NBER WORKING PAPER SERIES

THE INEXORABLE RECOVERIES OF UNEMPLOYMENT

Robert E. Hall
Marianna Kudlyak

Working Paper 28111
http://www.nber.org/papers/w28111

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2020, Revised February 2022

Hall's research was supported by the Hoover Institution. The opinions expressed are those of the authors and do not reflect those of the Federal Reserve Bank of San Francisco, the Federal Reserve System, or the National Bureau of Economic Research. We thank Stephane Dupraz, Emi Nakamura, and Jón Steinsson for providing the code for their business-cycle chronology software, and Marcelo Perlin for providing code for estimating hidden Markov models. We thank Markus Brunnermeier and Peter Diamond for comments. Backup data and code are available at Kudlyak's website.

The Inexorable Recoveries of Unemployment
Robert E. Hall and Marianna Kudlyak
NBER Working Paper No. 28111
November 2020, Revised February 2022
JEL No. E32,J64

ABSTRACT

Unemployment recoveries in the US have been inexorable. Between 1948 and 2019, the annual reduction in the unemployment rate during cyclical recoveries was fairly tightly distributed around 0.1 log points per year. The economy seems to have an irresistible force toward restoring full employment. In the aftermath of a recession, unless another crisis intervenes, unemployment continues to glide down. Occasionally, unemployment rises rapidly during an economic crisis, while most of the time, unemployment declines slowly and smoothly at a near-constant proportional rate. We show that similar properties hold for other measures of the US unemployment rate and for the unemployment rates of many other emerging and advanced countries.

Robert E. Hall
Hoover Institution
Stanford University
Stanford, CA 94305-6010
and NBER
rehall@gmail.com

Marianna Kudlyak
101 Market St
Federal Reserve Bank of San Francisco
San Francisco, CA 94105
mkudlyak@gmail.com

Contents

1 Introduction

We undertake a close examination of the behavior of US unemployment during cyclical recoveries, over the period from 1948 to 2019, using data from the Current Population Survey (CPS). We find that during this period, unemployment shot upward 10 times as the economy experienced economic crises. Following a crisis, the unemployment rate glided downward on a predictable but slow recovery path. In the longest recovery, from October 2009 to February 2020, unemployment reached 3.5 percent. Unemployment reached its historical minimal level over the entire period in the early 1950s, at 2.5 percent.

This paper is empirical and limited to the period from the beginning of modern unemployment measurement, in January 1948, to the end of the last completed recovery in February 2020. Further, we do not enter the thicket of general equilibrium models or Phillips curves. Rather, we study the behavior of unemployment in completed recoveries recorded in the CPS.

We find that the observed behavior of unemployment comprises (1) occasional sharp upward movements in times of economic crisis, and (2) an inexorable downward glide at a low but reliable proportional rate at other times.

We focus on recoveries. Our measurement starts in an economy that has been hit recently by an adverse shock that triggered a recession. The major recession that began in 1981 is generally viewed as the result of a sharp monetary contraction, while the major recession that began at the end of 2007 got much of its strength from the financial crisis of September 2008. This paper recognizes that the shocks that propel unemployment sharply upward have heterogeneous sources. The paper is about the homogeneity of historical recoveries.

Figure 1 shows our main evidence. It displays the log of the unemployment rate during the 10 completed recoveries since 1948, with the recession spells of sharply rising unemployment left blank. The key fact about recoveries is apparent in the figure: Unemployment declines smoothly but slowly throughout most recoveries most of the time, at close to the same proportional rate. In the log plot, the recoveries appear as impressively close to straight lines. In terms of levels rather than logs, this behavior implies that unemployment falls in a year by one tenth of its level at the beginning of the year. For example, in a year starting with 7 percent unemployment, the rate falls by 0.7 percentage points during the year. We document this regularity within the two main statistical approaches to business-cycle analysis and measurement: (1) construction of a chronology of turning points, and (2) estimation of a Markov regime-switching model. We also show that measures of US unemployment extended to include discouraged workers and others, not counted in the labor force, display the same

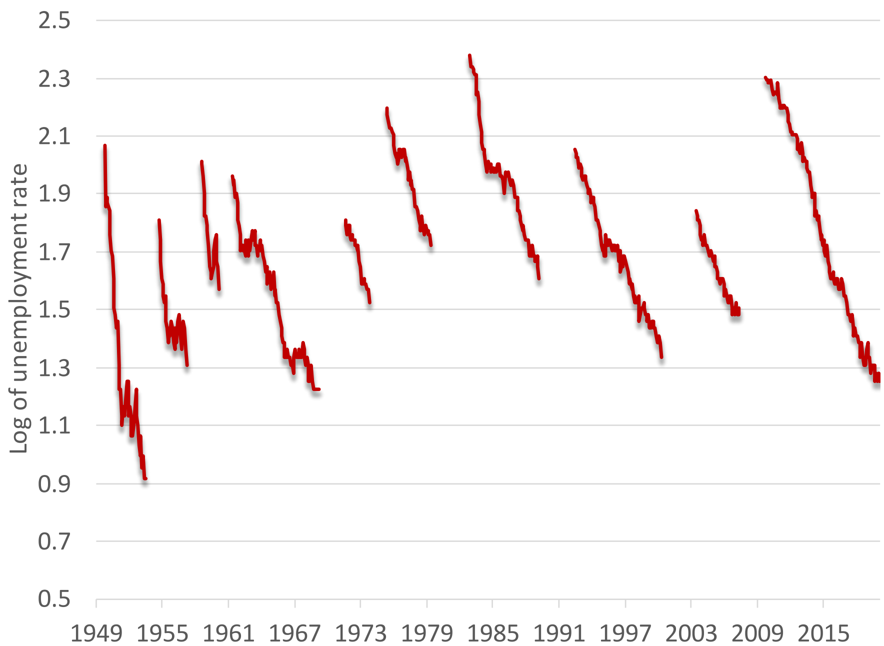

Figure 1: The Paths of Log-Unemployment during Recoveries

consistent pattern as the standard unemployment rate. And we show that unemployment in other advanced and emerging countries behaves in much the same way as in the US.

