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Haelim Anderson
Selman Erol
Guillermo Ordoñez

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ABSTRACT

Central banks provide public liquidity to traditional (regulated) banks with the intention of stabilizing the financial system. Shadow banks are not regulated, yet they indirectly access such liquidity through the interbank system. We build a model that shows how public liquidity provision may change the linkages between traditional and shadow banks, increasing systemic risk through three channels: reducing aggregate liquidity, expanding fragile short-term borrowing, and crowding out of private cross-bank insurance. We show that the creation of the Federal Reserve System and the provision of public liquidity changed the structure and nature of the U.S. interbank network in ways that are consistent with the model and its implications. We provide empirical evidence by constructing unique data on balance sheets and detailed disaggregated information on payments and funding connections in Virginia.

Haelim Anderson
Center for Financial Research
Federal Deposit Insurance Corporation
550 17th St NW
Washington, DC 20429
handerson@fdic.gov

Selman Erol
Carnegie Mellon University
Tepper School of Business
5000 Forbes Ave
Pittsburgh, PA 15213
erol@cmu.edu

Guillermo Ordoñez
University of Pennsylvania
Department of Economics
PCPSE - Room 505
133 South 36th Street
Philadelphia, PA 19104
and NBER
ordonez@econ.upenn.edu

1 Introduction

The recent large infusions of public liquidity by the Federal Reserve System (and several other central banks around the world) during the financial crises of 2007-2009 and the 2020 COVID-19 pandemic revitalized the discussion about the role that such infusions play in financial markets. If public liquidity needs were exogenous and its provision costless, its social desirability would be clear. Neither of those conditions, however, is likely to be met. This is the reason central banks grant access to public liquidity only to traditional banks that abide by several regulations. Yet the provision of public liquidity affects not only the behavior of individual banks but also the manner in which banks - regulated (traditional banks) and nonregulated (shadow banks) - interact with each other. The critical question, therefore, is: how does public liquidity affect financial fragility when some banks can avoid regulations and still obtain such liquidity indirectly from regulated banks?

This question is quite relevant, but also challenging, in modern times. In itself, though, the question is not new. It was also prominent during the early years of the Federal Reserve System, created by the 1913 Federal Reserve Act to offer liquidity to member banks through a discount window, with the precondition that members would accept stricter regulation. The Act made membership compulsory for national banks but voluntary for state banks, under the presumption that most state banks would choose to join. Most of them did not. Indeed, and perhaps inadvertently, the Federal Reserve Act may have created what we call a *shadow banking system*: a system of banks, commercial or other, that are not under federal regulation and do not have direct access to liquidity facilities or bailout promises. In the 1920s, this shadow banking system consisted of nonmember banks that operated under more-relaxed state regulations (relative to the ones imposed on member banks), yet had indirect access to public liquidity through correspondent banks that were members of the Federal Reserve System (CQ Researcher (1923)).[1]

In this paper, we first build a model to understand how introducing public liquidity affects the extent and intensity of banks' interactions, in particular between regulated members and unregulated nonmembers. We start by considering a financial system without public liquidity, which is motivated by the banking structure during the National Banking Era, the period preceding the introduction of the Federal Reserve System. We consider a peripheral country bank with access to liquidity and investment opportunities but also facing the possibility of sudden withdrawals. In response to the possibility of those shocks, the bank may choose to diversify its portfolio by placing deposits in a reserve-city bank - a core bank - that also has

[1]Banks placing deposits in other banks were called respondents and banks receiving deposits were called correspondents. Correspondent banks were generally located in financial centers.

investment opportunities and that serves as a hub to insure across regions (receiving deposits from several country banks, as New York banks did during the National Banking Era).

In the model we then introduce access to public liquidity by the core bank (now a member bank), motivated by the Federal Reserve Act. We show that as a response, the periphery bank (now a shadow nonmember bank) relies less on cash and deposits to insure against liquidity shocks and more on borrowing short-term funds from the member bank. In this way, the endogenous reaction of shadow banks to the public liquidity to which they lack direct access is not only to reduce cash holdings but also to accumulate more risks in their own portfolios. In short, the role of financial - center banks was transformed, as they went from being a *provider of productive diversification* to being a *nonproductive pass - through conduit* for shadow banks accessing cheaper liquidity without facing costlier regulations.

Beyond the fragility implied by the accumulation of risk in a single unregulated bank, the increase in interbank short-term borrowing between member and nonmember banks makes the overall network more complex and thereby strengthens the possibility of contagion. The system's vulnerability to shocks increases (without public liquidity ex-post, there would be more inefficient project liquidations), even though fragility (actual liquidations) declines by the umbrella provided by public liquidity. Naturally if public liquidity is not for free (an aspect we do not discuss in this paper), the reduction in fragility comes at the cost of an increase in vulnerability and effective use of public liquidity.

Finally, our simple model also shows that the provision of public liquidity can change the structure of the interbank system by decentralizing it geographically and thereby crowding out private insurance. Although during the National Banking Era the concentration of reserves in New York City had been deemed a source of financial instability, it had also allowed banks to smooth local liquidity shocks: since New York City banks pooled the reserves of a large number of banks across different regions, the interbank network was able to diversify regional shocks that were not correlated (Gilbert (1983)). With the introduction of public liquidity, however, country banks were induced to rely more on their local correspondents at lower costs (the costs were lower because distances were shorter, information was better, relations were stronger, and so forth), but the emergence of decentralized interbank relationships made the overall banking system more vulnerable to regional liquidity shocks. In short, the role of financial-center banks was transformed, as they went from being a *provider of private liquidity insurance* to being a *conduit for public liquidity insurance*.

The advantage to grounding the model on a counterfactual without central banks is the possibility of testing its implications against historical data. The challenge, however, is overcoming a lack of detailed balance sheet information on financial networks and banking connections at about the time the Federal Reserve Act was passed. Existing bank balance sheet data for the period in question report only the total amounts of interbank balance sheet items, without disaggregating them by individual debtor or creditor correspondent bank. In addition, commercial bank directories, such as *Rand McNally* and *Polk*, provide information on self-reported correspondent linkages but not on the types of interbank transactions or the amounts associated with these transactions. In other words, these directories provide names of counterparties but not information on the strength and nature of the relationships.

To overcome these limitations, we construct various datasets. First, we obtain yearly bank balance sheets for state banks (most of the banks we document are nonmembers) and for national banks (by construction, all national banks are members), aggregated at the state level from 1910 to 1929. This information gives us an aggregated view of (1) how the Federal Reserve Act's liquidity provision changed the aggregate amount of private liquidity in the system, and (2) what the differences in various balance sheets items were between members and nonmembers. This differentiation between members and nonmembers is important because the public liquidity provision would almost automatically reduce private liquidity for member banks, but its effect on nonmembers is less obvious. Our model strongly suggests that liquidity is reduced for nonmember banks as well, given their ability to access public liquidity through member banks. We show that this was indeed the case, even in states where state regulations become more stringent.

Although aggregate balance sheet data provide useful information, they still do not show how the creation of the Federal Reserve System changed the overall interbank structure. To solve this problem, we collect state bank examination reports for Virginia state banks for the years 1911 and 1922 (that is, before and after passage of the Federal Reserve Act). The examination reports provide assets and liabilities in detail for banks as well as detailed information on their correspondents. On the asset side, we obtain the the amounts of deposits between respondent and correspondent banks. Similarly, on the liability side, we obtain the amounts of short-term loans between respondent and correspondent banks. With this detailed information we document the nature and intensity of payment and funding relationships and investigate how the creation of the Federal Reserve changed the structure and nature of interbank relationships.

Consistent with the aggregate balance sheet findings - and with the model's implications - we find that the creation of the Federal Reserve System reduced the aggregate liquidity (cash and deposits) held by shadow banks in Virginia and strengthened their short-term borrowing. The ability of nonmember banks to borrow indirectly from the Federal Reserve through member banks increased nonmember banks' reliance on short-term funding, which in turn decreased their need to hold liquid assets. Furthermore, we find that the interbank system became more dispersed after the Federal Reserve Act. On the one hand, the interbank deposit network became more local, with banks reducing their number of correspondents in New York and increasing the number in local financial centers, such as Richmond and Norfolk. On the other hand, the interbank borrowing network also became more local, with banks borrowing from members in nearby towns instead of from larger banks in financial cities. In other words, the creation of the Federal Reserve System produced a decentralized interbank network, consistent with the endogenous network response predicted by our model.

Our study has important implications for policy today. The common view is that providing public liquidity to traditional banks insulates them from potential risks arising from unregulated shadow banks. That view, however, relies heavily on the assumption that the two types of banks do not interact, or that they cannot change their relationships in response to changes in public liquidity provision. Here we argue that unregulated banks can access public liquidity indirectly by changing their interbank operations, modifying the role of traditional banking and potentially creating a new source of systemic risk. Although the system is more stable when faced with small, "business as normal" shocks, it also creates room for a larger shock endogenously generated.

Related Literature: Our paper contributes to a rich literature that studies the consequences of the Federal Reserve Act. Previous work has found that the creation of the Federal Reserve reduced financial volatility by smoothing seasonal liquidity pressures on the banking system (See Miron (1986), Mankiw et al. (1987), Bernstein et al. (2010), Carlson and Wheelock (2018b)). We show that, even though the creation of the Federal Reserve may have stabilized the functioning of the system in "normal times," in the background it was building a shadow system that relied too much on public funds and guarantees, held too little liquidity, and connected too much on fragile borrowing. Creation of the Federal Reserve may have also increased systemic risk, planting the seed for larger collapses.

Indeed, recent empirical studies (Mitchener and Richardson (2019) and Calomiris et al. (2019)) have documented the importance of contagion and systemic risk in accounting for the Great Depression. Other studies have complemented this view, showing that the inability of nonmember banks for an immediate and direct access to central bank liquidity magnified the severity of banking crises during the Great Depression, leading to the creation of new and more extensive lending facilities, such as the Reconstruction Finance Corporation (Wicker (2000) and Anbil and Vossmeyer (2017)). Our paper argues that those preconditions may have been unanticipated consequences of the creation of the Federal Reserve System.

More recent work, which includes Mitchener and Richardson (2019) and Carlson and Wheelock (2018b), has documented changes in the structure of interbank deposit networks that resulted from creation of the Federal Reserve System. Jaremski and Wheelock (2019), for instance, using information about correspondent linkages from the *Rand McNally Directory*, document how banks established connections to correspondents that joined the Federal Reserve in cities with Federal Reserve offices, reducing concentration in the overall network. Our empirical work is consistent with these findings, but motivated by the theoretical implications of our model. In addition to considering changes at the extensive margin (tracing which banks were part of a connection) as these papers do, we constructed a new dataset with information at the intensive margin (the dollar amounts of the deposits and loans that were involved in a connection). Furthermore, we extended this analysis to funding relations (short-term loans) in addition to payment relations (deposits). These additions are critical for understanding the effect of the Federal Reserve System on systemic risk.

Our paper also contributes to the recent literature on the rise of shadow banking- both by regulatory arbitrage (Ordonez (2018)) and by restricted public liquidity provision (Bengui et al. (2019)). In this paper, we also argue these two factors have been critical to the structure and growth of perhaps the first shadow banking system in the U.S. We show this by using the fact that some banks chose to operate outside the Federal Reserve System (be a nonmenmber bank) and exploited indirect access to public liquidity through interbank connections. The modern application of our insights contribute to the recent literature on the transmission of monetary policy on the shadow banking sector (Adrian and Shin (2009), Freixas et al. (2011), Chen et al. (2018) and Bianchi and Bigio (2020)). According to our results, the monetary policy that introduces liquidity in the system has nontrivial effects on the relationship between traditional banks and shadow banks, the composition of their portfolios, and the stability of the financial system.[2]

In the area of theory, we apply a network structure to understand how interlinkages (both intensive, focusing on the degree of payment and funding, and extensive, focusing on the existence and anatomy of links) react to government interventions. There are recent studies that endogenize the effects of public interventions to the functioning of banking networks. Erol and Ordoñez (2017), for example, study the reaction of an interbank network to banking regulations. They show that liquidity and capital requirements that are intended to provide stability may also make the system more prone to collapse by discouraging the functioning of a network structure that insures against financial shocks. In this paper we study how facilities that lend to certain banks may also unintentionally harm both network functionality and total stability, and we provide evidence of those forces.

[2] Gorton and Ordonez (2020) studies the role of monetary policy on systemic risk through the impact on collateral information in the system, not through the impact on interbank relations.

In terms of financial network theory, the closest work to ours is Erol (2018), who argues that uncapped ex-post liquidity provision induces a more centralized network by mitigating the insolvency contagion through core banks. Instead we show that liquidity provision induces a less centralized network by reducing the value of liquidity coinsurance by core banks. In this sense, we also contribute to the literature of interbank networks and their effects on systemic risk, as in Allen et al. (2012), Acemoglu et al. (2015) and Chang and Zhang (2019). Empirically, Anderson et al. (2019) show how the concentration of interbank deposits affected systemic risk during the National Banking Era. This paper bridges the theoretical work with the empirical endeavor and focuses on how the provision of public liquidity changes interbank relations and systemic risk.

The remainder of the paper is organized as follows: Section 2 provides historical background of the interbank system functioning before and after passage of the Federal Reserve Act. Section 3 presents a model that not only introduces an external agent (central bank) that provides liquidity in a banking setting but also how the provision affects the holding of liquidity of a bank and the payment and funding relations across banks. Section 4 presents empirical evidence of (1) a reduction in aggregate liquidity (for both Federal Reserve members and nonmembers), (2) an increase in short-term borrowing and the possibilities of contagion, and (3) changes in the geographical properties of the core-periphery network. We conclude with some final remarks.

