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ABSTRACT

We provide a model-free framework to study the global factor structure of exchange rates. To this end, we propose a new methodology to estimate international stochastic discount factors (SDFs) that jointly price cross-sections of international assets, such as stocks, bonds, and currencies, in the presence of frictions. We theoretically establish a two-factor representation for the cross-section of international SDFs, consisting of one global and one local factor, which is independent of the currency denomination. We show that our two-factor specification prices a large cross-section of international asset returns, not just in- but also out-of-sample with R2s of up to 80%.

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One central finding of the large literature that studies the factor structure of exchange rates is that the co-movement of exchange rates can be described by a few factors. As a result, workhorse models in international finance posit international stochastic discount factors (SDFs) that are driven by a handful of global and local factors. These models are shown to fit many stylized facts in international macro-finance well. These results, however, rest on two pillars.

On the theoretical side, it is assumed that agents can trade in global international markets that are integrated, frictionless, and complete. While a natural starting point, such an assumption has non-trivial consequences for the factor structure of exchange rates. First, the degree of market segmentation (or the lack thereof) affects the factor structure of SDFs. For example, as we increase the degree of segmentation across countries, SDFs become less correlated and hence the factor structure weaker. In such cases, the importance of global factors may decrease while local factors become more important. Second, the assumption that markets are complete and frictionless also implies that the rate of appreciation of the exchange rate (X) is uniquely recovered from the ratio of the foreign and the domestic SDFs: $X = M_f/M_d$. This identity—which is known as the asset market view of exchange rates (AMV)—implies that shocks to foreign and domestic SDFs pin down the factor structure of exchange rates. However, it is well known that departures from the integrated and frictionless benchmark may cause AMV to be violated. As a result, the ensuing global factor structure would depend not only on the factor structure of SDFs but also on the *ex ante* assumptions about the degree of financial market integration and the presence of barriers to trade.

On the empirical side, the literature extracts global factors from the cross-section of exchange rates alone. However, assumptions about the menu of tradable assets can have first-order implications for the nature of extracted factors. For example, it is well known that FX and fixed income markets are intimately linked, as many intermediaries hedge their FX exposure in bond markets. Moreover, recent empirical evidence in Koijen and Yogo (2020) suggests significant substitution effects across international stocks, bonds, and currencies. It is hence natural to study global factors by focusing not just on exchange rates but also on a broader cross-section of international assets.

In this paper, we study the global factor structure of exchange rates in the presence of trade frictions and when global investors can trade a broad cross-section of international assets. We first develop a theoretical framework to study international SDFs while accommodating a wide class of trade frictions. Such barriers to trade generate market segmentation along assets and countries as an endogenous outcome rather than an assumption. As our main theoretical result, we show that, under fairly mild conditions and irrespective of the nature and extent of trade frictions, international SDFs exhibit a two-factor structure and the AMV continues to hold. As a result, two factors exactly span the cross-section of exchange rates. Empirically, we take the viewpoint of global investors who can trade stocks, bonds, and currencies across different numéraires and study their optimal portfolios. Extracting two factors from the cross-section of international SDFs of these global investors, we show that they price the cross-section of international assets not just in- but also out-of-sample. Moreover, we document a tight link between our two factors and those of Verdelhan (2018), who argues that two global factors, dollar and carry, account for a significant fraction of the variation in exchange rates.

Our paper contributes to the literature along at least three dimensions. First, we provide a novel theoretical framework to study model-free SDFs in the presence of frictions when the menu of assets available for trade is potentially large. Our framework is general, as it can incorporate various forms of frictions, such as proportional transaction costs, margin or collateral constraints, and short-sell constraints, while at the same time ensuring that model-free SDFs are consistent with the absence of arbitrage in asset markets with frictions.[1] It is well-known that in frictionless and arbitrage-free markets, asset prices can be fully characterized by an SDF that only depends on asset returns, which at the same time prices all assets exactly; see, e.g., Ross (1978) and Hansen and Richard (1987), among many others. However, in markets with frictions, linear SDF pricing implies in general non-zero pricing errors on some assets, which directly reflect the underlying structure of market frictions. Earlier literature has often treated pricing errors as evidence of SDF misspecification, by expressing them in terms of the least squares distance between an SDF and the family of SDFs that price correctly all assets. We instead explicitly work under the assumption of arbitrage-free markets with frictions and the resulting pricing error structures. In this setting, we characterize model-free SDFs in terms of the optimal portfolios of constrained global investors, which allows us to recover global model-free SDFs from asset return data alone.

Second, as we deviate from the complete and frictionless market assumption, violations of the AMV arise for the vast majority of international SDFs, see, e.g., Backus, Foresi, and Telmer (2001). This, however, implies a factor structure in international model-free SDFs that is not exact. While this is expected, we theoretically show that there exists a model-free SDF family—the family of minimum-entropy SDFs—which satisfies the AMV even in international asset markets with frictions, as long as frictions are internationally symmetric. Such symmetry requires that market frictions are identical across currency denominations.[2] We establish two powerful properties of minimum-entropy SDFs in markets with symmetric frictions. First, we show that these SDFs are by construction numéraire-invariant, meaning that an optimal SDF pricing assets well in one currency also prices assets well in any other currency. Second, this numéraire invariance results in an exact two-factor structure of international SDFs.

Third, using a large cross-section of short- and long-term bonds and equities in developed countries, we explore the global factor structure of exchange rates by estimating minimum-entropy SDFs under varying transaction cost features. When we assume that investors can trade internationally the full menu of assets in frictionless markets, we obtain volatile SDFs satisfying a virtually exact single-factor dynamics with nearly perfectly correlated SDFs. In such a setting, we find global investors' optimal portfolios to imply positions in single assets that may be hard to maintain in practice, without taking massively levered long and short positions. When instead we impose frictions, we obtain a number of zero optimal portfolio positions on some assets, i.e., endogenous

[1]Our setting also avoids the usual problems of standard approaches for estimating model-free SDFs from large cross-sections of asset returns, as it is well-known that they can lead to spurious estimates. For example, Kozak, Nagel, and Santosh (2020) address this concern using a model-free SDF that shrinks the coefficients of low variance principal components of characteristics-based factors via machine learning techniques. In contrast, we directly incorporate various forms of economically motivated international financial market frictions, which endogenously lead to more robust model-free SDFs.

[2]For instance, when we assume a symmetric market, we can either impose no frictions at all or we can impose the exact same friction across all countries. Asymmetric markets, on the other hand, imply differences in the portfolio weight dependent component of frictions of some countries, such as, e.g., lower trade barriers for investing in local assets than in foreign assets.

market segmentation. Moving from symmetric to asymmetric market settings, we find that SDF volatilities further drop by 20% and that the ensuing optimal portfolios become even more sparse. While lower than under the symmetric market setting, minimum entropy SDF correlations are still very high and common factor structures very strong.

The optimal portfolios of global investors in markets with frictions further uncover interesting insights into the global factor composition of international SDFs. We find that global investors always trade a carry, i.e., they are long high interest rate currencies and short low interest rate currencies, and are long US equity, while virtually ignoring long-term bonds. While these findings connect to a larger literature documenting priced systematic dollar and carry risks, our model-free approach allows us to interpret global factor compositions economically via the optimal portfolios of global investors.

From the cross-section of estimated international model-free SDFs, we identify the global factor as the average log SDF across currency denominations and the local currency basket factor as the average appreciation of a currency relative to the basket of remaining currencies. Using our cross-section of bonds and equities, we then study the pricing properties of our two factors, both in- and out-of-sample. As is well-known, in a frictionless market pricing errors of in-sample model-free SDFs are zero by construction. Hence, to control for such overfitting issues, out-of-sample analysis is more meaningful to compare the pricing abilities of SDFs across different market structures. Based on rolling training periods of ten years for SDF estimation and one year rolling windows for pricing evaluation, we find that our global factor alone explains up to 98% of the in-sample cross-sectional variation and up to 80% of the out-of-sample variation in the cross-section of currencies, stock, and bond returns across all denominations, in a way that is quite robust across different specifications of market frictions. We also find that market frictions are instrumental to improve the out-of-sample pricing performance as for example, the out-of-sample R^2 in the frictionless case amounts to a mere 40% when pricing the cross-section of FX. We also show that our two factors—the global SDF and the currency basket factor—are intimately linked to carry and dollar. This evidence provides model-free support to the model-based evidence in Lustig, Roussanov, and Verdelhan (2011, 2014) and Verdelhan (2018). This may seem surprising prima facie since our SDFs are estimated using information including international stock and bond returns, not just currencies. This, however, implies that the two-factor SDF structure proposed in these papers may have implications for asset returns more generally, not just exchange rates.

Finally, motivated by a large literature in international finance emphasizing the importance of international capital flows for exchange rate determination, we explore the link between the factors in our two-factor representation, gross capital flows, and financial intermediaries' constraints.[3] To explore this relationship more formally, we run regressions of our global SDF factor and local currency baskets, on proxies of capital flows and measures of financial intermediaries' constraints, such as implied volatility (see, e.g., Rey (2015)). Our results indicate that while the global SDF factor is strongly linked to proxies of intermediary constraints, such as implied volatilities and the intermediary capital

[3]For instance, Camanho, Hau, and Rey (2019) study a dynamic portfolio balancing model where exchange rates are determined by the net currency demand from portfolio balancing motives of global intermediaries. Since our framework establishes a unique mapping between the optimal portfolios of global investors in markets with frictions and model-free SDFs, it appears natural to study the links between our estimated SDF factors and international equity and bond flows.

proxy of He, Kelly, and Manela (2017), the local currency basket factors have no relation with these variables. For capital flows, we find instead the opposite result: While changes in capital flows induce a significant drop in the local currency basket factor, the global SDF is unaffected by them.

As a last exercise, we ask how big transaction costs have to be to match the equity and bond home bias typically observed in the data. In our analysis, we impose trading costs directly observable from markets (bid-ask spreads, leverage constraints, etc). However, it is reasonable to assume that these costs only represent a lower bound of the real costs that investors face when trading assets internationally.[4] To get a sense of the size of the cost needed to match the almost perfect home bias observed in the data, we take our asymmetric market friction setting and increase the cost to trade foreign assets. We find that in order to achieve almost perfect home bias, trading foreign assets needs to be around six times more expensive than trading local assets. Given that the average bid-ask spread is around 2bps, this implies that trading foreign assets entails a transaction cost of around 10bps in unobservable costs. Overall, different from earlier literature that has argued that transaction costs need to be unrealistically high to explain the home bias in the data, we conclude that these costs can be relatively small.

Literature Review: Our paper contributes to a growing literature in international finance studying global factors. The seminal work of Lustig, Roussanov, and Verdelhan (2011, 2014) and Verdelhan (2018) documents that two factors, carry and dollar, explain a significant share of the systematic variation in exchange rates. Panayotov (2020) studies global risk in an extended version of Lustig, Roussanov, and Verdelhan (2014), in which the US SDF has a larger exposure to global risk than all other international SDFs. Maurer, Tô, and Tran (2019) extract two principal exchange rate components from the cross-section of all cross-currency returns related to dollar and carry, in order to construct country specific SDFs. These SDFs are shown to price international equity returns well in-sample. Aloosh and Bekaert (2019) reduce the cross-section of currencies by means of currency baskets that measure the average appreciation of each currency against all other currencies. They then apply clustering techniques to the cross-section of currency baskets and identify two clusters, one related to the dollar and another related to the Euro. Lustig and Richmond (2020) model gravity in the cross-section of exchange rates and find factor structures related to physical, cultural, and institutional distances between countries. Jiang and Richmond (2019) link trade networks between countries to exchange rates comovement in order to explain the existence of the global dollar and carry factors.

Our paper is different from this literature along several dimensions. First, it relies on common SDF factors that are extracted from a family of model-free, numéraire invariant SDFs, which satisfy the AMV in arbitrage-free international asset markets with frictions. Second, our methodology allows us to extract global factors jointly from a cross-section of returns including international equities and bonds, in addition to exchange rate returns. Third, our factors are directly related to the optimal portfolios of global investors in international asset markets with frictions, which provide additional unique insights into the factor composition in terms of the portfolio exposure of these investors to various international assets. Fourth, the generality of our methodology allows us to incorporate

[4] For example, Coeurdacier and Rey (2013) argue that there could also be hedging costs and informational frictions which are very difficult to observe.

market frictions leading to endogenous market segmentation and to study the implication for global factor structures thereof. In this context, we find that our two factors which are extracted from the cross-section of SDFs are related to carry and dollar, in line with the findings of Lustig, Roussanov, and Verdelhan (2011, 2014). Finally, we show that our common factors jointly price cross-sections of returns including international equities and bonds, in addition to exchange rate returns, not just in- but also out-of-sample.

Our paper is also related to a more recent literature that studies priced risk factors across different asset classes. For example, Lettau, Maggiori, and Weber (2014) show that the difference between the unconditional and downside risk market betas can price the cross-section of expected returns across various asset classes and He, Kelly, and Manela (2017) document that shocks to intermediaries' equity capital ratio is a priced risk factor. Our paper is different from theirs as we provide a theoretical framework of how to extract model-free SDFs from cross-sections of assets in the presence of frictions and study their pricing ability not just in- but also out-of-sample.

Our work naturally extends an important literature estimating model-free SDFs that minimize various notions of stochastic dispersion. These SDFs are motivated by a need for powerful diagnostics when testing asset pricing models. For instance, Hansen and Jagannathan (1991), Stutzer (1995), and Almeida and Garcia (2017), among others, propose various such asset pricing bounds, while forcing exact pricing of all assets. In a different context, Ghosh, Julliard, and Taylor (2019) estimate minimum Kullback-Leibler divergence SDFs which price exactly all assets in-sample. Using these SDFs, they introduce a nonparametric empirical asset pricing model that performs better than standard factor models in pricing low dimensional cross-sections of assets out-of-sample. Sandulescu, Trojani, and Vedolin (2020) study international SDFs in frictionless markets when pricing errors are zero. The main focus of that paper is the relationship between market incompleteness and financial market structures to address international finance puzzles. Our approach significantly differs from this literature, by allowing for non-zero pricing errors on a subset of assets, where pricing errors are motivated by the presence of market frictions. Moreover, the portfolio penalizations induced by our approach allow us to accommodate large cross-sections of assets, leading to model-free SDFs with a reasonable dispersion. In our empirical analysis, we demonstrate that such penalizations are essential to obtain an improved pricing accuracy out-of-sample.

Finally, our paper is naturally linked to a smaller but important literature studying model-free SDFs in markets with frictions. He and Modest (1995) and Luttmer (1996) extend the Hansen and Jagannathan (1991) minimum variance SDF setting by incorporating various specifications of sublinear transaction costs that give rise to generalized diagnostics for asset pricing models. Hansen, Heaton, and Luttmer (1995) provide the econometric tools for the evaluation of asset pricing models in such settings. An important theoretical finding in this literature is that the pricing functional sublinearity gives rise to SDFs with non-zero pricing errors, which are tightly constrained by the given transaction cost structure. Korsaye, Quaini, and Trojani (2020) extend this theory to address minimum dispersion SDFs resulting from general convex pricing errors structures, which are characterized in terms of investors' optimal portfolios. We make use of this theory in order to identify financial market structures that deliver numéraire invariant model-free SDF families which satisfy the AMV. This result is key to identify parsimonious global exchange rate factors structures. Second, we exploit the theoretical relation between numéraire invariant model-free SDFs and the optimal portfolios of investors in markets with frictions, in order to study economic factor compositions in terms of the SDF exposures to international asset returns.

Outline of the paper: The rest of the paper is organized as follows. Section 1 provides the theoretical framework for studying model-free SDFs in international financial markets with frictions. Section 2 presents our main empirical findings, Section 3 reports some robustness checks, and Section 4 concludes. The Appendix provides proofs and derivations, while the Internet Appendix provides additional extensions and results omitted from the paper for brevity.

1 Global Stochastic Discount Factors and Market Frictions

In this section, we develop a model-free framework that allows us to study international SDFs in the presence of various forms of frictions, such as bid-ask spreads, short-selling, margin, or collateral constraints.

1.1 Model-Free SDFs in the Presence of Frictions

We start our analysis by developing a counterpart to the fundamental theorem of asset pricing in the presence of transaction costs, a result that enables us to characterize the set of stochastic discount factors in markets with frictions. For the time being, we focus on a single-country setting and drop all references to particular countries and currencies. We extend our framework to a multi-country setting in Subsection 1.3, where we study international SDFs.

Consider an economy consisting of n assets indexed by set $N = \{1, 2, \ldots, n\}$, with gross returns denoted by $\mathbf{R} = (R_1, \ldots, R_n)$. While a subset of assets, denoted by $S \subseteq N$, can be traded with no transaction costs, assets in set $F = N \setminus S$ are subject to trading frictions. We refer to these assets as *frictionless* and *frictional* assets, respectively. Throughout, we use n and $f = n - s$ to denote the number of frictionless and frictional assets in the economy, respectively.

We model transaction costs using a closed and sublinear transaction cost function h, which quantifies the costs associated with any portfolio based on the same common numéraire for returns. More specifically, we assume that implementing a portfolio with vector of portfolio weights $\boldsymbol{\theta} = [\boldsymbol{\theta}'_S \;\; \boldsymbol{\theta}'_F]' \in \mathbb{R}^n$ entails a cost equal to $h(\boldsymbol{\theta}_F)$, where $\boldsymbol{\theta}_F \in \mathbb{R}^f$ denotes the sub-vector of portfolio weights of frictional assets. The assumption that transaction cost function h is sublinear has two important implications. First, it implies that the cost of a portfolio implemented in a single execution is no greater than the cost of implementing the same portfolio in multiple executions. Second, it implies that the cost of implementing portfolio $\boldsymbol{\theta}^1$, which is a multiple of another portfolio $\boldsymbol{\theta}^2$ with a certain factor, is equal to the cost of implementing portfolio $\boldsymbol{\theta}^2$ multiplied by the same factor.[5] As we show in subsequent sections, this specification of transaction costs is general enough to nest the

[5]Formally, sublinearity of h ensures that $h(\boldsymbol{\theta}_F^1 + \boldsymbol{\theta}_F^2) \leq h(\boldsymbol{\theta}_F^1) + h(\boldsymbol{\theta}_F^2)$ for all pairs of portfolios $\boldsymbol{\theta}_F^1, \boldsymbol{\theta}_F^2 \in \mathbb{R}^f$ and $h(\lambda\boldsymbol{\theta}_F) = \lambda h(\boldsymbol{\theta}_F)$ for all $\lambda \geq 0$ and portfolios $\boldsymbol{\theta}_F \in \mathbb{R}^f$.

various market frictions relevant for our empirical analysis, including short-selling constraints, bid-ask spreads, and a general class of proportional transaction costs (such as leverage constraints).

Given the set of assets $N = S \cup F$ and transaction cost function h, we define the set of all portfolio returns that can be traded with finite transaction costs as

$$\Xi = \{x = \boldsymbol{\theta}'\mathbf{R} : \; h(\boldsymbol{\theta}_F) < \infty, \; \boldsymbol{\theta} \in \mathbb{R}^n\}.$$

We also define a pricing functional $\pi : \Xi \to \mathbb{R}$ as the minimum cost of replicating a given return when accounting for transaction costs:

$$\pi(x) = \inf_{\boldsymbol{\theta} \in \mathbb{R}^n} \left\{\boldsymbol{\theta}'\mathbf{1} + h(\boldsymbol{\theta}_F) \; : \; x = \boldsymbol{\theta}'\mathbf{R}\right\}.$$

Finally, as is standard in the literature, we say the pair (Ξ, π) is an *arbitrage-free* price system if $x \geq 0$ with $\mathbb{P}(x > 0) > 0$ implies $\pi(x) > 0$.

