preferences, as in Comin et al. (2021), which leads to structural change as income grows. Workers are forward-looking, have perfect foresight, and their dynamic migration decisions follow Caliendo et al. (2019). To infer bilateral migration costs between the 287 regions of the model, the paper relies on microdata from country censuses or population surveys and changes in bilateral migrant stocks across periods at the subnational level.

The climate side of the model is similar to the one described above, with the difference that the quantitative implementation has six sectors. To estimate sector-specific climate damage functions, the paper uses a panel fixed-effects model, with climate variables entering the regression in a nonlinear fashion. The estimates point to an inverted-U relationship between temperature and productivity, with productivity being sensitive to temperature especially in the manufacturing and the construction sectors. The author uses dynamic-hat algebra to evaluate the welfare effects of global warming. The results suggests a great degree of heterogeneity in terms of welfare effects across locations; for instance, workers in India suffer welfare losses 15 times larger than the global average, while some regions, such as states in Far East Russia, experience welfare gains. The heterogeneity of productivity responses to higher temperatures induces a reallocation of workers across economic sectors and geographic locations. Non-homotheticity plays a key role in the model: as global warming lowers income, it reallocates more workers to agriculture, in spite of it being the most vulnerable sector. Perversely, this is especially true in some of the world's poorest and warmest regions.

Rudik et al. (2023) develop a climate model with forward-looking migration decisions, trade, and input-output linkages. They apply the model to the U.S. economy and other countries across many industries. To estimate the effect of temperature on sector-specific productivity, they estimate an equation relating bilateral trade to temperature and productivity. Because temperature shocks affect productivity in different locations, it will affect trade flows. Hence, temperature-induced changes in trade flows are informative on the productivity damage function. Likewise, temperature-induced changes in migration flows are informative about the damage function for amenities. In their counterfactual analysis, climate change results in welfare gains in colder parts of Europe and North America, but significant losses elsewhere. The authors find that market-based adaptation from migration has moderate positive aggregate welfare effects in the United States.

Schmitz et al. (2024) study the labor market effects of a recent regulation of the U.S. Environmental Protection Agency that targets fine particle emissions. The introduction of this regulation had a differential impact across space: only counties in "non-attainment" of the national standards were forced to reduce emissions. Leveraging this variation, the authors find that, after the reform, employment growth in polluting industries declined more in non-attainment commuting zones. The paper then quantifies the aggregate effects of the environmental policy through the lens of a dynamic spatial framework that incorporates air pollution and abatement into Caliendo et al. (2019). Emission abatement by a firm reduces its productivity, but it also creates a positive externality on the productivity of all workers in a commuting zone, capturing the health benefits of lower fine particle pollution. They find that the policy lowered emissions by 11.1%, but destroyed approximately 250,000 jobs.

Bilal and Rossi-Hansberg (2023) develop a dynamic spatial framework in continuous time to analyze the anticipatory and adaptation responses to climate change in the United States. Their framework features forward-looking migration decisions and capital accumulation, and they apply it to more than 3,000 U.S. counties. To compute the model, the paper relies on a first-order perturbation of the Master Equation representation of the economy developed in Bilal (2023), which formulates the equilibrium in a state-space representation. Counterfactual analysis shows that the anticipation of future climate change plays a crucial role in mobility and investment. Without anticipation, climate-induced worker mobility is smaller because workers foresee the persistence of capital investments in locations that will become less attractive as the climate worsens. At the same time, anticipation has only small aggregate effects, as gains in some locations compensate for the losses in others.

Beyond analyzing climate change in a dynamic spatial setting, there other geographic aspects of interest to environmental economists. For instance, the quality of the environment may act as an amenity and influence the spatial distribution of economic activity; industrial pollution travels across space and can affects where people choose to live; and roads may impact where deforestation occurs. These and other examples are discussed in detail in the chapter by Balboni and Shapiro (2025) in this volume.

3.2 Migration and Trade Policy

Migration, trade and EU enlargement. Caliendo et al. (2021) examine the general equilibrium effects of both goods and labor market integration in the wake of the the 2004 European Union (EU) enlargement, when 10 countries with a combined population of almost 75 million joined the European Union, bringing the total number of member states from 15 to 25. The accession process involved a sequential elimination of migration restrictions. Regarding trade policy, tariffs between New Member States (NMS) and former EU members (EU-15) were reduced to zero starting in 2004. Due to the sequential changes in migration policy following the enlargement, migration decisions were inherently forward-looking and dynamic.

To capture the heterogeneity of countries, Caliendo et al. (2021) develop a model that incorporates households with varying skills and nationalities, who can be either employed or nonemployed. Each period, households make consumption choices and forward-looking migration decisions about whether to relocate to another location, as in Caliendo et al. (2019). The migration decision is influenced by factors such as the household's current location, nationality, skill level, employment status, migration costs (which are policy-dependent), and an idiosyncratic preference shock.