2 Historical Background

During the National Banking Era (1864-1912), the U.S. banking system exhibited seasonal spikes in loan interest rates and frequent episodes of banking panics. Short-term interest rates displayed strong seasonal fluctuations due to large increases in the supply of deposits during agricultural harvest seasons and the demand for credit during agricultural planting seasons. As a result, banks faced liquidity pressures in spring and fall, and panics occurred at times of the year in which these pressures peaked.

The interbank system of the period, through the network of correspondent deposits and short-term funding, played an important role in relaxing liquidity pressures. The interbank deposit network was characterized by a three-tier pyramid structure.[3] Country banks held

[3] The interbank system developed to overcome branching restrictions and facilitate interregional payments of goods and services. The National Banking Act institutionalized the interbank system by setting up a location-based three-tier system of national banks: central reserve-city banks (those located in New York City, Chicago, or St. Louis), reserve-city banks (banks in selected other large cities), and country banks (banks in all other locations). Central reserve-city banks were required to hold cash reserves equal to 25% of

deposits in reserve-city banks, which in turn kept deposits in New York City banks. The concentration of interbank deposits in New York City banks allowed these banks to reallocate liquidity across regions. When country banks in agricultural regions faced seasonal demands, they withdrew their interbank deposits from financial centers, with those funds coming from other banks in areas where seasonal demands were less pressing. The geographical regional differences in demand produced somewhat offsetting flows of interbank deposits in New York City banks, which effectively provided private insurance across regions (see, for instance, Kemmerer (1910)). The interbank system helped banks meet seasonal liquidity pressures not only by allowing banks to cross-share deposits but also by allowing them to borrow short-term funds from correspondents. Country banks borrowed the most, reserve-city banks borrowed rarely, and central reserve-city banks borrowed hardly at all.

But although the interbank system helped soften the seasonal demands on banks, it did not create additional liquidity. As a result, the cash demands of country banks drained cash balances from New York City banks and led to seasonal spikes in interest rates. Contemporaries thought these seasonal swings contributed to bank panics and instability, and this belief prompted calls for reform to create an *elastic currency* that would make the reallocation of funds across regions less dependent on interbank relationships (Sprague (1910)).

In response to this financial landscape, the Federal Reserve System was created in 1913 (under the Federal Reserve Act) with three primary objectives: to eliminate the concentration of bank reserves in New York City banks by establishing 12 regional reserve banks; to create an elastic currency and thereby reduce seasonal volatility; and to prevent panics (Calomiris (1994)). To achieve these goals, the Federal Reserve offered a discount window to member banks through the 12 regional Federal Reserve Banks, but required members to meet new reserve requirements by placing deposits in those Federal Reserve Banks instead of reserve-city and central reserve-city banks.[4]

The Federal Reserve Act retained for member banks the three-tier classification of central reserve-city banks, reserve-city banks, and country banks, but changed their reserve requirements. Member banks were required to hold 13%, 10% and 7%, respectively, of *demand*

their deposits. Reserve-city banks were also required to hold reserves equal to 25% of their deposits, of which one-half could be deposits with a correspondent bank in a central reserve city. Country banks were required to hold reserves equal to 15% of their deposits, but they could keep three-fifths of the 15% as deposits with a correspondent bank in reserve and/or central reserve cities. State bank regulators subsequently passed similar laws.

[4]Even though only member banks were given access to Federal Reserve services, including the discount window, the Act made it possible for the central bank to extend the discount window to nonmember banks in special circumstances with the approval of the Federal Reserve Board of Governors. Before 1923, for instance, the Board allowed member banks to discount eligible paper acquired from nonmember banks (See Hackley (1973), p. 119). But in general, the Board limited the extension of credit to nonmember banks in exceptional circumstances (See Carlson and Wheelock (2013)).

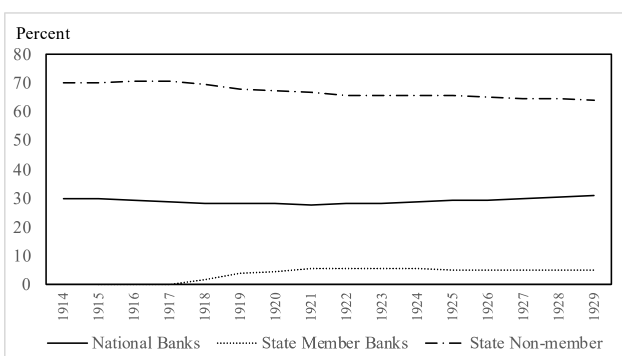
deposits and 3% of *time deposits* within the Federal Reserve Banks. The Federal Reserve did not pay interest on any of these deposits.[5]

Although the Federal Reserve Act succeeded in reducing the volume of interbank deposits, it failed to eliminate interbank network linkages. Deposits with the Federal Reserve Banks did not pay any interest, whereas deposits at city correspondents paid 2% interest. Hence, member banks continued to hold some deposits with correspondents, both to earn interest and to diversify their asset portfolios (CQ Researcher (1923), Carlson and Wheelock (2018b)).

Becoming a member, however, was partly voluntary. The Act made it compulsory for national banks to join, but for state banks, joining was voluntary. The creators of the Federal Reserve System hoped to bring state banks under a more unified system of regulation and supervision, but only a small fraction of state banks became members: by June 1915, only 17 state banks had chosen to join. This reluctance had two causes. First, even though the Act prohibited member banks from using interbank deposits to meet reserve requirements, state regulators allowed state banks to do so (CQ Researcher (1923)). Second, banks that did not become members could continue earning interest on *all* of their interbank deposits. Indeed, more banks joined after a 1917 revision that lowered capital and reserve requirements.

Figure 1 shows the proportions of national, state member, and state nonmember banks from 1914 to 1929. With only 5% of state banks choosing to become a member, more than 60% of all banks remained outside the Federal Reserve System. Membership grew slowly, eventually reaching a peak of 1,648 state banks (compared with 19,141 nonmember banks) in 1922 (Committee Branch Group (1935)).[6]

Figure 1: Percentages of Banks with respect to Federal Reserve Membership Status, 1914-1929



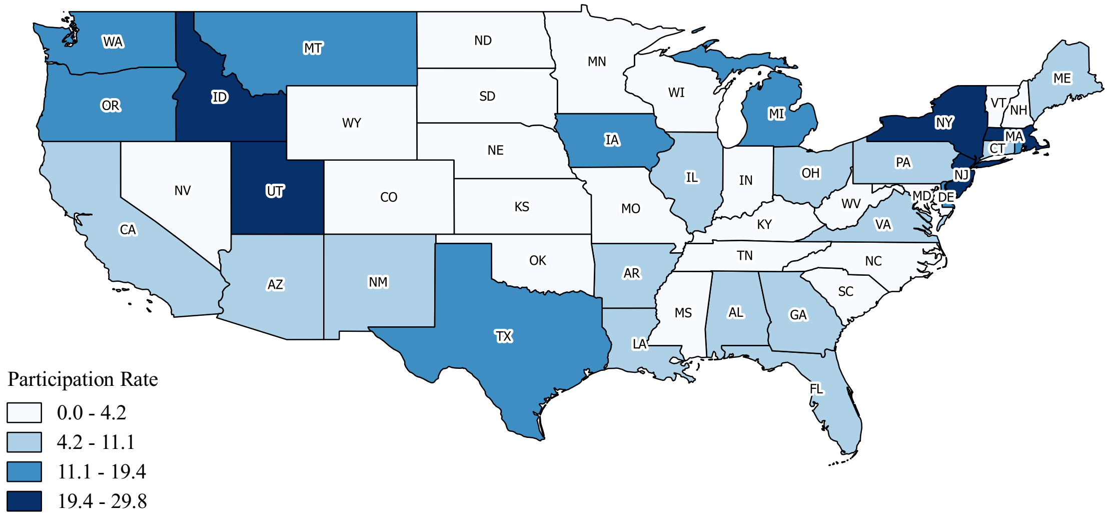
Source: *Committee on Branch, Group, and Chain Banking, 1932.*

[5]These reserve requirements were first introduced in 1913, took effect in 1914 and were amended in 1917.

[6]In terms of relative size, member banks tended to be larger than nonmembers but nonmembers still held a sizable fraction of total deposits. In 1923, for instance, nonmember banks held more than a third of total U.S. commercial bank deposits ($10.6 billion of a total of $37.7 in the whole system).

Figure 2 presents the rate of state bank membership by state in 1920, showing some heterogeneity across states, but still an overall low participation at the Federal Reserve Bank.

Figure 2: Federal Reserve System Participation Rate of State Banks - 1920



Source: Annual Report of the Board of Governors of the Federal Reserve System.

The unforeseen large fraction of state banks remaining outside the Federal Reserve System had major implications for the nature of the interbank system. In what follows we show that this led to a large number of banks accessing the System's discount window indirectly-through their correspondents. Before the Federal Reserve System was established, country banks borrowed short-term funds from their correspondents. After the System was established, country banks started relying more intensively on their correspondents in financial centers to borrow for short periods, with the understanding that when city correspondents ran out of funds, they would go to the Federal Reserve Bank and rediscount their own eligible paper to replenish their liquidity positions.

3 Model

As we have just noted, although the Federal Reserve System was to provide liquidity to the banking system, many state banks chose not to join the System. As we have also just noted, even though nonmember banks were not allowed to access the Federal Reserve's discount window directly, they did it indirectly through their relations with member banks in financial centers. The goal of our model is to understand how nonmember banks' indirect access to the central bank's liquidity affects (1) the aggregate liquidity of the banking system, (2) the nature of interbank exposures, and (3) the structure of the interbank network. The implications of the model provide us the compass to isolate the relevant variables to analyze in the data and the lenses to interpret them.

Our model not only helps us to understand the role of the Federal Reserve Bank creation on reshaping the interactions between member and nonmember banks, but its extrapolation is also informative about the behavior of modern unregulated financial intermediaries that are not considered banks in the traditional sense (the so-called shadow banks, such as money market funds, investment banks, etc.) and their interaction with regulated banks. Our model highlights the importance of understanding banking networks as a requisite to understand the effects of shadow banking in financial markets.

We begin with an environment with just two banks, a member and a nonmember, and analyze how the introduction of public liquidity affects aggregate liquidity and interbank relations. We then add more banks to study the structure of the interbank network.

3.1 Environment

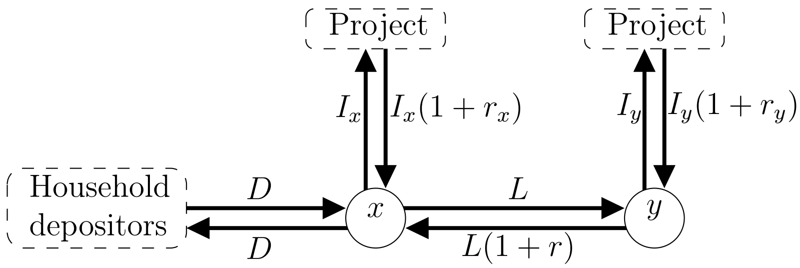
The economy is composed by two banks, x (nonmember bank) and y (member bank in a reserve city). Bank x accepts D household deposits and has access to a project that pays a net rate of return $r_x > 0$. Bank y does not have deposits and has a project that pays a net rate of return $r_y > 0$. Projects can be liquidated at any time to recover the original investment, but projects can only be liquidated in full (*no partial liquidation*).

Reserves and investments After investments, some depositors may need funds and withdraw from x before projects reach maturity –*liquidity shocks*. As projects can only be liquidated in full, x wants to maintain reserves to cover withdrawals, and may do so by holding cash or by depositing at bank y, earning net interest r, which we assume is low relative to the projects' returns.[7]

Denoting Φ_x the reserves that x keeps as cash, and L the amount that x deposits at y, bank x invests $I_x = D - \Phi_x - L$. Assuming bank y is subject to reserve requirements in the form of holding a fraction ϕ of liabilities in cash, and denoting Φ_y the reserves that y keeps in cash, $\Phi_y \geq \phi L$. This implies that y invests $I_y = L - \Phi_y$. We call I_x and I_y *investments*, Φ_x and Φ_y *cash reserves*, and L the *interbank deposits*. The transactions and obligations described thus far, absent liquidity shocks, are shown in Figure 3.

[7] During the National Banking Era, state regulators allowed state banks to keep reserves at reserve cities to meet reserve requirements, and reserve city banks paid 2% (and no more than 2%) interest on those deposits, which justify our assumption that r is exogenous (See James (1978)). We further discuss the rationale for these assumptions in Online Appendix C

Figure 3: Transactions absent Liquidity Shocks



Liquidity shocks Liquidity shocks caused by depositors withdrawing early can disrupt the previous flow of funds. We assume that full liquidation of projects always covers original investments, but projects can only be liquidated in full. This last assumption allows us to focus on liquidity crises and not solvency crises, as depositors can always recover D regardless of shocks, still maintaining the inefficiency of liquidations.

We denote early withdrawals by $\zeta \in [0, Z]$, where Z is the upper bound on possible withdrawals and ζ is drawn randomly from a distribution with CDF denoted by S. We call ζ the *liquidity shock*. Depending on the size of the liquidity shock and the size of investments, there are various scenarios that can materialize regarding liquidations. Next, we describe these scenarios for the case in which, facing a withdrawal that forces liquidation, bank x always withdraws its deposits from y before liquidating the own project. Formally, this happens when $I_x + \Phi_y > L$ and $I_x r_x > Lr$, which are conditions on endogenous variables that we prove later (in Lemma 1) that always occur on the path of play.[8]

1. If $\zeta \leq \Phi_x + \Phi_y$, the combined cash reserves from x and y are sufficient to meet the liquidity shock.

   (a) If $\zeta \leq \Phi_x$, withdrawals are met by x's cash in vault.