Our first result, which serves as the foundation of our subsequent analysis, establishes that absence of arbitrage in a frictional market is equivalent to the existence of SDFs inducing a corresponding set of closed-form constraints on non-zero pricing errors for frictional assets.

Proposition 1. *Price system (Ξ, π) is arbitrage free if and only if there exists a strictly positive stochastic discount factor M such that*

$$\mathbb{E}[M\mathbf{R}_S] - \mathbf{1} = 0 \qquad \textit{and} \qquad \mathbb{E}[M\mathbf{R}_F] - \mathbf{1} \in \mathcal{C}, \tag{1}$$

where

$$\mathcal{C} = \{\mathbf{y} \in \mathbb{R}^f \; : \; \mathbf{y}'\boldsymbol{\theta}_F \leq h(\boldsymbol{\theta}_F) \; \textit{for all} \; \boldsymbol{\theta}_F \in \mathbb{R}^f\} \tag{2}$$

and $\mathbf{R}_F$ and $\mathbf{R}_S$ are the vectors of gross returns of frictional and frictionless assets, respectively.

Proposition 1 is akin to the well-known fundamental theorem of asset pricing. It establishes the existence of strictly positive SDFs in markets with frictions by a standard no-arbitrage condition. Furthermore, it provides a characterization of the set of all admissible SDFs: according to equation (1), any such SDF prices all frictionless assets exactly but may result in non-zero pricing errors for assets subject to trading frictions, where the set of possible pricing errors, $\mathcal{C}$, is uniquely characterized in definition (2) by transaction cost function h. Intuitively, constraining the pricing error in set $\mathcal{C}$ ensures that the gain induced by mispricing via the SDFs for investing in any portfolio cannot exceed the transaction cost of implementing such a portfolio.

It is immediate to see that Proposition 1 nests the textbook case of arbitrage-free markets with no frictions as a special case: when all assets are frictionless, then $\mathbb{E}[MR_k] = 1$ for all $k \in N$. Proposition 1 also extends other results in the prior literature, such as Hansen, Heaton, and Luttmer (1995) and Luttmer (1996), which study markets with pricing errors constrained by a convex cone. For example, while Luttmer (1996) focuses on an economy in which $h(\boldsymbol{\theta}_F) \in \{0, \infty\}$, our more flexible formulation using a general sublinear transaction cost function allows us to incorporate a wider class of trading frictions, such as proportional transaction costs, which play a central role in our empirical application. We next provide a series of examples to illustrate how our setup can incorporate a variety of market frictions.

Example 1 (Short-sell constraints)**.** As a first example, we show that our framework can incorporate short-sell constraints in a straightforward manner. Consider the following transaction cost function,

$$h(\boldsymbol{\theta}_F) = \begin{cases} 0 & \boldsymbol{\theta}_F \geq 0 \\ \infty & \text{otherwise,} \end{cases}$$

according to which taking short positions in any of the frictional assets is infinitely costly, while taking long positions is costless. Under such a specification of transaction costs, Proposition 1 implies that, under no arbitrage, there exists a strictly positive stochastic discount factor M which induces non-positive pricing errors for assets subject to short-sell constraints, while pricing all frictionless assets exactly, i.e.,

$$\mathbb{E}[M\mathbf{R}_F] - \mathbf{1} \leq 0 \qquad \text{and} \qquad \mathbb{E}[M\mathbf{R}_S] - \mathbf{1} = 0.$$

Example 2 (Bid-ask spreads)**.** A natural way to incorporate bid-ask spreads into our framework is to consider long positions $\boldsymbol{\theta}_F^+$, when one buys an asset at ask price at time 0 and sells it at bid price at time 1, and short positions $\boldsymbol{\theta}_F^-$, when one buys an asset at bid price at time 0 and sells at ask price at time 1. This corresponds to a setting with (i) no short-selling constraints on long position $\boldsymbol{\theta}_F^+$ and (ii) no buying constraints on short positions $\boldsymbol{\theta}_F^-$. Denoting by $\boldsymbol{\theta}_F = [\boldsymbol{\theta}_F^{+\prime}, \boldsymbol{\theta}_F^{-\prime}]'$ the extended portfolio vector of long and short position on each asset, we can easily incorporate these market frictions with following transaction cost function:

$$h(\boldsymbol{\theta}_F) = \begin{cases} 0 & \boldsymbol{\theta}_F^+ \geq 0\,,\ \boldsymbol{\theta}_F^- \leq 0 \\ \infty & \text{otherwise.} \end{cases}$$

Under this specification of transaction costs, Proposition 1 implies that the pricing errors on the long positions have to be negative, while the pricing errors on short positions have to be positive, with no pricing errors for assets that are not subject to such constraints. That is,

$$\mathbb{E}[M\mathbf{R}_F^+] - \mathbf{1} \leq 0 \qquad \mathbb{E}[M\mathbf{R}_F^-] - \mathbf{1} \geq 0 \qquad \mathbb{E}[M\mathbf{R}_S] - \mathbf{1} = 0,$$

where $\mathbf{R}_F^+$ and $\mathbf{R}_F^-$ are the gross return vectors for long and short positions, respectively.

Example 3 (Proportional transaction costs)**.** As our final example, we consider transaction costs that are proportional to portfolio positions. We can model a general class of such frictions by assuming that $h(\boldsymbol{\theta}_F) = \lambda\|\boldsymbol{\theta}_F\|$ for some norm $\|\cdot\|$ and a constant $\lambda \geq 0$. A simple application of Proposition 1 then implies that, under no arbitrage, there exists a strictly positive SDF M such that:

$$\|\mathbb{E}[M\mathbf{R}_F] - \mathbf{1}\|_* \leq \lambda \qquad \text{and} \qquad \mathbb{E}[M\mathbf{R}_S] - \mathbf{1} = 0, \tag{3}$$

where $\|\cdot\|_*$ denotes the dual norm of $\|\cdot\|$.[6] As a useful special case, consider transaction costs specified by the l_1-norm, that is, $h(\boldsymbol{\theta}_F) = \lambda \sum_{k\in F} |\theta_k|$. Under such a specification, equation (3) implies that $|\mathbb{E}[MR_k] - 1| \leq \lambda$ for any asset $k \in F$ subject to transaction costs. In other words, proportional transaction costs impose a maximum bound of λ on frictional assets' pricing errors. As expected,

[6] The dual of norm $\|\cdot\|$ is defined as $\|\boldsymbol{y}\|_* = \sup\{\boldsymbol{\theta}'\boldsymbol{y} : \|\boldsymbol{\theta}\| \leq 1\}$.

this bound also implies that a decrease in transaction cost parameter λ results in a smaller set of admissible SDFs and smaller pricing errors.[7]

We conclude this discussion by noting that it is straightforward to accommodate different types of trading frictions for different subsets of assets. For example, one could introduce short-sell constraints for equities, while at the same time impose bid-ask spreads on the carry trade. In such settings, set $\mathcal{C}$ in Proposition 1 simply becomes the Cartesian product of the individual sets reflecting the trading frictions applied to each subset of assets.

1.2 Minimum-Entropy SDFs

Proposition 1 provides a characterization of the set of admissible SDFs in terms of the transaction cost function h. In general, however, there may exist multiple strictly positive SDFs that satisfy the restrictions in Proposition 1. As in frictionless economies, such multiplicity may arise as a result of market incompleteness. However, in our frictional framework, transaction costs may also be a source of SDF multiplicity, with a larger set $\mathcal{C}$ in equation (2) resulting in larger set of SDFs.

Due to such potential multiplicity, we focus next on a specific SDF, called the minimum-entropy SDF. As we show in subsequent results, this SDF satisfies a "numéraire-invariance" property, which plays a central role in our characterization of the factor structure of exchange rates when we extend our framework to an international setting. We start with the following definition:

Definition 1. Given transaction function h, the *minimum-entropy SDF M_0* is given by

$$\begin{aligned} M_0 = \arg\min_{M>0} \quad & \mathbb{E}[-\log M] \\ \text{s.t.} \quad & \mathbb{E}[M\mathbf{R}_S] - \mathbf{1} = 0 \\ & \mathbb{E}[M\mathbf{R}_F] - \mathbf{1} \in \mathcal{C}, \end{aligned} \tag{4}$$

where set $\mathcal{C}$ is given by equation (2).

As the minimum entropy label suggests, problem (4) picks the SDF with the smallest entropy from the set of all possible SDFs characterized by absence of arbitrage, while the constraints in (4) ensure that it respects the pricing constraints induced by the transaction cost function.[8] Working directly with problem (4) is inconvenient as it is an infinite-dimensional constrained optimization problem. Therefore, using convex duality theory, in the following proposition we characterize the minimum-entropy SDF with a solution to an unconstrained finite-dimensional optimization problem.

Proposition 2. *Given transaction cost function h, the minimum-entropy SDF satisfies*

$$M_0 = 1/\boldsymbol{\theta}_0'\mathbf{R}, \tag{5}$$

[7]Note that, unlike our previous examples, transaction cost $h(\boldsymbol{\theta}_F) = \lambda\|\boldsymbol{\theta}_F\|$ may take finite non-zero values. As a result, transaction costs within this class fall outside the framework studied by Luttmer (1996), who only considers market frictions in which $h(\boldsymbol{\theta}_F) \in \{0, \infty\}$.

[8]The minimum-entropy SDF belongs to the family of minimum-dispersion SDFs, which include several other well-known SDFs such as Hansen and Jagannathan's (1991) minimum-variance SDF and minimum Kullback-Leibler divergence SDFs, among many others.

where

$$\boldsymbol{\theta}_0 = \underset{\boldsymbol{\theta} \in \mathbb{R}^n}{\arg\min} \quad \mathbb{E}[-\log(\boldsymbol{\theta}'\mathbf{R})] + \boldsymbol{\theta}'\mathbf{1} + h(\boldsymbol{\theta}_F) \tag{6}$$

and $\mathbf{R}$ *denotes the vector of asset returns.*[9]

This proposition provides a closed-form expression for the minimum-entropy SDF that can be estimated directly from the data. In particular, it characterizes the minimum-entropy SDF in terms of the transaction cost function h and the solution of a penalized optimal growth portfolio problem. As a result, equations (5) and (6) will serve as the basis of our empirical analysis in Section 2.

As an example, consider the proportional transaction costs under the l_1-norm specified in Example 3. It is immediate from the above result that the minimum-entropy SDF can be obtained in terms of an optimal growth portfolio with a lasso-type penalty. Therefore, as is well known from the machine learning literature, such lasso penalization gives rise to an optimal portfolio with a sparse vector of portfolio weights for assets subject to market frictions.

1.3 International SDFs

We now extend our framework to a multi-country setting in which global investors trade assets internationally in markets with possibly different currency denominations. Formally, consider an economy consisting of m countries, denoted by $\{1, \ldots, m\}$. Investors in country i have access to a large set of n assets—consisting of both local and foreign assets—denominated in country i's currency. We use $\mathbf{R}^{(i)}$ to denote the vector of asset returns accessible to investors in country i, denominated in i's currency. Conversion of returns denominated in currency j to corresponding returns in currency i may be made accessible to investors in all countries through exchange rate markets:

$$\mathbf{R}^{(i)} = X^{(ij)}\mathbf{R}^{(j)} \tag{7}$$

for all pairs of countries i and j, where $X^{(ij)}$ denotes the gross exchange rate return, with the exchange rate defined as the price in country i currency of one unit of country j's currency.

As in our single-country framework, a subset of assets $F^{(i)} \subseteq N$ available to investors in country i is subject to trading frictions, with a sublinear transaction cost function $h^{(i)}$. Note that, in general, the set of frictional assets and their corresponding transaction costs may differ across countries. However, one particularly useful special case arises when all transaction cost are assumed to be identical:

Definition 2. International financial markets are *symmetric* if $F^{(i)} = F$ and $h^{(i)} = h$ for all i.

Irrespective of whether international markets are symmetric or not, Proposition 1 implies that they are arbitrage free with respect to each currency denomination if there exists a collection of strictly positive country-specific SDFs $\{M^{(i)}\}_{i=1}^m$ such that

$$\mathbb{E}[M\mathbf{R}_S^{(i)}] - \mathbf{1} = 0 \qquad \text{and} \qquad \mathbb{E}[M\mathbf{R}_F^{(i)}] - \mathbf{1} \in \mathcal{C}^{(i)}$$

for all countries i, where $\mathbf{R}_S^{(i)}$ and $\mathbf{R}_F^{(i)}$ denote the returns of the frictionless and frictional assets in country i, respectively, and $\mathcal{C}^{(i)}$ is the pricing error constraint set for investors in country i, defined

[9] This statement implicity assumes that the optimization problem (6) has a unique solution $\boldsymbol{\theta}_0$.

in equation (2). Finally, we can define the country-specific minimum-entropy SDFs $\{M_0^{(i)}\}_{i=1}^m$ as the solutions of optimization problems similar to (4) for each country i. Our next result relates international SDFs to exchange rates in markets with frictions.

Proposition 3 (Asset market view of exchange rates)**.** *If international financial markets are symmetric, then*

$$X^{(ij)} = M_0^{(j)}/M_0^{(i)} \tag{8}$$

for all pairs of countries $i \neq j$, where $M_0^{(i)}$ and $M_0^{(j)}$ are the minimum-entropy SDFs of country i and j.

The above result establishes that, as long as markets are symmetric, the exchange rate return between any pair of currencies is uniquely pinned down by the ratio of the two countries' minimum-entropy SDFs. While it is well-known that in complete and frictionless markets the rate of appreciation of the exchange rate has to be equal to the ratio of the two countries' SDFs, Proposition 3 shows that the same relationship holds for minimum-entropy SDFs under much more general conditions, irrespective of the extent of market incompleteness and the presence and nature of transaction costs.

The significance of equation (8), which is known as the *asset market view of exchange rates* (AMV), is threefold. First, it implies that, as long as markets are symmetric, the cross-section of $m(m-1)/2$ distinct log exchange rate returns is described *exactly* by a log linear transformation of the cross-section of m minimum entropy SDFs. Second, it implies a parsimonious factor structure of (minimum-entropy) international SDFs. This is a consequence of the fact that, under AMV, for international SDFs pairs to match the exchange rate volatility, they must be almost perfectly correlated (Brandt, Cochrane, and Santa-Clara, 2006). Third, the fact that Proposition 3 holds for any specification of transaction cost function h implies that one can uniquely pin down the factor structure of exchange rates without explicitly specifying the set of assets available to investors in each country: one can obtain the factor structure while allowing for the set of traded assets to be determined endogenously. This is in contrast to the extant literature that estimates factor structures by imposing a priori assumptions on the set of traded assets, which may lead to different exchange rate factor representations depending on the specification of the menu of the assets.

We now turn to another important property of international minimum-entropy SDFs, namely *numéraire-invariance*, according to which, international SDFs that price the cross-section of exchange rates are independent of their currency denomination. The following corollary to Propositions 2 and 3 formalizes this concept:

Corollary 1. *Suppose international financial markets are symmetric. Then, $\boldsymbol{\theta}_0^{(i)} = \boldsymbol{\theta}_0$ for all countries i, where $\boldsymbol{\theta}_0^{(i)}$ is the solution to i's penalized optimal growth portfolio problem in* (6).

The Corollary establishes that the optimal portfolio weights are the same for all countries. This is a consequence of the functional form of the minimum-entropy SDFs. The numéraire-invariance property of minimum-entropy SDFs then follows immediately: the optimal portfolio return in each country can be converted to the optimal portfolio return of another country via the corresponding exchange rate return, i.e., $\boldsymbol{\theta}_0^{(i)\prime}\mathbf{R}^{(i)} = X^{(ij)}\boldsymbol{\theta}_0^{(j)\prime}\mathbf{R}^{(j)}$. While a simple consequence of our previous results, Corollary 1 plays a central role in establishing the parsimonious nature of global factor structures, as we show next.

We conclude this discussion by noting that when international markets are not symmetric, the AMV usually does not hold. In general, for an arbitrary choice of a family of minimum-dispersion SDFs, the AMV may be violated and deviations from it can be captured by a family of Backus, Foresi, and Telmer (2001) stochastic exchange rate wedges $\{\eta^{(ij)}\}_{1\leq i,j\leq m}$, defined by:

$$X^{(ij)} = \frac{M_0^{(j)}}{M_0^{(i)}} \exp(\eta^{(ij)}). \tag{9}$$

1.4 The Global Factor Structure of Exchange Rates

Having developed the necessary theoretical framework, we can finally characterize the global factor structure of exchange rates with a model-free approach. In particular, we establish that, as long as international financial markets are symmetric—and irrespective of the nature of trade frictions—the cross-section of minimum-entropy SDFs has a two-factor representation consisting of a global factor and a local currency basket factor. The next theorem formalizes this result:

Theorem 1. *Suppose international markets are symmetric and let* $(w_1, \ldots, w_m)$ *denote a set of weights such that* $\sum_{i=1}^m w_i = 1$. *Then, the minimum-entropy SDF of country* i *satisfies*

$$\log M_0^{(i)} = \mathrm{G} + \mathrm{CB}^{(i)}, \tag{10}$$

where

$$\mathrm{G} = -\sum_{j=1}^{m} w_j \log \boldsymbol{\theta}_0^{(j)\prime}\mathbf{R}^{(j)} \quad \textit{and} \quad \mathrm{CB}^{(i)} = -\sum_{j\neq i} w_j \log X^{(ij)}$$

denote the global SDF factor and the local currency basket factor of country i*, respectively.*

Equation (10) thus establishes that every (log) minimum-entropy SDF can be written as the sum of (i) a common factor given by the average negative log return of the optimal portfolios across currency denominations and (ii) a currency basket factor $\mathrm{CB}^{(i)}$ measuring the average appreciation of currency i relative to all other currencies $j \neq i$.

The two-factor decomposition in Theorem 1 will serve as the basis of our empirical analysis in subsequent sections. In particular, from equation (10), we use the following two-factor approximation of the local SDF, consisting of both the global and the local currency basket factor:

$$M_0^{(i)} \approx \exp(\mathrm{G} + \mathrm{CB}^{(i)}) =: \widetilde{M}_0^{(i)}, \tag{11}$$

which by Theorem 1 is exact with $\boldsymbol{\theta}_0^{(1)} = \ldots = \boldsymbol{\theta}_0^{(n)}$ whenever international markets are symmetric.[10] Second, we can also study a one-factor representation of the local SDFs as an approximate global SDF:

$$M_0^{(i)} \approx \exp(\mathrm{G}) =: \widetilde{M}_0. \tag{12}$$

[10]The two-factor representation given in (11) is also related to Aloosh and Bekaert (2019) who decompose local SDFs into a global SDF and a component which is orthogonal to the global SDF. However, different from us, these authors construct global SDFs from equal-weighted currency baskets of all possible bilateral exchange rates with respect to a base currency.

We then test how the one- and two-factor approximations in equations (11) and (12) explain the cross-section of international asset returns.

We conclude by noting that the two-factor representation in equation (10) is closely related to the models of Lustig, Roussanov, and Verdelhan (2011, 2014) and Verdelhan (2018), who posit that country-level SDFs are driven by two factors: carry and dollar, where the latter measures the average appreciation with respect to the dollar. Our representation in Theorem 1 establishes that, in case of minimum-entropy SDFs in symmetric markets, not only does the cross-section of SDFs consists of a global factor and a local currency basket factor—akin to the dollar factor in the above-mentioned papers—but also that there cannot be any other factors. In asymmetric markets, more generally, the two-factor representation in equation (10) holds approximately, because of the presence of exchange rate wedges due to asymmetric frictions. Intuitively, the degree of accuracy of the approximation depends on the extent of the deviations from market symmetry. However, it is an empirical question how important these deviations are in the data.[11]

2 Empirical Analysis

Proposition 2 and in particular equation (5) allows us to estimate minimum dispersion international SDFs directly from returns data in the presence of trading frictions. We study two different market settings with varying transaction costs in both symmetric and asymmetric markets and explore the impact of frictions on the properties of SDFs. Using SDFs denominated in different currencies, we then extract the two factors from equation (10) in order to price cross-sections of currencies as well as long-term bonds, and stocks.