In the model, locations are indexed by i or j, nationalities by n, and skills by s. The lifetime utility of a representative household at time t in location i of nationality n, skill s, and labor force status ℓ (employed e, nonemployed ne), conditional on the aggregate state of the economy, and denoted by $V^{i\ell}_{n,s,t}$, is given by

$$V^{i\ell}_{n,s,t} = U(C^{i\ell}_{s,t}) + \nu \log \left[\sum_{j=1}^{N} \sum_{o=e,ne} \exp \left[\beta V^{jo}_{n,s,t+1} - m^{i\ell,jo}_{n,s,t} \right]^{1/\nu} \right] \tag{33}$$

where $C^{i\ell}_{s,t}$ is the consumption aggregator, β is the discount factor, ν is the dispersion of the idiosyncratic shock, and $m^{i\ell,jo}_{n,s,t}$ represents the migration cost from location i to location j at time t, which includes potential transitions between labor market statuses and depends on both non-policy and policy components. The location value (33) is determined by the current utility of households in country i, the expected value of remaining in that location for the next period and the option value of migrating to a different location. Importantly, this option value of migration varies by employment status, skill, and nationality, reflecting the fact that households of different nationalities living in the same country face different migration restrictions. The household's maximization problem determines the fraction of households of nationality n and skill s that migrate from country i to country j at time t, conditional on labor force status ℓ, and the total stock of households in each location. The demand side of the framework follows a production and trade structure as in Eaton and Kortum (2002), extended to incorporate agglomeration forces and congestion forces.

To apply their framework to the data, the authors construct gross migration flows by nationality, skill, and employment status for the period 2002-2014 using micro data from the European Labor Force Survey for both NMS and EU-15 countries, as well as trade and production data for EU countries and a constructed rest of the world.

Guided by theory, the authors develop an estimation strategy that considers migration flows before and after the policy change, while also exploiting cross-country variation in the timing of the adoption of the new migration policy. The identification strategy employs a difference-in-difference-in-differences approach, relying on the assumption that, in the absence of the EU enlargement, the difference in trends in migration costs between countries that changed their migration policy and those that did not would have been the same. Using estimated changes in migration costs due to the EU enlargement, along with observed changes in tariffs, Caliendo et al. (2021) quantify the effects of the EU enlargement using dynamic hat algebra. The authors solve for a counterfactual economy in which trade and migration policy are held constant relative to a baseline economy that reflects the actual evolution of policies.

Table 1: Migration and Welfare Effects of the EU Enlargement

(a) Migration Effects: Change in the Stock of NMS Nationals in EU-15

	NMS Low Skill		NMS High Skill	
	EU Enlargement	Trade Policy Unchanged	EU Enlargement	Trade Policy Unchanged
Short run (by 2015)	0.307	0.308	0.142	0.149
Steady state	1.797	1.810	1.074	1.106

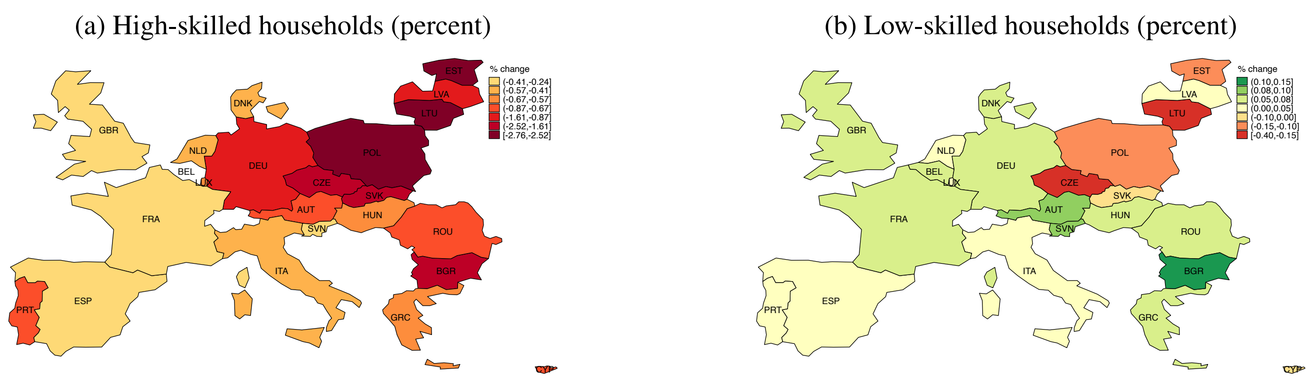
	EU Enlargement	Only Trade Policy	Only Migration Policy
EU-15:			
High skill	0.136	0.146	-0.014
Low skill	0.020	0.094	-0.076
NMS:			
High skill	1.701	0.552	1.079
Low skill	1.099	0.328	0.755

Note: Panel (a) shows the percentage point change in the share of low-skilled and high-skilled NMS nationals in EU-15 countries due to the 2004 EU enlargement, and in the absence of trade policy changes. Panel (b) shows the percentage change in welfare, measured as consumption equivalent, from changes to migration and trade policies. Source: Caliendo et al. (2021)

Panel (a) of Table 1 shows the short- and long-run changes in the share of NMS nationals in EU-15 countries due to the EU enlargement. The results show that the EU enlargement increased migration from NMS to EU-15 countries, particularly among low-skilled households. In the absence of trade liberalization, these migration flows would have been larger. In terms of welfare, Europe as a whole is better off with the EU enlargement, with the largest gains occurring in the NMS countries (Panel (b)). In the EU-15, households benefit from the enlargement due to access to cheaper goods resulting from tariff reductions. However, those gains are partly offset by welfare losses from changes in migration policy, especially for low-skilled households that face bigger competition from a larger increase in the supply of NMS low-skilled households. In contrast, in NMS countries, low- and high-skilled households benefit from both trade and migration policies. These findings underscore the importance of trade when quantifying the welfare and migration effects of labor market integration.