   (b) If $\Phi_x < \zeta \leq \Phi_x + \Phi_y$, x's cash reserves are not enough and x borrows $\zeta - \Phi_x$ short-term from y to cover the withdrawals.[9]

2. If $\zeta > \Phi_x + \Phi_y$, the combined cash reserves from x and y are not enough to cover the liquidity shocks, in which case x must either liquidate its own project or withdraw its deposits from y to an extent that y has to liquidate its project project. These are three possibilities:

[8]For expositional simplicity we focus on on-path scenarios, and we deal with off-path scenarios in the proof of Lemma 1.

[9]Such lending is risk-free so we assume that y does not charge an interest. Given this, whether x borrows $\zeta - \Phi_x$ or Φ_y is inconsequential. In what follows, we assume that x borrows the smallest amount that suffices for it to ride out the shock, which is robust to the existence of small borrowing costs.

(a) If $\Phi_x + \Phi_y < \zeta \leq \Phi_x + L$, the deposits of x at y are enough to cover the liquidity needs, together with x's cash. Then x withdraws L from y, who has to liquidate its project.[10]

(b) If $\Phi_x + L < \zeta \leq \Phi_x + \Phi_y + I_x$, x must liquidate its own project, as deposits at y are insufficient. In this case, x can keep its deposits at y. As the proceedings of liquidation may still be insufficient, x may need some of y's cash.

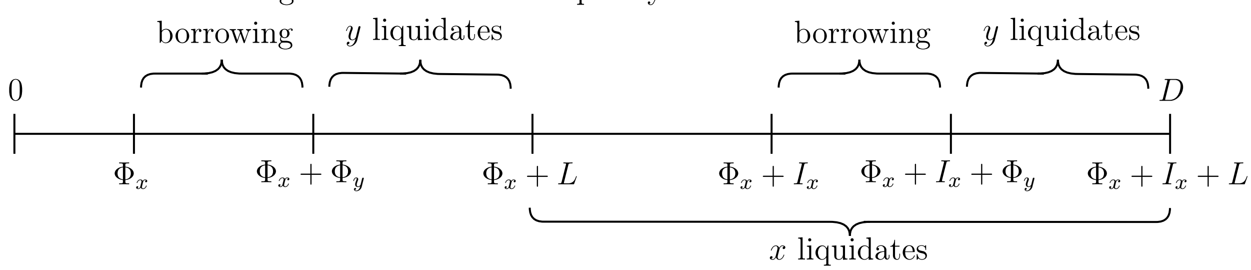
   i. If $\Phi_x + L < \zeta \leq \Phi_x + I_x$, x does not borrow short-term from y.

   ii. $\Phi_x + I_x < \zeta \leq \Phi_x + \Phi_y + I_x$, then x borrows $\zeta - \Phi_x - I_x$ short-term from y.

(c) If $\Phi_x + \Phi_y + I_x < \zeta$, neither I_x from the liquidation of the project, nor deposits L at y suffice by themselves, hence x liquidates its project and withdraws its deposits from y to cover the large withdrawal.

Figure 4 shows schematically all these scenarios.

Figure 4: Size of the Liquidity Shock and Transactions



These possible states determine ex-post short-term borrowing by bank x from bank y,

$$b = \begin{cases} \zeta - \Phi_x & \text{if } \Phi_x < \zeta \leq \Phi_x + \Phi_y \\ \zeta - \Phi_x - I_x & \text{if } \Phi_x + I_x < \zeta \leq \Phi_x + I_x + \Phi_y \\ 0 & \text{otherwise} \end{cases}$$

and bank x ex-post profits,

$$\pi_x = \begin{cases} I_x r_x + Lr & \text{if } \zeta \leq \Phi_x + \Phi_y \\ I_x r_x & \text{if } \Phi_x + \Phi_y < \zeta \leq \Phi_x + L \\ Lr & \text{if } \Phi_x + L < \zeta \leq \Phi_x + \Phi_y + I_x \\ 0 & \text{if } \Phi_x + \Phi_y + I_x < \zeta \end{cases}$$

[10]Since bank y is forced to liquidate the whole project upon withdrawal, we assume x withdraws the full amount L from y.

To obtain ex-ante short-term borrowing and ex-ante profits, we define

$$\Gamma \equiv S\left[\Phi_x + L\right]$$

the probability that x's project is not liquidated and

$$\Delta \equiv S[\Phi_x + \Phi_y] + \left(S\left[\Phi_x + \Phi_y + I_x\right] - S\left[\Phi_x + L\right]\right)$$

the probability that y's project is not liquidated. Bank x's expected profits are then

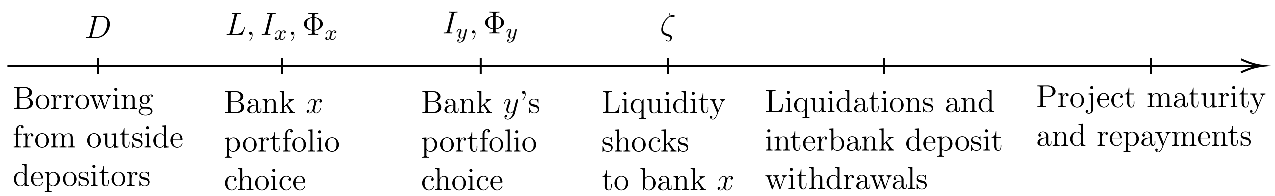
$$\Pi_x = \mathbb{E}[\pi_x] = \Gamma I_x r_x + \Delta L r. \tag{1}$$

Following similar arguments, bank y's expected profits are

$$\Pi_y = \Delta\left(I_y r_y - L r\right). \tag{2}$$

Timing Given the expected profits, bank x chooses investment I_x and deposits L, which determines its cash reserves Φ_x. Then y chooses investment I_y, which determines its cash reserves, which are subject to reserve requirements $\Phi_y \geq \phi L$.[11] Then, liquidity shocks materialize. This timeline is summarized in Figure 5.

Figure 5: Timeline of events



In this setting we can define upstream contagion as follows: Consider a realized shock ζ. If $\zeta \leq \Phi_x + \Phi_y$, there is no spillover from x to y. If $\Phi_x + L < \zeta \leq \Phi_x + I_x + \Phi_y$, x liquidates its own project. In these two cases, there is no contagion from x to y in terms of forcing y's project liquidation. If $\Phi_x + \Phi_y < \zeta \leq L + \Phi_x$, then x withdraws its deposits L from y. If $\Phi_x + \Phi_y + I_x < \zeta$, then both projects get liquidated. In both of these cases, y's project gets liquidated. We call this situation *upstream contagion from x to y*. The probability of upstream contagion is then $1 - \Delta$.

[11]Bank y always accept the deposit since its outside option is 0. We will assume that $(1-\phi)r_y > r$ and so y strictly prefers to accept the deposit.

3.2 Equilibrium Without Public Liquidity

To obtain clean implications from the model by closed-form solutions, we assume that with probability $\alpha \in [0,1]$, ζ is drawn from $U[0,Z]$ with $Z \geq D$. With probability $1-\alpha$ there is no liquidity withdrawal and $\zeta = 0$.[12] The expected profits of x and y from equations (1) and (2) can be rewritten as

$$\Pi_x = \underbrace{\left(1 - \alpha \frac{I_x}{Z}\right)}_{\Gamma} I_x r_x + \underbrace{\left(1 - \alpha \frac{2L - 2\Phi_y}{Z}\right)}_{\Delta} Lr$$

$$\Pi_y = \left(1 - \alpha \frac{2L - 2\Phi_y}{Z}\right) \left(L\left(r_y - r\right) - \Phi_y r_y\right).$$

While expected short-term borrowing is

$$B = \mathbb{E}[b] = \frac{\alpha \Phi_y^2}{Z}.$$

We solve for subgame perfect Nash equilibrium, in which bank x chooses I_x and L to maximize Π_x subject to $I_x, L \geq 0$ and $I_x + L \leq D$, given that bank y chooses $I_y \in [0, (1-\phi)L]$ to maximize Π_y.

In what follows we focus on a relatively low probability of early withdrawals, more specifically $\alpha \leq \overline{\alpha} = \frac{Z}{Z + \rho D}$, with $\rho = \max\left\{0, \frac{2}{2-\phi}\left(2(1-\phi) - \frac{r}{r_y}\right) - 1\right\}$. When α is not too large, equilibrium reserve requirements bind for y and $\Phi_y = \phi L$. Intuitively, banks are less prone to hold cash buffers in order to prevent the liquidation of projects.

Lemma 1. *For relatively low rate on deposits (this is, $2r < (1-\phi) r_x$ and $r < (1-\phi) r_y$, where $\phi < 0.5$), reserve requirements bind on the path of play: $\Phi_y = \phi L$. Moreover, $I_x + \Phi_y > L$ and $I_x r_x \geq Lr$.*

In order to save on otherwise cumbersome notation, we restrict attention to binding reserve requirements as described in Lemma 1. The next proposition shows how the allocation of funds changes in response to the probability of a liquidity shock α (taking into consideration the expectation of the withdrawal size in case it happens).

[12] That a bank faces more withdrawals than deposits implies additional legacy liabilities by an amount $Z - D \geq 0$. This extension avoids kinks in the solution once we introduce public liquidity, but $Z > D$ is irrelevant in this part of the paper, and one can simply assume $Z = D$ for now. Please see Online Appendix C for a detailed discussion of such $Z \geq D$.

Proposition 1. *Equilibrium Portfolios Without Public Liquidity Provision.*

If $\overline{\alpha} \geq \alpha \geq \alpha_2$*, the equilibrium quantities are*

$$L = \frac{D + Z_\alpha}{4(1-\phi)}, \qquad I_x = \frac{D + Z_\alpha}{2}, \qquad \Phi_x = D - I_x - L, \qquad \Phi_y = \phi L.$$

where $Z_\alpha \equiv \frac{Z(1-\alpha)}{\alpha}$ *and* $\alpha_2 \equiv \left(1 + \frac{D}{Z}\frac{1-2\phi}{3-2\phi}\right)^{-1}$.

If $\alpha_2 > \alpha > \alpha_1$*, the equilibrium quantities are instead*

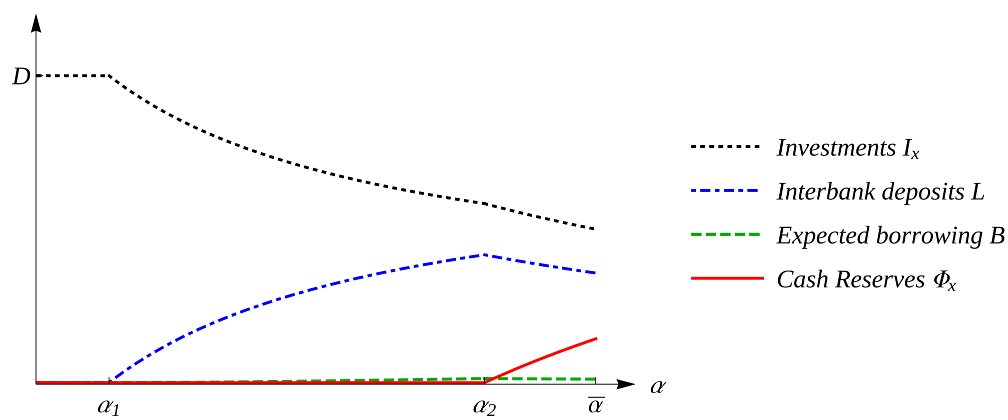
$$L = \frac{D(r_x + r) - Z_\alpha(r_x - r)}{2(r_x + 2(1-\phi)r)}, \qquad I_x = D - L, \qquad \Phi_x = 0, \qquad \Phi_y = \phi L.$$

where $\alpha_1 \equiv \left(1 + \frac{D}{Z}\frac{r_x + r}{r_x - r}\right)^{-1} < \alpha_2$.

Finally, if $\alpha_1 > \alpha$*, bank* x *does not deposit or hold cash, and* $I_x = D$*.*

Figure 6 illustrates the Proposition 1 as a function of the probability of a liquidity shock α. As α increases (that is, liquidity shocks become more likely), all instruments for dealing with these shocks increase (more cash reserve, more expected borrowing, and more interbank deposits). An increase in liquid assets is offset by a decline in illiquid investments.

Figure 6: Investments, Interbank Deposits, Short-Term Borrowing, and Cash Reserves



There is also a clear pecking order on holding liquid assets. When the risk of withdrawals is very low ($\alpha < \alpha_1$), the return of an additional unit of investment for bank x is larger than the risk of liquidating the whole project, and then x would rather invest fully in the project, without holding any cash or depositing in y. Once the risk of withdrawals increase enough ($\alpha_1 < \alpha < \alpha_2$), bank x reduces investment in the own project and places some deposits in

y. The reason is intuitive: by depositing in y bank x *diversifies its portfolio* such that, in case of withdrawals that are not too large there is no need to liquidate a single large project but instead a smaller one (either x's or y's). In other words, as banks cannot liquidate a fraction of projects, diversification works through investing in smaller ones. Finally, once the probability of withdrawals is large enough ($\alpha_2 < \alpha < \bar{\alpha}$), bank x also holds some cash and resorts more heavily on borrowing from y.

