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ABSTRACT

Economies routinely experience a variety of sector-specific supply and demand shifts. Yet, the distributional welfare consequences of these shifts are not well understood. We address this gap by developing an analytical framework that jointly integrates supply-side and demand-side heterogeneity without imposing specific functional forms on consumption and production. This enables us to identify the key forces that shape the distributional welfare impact of sector-specific supply and demand shifts—in terms of consumer preferences and sectoral production functions. We estimate key parameters and quantify the heterogeneous welfare effects of sectoral shifts, revealing significant variation in their impact.

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1. Introduction

Economies routinely experience a myriad of sector-specific supply and demand shifts. A prime example of the former includes sector-specific technical change, while changes in public sector expenditures and preference shifts are examples of the latter. In this paper, we propose a unified framework to characterize the determinants of the distributional welfare effects of sectoral shifts across workers with varying skill levels. Our general equilibrium framework integrates supply-side and demand-side heterogeneity imposing minimal structure on preferences and production, thereby illuminating how such sectoral shifts influence wages, goods prices, and, ultimately, welfare outcomes. Crucially, our analysis identifies the key factors driving welfare outcomes, which include consumption substitution patterns and total expenditure elasticities, consumption shares, and sector-specific skill intensities. Estimating these parameters, our empirical findings reveal substantial heterogeneity across high- and low-skill workers in the welfare impact of sectoral shifts.

We begin our analysis in Section 2 where we develop a general equilibrium multi-sector model featuring workers of heterogeneous skills. In our model, workers derive utility from consuming a bundle of goods, with the framework allowing for the possibility of non-homothetic preferences. On the production side, goods are produced in different sectors which vary in their skill intensity. We use this model to study the distributional welfare impact across high- and low-skilled workers, analyzing three different types of sectoral shifts: sectoral technical change, changes to public sector demand, and sectoral demand shifts driven by preference changes. In what follows, we describe each application in detail.

Our first application considers sectoral technical change, which have long been acknowledged as pivotal for economic growth.[1] Specifically, in Section 3 we analytically analyze what determines the distributional welfare impact of sectoral technical change. Our first theorem shows that this welfare impact can be decomposed into two distinct effects. The first effect, which we title the "Engel effect", stems from the fact that with non-homothetic preferences, consumers of different income levels consume bundles with different expenditure shares. As a result, price changes stemming from sectoral technical changes, benefit agents with different income differently. The second effect impacting welfare, titled the "Relative Wage" effect, arises because, in equilibrium,

[1] As vividly described in Harberger's 1998 AEA Presidential Address (Harberger (1998)), growth exhibits characteristics similar to that of "mushrooms" rather than "yeast"; That is, variation in sectoral total factor productivity (TFP) across different industries is substantial.

sectoral technical change causes demand to shift across sectors. As sectors vary in skill intensity, this demand shift alters the relative wages of high- and low-skilled workers and, hence, also the relative prices of goods.

We show that the change in the relative wage serves as a sufficient statistic for all equilibrium price changes, and analytically solve for the elasticity of the relative wage with respect to sectoral technical change. This leads to our second theorem, showing that the relative wage elasticity is determined by the extent to which demand moves towards or away from relatively more high skilled sectors. Our third theorem then identifies the model primitives that determine these demand shifts across sectors following sectoral technical change, showing that the key factors are the consumption expenditure and price elasticities, consumption expenditure shares, and the relative skill intensity of the sector undergoing technical change.

Crucially, all determinants of the distributional welfare impact derived in our theoretical analysis can be empirically estimated. Hence, our framework allows us to directly calculate the differential welfare effect of sectoral technical change. Guided by the analytical results, we use data from the Consumer Expenditure Survey (CEX) to estimate an Almost Ideal Demand System (AIDS) from which we recover price and total expenditure elasticities. We use the Current Population Survey (CPS) to recover labor shares by skill level, and KLEMS data for sectoral productivity dynamics. Finally, we use an input-output matrix to map the measures obtained from the CPS and KLEMS to consumption categories. Section 4 then presents five main quantitative findings.

First, sectoral technical change affects welfare of high- and low-skilled workers differently. Second, the differential welfare effect ranges from a 24% smaller welfare increase for high-skilled workers to a 75% larger welfare increase for high-skilled workers, depending on the sector experiencing technical change. Third, while both the Relative Wage effect and the Engel effect are quantitatively important in determining the overall distributional impact of technical change, it is the former that plays a more dominant role. Fourth, our demand system estimates reveal the presence of both complementarity and substitutability within the substitution matrix, which emphasizes the importance of allowing for flexibility in preferences when evaluating the distributional welfare impact of sectoral changes. In order to highlight the quantitative impact of such flexibility, we contrast our quantitative results with those derived from parametric non-homothetic preferences in which, as is commonly assumed in the literature, all goods are complements. We find significant differences in the estimates of the relative wage elasticity, and hence in the welfare impact. Finally, we calculate the overall differential welfare impact of sectoral technical change on high- versus low-skilled workers given the realized empirical changes in sectoral TFP over different decades, finding substantial variation in the distributional impact over different periods, depending on the realization of the sectoral TFP changes.

As discussed above, our analytical framework is general enough to be applicable to a broad range of applications. To illustrate this, we continue by analyzing two cases of demand-driven sectoral shifts: changes in public sector expenditure as well as changes in consumer preferences. In Section 5.1 we augment our baseline model to include a public sector and derive analytical results akin to those in Section 3. We show that the distributional welfare impact of shifts in public sector expenditures (financed by taxes levied on consumers) operates only through the Relative Wage effect – i.e., there is no Engel effect. This stems from the fact that unlike in the case of technical change, shifts in public sector expenditures do not affect directly the relative cost of good production. We then show that the distributional welfare impact of an increase in public expenditures is dictated by the skill-intensity of the public sector compared to that of the private sector, as well as by consumers' total expenditure elasticities, which govern the reshuffling of the private sector's demand across different goods due to the tax increase.

In Section 5.2 – our third and final application – we examine the distributional welfare impact of exogenous changes in consumer preferences across goods. As in the case of public expenditures, we show that this distributional impact operates only through the Relative Wage effect. As before, we show analytically that the relative skill intensity of the sectors to which demand shifts as a result of the preference change is a crucial factor in determining the welfare consequences. Guided by this analysis, we conclude by estimating the distributional welfare consequences of such preference shifts.

Related Literature Our framework identifies and quantifies the mechanisms through which sectoral demand and supply shifts affect the distributional welfare in the economy. As such, it is linked to several main strands of literature.

Our analysis of the distributive effects of sectoral technical change is related to three literature streams. First, a substantial body of literature, tracing back to "Baumol cost disease" (Baumol and Bowen (1965) and Baumol (1967)), studies the macroeconomic implications of sectoral shocks. Empirically, Nordhaus (2008) verifies a key prediction of the Baumol effect, showing that technologically stagnant sectors exhibit rising relative prices and declining relative real outputs.[2] Relatedly, the literature on production networks emphasizes the importance of interactions between sectors for the propagation of sectoral technical changes to aggregate productivity (see, for instance, Hulten (1978), Durlauf (1993), and more recently, Acemoglu et al. (2012) and Baqaee and Farhi (2019)). Broadly, this body of work focuses on the implications of sectoral changes to aggregate growth. In contrast, we analytically analyze the determinants of the heterogeneous welfare impact of sectoral technical change and estimate these welfare effects.

Second, our work is related to the structural change literature, which analyzes sectoral reallocation of economic activity between manufacturing, services, and agriculture (for recent contributions, see, e.g., Kongsamut, Rebelo and Xie (2001), Ngai and Pissarides (2007), Buera and Kaboski (2012), Herrendorf, Rogerson and Valentinyi (2013), Matsuyama (2019), Baqaee and Burstein (2021), Comin, Lashkari and Mestieri (2021), Alder, Boppart and Muller (2022), Kulish et al. (2023) as well as the review in Herrendorf, Rogerson and Valentinyi (2014)). In contrast to this literature, which focuses on the determinants of structural change, we provide a general framework that uncovers the general equilibrium mechanisms governing the differential welfare impact of sectoral technical changes. Within the structural change literature, closest to our paper is Buera et al. (2022), which explores the implications of structural change to the skill premium.[3] In addition to the focus on a different motivating question, the Buera et al. (2022) paper and our paper take a very different modelling approach. While they employ specific functional forms – both on the preferences and on the production side – our analytical framework imposes minimal structure. This allows us to derive analytical results that characterize the forces driving changes to relative wages in the economy in terms of model primitives; among others, these include the crucial role of flexible complementarity and substitutability patterns in demand across goods and their interaction with heterogeneity in skill-intensity across sectors.

Our section dedicated to sectoral technological change also relates to a third body of literature; This literature examines the impact of differential price changes on the welfare distribution in contexts where non-homothetic preferences lead consumers with different income levels to have differential consumption baskets. For example, the inflation inequality literature (see Jaravel (2021) for a comprehensive survey) shows how distinct inflation rates across goods, combined

[2] Hartwig (2011) shows similar results for Europe. Duernecker, Herrendorf and Valentinyi (2024) discuss the link between structural transformation and the Baumol cost disease.

[3] Ngai and Petrongolo (2017) and Ngai, Olivetti and Petrongolo (2024) explore the implications of structural transformation for gender differences in employment and hours worked, as well as for the gender wage gap.

with diverse consumption baskets among households, shape welfare outcomes. Similarly, the international trade literature examines the influence of trade on the relative prices of exported and imported goods and its implications on welfare when accounting for heterogeneous consumption baskets among households (see, for example, Fajgelbaum and Khandelwal (2016)). Our analysis shows that sectoral technical change also impacts the welfare distribution through such a price mechanism: Sectoral technical change affects the relative prices of goods which, given differential consumption shares, will affect the welfare distribution. As explained above, our decomposition result reveals a second, quantitatively more important mechanism through which sectoral technical change affects the welfare distribution – namely, through its impact on relative wage levels. As such, our findings emphasize that understanding the overall consequences of sectoral changes on the welfare distribution mandates a model that integrates both supply-side and demand-side heterogeneity.

Moving beyond sectoral technological change, our analysis also intersects with two additional literature strands, each distinct in its focus and contribution. First, a substantial body of empirical literature investigates the effects of government expenditure changes on the economy (See Ramey (2016), for a comprehensive survey). Our analysis emphasizes that understanding the distributional effects of changes in government expenditure requires considering not just the expenditure level but also its composition, as this affects labor demand, and consequently, relative wages in the economy. Second, consumer responses during the COVID-19 pandemic have spurred interest in sector-specific demand shifts (See for example Cox et al. (2020) and Beraja and Wolf (2021)). Our analysis underscores the distributional welfare impacts of such preference-driven demand shifts.

2. Model

We study a multi-sector general equilibrium model that imposes minimal structure on preferences and production. This allows us to identify the fundamental economic mechanisms that shape the distributional welfare impact of sectoral shifts without confining our analysis to restrictive parametric forms.

2.1. Setup

Workers derive utility from the consumption of a bundle of N different goods. Each of these N goods is produced by perfectly competitive firms that maximize profits and use two inputs: high- and low-skilled labor. Workers can move freely across sectors, and as such there are only two wages in the economy, one for each skill level. In what follows we formally present the model.

2.1.1. Production

The model comprises N sectors, each producing a different good. Sector i produces Y_i goods using the following constant returns to scale production function:

$$Y_i = A_i F^i(L_i, H_i), \forall i \in N,$$

where L_i and H_i denote low- and high-skilled labor inputs respectively, and A_i is a Hicks neutral productivity parameter. The representative firm in sector i solves:

$$\max_{L_i, H_i} P_i Y_i - W_L L_i - W_H H_i,$$

where P_i denotes the price of sector i's good, and W_L and W_H denote the wages of low- and high-skilled, respectively. Crucially, the production function F is indexed by i as well, allowing for differential production elasticities of the two inputs across sectors. Finally, we define α_i to be the equilibrium L labor share of sector i: $\alpha_i = \frac{W_L L_i}{P_i Y_i}$.

2.1.2. Workers

There are two types of workers, low-skilled and high-skilled, with a mass of L and H, respectively. Types are fixed (a worker cannot switch type), and we normalize the population so that $L + H = 1$. Throughout the paper, with a slight abuse of notation, we refer to workers of type L or H.

Workers derive utility from the bundle of goods and supply work inelastically. The maximization problem for an individual of type $m \in \{L,H\}$ is given by:

$$\max_{C_{m,1},\ldots,C_{m,N}} U\left(C_{m,1},\ldots,C_{m,N}\right)$$
$$\text{s.t. } \sum_{i=1}^{N} P_i C_{m,i} = W_m,$$

where $C_{m,i}$ is the consumption of good i by a worker of type m. Importantly, we do not restrict the utility function to be homothetic. As such, throughout the paper, we denote the expenditure shares of each type $m \in \{L,H\}$ for good i by $s_i(W_m, \mathbf{P})$, as these shares depend on the worker's wage, W_m, and the price vector $\mathbf{P}$.

2.1.3. Equilibrium

In equilibrium both firms and workers behave optimally and all markets clear. Given the lack of frictions, wages are equated across all sectors, and hence workers are indifferent over which sector to work in. Formally, the market for low-skilled labor is in equilibrium when the total supply of low-skilled labor input matches its demand across the N sectors, i.e.,

$$\sum_{i=1}^{N} L_i = L,$$

and analogously, for the high-skilled labor market, in equilibrium

$$\sum_{i=1}^{N} H_i = H.$$

For any sector $i \in N$, the production of its good must be equal to the sum of the demand from both low- and high-skilled workers, i.e.,

$$LC_{L,i} + HC_{H,i} = Y_i.$$

As discussed in the end of Appendix A.1, under regularity conditions the model has a unique equilibrium.

2.2. Welfare

Our welfare measure is a variant of the Equivalent Variation measure. Specifically, given any change that affects equilibrium prices and wages, the induced welfare change is defined as the incremental income necessary to obtain the post-change utility at the pre-change prices and wages. Given the expenditure function $e(\mathbf{P},u)$ for a vector of prices $\mathbf{P}$ and utility level u, the welfare change for type $m \in \{L,H\}$ is given by:

$$\widetilde{EV}_m = e(\mathbf{P}_0, u_{m,1}) - W_{m,0} \ ,$$

where $\mathbf{P}_0$ are the pre-equilibrium prices, $u_{m,1}$ is the post equilibrium utility for type m, and $W_{m,0}$ is the pre-change wage for type m.

Using Roy's identity, a well-known result is that, locally, the welfare change can be decomposed into changes in wages and changes in good prices:

$$\widetilde{EV}_m = dW_m - \Sigma_i C_i(W_{m,0}, \mathbf{P_0}) dP_i .$$

Normalizing the welfare change by the pre-change wage, and using a circumflex to denote percent deviations from pre-change levels, it follows that the percent change in welfare is given by:

$$EV_m := \frac{\widetilde{EV}_m}{W_{m,0}} = \widehat{W_m} - \Sigma_i s_i(W_{m,0}, \mathbf{P_0}) \widehat{P_i} = \Sigma_i s_i(W_{m,0}, \mathbf{P_0}) \widehat{\left(\frac{W_m}{P_i}\right)} . \tag{1}$$

The first equality in Equation (1) establishes that the change in welfare equals the change in the real wage of type j with a type-specific price index, where the weights are given by type-specific consumption shares $s_i(W_{m,0}, \mathbf{P_0})$. The second equality shows that this change in welfare can be also viewed as the share-weighted average change in the wage deflated by each good's price, $\widehat{\left(\frac{W_m}{P_i}\right)}$.

While Equation (1) is instrumental in understanding the determinants of welfare, it does not identify the economic forces dictating the endogenous changes in wages and sectoral prices in response to a sectoral technical change. To take a first step in addressing this, we re-write EV_L as[4]

$$EV_L = \Sigma_i \left(\widehat{F_L^i(1, \frac{H_i}{L_i})} \right) s_i(W_{L,0}, \mathbf{P_0}) + \Sigma_i \widehat{A}_i s_i(W_{L,0}, \mathbf{P_0}), \tag{2}$$

where F_L^i is the partial derivative with respect to L, and we have used the fact that from the firm's

[4] Similar derivations apply to EV_H.

optimization $\widehat{\left(\frac{W_L}{P_i}\right)} = \left(\widehat{F_L^i(1,\frac{H_i}{L_i})}\right) + \widehat{A}_i$. Hence, the change in welfare can be decomposed into two parts, the first stemming from the equilibrium reallocation of inputs (i.e., changes in $\frac{H_i}{L_i}$), and the second stemming from changes in productivity. The following lemma then shows that this reallocation effect can be expressed as a function of the relative wage. Its proof, as well as all other proofs, is provided in Appendix A.1.

Lemma 1. *The percentage change in the welfare of low-skilled workers satisfies*

$$EV_L = \widehat{\left(\frac{W_H}{W_L}\right)} \Sigma_i \left(\alpha_{i,0} s_i(W_{L,0}, \mathbf{P_0}) - 1\right) + \Sigma_i \widehat{A}_i s_i(W_{L,0}, \mathbf{P_0}). \tag{3}$$

Similarly, the percent change in the welfare of high-skilled workers is given by

$$EV_H = \widehat{\left(\frac{W_H}{W_L}\right)} \Sigma_i \left(\alpha_{i,0} s_i(W_{H,0}, \mathbf{P_0})\right) + \Sigma_i \widehat{A}_i s_i(W_{H,0}, \mathbf{P_0}), \tag{4}$$

where we define $\alpha_{i,0}$ as the pre-change low-skilled labor share in sector i.

In the sections that follow we show how the model's primitives determine the relative wage's response to supply and demand shifts. This, together with Lemma 1, allows us to analyze the forces that determine the distributional consequences of such shifts, which are captured by the difference in the welfare impact across types $\Delta EV := EV_H - EV_L$.

3. Sectoral Technical Change: Analytical Results

Our key objective in this section is to study the distributional welfare effects of sectoral technical change. Specifically, we consider a Hicks neutral change in productivity to a given sector, $k \in \{1...N\}$, denoted by $\widehat{A}_k$, while holding all other productivities constant. Throughout, without loss of generality, we normalize the price of the sector experiencing the change to 1. We denote $\eta_{W_H/W_L, A_k}$ to express the elasticity of the relative wage (the high-skill vs. low-skill) in relation to the aforesaid change in sectoral productivity, and throughout we refer to it as the "relative wage elasticity." Using Lemma 1, our analysis begins with the following theorem.

Theorem 1. *Given a productivity change in any sector k (while holding all other productivities constant), the difference in welfare elasticity between worker types in response to the productivity change is as follows*

$$\Delta\eta_{EV,A_k} := \frac{EV_H}{\widehat{A}_k} - \frac{EV_L}{\widehat{A}_k} = (s_k^H - s_k^L) + \eta_{W_H/W_L,A_k}\left[1 + \Sigma_i \alpha_{i,0}\left(s_i^H - s_i^L\right)\right], \tag{5}$$

where $\forall i$ *we define* $s_i^H := s_i(W_{H,0}, \mathbf{P_0})$ *and* $s_i^L := s_i(W_{L,0}, \mathbf{P_0})$.

Equation (5) shows that $\Delta\eta_{EV,A_k}$, which measures the distributional welfare impact of the productivity change, can be decomposed into two main objects. The first object, $s_k^H - s_k^L$, stems from non-homotheticity, reflected in the difference in consumption shares over types. A positive technical change to sector k directly results in a price decline for good k. Consequently, the type with a higher consumption share of this good experiences a larger welfare gain. In what follows, we refer to this effect as the *Engel effect*.

The second object is comprised of two factors: $\eta_{W_H/W_L,A_k}$, and an extra term that captures the covariance between the labor share and the difference over types in consumption shares. It is easy to show that the covariance term is positive, and hence the sign of the second term is determined by $\eta_{W_H/W_L,A_k}$.[5] Furthermore, empirically, the term $\Sigma_i \alpha_{i,0}\left(s_i^H - s_i^L\right)$ is very close to zero (see the discussion in footnote 18), implying that the magnitude of the second term is dominated by the relative wage elasticity. In what follows, we refer to this second effect as the *Relative Wage* effect.

3.1. Signing the Skill-Premium Elasticity

Given the significant role of the relative wage elasticity, $\eta_{W_H/W_L,A_k}$, we proceed by analyzing its determinants, beginning with the following lemma.