Economic impact of Ukrainian refugee influx. Following the 2022 Russian invasion of Ukraine, more than 4 million working-age refugees, with varying skill levels, were geographically dispersed across Europe, with the largest influx occurring in countries neighboring Ukraine. Caliendo et al. (2023) quantify the general equilibrium effects of this exogenous increase in the labor supply across 23 European countries. To allow for long-run adaptation, they extend Caliendo et al. (2021) by incorporating an endogenous process of capital accumulation as in Kleinman et al. (2023).

In the short run, the surge in labor supply strains the use of capital structures, which take time to develop. However, as countries expand their capital stock, output increases, leading to potential long-term benefits. The study also reveals significant distributional effects across countries and skill groups (Figure 4). High-skilled households lose due to the increased competition from a labor supply shock that is relatively high-skill intensive. In contrast, low-skilled households tend to benefit from the relatively larger increase in high-skilled labor and from the accumulation of capital structures over time.



Source: Caliendo et al. (2021)

Figure 4: Distributional Welfare Effects of the Refugees Crisis in Europe

Trade policy and skill upgrading. Recent work examines the labor market adjustment following Taiwan's accession to the WTO in 2002. This trade policy change is particularly interesting, as Taiwan is an open economy geographically close to China and entered the WTO at a similar time to China. Chang et al. (2022) study the dynamic labor market effects from 1995 to 2020. The authors depart from Caliendo et al. (2019) to incorporate different skill levels (low, middle, high) and the possibility of skill upgrading. Specifically, individuals make dynamic choices regarding sector and skill level in each period, influenced by sector-skill-specific wages, goods prices, switching costs, and idiosyncratic preference shocks. The study finds that tariff reductions during this period significantly contributed to the rapid growth of key sectors in Taiwan and the increasing share of high-skilled workers in the labor force. The bilateral tariff concessions between China and Taiwan play a major role in these outcomes, highlighting China's critical role in Taiwan's trade structure. The authors highlight that the skill-upgrading mechanism is important in explaining the substantial employment effects of Taiwan's WTO accession.

Unemployment effects from import competition with nominal rigidities. Recent work has also examined the impact of trade changes, beyond trade policy, on unemployment and labor force participation. Autor et al. (2013) documents higher unemployment effects and lower labor force participation in U.S. locations with greater exposure to import competition from China compared to less-exposed locations.

To examine the aggregate the effect of the China shock on labor force participation, unemployment and welfare, Rodriguez Clare et al. (2024) generalize the framework of Caliendo et al. (2019) along two dimensions. First, the authors add downward nominal wage rigidity as in Schmitt-Grohe and Uribe (2016), constraining the nominal wage in any period to be no less than a factor δ times the nominal wage in the previous period. This may lead to an employment level below labor supply—and hence to unemployment—when wages are constrained by the nominal rigidities. Second, they allow for a difference between the elasticity governing workers' mobility across sectors, denoted by $1/\nu$, and the elasticity governing mobility across local labor markets, denoted by $1/\kappa$.

The paper calibrates the key model parameters δ, ν, and κ to match the reduced-form evidence in Autor et al. (2013) on how labor force participation, unemployment, and population across U.S. labor markets are affected by the China shock. The authors then apply dynamic-hat algebra to simulate the effects of the China shock from the year 2000 onwards. They find that although the China shock improves the terms of trade for almost all states, employment actually falls in most states during the transition, both through an increase in unemployment and a decline in labor force participation. These employment effects lead to a two-thirds reduction in the U.S. welfare gains from the China shock.

Kim et al. (2023) develop a similar framework with forward-looking migration decisions and nominal rigidities, where trade imbalances are endogenously determined by consumption-savings decisions, and labor markets are defined at the industry level. The authors examine the impact of the interaction between the China shock and the currency peg on U.S. manufacturing employment, unemployment, and trade imbalances. The study finds that China's currency peg contributed to U.S. manufacturing job losses, accounted for one-third of the U.S. trade deficit, increased unemployment, and reduced U.S. welfare gains.[13]

In related work, Ulate et al. (2024) apply the dynamic spatial framework with downwards nominal wage rigidities in Rodriguez Clare et al. (2024) to study the labor market consequences of a temporary increase in international trade costs similar to the one observed during the COVID-19.

[13]Similarly, Dix-Carneiro et al. (2023) examine the role of endogenous trade imbalances in the effects of the China shock using a dynamic model where unemployment results from search frictions.

The authors find that the increase in trade costs leads to a temporary but prolonged decline in U.S. labor force participation, and a temporary increase in manufacturing employment as the United States is a net importer of manufactured goods.

Global welfare gains from free migration. An important consideration of changes in the spatial distribution of population is that productivity might change in response. As people relocate across space, new clusters of economic activity emerge, whereas other existing clusters may decline. Over time, this spatial reallocation affects productivity growth, and hence global welfare gains. This is exactly the type of question that a dynamic spatial model with endogenous innovation can address.

Using such a framework, Desmet et al. (2018) assess the welfare impact of removing all migration frictions in the world. With free migration, people tend to move to high-productivity places, hence increasing the correlation between density and productivity. In today's world, it is not uncommon for high density to coexist with low productivity—think, for example, of some regions in sub-Saharan Africa. When liberalizing migration restrictions, people move out of these regions, causing static productivity gains and making it less likely for low-productivity places to have high population density. At the same time, people also move to high-amenity places, such as Brazil, even if their initial productivity is not that high.