We are not the first to study the time-series properties of US unemployment. The basic asymmetry between the sharp rise of unemployment in contractions and the slow pace of expansions is well known, and studied carefully with new results and a thorough discussion of the earlier literature in Dupraz, Nakamura and Steinsson (forthcoming). We note that a well-documented property of the unemployment rate—most recently confirmed by those authors—is that unemployment rises rapidly in response to a significant aggregate adverse shock and then gradually recovers. Like fuel prices, unemployment rises like a rocket and falls like a feather. Our contribution to this literature is our demonstration of the reliability of the recovery process. We measure the rate of recovery of unemployment from recession-highs and demonstrate how uniform the rate is.

In a companion paper (Hall and Kudlyak (2021)), we consider resolutions of the puzzle of *slow decline* of unemployment in recoveries. Initially pointed out by Cole and Rogerson (1999), the puzzle is that unemployment declines much more slowly than the measured individual job finding rates would seem to indicate. In that paper, we discuss models in the Diamond-Mortensen-Pissarides tradition that can account for the puzzle.

2 Business Cycle Measurement

To study recoveries, we need a measure of the business cycle. Romer and Romer (2019) discuss cycle measures in detail. They conclude that the preferred defining characteristic of the measure is its ability to capture unused resources. In current business-cycle research, the primary alternative definition is based on extracting a higher-frequency component from real GDP or other output measure. That component is the higher-frequency series from the Hodrick-Prescott or other bandpass filter. We agree with the Romers that tying the business cycle to unused resources is conceptually superior to tying it to higher-frequency movements.

Our view further adopts the Romers' conclusion that the unemployment rate, or a similar measure derived from the unemployment data from the Current Population Survey, is the best available measure of the cycle. The unemployment rate appears to contain almost no movements associated with productivity or similar forces that would call for filtering out. A modest slow-moving demographic component of the unemployment rate is present—see Hornstein and Kudlyak (2019) and Crump, Eusepi, Giannoni and Sahin (2019).

We model log-unemployment in recoveries as the sum of a latent declining path component and a latent stationary component capturing survey sampling errors and other deviations from the path. The path is modestly downward. Our objective is to measure the central tendency and dispersion of the rate of decline of the latent systematic component of the monthly change of log-unemployment rate during recoveries.

We formalize the model as

$$\log u_t = \alpha - \beta t + \epsilon_t, \tag{1}$$

where $\alpha - \beta t$ is the systematic linear path component capturing the recovery phase of the business cycle, and ϵ_t is the random unsystematic component, taken to be uncorrelated with t. Later in the paper, we consider more elaborate parametrizations of the time path. In Figure 1, the approximate linearity of the recovery paths of log unemployment is plainly visible.

In the specification with $\log u_t$ on the left-hand side, the downward slope β is measured in log points, that is, percent declines in unemployment per unit of t. Where possible, we avoid stating the results in the potentially confusing terms of percents of percents, but that is the actual implication of the specification. We use the term "log points" and state them as decimals. For example, a typical finding is that unemployment declines during a recovery by 0.1 log points per year, which is 0.7 percentage points if the unemployment rate starts at 7 percent of the labor force.

The literature has focused on two general classes of specifications for the systematic component. One is *chronology-based* and proceeds by assigning turning points—dates when recessions end and recoveries begin, and dates when recoveries end and recessions begin. Chronologies are available from published sources, notably the National Bureau of Economic Research, which identifies monthly dates of turning points in a latent measure called economic activity. Chronologies can be created for a particular time series, such as the unemployment rate, as an exercise in human pattern recognition. And chronologies can be created by algorithms, such as the one described in Dupraz et al. (forthcoming). Given a chronology, we estimate the systematic component $\alpha - \beta t$ by standard econometric methods.

The other class of models focuses on *regime switching*, where the systematic component is modeled as a statistical time series that obeys one model in contractions and another in recessions. Hamilton (1989) launched the econometric literature on Markov-switching models in this class.

The key difference between these classes is that turning points are latent unobserved events in regime-switching models. These models yield a probability that a given month is a turning point, not an unambiguous turning-point date.

3 Estimation

3.1 Chronologies of US business-cycle recoveries

We consider three monthly business-cycle chronologies:

1. NBER: The chronology maintained by the National Bureau of Economic Research identifying turning points in economic activity, as described in detail at NBER.org.

2. DNS: The chronology produced by Dupraz et al. (forthcoming) (hereafter, DNS) algorithm based on US unemployment from January 1948 through February 2020, with size parameter 1.5.

3. HK: The chronology produced by us based on observed business cycle peaks and troughs, using the same data as DNS.

The NBER maintains a committee of business-cycle specialists to construct its chronology. The committee has developed a definition of economic activity, based primarily on real output and real income, together with definitions of contractions and recoveries. The chronology comprises turning points—activity reaches a trough in the transition month from contraction to recovery and it reaches a peak in the transition month from recovery to contraction. The committee operates in delayed real time—announcements of the determination of a turning point occurs around 6 to 18 months after the event.