Lemma 2. *The set of equilibrium conditions of the model, as outlined in Section 2.1.3, can be consolidated into a single market-clearing condition for high-skilled individuals. This condition is solely a function of the exogenous productivities and the relative wage:*

$$\mathscr{H}(\mathbf{A}, \frac{W_H}{W_L}) = 0. \tag{6}$$

[5] To see that the covariance term is positive, note that:

$$1 + \Sigma_i \alpha_{i,0}\left(s_i^H - s_i^L\right) = 1 + \Sigma_i \alpha_{i,0} s_i^H - \Sigma_i \alpha_{i,0} s_i^L > 0,$$

where the last inequality stems from the fact that $\alpha_{i,0} \in (0,1)\, \forall i$.

This single equation inherently encompasses all the equilibrium conditions of the other markets, and therefore functions as an excess demand equation specifically for the high-skilled labor market.

Equation (6) serves two key purposes. First, it enables us to ascertain what influences the *sign* of the relative wage's elasticity with respect to technical change, which we establish below in Theorem 2. Second, it elucidates how model primitives – preferences and production functions – determine this elasticity, as established in Theorem 3. As a first step, Theorem 2 signs the relative wage elasticity:

Theorem 2. *The sign of the relative wage elasticity with respect to a technical change in sector k (while holding all other productivities constant) is given by*

$$sign\left(\eta_{W_H/W_L,A_k}\right) = -sign\left\{\frac{d}{dA_k}\left[\sum_{i=1}^{N}\alpha_i\left(S_H s_i^H + S_L s_i^L\right)\right]\right\}, \tag{7}$$

where $S_H = \frac{W_H H}{(W_H H + W_L L)}$ *and* $S_L = \frac{W_L L}{(W_H H + W_L L)}$ *represent the aggregate expenditure shares of the high-skilled and low-skilled individuals in the economy, respectively.*

Recalling that α_i denotes the labor share of the low-skilled in sector i, Equation (7) illustrates the importance of whether demand moves either toward or away from the relatively high-skilled sectors in influencing the relative wage of high- versus low-skill workers. Specifically, when technical change reduces the share-weighted-α – i.e., when $\sum_{i=1}^{N}\alpha_i\left(S_H s_i^H + S_L s_i^L\right)$ decreases – the demand shifts towards more high-skill sectors. Equation (7) indicates that in such a case, the high-skilled relative wage increases.

3.2. The Determinants of the Relative Wage Elasticity

The next theorem characterizes the relative wage elasticity – a key determinant of the distributional welfare impact of sectoral technical change – in terms of model primitives.[6] Applying the implicit function theorem to Equation (6), and writing the result in terms of elasticities, we have:

[6]Henceforth, all parameters in the theoretical derivations refer to their pre-change values, and hence for ease of notation we drop the "0" subscript.

Theorem 3. *The elasticity of the equilibrium relative wage with respect to a technical change in sector k (while holding all other productivities constant) is given by*

$$\eta_{W_H/W_L,A_k} = -\frac{\eta_{\mathscr{H},A_k}\left(\mathbf{A},\frac{W_H}{W_L}\right)}{\eta_{\mathscr{H},W_H/W_L}\left(\mathbf{A},\frac{W_H}{W_L}\right)} = -\left(\frac{\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L \eta_{s_i^L,P_k} + S_H s_i^H \eta_{s_i^H,P_k}\right)}{Q\left(\boldsymbol{\alpha},\boldsymbol{\sigma},S_L,S_H,\mathbf{s}^L,\mathbf{s}^H,\boldsymbol{\eta}_{\mathbf{C},\mathbf{P}}^L,\boldsymbol{\eta}_{\mathbf{C},\mathbf{P}}^H,\boldsymbol{\eta}_{\mathbf{C},\mathbf{W}}^L,\boldsymbol{\eta}_{\mathbf{C},\mathbf{W}}^H\right)}\right), \tag{8}$$

where, $\eta_{\mathscr{H},A_k}$ *and* $\eta_{\mathscr{H},W_H/W_L}$ *denote the partial elasticity of* $\mathscr{H}$ *with respect to* A_k *and* W_H/W_L*, respectively.*[7] $\boldsymbol{\sigma}$ *is the vector of elasticities of substitution of the N production functions.* $\eta_{s_i^L,P_k}$ *is the uncompensated elasticity of the consumption share of good i with respect to* P_k *for the L types. Additionally,* $\boldsymbol{\eta}_{\mathbf{C},\mathbf{P}}^L$ *denotes the uncompensated price elasticity matrix with element* $\{i,j\}$ *equal to the uncompensated price elasticity of good i to price j for the L types.* $\boldsymbol{\eta}_{\mathbf{C},\mathbf{W}}^L$ *is the vector of Engel elasticities for the L types.* $\eta_{s_i^H,P_k}$, $\boldsymbol{\eta}_{\mathbf{C},\mathbf{P}}^H$ *and* $\boldsymbol{\eta}_{\mathbf{C},\mathbf{W}}^H$ *are defined analogously for the H types. Finally, Q is a real-valued function, explicitly defined as:*

$$\begin{aligned} Q = & \underbrace{-\sum_{i=1}^{N}\sigma_i(1-\alpha_i)\alpha_i\left(S_L s_i^L + S_H s_i^H\right)}_{\text{Substitution effect (Supply)}<0} \\ & \underbrace{+\frac{S_L}{W_L}(\boldsymbol{\alpha}\circ\mathbf{P})'\,\boldsymbol{Slutsky}^L(\boldsymbol{\alpha}\circ\mathbf{P}) + \frac{S_H}{W_H}(\boldsymbol{\alpha}\circ\mathbf{P})'\,\boldsymbol{Slutsky}^H(\boldsymbol{\alpha}\circ\mathbf{P})}_{\text{Substitution effect (Demand)}<0} \\ & \underbrace{-\left[\sum_{j=1}^{N}\alpha_j S_H s_j^H\right]\sum_{i=1}^{N}\alpha_i\left(\frac{\partial E_i^H}{\partial W} - \frac{\partial E_i^L}{\partial W}\right)}_{\text{Income effect (Non-homotheticity)}\lesseqgtr 0} \end{aligned} \tag{9}$$

where E_i^H *and* E_i^L *are expenditures of high- and low-skilled workers, respectively, and* $\boldsymbol{Slutsky}^H$ *and* $\boldsymbol{Slutsky}^L$ *are the Slutsky matrices for high- and low-skilled workers, respectively.*

Following the implicit function theorem intuition, the numerator in Equation (8) captures the partial equilibrium effect of technical change on excess demand for high-skilled labor while holding constant the relative wage. Similarly, the denominator captures the response required in the relative wage in order to maintain excess demand at zero given the change in demand induced by the technical change. We discuss each in turn below.

[7] The partial elasticity of $\mathscr{H}(A,\frac{W_H}{W_L})$ with respect to A is defined as $\frac{\partial\mathscr{H}}{\partial A}\frac{A}{\mathscr{H}}$, and analogously with respect to $\frac{W_H}{W_L}$.

3.2.1. How Excess Demand Responds to the Relative Wage: The Denominator

The denominator in Equation (8), Q, comprises three components. The first term captures a standard substitution effect in production between low- and high-skilled workers: when the relative wage of high-skilled workers increases, the demand for these workers declines. This effect is negative as long as at least one production function is not of the Leontief type (indicated by $\sigma_i \geq 0$). The second term reflects consumer substitution patterns across goods due to price changes from relative wage adjustments, holding income constant; As a weighted sum of quadratic forms using the low- and high-skilled Slutsky matrices, this term is negative. This is intuitive – when the relative wage of the high-skilled increases, the relative price of relatively high-skilled intensive goods rises. The substitution effects, as captured by the Slutsky matrix, imply that consumers shift away from these goods, leading to a decline in the relative demand for high-skilled workers.

The third term in the denominator reflects the varied income effects on demand due to non-homothetic preferences between low- and high-skilled workers. As the high-skilled relative wage increases, income effects operate in the opposite direction for the low- and high-skilled, and hence the sign of this effect is ambiguous. For instance, should the demand of high-skilled workers shift more *towards* sectors intensive in high-skilled labor (low-α) than the corresponding shift of low-skilled workers *away* from these sectors, the overall demand for high-skilled sectors will increase. Consequently, this would result in a positive sign for the third term.

Our empirical estimates of Q are always negative. This outcome primarily arises because Q can only be positive if the third term, stemming from varied income effects between low- and high-skilled workers is positive. In cases of homothetic preferences, where this term is zero, it is easy to see that Q is always negative. For non-homothetic preferences, we find that in all tested specifications, the sum of the first two negative terms significantly outweighs the third term, explaining the negative sign we obtain for Q.

3.2.2. The Partial Equilibrium Response to Technical Change: The Numerator

Returning to the numerator, Equation (8) points again, as in Theorem 2, to the importance of the share-weighted-α in determining the relative wage elasticity. Crucially, unlike in Theorem 2 where the weights were equilibrium objects, the weights in this equation are defined as functions of model primitives: preference parameters and specifically, the price elasticities. As such, given a sectoral technical change to a specific sector, the degree to which goods are complements or substitutes dictates the behavior of the share-weighted-α.

To gain intuition for the numerator, note that given a change in the price of sector k, $\widehat{P}_k$, the change in the economy's share-weighted-α is given by

$$\widehat{P}_k \sum_{i=1}^{N} \alpha_i \left(S_L s_i^L \eta_{s_i^L, P_k} + S_H s_i^H \eta_{s_i^H, P_k} \right), \tag{10}$$

i.e. the numerator in Equation (8) multiplied by $\widehat{P}_k$. Consider then an increase in A_k and its associated decline in the price of sector k, $\widehat{P}_k < 0$. Given that Q is negative, Theorem 3 shows that the relative wage increases with A_k when the numerator in Equation (8) is positive (recall the negative sign in Equation (8)), and thus the change in the share-weighted-α (i.e. 10) is negative. Put differently, the relative wage increases with sectoral technical change when the substitution matrix is such that a reduction in price in sector k shifts demand towards relatively high-skill intensive sectors.

The results from Theorem 3 also allow us to revisit Theorem 2 and establish conditions that determine the sign of the relative wage elasticity in terms of model primitives. This is done in the following lemma.

Lemma 3. *Assuming that* $Q < 0$*,*

a. If $\alpha_k - \Sigma_i \alpha_i w_{i,k}^m > 0$*, for* $m \in \{H, L\}$*, then* $\eta_{W_H/W_L, A_k} > 0$

b. If $\alpha_k - \Sigma_i \alpha_i w_{i,k}^m < 0$*, for* $m \in \{H, L\}$*, then* $\eta_{W_H/W_L, A_k} < 0$

with weights defined as $w_{i,k}^m = \frac{s_i^m \eta_{C_i^m, P_k}}{\Sigma_j s_j^m \eta_{C_j^m, P_k}}$.[8]

To understand the intuition behind Lemma 3, it is useful to decompose for each type $m \in \{L, H\}$ the change in the share-weighted-α in the following manner:

$$\widehat{P}_k \Sigma_{i=1}^N \alpha_i s_i^m \eta_{s_i^m, P_k} = \widehat{P}_k (s_k^m \alpha_k - s_k^m \Sigma_{i=1}^N \alpha_i w_{i,k}^m). \tag{11}$$

This decomposition stems from the fact that the own price elasticity of good k satisfies $\eta_{s_k^m, P_k} = 1 + \eta_{C_k^m, P_k}$, while the cross price elasticities satisfy $\eta_{s_i^m, P_k} = \eta_{C_i^m, P_k}$, for $i \neq k$. Naturally, the own price elasticity expression stems from the fact that a price change of good k has a direct effect on the expenditure share of good k. The impact of technical change on the change in the share-weighted-α (the left hand side of Equation (11)) is thus affected by two forces. The first element, $\widehat{P}_k s_k^m \alpha_k$, captures the direct price effect on the share-weighted-α: holding constant demand, following a

[8] If $\alpha_k - \Sigma \alpha_i w_{i,k}^m$ are of opposite signs for H and L, it is straightforward to derive an equivalent condition directly from the numerator of Theorem 3. This condition will also incorporate the weights of H and L in the population.

positive technical change to good k, the expenditure share of that good declines simply because its price declines; as such this direct price effect is always negative. The second element, $\widehat{P}_k s_k^m \Sigma \alpha_i w_{i,k}^m$, captures how the share-weighted-α changes due to the demand response of all goods (including k) to the price change. These shifts are governed by the complementary-substitution patterns with good k, as captured by the weights $w_{i,k}^m$.

From Theorem 3, we know that positive technical change causes the relative wage to increase when the change in the share weighted-α is negative, i.e., when $\widehat{P}_k \Sigma_{i=1}^N \alpha_i s_i^m \eta_{s_i^m, P_k} < 0$.[9] Thus, from Equation (11) it follows that following a positive technical change the relative wage rises when the sum of the direct price effect and the demand shift effect is negative — i.e., when $s_k^m \alpha_k - s_k^m \Sigma_i \alpha_i w_{i,k}^m > 0$, as stated in Lemma 3.

To further clarify the role of the substitution matrix, it is instructive to consider the case where sector k, experiencing the positive technical change, has the highest α. Assume first that all goods are gross complements.[10] In this case, it is easy to show that all the weights in Lemma 3 are positive and sum to 1. As such, $\alpha_k > \Sigma_i \alpha_i w_{i,k}$ and so condition (a) in the Lemma 3 always holds. In this scenario, technical positive change to the highest α sector always increases the high-skilled relative wage. Intuitively, when all goods are gross complements, positive technical change to a sector increases demand for all other sectors. If the sector experiencing the technical change has the highest α, this results in increasing overall demand for high-skilled labor in the economy.[11]

Consider now the scenario where not all goods are gross-complements, and in particular, assume that the demand for sector k, the highest-α sector, is sufficiently elastic, i.e. with an own-price elasticity satisfying $\eta_{C_k, P_k} < -1$.[12] In this scenario, positive technical change can cause the relative wage to *decrease*; Intuitively, the decline in P_k due to the technical change coupled with the high own-price elasticity shifts demand *towards* sector k, the sector with the lowest skill intensity – and hence the relative wage declines.

3.3. Taking Stock: Analytical Framework

To summarize, this section provides an analytic framework that characterizes the distributional impact of sectoral technical change and its underlying mechanisms. We started by showing in

[9]Recall that $\widehat{P}_k$<0 following positive technical change.

[10]This is relevant for many preference specifications used in the Macroeconomics literature. See, for example, estimates of the non-Homothetic CES preferences in Comin, Lashkari and Mestieri (2021).

[11]The reverse holds for the lowest α. See Corollary 1 in Appendix A.1 for a formal proof.

[12]In such a case there must be at least one sector that is a substitute with good k.

Theorem 1 that the distributional welfare effects are driven by two components – the Engel effect and the Relative Wage effect. The Engel effect is driven by the non-homotheticities and the resulting differential expenditure shares of high- and low-skilled individuals. With regard to the Relative Wage effect, Theorem 2 highlights that the change in the relative wage stems from the change in the share-weighted-α. Finally, Theorem 3 emphasizes the role of consumption price elasticities in shaping this share-weighted-α (through changes in demand), which in turn influences both the relative wage and the welfare distribution.

This discussion underscores that the effect of technical change on share-weighted-α and the elasticity of the relative wage is an empirical question which is contingent upon the extent to which sectors are substitutes or complements to the sector experiencing the productivity shift. Consequently, the ensuing Section 4 proceeds with estimating this substitution matrix. It then uses the results in Theorem 3 to estimate the differential welfare impact of sectoral technical change.

4. A Quantitative Evaluation of Sectoral Technical Change

In the previous section we characterized the distributional impact of sectoral technical change. Theorems 1 and 3 have another important implication: they provide a framework to assess the welfare effects of sectoral productivity changes by estimating key preference and production parameters, thereby circumventing the need for assuming specific functional forms.

Our theoretical analysis implies that the set of parameters required for estimation fall into two broad categories – one pertaining to consumption and one to production. On the consumption side we need to estimate for both high- and low-skilled workers (i) the aggregate expenditure shares, (ii) the expenditure shares by good categories, (iii) the matrix of uncompensated elasticities of consumption goods with respect to prices, and (iv) the Engel elasticities. On the production side, we need to estimate for each category (i) the equilibrium labor shares, and (ii) the elasticity of substitution of the N production functions. Finally, we need to measure for each category its specific technical change. In what follows, we discuss our estimation of these elements.

4.1. Estimation Framework

4.1.1. Consumption parameters

Given our analytical results, we do not need to constrain our analysis to a particular utility function and instead estimate an almost ideal demand system (AIDS) following Deaton and Muellbauer (1980). We then recover the required price and total expenditure elasticities from this estimated demand system. In contrast, confining to a particular utility function would impose restrictive assumptions on the price and total expenditure elasticities, which we have shown, in the previous section, are key determinants of the welfare impact of sectoral technical change.

4.1.2. Production parameters

The first set of production parameters consist of the equilibrium labor shares by good categories. As we describe below, we measure these good-category labor shares directly from the data using industry-level labor shares and a mapping between good-categories and industries. The second set of production parameters are the sectoral elasticity of substitutions. To obtain the vector of elasticities of substitution, we use variation across sectors in the growth rates of the ratio of high- to low-skilled workers (see Appendix A.2.2 for details).

4.2. Data

For the consumption side of the data we use the CEX. The CEX is a dataset produced by the U.S. Bureau of Labor Statistics that provides detailed information on the spending habits, income, and household characteristics of U.S. consumers. We use the dataset provided by Comin, Lashkari and Mestieri (2021), and keep their sample of urban households with a present household head aged between 25 and 64 for the years 1999-2010 and four CEX interviews.[13] We define low- and high-skilled workers in the CEX based on the household head's education level, with high-skilled defined as those with a Bachelor degree or above.

As in Comin, Lashkari and Mestieri (2021) we combine the CEX data with regional quarterly price series by consumption category from the BLS's urban CPI (CPI-U). To aggregate prices for a given good category, we use region-by-quarter aggregate expenditure shares.

[13]The data construction in Comin, Lashkari and Mestieri (2021) is based on Aguiar and Bils (2015). We follow Comin, Lashkari and Mestieri (2021) and drop households at the top and bottom ten percent of the total consumption distribution.

Our analysis requires income shares at the goods-category level, i.e. α_i. To obtain these data, we first calculate income shares by skill at the industry level using earnings data from the CPS over the years 1999-2010. We then use an input-output matrix to map industry cost shares to consumption category cost shares (similar to Bils, Klenow and Malin (2013)).[14] To obtain the income share of a given good-category, we calculate the weighted-average of the income shares of the industries associated with this goods-category, with the weights equal to the input costs. Finally, we use KLEMS U.S. data on industry-level TFP. We aggregate these TFP measures to a goods-category measure of TFP using an analogous process to that described for income shares.

4.3. Results

In what follows we report the results for our main quantitative analysis. In doing so we estimate the required set of preferences and production function parameters, and then use the results from the analytical analysis to calculate the distributional welfare impact of sectoral productivity changes.

Given our interest in sectoral technical changes and the importance of substitution patterns derived in the theoretical analysis, there is a concern that the three-sector categorization commonly employed in the structural change literature (Agriculture, Manufacturing, Services) is too coarse for our purposes and does not adequately capture the substitution patterns in which we are interested. Hence, in our main analysis, we consider a categorization which consists of the following seven sectors: Housing; Food Away, Entertainment, and Apparel; Transportation; Food at Home; Other Services; Durables; and Utilities. Appendix Table A1 reports the descriptive statistics for our sample. This finer categorization naturally gives rise to more complex substitution patterns, which the AIDS formulation easily captures. We return to this issue of the substitution patterns in the three vs. the seven sector categorization in Section 4.5.

Theorem 3 shows that estimating the distributional impact of sectoral change requires recovering price and total expenditure elasticities. To do so, we estimate an AIDS to recover the required elasticities. In estimating the demand system we use similar controls and instruments to Comin, Lashkari and Mestieri (2021).[15] We thus estimate separately the vectors of parameters of the demand system ($\boldsymbol{\theta} : \boldsymbol{\gamma}, \boldsymbol{\beta}_0, \boldsymbol{\beta}, \boldsymbol{\delta}$) for the low- and high-skilled types $m \in \{H, L\}$ using a standard

[14]See Appendix A.2 for a detailed discussion of how we link the various data sets.