Over time, the locations that attract people innovate more, because of dynamic agglomeration economies. This further motivates people to move there, further strengthening the correlation between productivity and density. As such, Europe, the U.S. and some places in Latin America become the world's densest and most productive regions. Because free migration tends to reallocate people to the world's most productive areas, which then innovate more because of increased density, removing migration restrictions raises the balanced-path growth rate of the world economy by about 0.5 percentage points. This underscores the importance of the spatial distribution of population for aggregate growth. It is an illustration of how spatial heterogeneity is key to understanding the macroeconomy. In present discounted value terms, full liberalization is estimated to increase world welfare by 306%, whereas real income increases by 126%.

3.3 Infrastructure Investment

Infrastructure projects have important consequences both for the aggregate economy and for the spatial distribution of economic activity. A number of studies have analyzed the impact of large transport infrastructure projects using static frameworks. Prominent examples include Donaldson (2018) who estimates the impact of the construction of colonial India's railroad network on real income and welfare, Donaldson and Hornbeck (2016) who estimate the impact of railroads in the U.S. on agricultural land values in 1890, and Allen and Arkolakis (2014) who assess the effect of the interstate highway system on the spatial distribution of the U.S. economy.

Of course, many infrastructure projects are long-lived, so that their effects are likely to be dynamic. For example, clusters of economic activity may emerge or get reinforced in locations that benefit from improved road connectivity. As this increases market access, it incentivizes local innovation, spurring growth, and attracting more people to move in. Desmet et al. (2017) use the framework of Desmet et al. (2018) to study the impact of improved infrastructure. They find that a worldwide drop in transport costs of 40% increases the present discounted value of real income by 71% globally. This effect is much larger than what is typically found in static quantitative trade models. As a comparison, Costinot and Rodríguez-Clare (2014) estimate that eliminating a worldwide tariff of 40% would increase welfare by a mere 3%. In a related study on optimal infrastructure investments in Latin America, Conte and Ianchovichina (2022) also find large differences between static and dynamic effects.

In another study, Nagy (2023) analyzes the importance of U.S. railroads on economic growth between 1830 and 1860. He focuses on the 'hinterland hypothesis' which claims that the abundance of land was essential for city formation and growth. Within the context of the model, a location with a larger hinterland attracts more farmers to its hinterland, is more likely to become a city, and innovates more. Because a larger local agricultural market implies higher returns to innovation in the local non-agricultural sector, towns develop into cities and aggregate growth improves. As in Desmet and Rappaport (2017), there is rapid growth in some of the rural counties, especially in the newly settled areas in the West. The introduction of the railroad further stimulates urban and aggregate growth, by lowering transport costs and expanding the size of a city's potential hinterland. Counterfactual analysis shows that railroads were responsible for 25% of real GDP growth between 1830 and 1860. An advantage of using quantitative spatial models to study historical episodes is that it facilitates testing the model. The calibration in Nagy (2023) targets moments of aggregate and city growth in the U.S. between 1790 and 1820, allowing him to check how well the model predicts the spatial evolution of the U.S. between 1820 and 1860. This lends credibility to the sometimes complex structure of dynamic spatial models.

Infrastructure development and adaptation to environmental risks may also be influenced by incentives arising from government intervention and commitment. For instance, Hsiao (2023) examines the construction of the West Flood Canal in Jakarta in 1918, which protected areas to the north but not to the south, and finds that this spatial discontinuity spurred increased development in the north. The author then estimates a dynamic spatial model in which developers and residents make investment and location decisions while accounting for flooding risks and shows that government commitment facilitates adaptation, particularly through the gradual management of retreat from flood-prone areas.

Another relevant paper is Balboni (2025) who uses a multi-region quantitative spatial equilibrium framework to examine the dynamic effects of environmental change on infrastructure allocation. The model features forward-looking location decisions as in Caliendo et al. (2019). At the end of each period, agents evaluate economic conditions and decide where to relocate, taking into account mobility costs and idiosyncratic preference shocks. The model is applied to 541 spatial units in

Vietnam, using trade, migration, and production data to calibrate the parameters and elasticities. Sea-level rise lowers the availability of land, increases transport cost in inundated areas of the road network, and affects productivity through a damage function. The paper finds that the net present value of aggregate welfare gains from realized road investments in Vietnam between 2000 and 2010 falls from 1.56% when ignoring the impact of future inundation to 1.37% when accounting for future sea level rise. The paper also shows that accounting for the dynamic effects of environmental change is key to making optimal decisions on where to allocate infrastructure today.

3.4 Dynamic Agglomeration Economies

Scale effects and productivity. The existence of dynamic agglomeration economies, emphasized since the early theories of Marshall (1890) and Jacobs (1969), imply a positive relationship between population size and innovation. In the dynamic spatial model of Desmet et al. (2018), this positive scale effect plays a key role in shaping the future spatial distribution of economic activity across the world. The basic force is that more dense locations become more productive over time because investing in local technologies is more profitable in larger markets. Depending on the level of migration frictions, this force can have a profound effect on the geography of development. Specifically, if migration frictions are high, people tend to stay where they are, and today's dense areas, which often coincide with developing countries, end up becoming the most productive areas of the future. In other words, the prediction is a productivity reversal, where sub-Saharan Africa and South Asia become highly productive in the distance future. If, instead, migration frictions are low, people can more easily move to today's productive regions, such as the United States and Europe. This allows today's developed countries to continue being tomorrow's productivity leaders. In the context of this discussion, an important question relates to the impact of size on long-term (steady-state) growth. In the calibrated version of the model of Desmet et al. (2018), the existence of strong enough congestion forces imply that the economy eventually converges to a balanced-growth path. Hence, denser places are more productive, but in the very long run all locations grow at the same rate.