2.1 Data

In our empirical analysis, we use monthly data between January 1988 and December 2015. We focus on the following developed markets: Australia, Canada, Euro Area, Japan, New Zealand, Switzerland, United Kingdom, and United States.[12] We collect data on exchange rates, short- and long-term interest rates, and MSCI country equity indices' prices from Datastream. When we analyze a specific currency denomination, we treat the corresponding market as the domestic and all other markets as foreign. Hence, we do not consider bilateral trades but a global economy where global investors can trade all possible assets.

We also calculate equity and FX volatility from the corresponding returns. To this end, we compute the standard deviation over one month of daily MSCI price index changes for each currency, and then the cross-sectional mean of these volatility series. Option-implied volatility on the S&P500, VIX, is available from the webpage of the CBOE. We also calculate a measure of gross capital flows. To this end, we follow Avdjiev, Hardy, Kalemli-Özcan, and Servén (2018) and construct measures of in- and outflows for the countries in our sample using quarterly data from the Balance of Payment (BOP) data available from the International Monetary Fund. The BOP data captures capital flows in and

[11] In our empirical analysis, we study various specifications of symmetric or asymmetric frictions in international asset markets and find that the approximation is very accurate also in asymmetric markets, i.e., stochastic wedges are negligible.

[12] Before the introduction of the Euro, we take the Deutsche Mark in its place.

out of a given country. We define inflows into any given country as the sum of direct investment into equity and debt, portfolio equity and debt, and other investment debt from the liability side. Similarly, outflows are defined the same way but from the asset side. To construct one variable of gross capital flows, we take averages across all countries.[13] Lastly, we use two variables which capture global risk aversion and intermediaries' capital constraints: the "Global Financial Cycle" variable of Miranda-Agrippino and Rey (2020) and intermediary capital of He, Kelly, and Manela (2017), both available from the authors' webpages.

2.2 Market Settings

We study two different market settings, symmetric and asymmetric markets, and in each market impose different types of transaction costs.

SYMMETRIC MARKETS: For the first symmetric market setting, we assume that global investors can trade the full menu of assets and there are no frictions to trade. The second symmetric market setting arises when investors face proportional transaction costs. We assume that investors incur no transaction costs when trading short-term bonds globally (i.e., investors can borrow and lend at the short-term interest rate without any frictions) but face transaction costs when trading long-term bonds and equity. More specifically, we assume that transaction costs, modeled with an l_1-norm, are proportional to their positions and in line with the size of bid-ask spreads.[14] To this end, the proportional transaction cost parameter λ is chosen such that we have comparable pricing errors implied on the returns based on mid-prices by (i) SDFs in an economy with proportional transaction costs and by (ii) SDFs in an economy where market frictions are quantified by bid-ask spreads.

ASYMMETRIC MARKETS: In the first asymmetric market setting, we assume that investors face bid-ask spreads when buying and selling international assets. To this end, we use average bid-ask spreads for exchange rates directly available from Datastream which are in the order of 2bps. For the long-term bonds, we also assume average bid-ask spreads of 2bps in line with Adrian, Fleming, and Vogt (2017) for the US and Bank of International Settlement (2016) for Japan and Germany ten-year bonds. For equity indices, we impose a 6bps spread.[15] The second asymmetric market setting assumes that local short-term bonds can be traded without any frictions whereas all foreign short-, as well as long-term bonds and equities face proportional transaction costs which are again consistent with the size of the bid-ask spread. In these settings the asymmetry is introduced through frictions that differentiate between home and foreign assets. To this end, we introduce an additional cost in the order of 2bps when trading forerign assets.

[13] Taking principal components across country-level gross flows and using the first principal component in our analysis instead leads to qualitatively and quantitatively similar results.

[14] This is a special case of equation (3), where $h(\boldsymbol{\theta}_F) = \lambda \sum_{k \in F} |\theta_k|$.

[15] It is in general impossible to know the exact bid-ask spread of assets. Luttmer (1996) uses bid-ask spreads of around 0.012% which corresponds to the tick size on the NYSE. Andersen, Bondarenko, Kyle, and Obizhaeva (2018) document that the bid-ask spread on E-Mini Futures on the S&P500, one of top two most liquid exchange traded futures in the world, is around 0.25 index points.

2.3 Properties of Model-Free International SDFs

As a first exercise, we study the properties of international SDFs, their comovement, and the corresponding optimal portfolio weights. To this end, we estimate equation (5) using the different transaction cost functions discussed before. Table 1 provides summary statistics for SDFs in each currency denomination for the four market structures.

[Insert Table 1 here.]

While average SDFs are the same across the different market settings, volatilities decrease significantly as we impose market frictions. For example, while the average volatility in markets with no frictions is around 0.36, the volatility in asymmetric markets with transaction costs is only around 0.2, a 45% drop. Recall that in asymmetric markets, deviations from the AMV can be captured by a stochastic wedge. To save space, we relegate summary statistics on stochastic wedges to the Internet Appendix. We find stochastic wedges to be minuscule and on average to be nearly zero for all currency pairs echoing the findings in Lustig and Verdelhan (2019) and Sandulescu, Trojani, and Vedolin (2020). Notice, however, while these authors study stochastic wedges in frictionless markets, we show that wedges are also small in the presence of frictions. Small stochastic wedges have immediate consequences for the cross-country correlations of international SDFs and hence factor structures.

We present cross-country correlations for the four market settings in the lower parts of each panel. The correlations are almost perfect in symmetric markets. This is intuitive, as under the assumption of market symmetry the AMV holds, which "enforces" a high correlation among international SDFs; see Proposition 3. As we move to an asymmetric market setting where transaction costs vary among the different countries, we notice that the correlations are slightly lower but still all above 90%. This may be more surprising, given that in this case we have violations of the AMV. The importance of this finding is twofold. First, the almost perfect correlation even in incomplete markets with asymmetric frictions implies that stochastic wedges are inconsequential for understanding the key properties of SDF factor structures. Second, as a consequence a strong factor structure of exchange rates emerges even in presence of market frictions, which implies that a low number of factors is sufficient to explain the cross-section of international asset returns.

2.4 Optimal Portfolio Weights

In order to shed more light on the optimal SDFs for different currencies, we now study optimal portfolios. Our theoretical framework allows us to exactly identify optimal portfolio weights from agents' Euler equations. Figure 1 plots the optimal portfolio weights for symmetric markets with no frictions (upper panel) and symmetric proportional transaction costs (lower panel).

[Insert Figure 1 here.]

Recall that a direct consequence of Proposition 3 is that portfolio holdings have to be identical in symmetric markets. Therefore, we only plot the USD denominated portfolio weights. We notice that whenever investors can trade without frictions, portfolio weights can be very large both long and short. The larger positions reveal that investors borrow in the lower interest rate currencies such as JPY, USD, CAD, and EUR and hold long positions in high interest rate currencies such as NZD and AUD. Most positions in long-term bonds are short, with the exception of USD, EUR, and JPY. Global investors hold large long positions in USD, AUD, and CHF equity indices. The large positions indicate that without taking large levered positions it may be hard to maintain this portfolio, which also contributes to the high volatilities documented in Panel A of Table 1.

When we impose symmetric proportional transaction costs on investors, given by an l_1-norm, some weights on assets are zero, portfolio holdings are sparse, and the size of the positions shrink, see the lower panel in Figure 1. More specifically, most portfolio weights on the long-term bonds are zero. This is not very surprising given that currency risk premia at the long-end of the term structure are small, see, e.g., Lustig, Stathopolous, and Verdelhan (2019). Investors also drop most of the equity indices except for a long position in the USD and CHF, and a short position in the JPY. Interestingly, most of the wealth is held in short-term bonds. In particular, as in the case with no transaction costs, global investors trade a "carry." Short positions are in typical funding currencies, whereas long positions are in investment currencies. Overall, we conclude that even small transaction costs which restrain investors' leverage can have significant impacts on the optimal portfolios held by global investors.

[Insert Figures 2 and 3 here.]

Figures 2 and 3 plot the cases where we assume asymmetric bid-ask spreads and proportional transaction costs, respectively. Because in asymmetric markets portfolio weights vary across the different currency denominations, we plot all currencies separately. As in the symmetric proportional transaction cost case, portfolios are more sparse and investors hold positions that resemble the carry and long equity. The most sparse portfolio coincides with the case when investors face asymmetric proportional transaction costs. The sparsity translates directly to the low SDF volatility reported in Table 1. Even though in asymmetric markets portfolio weights are not enforced to be the same, positions look almost identical. As in the symmetric market cases investors engage in carry trades: shorting the US dollar, CAD, EUR or CHF and go long in the NZD and AUD. In addition, investors trade long USD and CHF equity.

2.5 The Global Factor Structure of Exchange Rates

We now turn to the factor structure of exchange rates, the main forcus of our paper. Different from earlier literature, which primarily estimates SDF factors from the cross-section of currencies (or currency portfolios), our framework allows us to estimate the cross-section of international SDFs from international stock and bond data, in addition to currencies, under different assumptions for the underlying market frictions.

We leverage our findings thus far by noticing that the almost perfect correlation among international SDFs across all market settings naturally implies a very parsimonious SDF factor structure, not just in settings where the AMV holds but also in the presence of asymmetric frictions.

Indeed, recall that our two-factor representation decomposes each minimum entropy SDF into two conceptually distinct factors: A global factor given by the average return of a maximum growth portfolio, which is independent of the currency numéraire, and a local currency basket factor; see equation (10). Moreover, while equation (10) holds under market symmetry, the almost perfect correlation we find among international SDFs in asymmetric settings suggests that the resulting AMV deviations are rather small and that the two-factor approximation is quite accurate in these settings as well.

2.5.1 Global SDFs and Currency Baskets

Figure 4 plots the time-series of the global SDF risk factors from equation (10) for the four market settings. While we notice a much higher volatility for the global factor in market settings without any frictions, especially during crises, there is overall a high comovement among the different SDFs.[16] These SDF factors simultaneously increase during bad economic times, such as recessions, or during times of disruptions in financial markets, such as the dot com bubble burst or the Lehman default. Interestingly, we notice that global SDFs spike during US specific crisis events. For example, in all four market settings, we find that global SDFs exhibit a massive spike in August 2011 during the US downgrade from AAA to AA+ by S&P.

[Insert Figure 4 here.]

Figure 5 plots the time-series of the local currency baskets. While the global SDF is by construction currency independent, notice that local currency baskets are country specific. The global SDF factor correlates very differently to the local currency baskets. For example in the asymmetric proportional transaction cost setting, it correlates negatively to the high interest rate currency baskets, NZD (-43%) and AUD (-36%), and it positively correlates to typical funding currencies CHF (15%), EUR (26%), and JPY (31%). Global SDFs are instead almost uncorrelated to currency baskets USD (6%), GBP (3%) and CAD (-1%).

[Insert Figure 5 here.]

2.5.2 Factor Premia: Empirical Framework

We now turn to our main empirical results. To this end, we study the pricing ability of global SDF factors and local currency basket factors for the cross-section of international asset returns. Recall from equation (2) that in markets with frictions, minimum entropy SDFs always imply non-zero pricing errors even in-sample. One might therefore wonder whether there are any implications for running standard Fama and MacBeth (1973) cross-sectional regressions. To this end, notice that our setting implies that any vector of excess returns $\mathbf{R}_e^{(i)}$ in currency i has to satisfy the following decomposition:

$$\mathbb{E}[\mathbf{R}_e^{(i)}] = -\mathbb{C}ov\left(\mathbf{R}_e^{(i)}, \frac{M_0^{(i)}}{\mathbb{E}[M_0^{(i)}]}\right) + \mathbb{E}\left[\frac{M_0^{(i)}}{\mathbb{E}[M_0^{(i)}]}\mathbf{R}_e^{(i)}\right], \tag{13}$$

[16] The average correlation across the four settings is 94%.

where $M_0^{(i)}$ is the local SDF. Here, the second term on the RHS represents the vector of pricing errors induced by SDF $M_0^{(i)}$, while the first term represents the expected excess returns that is explained by a return covariance with this SDF.

The decomposition in equation (13) then gives rise to the following linear beta model for expected excess returns:

$$\mathbb{E}[\mathbf{R}_e^{(i)} - \overline{\mathbf{R}}_e^{(i)}\mathbf{1}] = \lambda^{(i)}(\boldsymbol{\beta}^{(i)} - \overline{\boldsymbol{\beta}}^{(i)}\mathbf{1}) + \mathbb{E}\left[\frac{M_0^{(i)}}{\mathbb{E}[M_0^{(i)}]}(\mathbf{R}_e^{(i)} - \overline{\mathbf{R}}_e^{(i)}\mathbf{1})\right] , \tag{14}$$

where we denote by $\overline{\boldsymbol{y}}$ the component-wise average of a vector $\boldsymbol{y}$, by $\lambda^{(i)}$ the (scalar) SDF factor premium and by $\boldsymbol{\beta}$ the vector of SDF betas:

$$\boldsymbol{\beta}^{(i)} = \frac{\mathbb{C}ov(\mathbf{R}_e^{(i)}, M_0^{(i)})}{\mathbb{V}ar(M_0^{(i)})} . \tag{15}$$

Note that in equation (14), the second term in the sum on the RHS is a vector of cross-sectionally centred pricing errors, which immediately leads to a zero-mean error term for a two-step cross-sectional analysis of excess returns. However, such an error term may not be cross-sectionally orthogonal to the vector of SDF betas, as this constraint is not part of the definition of a minimum-entropy SDF in markets with frictions. Moreover, recall that by construction the factor premium of SDF $M_0^{(i)}$ equals:

$$\lambda^{(i)} = -\frac{\mathbb{V}ar(M_0^{(i)})}{\mathbb{E}[M_0^{(i)}]} . \tag{16}$$

Taken together, this implies that the choice of factor premium (16) in linear model (14) may lead to a suboptimal cross-sectional fit. However, notice that an optimal fit is obtained via the factor premium estimated by a standard two-step Fama and MacBeth (1973) regression approach:

$$\lambda_{FMB}^{(i)} := \frac{\left(\boldsymbol{\beta}^{(i)} - \overline{\boldsymbol{\beta}}^{(i)}\mathbf{1}\right)' \left(\mathbb{E}[\mathbf{R}_e^{(i)} - \overline{\mathbf{R}}_e^{(i)}\mathbf{1}]\right)}{\left(\boldsymbol{\beta}^{(i)} - \overline{\boldsymbol{\beta}}^{(i)}\mathbf{1}\right)' \left(\boldsymbol{\beta}^{(i)} - \overline{\boldsymbol{\beta}}^{(i)}\mathbf{1}\right)} . \tag{17}$$

Here, differences between $\lambda_{FMB}^{(i)}$ and $\lambda^{(i)}$ provide information about the adjustments in SDF factor premia needed to obtain an optimal cross-sectional fit. Moreover, inference on the precise value of $\lambda_{FMB}^{(i)}$ is easily feasible with standard methods in a simple single-factor linear model of the form:

$$\mathbb{E}[\mathbf{R}_e^{(i)} - \overline{\mathbf{R}}_e^{(i)}\mathbf{1}] = \lambda^{(i)}(\boldsymbol{\beta}^{(i)} - \overline{\boldsymbol{\beta}}^{(i)}\mathbf{1}) + \boldsymbol{\epsilon}^{(i)} . \tag{18}$$

In the following, we therefore estimate factor premia via the standard two-step Fama and MacBeth (1973) estimator defined in equation (17) and we quantify the in- and out-of-sample pricing accuracy of international SDFs $M_0^{(i)}$, together with their two- and single-factor approximations in equations (11) and (12), respectively.

We present regression results both in- and out-of-sample for the cross-section of currencies and international stocks and bonds both separately and jointly. We focus on results denominated in USD (unless noted) and relegate results for all other currency denominations to the Internet Appendix.

2.5.3 Factor Premia: In-Sample Evidence

Tables 2 and 3 summarize in-sample estimated SDF factor premia (17) for the USD denominated cross-section of currencies (Table 2) and stocks and bonds (Table 3), using two-step Fama and MacBeth (1973) regressions. Column 1 of each table reports the price of risk with respect to the local minimum entropy SDF $M_0^{(usd)}$. Column 2 reports the risk premium relative to the single-factor SDF approximation $\widetilde{M}_0$ in equation (12). Column 3 instead reports the results for the two-factor SDF approximation $\widetilde{M}_0^{(usd)}$ in equation (11).

As by construction pricing errors are zero (i.e., expected asset returns are perfectly matched) in absence of market frictions, in-sample R^2s are 100% for minimum entropy SDFs, which are exactly reproduced by their two-factor SDF approximation: $M_0^{(usd)} = \widetilde{M}_0^{(usd)}$. Moreover, we find that in such settings, the pricing accuracy provided by the global single-factor SDF is also virtually perfect. As we introduce market frictions and asymmetry, the in-sample pricing ability of the minimum entropy SDF intuitively declines. However, we find that such decline is rather limited, with a lowest in-sample R^2 across market settings of about 97% (92%) for currencies (currencies, bonds, and stocks). All estimated factor risk premia in Tables 2 and 3 are negative and highly statistically significant for all SDFs and SDF factor specifications. Across all market settings, we find the global SDF explains virtually the same cross-sectional variation as the local minimum SDF. Using the two-factor representation instead, i.e., $\widetilde{M}_0^{(usd)}$, increases the adjusted R^2 only marginally relative to the global SDF for the cross-section of carry trades, while it even decreases the adjusted R^2 for the full cross-section of excess returns.

[Insert Tables 2 and 3 here.]

Overall, we conclude that one single factor can explain the majority of the variation of international asset returns. This finding is particularly noteworthy for currencies. For example, while our R^2s are similar in size to those reported in the literature, notice that our global SDF is not calculated from the cross-section of currencies itself, like for example the HML factor in Lustig, Roussanov, and Verdelhan (2011), the currency volatility factor in Menkhoff, Sarno, Schmeling, and Schrimpf (2012), or the currency baskets in Aloosh and Bekaert (2019). Moreover, while these papers price the cross-section of currency portfolios, we price the cross-section of individual currency returns together with long-term bond, and stock index returns.

It is also worth highlighting that the R^2s in Table 3 capture not only *with-in* asset class variation but also *across* asset class variation. To show this, Figure 6 plots the expected excess return as a function of exposure to the global SDF risk factor $\widetilde{M}_0$ for the USD denominated economy, together with the estimated price of risk (dashed line). In all four settings we find that the global factor model captures the cross asset class variation. Not surprisingly, as market frictions and asymmetry are introduced, the assets are less aligned across the dashed line compared to the case with no frictions. Moreover, in presence of asymmetry and market frictions, the variation in the risk exposure increases as can be gleaned from the x-axis of each figure. For example, while in the case of no transaction costs risk exposures vary between -0.05 and 0.01, risk exposures in a setting with asymmetric proportional transaction costs vary between -0.12 and 0.04. This feature is directly related to the properties of pricing errors induced by the various SDFs, as tighter pricing error constraints imply a higher minimum SDF variability and a weaker co-movement with asset excess returns.

[Insert Figure 6 here.]