[15]The controls include household age (by age groups), household size, and number of earners. Household expenditures are instrumented with after-tax annual household income and the income quintile of the household. Regional prices for each goods-category are instrumented using the average price of the good in the other regions, weighting these prices by the goods' regional expenditure share in the households' region.

AIDS specification:[16]

$$s_{i,j} = \beta_i^m + \boldsymbol{\gamma}_i^{m\prime} \log \mathbf{P}_j + \delta_i^m \left[\log E_j - b\left(\log \mathbf{P}_j, \boldsymbol{\theta}^m\right)\right] + u_{i,j}$$
$$b\left(\log \mathbf{P}_j, \boldsymbol{\theta}^m\right) = \boldsymbol{\beta}_0^{m\prime} \mathbf{X}_j + \boldsymbol{\beta}^{m\prime} \log \mathbf{P}_j + 0.5 \log \mathbf{P}_j^{\prime} \boldsymbol{\Gamma}^m \log \mathbf{P}_j,$$

where there is an equation for each sector i, $s_{i,j}$ is the expenditure share of good i for household j in a specific period, $\mathbf{P}_j$ is the vector of goods prices, E_j is total expenditure for household j, and $\mathbf{X}_j$ are household level controls. Estimation results are reported in Appendix Tables A2-A3.

The Differential Welfare Effects of Sectoral Technical Changes Figure 1 provides a first look at the differential welfare impact of sectoral technical change. For each of the seven sectors, the figure depicts the welfare elasticity with respect to technical change in sector k for high- and low-skilled workers, η_{EV_H,A_k} and η_{EV_L,A_k}, respectively, as implied by our estimates. For example, the figure shows that a 10% positive technical change in the "Food Away, Entertainment, and Apparel" sector leads to an approximately 2% welfare increase for high-skilled workers.

There are three key messages to the figure. First, the magnitude of the welfare impact of technical change varies across sectors. Second, within each sector, technical change affects welfare of high- and low-skilled workers differently. Third, this differential effect between high- and low-skilled workers varies across sectors in both magnitude and sign. Indeed, across the seven sectors, the implied differential effect varies from a 9p.p. (24%) *smaller* welfare increase for high- versus low-skilled workers (Housing) to a 9p.p. (75%) *larger* welfare increase for the high-skilled workers (Food Away, Entertainment, and Apparel).

Decomposing the Differential Welfare Effects Theorem 1 allows us to decompose the differential welfare change, $\Delta\eta_{EV,A_k}$, into its two underlying components – the Engel effect and the Relative Wage effect. The results are shown in Figure 2.

Panel A depicts the seven industries according to two attributes: their degree of low skill intensity, α, and their Engel elasticity. As implied by Section 3 and as discussed below, these two attributes are directly linked to the channels underlying the welfare analysis: First, the Engel elasticity determines the difference in expenditure shares between high- and low-skilled workers. Higher

[16] We use the package provided by Lecocq and Robin (2015), and use block bootstrap to obtain clustered standard errors at the household level. Following Banks, Blundell and Lewbel (1997) we also check the robustness of our results allowing for quadratic Engel curves and find similar results.

Engel elasticities imply that the difference in expenditure shares between high- and low-skilled workers is larger (i.e., $s^H - s^L$ rises), which means that the Engel effect stemming from positive technical change will be positive. Second, as shown in Section 3, the relative skill intensity of each sector, α, plays an important role in determining the relative wage elasticity.

Panel B of the figure decomposes the welfare impact of technical change into the Engel effect and the relative wage elasticity for the seven sectors. This decomposition is based on Equation (5), which we repeat here for convenience,

$$\Delta\eta_{EV,A_k} = (s_k^H - s_k^L) + \eta_{W_H/W_L,A_k}\left[1 + \Sigma_i \alpha_i \left(s_i^H - s_i^L\right)\right]. \tag{12}$$

The Engel Effect ($s_k^H - s_k^L$): Consider first technical change in the (i) Food at Home, (ii) Utilities, and (iii) Transportation sectors. Because all three are necessity sectors, expenditure shares of low-skilled workers are higher than that of high-skilled workers. As such, when these sectors experience a positive technical change, the associated price decrease benefits low skill workers by more. This is reflected in a negative Engel effect depicted for these sectors in Panel B of Figure 2. In contrast, the sectors (i) Food Away and Entertainment, (ii) Housing, and (iii) Other Services, are luxury sectors, so the expenditure shares of low-skilled workers are lower than that of high-skilled workers. Thus, the Engel effect of technical change in these sectors benefits high-skilled workers relatively more.[17]

The Relative Wage Effect ($\eta_{W_H/W_L,A_k}$): Panel B of Figure 2 shows that the Relative Wage effect is the dominant force in determining the overall distributional impact of technical change for five out of the seven sectors.[18] Further, the results show that in several instances, the Relative Wage effect is larger than the *total* effect of technical change; this is because in these cases the Engel effect is in the opposite directions to the Relative Wage effect.

To clarify the determinants of the Relative Wage effect's sign, recall that Lemma 3 shows that this sign results from comparing a sector's skill intensity, α_k, to the weighted average skill intensities, $\Sigma_i \alpha_i w_{i,k}$, with the weights based on price elasticities. As discussed in Section 3.2.2 this comparison

[17]We estimate the Durable sector's Engel curve to be slightly above one for low-skilled workers, and slightly below one for the high-skilled. As such, the Engel effect is about zero in this sector.

[18]We note that quantitatively the covariance term is negligible, given the fact that it is comprised of a product of share differentials multiplied by another share (the skill intensity). Hence, in what follows, references to the relative wage elasticity disregard this covariance term. Results remain almost identical given the small magnitude of the covariance term.

encapsulates the direct price effect and the indirect effects of demand shifts, as governed by sectoral complementarity and substitutability patterns. Figure 3 graphically illustrates this relation by comparing these two effects, depicting their impact on the Relative Wage effect.[19] As the figure shows, and consistent with Lemma 3, when $\alpha_k > \Sigma_i \alpha_{\mathrm{i}} w_{i,k}$ for both types, the Relative Wage effect is positive, while if the inequality is reversed then the Relative Wage effect is negative. Finally, Figure 3 shows that the Relative Wage effect broadly follows a pattern whereby when low-skilled sectors experience a positive technical change the relative wage rises, while when high-skilled sectors exhibit such technical change, relative wage declines.

4.4. The Overall Effect of TFP Changes

In the discussion above we analyzed the differential welfare elasticities to sectoral technical change and showed that there is a significant variation in the magnitude of these elasticities. It is therefore of interest to calculate the overall differential welfare impact on high- versus low-skilled workers given the empirical changes in sectoral TFP observed in the data. To do so, we calculate for each sector in a given time period, the product of the TFP growth and the difference in the welfare elasticities as calculated above, and then sum over all sectors. Our TFP data spans the years 1987 to 2019. We therefore analyze TFP growth rates over three different time periods: 1987–1997, 1997–2007, and 2010–2019.[20] Our results indicate that the overall welfare impact on high- and low-skilled workers differs by time period; Incorporating realized TFP growth in each sector, together with the sectoral differential welfare elasticities, shows that between 1987 and 1997, high-skilled workers enjoyed an overall 20% greater welfare increase compared to low-skilled workers. This differential impact declines to 15% between 1997 and 2007, and it reverses in sign so that the high-skilled gain about 6% less in the last period of 2010–2019.

4.5. The Importance of Being Flexible

The flexibility of our framework, which imposes minimal constraints on consumption and production functional forms, has two benefits. On the analytical side, it allows us to identify the key forces that

[19] The figure depicts the comparison for both L and H types; On the x-axis we depict the sector's α, while on the y-axis we depict the sector's $\Sigma\alpha_i w_{i,k}$ for the L-type (full circles) and for the H-type (hollow circles). The numbers in the graph are the relative wage elasticities.

[20] The third period excludes 2008–2010, the period of the Great Recession, but results are qualitatively similar when analyzing 2007–2019.

shape the distributional welfare impact of sector-specific technical change – in terms of consumer preferences and sectoral production functions. Confining the analysis to particular functional forms would have masked these forces.

Beyond the analytical aspects, the flexibility of our framework also has quantitative importance. To show this, we benchmark our results to those obtained when employing the Comin, Lashkari and Mestieri (2021) non-homothetic CES preferences – a prominent non-homothetic preference specification.[21]

Figure 4 compares for each sector the ratio of the elasticity of the relative wage to sectoral technical change obtained using our flexible AIDS specification, $\eta^{AIDS}_{W_H/W_L,A_k}$, to the same elasticity obtained under the non-homothetic CES preference specification, $\eta^{nh\,CES}_{W_H/W_L,A_k}$.[22] We focus on this elasticity as the Engel effect under the two specifications are always equal, since they rely solely on the expenditure shares of workers.

Panel A provides the elasticity ratio for our seven sector categorization, indicating the large discrepancies in the elasticity estimates under the two specifications. The figure shows wide variation in this ratio, ranging from almost -2 to almost 8. Put differently, the elasticity using the AIDS specification ranges from almost double and in opposite direction to almost eight times the elasticity of the non-homothetic CES. The fact that the two specifications significantly differ one from the other is not surprising. Non-homothetic CES preferences constrain demand so that goods are either all gross complements or all gross substitutes. In contrast, our price elasticity estimates obtained from the flexible AIDS estimation (see Appendix Tables A2 and A3) exhibit a significant presence of both complementarity and substitutability between goods – only 57% of the cross-price elasticities are negative. Our analytical results emphasize the importance of substitution patterns in determining the relative wage elasticity, and hence the discrepancy between the non-homothetic CES and flexible AIDS specifications.

Panel B undertakes the same exercise, but uses the three sector categorization common in the structural transformation literature. As is well known, with higher levels of aggregation, goods tend to be more complements. Indeed, using our AIDS specification in the three-sector classification, we find this to be the case as 92% of the cross-price elasticities are negative (see Appendix Table A4). Given this high degree of complementarity between goods, it is not surprising that the ratio between the two elasticities ($\eta^{AIDS}_{W_H/W_L,A_k}$ and $\eta^{nh\,CES}_{W_H/W_L,A_k}$) are closer to one and all in the same direction, ranging

[21] See Section 5.2 for a formal definition of this utility function.

[22] See Appendix A.2.3 for a discussion of this model estimation.

from roughly one to approximately three.

In summary, this analysis underscores the significance of flexibly capturing substitution patterns, particularly those that naturally occur when goods are more disaggregated.

5. Beyond Productivity Changes: Two Additional Applications

In this section, we demonstrate the versatility of our framework, emphasizing its applicability beyond the analysis of the welfare impact of sectoral technical change. Specifically, we explore two demand-driven changes: a shift in public sector expenditures and a change to consumer preferences.

We note that the welfare impact following these demand changes is bound to be smaller than the welfare impact following sectoral technical change; In the latter case, the supply change increases the economy's production capacity, while demand shifts do not. Yet, as shown below, changes in demand will have distributional consequences due to shifts in consumption across sectors and the variation of skill intensity across them.

5.1. Public Sector Demand Shifts

So far, our multi-sector model included only private-sector goods. In this section, we extend our model to include a public-sector good – for example, this sector could be thought of as government expenditure on health care or on military personnel.

To do so, we introduce a new sector (indexed as $N+1$) which produces a public sector good G, using a constant returns to scale production function $G = A_G F(L_G, H_G)$. For a given level of G, inputs are chosen optimally, taking the equilibrium wages as given. Furthermore, the total demand for the public sector good, G, is exogenously determined and does not enter the household maximization problem. The public sector is funded by a proportional tax on labor, implying that the flat tax rate τ satisfies $\tau = \frac{G}{(W_L L + W_H H)}$, where without loss of generality, we assume that the public good is the numeraire.

What is the distributional welfare impact of an exogenous change in the level of public sector expenditure, G? Using analogous arguments to those in Section 3, it can be shown that the difference in welfare elasticities with respect to G between high- and low-skilled workers is given by:

$$\Delta\eta_{EV,G} := \frac{EV^H}{\widehat{G}} - \frac{EV^L}{\widehat{G}} = \eta_{W_H/W_L,G}\left[1 + \Sigma_i \alpha_i \left(s_i^H - s_i^L\right)\right]. \tag{13}$$

Contrasting with Equation (5), it is apparent that the Engel effect is absent. This is because unlike the case of a productivity change, a shift to the size of the public sector has no direct effect on goods prices; It affects prices only indirectly through changes in the composition of demand for private sector goods and through the direct effect on the demand for labor in the public sector. Similar to Equation (5), these effects are captured by the change in the relative wage.

Following Equation (13), we continue by analyzing the elasticity of the relative wage with respect to the public sector expenditure level, $\eta_{W_H/W_L,G}$. It is straightforward to show that Equation (6), which encompasses all equilibrium conditions in the case of sectoral technical change, can be reformulated as

$$\widetilde{\mathscr{H}}(G, \frac{W_H}{W_L}) = 0. \tag{14}$$

As in the productivity case, the $\widetilde{\mathscr{H}}$ function represents an excess demand for high skilled workers, with the adjustment that demand for such workers also incorporates those required to produce the public good. Similar to the case of technical change, applying the implicit function theorem to Equation (14) yields:

Theorem 4. *The elasticity of the equilibrium relative wage with respect to the level of public sector expenditure G is given by*

$$\eta_{W_H/W_L,G} = \frac{\tau\eta_{\tau,G}\alpha_{N+1} + (1-\tau)\eta_{(1-\tau),G}\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L \eta_{C_i^L,W_L} + S_H s_i^H \eta_{C_i^H,W_H}\right)}{\widetilde{Q}\left(\boldsymbol{\alpha},\boldsymbol{\sigma},S_L,S_H,\mathbf{s}_L,\mathbf{s}_H,\boldsymbol{\eta}^L_{\mathbf{C},\mathbf{P}},\boldsymbol{\eta}^H_{\mathbf{C},\mathbf{P}},\boldsymbol{\eta}^L_{\mathbf{C},\mathbf{W}},\boldsymbol{\eta}^H_{\mathbf{C},\mathbf{W}},\tau\right)} \tag{15}$$

$$= \frac{\tau\left[\alpha_{N+1} - \sum_{i=1}^{N}\alpha_i\left(S_L s_i^L \eta_{C_i^L,W_L} + S_H s_i^H \eta_{C_i^H,W_H}\right)\right]}{\widetilde{Q}\left(\boldsymbol{\alpha},\boldsymbol{\sigma},S_L,S_H,\mathbf{s}_L,\mathbf{s}_H,\boldsymbol{\eta}^L_{\mathbf{C},\mathbf{P}},\boldsymbol{\eta}^H_{\mathbf{C},\mathbf{P}},\boldsymbol{\eta}^L_{\mathbf{C},\mathbf{W}},\boldsymbol{\eta}^H_{\mathbf{C},\mathbf{W}},\tau\right)} \tag{16}$$

where $\eta_{\tau,G}$ and $\eta_{(1-\tau),G}$ are the partial elasticities of τ and $1-\tau$ with respect to G, respectively, holding the relative wage constant. $\widetilde{Q}$ is a real-valued function with arguments as in Theorem 3, which is provided explicitly in Appendix A.1.

As in the case of technical change, we estimate the denominator to be negative.[23] The numerator in (15) captures the effect of a change in G on the economy-wide share-weighted-α. As G increases two forces are at play. First, the economy shifts towards a larger public sector and a smaller private

[23]The argument is similar to that in Theorem 3 – when the high-skilled relative wage rises, excess demand for high-skilled labor declines (see Section 3.2).

sector. This effect is captured by the term $\tau\eta_{\tau,G}$ which multiplies the low skilled-share of the public sector, α_{N+1}, and the term $(1-\tau)\eta_{(1-\tau),G}$ which multiplies the share-weighted-α in the private sector (clearly, the former is positive and the latter is negative). Second, the increase in G is financed through a tax increase. This implies that after-tax wages of both high- and low-skilled workers decline, which impacts the share-weighted-α in the private sector due to non-homotheticities. Rewriting (15) as (16) provides an additional insight. The skill intensity of the public good sector, α_{N+1}, relative to the equilibrium (i.e. post-change) skill intensity of the private sector, $\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L \eta_{C_i^L,W_L} + S_H s_i^H \eta_{C_i^H,W_H}\right)$, determines the sign of the impact on the relative wage.

Figure 5 depicts the differential welfare impact stemming from changes in public sector expenditure level, G, over the size of the public sector (pre-change) for three different levels of public sector skill intensity. The three skill intensity levels are the minimum, mean, and maximum low-skill intensity over the seven sectors previously discussed. The different elements required for the calculation of equations (13) and (16) are as in the case of technical change.

As can be seen in the figure, and reflecting the insights from the theoretical analysis, the distributional impact of government expenditure varies according to whether the public sector is high- or low-skill intensive. The high-skilled benefit from increases in G when the public sector is high-skilled intensive, while the low skilled benefit when it is low-skilled intensive. As expected, the figure also shows that the distributional welfare impact increases with the pre-change public sector's size.

As an example, consider two types of public sector expenditures, both depicted in Figure 5.[24] Military personnel as a fraction of private consumption in the U.S. is approximately 2.25% and its skill intensity is similar to the average in the economy. As such, the distributional impact of an increase in public expenditure in this sector is relatively small. In contrast, consider the public health expenditures. This sector's low-skill intensity is approximately 0.55, which is equal to the minimum α value over the seven sectors, and as a fraction of private consumption in the U.S. its share is approximately 12%. In this sector, for high-skilled workers, the elasticity of welfare to an expenditure change is 0.017, while for the low-skilled it is -0.01, resulting in a differential elasticity favoring the high-skilled of 0.027.

[24]See Appendix A.2.4 for a discussion of the construction of these public sector measures.

5.2. Changes to Consumer Preferences

In our final application we analyze the welfare impact of demand shifts driven by changes to consumer preferences. For ease of exposition, we return to the model without a public sector. All proofs in this section are analogous to the relevant proofs for the case of technical change.

Let β_k parameterize the demand shifter for the good produced by sector k. Then, the difference in welfare elasticities between high- and low-skilled workers with respect to β_k is given by

$$\Delta\eta_{EV,\beta_k} = \frac{EV_H}{\widehat{\beta}_k} - \frac{EV_L}{\widehat{\beta}_k} = \eta_{W_H/W_L,\beta_k}\left[1 + \Sigma_i \alpha_i \left(s^H_{i,\beta_{post}} - s^L_{i,\beta_{post}}\right)\right], \tag{17}$$

where we evaluate welfare using ex-post preferences denoted by β_{post}, and $s_{i,\beta_{post}}$ denotes the consumption share of good i when using the ex-post preferences.[25]

Similar to the public sector case, the Engel effect is absent. Again, this is due to the fact that, in contrast to productivity changes, preference shifters do not directly affect the price. Hence, the key determinant of the distributional welfare impact is the elasticity of the relative wage with respect to the sectoral preference shifter, $\eta_{W_H/W_L,\beta_k}$. Following the same strategy as in the technical change case, we derive the following theorem.

Theorem 5. *The elasticity of the equilibrium relative wage with respect to the sectoral preference demand shifter, β_k, is*

$$\eta_{W_H/W_L,\beta_k} = \frac{\sum_{i=1}^{N} \alpha_i \left(S_L s^L_i \eta_{s^L_i,\beta_k} + S_H s^H_i \eta_{s^H_i,\beta_k}\right)}{Q\left(\boldsymbol{\alpha}, \boldsymbol{\sigma}, S_L, S_H, \mathbf{s}_L, \mathbf{s}_H, \boldsymbol{\eta}^L_{\mathbf{C},\mathbf{P}}, \boldsymbol{\eta}^H_{\mathbf{C},\mathbf{P}}, \boldsymbol{\eta}^L_{\mathbf{C},\mathbf{W}}, \boldsymbol{\eta}^H_{\mathbf{C},\mathbf{W}}\right)}. \tag{18}$$

where Q is the same real-valued function defined in Theorem 3.

It is instructive to note the similarity here with Equation (8). The denominators in the two equations are identical, since both capture the required response of the relative wage in order to maintain excess demand at zero. In contrast, the numerator in Equation (8) features the share elasticities with respect to prices, which allows for a direct estimation of price elasticities without requiring an explicit model specification. This stands in contrast to the current case of demand shifters: indeed, Equation (18) features the share elasticities with respect to the sectoral demand shifter, β_k. This implies that we need to explicitly model how preference shifters affect demand.