A relevant study that more directly estimates the scale effect on productivity is Peters (2022) who examines the economic impact of the inflow of about 8 million ethnic Germans into West Germany after being expelled from Eastern Europe in the aftermath of the Second World War. This historical episode led to an increase in the German population by 20%. Empirically, locations that benefited from a larger population shock are shown to experience a gradual increase in income per capita. Specifically, a 10% increase in the share of refugees has a statistically insignificant effect after 2 years, but is estimated to raise income per capita by 5-6% after 15 years. This is consistent with the existence of dynamic agglomeration economies.

Guided by these reduced-form findings, the paper develops a theory that can rationalize this evidence. The model considers an economy with many locations and two sectors, agriculture and manufacturing. Individuals are myopic, and face a consumption choice, a sector labor supply choice, and a migration choice. Productivity in the manufacturing sector benefits from using a variety of inputs, as in Romer (1990a). The labor requirement to start a firm that produces a new input variety in region r and period t is given by $h_{rt} = f_E N_{rt-1}^{-\lambda}$, where f_E is a fixed cost parameter, N_{rt-1} is the measure of local input varieties in the previous time period, and $\lambda \leq 1$ governs the dynamic spillovers that make entry cheaper when the local market is larger. The free entry condition gives rise to an expression for the evolution of N_{rt}:

$$N_{rt} = \frac{1}{f_E} \frac{1}{\rho - 1} \times H_{rPt} \times (N_{rt-1})^{\lambda} \tag{34}$$

where $\rho > 1$ is the elasticity of substitution across inputs, and H_{rPT} denotes the mass of production workers. The second term captures the standard market size effect: a larger market, as measured by the number of workers, can sustain more varieties. The third term represents the dynamic agglomeration force: as long as $\lambda > 0$, any positive shock to the size of the market will lead to a positive growth rate along the transition path.

The evolution of the mass of varieties in expression (34) determines the growth rate of the economy along the balanced growth path, and nests different models. If $\lambda = 1$, we get the fully endogenous growth model of Romer (1990b) which implies a linear relationship between long-run growth and the size of population. If $0 < \lambda < 1$, we obtain the semi-endogenous growth model of Jones (1995), where a positive shock to population increases the growth rate along the transition path, but growth eventually converges to zero in the absence of long-run population growth. In other words, in steady state larger locations will have a higher level of productivity, but not a higher growth rate. Using an indirect-inference strategy, Peters (2022) structurally estimates the model's parameters, and finds a value of $\lambda = 0.71$, consistent with growth being semi-endogenous.

History, persistence, and path dependence. Agglomeration forces tend to lock in the spatial distribution of economic activity. At the same time, where exactly such clusters emerge and persist often seems be due to historical chance. Past shocks may affect the current location of economic activity either because shocks have high persistence or because of path dependence. To study uniqueness, persistence and path dependence, Allen and Donaldson (2022) develop a dynamic spatial framework with trade and forward-looking migration decisions in the presence of static and dynamic agglomeration economies.

The framework consists of an overlapping generations model in discrete time. The world consists of multiple locations, inhabited by forward-looking dynastic families where each individual lives for two periods, first as a child and then as an adult. At the beginning of the second period, an individual experiences an idiosyncratic preference shock and decides where to live. In making this decision, the individual is forward-looking as in Caliendo et al. (2019), and considers both her own benefits and the anticipated benefits for all future generations of her family.

In each location, a continuum of firms use labor to produce a homogeneous good under perfect competition. The productivity level in location i and time t is given by

$$A_{it} = \bar{A}_{it} L_{it}^{\alpha_1} L_{it-1}^{\alpha_2}, \tag{35}$$

where $\bar{A}_{it}$ is the exogenous component of productivity in location i at time t. Productivity is also a function of contemporaneous agglomeration spillovers whose strength is governed by α_1 and potential historical spillovers whose strength depends on α_2. Similarly, a location's amenity value, denoted by u_{it}, is also subject to spillovers:

$$u_{it} = \bar{u}_{it} L_{it}^{\beta_1} L_{it-1}^{\beta_2}, \tag{36}$$

where $\bar{u}_{it}$ represents an exogenous amenity component, and β_1 and β_2 govern the strength of contemporaneous and historical spillovers. An important difference with productivity spillovers is that β_1 and β_2 can be positive or negative, potentially reflecting positive impacts from infrastructure projects, negative congestion effects, and so on.

The paper then studies parameter restrictions that guarantee that the economy exhibits a unique equilibrium, and it also discusses when temporary shocks are highly persistent and under which conditions there is path dependence. The conditions depend crucially on the magnitude of the spillover elasticities. Intuitively, when contemporaneous agglomeration economies are weaker than congestion forces, the equilibrium is unique. However, when forward-looking behavior is strong enough, this condition becomes more complex. When local agglomeration economies become strong enough, we get multiple equilibria, and hence path dependence. When the spillovers are close to the boundary at which uniqueness can no longer be guaranteed, temporary shocks may exhibit strong persistence, without leading to long-run path dependence.

Empirically, the authors rely on U.S. historical data from 1800 to 2000 to estimate the spillover parameters. When focusing on contemporaneous spillovers, they conclude that the U.S. economy is in the parameter region where the equilibrium is unique, but with very persistent historical shocks. When taking into account both contemporaneous and historical spillovers, there is the possibility of the U.S. economy exhibiting multiple steady states. This implies that temporary events may have long-lived, and sometimes even permanent, effects on the spatial distribution of economic activity.