DNS developed an algorithm that maps a time series into another time series taking on three discrete values: trough, peak, and neither. For unemployment, most months are classified as neither a trough nor a peak, but rather a continuation of a previous path. The DNS algorithm is based on its creators' judgments about how to extract turning points from time-series data, but its application banishes human judgment from the actual determination. The algorithm is a filter that applies prior beliefs embodied in the algorithm to determine turning points. The algorithm makes it cheap to extract a chronology, and, because it is a function, producing a unique chronology from any particular input, it is a suitable basis for experimenting with the use of a chronology.

Our procedure (HK) uses informed human judgment to identify turning points based on criteria similar to those of the NBER, but using data from just one cyclical indicator, the unemployment rate compiled by the Bureau of Labor Statistics. Specifically, it is the standard monthly unemployment rate, called U-3 in the BLS's nomenclature. The main purpose of the HK chronology is to validate our use of the pure algorithm of DNS as a proxy for human judgment.

The HK chronology has turning points very similar to the DNS algorithm—the validation is successful. However, we pick some later dates for peaks and troughs. The statistical results based on the HK chronology are quite similar to those based on the DNS chronology. Accordingly, we are confident that our application of the DNS algorithm to other measures of US unemployment, and to unemployment data from other countries, generates chronologies that resemble reasonably closely those might have been generated by human judgment.

Figure 2 shows the three chronologies. One disagreement is immediately apparent—the NBER chronology has a recovery beginning in July 1980 and ending 12 months later in July 1981. There is no comparable recovery in the other two chronologies. This disagreement reflects the small rise and fall of unemployment in the 1980 cycle. In general, DNS and HK are similar to one another and differ from NBER. The reason is that DNS and HK are chronologies for unemployment alone, while NBER is a chronology for latent economic activity.

Although the NBER has determined that April 2020 was a turning point from recession to recovery, we do not include that recovery because it is incomplete as we write, and because of the explosion of temporary-layoff unemployment, discussed briefly later in this paper.

3.2 Modeling recoveries using a chronology

We develop a separate model with its own parameters for each recovery in a chronology. For a recovery running from an initial high point of unemployment, which we number as $t = 0$,

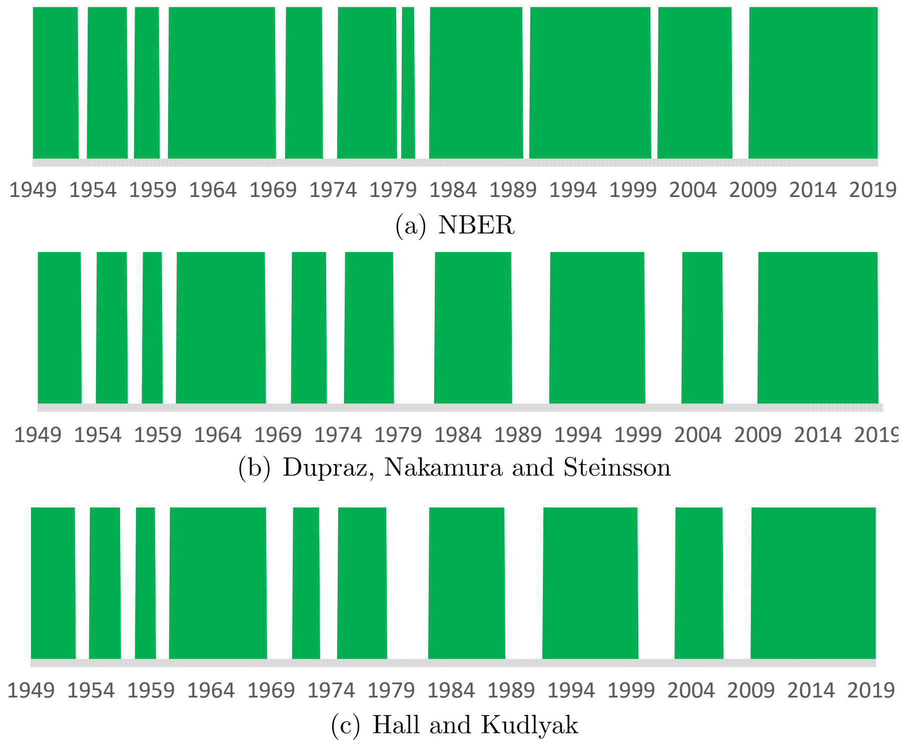

Figure 2: Three Chronologies for the US Unemployment Rate

to the following low point, which we number as T, our model for a single recovery is

$$\log u_t = \alpha - \beta\, t + \epsilon_t. \tag{2}$$

The residual, ϵ_t, follows an AR(1) process,

$$\epsilon_t = \rho \epsilon_{t-1} + \eta_t. \tag{3}$$

The innovation η_t is white noise. We apply an autoregressive transformation to obtain

$$\log u_t = \rho \log u_{t-1} + (1-\rho)\alpha - \beta[t - \rho \cdot (t-1)] + \eta_t. \tag{4}$$

We rewrite this equation in terms of a new parametrization: $\kappa = (1-\rho)\alpha - \rho\beta$ and $\gamma = (1-\rho)\beta$, so that the equation becomes

$$\log u_t = \rho \log u_{t-1} + \kappa - \gamma t + \eta_t. \tag{5}$$

We calculate the implied value of the recovery-rate parameter as

$$\beta = \gamma/(1-\rho). \tag{6}$$

The autoregressive parameter ρ indexes a range of estimators of the key parameter β, the recovery rate. If the random part of the unemployment path is serially uncorrelated, with $\rho = 0$, the equation conditional on ρ is

$$\log u_t = \alpha - \beta t + \eta_t, \tag{7}$$

and the estimated recovery rate is the coefficient of $-t$ in a simple regression.