[25]The quantitative results are almost identical to when we use the ex-ante values.

Recognizing this fundamental difference between the two cases (exogenous changes to supply and preference changes), in our analysis of this case we assume that demand follows the non-Homothetic CES demand model as in Comin, Lashkari and Mestieri (2021).[26] Specifically, the demand for good k is given by

$$C_k = (\beta_k U^{\varepsilon_k}) \left(\frac{W}{P_k}\right)^{\nu}, \tag{19}$$

where W is the individual's income, and U is implicitly defined as

$$\sum_{i=1}^{N} (\beta_i U^{\varepsilon_i})^{\frac{1}{\nu}} C_i^{\frac{\nu-1}{\nu}} = 1. \tag{20}$$

Under this functional form, β_k acts as a demand shifter for consumption good k.

It follows then, that the elasticities of the consumption shares with respect to a sectoral change in demand, denoted as $\eta_{s_i^L,\beta_k}$ and $\eta_{s_i^H,\beta_k}$ in the numerator, can be expressed as follows. First, for the L types, the elasticity of the share of good k with respect to its own demand change, β_k, is given by $\eta_{s_k^L,\beta_k} = 1 - s_k^L \frac{\eta_{C_k^L,W_L} - \nu}{1-\nu}$, where we remind the reader that $\eta_{C_k^L,W_L}$ is the Engel elasticity for the L types for good k. Second, the elasticity of the share of good $i \neq k$ with respect to β_k is given by $\eta_{s_i^L,\beta_k} = -s_k^L \frac{\eta_{C_k^L,W_L} - \nu}{1-\nu}$. Analogous expressions apply to the H type. Therefore, given our Engel elasticity estimates and a value for ν we can proceed to evaluate Equation (18).[27]

Section 4.5 showed that the seven-sector goods categorization features an almost equal share of goods that are complements and goods that are substitutes while the three sector classification exhibited almost uniformly complementarities between goods. As such, because with non-Homothetic CES preferences all goods are either complements or all substitutes, and because our estimates imply that all goods are complements (as in Comin, Lashkari and Mestieri (2021)), we use the three sector classification system of Agriculture, Manufacturing, and Services.

Our quantitative results are depicted in Figure 6.[28] The differential welfare impact of sectoral demand change is intuitive. When a sector that is more low skill intensive (e.g., Agriculture or Manufacturing) experiences an increase in its demand, the high-skilled relative wage declines.

[26] We use this specification and not AIDS, since there is no natural scale invariant method to capture demand shifters within the AIDS specification.

[27] See Appendix A.2.3 for a discussion of how the non-Homothetic CES preferences is parameterized.

[28] Note that the shares in Equation (17) are post change. We approximate these as before with the average shares over our sample period. Given that the covariance term $\Sigma_i \alpha_i \left(s_{i,\beta_{post}}^H - s_{i,\beta_{post}}^L\right)$ is approximately zero, this assumption has no quantitative consequences.

Because in the case of demand changes, the relative wage is the only operational channel, this directly maps into the welfare of the two groups: the welfare of the low-skill rises, while that of the high-skill decreases. In contrast, when a high-skill intensity sector (e.g. Services) experiences an exogenous increase in demand the reverse holds: the high-skilled relative wage rises, increasing the welfare of the high-skilled while reducing the welfare of the low-skilled.

Quantitatively, we find that when the Agriculture or Manufacturing sectors (both low-skill intensive relative to the aggregate economy) experience a positive demand change, the low-skill welfare gain is approximately 60% of the high-skill welfare loss. In contrast, when the Services sector (high-skill intensive) experiences a positive demand change, the high-skill gain is approximately 167% of the low-skill loss.

6. Conclusions

In this paper we develop a general analytical framework that analyzes the distributional welfare implications of sectoral supply and demand changes. Integrating supply-side and demand-side heterogeneity into our model, we show that the distributional welfare impact of sectoral shifts can be decomposed into two effects: an Engel effect and a Relative Wage effect. The former is driven by non-homothetic consumption patterns, wherein consumers at varied income levels allocate their expenditure differently across goods, leading to differential gains when prices change due to sectoral productivity changes. The Relative Wage effect captures equilibrium reallocation in the economy, which is driven by how sectoral supply and demand shifts change the composition of demand across sectors.

In our theoretical analysis, we show how the patterns of consumption substitution and their interaction with skill intensities across sectors determine the Relative Wage effect. By not confining the analysis to specific functional forms for the utility and production functions we are able to identify the key factors that shape the distributional welfare impact of sectoral supply and demand shifts. These key factors are all objects that can be estimated in the data, allowing us to quantify the welfare effects of sectoral shifts.

We apply our estimation framework to three different sectoral shifts: sectoral technical change, public sector demand shifts, and changes in consumer preferences. By combining various data sources, our quantitative results reveal significant welfare disparities between high- and low-skilled workers as a consequence of sectoral demand and supply changes. In the case of sectoral technical change, the differential welfare effect between high- and low-skilled workers ranges between negative 24% and positive 75%, depending on the sector experiencing the technical change. Moreover, our findings emphasize the prominence of the Relative Wage effect in influencing the overall distributional impact of technical change. With respect to public sector demand shifts, we quantify the differential welfare impact of changes across two distinct public sectors – public health and military personnel – showing that the relative size and skill intensity of these sectors vis-à-vis the private sectors shape the welfare impact. Finally, we quantify the distributional welfare impact of changes in consumer preferences across sectors.

References

Acemoglu, Daron, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The network origins of aggregate fluctuations." *Econometrica*, 80(5): 1977–2016.

Aguiar, Mark, and Mark Bils. 2015. "Has consumption inequality mirrored income inequality?" *American Economic Review*, 105(9): 2725–56.

Alder, Simon, Timo Boppart, and Andreas Muller. 2022. "A theory of structural change that can fit the data." *American Economic Journal: Macroeconomics*, 14(2): 160–206.

Banks, James, Richard Blundell, and Arthur Lewbel. 1997. "Quadratic Engel curves and consumer demand." *Review of Economics and statistics*, 79(4): 527–539.

Baqaee, David, and Ariel T Burstein. 2021. "Welfare and output with income effects and taste shocks." NBER.

Baqaee, David Rezza, and Emmanuel Farhi. 2019. "The macroeconomic impact of microeconomic shocks: Beyond Hulten's theorem." *Econometrica*, 87(4): 1155–1203.

Baumol, William J. 1967. "Macroeconomics of unbalanced growth: the anatomy of urban crisis." *American Economic Review*, 57(3): 415–426.

Baumol, William J, and William G Bowen. 1965. "On the performing arts: the anatomy of their economic problems." *American Economic Review*, 55(1/2): 495–502.

Beraja, Martin, and Christian K Wolf. 2021. "Demand composition and the strength of recoveries." NBER.

Bils, Mark, Peter J Klenow, and Benjamin A Malin. 2013. "Testing for Keynesian labor demand." *NBER Macroeconomics Annual*, 27(1): 311–349.

Buera, Francisco J, and Joseph P Kaboski. 2012. "The rise of the service economy." *American Economic Review*, 102(6): 2540–69.

Buera, Francisco J, Joseph P Kaboski, Richard Rogerson, and Juan I Vizcaino. 2022. "Skill-biased structural change." *The Review of Economic Studies*, 89(2): 592–625.

Comin, Diego, Danial Lashkari, and Martí Mestieri. 2021. "Structural change with long-run income and price effects." *Econometrica*, 89(1): 311–374.

Cox, Natalie, Peter Ganong, Pascal Noel, Joseph Vavra, Arlene Wong, Diana Farrell, Fiona Greig, and Erica Deadman. 2020. "Initial impacts of the pandemic on consumer behavior: Evidence from linked income, spending, and savings data." *Brookings Papers on Economic Activity*, 2020(2): 35–82.

Deaton, Angus, and John Muellbauer. 1980. "An almost ideal demand system." *American Economic Review*, 70(3): 312–326.

Duernecker, Georg, Berthold Herrendorf, and Akos Valentinyi. 2024. "Structural change within the services sector and the future of cost disease." *Journal of the European Economic Association*, 22(1): 428–473.

Durlauf, Steven N. 1993. "Nonergodic economic growth." *The Review of Economic Studies*, 60(2): 349–366.

Fajgelbaum, Pablo D, and Amit K Khandelwal. 2016. "Measuring the unequal gains from trade." *The Quarterly Journal of Economics*, 131(3): 1113–1180.

Harberger, Arnold C. 1998. "A vision of the growth process." *The American Economic Review*, 88(1): 1–32.

Hartwig, Jochen. 2011. "Testing the Baumol–Nordhaus model with EU KLEMS data." *Review of Income and Wealth*, 57(3): 471–489.

Herrendorf, Berthold, Richard Rogerson, and Akos Valentinyi. 2013. "Two perspectives on preferences and structural transformation." *American Economic Review*, 103(7): 2752–89.

Herrendorf, Berthold, Richard Rogerson, and Akos Valentinyi. 2014. "Growth and structural transformation." *Handbook of economic growth*, 2: 855–941.

Hulten, Charles R. 1978. "Growth accounting with intermediate inputs." *The Review of Economic Studies*, 45(3): 511–518.

Jaravel, Xavier. 2021. "Inflation inequality: Measurement, causes, and policy implications." *Annual Review of Economics*, 13: 599–629.

Kongsamut, Piyabha, Sergio Rebelo, and Danyang Xie. 2001. "Beyond balanced growth." *The Review of Economic Studies*, 68(4): 869–882.

Krusell, Per, Lee E Ohanian, José-Víctor Ríos-Rull, and Giovanni L Violante. 2000. "Capital-skill complementarity and inequality: A macroeconomic analysis." *Econometrica*, 68(5): 1029–1053.

Kulish, Mariano, James Morley, Nadine Yamout, and Francesco Zanetti. 2023. "Dutch Disease, Unemployment and Structural Change." CAMA Working Paper 38/2023.

Lecocq, Sébastien, and Jean-Marc Robin. 2015. "Estimating almost-ideal demand systems with endogenous regressors." *The Stata Journal*, 15(2): 554–573.

Matsuyama, Kiminori. 2019. "Engel's law in the global economy: Demand-induced patterns of structural change, innovation, and trade." *Econometrica*, 87(2): 497–528.

Ngai, L Rachel, and Barbara Petrongolo. 2017. "Gender gaps and the rise of the service economy." *American Economic Journal: Macroeconomics*, 9(4): 1–44.

Ngai, L Rachel, and Christopher A Pissarides. 2007. "Structural change in a multisector model of growth." *American Economic Review*, 97(1): 429–443.

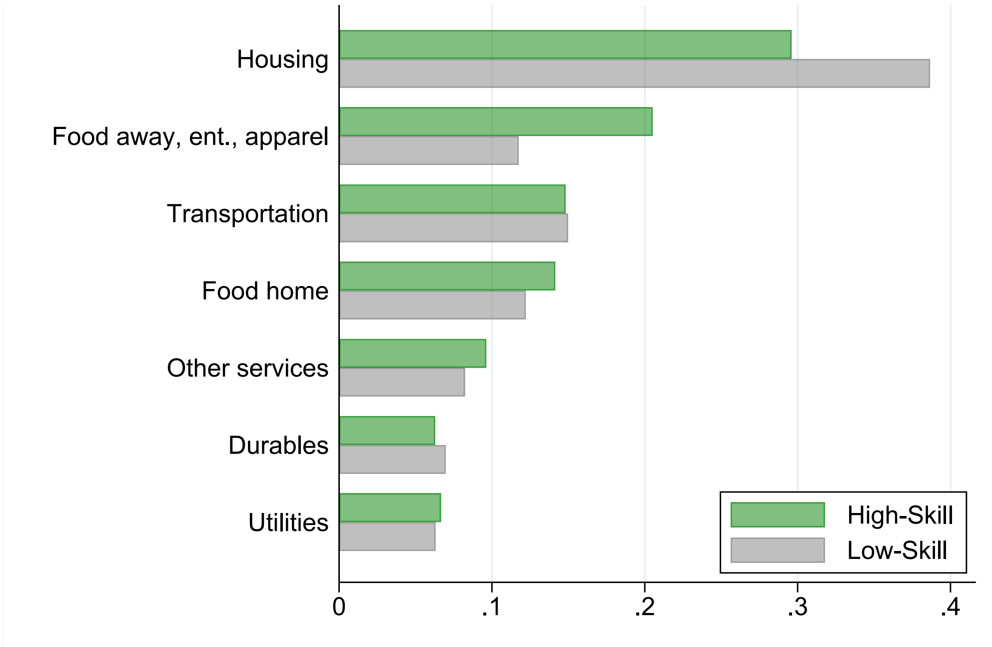
Ngai, Rachel, Claudia Olivetti, and Barbara Petrongolo. 2024. "Structural Transformation over 150 years of Women's and Men's Work." *Unpublished Working Paper*.

Nicholson, Walter, and Christopher M Snyder. 2016. *Microeconomic theory: Basic principles and extensions.* Cengage Learning.

Nordhaus, William D. 2008. "Baumol's diseases: a macroeconomic perspective." *The BE Journal of Macroeconomics*, 8(1).

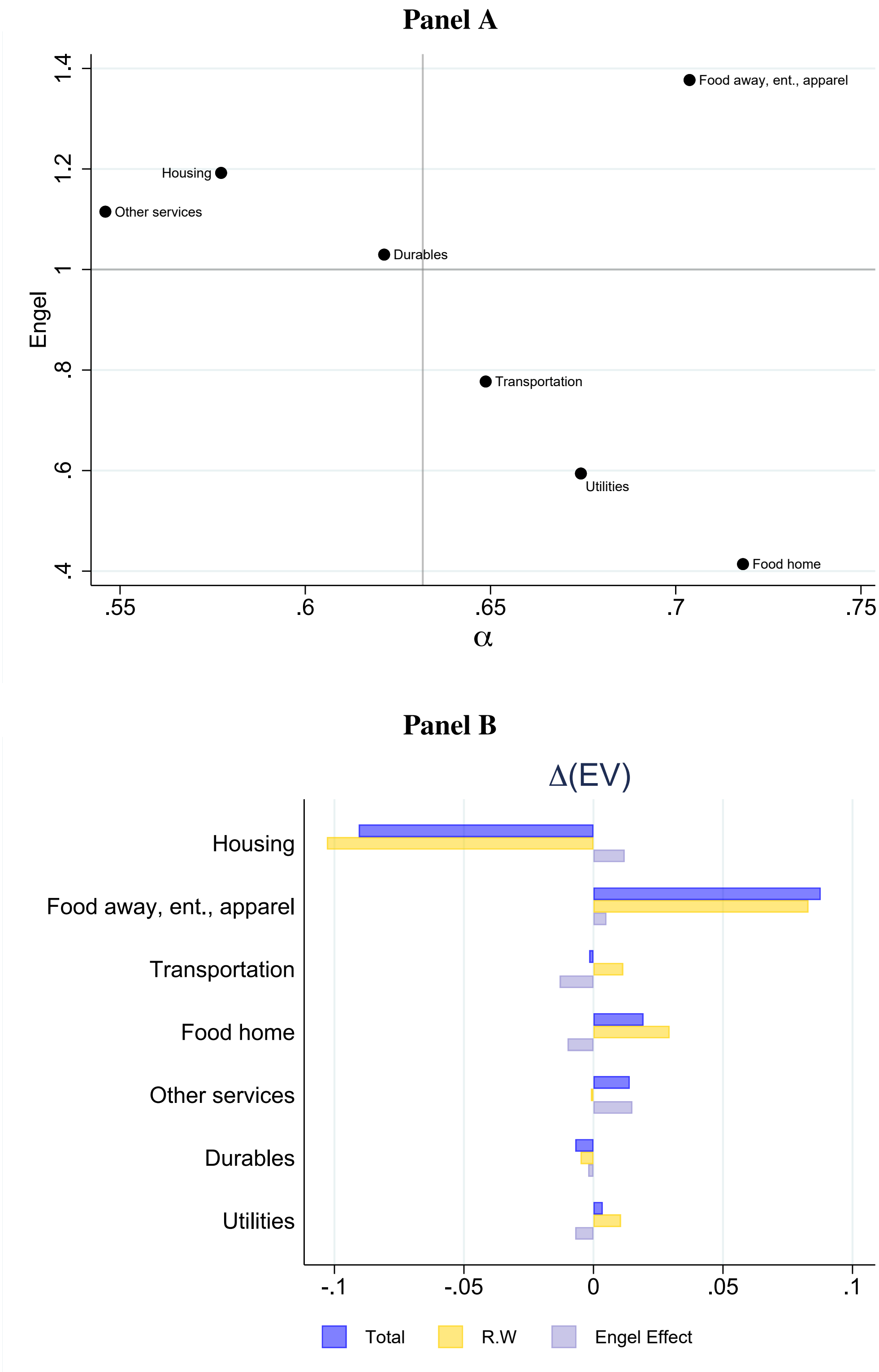
Ramey, Valerie A. 2016. "Macroeconomic shocks and their propagation." *Handbook of macroeconomics*, 2: 71–162.

Figure 1: Differential Welfare Impact of Sectoral Technical Change



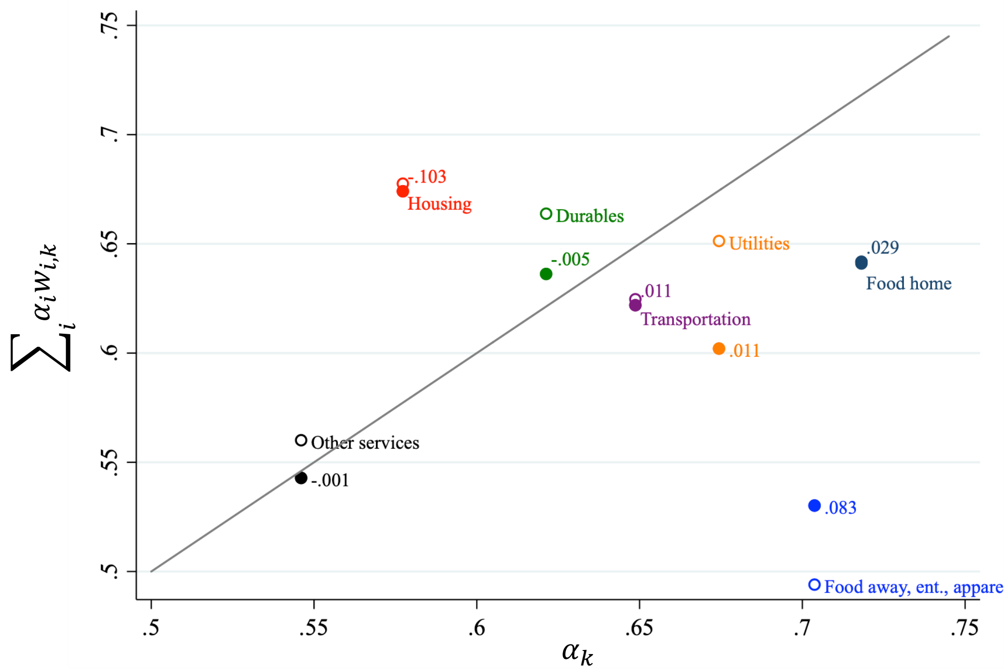
Notes: Bars depict welfare elasticities with respect to technical change in a specific sector for high- and low-skilled workers, η_{EV_H,A_k} and η_{EV_L,A_k}, respectively. Sectors are sorted by their aggregate share.