3.5 Structural Change

Long-run growth and development lead to structural change, first from agriculture to manufacturing, and later to services. Structural change is inherently spatial. One reason for this is the link between the spatial distribution of economic activity and innovation. The relative geographic clustering of different sectors affects their relative productivity growth, thereby shaping the process of structural transformation. Another reason is comparative advantage: as some sectors decline while others expand, we would expect regions with comparative advantage in expanding sectors to benefit. This process reallocates economic activity across space. Of particular relevance in this context is the role of transport costs. By changing the incentives to spatially concentrate, transport costs influence the incentives to innovate in different sectors, and thus the timing of structural transformation.

Structural change, innovation, and spatial concentration. Over the last half-century, the service industry in the U.S. has become increasingly concentrated in space, whereas manufacturing has become geographically more dispersed. Using a two-sector spatial dynamic model, Desmet and Rossi-Hansberg (2015) argue that endogenous changes in the relative rates of innovation in manufacturing and services drive this spatial reorganization of the U.S. economy.

Their model is a simplified one-dimensional version of the spatial dynamic model described in Section 2.1. To illustrate the workings of the model, assume that initially productivity growth is faster in manufacturing than in services. As this reallocates labor towards services, the service sector expands. The larger market size in services endogenously accelerates innovation in that sector, allowing the economy to avoid Baumol's disease. Because structural change tends to shift labor from high-productivity to low-productivity growth sectors, Baumol (1967) predicted that the economy was inexorably headed towards stagnation. That dismal prediction does not occur in Desmet and Rossi-Hansberg (2015), because as labor moves towards the low-productivity growth sector, eventually that sector becomes large enough for innovation to endogenously take off. As this process unfolds, services become spatially more concentrated due to the positive relation between employment density and innovation.

A calibrated version of the model generates the observed reduction in the manufacturing employment share in the second half of the 20$^{\text{th}}$ century, as well as the growth in service productivity growth starting in the mid-1990s. Consistent with the data, the model also shows the service sector becoming spatially more concentrated. This geographic clustering makes services and manufacturing compete for the same land, leading to a rise in the dispersion of land rents. As such, the model successfully matches some of the key macroeconomic and spatial stylized facts of the U.S. economy over the last half-century.

Structural change and the geography of comparative advantage. Another relevant paper on the spatial consequences of structural change is Eckert and Peters (2023). Focusing on the period 1880-1920, they analyze how the shift from agriculture to non-agriculture reshaped the economic geography of the U.S. In principle, structural change should have a bias against regions that have a comparative advantage in agriculture. As the U.S. economy shifted away from agriculture, this should have hurt agricultural regions.[14] However, this prediction is not borne out in the data. In fact, economic growth was on average faster in regions specialized in agriculture. They hypothesize that another force may be at work: catch-up growth that tends to benefit backward regions. Because agricultural regions were on average poorer, this should have helped rural areas. Using a two-sector

[14] For an early discussion of how the rise and decline of regions may track the rise and decline of the industries they are specialized in, see Brezis et al. (1993). However, in the presence of technological spillovers between sectors, this mapping becomes more complicated (Desmet, 2002; Berkes et al., 2025).

spatial growth model, counterfactual exercises show that catch-up growth is key to understanding the convergence of rural America with the rest of the country during the late 19$^{\text{th}}$ and early 20$^{\text{th}}$ centuries.

Structural change and transport costs. Transport costs affect the incentives to geographically concentrate, and hence the process of structural change. Desmet and Rossi-Hansberg (2015) show that if transport costs are high, sectors benefit from colocating. Specifically, if one sector (manufacturing) is already somewhat clustered, the other sector (services) saves transport costs by clustering around it. Because this leads to more spatial concentration, higher transport costs tend to speed up the take-off of the service industry.

Trew (2020) extends the spatial growth model of Desmet and Rossi-Hansberg (2014) to allow for endogenous investment in transport infrastructure by local landowners. His goal is to analyze the role of transport infrastructure in England for the timing of the Industrial Revolution. The model is initialized using data on occupational structure and transport infrastructure for England and Wales in 1710, and is then simulated forward until 1881. The model endogenously generates agricultural productivity growth in the South of England, making demand shift increasingly towards manufacturing. This increases the incentives to innovate in manufacturing, leading to the emergence of industrial hotspots in the North.

Trew (2020) then uses counterfactual analysis to explore the role of endogenous transport development. He makes two key assumptions regarding transport. First, in addition to standard iceberg transport costs, local landowners charge a toll for transportation services produced by transportation workers. Second, an improvement in transport infrastructure not only reduces iceberg transport costs, it also lowers tolls because less transport workers are needed. In this case, bringing infrastructure development forward in time has two opposing effects: on the one hand, lower transport costs can reduce agglomeration incentives, delaying industrial takeoff, as in Desmet and Rossi-Hansberg (2014); on the other hand, lower transport costs also releases transport labor to the manufacturing sector, speeding up industrial takeoff. On balance, the second effect dominates: earlier infrastructure development leads to the Industrial Revolution occurring earlier.[15]

3.6 Other Extensions and Applications

In this subsection we present a more concise description of recent work that extends the dynamic spatial model with forward-looking decisions to other settings and applications.