If the random part of the unemployment path is a random walk, with $\rho = 1$, the conditional equation is

$$\log u_t - \log u_{t-1} = -\beta + \eta_t, \tag{8}$$

and the estimated recovery rate is the mean decline in log unemployment over the recovery, which can also be expressed as estimated $\beta = (\log u_0 - \log u_T)/T$. Thus, the parameter of interest, the recovery rate β, is identified when unemployment is a random walk, but the constant, α, is not.

Our estimates of ρ are generally positive and, in some cases, close to 1. Equation (6) shows that the estimator based on $\rho < 1$ blows up if ρ is really close to 1. We proceed by switching to the $\rho = 1$ estimator of β if our estimate of ρ is above a threshold value, which we take to be 0.9.

Although the key parameter in the model is identified even in the random-walk case, there is some interest in the special problems that arise in that case. For example, as equation (6) shows, the lack of identification of α rules out separation of that parameter from the initial level of unemployment. And testing the null hypothesis $\rho = 1$ uses the standard t statistic, but its distribution is distinctly more dispersed than the t distribution.

Our specification fits the analysis of Dickey and Fuller (1979) as explained in Hamilton (1994), pages 497-502 (case 4), where there is a constant and a trend on the right side of the equation, as in equation (5). In the range of sample sizes in our work, the critical value of the Dickey-Fuller test statistic at the 0.05 level, is 3.7. The statistic is the ratio of the estimated value of $1 - \rho$ from OLS applied to equation (5) divided by the OLS standard error (the "t"-statistic).

To approximate the standard error of the rate of decline of unemployment, β, we carry out a bootstrap-style simulation. For each of the historical recoveries, we generate 100,000 vectors of values of the innovation η_t by re-sampling the innovations from our estimation, with replacement. Then we calculate the implied vectors of $\log u_t$ from equation (3) and equation (2), using the estimated values of the parameters. Finally, we re-estimate β from the bootstrapped data following the same steps as for the earlier estimation from the actual data. The bootstrap standard error is the standard deviation of the re-estimated values of β.

We include the switch to the first-difference estimator when the estimate of ρ exceeds 0.9 in the bootstrap. Absent this procedure, the bootstraps sometimes involve a small fraction of values of ρ almost infinitesimally close to 1, which result in substantial exaggeration of the sampling dispersion of the estimate of β.

The dispersion measures are conditional on the chronologies; that is, they presume knowledge of the turning-point dates when in fact the dates are subject to sampling error. A sufficiently large realization of η_t can bump the assigned turning point date by a month away from its true value. The bootstrap procedure reveals the contribution of the random innovations η_t to sampling errors in β, but not the contribution of the sampling error in the locations of the turning points in the chronology. Our finding of essentially identical results from the HK and DNS chronologies suggests that noise in the dates is not an important source of sampling error in the estimates of β. Even the large differences in cycle dates between the NBER chronology on the one hand and the HK and DNS chronologies on the other has little effect on the sampling error and a moderate downward bias in the recovery rates estimated from the NBER chronology.

True bootstrap estimation of the standard errors of the estimates of β is not available with time series, because the observations are not independent, but re-sampling estimated innovations is a recognized approximation—see Chernick (2008), page 99.

3.3 Estimation using the hidden Markov approach

Our second approach to modeling business cycles draws on a class of models launched in Hamilton (1989). The paper derived the likelihood function in a computationally convenient form. Marcelo Perlin provided the Matlab package for estimating hidden Markov models that we used (Perlin (2015)).

Our implementation has two regimes, recession and recovery, one endogenous variable, log unemployment, and 6 parameters controlling the state of the economy:

- ϕ_1, the average monthly change in log unemployment during recessions (positive),
- ϕ_2, the average monthly change in log unemployment in recoveries (negative),
- τ_1, the monthly probability of a turning point to recovery while in a recession,
- τ_2, the monthly probability of a turning point to recession while in a recovery,
- σ_1 and σ_2, the standard deviations of the changes in unemployment around their means in recessions and recoveries.

This paper is mostly about a single parameter, ϕ_2, the expected change in unemployment during a recovery. We are also concerned with the dispersion of unemployment change in recoveries around its mean, σ_2. To avoid tripping over negative numbers in our discussion, we define $\beta = -\phi_2$, as the average rate of decline of unemployment during recoveries. It has the same interpretation as the β in the chronology-based approach.

To compare the regime-switching approach to the chronology approach, we rewrite the model for recoveries as

$$\log u_t = x_t + \epsilon_t, \tag{9}$$

where x_t is unobserved, but switches between positively- and negatively-sloped segments at random, according to the Markov process involving the transition probabilities τ_1 and τ_2. Under the assumption that the disturbance is a random walk, $\Delta\epsilon_t = \eta_t$, with η_t being white noise, the model becomes

$$\Delta \log u_t = -\beta + \eta_t. \tag{10}$$

We note that the property that ϵ is a random walk is an assumption of our application of the hidden Markov model. We need the assumption to justify taking first-differences, which has the effect of isolating β on the right-hand side of the equation. This step also puts the iid innovation η_t on the right-hand side, a property that is the starting point for the regime-change class of models. In the results for the chronology-based model, ϵ generally has an AR(1) parameter ρ less than 1. Thus, first-differencing may not yield the true innovation η, but only something approximating it. The Dickey-Fuller test fails to reject the random-walk hypothesis in the majority of US recoveries, however.

For the hidden Markov estimates, Hamilton's approach is an application of maximum likelihood, so the information matrix is the basis of an estimator of the covariance matrix of the estimated parameters.

We note one important difference between the regime-switching approach and the chronology approach: In the former, we pool all of the data over numerous recoveries, whereas in the latter, we treat each recovery as a separate body of data, without pooling.

We refer to regime switching model as "hidden Markov" because the occurrence of a peak or trough is not known for sure to the econometrician. Rather, estimation deals with the probability that any given month is a turning point.