Figure 2: Decomposing the Welfare Impact of Sectoral Technical Change



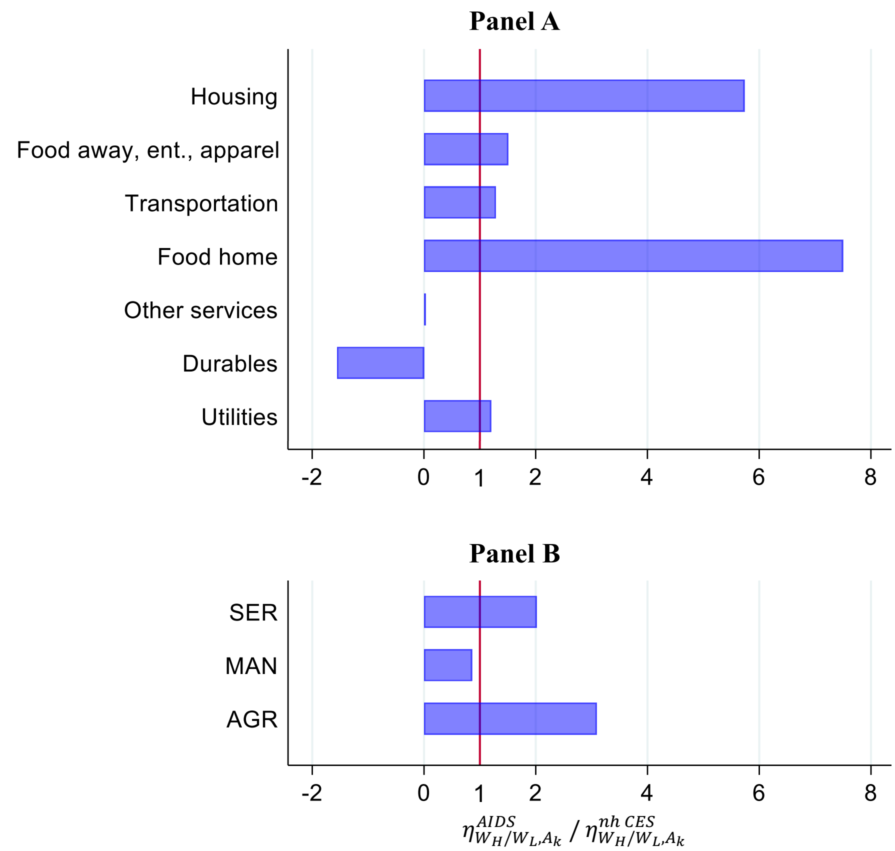
Notes: Panel A depicts the seven industries according to two attributes: their degree of low skill intensity, α, and their Engel elasticity. Panel B decomposes the welfare impact of technical change into the Engel effect and the relative wage elasticity for the seven sectors. This decomposition is based on Equation (5).

Figure 3: Direct and Indirect Demand



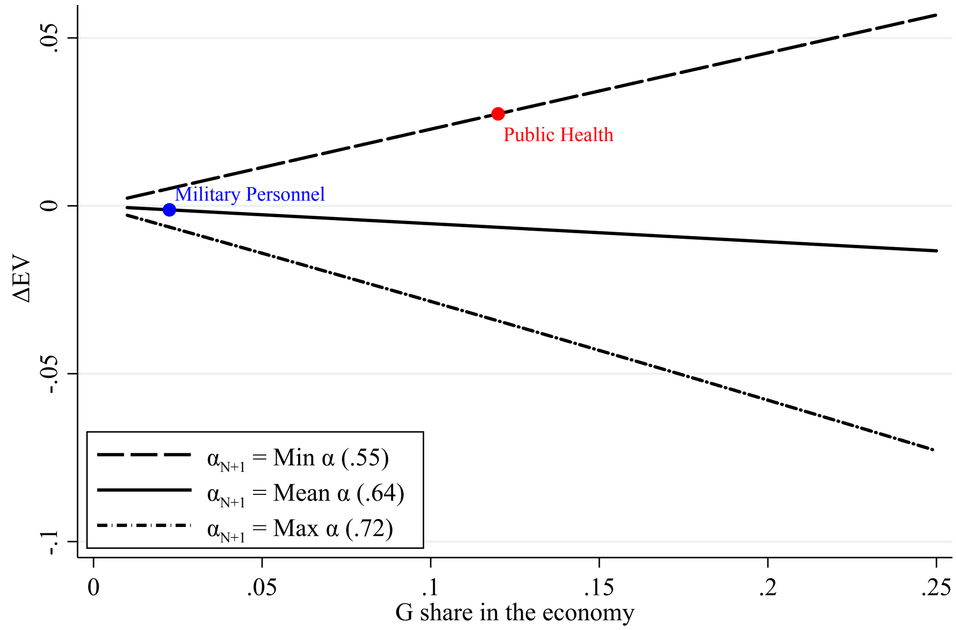
Notes: The x-axis denotes the sector's α, while the y-axis depicts the sector's $\Sigma\alpha_i w_{i,k}$ for the L-type (full circles) and H-type (hollow circles). The numbers in the graph denote the estimated Relative Wage elasticity for each sector.

Figure 4: AIDS vs. non-Homothetic CES Relative Wage Elasticities



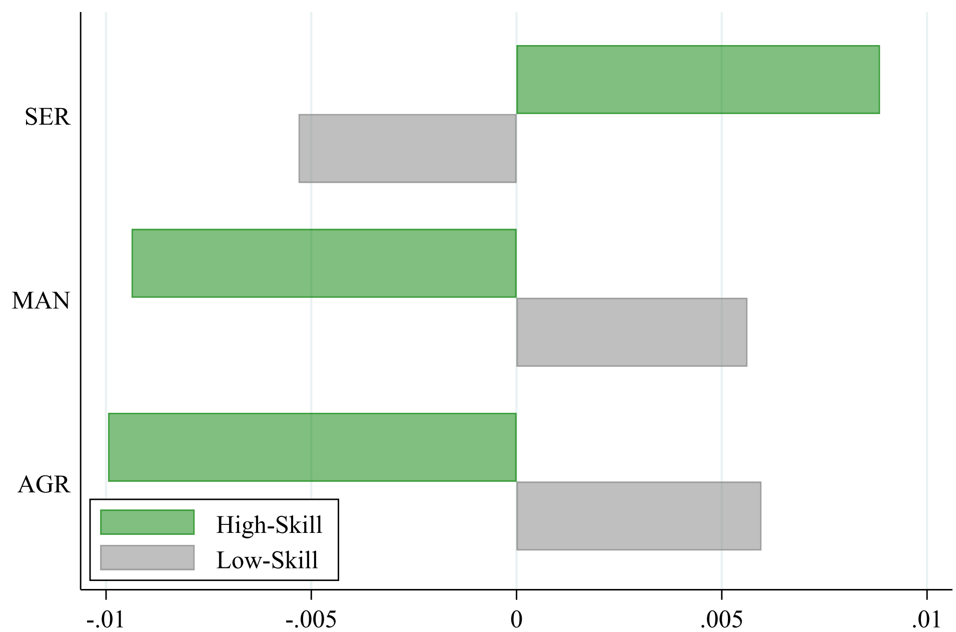
Notes: The x-axis denotes the ratio between $\eta^{AIDS}_{W_H/W_L,A_k}$ (the elasticity of the relative wage with respect to a technical change in sector k obtained applying the AIDS estimation) to $\eta^{nh\,CES}_{W_H/W_L,A_k}$ (the analogous elasticity obtained constraining preferences to non-homothetic CES). Panel A provides this ratio for the seven sectors categorization, while Panel B provides it for the three sectors categorization. The vertical red-line is marked at the value of one, i.e. when the two elasticities coincide.

Figure 5: Differential Welfare Impact of Public Sector Shifts



Notes: the figure depicts the differential welfare impact stemming from changes in public sector expenditure level, G, over the size of the public sector for three different levels of public sector skill intensity. The three skill intensity levels are the minimum, mean, and maximum low-skill intensity over the seven sectors previously discussed.

Figure 6: Differential Welfare Impact of Sectoral Preference Change



Notes: Bars depict welfare elasticities with respect to sectoral demand shifts for high- and low-skilled workers, η_{EV_H,β_k} and η_{EV_L,β_k}, respectively. Sectors are sorted by their aggregate share.

A. Online Appendix not for Publication

A.1. Theoretical Derivations

Lemma 1

Before providing the proof for Lemma 1, it is useful to derive the following result:

Lemma A.1. *$\sigma_i \eta_{F_L^i(1,\frac{H_i}{L_i}),\frac{H_i}{L_i}} = 1-\alpha_i$, and $\sigma_i \eta_{F_H^i(1,\frac{H_i}{L_i}),\frac{H_i}{L_i}} = -\alpha_i$, where $\sigma_i = \eta_{\frac{H_i}{L_i},\frac{W_L}{W_H}}$ is the elasticity of substitution in sector i at the point $\left(\frac{H_i}{L_i},\frac{W_L}{W_H}\right)$, and α_i is the low-skill labor share in sector i. $F_L^i(1,\frac{H_i}{L_i})$ is the marginal product of $F^i(1,\frac{H_i}{L_i})$ with respect to the first element (low-skilled), and $F_H^i(1,\frac{H_i}{L_i})$ is the marginal product with respect to the second element (high-skilled). $\eta_{x,y}$ denotes the elasticity of x with respect to y, and all elasticities are calculated at the pre-change equilibrium values.*

Proof. We have that

$$\begin{aligned}
\sigma_i \eta_{F_L^i(1,\frac{H_i}{L_i}),\frac{H_i}{L_i}} &= \sigma_i \left(\frac{\partial F_L^i(1,\frac{H_i}{L_i})}{\partial \frac{H_i}{L_i}} \frac{\frac{H_i}{L_i}}{F_L^i(1,\frac{H_i}{L_i})} \right) \\
&= \left(\frac{F_L^i(L_i,H_i) F_H^i(L_i,H_i)}{F_{LH}^i(L_i,H_i) F^i(L_i,H_i)} \right) \left(\frac{\partial F_L^i(1,\frac{H_i}{L_i})}{\partial \frac{H_i}{L_i}} \frac{\frac{H_i}{L_i}}{F_L^i(1,\frac{H_i}{L_i})} \right) \\
&= \left(\frac{F_H^i(L_i,H_i)}{F_{LH}^i(L_i,H_i) F^i(L_i,H_i)} \right) \left(\frac{\partial F_L^i(1,\frac{H_i}{L_i})}{\partial \frac{H_i}{L_i}} \frac{H_i}{L_i} \right) \\
&= \left(\frac{F_H^i(L_i,H_i)}{F_{LH}^i(L_i,H_i) F^i(L_i,H_i)} \right) \left(F_{LH}^i(1,\frac{H_i}{L_i}) \frac{H_i}{L_i} \right),
\end{aligned}$$

where F_{LH}^i stands for the cross-derivative. The first line is from the definition of the elasticity, and the second line stems from the fact the production is constant returns to scale (see for example footnote 6 on page 308 of Nicholson and Snyder (2016)). F_{LH} is homogeneous of degree -1, hence:

$$\begin{aligned}
\sigma_i \left(\eta_{F_L^i(1,\frac{H_i}{L_i}),\frac{H_i}{L_i}} \right) &= \left(\frac{F_H^i(L_i,H_i)}{F_{LH}^i(L_i,H_i) F^i(L_i,H_i)} \right) \left(L_i F_{LH}^i(L_i,H_i) \frac{H_i}{L_i} \right) \\
&= \left(\frac{F_H^i(L_i,H_i)}{F^i(L_i,H_i)} \right) H_i \\
&= \left(\frac{P_i A_i F_H^i(L_i,H_i)}{P_i A_i F^i(L_i,H_i)} \right) H_i \\
&= \left(\frac{W_H H_i}{P_i A_i F^i(L_i,H_i)} \right) = 1-\alpha_i \equiv \text{H labor share},
\end{aligned}$$

where the last two lines use the first order conditions of the firm.

Similarly, it is straightforward to show that:

$$\sigma_i \eta_{F_H^i(1,\frac{H_i}{L_i}),\frac{H_i}{L_i}} = -\alpha_i$$

□

Armed with this lemma, we can now turn to the proof of Lemma 1:

Proof. Consider first the L type. From equation (1):

$$\begin{aligned}
EV^L &= \widehat{W^L} - \Sigma_i s_i^L(P_0, W_0^L)\widehat{P_i} \\
&= \Sigma_i s_i^L(P_0, W_0^L)\widehat{W^L} - \Sigma_i s_i^L(P_0, W_0^L)\widehat{P_i} \\
&= \Sigma_i \widehat{\left(\frac{W^L}{P_i}\right)} s_i^L(P_0, W_0^L) \\
&= \Sigma_i \left(A_i \widehat{F_L^i(1, \frac{H_i}{L_i})}\right) s_i^L(P_0, W_0^L) \\
&= \Sigma_i \left(\widehat{F_L^i(1, \frac{H_i}{L_i})}\right) s_i^L(P_0, W_0^L) + \Sigma_i \widehat{A}_i s_i^L(P_0, W_0^L),
\end{aligned}$$

where the penultimate line stems from the first order condition for the firm. Writing each term $\left(\widehat{F_L^i(1, \frac{H_i}{L_i})}\right)$ as a function of $\widehat{\left(\frac{W_L}{W_H}\right)}$ and evaluating elasticities at pre-change values, we have:

$$\begin{aligned}
EV^L &= \Sigma_i \left(\eta_{F_L^i(1,\frac{H_i}{L_i}),\frac{H_i}{L_i}}\right)\left(\eta_{\frac{H_i}{L_i},\frac{W_L}{W_H}}\right)\widehat{\left(\frac{W_L}{W_H}\right)} s_i^L(P_0, W_0^L) + \Sigma_i \widehat{A}_i s_i^L(P_0, W_0^L) = \\
&= \Sigma_i \left(\eta_{F_L^i(1,\frac{H_i}{L_i}),\frac{H_i}{L_i}}\right)(\sigma_i)\widehat{\left(\frac{W_L}{W_H}\right)} s_i^L(P_0, W_0^L) + \Sigma_i \widehat{A}_i s_i^L(P_0, W_0^L) = \\
&= \Sigma_i \left(\eta_{F_L^i(1,\frac{H_i}{L_i}),\frac{H_i}{L_i}}\right)(-\sigma_i)\widehat{\left(\frac{W_H}{W_L}\right)} s_i^L(P_0, W_0^L) + \Sigma_i \widehat{A}_i s_i^L(P_0, W_0^L) = \\
&\overset{\text{Lemma A.1}}{=} \widehat{\left(\frac{W_H}{W_L}\right)} \Sigma_i(\alpha_{i,0} - 1) s_i^L(P_0, W_0^L) + \Sigma_i \widehat{A}_i s_i^L(P_0, W_0^L) = \\
&= \widehat{\left(\frac{W_H}{W_L}\right)} \Sigma_i \left(\alpha_{i,0} s_i^L(P_0, W_0^L) - 1\right) + \Sigma_i \widehat{A}_i s_i^L(P_0, W_0^L).
\end{aligned}$$

The proof for EV^H is analogous. □

Theorem 1

Proof. Consider a change to the productivity of sector k, $\widehat{A_k}$, holding productivity in all other sectors constant. Based on Lemma 1 we have:

$$\frac{EV^L}{\widehat{A_k}} = \eta_{W_H/W_L,A_k}\left[\Sigma_i \alpha_i s_i^L(P_0,W_0^L)-1\right] + s_k^L(P_0,W_0^L)$$

Applying a similar derivation for the H type implies:

$$\frac{EV^H}{\widehat{A_k}} = \eta_{W_H/W_L,A_k}\Sigma_i \alpha_i s_i^H(P_0,W_0^H) + s_k^H(P_0,W_0^H)$$

Taking the difference:

$$\Delta\eta_{EV,A_k} := \frac{EV^H}{\widehat{A_k}} - \frac{EV^L}{\widehat{A_k}} = s_k^H(P_0,W_0^H) - s_k^L(P_0,W_0^L) + \eta_{W_H/W_L,A_k}\left\{1+\Sigma_i\alpha_i\left[s_i^H(P_0,W_0^H)-s_i^L(P_0,W_0^L)\right]\right\}.$$

□

Lemma 2

Proof. Market clearing for sector i implies:

$$LC_i(W_L,\mathbf{P}) + HC_i(W_H,\mathbf{P}) = Y_i$$

$$\begin{aligned} LP_iC_i(W_L,\mathbf{P}) + HP_iC_i(W_H,\mathbf{P}) = P_iY_i &= W_LL_i + W_HH_i \\ &= L_i(W_L + W_H\frac{H_i}{L_i}), \end{aligned}$$

where the last two equations stem from production being CRS. We thus have:

$$L_i = \frac{LP_iC_i(W_L,\mathbf{P}) + HP_iC_i(W_H,\mathbf{P})}{W_L(1+\frac{W_HH_i}{W_LL_i})} \tag{A.1}$$

Using firms' first order conditions:

$$\frac{W_H}{W_L} = \frac{F_H^i(1,\frac{H_i}{L_i})}{F_L^i(1,\frac{H_i}{L_i})}.$$

Applying Euler's homogeneous function theorem to $F(1,\frac{H_i}{L_i})$ and its partial derivatives, it is easy to show that with CRS, $\frac{F_H^i(1,\frac{H_i}{L_i})}{F_L^i(1,\frac{H_i}{L_i})}$ is invertible, and hence we can define functions Ω_i with:

$$\frac{H_i}{L_i} = \Omega_i(\frac{W_H}{W_L}) \tag{A.2}$$

i.e., in equilibrium, each $\frac{H_i}{L_i}$ is a function solely of the relative wage $\frac{W_H}{W_L}$.

Now from equation (A.2), we have that $H_i = L_i\Omega_i(\frac{W_H}{W_L})$, and so from labor market clearing for the high and low types we get:

$$\begin{aligned} H = \sum_{i=1}^{N} H_i &= \sum_{i=1}^{N-1} \frac{H_i}{L_i} L_i + \frac{H_N}{L_N}\left(L - \sum_{i=1}^{N-1} L_i\right) \\ &= \sum_{i=1}^{N-1} \frac{H_i}{L_i} L_i + \frac{H_N}{L_N}\left(L - \sum_{i=1}^{N-1} L_i\right) \end{aligned}$$

Plugging equation (A.1) we then have:

$$H = \sum_{i=1}^{N-1} \frac{H_i}{L_i}\left(\frac{LP_iC_i(W_L,\mathbf{P}) + HP_iC_i(W_H,\mathbf{P})}{W_L(1+\frac{W_H H_i}{W_L L_i})}\right) + \frac{H_N}{L_N}\left(L - \sum_{i=1}^{N-1}\left(\frac{LP_iC_i(W_L,\mathbf{P}) + HP_iC_i(W_H,\mathbf{P})}{W_L(1+\frac{W_H H_i}{W_L L_i})}\right)\right)$$

which after some manipulation yields:

$$\frac{H}{L} = \frac{H_N}{L_N} + \sum_{i=1}^{N-1}\left(\frac{H_i}{L_i} - \frac{H_N}{L_N}\right)\left(\frac{s_i(W_L,\mathbf{P}) + \frac{H}{L}\frac{W_H}{W_L}s_i(W_H,\mathbf{P})}{(1+\frac{W_H}{W_L}\frac{H_i}{L_i})}\right) \tag{A.3}$$

Since for all i we have that $\frac{H_i}{L_i}$ are functions of $\frac{W_H}{W_L}$, it is left to show that W_L, W_H, and $\mathbf{P}$ are also functions of only $\frac{W_H}{W_L}$ and $\mathbf{A}$. This is seen from firms' FOCs. Normalizing the price of the Nth good to one, exploiting the fact that the marginal products are homogeneous of degree zero, and using the Nth sector's first order condition, we have that

$$W_L = A_N F_L^N(L_N,H_N) = A_N F_L^N(1,\frac{H_N}{L_N})$$

$$W_H = A_N F_H^N(L_N,H_N) = A_N F_H^N(1,\frac{H_N}{L_N})$$

In regards to the price vector, **P**, using the first-order-condition again we obtain:

$$P_i = \frac{W_L}{A_i F_L^i(1, \frac{H_i}{L_i})} = \frac{A_N F_L^N(1, \frac{H_N}{L_N})}{A_i F_L^i(1, \frac{H_i}{L_i})}.$$

Hence, we have shown that (A.3) is a market-clearing condition which consolidates goods and labor market clearing conditions as a function of only $\frac{W_H}{W_L}$ and **A**.