Finally, we can also compare the pricing ability of the one- versus two-factor SDF representation across currency denominations different from the US dollar. To this end, we plot in Figure 7 (Figure 8) the cross-sectional R^2s for different economies in the symmetric (asymmetric) market settings. In the case of no transaction costs (upper four panels in Figure 7), adding the currency basket always leads to R^2s which are 100%, independent of the currency denomination and the asset class priced. This is expected, as in this setting, the two-factor SDF approximation is exact and the local currency minimum entropy SDFs produce by construction a perfect cross-sectional fit. When we add symmetric frictions (lower four panels), we notice that while for the cross-section of currencies we still see a 100% in-sample R^2 by construction, differences between the R^2s when using the global factor alone and adding the local currency basket diminish. A similar picture emerges in the asymmetric market settings, see Figure 8. The global SDF uniformly explains a large fraction of the cross-sectional variation across all asset classes. The lowest R^2 is about 58% for Australian dollar denominated long-term bonds. One observation worth highlighting is the following. Interestingly, we find that the local currency basket provides consistent additional explanatory power for the so-called commodity currencies, AUD, CAD, and NZD. This is the case across different specifications of frictions. This echoes findings in Aloosh and Bekaert (2019) who document evidence for a priced commodity risk factor for the cross-section of currencies. We show that this is also true for long-term bonds and equities more generally.

[Insert Figures 7 and 8 here.]

2.5.4 Factor Premia: Out-Of-Sample Evidence

An economically more interesting question asks whether our SDF factor models are able to price the cross-section of international assets out-of-sample. To this end, we use a training window of ten-years of monthly returns up to say, month y, to estimate the optimal weight $\hat{\boldsymbol{\theta}}_y$ solving problem (6) by maximizing a penalized Sharpe ratio instead of a penalized expected log utility criterion.[17] For the following twelve monthly return observations $\boldsymbol{R}_{y+m}$, we compute the sequence of out-of-sample monthly SDFs $\hat{M}_{y+m} = \max\{-\hat{\boldsymbol{\theta}}_y' \boldsymbol{R}_{y+m}, 0\}$. We then construct the out-of-sample SDF time-series $\{\hat{M}_{y+m}\}$ in a rolling window manner, by updating the estimation window at the yearly frequency. Here, the out-of-sample global SDF is computed using the local out-of-sample SDFs together with equation (12). We finally evaluate the out-of-sample pricing performance, by calculating out-of-sample cross-sectional R^2s with two-step Fama and MacBeth (1973) regressions.

[17]While all our in-sample results are derived for the minimum-entropy SDF for reasons explained in the theory section, we now compute minimum-variance SDFs to ensure non-negative out-of-sample SDFs, which cannot be guaranteed for out-of-sample minimum-entropy SDFs. In general, we find our results not to be dependent on the dispersion measured used. For example, even though minimum variance SDFs do not imply the AMV even in symmetric markets, we find the ensuing factor structures to be very similar to those for minimum entropy SDFs. We therefore conclude that the results on the factor structure of international assets does not depend on the dispersion measure used in a significant way. We gather all results for in-sample minimum variance SDFs in the Internet Appendix to save space.

Tables 4 and 5 present the out-of-sample results for the four market settings. The results are mixed for the cross-section of FX returns: While estimated SDF factor premia are significant in specifications with transaction costs, they are insignificant in the no frictions case. This, however, is not very surprising given the volatile nature of the no friction SDFs documented in Table 1 and highlights the need to impose frictions for superior out-of-sample pricing. To see this more clearly, we can gauge the estimated factor premia in symmetric market settings but with proportional transaction costs. In this case, we significantly improve on the cross-sectional pricing abilities of both the local and global SDFs, with significantly increased R^2s. We can further improve upon the pricing performance by imposing asymmetric market settings, as the ensuing SDFs feature more robust properties, e.g., lower volatilities and lower sensitivities to currency returns. Indeed, when pricing only the currency returns, proportional transaction costs lead to further improved R^2s and statistical significance in estimating the risk premium. For instance, when we assume that markets are asymmetric and global investors face proportional transaction costs, the global SDF alone explains 80% of the variation in currency returns, which is more than twice the R^2 in absence of market frictions. When pricing the full cross-section of assets, we obtain similarly large R^2 of about 80% for the global SDFs. Finally, in settings with transaction costs, we find that the SDF currency basket factor is typically significant and that it adds to cross-sectional pricing accuracy especially when pricing currency returns.

[Insert Tables 4 and 5 here.]

Figure 9 plots the expected excess return as function of the exposure to out-of-sample global SDF risk factor for the USD denominated economy. The ability of global SDF alone to capture the variation within and across asset classes increases as frictions and asymmetry are introduced. Moreover, as in the in-sample case, the variation of the exposure to the global SDF risk factor increases with market frictions and asymmetry.

[Insert Figure 9 here.]

2.5.5 Relation To Verdelhan (2018)

In our setting, SDFs are fully characterized by two factors: the average negative log return of global optimal portfolios and a currency basket measuring the average appreciation relative to all other currencies. It is natural to ask how these factors relate to the carry and dollar risk factors in Verdelhan (2018) given that ours are estimated from stock and bond data.

Figure 10 plots the global SDF together with carry (upper panel) and the currency basket risk factor together with dollar (lower panel) in a symmetric market with proportional transaction costs. The evidently very high correlation is important for at least two reasons.[18] First, recall that in our two-factor SDF representation from Proposition 1, the decomposition of the SDFs into global SDF and currency basket is exact under the assumption of symmetric markets whenever we use minimum entropy SDFs. This implies that, given the global SDF and currency basket factors, there cannot exist

[18]By construction, the dollar factor and our currency basket should be the same, the reason for the less than perfect correlation is due to the fact that the dollar factor in Figure 10 uses a larger cross-section of currencies than we do.

any other factors. Therefore, two factors are sufficient to describe in-sample risk premia not only for currencies, but also stocks, and bonds. Second, in our framework, global investors can buy and sell a broad menu of assets, including stocks, as well as short- and long-term bonds. Hence, our risk factors are not extracted from currencies, as in the case of Verdelhan (2018), but from international asset prices. Here, the global SDF can be interpreted as a function of the optimal portfolios of these global investors. Despite these differences, the two factors are highly correlated, implying that the global factors in Verdelhan (2018) may not just be important drivers of bilateral exchange rates, but also more broadly of international asset returns.

[Insert Figure 10 here.]

2.6 Capital Flows, International SDFs, and Home Bias

A large literature in international finance emphasizes the importance of international capital flows for exchange rate determination. For example, in the model of Hau and Rey (2006), incomplete hedging of FX risk of global investors induces capital flows which are key drivers of exchange rates. More recently, Camanho, Hau, and Rey (2019) study a dynamic portfolio balancing model where exchange rates are determined by the net currency demand from portfolio balancing motives of global intermediaries.[19] One defining feature of international capital flows is that they are driven by a low number of global factors, see, e.g., Milesi-Ferretti and Tille (2011), Forbes and Warnock (2012), Rey (2015), and Davis, Valente, and van Wincoop (2019).

Recall that our framework provides a unique mapping between the optimal portfolios of global investors trading in markets with frictions and SDFs. It is therefore natural to assume that our estimated SDFs are linked to equity and bond flows internationally. In the following, we study the relation between our two factors—global SDF and local CB—and measures of capital flows or intermediaries' constraints. To this end, we run regressions from changes in global SDF and local CB on changes in capital flows, volatilities, global cycle, and He, Kelly, and Manela (2017) intermediary capital:

$$\Delta \text{G} \quad \text{or} \quad \Delta \text{CB}^{(i)} \quad = \beta_0 \times \Delta X_t + \epsilon_t,$$

where ΔX_t are either changes in world equity, the Verdelhan (2018) carry or dollar factor, changes in VIX or FX volatility, changes in the Miranda-Agrippino and Rey (2020) global cycle, changes in our proxy of capital flows, and changes in intermediary capital.[20] The results are presented in Table 6 for the global SDF (Panel A) and the local CB (Panel B).

We find that changes in carry, VIX and FX volatility lead to positive and significant increases in the global SDF. This makes intuitively sense since high states of the global SDF coincide with globally bad times, i.e., times of high global volatility. Miranda-Agrippino and Rey (2020)'s global cycle, on the other hand, significantly reduces global SDFs. As the authors show, their factor is inversely related

[19] Relatedly, instead of asset flows, Gabaix and Maggiori (2015) assume that global demand of goods results in capital flows which are intermediated by global financiers. Since intermediaries' SDFs are a function of financial constraints, the tightness of these constraints determine asset prices and exchange rates in equilibrium.

[20] To make all coefficients comparable, we standardize each variable, meaning we de-mean and divide each variable by its standard deviation.

to time-varying risk aversion of heterogeneous global investors. As risk aversion increases, so does the global SDF. Changes in capital flows, on the other hand, do not have any significant effect on the global SDF.

Panel B reveals a different picture, as none of the previous factors has a significant coefficient anymore. Not very surprisingly, dollar is highly statistically significant here, with an associated R^2 of 88%. The coefficient on changes in capital flows is now significant and negative, indicating that a one standard deviation drop in capital flows leads to a 0.2 standard deviation drop in the local CB factor. Overall, we conclude that the two components of country-level SDFs load differently on intermediary constraints and capital flows: while the global SDF is linked to measures of global volatility and risk aversion, local CB captures changes in capital flows.

[Insert Table 6 here.]

As a last exercise, we use our framework to study home bias in global portfolios. It is well-known that capital flows follow a distinct pattern during crises: domestic capital inflows increase during periods of domestic or global crises (retrenchment), while investors withdraw capital from foreign markets during periods of foreign crises (fickleness), see, e.g., Forbes and Warnock (2012). This pattern leads to an increase in the home bias in bonds and equities.[21] As noted by Broner, Didier, Erce, and Schmukler (2013), these patterns are difficult to explain in models without frictions. In our main analysis, we impose average transaction costs observable in stock and bond markets. It is, however, reasonable to assume that these costs only represent a lower bound to the true cost associated with international trade. For example, Coeurdacier and Rey (2013) emphasize the potentially important role of hedging costs and informational frictions, which are very difficult to observe.[22]

While we can measure transaction costs such as bid-ask spreads, it is harder to quantify other types of asset trade costs such as the cost to trade via an intermediary or the role of international taxation. Intangible costs such as information frictions and behavioral biases are even harder to quantify, see, e.g., Coeurdacier and Rey (2013) for a discussion.[23] It is therefore natural to assume that the observed costs such as bid-ask spreads that we impose to extract international SDFs represent a lower bound to the true costs of trading foreign assets.

In the following, we can use our framework to estimate the unobservable cost of trading foreign assets such that portfolio holdings line up with the home bias observed in the data. To this end, we incrementally increase foreign transaction costs relative to local transaction costs and study the effect on the optimal portfolios of model-free SDFs.

[21] For example, French and Poterba (1991) and Lewis (1999), and more recently Camanho, Hau, and Rey (2019), show that the proportion of domestic stocks invested in portfolios exceeds their country's relative market capitalization in the world. This home bias phenomenon extends to bonds and is found to be even more pronounced, see, e.g., Maggiori, Neiman, and Schreger (2020).

[22] Most studies that empirically explore the effect on asset trade costs conclude that the costs would need to be unrealistically high to explain the level of home bias observed in the data. For example, French and Poterba (1991) argue in a mean-variance framework that these costs must be several hundred basis points. However, a different strand of the literature argues that if diversification benefits are small across countries, then these costs can be small and still explain home bias, see, e.g., Martin and Rey (2004) and Bhamra, Coeurdacier, and Guibaud (2014).

[23] Van Nieuwerburgh and Veldkamp (2009) study a model of home bias with informational frictions in international markets and link information asymmetry to earning forecasts, investors behavior, or pricing errors.

[Insert Figure 11 here.]

Figure 11 plots the optimal portfolio weights for each currency assuming that transaction costs on the foreign assets are six times as big as on the local assets. As we note, there is an almost perfect home bias in equities and long-term bonds for each currency. For nearly all currencies, local investors short the local long-term bond and have a long position in the local equity index. At the same time, we notice that across all currency denominations, investors trade the carry. For example, US investors short the US short-term bond while holding a long position in the Australian short-term bond. Australian global investors, on the other hand, short the Japanese Yen and buy New Zealand short-term bonds. This implies that independent of the foreign transaction cost, investors trade carry.

To get a sense of the implied cost to achieve home bias, recall that the average bid-ask spread is around 2bps in our data sample, which implies that the "hidden" costs are around 10bps. As mentioned earlier, these costs can include differences in taxation, behavioral or informational costs, as well as intermediation costs. Therefore, we conclude that even small transaction costs can lead to highly currency biased portfolios.

3 Robustness

In this section, we perform two robustness checks on our main results. First, we study the robustness of our results with respect to the weighting scheme used to calculate the global SDF and currency basket factors. Second, we explore potential weak identification issues in our two-step regressions.

3.1 Weighting Scheme

Recall from Theorem 1 that in order to calculate global factors, we need weights $(w_1, \dots, w_m)$ and we require $\sum_{i=1}^{m} w_i = 1$. Our main results rely on equal weights for each currency i, however, one might wonder how the weighting scheme affects our results. A priori, we do not expect the specific weighting scheme to have a big effect on our results for at least two reasons. First, in symmetric market settings, SDFs are the same, irrespective of the currency numéraire. However, even in asymmetric markets, our results show that international optimal portfolios are very similar across currencies. Second, as is well-known from the exchange rate volatility puzzle, the volatility of exchange rates is significantly smaller than the volatility of international SDFs. However, in order to check for robustness, we employ GDP-weights instead of equal weights and re-run all our analysis.

To save space, we defer all results to the Internet Appendix. Our findings indicate that there is virtually no difference whether we use equal weights or GDP weights as all results are the same. We therefore conclude that the particular weighting scheme does not affect our results.

3.2 Weak Identification

The identification of factors describing the cross-section of asset returns and the estimation of their factor premia may be challenged by several problems, such as model misspecification, small time-series sample sizes relative to the number of test assets, or poor identification. Our analysis is unlikely to be plagued by the first two, given (i) that we work with a linear SDF factor model that is by construction correctly specified (see equation (18)) and (ii) a sufficiently long time-series compared to the cross-sectional dimension. However, since we jointly price under a single-factor approach various asset classes (stocks, currencies, and bonds) having different volatilities and potentially heterogeneous factor structures, one may nevertheless be worried about identification issues. Such a concern may arise, e.g., when estimated betas in the first step of the two-stage Fama and MacBeth (1973) methodology do not display enough variation, leading to weak identification of the factor prices of risk in the second stage, invalid asymptotic properties of the two-stage estimator of the price of factor risk, and spuriously large cross-sectional adjusted R^2s; see, e.g., Kleibergen and Zhan (2020), among many others.

Intuitively, the cross-section of factor betas is linked to the variance of SDFs via the underlying pricing constraints satisfied by these SDFs. An excessive SDF variance under exact pricing constraints (as is for example the case with no frictions) may therefore produce low average SDF correlations with returns and hence a degenerate or nearly degenerate cross-section of SDF betas. These issues, however, are naturally mitigated by minimum dispersion SDFs allowing for non-zero pricing errors (as in our market settings with frictions), because of the lower variance of these SDFs, induced by the less tight pricing error constraints.

To explore a possible weak identification problem in our empirical framework in Section 2.5.2, we borrow from the recent methodology proposed in Kleibergen and Zhan (2020). Their methodology includes a simple test of weak identification for Fama and MacBeth (1973) two-step empirical asset pricing frameworks and an extended Gibbons, Ross, and Shanken (1989) test, which is robust against weak identification, for jointly testing correct specification and parametric hypotheses for the vector of factor risk premia.

[Insert Figure 12 here.]

Figure 12 reports the $p-$values of the Kleibergen and Zhan (2020) test of weak identification for the various in-sample Fama and MacBeth (1973) settings considered in our analysis. Overall, we never find evidence of weak identification for all asset pricing frameworks based on model-free SDFs in markets with frictions, since all $p-$values for the test are minuscule. In contrast, for the asset pricing frameworks based on model-free SDFs in frictionless markets, we find that the results for the cross-section of long-term bond returns may suffer from a weak identification issue for different currency denominations. This finding is not very surprising, given our earlier finding that the cross-sectional distribution of SDF betas for long-term bonds is nearly degenerate in frictionless settings.[24]

In summary, we conclude that our main empirical findings on the in- and out-of-sample cross-sectional pricing accuracy of our model-free SDFs in international markets with frictions is not driven by weak identification issues.

[24]We report the same statistics for the out-of-sample Fama and MacBeth (1973) two-step empirical asset pricing estimates in the Internet Appendix. We again find no evidence of weak identification in markets with frictions.

4 Conclusions

This paper develops a theoretical model-free framework that allows us to identify global risk factors from cross-sections of international assets, such as stocks, bonds, and currencies when investors face barriers to trade. Intuitively, limiting the allocation of wealth that is invested into risky assets leads to more sparse portfolios, endogenously segmented markets, and hence more robust properties of international SDFs.

Our main theoretical contribution is twofold. First, under the assumption of market symmetry, we show that we can always uniquely recover the exchange rate appreciation from the ratio of foreign and domestic numéraire-invariant SDFs even in the presence of frictions. The numéraire-invariance makes our SDFs uniquely suited to extract global risks. Second, our main result characterizes international SDFs using a two factor representation. The first factor, *global SDF*, is currency-independent and pertains to the average optimal portfolio return of global investors. The second factor, *currency basket*, is given by the average appreciation of a local currency against the foreign currencies.

We then use this framework to estimate international SDFs from the cross-section of stocks, and short- and long-term bonds for developed countries. When international agents face no barriers to trade, SDFs need to exactly price all assets, which leads to volatile SDFs and perfect correlation, because the AMV holds. Imposing market frictions significantly reduces the SDF volatility, but we find again correlations to be nearly perfect, the reason being that investors hold almost identical portfolios. Closer inspection of these portfolios reveals that global investors take their biggest exposures in the classical carry trade, together with long positions in USD and CHF equities and short position in JPY equity.

We analyze the ability of our factor representation to price separately and jointly the cross-section of currency returns, bonds, and equities. Both the in- and out-of sample results suggest that the numéraire-invariant global SDF risk factor alone captures most of the cross-sectional variation both within and across asset classes. Moreover, introducing market frictions and market asymmetries is instrumental to generate a better out-of sample fit of the cross-section of international assets. Indeed, when international investors face asymmetric proportional transaction costs, the ensuing SDF captures up to 95% of the in- and 80% of the out-of-sample cross-sectional variation across all denominations. Finally, we link our two factors to variables proposed in the literature and find them to be strongly related to measures of volatility and capital flows.

In our paper, we explore the source and nature of common factors across international assets from asset return data alone, but we ignore international portfolio holdings data. Future research could link the observed prices directly to optimal portfolios held by international investors. For example, in the demand systems approach of Koijen and Yogo (2020) global factors of stocks, bonds, and currencies are determined by matching not just asset prices but also holdings data. Moreover, our framework naturally accommodates the latent demand highlighted in Koijen and Yogo (2020), due to the presence of wedges in case of asymmetric markets. We leave this exciting research for future work.