	Chronology			*Hidden Markov*
	NBER	*Dupraz-Nakamura-Steinsson*	*Hall-Kudlyak*	
Annual recovery rate, log points	0.089	0.108	0.107	0.066
(Average of bootstrap standard errors)	(0.028)	(0.028)	(0.023)	
(Information matrix standard error)				(0.015)

Table 1: Summary of the Estimation Results

4 Estimates of the Unemployment Recovery Rate

4.1 Main results

Table 1 summarizes the statistical results for the chronology and for hidden Markov estimation approaches. Here and in the rest of the paper, we report recoveries at annual rates, 12 times the monthly rates from the estimation. The table shows the estimates of the key result in this study: the annual recovery rate in log points, β. The sample period is October 1949 through February 2020. The left three values are the estimated recovery rate β in log points per year using the chronology approach, together with the average of their standard errors over recoveries, for each of the three chronologies. The rightmost value is the recovery rate estimated by the hidden Markov approach, together with its standard error.

The left panel of Table 1 reports the equally weighted average recovery rate across the ten0 (in the case of DNS or HK chronologies) or eleven (in the case of the NBER chronology) recoveries, along with the average of their bootstrap standard errors. Note that it is the average of the standard errors, not the standard error of the average. We do not regard the recovery-specific rates as draws from a homogeneous population, so the entries in the left panel are not the result of pooling across the recoveries. With the chronology approach, we consider vectors of results across recoveries without statistical pooling. Most of the recoveries are long enough and have sufficiently small variances of the innovations that the sampling dispersion for β is quite small for individual recoveries.

For the NBER chronology, the estimated average decline rate across recoveries is 0.089 log points per year. Recovery rates for the DNS and HK chronologies are similar to each other and are above the NBER level, at 0.108 and 0.107. The DNS and HK chronologies, constructed from unemployment alone, are more successful at capturing the movements of unemployment during recoveries, because they are better synchronized with the actual movements. Of course, DNS and HK would be correspondingly poorer at tracking economic activity, the concept behind the NBER chronology.

We illustrate the interpretation of the annual decline figures in the table with an example from the recovery rates based on the HK chronology. Consider the situation just after a severe recession, with the unemployment rate starting at 10 percent. The expected unemployment rate a year later is 10 $\exp(-0.107) = 9.0$ percent. With the recovery rate based on DNS, the rate a year later would be essentially the same as with the HK chronology.

The right-hand result in Table 1 pertains to the hidden Markov model. The estimated annual recovery rate is 0.066, below the results for the chronology-based estimates, especially in the case of DNS and HK. We attribute the lower estimate to the uncertainty about the timing of the transitions. This bias is a relative of the classic errors-in-variables downward bias on the magnitude of a regression coefficient for a variable measured with uncorrelated errors.

One reason for the disagreement between the two estimators is that the theory of the application of the hidden-Markov setup to our problem requires the assumption that the disturbance is a random walk, whereas it is actually an AR(1) process with coefficient somewhat less than one. The chronology-based estimator can be considered an application of Bayesian thinking, in that it imposes prior beliefs about the process. The posterior, so to speak, may involve a higher implied value of the recovery rate because the prior belief pushes the posterior in that direction, relative to the likelihood.

The right panel of the table is based on the hidden Markov model, which does treat recoveries as draws from a homogeneous population. Note that the standard error of the estimated β is based on pooling across the entire body of data and thus is correspondingly smaller than the average of standard errors across the individual recoveries in the left panel.

4.2 Estimates by recovery

Figure 3 shows the separate results for the ten recoveries in the HK chronology. Estimated rates for the first three recoveries are more variable than the remaining seven recoveries and have much higher standard errors. The estimates from 1961 to 2020 cluster around 0.10 with small standard errors. Over that 60-year period, with seven recoveries, some mild and two quite severe, the recovery rates are similar. The standard deviation of the ten annual recovery rates over the entire sample period is 0.034 log points, while the standard deviation for the seven recoveries starting in the 1960s is 0.020 log points. These results nail down the primary thesis of our study—the uniformity of recovery rates over the past 60 years and their remarkably low levels.

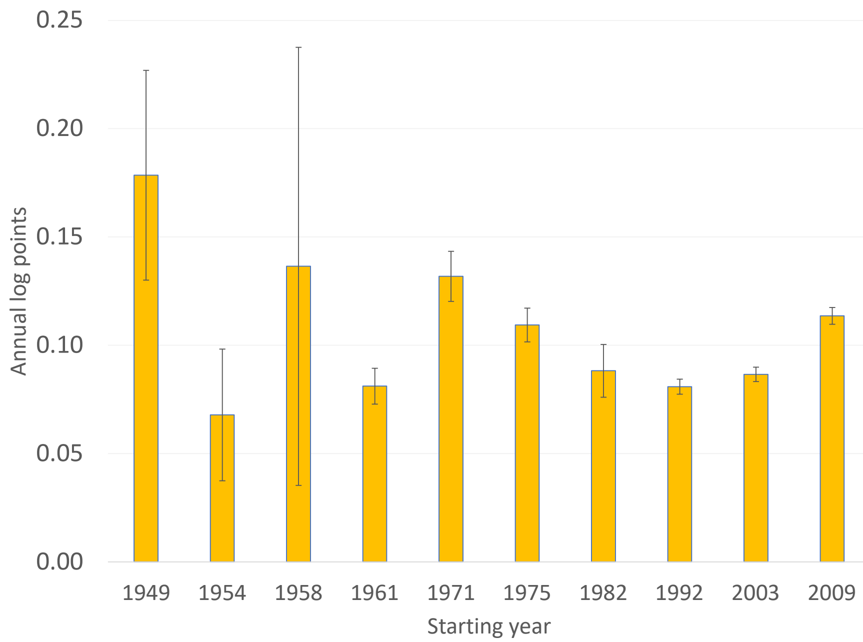

Figure 3: Estimated Recovery Rates by Recovery

The results reveal accurately—as is evident from the small standard errors—that the values of β from the seven later recoveries are similar. Note that it is obvious that a standard statistical test would reject the hypothesis that the values of β are the same across the seven later recoveries. Our choice not to pool across recoveries is powerfully supported.