Caliendo and Parro (2020) introduce forward-looking firms' location decisions and endogenous entry and exit into a tractable dynamic quantitative framework with forward-looking migration decisions and capital accumulation. Inactive firms choose their production location for the next

[15] Fajgelbaum and Redding (2022) also study the importance of transport costs for structural transformation, in this case between non-traded and traded goods late 19th century. Argentina. Their spatial model is not dynamic, though.

period and must pay an entry cost in units of capital to start producing in the location with the highest value. A distinctive feature of the model is that the equilibrium conditions feature endogenous entry, exit, and distribution of firms across locations—equilibrium objects that can be mapped to data. The effects of trade protectionism balance direct price effects from trade policy, firm's location effects on the price index, as well as general equilibrium interactions through trade, migration, and production decisions. The authors use the model to examine the effect of trade policy changes resulting from the 2018 U.S.-China trade war. They find that increased trade protectionism leads to higher firm entry in the United States, giving consumers access to cheaper goods. However, this positive effect takes time to materialize and is not large enough to offset the direct price effect of higher tariffs on imported goods. This result contrasts with the price effect in Venables (1987), where an import tariff stimulates the entry of domestic firms, leading to a decline in domestic prices through enhanced competition.

Donald et al. (2023) studies optimal migration policy in a model with forward-looking migration decisions. The authors build on Artuc et al. (2010) and Caliendo et al. (2019) but allow for a more flexible structure of idiosyncratic shocks that can accommodate arbitrary correlations across potential destinations. Locations differ in their fundamental components, including productivity and amenities, as well as bilateral trade and migration frictions. Their framework also features localized production and amenity externalities. In this environment, the Planner cannot directly regulate migration flows because transfers cannot be contingent on the idiosyncratic preference shocks. The Planner faces a fundamental trade-off: a negative productivity shock in a location implies a higher marginal utility, creating an incentive to increase consumption there. However, this decision increases migration flows into that location. The paper's main theoretical results is a recursive expression of the optimal spatial transfers that highlight the balance between these two forces. The formula equates the marginal benefit of providing consumption insurance, which depends on the marginal utility from a transfer in each location and time, with the marginal efficiency loss, which depends on a fiscal externality arising from distorting migration decisions.

Ahlfeldt et al. (2020) develop a dynamic spatial model with forward-looking location choices, incorporating group-specific quality of life differences across regions. Groups in the model depend on sex, skill and age. Using German microdata, quality-of-life estimates are inferred by inverting the model. As an application, the paper studies the spatial effects of a reduction in air pollution Germany's most polluted regions. Suzuki (2023) incorporates demographic dynamics (e.g. aging, fertility) into a dynamic spatial framework with forward-looking migration decisions to study how the economy adjusts to shocks to economic fundamentals. The author takes the model to Japanese data and studies the spatial effects of earthquakes under different demographic scenarios. Takeda and Yamagishi (2024) introduce agglomeration forces into a model with forward-looking migration decisions and Calvo-type migration frictions to study Hiroshima's recovery after the bombing. Agglomeration forces induce multiple equilibria; in some equilibrium, the city center fails to recover, in contrast to what occurred in reality. The authors find that self-fulfilling expectations of recovery play a crucial role in achieving the recovery equilibrium.

Other recent studies have leveraged dynamic spatial frameworks to address additional questions related to the local labor market effects of trade shocks. Seo and Oh (2024) develop a sufficient statistic approach to evaluate the welfare effects of sectoral shocks in a dynamic discrete choice model that incorporates forward-looking mobility choices and allows for time-invariant worker heterogeneity based on both observable and unobserved characteristics. The sufficient statistics require inputs on the dispersion of idiosyncratic shocks and steady-state workers, summarizing the impact of worker heterogeneity on mobility and welfare. The authors find that the negative welfare effects of increased import competition from China on manufacturing workers are amplified by persistent heterogeneity among workers. Benguria et al. (2024) use dynamic-hat algebra to examine the role of spatial linkages in explaining the decline of the skill premium following a commodity boom in Brazil.

Atalay et al. (2023) investigate the effects of a place-based policy implemented in Turkey in 2012, which introduced investment and wage subsidies varying by province and industry. The policy allowed firms to benefit from a combination of reduced corporate income tax rates, assistance with social security payments, and interest rate subsidies on private loans. The analysis shows that industry-province pairs receiving higher levels of subsidies experienced stronger growth in firm entry, employment, and revenues. The paper also investigates the propagation of spillover effects within the firm network, finding that the indirect effects through the production network are substantial. Using a dynamic multi-region, multi-industry general equilibrium model based on Caliendo et al. (2019), they quantify the aggregate effects of the subsidy program, finding it was moderately effective in reducing inequality between regions over the long term. These modest long-term effects are attributed to the mobility of households in response to the subsidy program and to input-output linkages between Turkish regions.

Another study that evaluates placed-based policies through the lens of dynamic spatial frameworks is Hyunh (2024). Building on Caliendo and Parro (2020), the paper examines the impact of place-based tax incentives and a reform easing migration restrictions in Vietnam on firm entry and spatial inequality. The paper finds moderate effects from the place-based policy and a small impact from the migration reform.

4 Further Discussion and Conclusion

In this chapter, we have discussed recent methodological advances in the dynamic spatial literature that have enabled researchers to quantitatively examine many important questions that have both a spatial and a time dimension. A key achievement of this literature has been the tractable integration of dynamic economic mechanisms into frameworks with complex economic geography. In particular, we have explained how these models address the dimensionality challenges inherent in spatial economies characterized by numerous locations with heterogeneous fundamentals.

In this concluding section, we turn to two remaining topics. First, we discuss how well these dynamic spatial models perform in terms of the credibility of counterfactual predictions, despite some of the simplifying assumptions. Second, we examine some open questions in the dynamic spatial literature that present valuable opportunities for future research.