3.3 Equilibrium with Public Liquidity

In this section, we show a public liquidity provision reduces aggregate private liquidity in the banking system, including the private liquidity of banks that do not have direct access to public liquidity. In addition, a public liquidity provision can make the banking system more vulnerable to regional shocks because banks reduce their connectivity to core banks, and such connectivity provides a private tool to smooth out cross-regional liquidity shocks.

Suppose there is a *central bank* that provides short-term liquidity only to y (a member bank), for a maximum amount m, which we refer to as the public liquidity provision ($m = 0$ is the baseline case of no liquidity provision of the previous section). Although bank x is not a member of the Federal Reserve System, it can indirectly access the Federal Reserve's liquidity facilities though its interbank relationship with y. We are interested in how the ability of x to indirectly access the central bank's liquidity affects x's reserve holdings, and in turn affects contagion and systemic risk.

Regardless of the amount of public liquidity m, bank y does not want to keep reserves and $\Phi_y = \phi L$. For bank x, using idle reserves Φ_x or borrowing at most m from the central bank via y are substitutes. For bank x, therefore, any shock ζ below m can be met at no cost just by borrowing short-term from the member bank. In contrast, a shock above m will require banks to use their own reserves or to liquidate projects, as above.

Formally, from the viewpoint of bank x, future shocks become $\zeta' = \max\{0, \zeta - m\}$, with ζ' equal to 0 with probability $1 - \alpha + \alpha\frac{m}{Z}$, and drawn from $U[0, Z - m]$ with probability $\alpha\frac{Z-m}{Z}$. We focus on the values of $m < Z - D$ so that public liquidity does not eliminate liquidity risk in the financial sector when liquidity shocks are large.

We can rewrite the ex-post profit of bank x as

$$\Pi_{x,m} = \underbrace{\left(1 - \alpha\frac{I_x - m}{Z}\right)}_{\Gamma_m} I_x r_x + \underbrace{\left(1 - \alpha\frac{2L - 2\Phi_y - m}{Z}\right)}_{\Delta_m} Lr$$

and the following proposition extends Proposition 1 with public liquidity provision.

Proposition 2. *Equilibrium Portfolios With Public Liquidity Provision.*

If $\overline{\alpha} \geq \alpha \geq \widehat{\alpha}_2$*, the equilibrium quantities are*

$$L = \frac{D + Z_\alpha + m}{4\,(1-\phi)}, \qquad I_x = \frac{D + Z_\alpha + m}{2}, \qquad \Phi_x = D - I_x - L, \qquad \Phi_y = \phi L.$$

where $Z_\alpha \equiv \frac{Z(1-\alpha)}{\alpha}$ *and* $\widehat{\alpha}_2 \equiv \left(1 + \frac{D}{Z}\frac{1-2\phi}{3-2\phi} - \frac{m}{Z}\right)^{-1}$.

If $\widehat{\alpha}_2 > \alpha > \widehat{\alpha}_1$*, the equilibrium quantities are instead*

$$L = \frac{D\,(r_x + r) - (Z_\alpha + m)\,(r_x - r)}{2\,(r_x + 2(1-\phi)r)}, \qquad I_x = D - L, \qquad \Phi_x = 0, \qquad \Phi_y = \phi L.$$

where $\widehat{\alpha}_1 \equiv \left(1 + \frac{D}{Z}\frac{r_x+r}{r_x-r} - \frac{m}{Z}\right)^{-1} < \widehat{\alpha}_2$.

Finally, if $\widehat{\alpha}_1 > \alpha$*, bank* x *does not deposit or hold cash, and* $I_x = D$.

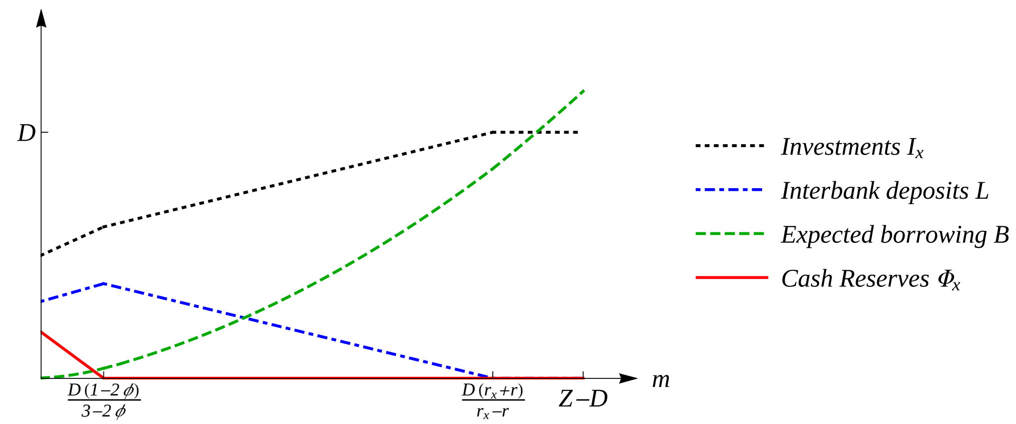
Figure 7 shows how bank x's choices change with public liquidity m for a level of α that justified x holding cash with $m = 0$ in Figure 6. For low levels of m (first parametric case in the previous proposition), I_x and L increase with m because both are treated as investments. This leads to a steep reduction in cash reserves. When m becomes large enough (second parametric case), x will not keep any cash reserves and will keep only interbank deposits. Then, as m goes up, x starts reducing interbank deposits L as it shifts its asset portfolio from low paying investment L to high paying investment I_x. All in all, the combined reserves of bank x, $\Phi_x + L$, decrease in m. Intuitively, indirect access to public liquidity reduces the need for holding reserves privately and diversifying its portfolio.

Next we describe how short-term borrowing reacts to m. The ex-post amount of x's short-term borrowing from y is

$$b = \begin{cases} \zeta - \Phi_x & \text{if } \Phi_x < \zeta \leq \Phi_x + \Phi_y + m \\ \zeta - L - \Phi_x & \text{if } \Phi_x + \max\{L, \Phi_y + m\} < \zeta \leq \Phi_x + L + m \\ \zeta - I_x - \Phi_x & \text{if } \Phi_x + \max\{I_x, L + m\} < \zeta \leq \Phi_x + I_x + \Phi_y + m \\ 0 & \text{otherwise} \end{cases}$$

These cases lead to the following proposition.

Figure 7: Investments, Interbank Deposits, Short-Term Borrowing, and Cash Reserves



Equilibrium allocation as a function of central bank liquidity m in line with Propositions 2 and 3.

Proposition 3. *Equilibrium Short-Term Borrowing*

Expected short term borrowing is

$$B = \frac{\alpha}{2Z}\left(2(m+\Phi_y)^2 + m^2 - \max\{0, m+\Phi_y - L\}^2 - \max\{0, m+L-I_x\}^2\right)$$

which is strictly increasing in m in equilibrium.

Figure 7 also illustrates this proposition, with short-term borrowing increasing in m, simply because bank x relies more on public liquidity provided through bank y. Bank x holds more illiquid assets and a less diversified portfolio to meet withdrawals.

This simple analysis highlights the effect of public liquidity provision on the investments and private reserves of shadow banks. Compared with the case of no public provision of liquidity ($m = 0$), shadow banks always invest more in illiquid assets and hold less in cash reserves, relying more on member banks, not to diversify projects but instead as a simple conduit to access public liquidity, disintermediating the system.

3.4 Systemic Risk: Fragility and Vulnerability

We have shown how banks adjust their portfolios in response to public liquidity m. Even though they reduce private liquidity as m increases, its potential negative effects is offset by higher public liquidity. Hence, they will not need to liquidate projects when they face liquidity shocks as often as they would in the absence of public liquidity. If public liquidity is costless, central banks may provide an unlimited amount of public liquidity, which would implement the first best in our setting, which is given by investing all in the most productive project without the need to liquidate it.

To allow for the possibility of liquidations on path, we assume that, although banks expect public provision of liquidity by the central banks, they do not know the exact amount of such public liquidity. If they overestimate the availability of public liquidity, they will hold too much in illiquid assets and may have to liquidate their investments. We model this uncertainty with stochastic m. Suppose that m is random between 0 and $Z - D$. Then regardless of the level of public liquidity, there is always a shock high enough to require the liquidation of both projects. Therefore, all of our earlier analyses go through simply by replacing m with $\mathbb{E}[m]$ in the equilibrium quantities.

There are different ways to categorize risks in the financial system. A first category involves the identity of projects that need liquidation. *Direct risk* refers to the probability that the project of x gets liquidated as a consequence of the (direct) liquidity shock to x. *Contagion risk* refers to the probability that the project of y gets liquidated as a consequence of x withdrawing its interbank deposits from y. *Systemic risk* refers to the probability that all projects get liquidated.

A second category is based on banks' demand for public liquidity and their use of it. On the one hand *Fragility* refers to the liquidation risk of portfolios, where these portfolios are chosen based on expected liquidity shocks and expected public liquidity. Fragility takes into account all sources of liquidity. A fragile economy, then, is an economy that is more likely to have less than expected public liquidity (for political or macroeconomic shocks that imply less than expected m) that forces project liquidation. On the other hand, *Vulnerability* refers to the liquidation risk of portfolios if there were no public liquidity available ex-post, where these portfolios are chosen based on expected liquidity shocks and expected public liquidity. Vulnerability takes into account only private liquidity. A vulnerable economy, for instance, would be one with very large projects and very few private reserves. A large provision of public liquidity would make an economy highly vulnerable but not fragile.

To obtain closed-form results that clarify comparisons, suppose that m is 0 with probability β and $U[0, \frac{2m^*}{1-\beta}]$ with probability $1 - \beta$, where $m^* < \frac{1-\beta}{2}(Z - D)$. This distribution implies that m has mean m^* and mass β at no public liquidity. The next Proposition characterizes the effect of expected public liquidity m^* on the different categories of risk defined above,

Proposition 4. *Systemic Risk*

Direct vulnerability is increasing in m^*. Systemic vulnerability and contagion vulnerability are increasing in m^* under $m^* < D\frac{1-2\phi}{3-2\phi} - Z_\alpha$ and decreasing in m^* under $m^* > D\frac{1-2\phi}{3-2\phi} - Z_\alpha$. All notions of fragility are decreasing in m^*.

When there is no expectation that central banks will provide liquidity support (this is $m^* = 0$), fragility and vulnerability are the same. In that situation, projects that are vulnerable *because* they may be liquidated without public liquidity support will indeed be liquidated when there is no expectation that central banks will provide liquidity support. The larger the expected injection of public liquidity, the lower is the fragility given a level of system vulnerability.

The effect of expected public liquidity on fragility then has two components. An *injection effect* – more expected public liquidity always reduces the need for liquidation – and an *equilibrium effect* – more expected public liquidity reduces private liquidity and increases the need for liquidation. Notice that the equilibrium effect in the evaluation of fragility is, in fact, what we referred to as vulnerability.

$$\text{Fragility} = \text{Vulnerability - Injection Effect}$$

Intuitively, this explains why all measures of fragility decline given a level of vulnerability in Proposition 4 (all projects are less likely to be liquidated when there are large amounts of public liquidity in the system). Vulnerability, however, measures the exposure of the system to the need for liquidation. Direct vulnerability is increasing in m^* because bank x reduces the buffer $L + \Phi_x$ that protects I_x when it expects large public liquidity support. As the project of bank x becomes more reliant on public liquidity, its direct vulnerability increases.

3.5 Networks

In this section, we extend our framework to study how the structure of the interbank network changes in response to the provision of public liquidity. We show that banks move their interbank relations towards counterparts that are less costly to maintain. If it is less costly to maintain relationships with correspondents close in geographic proximity, banks choose to connect less to central reserve cities and more to regional reserve cities. Hence, public insurance crowds out the private insurance that smooths out cross-regional liquidity shocks.

We extend our analysis to several banks. As a first step we focus on four banks in two pairs. More specifically, banks x_1 and y_1 are linked as described in the baseline, and the same is true for banks x_2 and y_2. We assume that banks x_1 and x_2 have household deposits and projects. In contrast, banks y_1 and y_2 have interbank deposits received from x_1 and x_2, and projects. We call $\{x_1, x_2\}$ *the periphery* and $\{y_1, y_2\}$ *the core.* As a next step we generalize the functioning of banks in the core. We introduce these generalizations in Section 3.5.1. We study how, in the absence of public liquidity, core banks coinsure each other through forming a sort of clearinghouse, as large New York banks historically did before the Federal Reserve Act. Finally, in Section 3.5.2, we allow periphery banks located in different regions to choose their correspondents from among two groups of banks: those that have greater coinsurance possibilities but may be farther away (say, banks in New York) and those that have fewer coinsurance possibilities but may be closer (say, banks located in regional reserve cities). This allows us to study the effect of the central bank's liquidity provision m on the network structure. We show that central bank liquidity induces a shift of links from the far core (New York City) to the close core (regional reserve cities), thereby crowding out the private insurance that the system is able to provide.

3.5.1 Liquidity coinsurance

We assume that each of the core banks y_1 and y_2 has access to central bank liquidity, capped at (deterministic) m in total. We also assume that the shocks faced by x_1 and x_2 are negatively correlated, so we rule out competition over central bank liquidity.