□

Theorem 2

Proof. We start from equation (A.3), and for convenience omit the explicit reference of the share function (s_i^L and s_i^H) to their arguments (W and **P**):

$$\frac{H}{L} = \frac{H_N}{L_N} + \sum_{i=1}^{N-1} \left(\frac{H_i}{L_i} - \frac{H_N}{L_N} \right) \left(\frac{s_i^L + \frac{H}{L}\frac{W_H}{W_L} s_i^H}{(1 + \frac{W_H}{W_L}\frac{H_i}{L_i})} \right).$$

Recalling that α_i is the labor share of the low-skilled, multiplying both sides by $\frac{LW_H}{HW_L}$, and denoting $S_H = \frac{HW_H}{LW_L + HW_H}$ and $S_L = \frac{LW_L}{LW_L + HW_H}$, we obtain:

$$\begin{aligned}
\frac{W_H}{W_L} &= \frac{L}{H}\frac{W_H}{W_L}\frac{H_N}{L_N} + \frac{L}{H}\sum_{i=1}^{N-1} \left(\frac{W_H}{W_L}\frac{H_i}{L_i} - \frac{W_H}{W_L}\frac{H_N}{L_N} \right) \left(\frac{s_i^L + \frac{H}{L}\frac{W_H}{W_L} s_i^H}{(1 + \frac{W_H}{W_L}\frac{H_i}{L_i})} \right) \\
&= \frac{L}{H}\frac{1-\alpha_N}{\alpha_N} + \frac{L}{H}\sum_{i=1}^{N-1} \alpha_i \left(\frac{1-\alpha_i}{\alpha_i} - \frac{1-\alpha_N}{\alpha_N} \right) \left(s_i^L + \frac{S_H}{S_L} s_i^H \right) \\
&= \frac{L}{H}\frac{1-\alpha_N}{\alpha_N} + \frac{L}{H}\sum_{i=1}^{N-1} \left(1 - \frac{\alpha_i}{\alpha_N} \right) \left(s_i^L + \frac{S_H}{S_L} s_i^H \right) \\
&= \frac{L}{H}\frac{1-\alpha_N}{\alpha_N} + \frac{L}{H}\sum_{i=1}^{N} \left(1 - \frac{\alpha_i}{\alpha_N} \right) \left(s_i^L + \frac{S_H}{S_L} s_i^H \right) \\
&= \frac{1}{\alpha_N}\frac{L}{H} + \frac{L}{H}\frac{S_H}{S_L} - \frac{L}{H}\sum_{i=1}^{N} \left(\frac{\alpha_i}{\alpha_N} \right) \left(s_i^L + \frac{S_H}{S_L} s_i^H \right) \\
&= \frac{1}{\alpha_N}\frac{L}{H} + \frac{W_H}{W_L} - \frac{1}{\alpha_N}\frac{1}{S_L}\frac{L}{H}\sum_{i=1}^{N} \alpha_i \left(S_L s_i^L + S_H s_i^H \right)
\end{aligned}$$

And therefore:

$$\frac{1}{\alpha_N}\frac{L}{H} - \frac{1}{\alpha_N}\frac{1}{S_L}\frac{L}{H}\sum_{i=1}^{N} \alpha_i \left(S_L s_i^L + S_H s_i^H \right) = 0.$$

Additional manipulation thus yields that equation (A.3) can be written as:

$$S_L - \sum_{i=1}^{N} \alpha_i \left(S_L s_i^L + S_H s_i^H \right) = 0 \tag{A.4}$$

Since $S_L = \frac{LW_L}{LW_L + HW_H}$, this is equivalent to:

$$\frac{W_H}{W_L} = \frac{L}{H} \left[\frac{1}{\sum_{i=1}^{N} \alpha_i \left(S_L s_i^L + S_H s_i^H \right)} - 1 \right],$$

which directly implies that in equilibrium, the relative wage, $\frac{W_H}{W_L}$, is inversely related to $\sum_{i=1}^{N} \alpha_i \left(S_L s_i^L + S_H s_i^H \right)$. Given an exogenous change in productivity in the kth sector, A_k, we therefore have:

$$sign(\eta_{\frac{W_H}{W_L},A_k}) = -sign\left\{ \frac{d}{dA_k} \left[\sum_{i=1}^{N} \alpha_i \left(S_L s_i^L + S_H s_i^H \right) \right] \right\}.$$

□

Theorem 3

Before we proceed to Theorem 3, we establish the following Lemma:

Lemma A.2. *Consider y, a function of x, defined implicitly by the equation* $\mathscr{H}(x,y) = 0$*, where* $\mathscr{H}(x,y) := f(x,y) + g(x,y)$*. Then, the full elasticity* $\eta_{y,x}$ *is given by* $-\frac{f(x,y)\eta_{f,x} + g(x,y)\eta_{g,x}}{f(x,y)\eta_{f,y} + g(x,y)\eta_{g,y}}$*, where* $\eta_{f,x}, \eta_{f,y}, \eta_{g,x}$ *and* $\eta_{g,y}$ *are all partial elasticities.*

Proof. Apply the impicit function theorem:

$$\eta_{y,x} = \left(-\frac{\frac{\partial \mathscr{H}}{\partial x}}{\frac{\partial \mathscr{H}}{\partial y}} \right) \frac{x}{y} = -\frac{\frac{\partial \mathscr{H}}{\partial x} x}{\frac{\partial \mathscr{H}}{\partial y} y}$$

By the definition of H we thus have:

$$\begin{aligned} \eta_{y,x} &= -\frac{\frac{\partial f(x,y)}{\partial x} x + \frac{\partial g(x,y)}{\partial x} x}{\frac{\partial f(x,y)}{\partial y} y + \frac{\partial g(x,y)}{\partial y} y} \\ &= -\frac{f(x,y) \frac{\partial f(x,y)}{\partial x} \frac{x}{f(x,y)} + g(x,y) \frac{\partial g(x,y)}{\partial x} \frac{x}{g(x,y)}}{f(x,y) \frac{\partial f(x,y)}{\partial y} \frac{y}{f(x,y)} + g(x,y) \frac{\partial g(x,y)}{\partial y} \frac{y}{g(x,y)}} = -\frac{f(x,y)\eta_{f,x} + g(x,y)\eta_{g,x}}{f(x,y)\eta_{f,y} + g(x,y)\eta_{g,y}} \end{aligned}$$

□

Armed with this lemma, we can now turn to the proof of Theorem 3:

Proof. As a first step we show the following preliminaries:

A. Holding constant the relative wage, the partial elasticities satisfy $\eta_{\alpha_i,A_k} = \eta_{S_L,A_k} = \eta_{S_H,A_k} = 0$. The proof is trivial given that all three objects are only a function of the relative wage:

$$\alpha_i = \frac{L_i W_L}{L_i W_L + H_i W_H} = \frac{1}{1 + \frac{H_i}{L_i}\frac{W_H}{W_L}}$$

$$S_H = \frac{W_H H}{W_L L + W_H H} = \frac{1}{\frac{W_L L}{W_H H} + 1}$$

$$S_L = \frac{W_L L}{W_L L + W_H H} = \frac{1}{1 + \frac{W_H}{W_L}\frac{H}{L}}$$

B. Holding constant the relative wage, the partial elasticities satisfy $\eta_{W_L,A_k} = \eta_{W_H,A_k} = \eta_{P_i,A_k} = 1$, for all $i \neq k$. To see this, note from the first order condition in the proof to Lemma 2 that

$$\eta_{W_L,A_k} = \eta_{A_k F_L^k(1,\frac{H_k}{L_k}),A_k} = 1,$$

and also for all $i \neq k$:

$$\eta_{P_i,A_k} = \eta_{\frac{W_L}{A_i F_L^i(1,\frac{H_i}{L_i})},A_k} = \eta_{\frac{A_k F_L^k(1,\frac{H_k}{L_k})}{A_i F_L^i(1,\frac{H_i}{L_i})},A_k} = 1.$$

C. Holding constant **A**, the partial elasticities satisfy $\eta_{\alpha_i,W_H/W_L} = -(1-\alpha_i)(1-\sigma_i)$, $\eta_{S_{L,W_H/W_L}} = -S_H$, and $\eta_{S_H,W_H/W_L} = S_L$. To see this, note that:

$$\begin{aligned}\eta_{\alpha_i,W_H/W_L} &= \eta_{\frac{1}{1+\frac{H_i}{L_i}\frac{W_H}{W_L}},W_H/W_L} \\ &= -\eta_{1+\frac{H_i}{L_i}\frac{W_H}{W_L},W_H/W_L} \\ &= -\frac{\frac{H_i}{L_i}\frac{W_H}{W_L}}{1+\frac{H_i}{L_i}\frac{W_H}{W_L}}\eta_{\frac{H_i}{L_i}\frac{W_H}{W_L},W_H/W_L} \\ &= -(1-\alpha_i)(1-\sigma_i)\end{aligned}$$

$$
\begin{aligned}
\eta_{S_{L},W_H/W_L} &= \eta_{\frac{1}{1+\frac{W_H}{W_L}\frac{H}{L}},W_H/W_L} \\
&= -\eta_{1+\frac{W_H}{W_L}\frac{H}{L},W_H/W_L} \\
&= -\frac{\frac{W_H}{W_L}\frac{H}{L}}{1+\frac{W_H}{W_L}\frac{H}{L}}\eta_{\frac{W_H}{W_L}\frac{H}{L},W_H/W_L} \\
&= -S_H
\end{aligned}
$$

$$
\begin{aligned}
\eta_{S_H,W_H/W_L} &= \eta_{\frac{1}{\frac{W_L L}{W_H H}+1},W_H/W_L} \\
&= -\eta_{\frac{W_L L}{W_H H}+1,W_H/W_L} \\
&= -\frac{\frac{W_L L}{W_H H}}{\frac{W_L L}{W_H H}+1}\eta_{\frac{W_L L}{W_H H},W_H/W_L} \\
&= S_L
\end{aligned}
$$

D. Holding constant **A**, the partial elasticities satisfy $\eta_{W_L,\frac{W_H}{W_L}} = \alpha_k - 1$, $\eta_{W_H,\frac{W_H}{W_L}} = \alpha_k$, and $\eta_{P_i,\frac{W_H}{W_L}} = \alpha_k - \alpha_i$ (where recall that k is the numeraire). The proofs are given below:

$$
\begin{aligned}
\eta_{W_L,\frac{W_H}{W_L}} &= \eta_{A_k F_L^k(1,\frac{H_k}{L_k}),\frac{W_H}{W_L}} \\
&= \eta_{F_L^k(1,\frac{H_k}{L_k}(\frac{W_H}{W_L})),\frac{W_H}{W_L}} \\
&= \eta_{F_L^k(1,\frac{H_k}{L_k}),\frac{H_k}{L_k}}\eta_{\frac{H_k}{L_k},\frac{W_H}{W_L}} \\
&= \frac{1-\alpha_k}{\sigma_k} * (-\sigma_k) = \alpha_k - 1
\end{aligned}
$$

where the last row is directly from the definition of σ_k and from Lemma 1. Similarly:

$$
\eta_{W_H,\frac{W_H}{W_L}} = \eta_{W_L\frac{W_H}{W_L},\frac{W_H}{W_L}} = \alpha_k - 1 + 1 = \alpha_k
$$

$$
\begin{aligned}
\eta_{P_i,\frac{W_H}{W_L}} &= \eta_{\frac{W_L}{A_i F_L^i\left(1,\frac{H_i}{L_i}\right)},\frac{W_H}{W_L}} \\
&= \eta_{W_L,\frac{W_H}{W_L}} - \eta_{F_L^i\left(1,\frac{H_i}{L_i}\right),\frac{W_H}{W_L}} \\
&= \alpha_k - 1 - \left(\eta_{F_L^i\left(1,\frac{H_i}{L_i}\right),\frac{H_i}{L_i}} \eta_{\frac{H_i}{L_i},\frac{W_H}{W_L}}\right) \\
&= \alpha_k - 1 - (\alpha_i - 1) \\
&= \alpha_k - \alpha_i
\end{aligned}
$$

Having established the preliminaries we proceed with the proof. Following Equation (A.4) Theorem 2, define :

$$
\mathscr{H}\left(\mathbf{A},\frac{W_H}{W_L}\right) := S_L - \sum_{i=1}^{N} \alpha_i \left(S_L s_i\left(W_L,\mathbf{P}\right) + S_H s_i\left(W_H,\mathbf{P}\right)\right) = 0,
$$

where $S_L = \frac{W_L L}{W_H L + W_L L}, S_H = \frac{W_H H}{W_H L + W_L L}, \alpha_i = \frac{W_L L_i}{P_i Y_i}$, and we have shown in Lemma 2 that all the objects in these equation can be written solely as functions of $\mathbf{A}, \frac{W_H}{W_L}$.

We apply Lemma A.2, starting from the numerator (all η operators signify partial elasticities):

$$
\begin{aligned}
-\left(f(x,y)\eta_{f,x} + g(x,y)\eta_{g,x}\right) &= -\left[S_L \eta_{S_L,A_k} - \left(\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right)\right)\eta_{\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right),A_k}\right] \\
&= \left(\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right)\right)\eta_{\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right),A_k} \\
&= \left(\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right)\right)\sum\frac{\alpha_i\left(S_L s_i^L + S_H s_i^H\right)}{\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right)}\eta_{\alpha_i\left(S_L s_i^L + S_H s_i^H\right),A_k} \\
&= \sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right)\left(\eta_{\alpha_i,A_k} + \eta_{\left(S_L s_i^L + S_H s_i^H\right),A_k}\right) \\
&= \sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right)\eta_{\left(S_L s_i^L + S_H s_i^H\right),A_k}
\end{aligned}
$$

where the second and last line stem from preliminary A.

Applying the rules for the elasticity of a sum, we therefore obtain:

$$
\begin{aligned}
&-\left(f(x,y)\eta_{f,x} + g(x,y)\eta_{g,x}\right) \\
&= \sum_{i=1}^{N}\alpha_i\left(S_L s_i(W_L,P) + S_H s_i(W_H,P)\right)\left(\frac{S_L s_i(W_L,P)}{S_L s_i(W_L,P) + S_H s_i(W_H,P)}\eta_{S_L s_i,A_k} + \frac{S_H s_i(W_H,P)}{S_L s_i(W_L,P) + S_H s_i(W_H,P)}\eta_{S_H s_i,A_k}\right) \\
&= \sum_{i=1}^{N}\alpha_i\left(S_L s_i(W_L,P)\,\eta_{s_i(W_L,P),A_k} + S_H s_i(W_H,P)\,\eta_{s_i(W_H,P),A_k}\right) \qquad \text{(A.5)}
\end{aligned}
$$

where we apply preliminary A again to obtain the second line.

Applying preliminary B, we obtain: $\eta_{s_i(W_L,\mathbf{P}),A_k} = \eta_{\frac{P_i}{W_L}C_i(W_L,\mathbf{P}),A_k} = \eta_{C_i(W_L,\mathbf{P}),A_k}, \forall i \neq k$. Applying the chain rule for all $i \neq k$ (and recalling that k is the numeraire):

$$\begin{aligned}
\eta_{s_i(W_L,\mathbf{P}),A_k} = \eta_{C_i(W_L,\mathbf{P}),A_k} &= \left(\Sigma_{j\neq k}\eta_{C_i(W_L,\mathbf{P}),P_j}\eta_{P_j,A_k}\right) + \eta_{C_i(W_L,\mathbf{P}),W_L}\eta_{W_L,A_k} \\
&= \left(\Sigma_{j\neq k}\eta_{C_i(W_L,\mathbf{P}),P_j}\right) + \eta_{C_i(W_L,\mathbf{P}),W_L} \\
&= -\eta_{C_i(W_L,\mathbf{P}),P_k},
\end{aligned}$$

where the second equality is again applying preliminary B, and the last equality stems from the homogeneity of degree zero of the uncompensated demand function, impliying that $\eta_{C_i,W} + \Sigma_{j=1}^{N}\eta_{C_i,P_j} = 0$. For the case of $i = k$:

$$\eta_{s_k(W_L,P),A_k} = \eta_{C_k(W_L,P),A_k} - \eta_{W_L(W_L,P),A_k} = -\eta_{C_k(W_L,P),P_k} - 1.$$

It is easy to derive the analogous expressions for the H type.

Plugging back into (A.5), we get:

$$\begin{aligned}
&= -\sum_{i\neq k}\alpha_i\left(S_L s_i(W_L,P)\,\eta_{C_i(W_L,P),P_k} + S_H s_i(W_H,P)\,\eta_{C_i(W_H,P),P_k}\right) - \alpha_k\begin{bmatrix} S_L s_k(W_L,P)\left(\eta_{C_k(W_L,P),P_k}+1\right) \\ +S_H s_k(W_H,P)\left(\eta_{C_k(W_H,P),P_k}+1\right)\end{bmatrix} \\
&= -\sum_{i=1}^{N}\alpha_i\left(S_L s_i(W_L,P)\,\eta_{s_i(W_L,P),P_k} + S_H s_i(W_H,P)\,\eta_{s_i(W_H,P),P_k}\right),
\end{aligned}$$

where the last equality stems from $\eta_{s_i(W_L,P),P_k} = \eta_{C_i(W_L,P),P_k}$ for $i \neq k$ and $\eta_{s_k(W_L,P),P_k} = \eta_{C_k(W_L,P),P_k} + 1$. This completes the proof of the numerator.

Next, we apply Lemma A.2 to the denominator (all η operators signify again partial elasticities):

$$f\eta_{f,y} + g\eta_{g,y} = \left[S_L\eta_{S_{L,W_H/W_L}} - \left(\sum_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right)\right)\eta_{\Sigma_{i=1}^{N}\alpha_i\left(S_L s_i^L + S_H s_i^H\right),W_H/W_L}\right].$$

Define:

$$Q_1 := S_L \eta_{S_{L,W_H/W_L}} = -S_L S_H$$
$$Q_2 := -\left(\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right)\right) \eta_{\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right), W_H/W_L},$$

where the equality in the first row stems from preliminaries C.

We have:

$$\begin{aligned}
Q_2 &= -\left(\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right)\right) \eta_{\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right), W_H/W_L} \\
&= -\left(\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right)\right) \left[\frac{\alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right)}{\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right)} \eta_{\alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right), W_H/W_L}\right] \\
&= -\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right) \eta_{\alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right), W_H/W_L} \\
&= -\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right) \left[\eta_{\alpha_i, W_H/W_L} + \eta_{\left(S_L s_{i,L} + S_H s_{i,H}\right), W_H/W_L}\right] \\
&= -\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right) \left[\eta_{\alpha_i, W_H/W_L} + \frac{S_L s_{i,L}}{\left(S_L s_{i,L} + S_H s_{i,H}\right)} \eta_{\left(S_L s_{i,L}\right), W_H/W_L} + \frac{S_H s_{i,H}}{\left(S_L s_{i,L} + S_H s_{i,H}\right)} \eta_{\left(S_H s_{i,H}\right), W_H/W_L}\right] \\
&= -\sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right) \eta_{\alpha_i, W_H/W_L} - \sum_{i=1}^{N} \alpha_i \left[S_L s_i \eta_{\left(S_L s_{i,L}\right), W_H/W_L} + S_H s_i \eta_{\left(S_H s_{i,H}\right), W_H/W_L}\right]
\end{aligned}$$

And applying preliminary C again, we obtain:

$$\begin{aligned}
Q_2 &= \sum_{i=1}^{N} \alpha_i \left(S_L s_{i,L} + S_H s_{i,H}\right) \left(1-\alpha_i\right)\left(1-\sigma_i\right) - \sum_{i=1}^{N} \alpha_i \left[S_L s_{i,L}\left(-S_H + \eta_{s_{i,L}, W_H/W_L}\right) + S_H s_{i,H}\left(S_L + \eta_{s_{i,H}, W_H/W_L}\right)\right] \\
&= \sum_{i=1}^{N} \alpha_i \left\{S_L s_{i,L}\left[\left(1-\alpha_i\right)\left(1-\sigma_i\right) + S_H - \eta_{s_{i,L}, W_H/W_L}\right] + S_H s_{i,H}\left[\left(1-\alpha_i\right)\left(1-\sigma_i\right) - \left(S_L + \eta_{s_{i,H}, W_H/W_L}\right)\right]\right\}
\end{aligned} \tag{A.6}$$

For ease of exposition, it is useful to re-order the goods such that the numeraire (which is also the sector expriencing the change) is good N.[29]

We turn now, to the derivation of $\eta_{s_i(W_L,\mathbf{P}), \frac{W_H}{W_L}}$ and $\eta_{s_i(W_H,\mathbf{P}), \frac{W_H}{W_L}}$:

[29]This is imaterial, because, as we show below, the denominator is independent of the good experiencing the change.

$$\begin{aligned}
\eta_{s_i(W_L,\mathbf{P}),\frac{W_H}{W_L}} &= \eta_{P_i} - \eta_{W_L,\frac{W_H}{W_L}} + \eta_{C_i(W_L,\mathbf{P}),\frac{W_H}{W_L}} \\
&= 1 - \alpha_i + \eta_{C_i(W_L,\mathbf{P}),\frac{W_H}{W_L}} \\
&= 1 - \alpha_i + \begin{pmatrix} . & \eta_{C_i^L,P_j} & . & \eta_{C_i^L,W_L} \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_N - \alpha_j \\ . \\ \alpha_N - 1 \end{pmatrix}_{N\times 1}
\end{aligned}$$

where j runs from 1 to $N-1$ (recall that good N is the numeraire). In the derivation above, the second line stems from preliminary D, and the last line from applying the chain rule and once again using preliminary D.