Figures

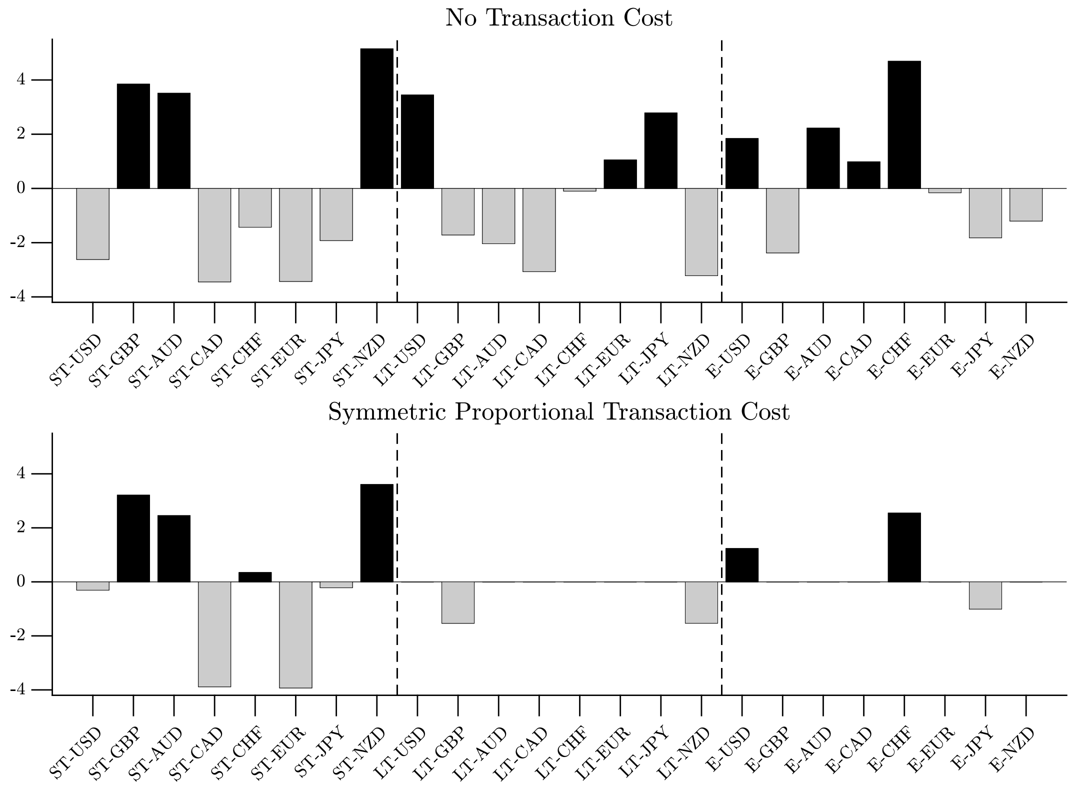


Figure 1. **Optimal Weights: Symmetric Markets.** The upper panel plots the portfolio weights in each asset denominated in USD assuming that investors face no trade barriers. The lower panel plots the portfolio weights in each asset denominated in USD assuming that investors face symmetric proportional transaction costs. ST-XXX is the short-term bond for currency XXX, LT-XXX is the long-term bond for currency XXX, and E-XXX is the equity index for currency XXX. Data is monthly and runs from January 1988 to December 2015.

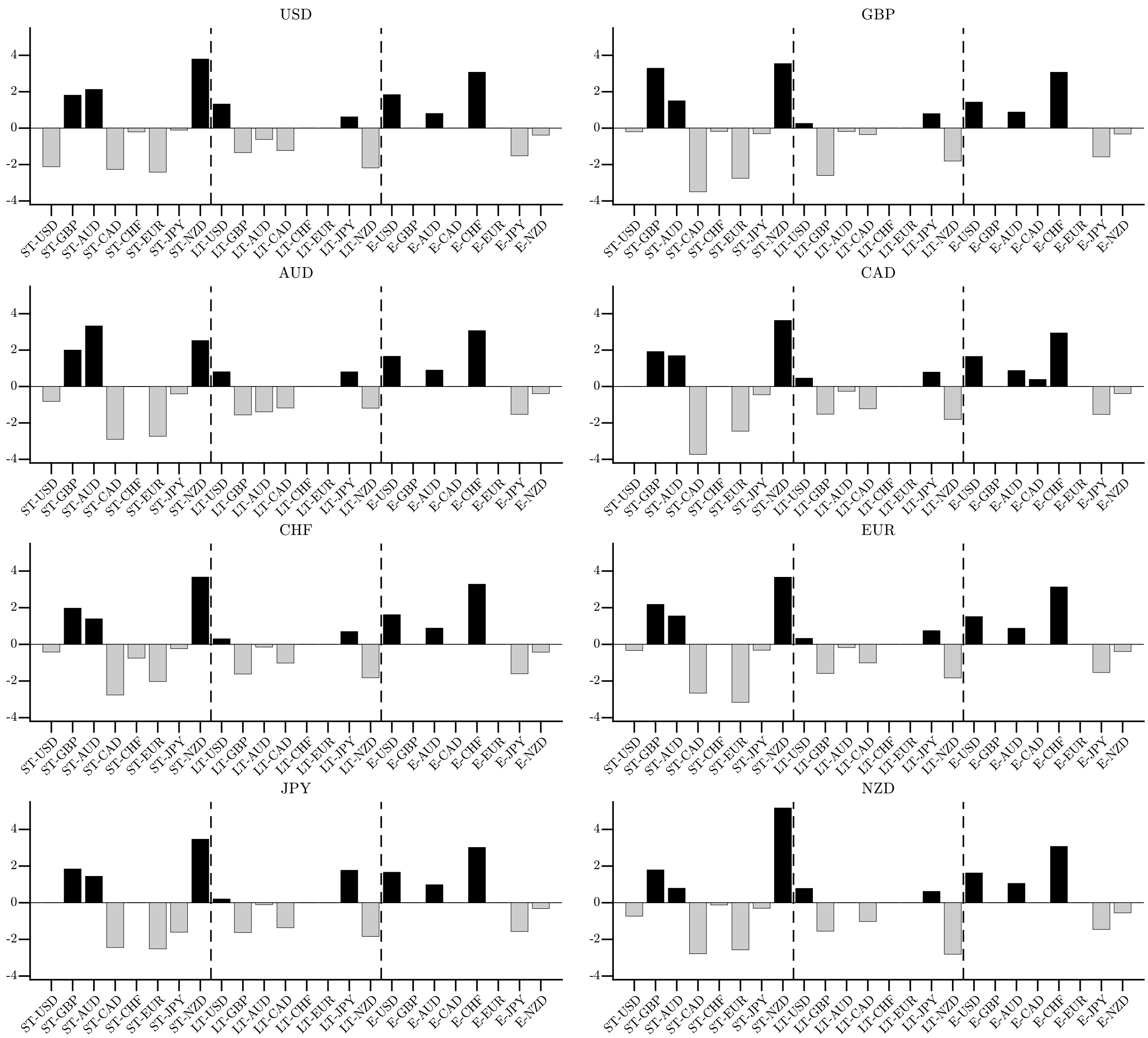


Figure 2. **Optimal Weights: Bid-Ask Spreads.** This figure plots the portfolio weights in each asset for all currency denominations assuming that investors face bid-ask spreads. ST-XXX is the short-term bond for currency XXX, LT-XXX is the long-term bond for currency XXX, and E-XXX is the equity index for currency XXX. Data is monthly and runs from January 1988 to December 2015.

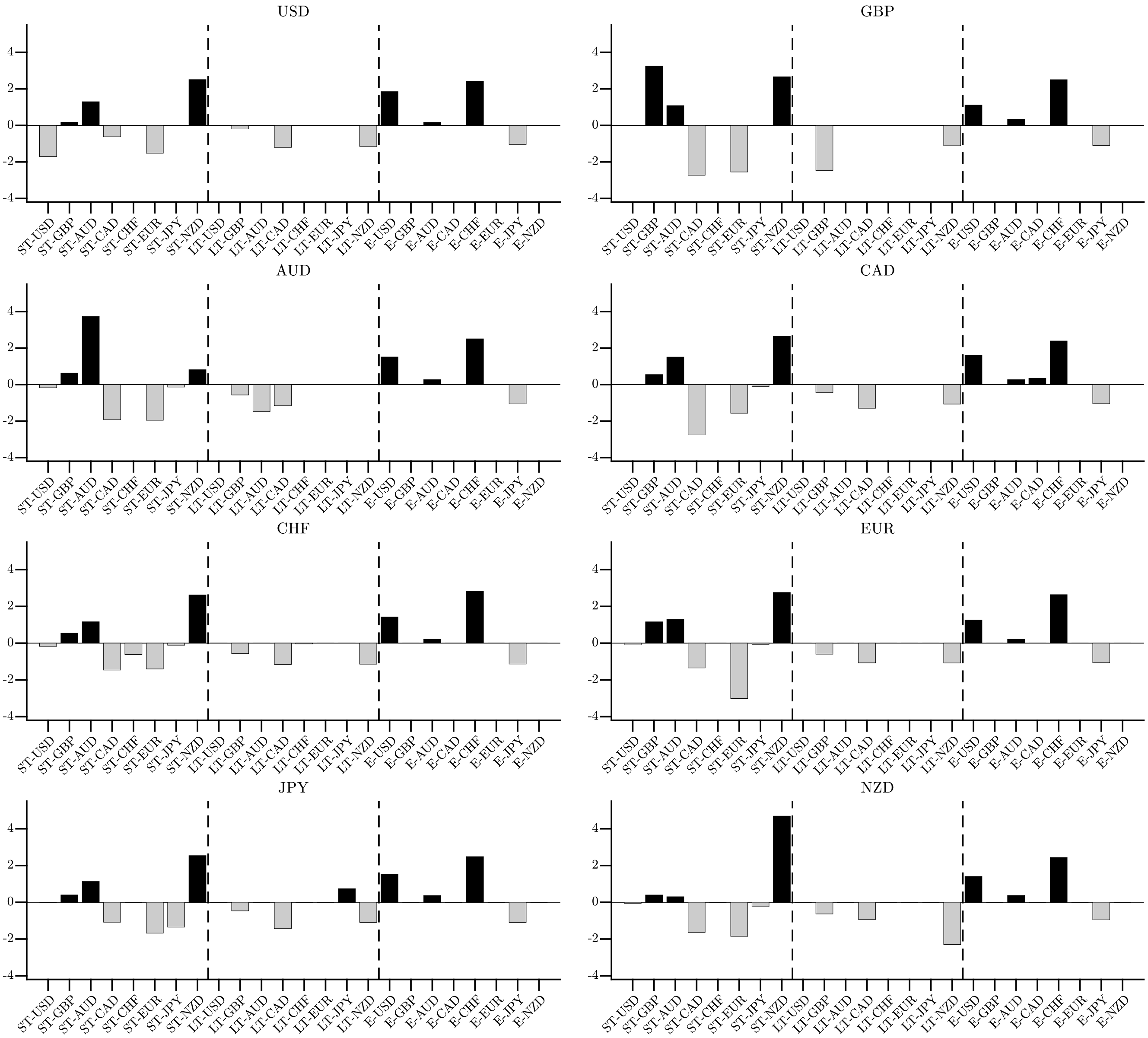


Figure 3. **Optimal Weights: Asymmetric Proportional Transaction Costs.** This figure plots the portfolio weights in each asset for all currency denominations assuming that investors face proportional transaction costs. ST-XXX is the short-term bond for currency XXX, LT-XXX is the long-term bond for currency XXX, and E-XXX is the equity index for currency XXX. Data is monthly and runs from January 1988 to December 2015.

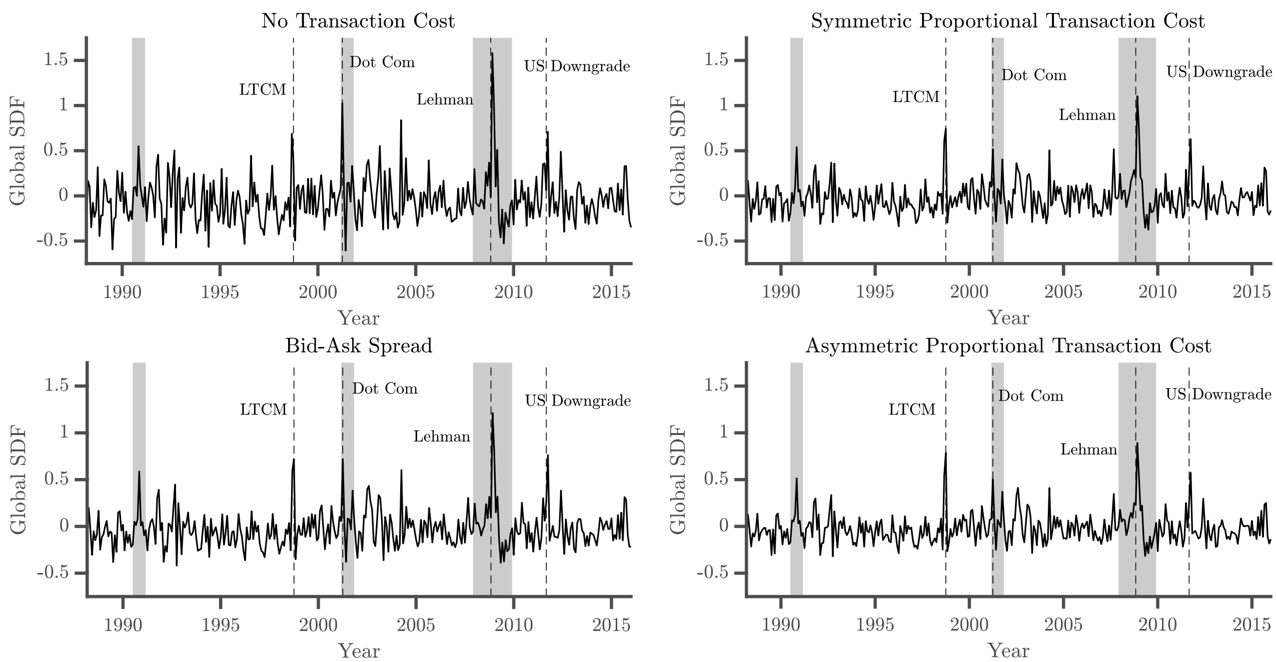


Figure 4. **Global SDF Factor.** This figure plots the numéraire-invariant global SDF factor, i.e., $G = -\frac{1}{m}\sum_{j=1}^{m}\log \boldsymbol{\theta}_0' \boldsymbol{R}^{(j)}$, estimated with no frictions (upper left), with symmetric proportional transaction costs (upper right), with bid-ask spreads (lower left), and asymmetric proportional transaction cost (lower right). Gray bars indicate recessions according to NBER. Data is monthly and runs from January 1988 to December 2015.

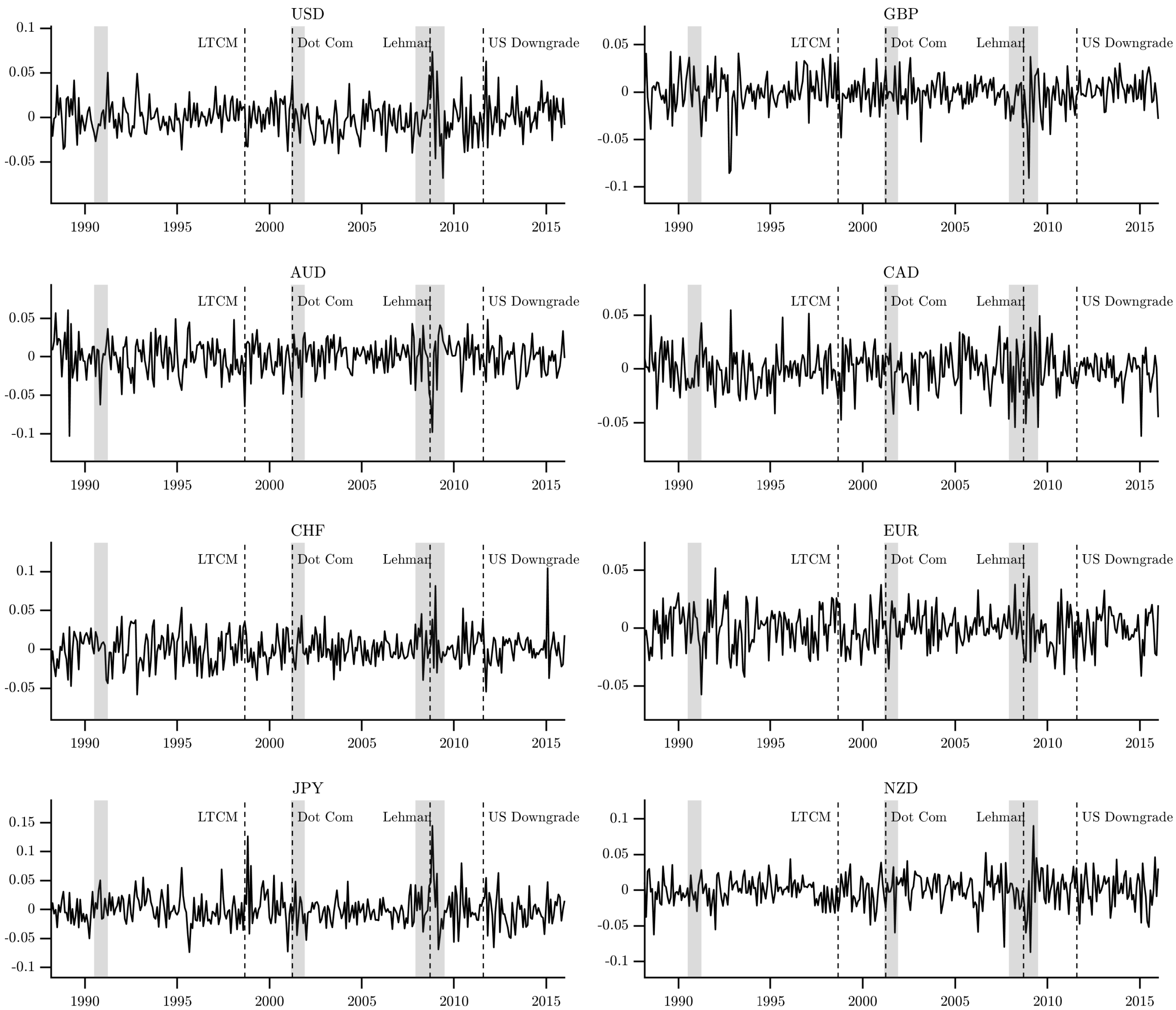


Figure 5. **Local Currency Baskets.** This figure plots the country-specific currency basket, i.e., the average appreciation of each local currency with respect to the remaining currencies. Gray bars indicate recessions according to NBER. Data is monthly and runs from January 1988 to December 2015.