There is $\theta = \frac{\alpha}{2} \leq 0.5$ probability that the shock ζ_1 is drawn from $U[0, Z]$ and the shock $\zeta_2 = 0$. The parameter θ is also the symmetric probability that the shock ζ_2 is drawn from $U[0, Z]$ and the shock $\zeta_1 = 0$. There is, then a probability $1 - 2\theta = 1 - \alpha$ that there is no shock, and $\zeta_1 = \zeta_2 = 0$. This specification implies that only one bank needs liquidity at a time and that we do not need to model the priorities of the central bank over which bank to provide liquidity to, and how much. In other words, we abstract from aggregate liquidity shocks in the system such that the central bank has to rescue both pairs of banks.

We further allow core banks y_1 and y_2 to insure each other against liquidity shocks coming from bank x by reallocating liquidity between the two. When x_i faces a liquidity shock, it can borrow from y_i, which can borrow from y_j as well as from the central bank.

Without the liquidity coinsurance possibility, the ex-ante profit of x_i is

$$\Pi_{x_i} = \left(1 - \theta \frac{I_{x_i} - m}{Z}\right) I_{x_i} r_x + \left(1 - \theta \frac{2(1-\phi)L_i - m}{Z}\right) L_i r$$

whereas with the liquidity coinsurance, the ex-ante profit is given by

$$\Pi_{x_i} = \left(1 - \theta \frac{I_{x_i} - m - \phi L_j}{Z}\right) I_{x_i} r_x + \left(1 - \theta \frac{2(1-\phi)L_i - m - \phi L_j}{Z}\right) L_i r$$

Proposition 5. *Equilibrium Portfolios and Liquidity Co-Insurance*

Suppose that $m < D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$. *The equilibrium level of interbank deposits with and without* *liquidity co-insurance is given by*

$$L_i^{no\ ins} = \frac{D\left(r_x + r\right) - \left(Z_\theta + m\right)\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)} > L_i^{ins} = \frac{D\left(r_x + r\right) - \left(Z_\theta + m\right)\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right) + \phi(r_x - r)}$$

whereas $I_{x_i} = D - L_i$, $\Phi_{x_i} = 0$, *and* $\Phi_{y_i} = \phi L_i$ *for both cases.*

If $m > D\frac{r_x+r}{r_x-r} - Z\frac{1-\theta}{\theta}$, $I_{x_i} = D$ *for both cases.*

This proposition shows that in the presence of coinsurance possibilities, interbank lending allows banks to hedge against liquidity shocks and allows them to increase its own investments. Hence, banking network at the core acts as an additional source of private liquidity on top of a public liquidity provision.

3.5.2 Endogenous network

Here we extend the framework to show that the provision of public liquidity insurance crowds out the provision of private liquidity insurance. This happens when the periphery banks' choices of correspondents changes under the central bank liquidity provision and leads to the formation of a new network structure.

Let x_i represent a bank in region i which can place deposits in a local reserve-city bank y_i^C or a New York City bank y_i^N. Similarly, let x_j represent a bank in region j, which can place deposits in a local reserve-city bank y_j^C or a New York City bank y_j^N. For both banks x_i and x_j, placing deposits in New York City banks incurs a higher cost than placing deposits in regional reserve-city banks because of the geographical distance between respondents and correspondents. As discussed above, two New York City banks y_i^N and y_j^N insure each other against liquidity shocks by reallocating liquidity in the system. In the absence of the central bank x_i and x_j will choose y_i^N and y_j^N in order to reduce their exposure to local liquidity shocks. Since liquidity shocks are not perfectly correlated between regions i and j, x_i and x_j can smooth local liquidity shocks by adjusting their interbank deposits in New York City.

Now, we introduce central bank liquidity, m. Since x_i can mitigate local liquidity shocks by borrowing from a regional correspondent y_i^C directly, we study conditions under which it will choose to connect to y_i^C rather than y_i^N, which is more expensive. Similarly, x_j will choose to connect to y_j^C rather than y_j^N. There are two options for equilibria. Banks can either connect to New York City banks for private insurance but pay higher costs, or they can connect to regional reserve city banks.

From the analysis in Section 3.5.1, if both banks connect to their regional correspondents,

$$\Pi_{x_i}^C = \left(1 - \theta \frac{(D - L_C) - m}{D}\right)(D - L_C)\, r_x + \left(1 - \theta \frac{2\,(1-\phi)\, L_C - m}{D}\right) Lr$$

where L_C is given by $L^{no\ ins}$ in Proposition 5. If both banks connect to New York City banks,

$$\Pi_{x_i}^N = \left(1 - \theta \frac{D - L_N - \phi L_N - m}{D}\right)(D - L_N)\, r_x + \left(1 - \theta \frac{2\,(1-\phi)\, L_N - \phi L_N - m}{D}\right) L_N r - c$$

where L_N is given by L^{ins} in Proposition 5.

The next Lemma shows that the relative gain to connect with core banks decline with the volume of public liquidity offered by the Federal Reserve System.

Lemma 2. *If* $0 \leq m < D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$

$$\frac{d\left(\Pi_{x_i}^C - \Pi_{x_i}^N\right)}{dm} > \frac{r(r_x - r)\phi}{2\left(r_x + 2(1-\phi)r\right) + \phi(r_x - r)} \frac{\theta D}{Z} > 0.$$

This characterization leads to the following proposition.

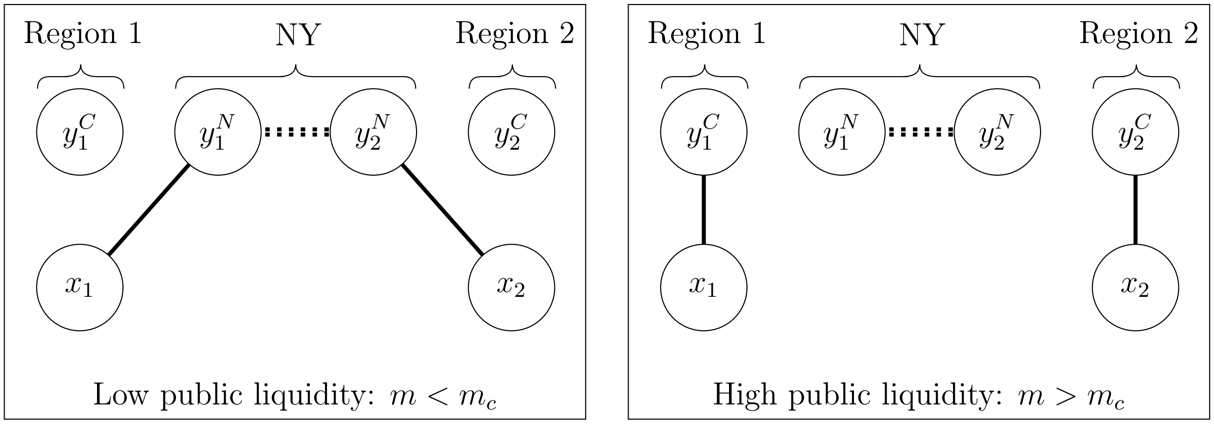
Proposition 6. *Network Geographic Concentration*

There exists m_c such that, for $m < m_c$, banks both regions deposit their reserves at New York City banks, and for $m > m_c$ banks in both regions deposit their reserves in their corresponding reserve cities.[13]

Under high enough public liquidity (more specifically, when $m > m_c$), there are no deposits placed in New York City banks, as periphery banks do not rely explicitly on the cross-regional insurance services they provide. With lower levels of public liquidity, however (more specifically, $m < m_c$), the extensive margin of lending changes. Even after accounting for endogenous deposit levels, the marginal benefit of connecting to New York City banks decreases as the amount of central bank liquidity increases. Because public liquidity increases the ability of banks to absorb local liquidity shocks, x_i and x_j reduce their reliance on New York City banks and rely on banks in regional reserve cities. A new network structure emerges as the concentration of links decreases. These changes are illustrated in Figure 8.

[13]We use stability as our equilibrium concept, which allows for x_1 and x_2 to deviate together.

Figure 8: Network Reactions to Public Liquidity Provision



Change in the structure of the regional interbank network.

3.6 Summary

Our simple model generates three testable predictions of public liquidity provision, m, for the allocation of funds and the shape of interbank linkages. These are:

1. *An increase in public liquidity provision (m) reduces aggregate private liquidity.* Private liquidity holdings (cash and interbank deposits) decline for both member and nonmember banks.

2. *An increase in public liquidity provision (m) intensifies interbank relations.* With more interconnections (in terms of short-term borrowing) there is an increase in the possibility of contagion, which increases the system's vulnerability to regional liquidity shocks.

3. *An increase in public liquidity provision (m) dissipates the overall interbank network.* The network structure changes from a geographically concentrated core to a dissipated core, crowding out private insurance for cross-regional shocks.

4 Empirical Evidence

Here we provide empirical evidence for our theoretical predictions. We document how the advent of the Federal Reserve's discount window changed aggregate liquidity in the banking system as well as the nature of interbank relations and the structure of interbank networks.

4.1 Data Sources

We collect data from two sources. The first is aggregate, and allows us to test the implications in terms of aggregate private liquidity and short-term borrowing in the nation as a whole. The second is bank-specific, and allows us to test the interbank network implications.

4.1.1 Aggregate Balance Sheet Data

Using the *Annual Report of the Comptroller of the Currency*, we collect balance sheet information for national and state banks from 1910 to 1921 and construct balance sheet data aggregated at the state level. During this period, banks were divided into three subcategories based on size and location: central reserve city banks, reserve city banks, and country banks. For national banks, the OCC report provides data for three groups of banks separately, but for state banks it does not.[14]

With these aggregate balance sheet data, we examine the effect that the creation of the Federal Reserve had on member versus nonmember banks at the aggregate level. Because all national banks were members of the Federal Reserve System and few state banks became members (Figures 1 and 2), we treat national banks generically as a proxy for member banks and state banks as a proxy for nonmember banks.

As noted, we examine national and state banks from 1910 to 1929-but with a gap between 1918 and 1920 for three reasons. First, in 1917 the Federal Reserve provided a three-year phase-in period allowing member banks to adjust to new reserve requirements. Second, in 1917 Congress amended the 1913 legislation and lowered reserve requirements in order to attract more state banks. Third, after the nation's entrance into World War I (April 1917), the Federal Reserve offered a preferential discount rate on loans secured by government debt to support the war effort, but between 1920 and 1921 it removed this preferential rate, raising its discount rate and tightening banks' access to the discount window.

4.1.2 Bank-Level Balance Sheet Data

To capture the testable implications in terms of the structure and role of interbank relations, we collect state bank examination reports for all state-chartered banks in Virginia for the

[14] For 3 central reserve cities (New York, Chicago, and St. Louis), the OCC constructs data at the city level, and it does likewise for 17 reserve cities (Albany, NY; Baltimore, MD; Boston, MA; Cincinnati, OH; Cleveland, OH; Detroit, MI; Kansas City, MO; Louisville, KY; Milwaukee, WI; Minneapolis, MN; New Orleans, LA; Omaha, NE; Philadelphia, PA; Pittsburgh, PA; Saint Joseph, MO; Saint Paul, MN; and San Francisco, CA). For the country banks it regulates, however, the OCC constructs data at the state-level.

years 1911 and 1922-thus, before and after passage of the Federal Reserve Act. Virginia State bank examiners inspected all banks and trust companies with a state charter and filed reports once or twice a year. We collected 222 of these examination reports for 1911 and 327 examination reports for 1922. In Virginia there were 248 and 334 state banks in 1911 and 1922, respectively. Hence, our dataset provides comprehensive information on Virginia state banks-most especially, for our purposes, on their balance sheets and their counterparts-before and after passage of the Federal Reserve Act. We focus on nonmember banks, however, because in Virginia only 11 state banks joined the Federal Reserve System.[15]

The banks' balance sheet statements and detailed information on interbank relationships (see Appendix Figure A1) allow us to examine the connections that exist between the role of interbank relationships for payments and funding. For a given bank, the dataset reports three types of interbank relationships: deposits *due from* other bank, deposits *due to* other bank, and short-term borrowing from another bank.

The examiners recorded detailed information on interbank deposits to verify whether state banks were holding enough interbank deposits to meet regulatory reserve requirements. As mentioned in the section on historical background, above, state bank regulators allowed state nonmember banks to hold interbank deposits to satisfy reserve requirements, which member banks were not allowed to do. In Virginia, nonmember banks could hold up to 7/12 of required reserves in the form of interbank deposits with approved reserve agents. Virginia state bank regulators did not make differentiated reserve requirements for Richmond and country banks so we can analyze them jointly.[16]

The examiner reported the amount that was *due from* each correspondent bank and the name of each of those correspondent banks. "Due from" deposits are then assets. Deposits *due to* other banks are deposits that other banks hold with a correspondent bank and are thus liabilities of the bank. Although only balances with reserve agents could be used to satisfy the legal reserve requirement, the examiner reported *all* the balances due from other banks. Because due-to deposits constituted a small fraction of country banks' liabilities, we focus throughout on due-from deposits, and this information provides us with a complete picture of the *payment networks* of state banks in Virginia during the two years in question.

The examination reports also provide information on whether a bank borrowed on a collateralized basis from its correspondents, the amounts of the loan, and the identity of the lender. These short-term borrowings took the form of rediscounts and bills payable. "Bills rediscounted" were loans sold with recourse. "Bills payable" consisted of either promissory

[15] Although we collected information on 2 of these 11 banks, we drop them from our analysis.
[16] Richmond was a reserve city, and in 1922 it became home to the district's Federal Reserve Bank.

notes of the borrowing bank or borrowing from Federal Reserve Banks. This information provides us with a complete picture of the *funding networks* of Virginia state banks.