By an analagous line of reasoning, it is possible to show that

$$\eta_{s_i(W_H,\mathbf{P}),\frac{W_H}{W_L}} = -\alpha_i + \begin{pmatrix} . & \eta_{C_i^H,P_j} & . & \eta_{C_i^H,W_H} \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_N - \alpha_j \\ . \\ \alpha_N \end{pmatrix}_{N\times 1}$$

Plugging these into (A.6), we get:

$$Q_2 = \sum_{i=1}^{N} \alpha_i \left\{ \begin{array}{l} S_L s_i(W_L,P)\left[(1-\alpha_i)(1-\sigma_i)+S_H-\left(1-\alpha_i+\begin{pmatrix} . & \eta_{C_i^L,P_j} & . & \eta_{C_i^L,W_L}\end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_N-\alpha_j \\ . \\ \alpha_N-1 \end{pmatrix}_{N\times 1}\right)\right] \\ +S_H s_i(W_H,P)\left[(1-\alpha_i)(1-\sigma_i)-\left(S_L-\alpha_i+\begin{pmatrix} . & \eta_{C_i^H,P_j} & . & \eta_{C_i^H,W_H}\end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_N-\alpha_j \\ . \\ \alpha_N \end{pmatrix}_{N\times 1}\right)\right] \end{array} \right\}$$

$$= \sum_{i=1}^{N} \alpha_i \left\{ \begin{array}{l} \underbrace{S_L s_i(W_L,P)\left[-\sigma_i(1-\alpha_i)+S_H-\begin{pmatrix} . & \eta_{C_i^L,P_j} & . & \eta_{C_i^L,W_L}\end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_N-\alpha_j \\ . \\ \alpha_N-1 \end{pmatrix}_{N\times 1}\right]}_{\text{Part A}} \\ \underbrace{+S_H s_i(W_H,P)\left[-\sigma_i(1-\alpha_i)-S_L+1-\begin{pmatrix} . & \eta_{C_i^H,P_j} & . & \eta_{C_i^H,W_H}\end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_N-\alpha_j \\ . \\ \alpha_N \end{pmatrix}_{N\times 1}\right]}_{\text{Part B}} \end{array} \right\}$$

Starting from Part A, we get:

$$\text{Part A} = S_L s_i(W_L,P)\left[-\sigma_i(1-\alpha_i)+S_H-\begin{pmatrix} . & \eta_{C_i^L,P_j} & . & \eta_{C_i^L,W_L}\end{pmatrix} * \begin{pmatrix} . \\ \alpha_N \\ . \\ \end{pmatrix}_{N\times 1} + \begin{pmatrix} . & \eta_{C_i^L,P_j} & . & \eta_{C_i^L,W_L}\end{pmatrix} * \begin{pmatrix} . \\ \alpha_j \\ . \\ 1 \end{pmatrix}_{N\times 1}\right]$$

$$= S_L s_i(W_L,P)\left[-\sigma_i(1-\alpha_i)+S_H-\alpha_N\left(\sum_{j=1}^{N-1}\eta_{C_i^L,P_j}+\eta_{C_i^L,W_L}\right)+\begin{pmatrix} . & \eta_{C_i^L,P_j} & . & \eta_{C_i^L,W_L}\end{pmatrix} * \begin{pmatrix} . \\ \alpha_j \\ . \\ 1 \end{pmatrix}_{N\times 1}\right]$$

From homogeneity degree zero of the demand function, $\sum_{j=1}^{N-1} \eta_{C_i,P_j} + \eta_{C_i,W_L} = -\eta_{C_i,P_N}$, hence:

$$\text{Part A} = S_L s_i(W_L,P)\left[-\sigma_i(1-\alpha_i) + S_H + \begin{pmatrix} . & \eta_{C_i(W_L,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \\ \alpha_N \end{pmatrix}_{N\times 1} + \eta_{C_i(W_L,P),W_L}\right]$$

A similar argument shows that:

$$\text{Part B} = S_H s_i(W_H,P)\left[-\sigma_i(1-\alpha_i) - S_L + 1 + \begin{pmatrix} . & \eta_{C_i(W_H,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \\ \alpha_N \end{pmatrix}_{N\times 1}\right]$$

Therefore:

$$Q_2 = \sum_{i=1}^{N} \alpha_i \left\{ \begin{array}{l} S_L s_i(W_L,P)\left[-\sigma_i(1-\alpha_i) + S_H + \begin{pmatrix} . & \eta_{C_i(W_L,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \\ \alpha_N \end{pmatrix}_{N\times 1} + \eta_{C_i(W_L,P),W_L}\right] \\ +S_H s_i(W_H,P)\left[-\sigma_i(1-\alpha_i) - S_L + 1 + \begin{pmatrix} . & \eta_{C_i(W_H,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \\ \alpha_N \end{pmatrix}_{N\times 1}\right] \end{array} \right\}$$

$$= S_L S_H + \sum_{i=1}^{N} \alpha_i \left\{ \begin{array}{c} S_L s_i(W_L,P) \left[-\sigma_i(1-\alpha_i) + \begin{pmatrix} . & \eta_{C_i(W_L,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1} + \eta_{C_i(W_L,P),W_L} \right] \\ + S_H s_i(W_H,P) \left[-\sigma_i(1-\alpha_i) + \begin{pmatrix} . & \eta_{C_i(W_H,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1} \right] \end{array} \right\}$$

where the last equality stems from Equation (A.4).

Putting together Q_1 and Q_2, the denominator (Q_1+Q_2) becomes:

$$Q_1 + Q_2 = \sum_{i=1}^{N} \alpha_i \left\{ \begin{array}{c} S_L s_i(W_L,P) \left[-\sigma_i(1-\alpha_i) + \begin{pmatrix} . & \eta_{C_i(W_L,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1} + \eta_{C_i(W_L,P),W_L} \right] \\ + S_H s_i(W_H,P) \left[-\sigma_i(1-\alpha_i) + \begin{pmatrix} . & \eta_{C_i(W_H,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1} \right] \end{array} \right\}$$

$$\begin{aligned} &= -\sum_{i=1}^{N} \sigma_i (1-\alpha_i)\, \alpha_i \left\{ S_L s_i(W_L,P) + S_H s_i(W_H,P) \right\} \qquad \text{(A.7)} \\ &+ \sum_{i=1}^{N} \alpha_i S_L s_i(W_L,P) \left[\begin{pmatrix} . & \eta_{C_i(W_L,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1} + \eta_{C_i(W_L,P),W_L} \right] \\ &+ \sum_{i=1}^{N} \alpha_i S_H s_i(W_H,P) \left[\begin{pmatrix} . & \eta_{C_i(W_H,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1} \right] \end{aligned}$$

Next, we focus on the second element in the summation of (A.7). Applying the Slutsky equation, $\frac{\partial C_i(W_L,P)}{\partial P_j} = \frac{\partial C_i^{hicks}(W_L,P)}{\partial P_j} - \frac{\partial C_i(W_L,P)}{\partial W_L} C_j(W_L,P)$:

$$\sum_{i=1}^{N} \alpha_i S_L s_i(W_L,P) \left[\begin{pmatrix} . & \eta_{C_i(W_L,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1} + \eta_{C_i(W_L,P),W_L} \right]$$

$$= \frac{S_L}{W_L} \sum_{i=1}^{N} \alpha_i \left\{ P_i \left[\begin{pmatrix} . & \frac{\partial C_i(W_L,P)}{\partial P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j P_j \\ . \end{pmatrix}_{N\times 1} + C_i(W_L,P)\, \eta_{C_i(W_L,P),W_L} \right] \right\}$$

$$= \frac{S_L}{W_L} \sum_{i=1}^{N} \alpha_i \left\{ P_i \left[\begin{pmatrix} . & \frac{\partial C_i^{hicks}(W_L,P)}{\partial P_j} - \frac{\partial C_i(W_L,P)}{\partial W_L} C_j(W_L,P) & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j P_j \\ . \end{pmatrix}_{N\times 1} + C_i(W_L,P)\, \eta_{C_i(W_L,P),W_L} \right] \right\}$$

$$= \frac{S_L}{W_L} (\alpha \circ \mathbf{P})' \boldsymbol{Slutsky}^L (\alpha \circ \mathbf{P}) + \frac{S_L}{W_L} \sum_{i=1}^{N} \alpha_i P_i \begin{pmatrix} . & -\frac{\partial C_i(W_L,P)}{\partial W_L} C_j(W_L,P) & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j P_j \\ . \end{pmatrix}_{N\times 1} + \frac{S_L}{W_L} \sum_{i=1}^{N} \alpha_i C_i(W_L,P) P_i \eta_{C_i(W_L,P),W_L}$$

Similarly, the third element of (A.7) can be written as:

$$\sum_{i=1}^{N} \alpha_i S_H s_i(W_H,P) \left[\begin{pmatrix} . & \eta_{C_i(W_H,P),P_j} & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1} \right]$$

$$= \frac{S_H}{W_H} (\alpha \circ \mathbf{P})' \boldsymbol{Slutsky}^H (\alpha \circ \mathbf{P}) + \frac{S_H}{W_H} \sum_{i=1}^{N} \alpha_i P_i \begin{pmatrix} . & -\frac{\partial C_i(W_H,P)}{\partial W_H} C_j(W_H,P) & . \end{pmatrix}_{1\times N} * \begin{pmatrix} . \\ \alpha_j P_j \\ . \end{pmatrix}_{N\times 1}$$

Hence,

$$Q_1+Q_2=\frac{S_L}{W_L}(\alpha\circ\mathbf{P})'\,\boldsymbol{Slutsky}^L(\alpha\circ\mathbf{P})+\frac{S_H}{W_H}(\alpha\circ\mathbf{P})'\,\boldsymbol{Slutsky}^H(\alpha\circ\mathbf{P}) \tag{A.8}$$
$$-\sum_{i=1}^{N}\sigma_i(1-\alpha_i)\alpha_i\left[S_L s_i(W_L,P)+S_H s_i(W_H,P)\right]$$
$$+\frac{S_L}{W_L}\sum_{i=1}^{N}\alpha_i\left\{P_i\left[\begin{pmatrix} . & -\frac{\partial C_i(W_L,P)}{\partial W_L}C_j(W_L,P) & . \end{pmatrix}_{1\times N}*\begin{pmatrix} . \\ \alpha_j P_j \\ . \end{pmatrix}_{N\times 1}+C_i(W_L,P)\,\eta_{C_i(W_L,P),W_L}\right]\right\}$$
$$+\frac{S_H}{W_H}\sum_{i=1}^{N}\alpha_i P_i\left[\begin{pmatrix} . & -\frac{\partial C_i(W_H,P)}{\partial W_H}C_j(W_H,P) & . \end{pmatrix}_{1\times N}*\begin{pmatrix} . \\ \alpha_j P_j \\ . \end{pmatrix}_{N\times 1}\right]$$

Finally, focusing on the last two lines of (A.8):

$$\frac{S_L}{W_L}\sum_{i=1}^{N}\alpha_i\left\{P_i\left[\begin{pmatrix} . & -\frac{\partial C_i(W_L,P)}{\partial W_L}C_j(W_L,P) & . \end{pmatrix}\begin{pmatrix} . \\ \alpha_j P_j \\ . \end{pmatrix}_{N\times 1}+C_i(W_L,P)\,\eta_{C_i(W_L,P),W_L}\right]\right\}+\frac{S_H}{W_H}\sum_{i=1}^{N}\alpha_i P_i\left[\begin{pmatrix} . & -\frac{\partial C_i(W_H,P)}{\partial W_H}C_j(W_H,P) & . \end{pmatrix}\begin{pmatrix} . \\ \alpha_j P_j \\ . \end{pmatrix}_{N\times 1}\right]$$

$$=\sum_{i=1}^{N}\alpha_i P_i\left[\begin{pmatrix} . & -\frac{\partial C_i(W_L,P)}{\partial W_L}S_L s_j(W_L,P) & . \end{pmatrix}\begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1}+\begin{pmatrix} . & -\frac{\partial C_i(W_H,P)}{\partial W_H}S_H s_j(W_H,P) & . \end{pmatrix}\begin{pmatrix} . \\ \alpha_j \\ . \end{pmatrix}_{N\times 1}+\frac{S_L}{W_L}C_i(W_L,P)\,\eta_{C_i(W_L,P),W_L}\right] \tag{A.9}$$

But note that from equation (A.4):

$$\frac{S_L}{W_L}C_i(W_L,P)\,\eta_{C_i(W_L,P),W_L}=\frac{\sum_{j=1}^{N}\alpha_j\left(S_L s_j(W_L,P)+S_H s_j(W_H,P)\right)}{W_L}C_i(W_L,P)\frac{\partial C_i(W_L,P)}{\partial W_L}\frac{W_L}{C_i(W_L,P)}$$
$$=\sum_{j=1}^{N}\alpha_j\left(S_L s_j(W_L,P)+S_H s_j(W_H,P)\right)\frac{\partial C_i(W_L,P)}{\partial W_L}$$

and plugging back into (A.9):

$$= \sum_{i=1}^{N} \alpha_i P_i \left[\begin{array}{c} \left(\cdot \quad -\frac{\partial C_i(W_L,P)}{\partial W_L} S_L s_j(W_L,P) \quad \cdot \right)_{1\times N} * \begin{pmatrix} \cdot \\ \alpha_j \\ \cdot \end{pmatrix}_{N\times 1} + \left(\cdot \quad -\frac{\partial C_i(W_H,P)}{\partial W_H} S_H s_j(W_H,P) \quad \cdot \right)_{1\times N} * \begin{pmatrix} \cdot \\ \alpha_j \\ \cdot \end{pmatrix}_{N\times 1} \\ +\sum_{j=1}^{N} \alpha_j \left(S_L s_j(W_L,P) + S_H s_j(W_H,P)\right) \frac{\partial C_i(W_L,P)}{\partial W_L} \end{array} \right]$$

$$= \sum_{i=1}^{N} \alpha_i P_i \left[\begin{array}{c} -\sum_{j=1}^{N} \alpha_j \left(\frac{\partial C_i(W_L,P)}{\partial W_L} S_L s_j(W_L,P) \right) - \sum_{j=1}^{N} \alpha_j \left(\frac{\partial C_i(W_H,P)}{\partial W_H} S_H s_j(W_H,P) \right) \\ +\sum_{j=1}^{N} \alpha_j \left(S_L s_j(W_L,P) \frac{\partial C_i(W_L,P)}{\partial W_L} \right) + \sum_{j=1}^{N} \alpha_j \left(S_H s_j(W_H,P) \frac{\partial C_i(W_L,P)}{\partial W_L} \right) \end{array} \right]$$

$$= \sum_{i=1}^{N} \alpha_i P_i \left[\sum_{j=1}^{N} \alpha_j \left(S_H s_j(W_H,P) \left(\frac{\partial C_i(W_L,P)}{\partial W_L} - \frac{\partial C_i(W_H,P)}{\partial W_H} \right) \right) \right]$$

$$= \left[\sum_{j=1}^{N} \alpha_j \left(S_H s_j(W_H,P)\right) \right] \sum_{i=1}^{N} \alpha_i \left(\frac{\partial E_i(W_L,P)}{\partial W_L} - \frac{\partial E_i(W_H,P)}{\partial W_H} \right)$$

To summarize, the denominator can be decomposed as follows:

$$Q := Q_1 + Q_2 = \underbrace{\frac{S_L}{W_L} (\alpha \circ \mathbf{P})' \boldsymbol{Slutsky}^L (\alpha \circ \mathbf{P}) + \frac{S_H}{W_H} (\alpha \circ \mathbf{P})' \boldsymbol{Slutsky}^H (\alpha \circ \mathbf{P})}_{\text{Substitution effect (Demand)}<0}$$

$$\underbrace{-\sum_{i=1}^{N} \sigma_i (1-\alpha_i) \alpha_i \left[S_L s_i(W_L,P) + S_H s_i(W_H,P)\right]}_{\text{Substitution effect (Supply)}<0}$$

$$\underbrace{-\left[\sum_{j=1}^{N} \alpha_j \left(S_H s_j(W_H,P)\right) \right] \sum_{i=1}^{N} \alpha_i \left(\frac{\partial E_i(W_H,P)}{\partial W_H} - \frac{\partial E_i(W_L,P)}{\partial W_L} \right)}_{\text{Income effect (Non-homotheticity)} \lesseqgtr 0}$$

□

Lemma 3

Proof. To prove part (a), note that because the denominator in Equation 8 is negative, Theorem 3 implies that a sufficient condition for $\eta_{\frac{W_H}{W_L},A_k}$ to be positive is that for each type $m \in \{H, L\}$:

$$0 < \Sigma_{i=1}^{N} \alpha_i s_i^m \eta_{s_i^m,P_k} \tag{A.10}$$

Since $s_i = \frac{C_i P_i}{W_i}$, for all $i \neq k$ we have that $\eta_{s_i^m, P_k} = \eta_{C_i^m, P_k}$, whereas for $i = k$ we have $\eta_{s_i^m, P_k} = \eta_{C_i^m, P_k} + 1$. Therefore:

$$\begin{aligned}
\Sigma_{i=1}^N \alpha_i s_i^m \eta_{s_i^m, P_k} &= \alpha_k s_k^m + \Sigma_{i=1}^N \alpha_i s_i^m \eta_{C_i^m, P_k} \\
&= \alpha_k s_k^m - s_k^m \Sigma_{i=1}^N \alpha_i \frac{s_i^m \eta_{C_i^m, P_k}}{(-s_k^m)} \\
&= \alpha_k s_k^m - s_k^m \Sigma_{i=1}^N \alpha_i \frac{s_i^m \eta_{C_i^m, P_k}}{\Sigma_{j=1}^N s_j^m \eta_{C_j^m, P_k}},
\end{aligned}$$

where the last line stems from Cournot aggregation.

Defining $w_{i,k}^m := \frac{s_i^m \eta_{C_i^m, P_k}}{\sum_j s_j^m \eta_{C_j^m, P_k}}$ and plugging into Inequality (A.10), we have that a sufficient condition for the elasticity $\eta_{\frac{W_H}{W_L}, A_k}$ to be positive is that for each type $m \in \{H, L\}$:

$$0 < \alpha_k s_k^m - s_k^m \Sigma_{i=1}^N \alpha_i w_{i,k}^m.$$

Dividing by s_k^m completes the proof of part (a) of the Lemma. Part (b) is proved in an analogous manner.

□

Corollary 1. *Assume that $Q < 0$ and that all goods are gross-complements. Then: $\eta_{W_H/W_L, A_k}$ will be positive when the lowest skill intensity sector ($\max(\alpha_i)$) experiences positive technical change and negative when the highest skill intensity sector ($\min(\alpha_i)$) experiences positive technical change.*

Proof. From Lemma 3 it is easy to see that since the denominator in Equation (8) is negative, $\eta_{\frac{W_H}{W_L}, A_k}$ will be positive when for each type $m \in \{H, L\}$:

$$\Sigma_{i=1}^N \alpha_i w_{i,k}^m < \alpha_k$$

Now, if all goods are gross complements, all $\{w_{i,k}^m\}_{i=1}^N$ are non-negative. To see this note that

$$\begin{aligned}
w_{i,k}^m &:= \frac{s_i^m \eta_{C_i^m, P_k}}{\sum_i s_i^m \eta_{C_i^m, P_k}} \\
&= \frac{s_i^m (-\eta_{C_i^m, P_k})}{-\sum_i s_i^m \eta_{C_i^m, P_k}} \\
&= \frac{s_i^m (-\eta_{C_i^m, P_k})}{s_k^m} \geq 0
\end{aligned}$$

where the last equality again stems form Cournot aggregation, and the inequality stems from the fact that goods are gross complements.