Why is there a widespread impression that the recovery from the 2007 recession and financial crisis of 2008 was slower than previous recoveries? The answer is that recoveries tend to be judged in terms of output. Both actual growth of real GDP and growth of potential GDP were lower for a number of reasons, including especially the decline in the rate of productivity growth—see Fernald, Hall, Stock and Watson (2017). The facts are that output growth was substandard during the recovery but the decline in unemployment was at the normal rate for recoveries.

4.3 Additional results

Table 2 provides additional information about the ten recoveries in the HK chronology. The topmost two rows show the estimated annual recovery rates and their bootstrap standard errors, as graphed in Figure 3. The next two rows show the monthly serial correlation parameters ρ, and their conventional regression standard errors. The parameters are all unambiguously positive and half of them are over 0.8. None of them exceeds 0.9, so our

Recovery number	1	2	3	4	5	6	7	8	9	10
ß, annual rate of recuvery	0.179	0.068	0.136	0.081	0.132	0.109	0.088	0.081	0.087	0.114
Standard error	(0.048)	(0.030)	(0.101)	(0.008)	(0.012)	(0.008)	(0.012)	(0.003)	(0.003)	(0.004)
ρ, serial correlation coefficient of random component of recovery path	0.834	0.672	0.813	0.848	0.473	0.664	0.880	0.705	0.369	0.823
Standard error	(0.069)	(0.103)	(0.127)	(0.053)	(0.186)	(0.110)	(0.047)	(0.073)	(0.136)	(0.052)
Dickey-Fuller statistic	2.41	3.20	1.48	2.85	2.84	3.07	2.53	4.05	4.63	3.43
t-statistic for curvature, log specification	-0.702	-0.139	0.114	-0.286	1.152	0.764	0.917	-0.821	-1.639	-1.433
t-statistic for curvature, level specification	-0.837	-0.061	0.099	-0.675	0.901	0.167	0.541	-1.376	-2.050	-1.102
R2, log specification restated as level	0.589	0.363	0.330	0.917	0.931	0.954	0.845	0.968	0.969	0.985
R2, level specification	0.707	0.328	-0.100	0.915	0.944	0.950	0.677	0.969	0.975	0.978

Table 2: Additional Results by Recovery

policy of switching to the random-walk estimator has no bearing on our main estimated β values. It only improves the performance of the bootstrap.

The next row in the table shows the Dickey-Fuller statistics—these are calculated as the usual t-statistics for the hypothesis that a coefficient is one, but the critical value is 3.7 rather than the value for the actual t distribution of about 2.0. The random-walk hypothesis is rejected only for recoveries 8 and 9. However, the test statistic exceeds 3 for three other recoveries, indicating stationarity with considerable but not overwhelming confidence. We emphasize that stationarity is not an assumption of our approach, so a finding of high serial correlation or a random walk is not evidence against our view. It supports our use of the hidden Markov model.

4.4 Evidence of departures from the log-linear specification

To study the possibility that the paths of log-unemployment are not straight lines, we add a curving quadratic term to the model, controlled by a parameter χ, so the model for a recovery becomes

$$\log u_t = \alpha - \beta\, t - \chi t^2 + \epsilon_t. \tag{11}$$

Positive values of χ mean that the recovery path of unemployment is concave to the origin. Results for this specification are summarized in the last four lines of Table 2. The first of the four shows the t-statistics for the hypothesis of no curvature in our baseline specification with the log of unemployment. The evidence for curvature around the log specification is fairly weak. Only three of the ten t-statistics exceed 1 in magnitude and none exceeds 2. The evidence gives little support to the idea that there is systematic curvature in the same direction, as four of the estimates of the χ-parameters are positive and the other six are negative.

We also consider an alternative to the baseline, in which the basic specification is a straight line rather than the curve implied by the log specification. The results for curvature are somewhat stronger. Thus, it appears that the log specification uses the χ-parameters to explain some of the curvature in the data that is built into the log.

To perform a direct comparison of the success of the two versions, we calculate the fitted values in levels from the log specification restated in level form (the exponential of the fitted value), to those from the level specification. We use R^2 as the metric of fit. In principle, the level specification has an advantage in this comparison, because it uses the fitted values that maximize R^2, whereas the calculation for the log specification uses values that maximize R^2 in logs, not levels, but we judge it after exponentiation to levels. The bottom two rows in Table 2 show that, for five of the ten recoveries, the fit is better in the log-based specification than in the level-based one. However, the average R^2 for the log-based model is 0.05 higher than the R^2 for the level-based model.

We conclude that the evidence confirms the visual impression in Figure 1 that unemployment in recoveries basically follows a log-linear path. That said, for most of the recoveries that graph would convey much the same impression if it were presented in levels. None of our substantive conclusions would be affected if our baseline specification were linear rather than log-linear.

5 Alternative Measures of US Unemployment

For the US, the BLS provides compilations of many alternative measures of unemployment. In this section, we investigate the behavior of some of these measures in recoveries, using the framework of this paper. We form chronologies using the DNS software and then estimate recovery rates as described earlier in this paper, using both the chronology and hidden Markov approaches.