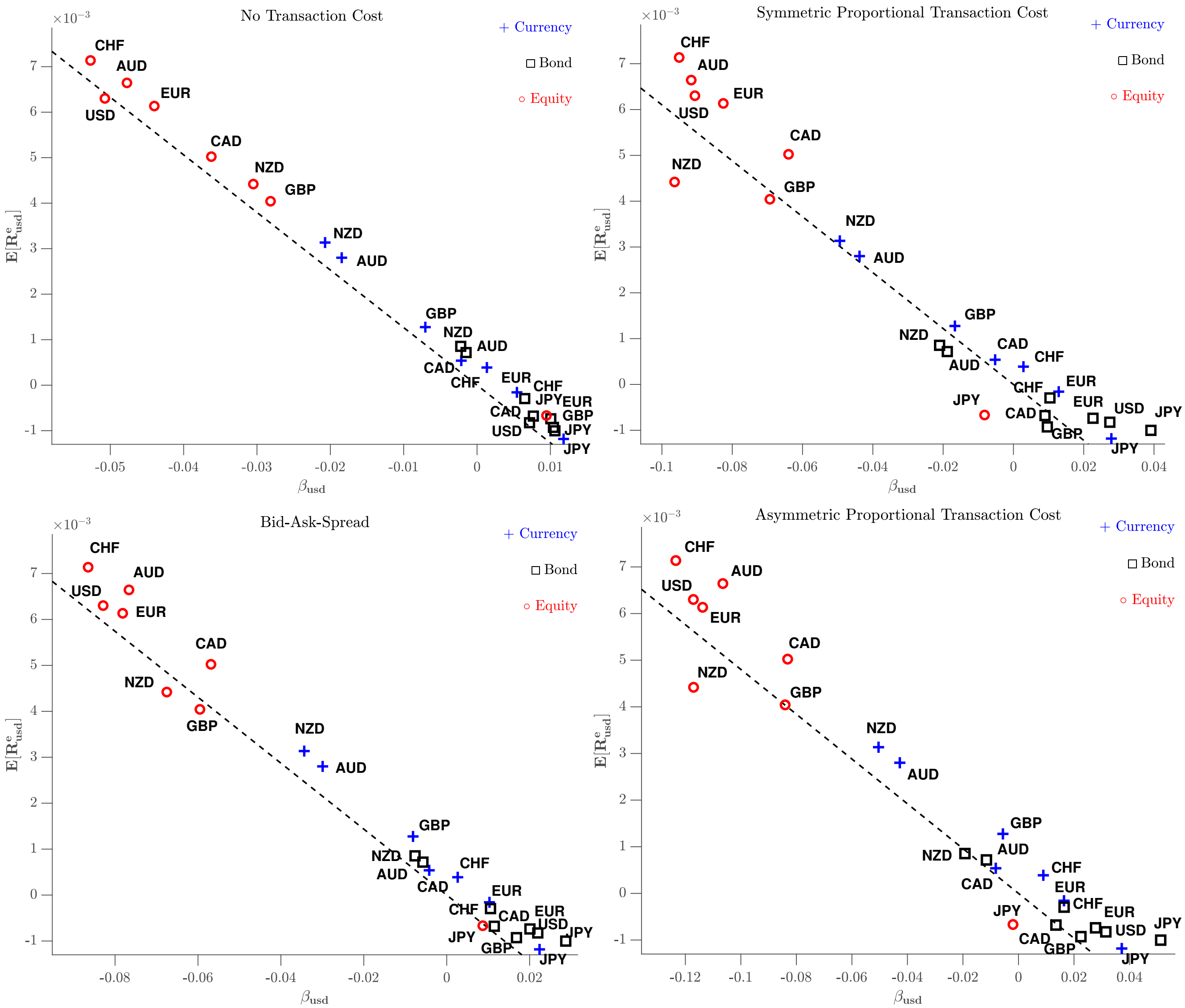
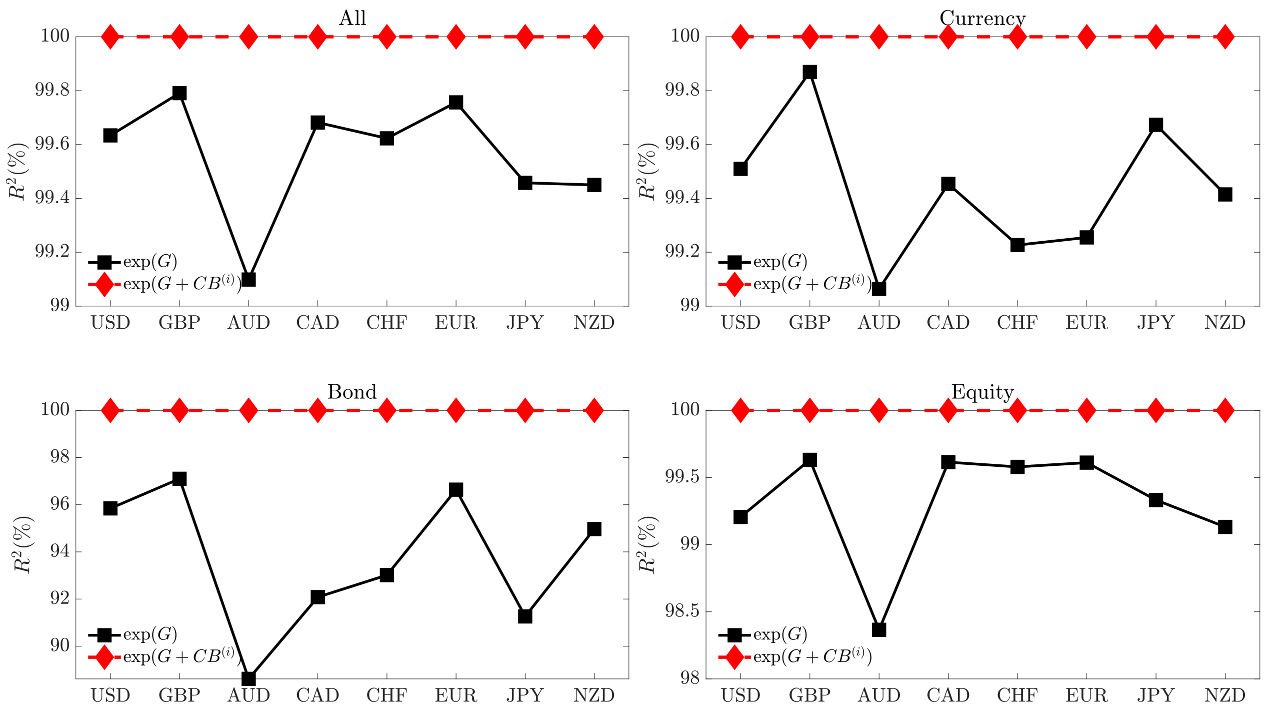
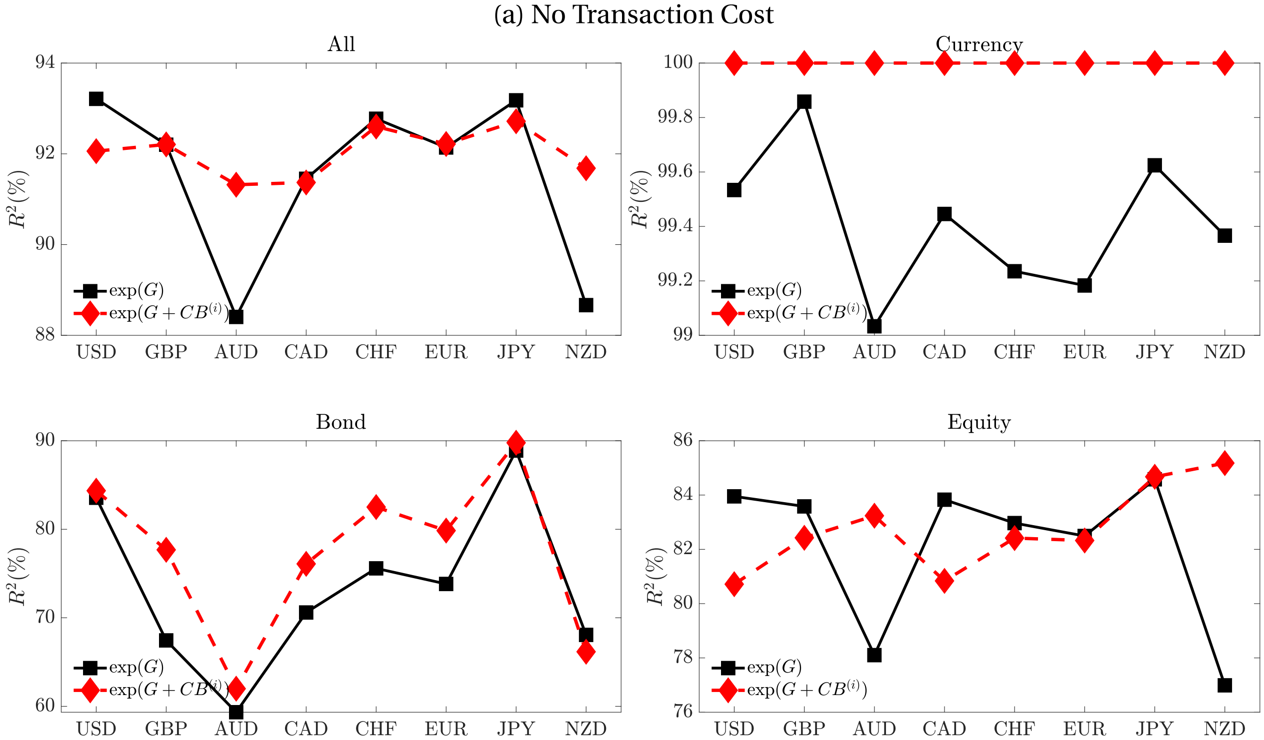


Figure 6. **In-Sample Risk-Return Relationship with Global SDF.** The upper panel reports the in-sample risk-return relation for USD-denominated currencies, bonds, and equities in symmetric market settings, no transaction costs (left) and symmetric proportional transaction costs (right). The lower panel corresponds to asymmetric market settings, Bid-Ask spreads (left) and asymmetric proportional transaction costs (right). The figures report the relation between the expected excess returns (y-axis) and the risk factor exposures (x-axis), where the factor depends only on the global SDF, i.e., $\widetilde{M}_0 := \exp(G)$. The dashed line corresponds to the coefficient λ in the cross-sectional regression (18) and the factor loading β_{USD} is given in equation (15). Data is monthly and runs from January 1988 to December 2015.

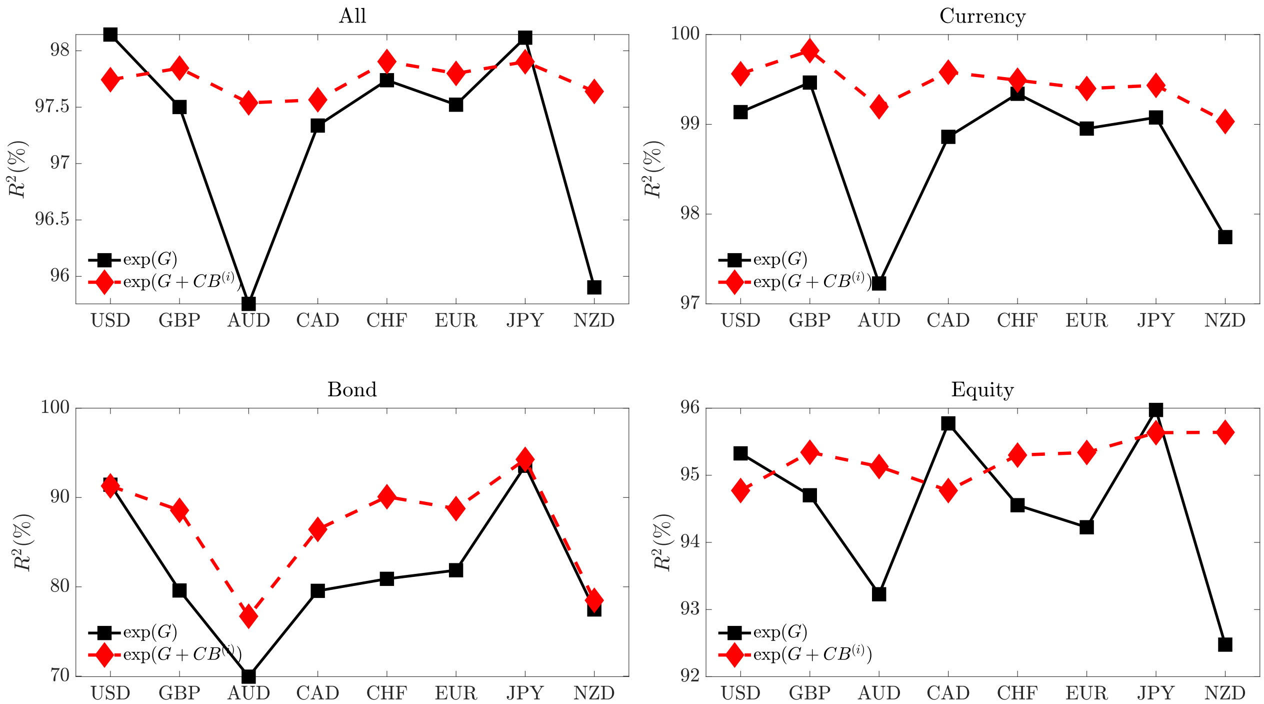


(a) No Transaction Cost

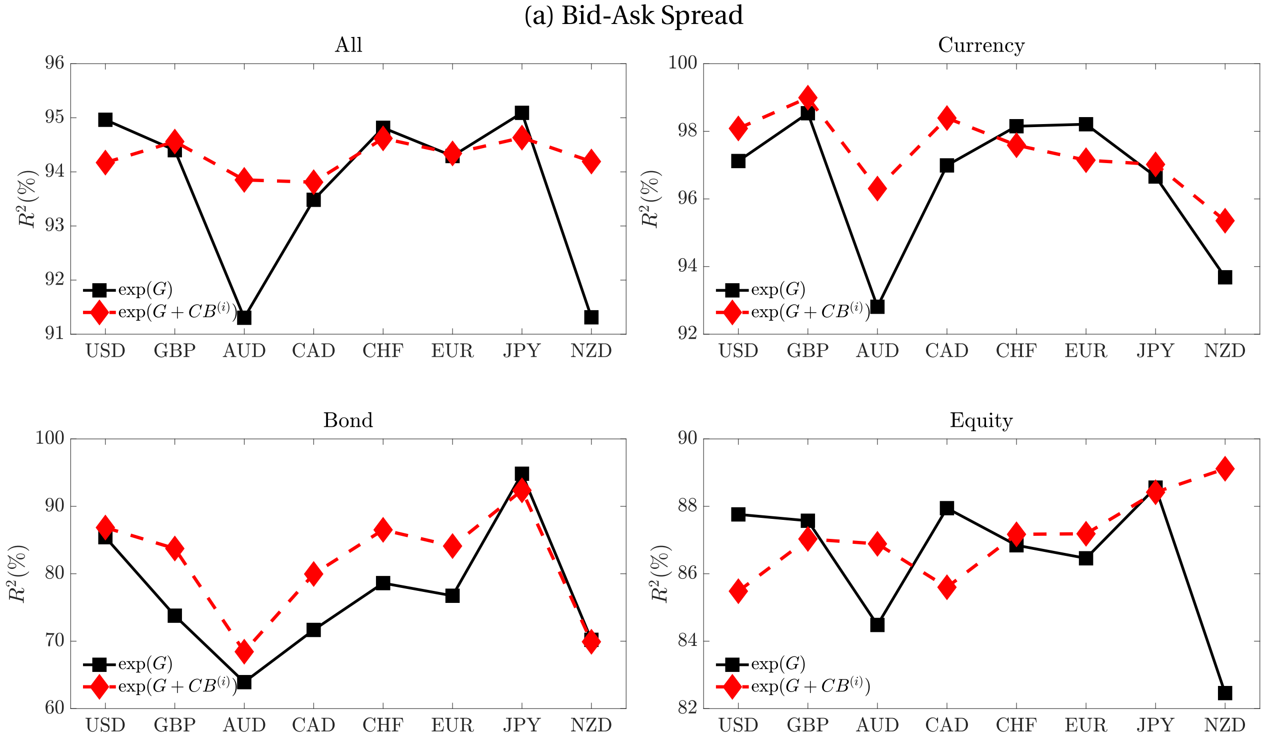


(b) Symmetric Proportional Transaction Cost

Figure 7. **In-sample R^2s Across All Denominations Symmetric Markets.** This figure reports the cross-sectional variation explained in an asymmetric proportional transaction cost setting by a factor model using only the global factor, $\widetilde{M}_0$ (black line) and a factor model using both the global factor and the local currency basket factor, $\widetilde{M}_0^{(i)}$, (dashed red line). Cross-sectional R^2s are reported when pricing all assets (top-left), currency returns (top-right), long-term bonds (bottom-left), and international equity indices (bottom-right). Data is monthly and runs from January 1988 to December 2015.



(a) Bid-Ask Spread



(b) Asymmetric Proportional Transaction Cost

Figure 8. **In-sample R^2s Across All Denominations Asymmetric Markets.** This figure reports the cross-sectional variation explained in an asymmetric proportional transaction cost setting by a factor model using only the global factor, $\widetilde{M}_0$ (black line) and a factor model using both the global factor and the local currency basket factor, $\widetilde{M}_0^{(i)}$, (dashed red line). Cross-sectional R^2s are reported when pricing all assets (top-left), currency returns (top-right), long-term bonds (bottom-left), and international equity indices (bottom-right). Data is monthly and runs from January 1988 to December 2015.

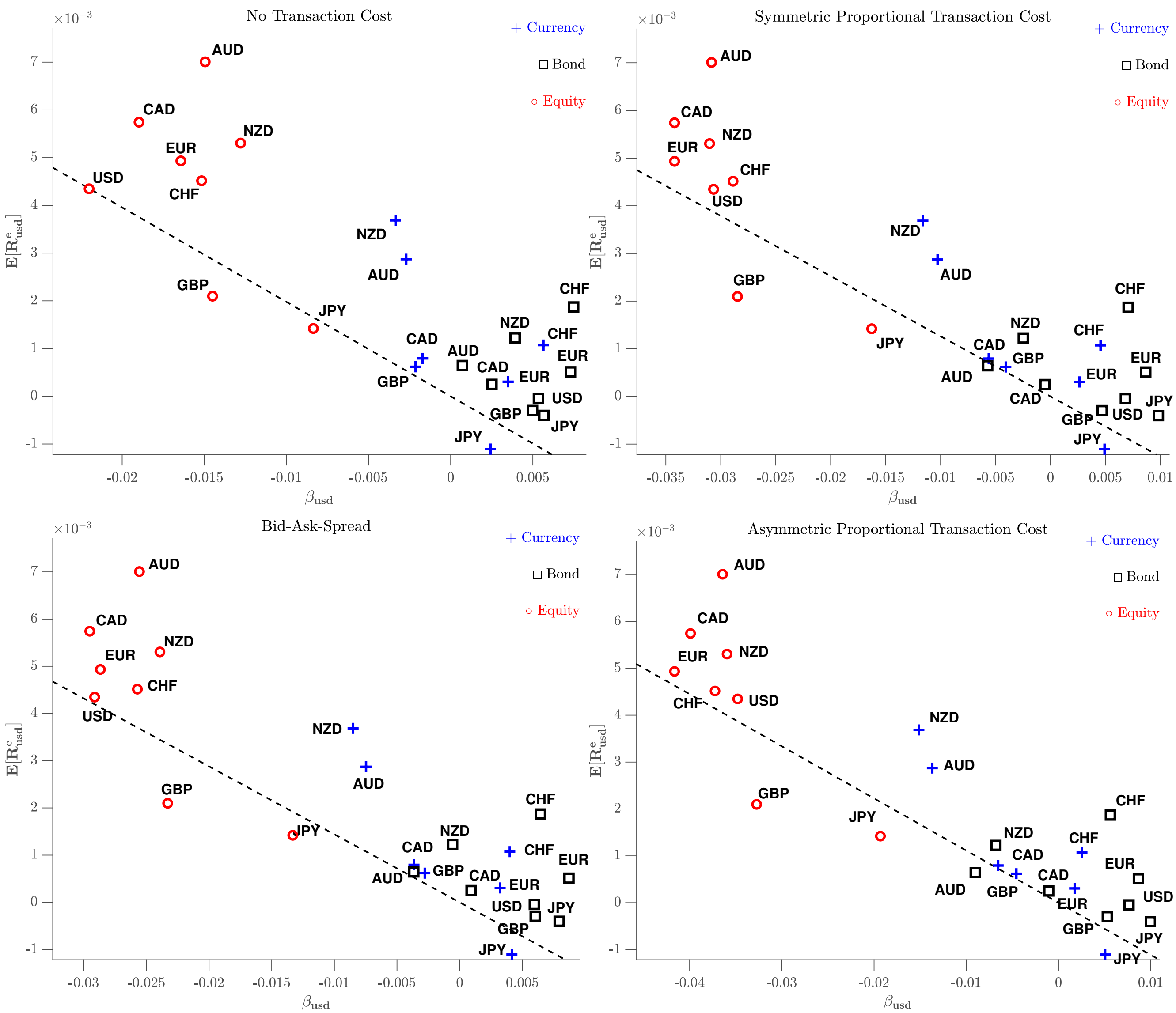


Figure 9. **Out-of-Sample Risk-Return Relationship with Global SDF.** The upper panel reports the out-of-sample risk-return relation for USD-denominated currencies, bonds, and equities in symmetric market settings, no transaction costs (left) and symmetric proportional transaction costs (right). The lower panel corresponds to asymmetric market settings, Bid-Ask spreads (left) and asymmetric proportional transaction costs (right). The figures report the relation between the expected excess returns (y-axis) and the risk factor exposures (x-axis), where the factor depends on only the global SDF, i.e., $\widetilde{M}_0 := \exp(G)$. The dashed line corresponds to the coefficient λ in the cross-sectional regression (18) and the factor loading β_{USD} is given in equation (15). Data is monthly and runs from January 1988 to December 2015.