Because $\{w^m_{i,k}\}_{i=1}^N$ sum to one (by their definition) and are non-negative, under the conditions of Corollary 2 they are weights. As such, when the lowest-skill sector experiences the technical change, i.e. $\alpha_k = max\{\alpha_i\}$, then the inequality $\Sigma_{i=1}^N \alpha_i w^m_{i,k} < \alpha_k$ will hold for $m \in \{H, L\}$, and $\eta_{\frac{W_H}{W_L}, A_k}$ will therefore be positive.

By an analogous argument it is easy to show that when the highest-skill sector experiences the technical change, i.e. $\alpha_k = min\{\alpha_i\}$, then $\eta_{\frac{W_H}{W_L}, A_k}$ will be negative.

We note that with further restrictions on preferences, the results in this corollary extend beyond the highest and lowest α sectors. Indeed, if preferences are Non-homothetic CES and all goods are gross-complements (as in Comin, Lashkari and Mestieri (2021)), it can be shown that there exist $\overline{\alpha} > \underline{\alpha}$ such that:

$$\forall \alpha_k > \overline{\alpha} : \ \eta_{W_H/W_L, A_k} > 0$$
$$\forall \alpha_k < \underline{\alpha} : \ \eta_{W_H/W_L, A_k} < 0$$

Furthermore, if preferences are homothetic CES and all goods are gross-complements, then $\overline{\alpha} = \underline{\alpha}$. □

Theorem 4

Proof. The proof, which is available upon request, applies the implicit function theorem analogously to the proof of Theorem 3. Doing so implies that the the modified denominator is:

$$\tilde{Q} = \tau\alpha_{N+1}\left(\eta_{\tau, W_H/W_L} - (1-\alpha_{N+1})(1-\sigma_{N+1})\right) + S_L S_H + (S_L - \tau\alpha_{N+1})\,\eta_{(1-\tau), W_H/W_L} + (1-\tau)\left[\sum_{i=1}^{N}\{\alpha_i\,(S_L s_i(W_L(1-\tau), \mathbf{P}) + S_H s_i(W_H(1-\tau), \mathbf{P}))\}\,\eta_{\alpha_i, W_H/W_L}\right] + (1-\tau)\left[\sum_{i=1}^{N}\alpha_i\left\{S_L s_i(W_L(1-\tau), \mathbf{P})\left(-S_H + \eta_{s_i(W_L(1-\tau), \mathbf{P}), W_H/W_L}\right) + S_H s_i(W_H(1-\tau), \mathbf{P})\left(S_L + \eta_{s_i(W_H(1-\tau), \mathbf{P}), W_H/W_L}\right)\right\}\right]$$

where similar to the proofs of the preliminaries in the proof of 3, one can show that:

- $\eta_{\tau, W_H/W_L} = S_L - \alpha_{N+1}$
- $\eta_{(1-\tau), W_H/W_L} = -\frac{\tau}{1-\tau}(S_L - \alpha_{N+1})$

- $\eta_{\alpha_i, W_H/W_L} = -(1-\alpha_i)(1-\sigma_i)$
- $\eta_{s_i(W_L(1-\tau),\mathbf{P}),W_H/W_L} = (1-\alpha_i) - \eta_{(1-\tau),W_H/W_L} + \eta_{C_i(W_L(1-\tau),\mathbf{P}),W_H/W_L}$
- $\eta_{s_i(W_H(1-\tau),\mathbf{P}),W_H/W_L} = -\alpha_i - \eta_{(1-\tau),W_H/W_L} + \eta_{C_i(W_H(1-\tau),\mathbf{P}),W_H/W_L}$
- $\eta_{C_i(W_L(1-\tau),\mathbf{P}),\frac{W_H}{W_L}} = \eta_{C_i(W_L(1-\tau),\mathbf{P}),W_L}\left(\eta_{(1-\tau),\frac{W_H}{W_L}} - 1\right) - \sum_{j=1}^{N}\alpha_j\eta_{C_i(W_L(1-\tau),\mathbf{P}),P_j}$
- $\eta_{C_i(W_H(1-\tau),\mathbf{P}),\frac{W_H}{W_L}} = \eta_{C_i(W_H(1-\tau),\mathbf{P}),W_H}\left(\eta_{(1-\tau),\frac{W_H}{W_L}}\right) - \sum_{j=1}^{N}\alpha_j\eta_{C_i(W_H(1-\tau),\mathbf{P}),P_j}$

□

Uniqueness.

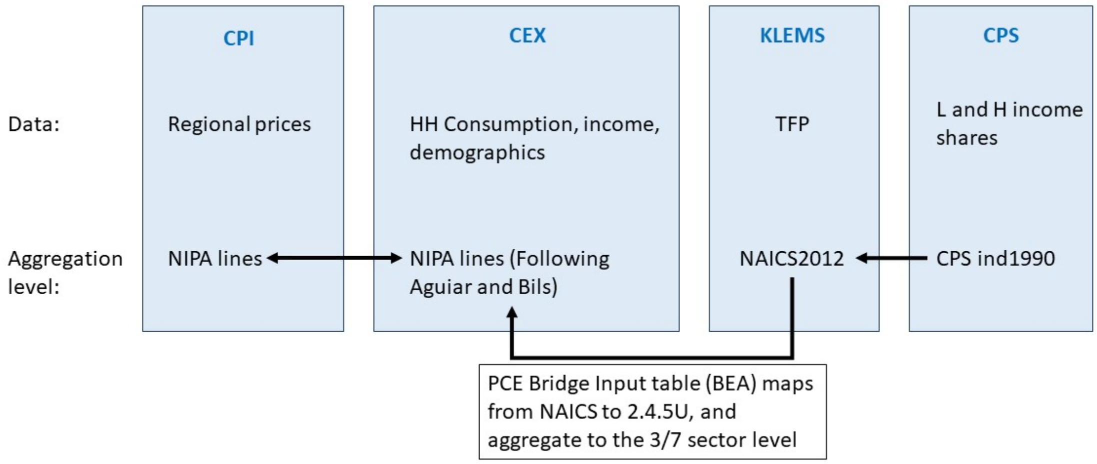
The discussion herein relies on the results from Lemma 2 and Theorem 3. In particular, Lemma 2 shows that all equilibrium conditions can be consolidated into a single market-clearing condition for high-skilled individuals given by $\mathscr{H}(\mathbf{A}, \frac{W_H}{W_L}) = 0$. Thus, a sufficient condition to have a unique equilibrium relative wage $\frac{W_H}{W_L}$, is that $\mathscr{H}(\mathbf{A}, \frac{W_H}{W_L})$ is strictly monotone in $\frac{W_H}{W_L}$. From the proof to Lemma 2, it is easy to show that this unique $\frac{W_H}{W_L}$ pins down all other prices and a unique allocations of high and low skilled labor across sectors.

Finally, we remind the reader that, as shown in Theorem 3, the partial elasticity of $\mathscr{H}$ w.r.t W_H/W_L is given by $Q\left(\boldsymbol{\alpha}, \boldsymbol{\sigma}, S_L, S_H, \mathbf{s}^L, \mathbf{s}^H, \boldsymbol{\eta}^L_{\mathbf{C},\mathbf{P}}, \boldsymbol{\eta}^H_{\mathbf{C},\mathbf{P}}, \boldsymbol{\eta}^L_{\mathbf{C},\mathbf{W}}, \boldsymbol{\eta}^H_{\mathbf{C},\mathbf{W}}\right)$. Thus, so long as Q is always negative, the equilibrium is unique. As discussed in Theorem 3, Q is always negative for the case of homothetic preferences. In the case that preferences are non-homothetic, it is negative as long as the income effect in (9) does not dominate the two substitutions effects, which is always the case in our application.

A.2. Empirical Analysis

A.2.1. Data Construction and Estimation Details

The figure below provides a schematic representation of our data linking process.



A.2.2. Calculating Sectoral EOS

In our analysis, we aim for an average elasticity of substitution of 1.4, as indicated by the estimates in Krusell et al. (2000). Under our model's assumption that wages specific to different skill levels are uniform across all sectors, the relationship between any two sectors elasticities of substitution equals the ratio of the sector-specific changes in the relative proportions of low-to-high skilled inputs. This relationship, combined with our target elasticity of 1.4 and the respective sectoral weights, allows us to calculate the varying degrees of labor substitution across different sectors. The values we obtain are as follows; $\sigma_{\text{Housing}} = 1.61$, $\sigma_{\text{Food Away and Entertainment}} = 1.57$, $\sigma_{\text{Transportation}} = 1.52$, $\sigma_{\text{Food at Home}} = 1.22$, $\sigma_{\text{Other Services}} = 1.14$, $\sigma_{\text{Durables}} = 1.59$, and $\sigma_{\text{Utilities}} = 1.21$.

A.2.3. Estimating the Relative Wage Elasticities for the non-Homothetic CES case

To obtain estimates of $\eta^{nh\,CES}_{W_H/W_L,A_k}$, we need to apply the results from Theorem 3 to a set of income and price elasticities obtained while imposing the non-homothetic CES functional form restrictions. The non-homothetic CES function is characterized by a vector of N parameters ($\boldsymbol{\varepsilon}$) that directly discipline the total expenditure elasticities, and another parameter that disciplines the elasticity of substitution between goods (ν). We set the parameters such that the total expenditure elasticities are perfectly matched, and the quadratic distance between the AIDS estimated cross-price elasticities and the non-homothetic CES implied elasticities is set to a minimum. For the 3 categories case, we obtain $\nu = 0.25$, which is very similar to the one reported in Comin, Lashkari and Mestieri (2021). For the 7 categories case, we obtain $\nu = 0.34$.

A.2.4. Data Construction for Public Sectors

The military personnel measures are taken from The center for strategic and budgetary assessment. In order to construct the skill intensity of this sector we calculate the share of workers that are low-vs-high skill in this sector. We then attribute to them the average low and high skill wages in the economy.

For the public health sector, we note that the overall expenses on Medicare and Medicaid are approximately 12% of private consumption. We then calculate from the CPS the skill intensity of the health sector.

A.2.5. Appendix Tables

Table A1: Descriptive Statistics

	Low skill		High skill		Low-skilled labor share
	Mean	S.D.	Mean	S.D.	
A. Expenditure shares: 3 goods					
Agriculture	0.14	0.07	0.11	0.06	0.72
Manufacturing	0.26	0.12	0.25	0.11	0.66
Services	0.6	0.12	0.64	0.11	0.59
B. Expenditure shares: 7 goods					
Food home	0.13	0.06	0.11	0.05	0.72
Housing	0.35	0.11	0.37	0.11	0.58
Utilities	0.07	0.03	0.05	0.03	0.67
Transportation	0.16	0.1	0.15	0.1	0.65
Food away, ent., apparel	0.14	0.07	0.15	0.07	0.7
Other services	0.09	0.07	0.1	0.07	0.55
Durables	0.06	0.04	0.06	0.04	0.62
C. Household aggregates and controls					
Nominal household expenditures	37,644	20,188	48,420	25,716	
Nominal household after tax income	53,578	29,974	73,766	35,792	
Age	45.5	10.4	44.3	10.5	
Number of family members	3	1.5	2.7	1.4	
A two earner household	0.6	0.5	0.6	0.5	
Number of households	18,045		9,226		
Number of observations	56,018		29,509		

Notes: Descriptive Statistics for CEX sample used in the estimation of the AIDS. The low-skilled labor share column is calculated using CPS data.

Table A2: Expenditure and price elasticities: 7 categories, low-skilled

	Expenditure Elasticity	Price Elasticities						
		Food home	Housing	Utilities	Transportation	Food away, ent., apparel	Other services	Durables
Food home	0.362***	-0.343	-0.294	0.095	0.059	-0.07	0.057	0.134*
	(0.018)	(0.255)	(0.189)	(0.084)	(0.086)	(0.182)	(0.061)	(0.071)
	[0.332,0.391]	[-0.828,0.022]	[-0.557,0.055]	[-0.035,0.241]	[-0.069,0.194]	[-0.332,0.233]	[-0.043,0.165]	[0.007,0.245]
Housing	1.162***	-0.219***	-0.236*	-0.13***	-0.005	-0.553***	0.018	-0.038
	(0.014)	(0.072)	(0.131)	(0.034)	(0.047)	(0.064)	(0.039)	(0.032)
	[1.14,1.185]	[-0.322,-0.086]	[-0.442,-0.028]	[-0.189,-0.075]	[-0.081,0.068]	[-0.656,-0.444]	[-0.045,0.077]	[-0.09,0.012]
Utilities	0.568***	0.163	-0.471***	-0.375***	-0.075	0.078	0.09	0.023
	(0.023)	(0.166)	(0.18)	(0.097)	(0.091)	(0.155)	(0.07)	(0.082)
	[0.531,0.6]	[-0.098,0.457]	[-0.793,-0.181]	[-0.535,-0.207]	[-0.235,0.076]	[-0.213,0.318]	[-0.016,0.206]	[-0.108,0.17]
Transportation	0.828***	-0.011	0.106	-0.05	-0.662***	0.033	-0.199***	-0.045
	(0.023)	(0.074)	(0.107)	(0.039)	(0.085)	(0.074)	(0.052)	(0.042)
	[0.786,0.866]	[-0.12,0.098]	[-0.062,0.282]	[-0.116,0.015]	[-0.792,-0.503]	[-0.078,0.16]	[-0.279,-0.098]	[-0.118,0.017]
Food away, ent., apparel	1.374***	-0.197	-1.381***	-0.019	-0.05	0.465**	0.015	-0.207***
	(0.02)	(0.165)	(0.15)	(0.07)	(0.078)	(0.183)	(0.058)	(0.06)
	[1.343,1.408]	[-0.433,0.084]	[-1.627,-1.126]	[-0.153,0.091]	[-0.165,0.086]	[0.137,0.744]	[-0.081,0.108]	[-0.301,-0.103]
Other services	1.29***	-0.032	0.032	0.025	-0.444***	0.04	-1.023***	0.112**
	(0.04)	(0.1)	(0.166)	(0.057)	(0.098)	(0.105)	(0.113)	(0.055)
	[1.227,1.354]	[-0.193,0.146]	[-0.243,0.271]	[-0.06,0.121]	[-0.595,-0.264]	[-0.129,0.206]	[-1.202,-0.818]	[0.032,0.202]
Durables	1.069***	0.169	-0.161	-0.01	-0.139	-0.406***	0.153**	-0.673***
	(0.028)	(0.14)	(0.161)	(0.08)	(0.094)	(0.131)	(0.066)	(0.089)
	[1.024,1.116]	[-0.085,0.386]	[-0.417,0.101]	[-0.14,0.135]	[-0.303,0.001]	[-0.607,-0.178]	[0.057,0.259]	[-0.82,-0.517]

Notes: Expenditure and uncompensated price elasticities implied by the AIDS estimates. Bootstrapped standard errors in parentheses and 90% confidence intervals in square brackets.

Table A3: Expenditure and price elasticities: 7 categories, high-skilled

	Expenditure Elasticity	Price Elasticities						
		Food home	Housing	Utilities	Transportation	Food away, ent., apparel	Other services	Durables
Food home	0.503***	0.035	-0.216	-0.178*	0.095	-0.275	0.064	-0.028
	(0.023)	(0.35)	(0.23)	(0.105)	(0.115)	(0.26)	(0.068)	(0.086)
	[0.463,0.537]	[-0.591,0.613]	[-0.568,0.159]	[-0.329,0.019]	[-0.112,0.252]	[-0.646,0.203]	[-0.046,0.178]	[-0.162,0.12]
Housing	1.244***	-0.164**	-0.4***	0.002	-0.065	-0.725***	0.059	0.049
	(0.025)	(0.078)	(0.136)	(0.037)	(0.06)	(0.089)	(0.049)	(0.042)
	[1.208,1.28]	[-0.287,-0.039]	[-0.628,-0.185]	[-0.067,0.059]	[-0.166,0.04]	[-0.879,-0.581]	[-0.015,0.145]	[-0.028,0.114]
Utilities	0.639***	-0.382*	0.23	-0.698***	0.082	0.558***	-0.041	-0.387***
	(0.031)	(0.214)	(0.226)	(0.115)	(0.118)	(0.203)	(0.089)	(0.101)
	[0.589,0.686]	[-0.684,0.022]	[-0.187,0.578]	[-0.865,-0.51]	[-0.098,0.278]	[0.232,0.921]	[-0.209,0.1]	[-0.549,-0.229]
Transportation	0.69***	0.06	0.032	0.032	-0.722***	-0.064	-0.163**	0.134**
	(0.036)	(0.099)	(0.154)	(0.05)	(0.106)	(0.109)	(0.075)	(0.056)
	[0.637,0.754]	[-0.118,0.192]	[-0.206,0.301]	[-0.043,0.116]	[-0.895,-0.551]	[-0.247,0.111]	[-0.289,-0.057]	[0.05,0.229]
Food away, ent., apparel	1.382***	-0.328	-1.761***	0.174**	-0.156	0.801***	-0.019	-0.092
	(0.036)	(0.209)	(0.217)	(0.079)	(0.099)	(0.242)	(0.082)	(0.085)
	[1.331,1.443]	[-0.62,0.057]	[-2.146,-1.409]	[0.051,0.316]	[-0.317,0.008]	[0.431,1.168]	[-0.158,0.115]	[-0.203,0.075]
Other services	0.815***	0.043	0.376**	-0.036	-0.255**	0.056	-0.975***	-0.025
	(0.045)	(0.087)	(0.186)	(0.056)	(0.108)	(0.129)	(0.132)	(0.069)
	[0.732,0.888]	[-0.092,0.189]	[0.107,0.69]	[-0.134,0.055]	[-0.424,-0.097]	[-0.155,0.267]	[-1.192,-0.756]	[-0.134,0.093]
Durables	0.962***	-0.109	0.37	-0.37***	0.247**	-0.149	-0.051	-0.9***
	(0.043)	(0.158)	(0.233)	(0.092)	(0.117)	(0.196)	(0.1)	(0.117)
	[0.895,1.035]	[-0.349,0.17]	[-0.055,0.711]	[-0.521,-0.225]	[0.067,0.453]	[-0.397,0.217]	[-0.215,0.118]	[-1.091,-0.726]

Notes: Expenditure and uncompensated price elasticities implied by the AIDS estimates. Bootstrapped standard errors in parentheses and 90% confidence intervals in square brackets.

Table A4: Expenditure and price elasticities: 3 categories

A. Low-skilled:

	Expenditure Elasticity	Price Elasticities		
		Agriculture	Manufacturing	Services
Agriculture	0.354***	-0.381**	0.135***	-0.108
	(0.017)	(0.171)	(0.042)	(0.153)
	[0.325,0.382]	[-0.647,-0.08]	[0.07,0.214]	[-0.384,0.147]
Manufacturing	0.934***	-0.009	-0.463***	-0.462***
	(0.015)	(0.021)	(0.029)	(0.033)
	[0.909,0.959]	[-0.041,0.032]	[-0.511,-0.419]	[-0.515,-0.401]
Services	1.17***	-0.131***	-0.264***	-0.775***
	(0.007)	(0.033)	(0.015)	(0.033)
	[1.159,1.18]	[-0.187,-0.079]	[-0.286,-0.239]	[-0.826,-0.716]

B. High-skilled:

	Expenditure Elasticity	Price Elasticities		
		Agriculture	Manufacturing	Services
Agriculture	0.502***	-0.235	-0.008	-0.259
	(0.025)	(0.225)	(0.047)	(0.205)
	[0.463,0.54]	[-0.639,0.144]	[-0.074,0.071]	[-0.574,0.108]
Manufacturing	0.874***	-0.049**	-0.298***	-0.526***
	(0.024)	(0.023)	(0.044)	(0.045)
	[0.831,0.909]	[-0.08,-0.011]	[-0.369,-0.221]	[-0.602,-0.453]
Services	1.147***	-0.129***	-0.278***	-0.74***
	(0.011)	(0.04)	(0.018)	(0.041)
	[1.128,1.165]	[-0.194,-0.056]	[-0.309,-0.247]	[-0.804,-0.676]

Notes: Expenditure and uncompensated price elasticities implied by the AIDS estimates. Bootstrapped standard errors in parentheses and 90% confidence intervals in square brackets.