5.1 More inclusive overall measures of unemployment

Some of the alternative measures include more individuals than does the standard unemployment rate. We report results in the framework of this paper for the three extended unemployment rates, called U-4, U-5, and U-6, over the period of publication, which began in 1994, after a comprehensive revision of the CPS. The results are quite similar to those graphed in Figure 3. We believe that this evidence demonstrates that our findings are robust

	Chronology approach		Hidden Markov approach
Unemployment measure	*Number of recoveries*	*Average annual recovery rates with average standard errors*	*Annual recovery rate with standard error*
Standard unemployment for last 3 recoveries	3	0.094 (0.004)	0.069 (0.018)
Standard unemployment plus discouraged workers	3	0.094 (0.003)	0.069 (0.017)
Above plus marginally attached to labor force	3	0.089 (0.003)	0.061 (0.015)
Above plus part time for economic reasons	3	0.084 (0.003)	0.054 (0.015)
Jobless unemployment--standard unemployment minus temporary layoffs	5	0.096 (0.007)	0.055 (0.019)

Table 3: Results for Other Measures of Unemployment

across measures of unemployment and are not artifacts of the specific choices embodied in the standard unemployment rate.

Table 3 shows the results for four measures of unemployment with ascending scope. The table gives the average annual recovery rates in log points and the averages of their standard errors. The first row shows the average results for the standard unemployment rate, for the last three recoveries. They are similar to but more precise than the averages over the full span of 10 recoveries reported earlier in Table 1.

The second row of the table adds people to the standard unemployment count who do not satisfy the requirement of active job search in the four weeks prior to the survey, because they have become discouraged, but who indicate a desire to work and are available to work. The BLS calls this group U-4. The recovery rates and their standard errors remain almost exactly the same.

Adding another group, called marginally attached workers, whose lack of recent job-seeking effort arises from a variety of other causes, results in a slightly smaller recovery rate of the measure called U-5. Another small decrement in the recovery rate applies to group U-6, shown in the fourth line in the table. This group adds those working part-time for economic reasons. These people are working fewer than their desired hours as a result of adverse economic conditions.

As a general matter, the recovery rates for extended measure of unemployment are quite similar to those classified in the standard way. The results of this paper are robust to inclusion of unemployed individuals with lower job-seeking efforts or other reasons that they are not included in the standard measure.

5.2 Temporary-layoff unemployment

As we write, the United States is recovering from a major pandemic and resulting sharp recession. The recovery of the US unemployment rate has been vastly speedier so far than its low historical value, dropping from its maximum of 14.7 percent in April 2020 to 3.9 percent in December 2021. In a separate paper (Hall and Kudlyak (2020)), we discuss how a completely unprecedented volume of temporary layoffs accounts for the highly unusual rate of decline of unemployment. A substantial fraction of workers on temporary layoff are recalled to their previous positions—in effect, these individuals are on leave from jobs that they continue to hold. See Fujita and Moscarini (2017) on recalls in general and Gregory, Menzio and Wiczer (2020) on the role of recalls in the recovery from the pandemic.

Starting in 1967, the CPS included questions that identify workers on temporary layoff. In recessions in the 1970s and 1980s, temporary layoffs spiked to just over two percent of the labor force, but subsequently declined to under one percent in recent decades and around 0.5 percentage points during the long recovery after 2009. In our related research, we make the case that unemployment analysis should distinguish temporary-layoff unemployment from what we call jobless unemployment—individuals who are searching actively and do not hold existing jobs. Accordingly, we have repeated our measurement of recovery rates using data starting in 1967 that excludes workers classified as temporarily laid off. The results appear at the bottom of Table 3. The recoveries during 1967 through 2019 of the jobless unemployment were at the same rate of about 0.1 log points per year as we have found for using the chronology approach for standard unemployment. The rate found by the hidden Markov approach is slightly lower than we found with standard unemployment, at 0.055 log points per year.

6 Results for Advanced and Emerging Economies

We also perform our basic analysis on the unemployment rates from a considerable number of advanced and emerging countries. Again, we form chronologies using the DNS software and then estimate recovery rates and their standard errors as described earlier in this paper, using both the chronology and hidden Markov approaches.

The Organisation for Economic Co-operation and Development compiles harmonized unemployment data for many countries. Table 4 reports results based on the two statistical approaches of this paper. The countries are sorted in the order of the number of recoveries

	Chronology approach			Hidden Markov approach	
Country	*Number of recoveries*	*Average annual recovery rate*	*Average standard errors*	*Annual recovery rate*	*Standard error*
Canada	9	0.139	(0.015)	0.047	(0.014)
Chile	5	0.184	(0.023)	0.077	(0.019)
Estonia	5	0.354	(0.066)	0.114	(0.042)
Belgium	4	0.125	(0.026)	0.294	(0.041)
Finland	4	0.145	(0.014)	0.043	(0.030)
Netherlands	4	0.152	(0.012)	0.120	(0.011)
Austria	3	0.113	(0.021)	*	
Czech Republic	3	0.165	(0.038)	0.033	(0.110)
Denmark	3	0.112	(0.017)	0.061	(0.014)
France	3	0.047	(0.006)	0.040	(0.008)
Greece	3	0.072	(0.006)	0.056	(0.028)
Ireland	3	0.138	(0.009)	0.062	(0.011)
Italy	3	0.054	(0.012)	0.044	(0.017)
Japan	3	0.074	(0.005)	0.001	(0.010)
Norway	3	0.191	(0.033)	0.370	(0.107)
Poland	3	0.178	(0.010)	0.230	(0.025)
Portugal	3	0.137	(0.020)	0.162	(0.021)
Slovak Republic	3	0.109	(0.018)	0.105	(0.011)
Spain	3	0.082	(0.010)	0.075	(0.009)
Sweden	3	0.116	(0.006)	*	
Costa Rica	2	0.017	(0.011)	*	
Germany	2	0.085	(0.007)	0.094	(0.008)
Hungary	2	0.143	(0.015)	0.044	(0.038)
Korea	2	0.141	(0.024)	0.036	(0.031)
Lithuania	2	0.177	(0.019)	0.099	(0.020)
Luxemburg	2	0.101	(0.020)	0.055	(0.019)
Mexico	2	0.154	(0.009)	0.008	(0.054)
Slovenia	2	0.107	(0.029)	0.112	(0.024)
Turkey	2	0.196	(0.014)	0.039	(0.022)
United Kingdom	2	0.087	(0.013)	0.068	(0.008)
Colombia	1	0.058	(0.003)	0.005	(0.046)
Iceland	1	0.123	(0.016)	0.004	(0.020)
Israel	1	0.088	(0.004)	0.090	(0.053)
Latvia	1	0.129	(0.022)	0.110	(0.020)