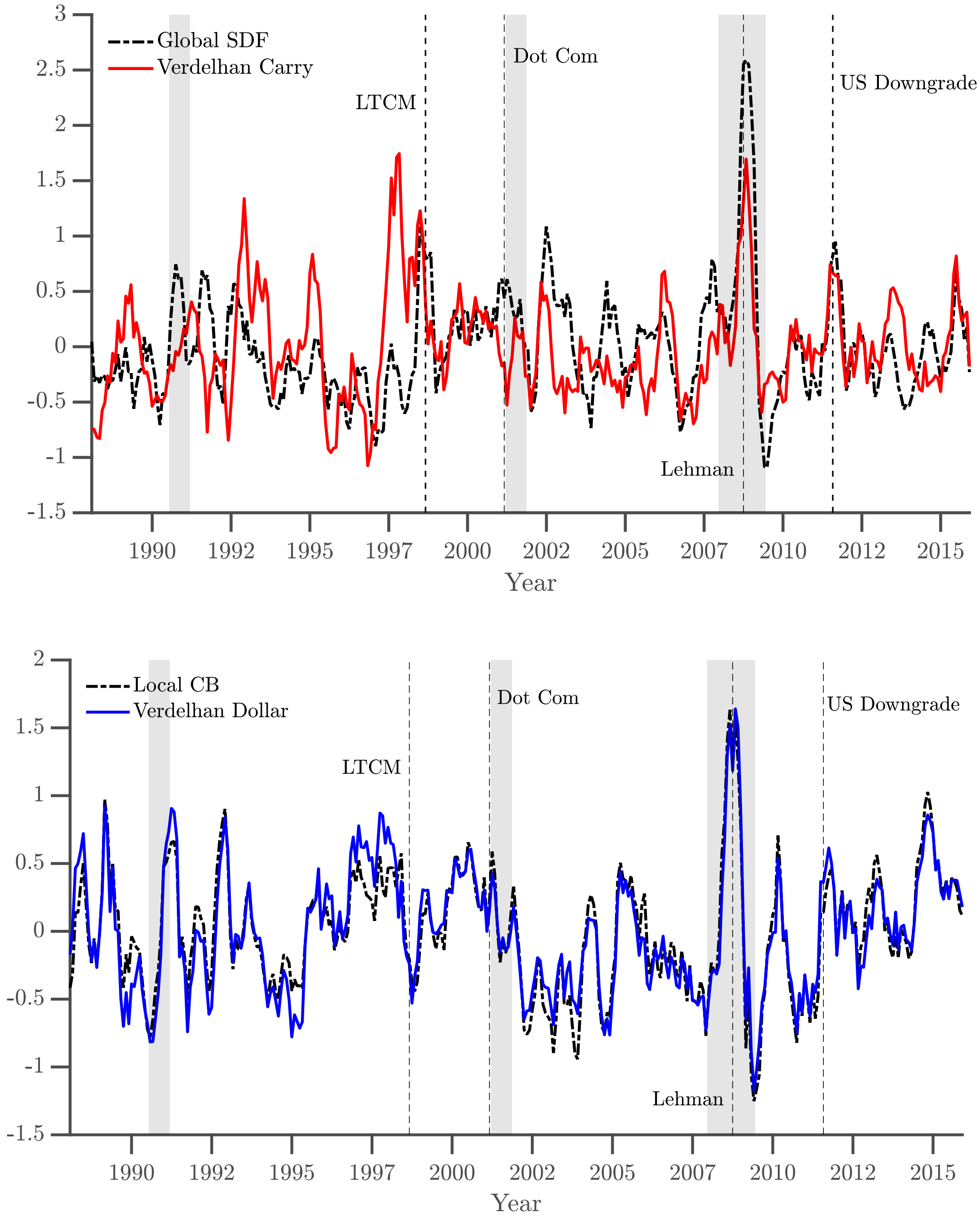


Figure 10. **Global SDF, Local CB and Verdelhan (2018) Risk Factors.** The upper panel plots the global SDF factor, G, together with Verdelhan (2018)'s Carry factor. The lower panel plots the local currency basket, CB^{usd}, together with Verdelhan (2018)'s Dollar factor. Time-series are six-month moving averages calculated from monthly data running from January 1988 to December 2015.

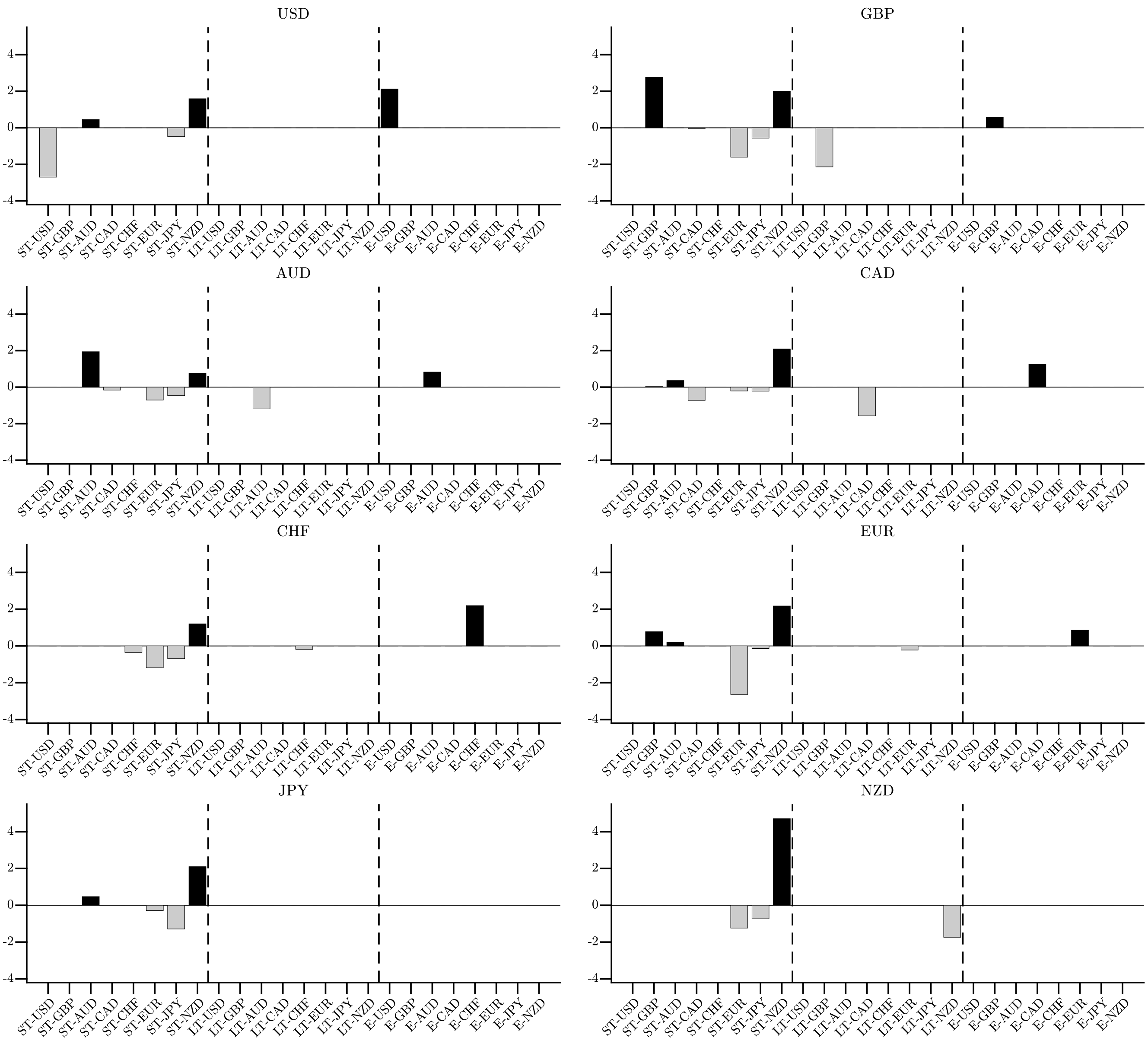
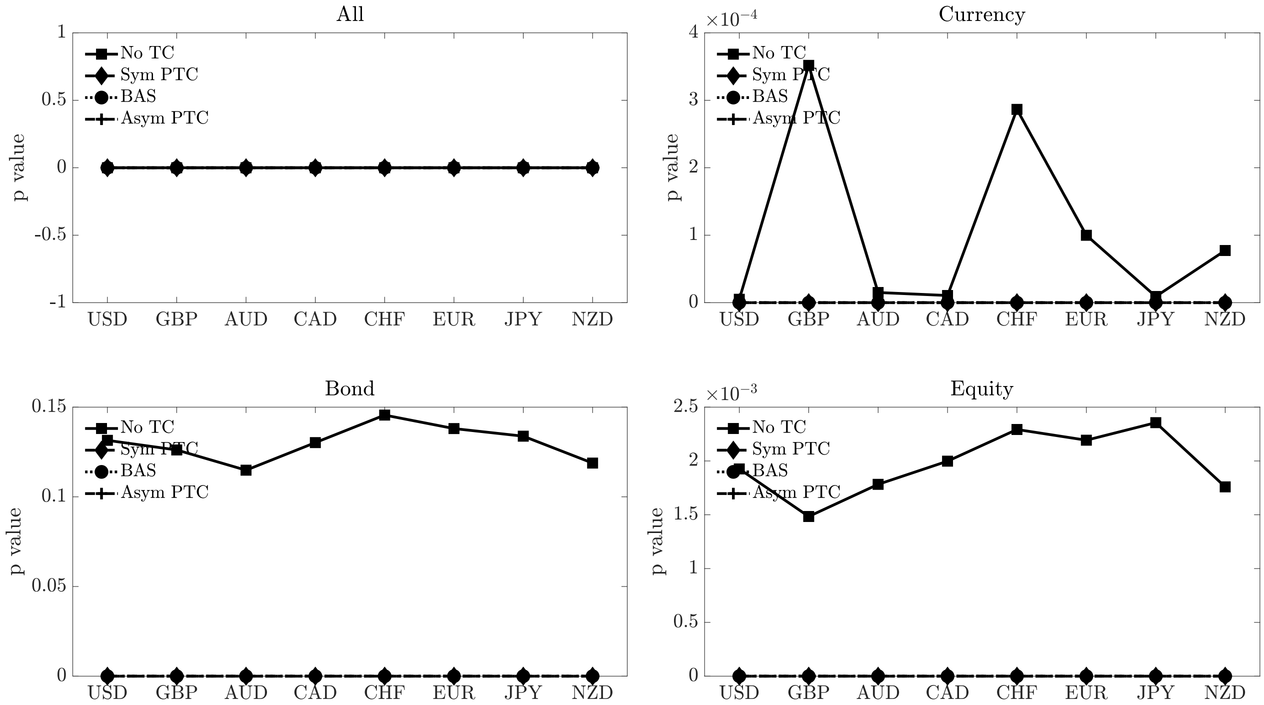
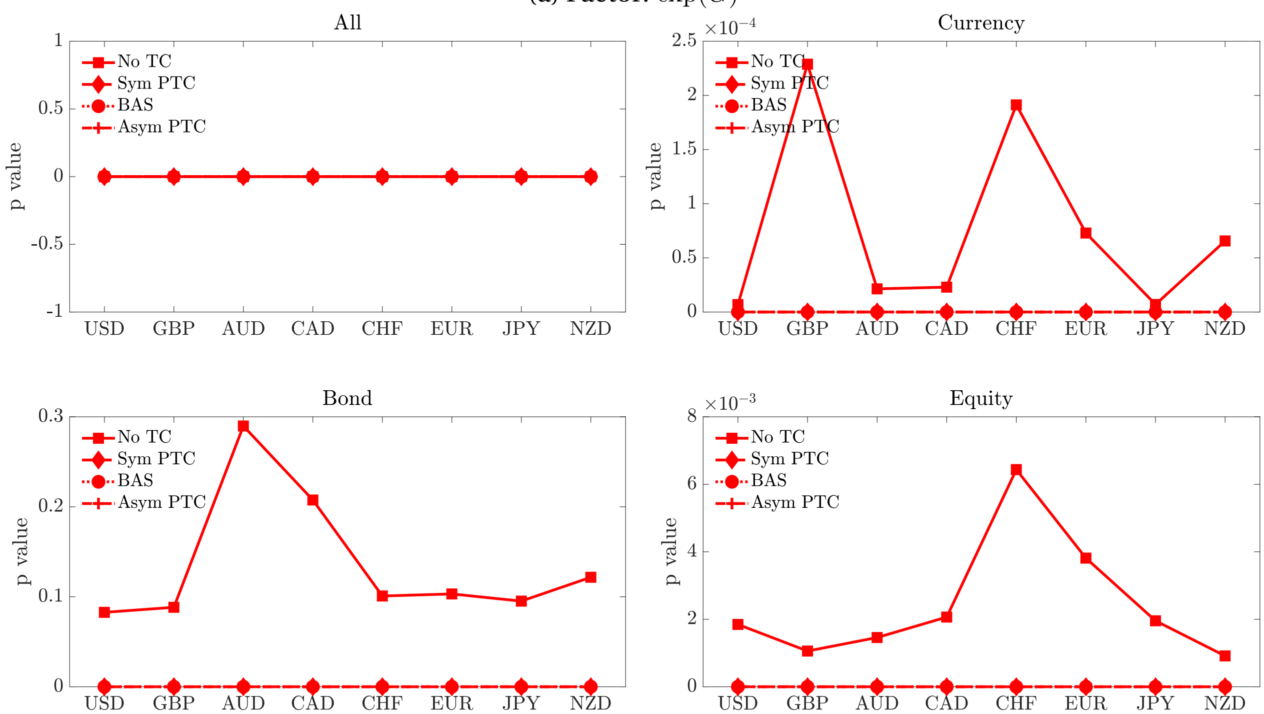


Figure 11. **Home Bias.** This figure plots the optimal portfolio weights in an asymmetric market setting where transaction costs on foreign assets are six times larger than in local markets. Data is monthly and runs from January 1988 to December 2015.



(a) Factor: $\exp(G)$



(b) Factor: $\exp(G + CB^{(i)})$

Figure 12. **Test of Weak Identification.** This figure reports the p-values of Kleibergen and Zhan (2020) test of weak identification, which tests with a χ^2-statistic the null hypothesis $\boldsymbol{\beta} - \overline{\boldsymbol{\beta}} = \mathbf{0}$ in linear model (18). The null hypothesis is tested for two single-factor models: Panel (a), where the factor is given by only the global SDF, i.e., $\widetilde{M}_0 = \exp(G)$, and Panel (b), where the factor is constructed from the global factor and the currency basket, i.e., $\widetilde{M}_0^{(i)} = \exp(G + CB^{(i)})$. p-values are reported when pricing all assets (top-left), currency returns (top-right), long-term bonds (bottom-left), and international equity indices (bottom-right). Data is monthly and runs from January 1988 to December 2015.

Tables

Table 1. **Summary Statistics Global SDFs**

This table reports summary statistics: mean and standard deviation and average correlations for minimum entropy SDFs denominated in different currencies assuming four different market structures: no transaction costs, symmetric transaction costs, bid-ask spreads, and asymmetric transaction costs. Data is monthly and runs from January 1988 to December 2015.

	Panel A: No Transaction Cost								*Panel B: Symmetric Proportional Transaction Cost*							
	USD	GBP	AUD	CAD	CHF	EUR	JPY	NZD	USD	GBP	AUD	CAD	CHF	EUR	JPY	NZD
mean	0.997	0.996	0.995	0.996	0.998	0.997	0.999	0.995	0.997	0.996	0.995	0.996	0.998	0.997	0.999	0.995
stdev	0.359	0.347	0.348	0.352	0.355	0.361	0.369	0.341	0.234	0.225	0.221	0.230	0.233	0.237	0.245	0.217
USD	1.000								1.000							
GBP	0.996	1.000							0.992	1.000						
AUD	0.995	0.993	1.000						0.988	0.985	1.000					
CAD	0.998	0.996	0.997	1.000					0.995	0.991	0.993	1.000				
CHF	0.993	0.995	0.993	0.994	1.000				0.987	0.991	0.986	0.987	1.000			
EUR	0.996	0.997	0.996	0.996	0.998	1.000			0.990	0.994	0.991	0.991	0.997	1.000		
JPY	0.996	0.994	0.992	0.994	0.993	0.995	1.000		0.992	0.988	0.984	0.988	0.988	0.989	1.000	
NZD	0.994	0.994	0.997	0.997	0.995	0.996	0.993	1.000	0.988	0.987	0.994	0.992	0.990	0.993	0.986	1.000
	Panel C: Bid-Ask Spreads								*Panel D: Asymmetric Proportional Transaction Cost*							
	USD	GBP	AUD	CAD	CHF	EUR	JPY	NZD	USD	GBP	AUD	CAD	CHF	EUR	JPY	NZD
mean	0.997	0.996	0.995	0.996	0.998	0.997	0.999	0.995	0.997	0.996	0.995	0.996	0.998	0.997	0.999	0.995
stdev	0.265	0.256	0.248	0.255	0.257	0.264	0.270	0.246	0.206	0.202	0.186	0.200	0.202	0.206	0.210	0.190
USD	1.000								1.000							
GBP	0.981	1.000							0.944	1.000						
AUD	0.983	0.986	1.000						0.952	0.956	1.000					
CAD	0.978	0.985	0.986	1.000					0.958	0.949	0.971	1.000				
CHF	0.966	0.983	0.975	0.978	1.000				0.943	0.952	0.964	0.956	1.000			
EUR	0.977	0.991	0.986	0.982	0.988	1.000			0.940	0.967	0.962	0.950	0.979	1.000		
JPY	0.975	0.981	0.978	0.976	0.972	0.978	1.000		0.957	0.949	0.954	0.948	0.955	0.949	1.000	
NZD	0.979	0.986	0.987	0.986	0.983	0.988	0.977	1.000	0.945	0.953	0.968	0.960	0.965	0.962	0.950	1.000

Table 2. **Risk Prices FX: In-Sample**

This table reports estimated in-sample risk prices in the two-step Fama and MacBeth (1973) regressions of USD denominated currency excess returns on estimated factor loadings of (1) the corresponding local SDF (M_0^{usd}), (2) approximation of the latter with the global SDF, i.e., $\widetilde{M}_0 := \exp(\text{G})$, and (3) approximation with both the global SDF and the local currency basket factor, i.e., $\widetilde{M}_0^{usd} := \exp(\text{G+CB}^{usd})$. The global SDF factor is the average SDF calculated from the cross-section of all local SDFs. The currency basket factor is the average appreciation of the local currency. Shanken (1992)-corrected standard errors are reported in brackets. Labels (***), (**) and (*) denote significance at the 1%, 5%, and 10% level, respectively. Data runs from January 1988 to December 2015.

Panel A: No Transaction Cost

M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.129^{**}			100.000	0.000
[0.062]				
	-0.129^{**}		99.510	0.011
	[0.063]			
		-0.129^{**}	100.000	0.000
		[0.062]		

Panel B: Symmetric Proportional Transaction Cost

M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.055^{**}			100.000	0.000
[0.026]				
	-0.054^{**}		99.534	0.011
	[0.026]			
		-0.055^{**}	100.000	0.000
		[0.026]		

Panel C: Bid-Ask Spreads

M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.079^{**}			99.370	0.012
[0.038]				
	-0.075^{**}		99.136	0.014
	[0.037]			
		-0.075^{**}	99.563	0.010
		[0.036]		

Panel D: Asymmetric Proportional Transaction Cost

M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.053^{**}			97.187	0.026
[0.026]				
	-0.049^{**}		97.121	0.026
	[0.024]			
		-0.049^{**}	98.085	0.022
		[0.024]		

Table 3. **Risk Prices All: In-Sample**

This table reports estimated in-sample risk prices in the two-step Fama and MacBeth (1973) regressions of USD denominated currency, bond and equity excess returns on estimated factor loadings of (1) the corresponding local SDF (M_0^{usd}), (2) approximation of the latter with the global SDF, i.e., $\widetilde{M}_0 := \exp(\text{G})$, and (3) approximation with both the global SDF and the local currency basket factor, i.e., $\widetilde{M}_0^{usd} := \exp(\text{G} + \text{CB}^{usd})$. The global SDF factor is the average SDF calculated from the cross-section of all local SDFs. The currency basket factor is the average appreciation of the local currency. Shanken (1992)-corrected standard errors are reported in brackets. Labels (***), (**) and (*) denote significance at the 1%, 5%, and 10% level, respectively. Data runs from January 1988 to December 2015.