The data for 1911 (the first year Virginia's banking department released examination reports) capture bank behavior before passage of the Act. The data for 1922 capture bank behavior after passage of the Act. We work under the hypothesis that, by 1922 the structure of the interbank network stabilized after the end of the war.

4.2 Balance Sheet Analysis

To determine how the advent of the Federal Reserve System affected the liquidity of the banking system, we begin with an analysis of balance sheet ratios of national (member) and state (nonmember) banks using state-level aggregate bank balance sheets. Then we drill down to bank-level balance sheet data to compare macro- and micro-level patterns. We focus on trends in our testable counterparts: cash, due from other banks (deposits in other banks), and borrowed money (short-term borrowing).

4.2.1 Aggregate balance sheet analysis

In Table 1, we show the effect that creation of the Federal Reserve System had on member versus nonmember banks at the aggregate level.

We begin by examining the volume of short-term borrowing by banks (B in our model). Before the advent of the Federal Reserve, short-term borrowing was not large, with national banks borrowing less than state banks (roughly 1% versus 2%).[17] After the advent of the Federal Reserve, both types of banks increased their borrowing significantly, but national banks increased theirs more than state banks did: whereas state banks almost doubled their relative borrowing, national banks more than tripled theirs. This is consistent with our model in that after public liquidity was provided, banks relied more on short-term borrowing to face short-term liquidity needs.

Now we turn attention to the most liquid asset on the balance sheet-cash (Φ_x in our model). We look at the share of bank assets held in the form of vault cash for both national and state banks from 1910 to 1929. Although all banks held less cash after the Federal Reserve was created, the reduction was larger for national banks (from 6.1% to 1.8%) than for state banks (from 4.3% to 2.4%). While the reduction for national banks resulted directly from

[17]These patterns are driven by the fact that reserve-city and central reserve-city banks generally borrowed less often than country banks (See Carlson and Wheelock (2018b)).

Table 1: Balance Sheet Ratios, U.S. Aggregate, 1910-1917 and 1921-1929

	National Banks		State Banks	
	1910-1917	1921-1929	1910-1917	1921-1929
Cash to assets	6.056	1.769	4.257	2.392
	(2.887)	(0.705)	(1.688)	(1.442)
Duefroms to assets	14.36	7.615	14.26	7.914
	(6.167)	(3.487)	(4.915)	(5.524)
Equity to liabilities	18.9	13.2	19.84	13.86
	(9.467)	(2.92)	(5.776)	(3.202)
Deposits to liabilities	59.3	69.94	72.57	77.75
	(24.16)	(9.05)	(8.823)	(6.706)
Duetos to liabilities	13.76	8.39	3.062	2.452
	(13.91)	(8.545)	(2.237)	(2.164)
Borrowing to liabilities	1.142	3.422	2.165	3.553
	(2.359)	(3.544)	(2.989)	(3.395)
Obs.	567	630	384	432

Table 1 displays summary statistics for national and state banks during the period 1910-1929. Cash is composed of specie and legal tender notes. Duefroms are interbank deposits due from other banks. Equity is composed of paid in capital and surplus. Duetos are interbank deposits due to other banks. Borrowing is short-term borrowing from other banks or the Federal Reserve Bank.
Source: *Annual report of the Comptroller of the Currency*.

access to public liquidity and lower reserve requirements, the reduction for state banks is largely explained by indirect access to public liquidity, which is consistent with our model. Even though state banks were not members, they held less liquidity because they were able to access public liquidity through their member correspondents, even in states where reserve requirements were increased, as we discuss in Appendix B.

Next, we examine the movement of interbank deposits (L in our model). On the asset side, for both national and state banks the relative deposits in other banks declined roughly by 50% (in both cases, dropping from around 14% to around 8%). The liability side of the balance sheets shows that deposits due to other banks decreased as well. In this case, the large decline was mostly experienced by national banks that used to be correspondents and received most of the interbank deposits in the system. For national banks, a reduction in the volume of interbank deposits due from and due to other banks was likely a direct consequence of the National Banking Act, which prohibited the use of interbank deposits to meet reserve requirements. For state banks, in contrast, a decline in the volume of due-from deposits was less mechanical, for those banks were still able to meet reserve requirements by holding interbank deposits.

Because interbank deposits were an important source of financial contagion, we decompose the holding of interbank deposits by national banks. In Figure 9, we plot interbank deposits for national banks by geographical classification and examine separately the relationships with national and with state banks.[18] Panel (a) of Figure 9 shows national banks' deposits due from other national banks and due from state banks. Until 1914, national banks held most of their deposits in other national banks and very little in state banks. After creation of the Federal Reserve, for all three groups of national banks the share of bank assets held in the form of deposits due from other national banks declined significantly. This suggests that among national banks, private insurance was reduced after creation of the central bank.

In Panel (b) of Figure 9, we look at national banks' deposits due to other national banks and due to state banks. After creation of the Federal Reserve, national banks in both reserve and central reserve cities saw a large reduction in the volume of their deposits due to other national banks. However, they did not see a significant decline in the volume of their deposits due to state banks. In other words, despite the reduction of private cross-insurance, national banks in financial centers were still vulnerable to runs by state banks.

4.2.2 Bank-Level balance sheet analysis

In this section, we use individual balance sheets of Virginia state banks to examine the consistency of the results between state-level balance sheet data and aggregated bank-level balance sheet data, and we focus on the behavior of nonmember banks.

In Table 2, we show balance sheet ratios for the years 1911 and 1922. First, we examine the share of short-term borrowing by banks. In 1911, before creation of the Federal Reserve, country banks' short-term borrowing accounted for 4% of country bank liabilities. In 1922, after creation of the central bank, country banks' short-term borrowing increased to 6%. Virginia state banks also reduced liquid asset holdings. The share held in the form of vault

[18]Note that the gap is larger in these two figures when compared with the rest because the OCC did not separate "deposits due from other banks" into "deposits due from other national banks" and "deposits due from other state banks" between 1915 and 1917. Similarly, it did not report "deposits due to other national banks" and "deposits due to other state banks" separately.

Table 2: Balance Sheet Ratios, Virginia State Banks, 1911 and 1922

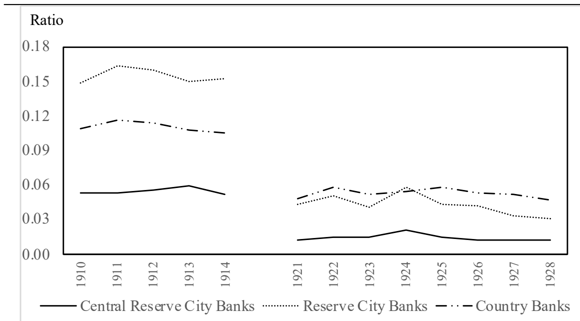
	1911	1922
Cash to assets	4.607	3.159
	(2.913)	(2.782)
Duefroms to assets	12.38	8.880
	(7.597)	(7.018)
Equity to liabilities	25.65	22.22
	(10.63)	(11.85)
Deposits to liabilities	68.38	70.19
	(14.80)	(16.61)
Duetos to liabilities	1.608	1.224
	(6.972)	(6.015)
Borrowing to liabilities	3.805	5.764
	(6.309)	(7.675)
Obs.	220	320

Cash is composed of specie and legal tender notes. Duefroms are interbank deposits due from other banks. Equity is composed of paid in capital and surplus. Duetos are interbank deposits due to other banks. Borrowing is short-term borrowing from other banks.
Source: *Virginia State Bank Examination Reports*.

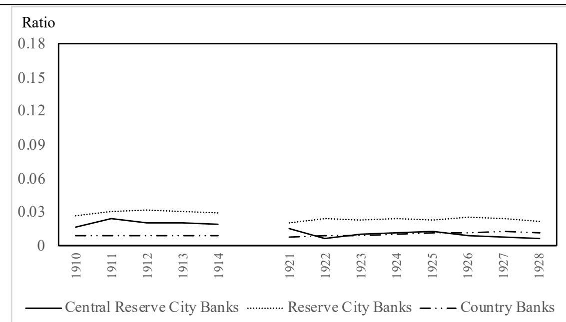
Figure 9: Interbank Deposits, National Banks in Tier Groups, 1910-1914 and 1921-1928

(a): Deposits due from other Banks

Due from National Banks

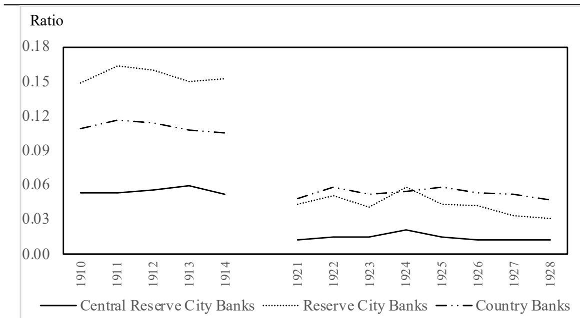


Due from State Banks



(b): Deposits due to other Banks

Due to National Banks



Due to State Banks

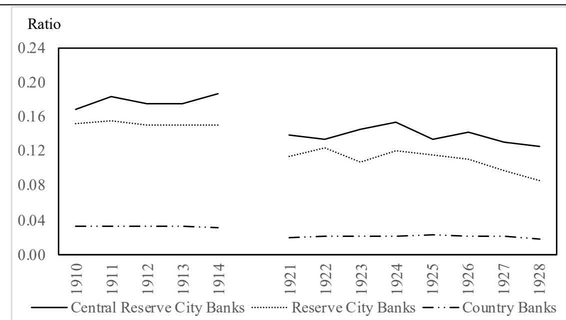


Figure 9 plots the ratio of deposits to other banks against total liabilities for national banks in each tier group. The table examines national banks' deposits due to other national banks and their deposits due to other state banks separately. Data for country banks are aggregated across all U.S. states; data for reserve city banks are aggregated across 17 reserve cities; and data for central reserve cities are aggregated across the three central reserve cities.
Source: *Annual report of the Comptroller of the Currency*.

cash (specie and legal tender notes) declined from 4.7% in 1911 to 3.1% in 1922. In addition, the share of correspondent deposits in other banks declined roughly by 30% (from around 12% to around 8%). These patterns are consistent with findings from the aggregated data and also with the prediction of the model.

To summarize, both aggregate balance sheet data and more-detailed balance sheet data from Virginia state banks indicate that the advent of the Federal Reserve reduced liquidity (in the form of cash and interbank deposits) and intensified the funding role of interbank relationships (in the form of short-term borrowing) between member and nonmember banks.

4.3 Network Analysis

The effect we have seen on the allocation of funds at the bank level does not, however, provide insights into the concentration of interbank relations at the geographical level. In this section, we study how the creation of the Federal Reserve System affected the nature and structure of the interbank system in Virginia, a state where we have detailed extensive and intensive margins of banking relations.

Figure 10 maps all respondent banks (Virginia state banks) and correspondent banks for the years 1911 and 1922. The respondent (corresponding) banks that *only* placed (received) deposits are in blue, while banks that *both* placed (received) deposits and borrowed (lent) short-term funds are in red. There were 1,033 and 1,056 unique due-from relationships in 1911 and 1922, respectively. In addition, there were 150 and 309 unique short-term borrowing relationships in 1911 and 1922. These maps provide the first clues that the funding roles of the network became more prevalent after passage of the Federal Reserve Act.

Table 3 shows the number of correspondent relationships in 1911 and 1922. Three patterns can be discerned. First, state banks held multiple correspondent accounts in multiple banks, whereas they borrowed from one or two banks each. Second, the average number of correspondents in which a bank had deposits decreased from 4.8 to 3.3. Third, the average number of correspondents from which a bank borrowed short term remained at around 1.8.

Table 3: Number of Correspondent Relationships, 1911 and 1922

	1911			1922		
	Number of Banks	Mean	SD	Number of Banks	Mean	SD
Due-from	218	4.71	4.13	323	3.27	2.41
Borrowing	89	1.71	1.05	172	1.80	1.06

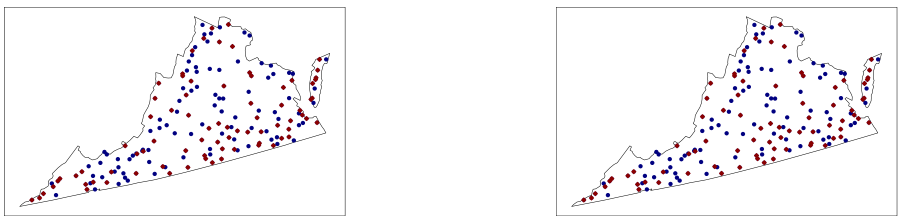
Table 3 displays the average number of correspondent relationships per bank. “Due-from" indicates the average number of other banks from which a bank had amounts due. Similarly, “Borrowing" indicates the average number of correspondent banks that lent short-term funds to a respondent bank.
Source: *Virginia State Bank Examination Reports*.