Note: * indicates that the likelihood estimation converged to a pair of equal wrong-signed rates of unemployment change.

Table 4: Results for Other Countries Based on OECD Harmonized Data

recorded for each. The right-hand column shows the average annual recovery rate and the average annual standard errors of the underlying estimates.

Only Canada has a record almost as long as the US, with nine recoveries. Its average recovery rate is somewhat above our finding for the US. It is estimated with good precision.

Note that our general application of the chronology approach, of treating each recovery as a separate object, implies that countries that only recently began data collection can achieve high precision. Among the four countries that have only a single recovery, two have standard errors less than one-tenth of a log point.

The unemployment recovery rates for advanced and emerging economies cluster in the range of the US rates for more recent recoveries, around 0.1 log points per year. Slow but sure is not limited to the US. The findings of this paper are robust to inclusion a great many other countries.

7 Concluding Remarks

We have developed a parsimonious statistical model of the behavior of unemployment in cyclical recoveries. In economies subject to occasional major negative shocks, it describes an inexorable downward glide at a low but reliable proportional rate during quiescent times. That rate is around 0.1 log points per year. Our model describes: (1) occasional sharp upward movements in unemployment in times of economic crisis, and (2) an inexorable downward glide at a low but reliable proportional rate at all other times. The glide continues until another economic crisis interrupts the glide. The behavior of unemployment has a similar character in alternative measures of US unemployment and in many other advanced and emerging countries.

Our companion paper, Hall and Kudlyak (2021), considers explanations of the mechanisms that underlie the movements we document in this paper. We show that the immediate victims of job loss in a crisis tend to have downstream unemployment lasting several years, but not long enough to account for more than a fraction of the persistence documented in this paper. And the evidence shows that the long bulge in unemployment following a crisis involves recruitment of additional victims who did not lose jobs in the crisis itself.

In view of these findings, in our companion paper, we seek a mechanism that delivers consistent but slow recoveries of unemployment during the last seven decades, in the US and other advanced economies. We argue that such a mechanism generates self-recovery in the labor market. Self-recovery is present in the standard Diamond-Mortensen-Pissarides model of unemployment, but it is faster than in the data. We propose a mechanism whereby a negative feedback from high unemployment to job creation early in the recovery generates reliable but slow recoveries, as in the data. Models of congestion are a leading example of the mechanisms we discuss.

References

Chernick, Michael R., *Bootstrap Methods: A Guide for Practitioners and Researchers*, Wiley, 2008.

Cole, Harold L. and Richard Rogerson, "Can the Mortensen-Pissarides Matching Model Match the Business-Cycle Facts?," *International Economic Review*, 1999, *40* (4), 933–959.

Crump, Richard K., Stefano Eusepi, Marc Giannoni, and Aysegul Sahin, "A Unified Approach to Measuring u," *Brookings Papers on Economic Activity*, 2019, pp. 143–214.

Dickey, David and Wayne Fuller, "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association*, 1979, *74* (366), 427–431.

Dupraz, Stephane, Emi Nakamura, and Jon Steinsson, "A Plucking Model of Business Cycles," *Journal of Political Economy*, forthcoming.

Fernald, John., Robert E. Hall, James H. Stock, and Mark W. Watson, "The Disappointing Recovery of Output after 2009," *Brookings Papers on Economic Activity*, 2017, *Spring.*

Fujita, Shigeru and Giuseppe Moscarini, "Recall and Unemployment," *American Economic Review*, 2017, *107* (12), 3875–3916.

Gregory, Victoria, Guido Menzio, and David Wiczer, "Pandemic Recession: L or V-Shaped?," *Quarterly Review*, 2020, (4011). Federal Reserve Bank of Minneapolis, May 27.

Hall, Robert E. and Marianna Kudlyak, "The Unemployed With Jobs and Without Jobs," Working Paper No. 27886, National Bureau of Economic Research 2020.

____ and ____ , "Why Has the US Economy Recovered So Consistently from Every Recession in the Past 70 Years?," *NBER Macro Annual*, 2021, *36.*

Hamilton, James D., "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica*, 1989, *57*, 357–384.

____ , *Time Series Analysis*, Princeton University Press, 1994.

Hornstein, Andreas and Marianna Kudlyak, "Aggregate Labor Force Participation and Unemployment and Demographic Trends," 2019. Federal Reserve Bank of San Francisco, Working Paper No. 2019-07.

Perlin, Marcelo, “MS Regress - The MATLAB Package for Markov Regime Switching Models,” 2015. Available at SSRN.

Romer, Christina D. and David H. Romer, “NBER Business Cycle Dating: History and Prospect,” December 2019. University of California, Berkeley.