Panel A: No Transaction Cost					*Panel B: Symmetric Proportional Transaction Cost*				
M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)	M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.129^{***}			100.000	0.000	-0.061^{***}			92.059	0.081
[0.040]					[0.021]				
	-0.127^{***}		99.634	0.017		-0.061^{***}		93.211	0.074
	[0.039]					[0.020]			
		-0.129^{***}	100.000	0.000			-0.061^{***}	92.059	0.081
		[0.040]					[0.021]		
Panel C: Bid-Ask Spreads					*Panel D: Asymmetric Proportional Transaction Cost*				
M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)	M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.076^{***}			97.815	0.042	-0.051^{***}			94.051	0.070
[0.024]					[0.017]				
	-0.072^{***}		98.144	0.039		-0.048^{***}		94.963	0.064
	[0.023]					[0.016]			
		-0.073^{***}	97.744	0.043			-0.048^{***}	94.172	0.069
		[0.023]					[0.016]		

Table 4. **Risk Prices FX: Out-of-Sample**

This table reports estimated out-of-sample risk prices in the two-step Fama and MacBeth (1973) regressions of USD denominated currency excess returns on estimated factor loadings of(1) the corresponding local SDF (M_0^{usd}), (2) approximation of the latter with the global SDF, i.e., $\widetilde{M}_0 := \exp(\mathrm{G})$, and (3) approximation with both the global SDF and the local currency basket factor, i.e., $\widetilde{M}_0^{usd} := \exp(\mathrm{G} + \mathrm{CB}^{usd})$. The global SDF factor is the average SDF calculated from the cross-section of all local SDFs. The currency basket factor is the average appreciation of the local currency. Shanken (1992)-corrected standard errors are reported in brackets. Labels (***), (**) and (*) denote significance at the 1%, 5%, and 10% level, respectively. Data runs from January 1988 to December 2015.

Panel A: No Transaction Cost					*Panel B: Symmetric Proportional Transaction Cost*				
M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)	M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.270			36.262	0.129	-0.196^{*}			71.560	0.086
[0.245]					[0.141]				
	-0.269		35.030	0.130		-0.196^{*}		70.478	0.088
	[0.248]					[0.142]			
		-0.297	45.245	0.119			-0.194^{*}	76.307	0.079
		[0.253]					[0.137]		
Panel C: Bid-Ask Spreads					*Panel D: Asymmetric Proportional Transaction Cost*				
M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)	M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.245^{*}			72.722	0.084	-0.184^{*}			81.890	0.069
[0.175]					[0.126]				
	-0.249^{*}		69.940	0.088		-0.182^{*}		80.936	0.070
	[0.180]					[0.125]			
		-0.247^{*}	76.716	0.078			-0.176^{*}	85.322	0.062
		[0.173]					[0.119]		

Table 5. **Risk Prices All: Out-of-Sample**

This table reports estimated out-of-sample risk prices in the two-step Fama and MacBeth (1973) regressions of USD denominated currency, bond and equity excess returns on estimated factor loadings of(1) the corresponding local SDF (M_0^{usd}), (2) approximation of the latter with the global SDF, i.e., $\widetilde{M}_0 := \exp(\text{G})$, and (3) approximation with both the global SDF and the local currency basket factor, i.e., $\widetilde{M}_0^{usd} := \exp(\text{G} + \text{CB}^{usd})$. The global SDF factor is the average SDF calculated from the cross-section of all local SDFs. The currency basket factor is the average appreciation of the local currency. Shanken (1992)-corrected standard errors are reported in brackets. Labels (***), (**) and (*) denote significance at the 1%, 5%, and 10% level, respectively. Data runs from January 1988 to December 2015.

Panel A: No Transaction Cost					*Panel B: Symmetric Proportional Transaction Cost*				
M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)	M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.199^*			69.066	0.127	-0.127^*			78.018	0.107
[0.139]					[0.086]				
	-0.198^*		68.627	0.128		-0.126^*		77.653	0.108
	[0.138]					[0.086]			
		-0.199^*	71.824	0.121			-0.125^*	79.339	0.103
		[0.138]					[0.085]		

Panel C: Bid-Ask spreads					*Panel D: Asymmetric Proportional Transaction Costs*				
M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)	M_0^{usd}	$\widetilde{M}_0$	$\widetilde{M}_0^{usd}$	R^2(%)	RMSE(%)
-0.144^*			77.616	0.108	-0.113^*			79.461	0.103
[0.096]					[0.076]				
	-0.144^*		76.844	0.110		-0.111^*		79.335	0.103
	[0.096]					[0.074]			
		-0.143^*	78.943	0.104			-0.110^*	80.763	0.100
		[0.095]					[0.073]		

Table 6. **Global SDF and Local CB: Regressions**

This table reports estimates of regressions from changes in Global SDF and USD Local CB in symmetric markets with transaction costs on changes in world equity, Verdelhan (2018)'s Carry and Dollar, FX volatility, VIX, Miranda-Agrippino and Rey (2020) global cycle, gross capital flows, and He, Kelly, and Manela (2017) intermediary capital. All variables are standardized, meaning we de-mean and divide each variable by its standard deviation. Newey and West (1987) adjusted t-statistics are reported in brackets. Labels (***), (**) and (*) denote significance at the 1%, 5%, and 10% level, respectively. Data is quarterly and runs from January 1988 to December 2015.

Panel A: Global SDF								
world equity	-0.378^{***}							
	[-3.114]							
carry		0.353^{**}						
		[2.096]						
dollar			-0.100					
			[-0.725]					
FX vol				0.353^{***}				
				[2.715]				
VIX					0.388^{***}			
					[3.223]			
global cycle						-0.270^{***}		
						[-3.405]		
capital flow							0.079	
							[1.196]	
intermediary capital								-0.311^{***}
								[-2.682]
R^2 (%)	14.273	12.438	1.010	12.483	15.093	7.312	0.629	9.680
Panel B: Local CB								
world equity	-0.305^{***}							
	[-3.087]							
carry		0.145						
		[1.233]						
dollar			0.940^{***}					
			[20.103]					
FX vol				0.035				
				[0.278]				
VIX					0.105			
					[0.914]			
global cycle						-0.050		
						[-0.434]		
capital flow							-0.191^{**}	
							[-2.040]	
intermediary capital								-0.098
								[-0.901]
R^2 (%)	9.284	2.100	88.421	0.125	1.108	0.253	3.634	0.968

Appendix A Proofs

Throughout the proofs, we assume that each component of the vector of returns $\mathbf{R}$ belongs to the space $L^q(\Omega, \mathcal{F}, \mathbb{P})$ of random variables on $(\Omega, \mathcal{F}, \mathbb{P})$ with finite q-th moment, for some $1 < q < \infty$, and denote the dual space of L^q by L^p, where $1/p + 1/q = 1$.

Proof of Proposition 1

Sublinearity of transaction cost function h implies sublinearity of pricing functional π and a set Ξ of traded payoffs that is a convex cone. Therefore, by Chen (2001, Theorem 5) and Clark (1993, Theorem 6), the no-arbitrage condition is equivalent to the no-free-lunch condition (Harrison and Kreps (1979)). On the other hand, by Chen (2001, Theorem 1), the no-free-lunch condition is equivalent to the existence of a strictly positive, continuous linear functional ψ defined on L^q, such that $\psi|_\Xi \leq \pi$.[25] Therefore, by the Riesz Representation Theorem, this is equivalent to the existence of a strictly positive element M in dual space L^p, such that $\psi(x) = \mathbb{E}[Mx]$. Hence, $\mathbb{E}[Mx] \leq \pi(x)$ for all $x \in \Xi$. By the definition of pricing functional π, these inequalities holds if and only if

$$\boldsymbol{\theta}'\mathbb{E}[M\mathbf{R}] \leq \boldsymbol{\theta}'\mathbf{1} + h(\boldsymbol{\theta}_F)$$

for all $\boldsymbol{\theta} \in \mathbb{R}^n$, where $\boldsymbol{\theta}_F \in \mathbb{R}^f$ denotes the sub-vector of portfolio weights of frictional assets, i.e.:

$$\boldsymbol{\theta}_F'\mathbb{E}[M\mathbf{R}_F] + \boldsymbol{\theta}_S'\mathbb{E}[M\mathbf{R}_S] \leq \boldsymbol{\theta}_F'\mathbf{1} + \boldsymbol{\theta}_S'\mathbf{1} + h(\boldsymbol{\theta}_F).$$

Since this inequality holds for all $\boldsymbol{\theta} \in \mathbb{R}^n$, it can be stated as the intersection of the two following inequalities:

$$\begin{aligned} \boldsymbol{\theta}_S'\mathbb{E}[M\mathbf{R}_S] &\leq \boldsymbol{\theta}_S'\mathbf{1} && \text{for all } \boldsymbol{\theta}_S \in \mathbb{R}^s \\ \boldsymbol{\theta}_F'\mathbb{E}[M\mathbf{R}_F] &\leq \boldsymbol{\theta}_F'\mathbf{1} + h(\boldsymbol{\theta}_F) && \text{for all } \boldsymbol{\theta}_F \in \mathbb{R}^f. \end{aligned}$$

The first of these two inequalities, given that it holds for every $\boldsymbol{\theta}_S \in \mathbb{R}^s$, is equivalent to constraint $\mathbb{E}[M\mathbf{R}_S] = \mathbf{1}$, while the second inequality, by Bauschke and Combettes (2011, Prop. 13.10 (i)) is equivalent to the constraint $h^*(\mathbb{E}[M\mathbf{R}_F] - \mathbf{1}) \leq 0$, where h^* is the convex conjugate of transaction cost function h. Since h is closed and sublinear, by Hiriart-Urruty and Lemaréchal (2012, Theorem 3.1.1), it is a support function of set $\mathcal{C}$ in equation (2). Recalling that the convex conjugate of a support function of a set $\mathcal{C}$ is the set's indicator function (i.e., $h^*(\eta) = 0$ if $\eta \in C$ and $= \infty$ otherwise), the second inequality above is equivalent to the condition $\mathbb{E}[M\mathbf{R}_F] - \mathbf{1} \in \mathcal{C}$. This concludes the proof. □

Proof of Proposition 2

Recall from Definition 1 that the minimum-entropy SDF is the solution to the following optimization problem:

$$\begin{aligned} M_0 = \arg\min_{M>0} \quad & \mathbb{E}[-\log M] \\ \text{s.t.} \quad & \mathbb{E}[M\mathbf{R}_S] - \mathbf{1} = 0 \\ & \delta_{\mathcal{C}}(\mathbb{E}[M\mathbf{R}_F] - \mathbf{1}) \leq 0, \end{aligned} \tag{A.1}$$

[25] $\psi|_\Xi$ denotes the restriction of functional ψ to Ξ.

where set $\mathcal{C}$ is given by equation (2) and

$$\delta_{\mathcal{C}}(x) = \begin{cases} 0 & \text{if } x \in \mathcal{C} \\ +\infty & \text{if } x \notin \mathcal{C} \end{cases}$$

is the indicator function. Therefore, by Proposition 2 of Korsaye, Quaini, and Trojani (2020),

$$M_0 = 1/(\boldsymbol{\theta}_0'\mathbf{R}),$$

where we are using the fact that the convex conjugate of $\phi(x) = -\log(x)$ restricted to the positive reals is given by $\phi_+^*(x) = -\log(-x)$ and $\boldsymbol{\theta}_0$ is the solution to the dual optimal portfolio problem:

$$\boldsymbol{\theta}_0 = \arg\min_{\boldsymbol{\theta}\in\mathbb{R}^n} \left\{ -\mathbb{E}[\log(\boldsymbol{\theta}'\mathbf{R})] + \boldsymbol{\theta}'\mathbf{1} + \sup_{\boldsymbol{\eta}}\{\boldsymbol{\eta}'\boldsymbol{\theta}_F : \delta_{\mathcal{C}}(\boldsymbol{\eta}) \leq 0\} \right\}. \tag{A.2}$$

The proof is therefore complete once we show that the optimal portfolio problem in (A.2) is identical to the problem in (6). To this end, note that $\sup_{\boldsymbol{\eta}}\{\boldsymbol{\eta}'\boldsymbol{\theta}_F : \delta_{\mathcal{C}}(\boldsymbol{\eta}) \leq 0\} = \sup_{\boldsymbol{\eta}\in\mathcal{C}}(\boldsymbol{\eta}'\boldsymbol{\theta}_F)$, which is the support function of set $\mathcal{C}$. Therefore, by Corollary 3.1.2 of Hiriart-Urruty and Lemaréchal (2012), $\sup_{\boldsymbol{\eta}}\{\boldsymbol{\eta}'\boldsymbol{\theta}_F : \delta_{\mathcal{C}}(\boldsymbol{\eta}) \leq 0\} = h(\boldsymbol{\theta}_F)$. Consequently, the problems in (6) and (A.2) are identical. □

Proof of Proposition 3

Let $M_0^{(i)}$ denote the minimum-entropy SDF in country i. From Definition 1,

$$\begin{aligned} M_0^{(i)} = \arg\min_{M\in L_+^p} \quad & \mathbb{E}[-\log(M)] \\ \text{s.t.} \quad & \mathbb{E}[M\mathbf{R}_S^{(i)}] - \mathbf{1} = 0\,,\ \mathbb{E}[M\mathbf{R}_F^{(i)}] - \mathbf{1} \in \mathcal{C}, \end{aligned}$$

where $\mathcal{C} = \{\mathbf{y} \in \mathbb{R}^f : \mathbf{y}'\boldsymbol{\theta}_F \leq h(\boldsymbol{\theta}_F) \text{ for all } \boldsymbol{\theta}_F \in \mathbb{R}^f\}$. Note that under market symmetry set $\mathcal{C}$ is independent of country index i. Equation (7) then gives for any pair of indices $i \neq j$:

$$\begin{aligned} M_0^{(i)} = \arg\min_{M\in L_+^p} \quad & \mathbb{E}[-\log(M)] \\ \text{s.t.} \quad & \mathbb{E}[MX^{(ij)}\mathbf{R}_S^{(j)}] - \mathbf{1} = 0\,,\ \mathbb{E}[MX^{(ij)}\mathbf{R}_F^{(j)}] - \mathbf{1} \in \mathcal{C}. \end{aligned}$$

The solution of this optimization problem is unaffected if we replace the objective function with $\mathbb{E}[-\log(M)] - \mathbb{E}[\log X^{(ij)}]$:

$$\begin{aligned} M_0^{(i)} = \arg\min_{M\in L_+^p} \quad & \mathbb{E}[-\log(MX^{(ij)})] \\ \text{s.t.} \quad & \mathbb{E}[MX^{(ij)}\mathbf{R}_S^{(j)}] - \mathbf{1} = 0\,,\ \mathbb{E}[MX^{(ij)}\mathbf{R}_F^{(j)}] - \mathbf{1} \in \mathcal{C}. \end{aligned}$$

However, since $X^{(ij)} > 0$, it also follows for any pair of indices $i \neq j$:

$$\begin{aligned} M_0^{(i)}X^{(ij)} = \arg\min_{M\in L_+^p} \quad & \mathbb{E}[-\log(M)] \\ \text{s.t.} \quad & \mathbb{E}[M\mathbf{R}_S^{(j)}] - \mathbf{1} = 0\,,\ \mathbb{E}[M\mathbf{R}_F^{(j)}] - \mathbf{1} \in \mathcal{C}\,. \end{aligned}$$

Noting that the right-hand side of this last identity defines the minimum-entropy SDF for country j, we finally obtain $M_0^{(i)}X^{(ij)} = M_0^{(j)}$. This concludes the proof. □

Proof of Corollary 1

Let $\boldsymbol{\theta}_0^{(i)}$ denote the solution to country i's penalized optimal portfolio problem in equation (6), i.e.,

$$\begin{aligned}\boldsymbol{\theta}_0^{(i)} = \arg\min_{\boldsymbol{\theta}\in\mathbb{R}^n} \quad & \mathbb{E}[-\log(\boldsymbol{\theta}'\mathbf{R}^{(i)})] + \boldsymbol{\theta}'\mathbf{1} + h^{(i)}(\boldsymbol{\theta}_{F^{(i)}}) \\ \text{s.t.} \quad & \boldsymbol{\theta}'\mathbf{R}^{(i)} > 0\,.\end{aligned}$$

By assumption, financial markets are symmetric, i.e. $h^{(i)} = h$ and $F^{(i)} = F$. Therefore, for any country index $j \neq i$ market symmetry and equation (7) yield:

$$\begin{aligned}\boldsymbol{\theta}_0^{(i)} = \arg\min_{\boldsymbol{\theta}\in\mathbb{R}^n} \quad & \mathbb{E}[-\log(\boldsymbol{\theta}'\mathbf{R}^{(j)})] - \mathbb{E}[\log X^{(ij)}] + \boldsymbol{\theta}'\mathbf{1} + h(\boldsymbol{\theta}_F) \\ \text{s.t.} \quad & X^{(ij)}\boldsymbol{\theta}'\mathbf{R}^{(j)} > 0\,.\end{aligned}$$

Since $X^{(ij)} > 0$, the left hand side of the above identity also reads

$$\begin{aligned}\boldsymbol{\theta}_0^{(i)} = \arg\min_{\boldsymbol{\theta}\in\mathbb{R}^n} \quad & \mathbb{E}[-\log(\boldsymbol{\theta}'\mathbf{R}^{(j)})] + \boldsymbol{\theta}'\mathbf{1} + h(\boldsymbol{\theta}_F) \\ \text{s.t.} \quad & \boldsymbol{\theta}'\mathbf{R}^{(j)} > 0\,.\end{aligned}$$

The right-hand side of this last identity defines $\boldsymbol{\theta}_0^{(j)}$. Therefore, $\boldsymbol{\theta}_0^{(i)} = \boldsymbol{\theta}_0^{(j)}$ for all pairs of countries with indices $i \neq j$. This concludes the proof. □

Proof of Theorem 1

From Proposition 2, the minimum-entropy SDF of country j satisfies $\log M_0^{(j)} = -\log \boldsymbol{\theta}_0'\mathbf{R}^{(j)}$, where $\boldsymbol{\theta}_0$ is the solution to j's penalized optimal portfolio problem (6). Moreover, by Corollary 1 this solution is independent of the country index j. On the other hand, Proposition 3 guarantees that $\log M_0^{(i)} = \log M_0^{(j)} - \log X^{(ij)}$ for all country pairs $i \neq j$. Together, we obtain:

$$\log M_0^{(i)} = -\log \boldsymbol{\theta}_0'\mathbf{R}^{(j)} - \log X^{(ij)}\,.$$

Multiplying both sides of this equation by w_j, summing over all $j \neq i$ and using the fact that $\sum_{j=1}^m w_j = 1$, we obtain:

$$\log M_0^{(i)} = -\sum_{j=1}^{m} w_j \log \boldsymbol{\theta}_0'\mathbf{R}^{(j)} - \sum_{j\neq i} w_j \log X^{(ij)},$$

thus establishing equation (10). This concludes the proof. □

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