Before providing detailed information on the overall network before and after passage of the Federal Reserve Act, we present a specific example to illustrate the change in a bank's relation after the advent of the Federal Reserve Bank. The banking relations of the Bank of Warm Springs in Warm Springs, Virginia, are depicted in Figure 11. The correspondent banks that received *only* deposits from the Bank of Warm Springs are in blue and the ones that *both* received deposits and lent short-term to the Bank of Warm Springs are in red. In the

Figure 10: Respondent and Correspondent Banks, 1911 and 1922

Respondent banks

1911	1922



Correspondent banks

1911	1922

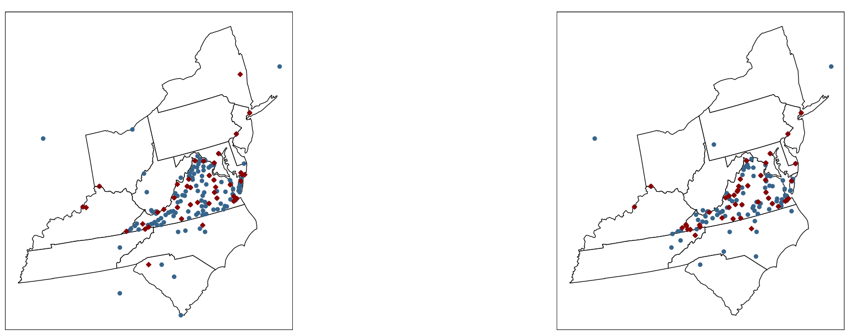


Figure 10 maps all respondent banks (Virginia state banks) and correspondent banks for the years 1911 and 1922. The respondent (corresponding) banks that *only* placed (received) deposits are in blue, while banks that *both* placed (received) deposits and borrowed (lent) short-term funds are in red.
Source: *Virginia State Bank Examination Reports*.

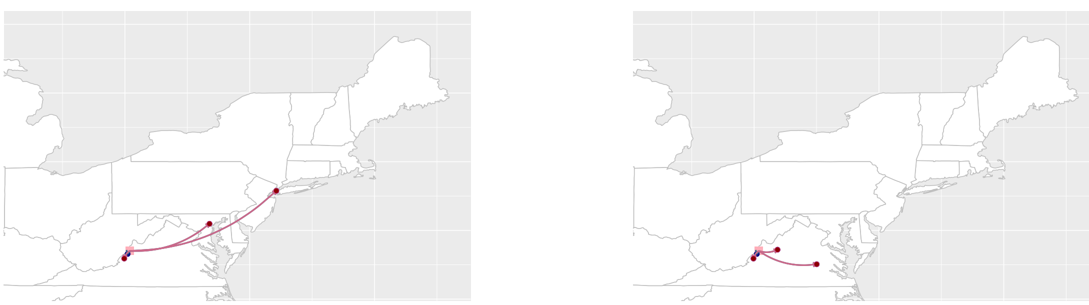
tabular component of the map, we provide detailed information about these correspondent relationships. Columns (1) and (2) provide the names and locations of the correspondent banks of the Bank of Warm Springs. Columns (3) and (4) show the amount of interbank deposits due from these banks and the amount of short-term funds borrowed from them.

Figure 11 shows how the structure and nature of the bank network for that specific bank changed after passage of the Federal Reserve Act. First, correspondent relationships became more local. In 1911, Bank of Warm Springs maintained correspondent banking relationships in New York and Baltimore, but by 1922 it had dissolved these relationships and opened new ones with banks in Richmond and Staunton, which were in close proximity. In addition, after the Act was passed the bank placed large correspondent deposits in local banks, whereas previously it had held a majority of its interbank deposits in Baltimore. Similarly, after passage of the Act the bank relied on Virginia banks for short-term borrowing, whereas previously it had borrowed from a New York City correspondent.

Figure 11: Bank Network for Bank of Warm Springs

Bank of Warm Springs in Warm Springs

1911 | 1922



Correspondents	Town	State	Duefrom	Borrowed Money	Correspondents	Town	State	Duefrom	Borrowed Money
Chase National Bank	New York	NY	809.28	10000	Covington National Bank	Covington	VA	2562.25	21500
National Exchange Bank	Baltimore	MD	2459.28	5000	Bath County National Bank	Hot Springs	VA	1376.53	
Covington National Bank	Covington	VA	509.07	5000	Merchants National Bank	Richmond	VA	2129.64	25000
Bath County National Bank	Hot Springs	VA	237.61		National Valley Bank	Staunton	VA	1091.03	15000

Notes: Figure 11 provides information for the Bank of Warm Springs in Warm Springs. Columns (1) and (2) provide information about the names and locations of correspondent banks. Columns (3) and (4) provide information about the amount of interbank deposits due from these banks and the amount of short-term funds borrowed from them.
Source: *Virginia State Bank Examination Reports*.

The changes made by Bank of Warm Springs are representative of the general patterns that characterize interbank networks before and after creation of the Federal Reserve System. In Tables 4 through 6 we present more systematically the interbank relationships for all

Virginia state banks. Tables 4 and 5 show the structure of the interbank system at extensive and intensive margins, and Table 6 shows the distance in miles between respondent and correspondent banks.

Table 4 shows the distribution of state banks' due-from deposits (payment network). We find that the creation of the central bank encouraged banks to rely more heavily on local correspondents. Before the advent of the Federal Reserve, banks relied more on correspondent relationships with banks outside Virginia; for example, many banks had city correspondents in New York City and Baltimore. After the Federal Reserve's creation, however, the due-from deposit network became more dispersed. Banks also shifted their relationships away from New York and Baltimore and into other country banks in Virginia. These results are consistent at both extensive and intensive margins.

Table 4: "Due from" Relationships, Virginia State Banks, 1911 and 1922.

	Extensive Margin (Links)		Intensive Margin (Amount)	
	1911	1922	1911	1922
New York	19.37	12.47	10.63	6.884
	(19.02)	(16.55)	(16.93)	(13.91)
Chicago	0.105	0.0163	0.0564	0.01
	(0.791)	(0.293)	(0.471)	(0.0908)
Baltimore	9.126	6.816	10.33	6.948
	(17.73)	(16.39)	(23.45)	(19.70)
Washington, DC	2.226	1.719	1.812	1.556
	(7.787)	(9.725)	(7.506)	(10.73)
Richmond	20.81	22.22	28.09	26.85
	(20.14)	(27.98)	(32.56)	(34.53)
Reserve Cities in Other States	2.592	3.404	2.554	3.987
	(7.774)	(13.46)	(8.465)	(16.68)
Country Banks in VA	43.07	50.59	42.46	51.76
	(29.04)	(34.87)	(37.56)	(40.74)
Country Banks in Other States	2.701	2.769	3.174	1.704
	(10.70)	(10.25)	(14.80)	(9.389)
Obs.	218	323	218	323

Notes: Rows indicate the location of correspondent banks. New York was a central reserve city. Baltimore and Washington, DC, were reserve cities. Richmond was not a reserve city in 1911 but became one in 1922. Columns indicate the location of respondent banks. Extensive margins are the proportions of links in each location against total links. Intensive margins are proportions of correspondent deposits held at different locations against total due-from deposits.
Source: *Virginia State Bank Examination Reports*.

Table 5 shows the nature of the short-term borrowing network (funding network). We find that creation of the central bank encouraged more local short-term borrowing relationships. Before the agency's creation, 40% of country banks borrowed short-term funds from their correspondents, particularly from Richmond banks. After the agency's creation, banks borrowed more heavily from other country banks in Virginia instead of from Richmond banks.

Table 5: "Short-term Borrowing" Relationships, Virginia State Banks, 1911 and 1922.

	Extensive Margin (Links)		Intensive Margin (Amount)	
	1911	1922	1911	1922
New York	8.427 (23.11)	8.992 (22.35)	7.661 (22.40)	8.376 (22.57)
Baltimore	11.33 (28.43)	7.045 (22.98)	11.35 (29.23)	6.936 (23.18)
Washington, DC	2.060 (11.74)	1.599 (11.52)	1.814 (11.20)	1.589 (11.64)
Richmond	32.68 (41.73)	21.57 (35.19)	32.14 (42.40)	21.10 (35.41)
Reserve Cities in Other States	3.464 (13.92)	4.186 (17.39)	3.684 (15.14)	4.162 (17.39)
Country Banks in VA	38.58 (42.67)	52.42 (43.64)	36.56 (42.87)	49.66 (44.43)
Country Banks in Other States	3.464 (14.37)	4.186 (18.05)	2.340 (12.51)	4.127 (18.04)
Obs.	89	172	91	172

Notes: Rows indicate the location of correspondent banks. Extensive margins provide information on the proportions of links in each location against total links. Intensive margins provide information on the proportions of borrowed money from correspondents at different locations against total borrowed money.
Source: *Virginia State Bank Examination Reports*.

To identify more clearly the change in the geographical concentration of the interbank system, we compute the distances in miles between respondent and correspondent banks. Table 6 shows the distance between state banks and the correspondent banks with which they placed deposits and the distance between state banks and the correspondent banks from which they borrowed short-term funds. In placing deposits after creation of the central bank, Virginia state banks chose to reduce their connectivity to New York City and placed their deposits in local banks. Likewise in borrowing short-term funds after creation of the central bank, those same banks chose to borrow from banks located in close geographic proximity, for any member bank independent of location could access the central bank's discount window.

The shift in the network structure toward geographically closer links suggests that the existence of the Federal Reserve System enabled banks to rely more on the provision of public

Table 6: Distance between Respondent and Correspondent Banks

	Due-froms		Short-term borrowing	
	1911	1922	1911	1922
Longest Distance	347.1	253.3	187.9	167.7
	(156.8)	(267.5)	(147.5)	(427.0)
Shortest Distance	20.76	31.08	64.16	67.68
	(34.29)	(224.1)	(83.45)	(414.9)
Mean Distance	134.4	111.0	118.3	108.2
	(63.72)	(227.7)	(95.23)	(415.0)
Median Distance	105.8	84.64	107.2	91.78
	(74.10)	(229.0)	(97.07)	(416.1)
Total Distance	1107.8	587.1	307.2	238.5
	(1125.3)	(698.9)	(313.9)	(457.3)
Number of Banks	218	323	86	169
Obs.	997	1047	145	303

Table 6 provides information on geographical distance between respondent and correspondent banks in miles. It shows that the existence of the Federal Reserve led banks to choose correspondents located in close geographic proximity.
Source: *Virginia State Bank Examination Reports*.

liquidity through closer banks and less on the provision of private liquidity through farther banks. The importance of New York City banks in providing private insurance arrangements for regional liquidity shocks before the advent of the Federal Reserve has been well documented (Carlson and Wheelock (2018a)). By pooling bank reserves from banks in different regions, New York City banks had been able to accommodate liquidity transfers between regions, thereby smoothing interregional flows (Gilbert (1983), James and Weiman (2010)). As our analysis shows, however, the provision of liquidity by the central bank reduced the relevance of New York City banks in the U.S. interbank network. In this way, the central bank liquidity provision crowded out previous private liquidity insurance, plausibly at the cost of using public funds to cover such public insurance.

To summarize, the introduction of liquidity provided by a central bank changed the structure of the interbank system for nonmember banks as well as for member banks. By injecting public liquidity into the banking system, the existence of the Federal Reserve reduced the need for nonmember banks to maintain correspondent relationships across multiple cities and outside the state. In addition, the provision of public liquidity eliminated the role of New York City banks as the ultimate liquidity provider and allowed country banks to rely on local banks to access liquidity. The shift of correspondent relationships away from New York and toward local banks transformed what had been a *national core-periphery structure* based in New York City into a *regional core-periphery structure* based in reserve cities.

5 Conclusion

The provision of public liquidity by the Federal Reserve System, revitalized by the financial crisis of 2007-2009 and the COVID-19 pandemics, is a subject of heated debate among academics and policymakers. Answers to questions like How much? To whom? For how long? are usually "contaminated" by the closeness to the events and by the complexity and variety of modern banking. This paper takes a different approach as we study, both theoretically and empirically, the role of public liquidity provision at its inception, during the creation of the Federal Reserve System.

In 1913 the Federal Reserve Act was passed to provide liquidity to member banks that satisfied stringent reserve requirements. While this public insurance came at the social cost of taxation, it also brought the benefit of regulating and supervising members. We provide a model that suggests, in addition to the intended benefit, three possible unintended consequences of public liquidity. That all three were in fact realized is borne out by our novel historical dataset on banks' relations.

The first unintended consequence was that many banks decided against joining; though this enabled them to bypass regulations, by using the interbank network they could still access public liquidity. This led to less private liquidity holding, less diversification, and more systemic risk. The second unintended consequence was that interbank borrowing increased, with a concomitant increase in the intensity of interbank relations. This increased intensity made the entire network more vulnerable to shocks and therefore more exposed to contagion. The third unintended consequence was that private insurance across regions was crowded out. Before creation of the Federal Reserve System, New York City had been at the center of the interbank system, and its centrality ensured private cross-regional liquidity insurance. After the Federal Reserve's creation, however, the interbank system with a common center was transformed into a system with a diffuse set of relationships, and the role of cross-regional insurance was transferred to the public sphere.

Thus, while in "normal times" the provision of public liquidity may have stabilized the system as intended (perhaps at a cost in terms of taxation and distortions), these three unintended consequences were lurking in the background to make the system more prone to endogenous larger collapses that might require larger interventions (perhaps at even large social costs).

These results have natural implications for current policy discussion and for assessing post-reform attempts to prevent non-bank financial institutions from accessing public liquidity. As our results show, restricting "official" access to public liquidity does not prevent "real" access to public liquidity, and an attempt to prevent real access by restricting official access may indeed backfire by creating a landscape favorable to the flourishing of shadow banks that operate with illiquid assets and generate systemic risk.