Model performance. Evaluating the counterfactual predictions of a model is not a straightforward task. It requires distinguishing between which predictions are factual and which are counterfactual, especially when economies have experienced multiple shocks during the period of analysis. For instance, counterfactual predictions about the spatial effects of a railroad project may not align with observed outcomes if locations simultaneously experience productivity changes that are unrelated to the infrastructure project. A possible approach in such cases might be to incorporate these shocks into the model, which would require sufficient variation in the data to infer a potentially large number of fundamentals. Additionally, there is the issue of determining whether some of the inferred fundamentals should be interpreted as structural residuals that fit the data but lack external validation. In what follows, we discuss alternative approaches to assess the credibility of the counterfactual predictions of the different dynamic spatial frameworks discussed in this chapter.

The first set of models, discussed in Section 2.1, addresses the dimensionality issue that arises when solving spatial endogenous growth models with many locations by proposing a structure that simplifies the dynamic forward-looking problem into a sequence of static problems (Desmet and Rossi-Hansberg, 2012, 2015; Desmet et al., 2018). Essentially, these models impose a structure so that forward-looking migration and innovation decisions respond only to current conditions. It is worth emphasizing that when changes in economic conditions across space are gradual, this seems a reasonable simplification. For example, global warming is a slow-moving process, so the effect of a future when temperature is 2 to 4°C higher on today's decisions may be limited. Furthermore, any long-run trends that are not captured by current conditions but might still affect decisions are likely to be partly incorporated when calibrating the model. For instance, suppose long-run expectations are that Europe faces a slow secular productivity decline. This would lead to slower migration into Europe if agents take into account future conditions. When using population changes to calibrate mobility costs in a model where agents do not take into account future conditions, this slower observed migration would show up as higher calibrated costs of entering Europe. In other words, even though the model consists of a sequence of static problems, it would be still be able to match the slower migration into Europe.

A more direct way of assessing whether the simplifying assumptions come at a cost in terms of realism is to evaluate how well the model performs in predicting the spatial distribution of economic activity over time. To that effect, Desmet et al. (2018) do a backcasting exercise: starting in the year 2000, they run their model backwards, and compare the model-generated spatial distribution of population to the actual spatial distribution of population. Although there are many unmodeled shocks and events that occurred during the 20$^{\text{th}}$ century, including two World Wars, the model performs well. The country-level correlation in population levels between the actual data and the model is 0.96 in 1950. In terms of population growth rates, the correlations are of course lower, but they are still relatively high. For example, the correlation between data and model for country-level population growth over the period 1950-2000 is 0.76. From this the authors conclude that, in spite of the simplifying assumptions, the model captures the main forces that affected the changing economic geography of the world economy over the last half-century.

The second set of models, discussed in Section 2.2, incorporate forward-looking decisions, and use dynamic-hat algebra for counterfactual analysis, leveraging both cross-sectional and time-series data. This approach avoids the need to identify a potentially large set of model fundamentals, and perfectly matches the cross-sectional and time-series data. The counterfactual predictions in this class of models are also shaped by the model's relevant elasticities. Specifically, the assumptions about the behavior of individual agents lead to predictions of outcomes for representative agents across locations, which are influenced by migration and other relevant elasticities. Therefore, leveraging exogenous variation to identify these elasticities is crucial for credible counterfactual predictions (e.g., Caliendo et al., 2019; Artuc and McLaren, 2015; Rodriguez Clare et al., 2024, among others).

Another way to validate counterfactual predictions is to demonstrate that the implied differential effects in the model align with those from difference-in-differences estimates, as done, for instance, in Caliendo et al. (2019) and Rodriguez Clare et al. (2024). Alternatively, Balboni (2025) estimates fundamentals, such as productivities and trade costs, using the model's structure and then performs rigorous over-identification tests using data external to the model (e.g., firm-level data).

Future agenda. While significant progress has been made in developing dynamic spatial models over the past decade, ample opportunities remain for future research. First, further studying how the interactions between the different sources of dynamics discussed in this chapter—including endogenous innovation, dynamic migration decisions, and the accumulation of physical and human capital—shapes transitional dynamics and long-term outcomes of economic shocks represents an important avenue for future work. In addition, while the dynamic spatial frameworks presented in this chapter have tractably incorporated several key sources of dynamics, the modeling of the housing supply remains somewhat stylized. A strand of the literature has emphasized the significance of housing market dynamics for spatial outcomes (e.g., Glaeser and Gyourko, 2018). Linking the spatial dynamic models described in this chapter with the literature on housing supply dynamics, such as that reviewed in Baum-Snow and Duranton (2025), would broaden the scope of dynamic spatial models and offer another promising direction for future research.

Second, further work on developing systematic approaches to contrast the dynamic predictions of these models with data would be a valuable contribution to the spatial dynamics literature. As highlighted earlier in this section, it is crucial that such approaches to test counterfactual predictions provides guidance on how to distinguish factual predictions (that can be compared to observed data) from counterfactual predictions (that assume certain fundamentals). A method to test counterfactual predictions must also be portable across a general class of dynamic spatial frameworks and empirical settings.

Third, continuing to incorporate elements that have been widely studied in the macroeconomic literature—such as the effects of monetary policy across space, the role of stochastic fundamentals over the business cycle, among others—into dynamic spatial economies would also be a fruitful area for future research. Bringing the spatial dimension into the business cycle literature would enhance our understanding of how regional business cycles shape aggregate fluctuations and provide deeper insights into the spatial effects of macroeconomic policies. Advancing on this front also requires improved access to business cycle data at more granular geographic levels.

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