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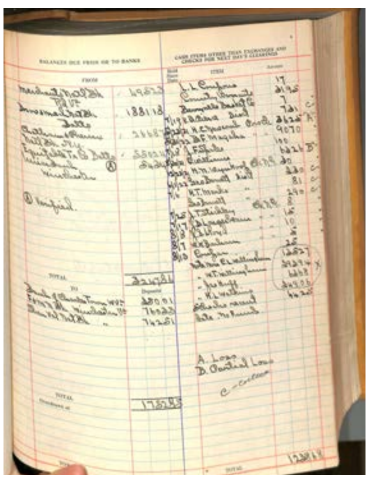
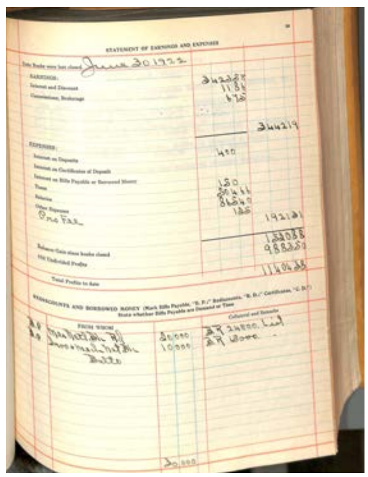
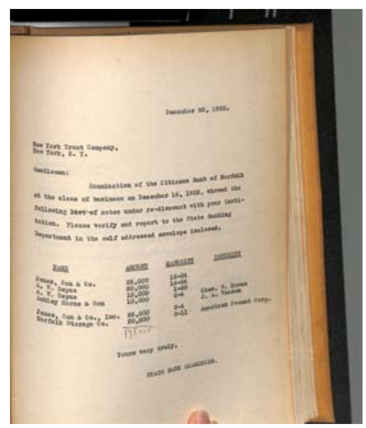
A Virginia State Bank Examination Reports

In Figure A1 we present images of representative pages in the state bank examination reports used for this study. The reports provide information on three types of interbank relationships: on the asset side of the balance sheet, the amounts due from other banks by individual debtor banks; on the liability side of the balance sheet, the amounts due to other banks by individual creditor banks; and the amounts of borrowed money and the provider of these short-term loans. In some cases, the reports provide information on collateral used for securing short-term funds.

Figure A1: Virginia State Bank Examination Reports

Interbank Deposits	Short-term Borrowing	Collateral for Borrowing
		

B Aggregate Balance Sheet Analysis in More Detail

In the main text, we provide summary statistics of the balance sheet data aggregated at the state level. In Figure B2, we plot the movement of balance sheet ratios from 1910 to 1929. Figure B2 shows that in the 1920s, short-term borrowing increased and liquid assets declined.

Figure B2: Aggregate Balance Sheet Ratios, 1910-1929

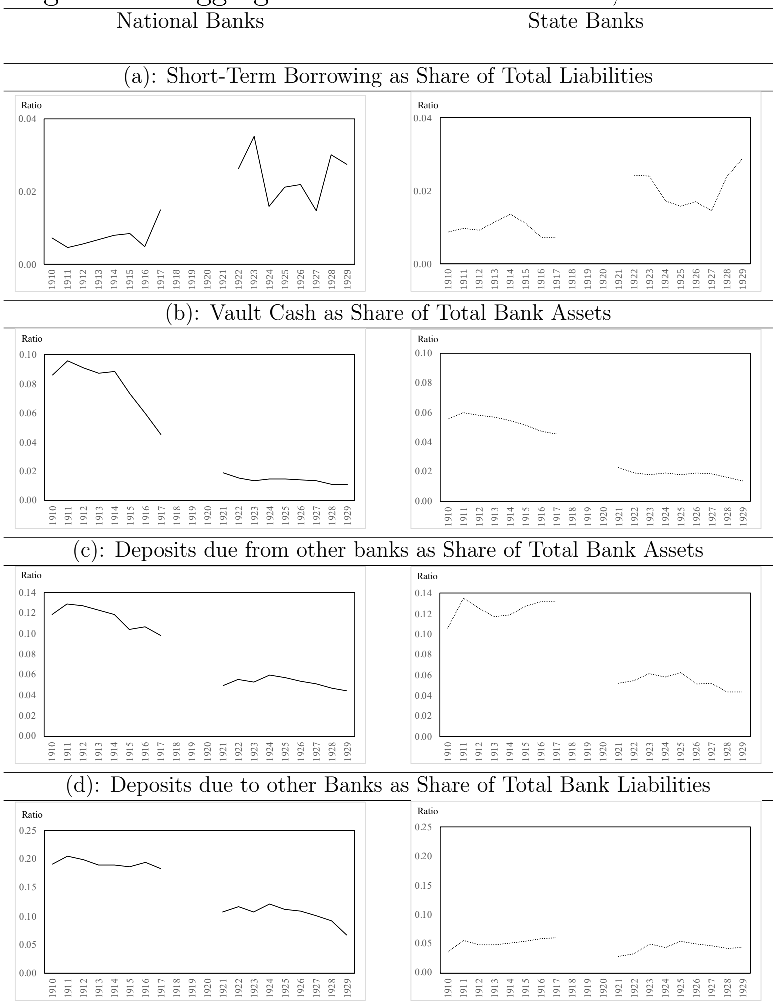


Figure B2 plots the ratio of short-term borrowing to total liabilities for national and state banks. All data are aggregated by the OCC: data for national banks are aggregated across all states, 17 reserve cities, and 3 central-reserve cities; data for state banks are aggregated across all states, all reserve cities, and reserve cities.
Source: *Annual Report of the Comptroller of the Currency*.

In addition, we check the robustness of our findings by restricting the data in two dimensions. First, we restrict our sample using state bank participation rate. As shown in Figure 2, states with financial and manufacturing sectors displayed a higher proportion of state bank membership than agricultural states. Given the irregular geographic distribution of membership, one might be concerned that the described changes were generated by state member banks and that therefore our classifying all state banks as nonmembers clutters the analysis. To alleviate this concern, we restrict our sample and compare the asset composition of member and nonmember banks only in states where the membership ratio of state banks was under 10% in 1920.

Second, we restrict our sample using state-level reserve requirements. Changes in the liquidity of the state banking system might be driven by changes in reserve requirements by state regulators rather than by voluntary liquidity changes. To rule out this possibility, we divided states into three groups: (1) states that decreased their reserve requirements, (2) states that increased their reserve requirements, and (3) states that did not change their reserve requirements. Between 1910 and 1929, 22 states reduced reserve requirements, 10 states increased reserve requirements, and 16 states kept reserve requirements unchanged.[19]

For states where the state bank participation rate was below 10%, Figure B3 plots the fraction of total assets that state banks in those states held in borrowing, cash, and interbank deposits. In all cases, and regardless of the change in reserve requirements, nonmember banks reduced cash and interbank deposits and increased borrowing after the Federal Reserve came into existence (in 1914).

To summarize, we find that the existence of the Federal Reserve reduced liquidity (in the form of cash and interbank deposits) and intensified interbank relations (in the form of higher short-term borrowing) for both member and nonmember banks. Furthermore, member banks significantly reduced their relations with other member banks, but not their relations with nonmember banks. These factors suggest less private cross-insurance but still exposure to withdrawals, which contributed to the possibility of more contagion and greater vulnerability of the financial system.

[19]See White (2014) for information on state reserve requirements. We classify CA, DE, GA, IN, KS, KY, LA, MI, MN, MT, NM, NY, OK, OR, PA, SD, TX, VA, WA, WI, WV as states with decreasing reserve requirements. In addition, we classify AR, CO, IA, MD, MS, NH, SC, TN, VT, WY as states with increasing reserve requirements. Last, we classify AL, CT, FL, ID, IL, MA, ME, MO, NC, ND, NE, NJ, NV, OH, OK, UT as states that did not change reserve requirements.

Figure B3: Bank Liquidity and Changes in State-Level Reserve Requirements, 1910-1929

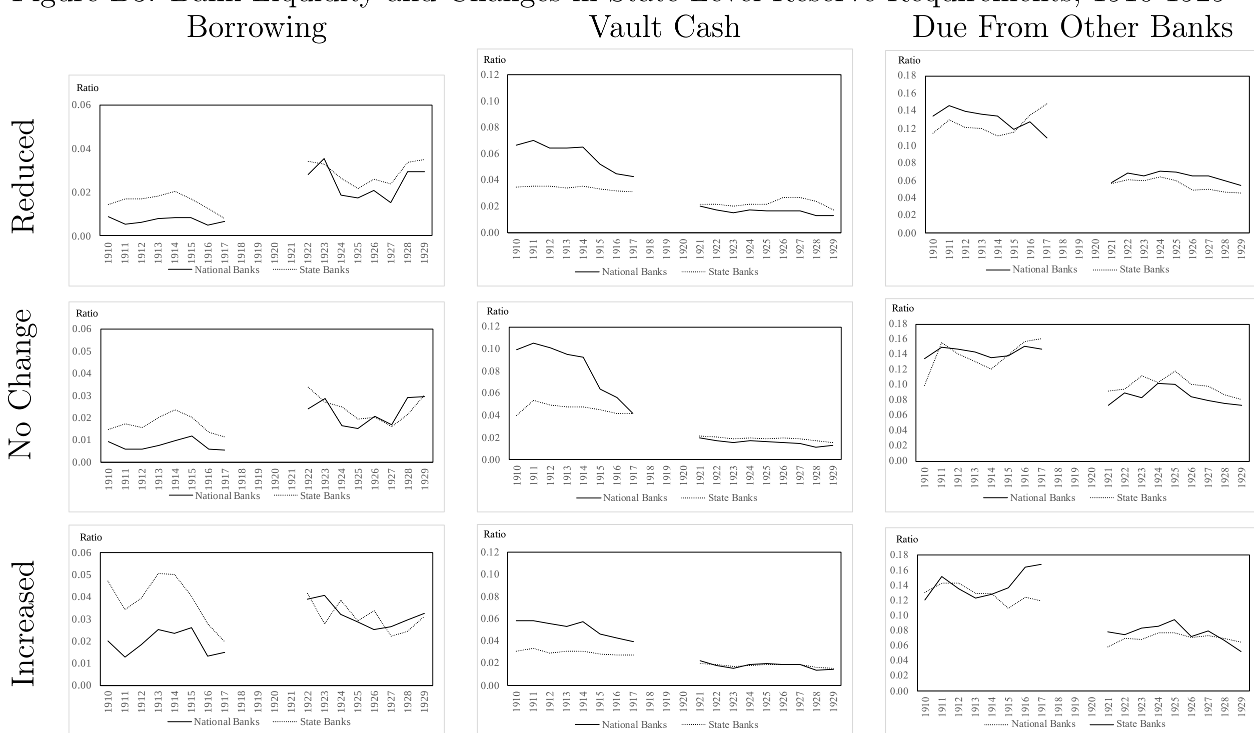


Figure B3 the share of short-term borrowing against total liabilities, the share of vault cash against total assets, and the share of deposits due from other banks against total assets for states with different reserve requirements. Data are further restricted for states where the Federal Reserve membership ratio of state banks was under 10% in 1920. All data are aggregated by the OCC: data for national banks are aggregated across all states, 17 reserve cities, and 3 central-reserve cities; data for state banks are aggregated across all states, all reserve cities, and reserve cities.
Source: *Annual Report of the Comptroller of the Currency*.

ONLINE APPENDIX

C Remarks on the model and assumptions

The size of the liquidity shocks. We assume that the liquidity shock can exceed D so we do not deal with the corner solutions. In particular, the liquidity shock ζ is 0 w.p. $1-\alpha$ and $U[0,Z]$ w.p. α where $Z > D$. The story is as follows. There are legacy assets and liabilities. M captures the sum of legacy liabilities and K captures the sum of returns from illiquid legacy assets. These are safe but the the return time for legacy assets and withdrawal time for legacy liabilities are random. $K \geq M$ so there is no solvency issue. There can be an illiquidity issue. At the time of the liquidity shock, if the return so far from legacy assets is k and the amount of legacy liabilities realized so far is m, and the realized liquidity withdrawal from depositors (who have seniority) is $d \in [0, D]$ then the actual liquidity need at the time of the liquidity shock is $l = d + m - k$. We assume that l has distribution $U[-K, D + M]$. Now denote $\alpha = \frac{D+M}{D+M+K}$ and $Z = D + M$. Then $l \leq 0$ w.p. $1 - \alpha$ and $l \sim U[0, Z]$ w.p. α. Now let $\zeta = l_+$ the private liquidity need. (We use the notation $z_+ = \max\{z, 0\}$.) Then $\zeta = 0$ w.p. $1 - \alpha$ and $U[0, Z]$ w.p. α. When there is central bank liquidity m, the effect of m will be to make the private liquidity need $(\zeta - m)_+$.

Notation. Going forward, the fundamentals of the model are r_x, r_y, r for the return rates, α, Z, ζ, for shocks, D, m for liquidity. Denote $Z_{\alpha,m} = \frac{Z(1-\alpha)}{\alpha} + m$. For a random variable X, F_X denotes its CDF. Also, $f \underset{z}{\propto} g$ means that f and g are monotone transformations of each other as functions of z.

Discussion of parametric assumptions. We will take Z to be large enough compared to D and m in order to avoid corner issues in the algebra. In particular, $Z > m + D$ so that even the entire liquidity in the system may not suffice, although this event has small probability. This way, we do not need to worry about cumbersome corner solutions in the algebra. This, in a way, "convexifies" the problem.

Assumption 1. $0 \leq m \leq Z - D$.

Also, for technical reasons and for the simplicity of algebra, we will restrict attention to α that is not too large.

Assumption 2. $\alpha \leq \overline{\alpha} = \frac{Z}{Z+\rho D}$ *where*

$$\rho = \max\left\{0, \frac{2}{2-\phi}\left(2(1-\phi) - \frac{r}{r_y}\right) - 1\right\}$$

The major role of this assumption is to make sure that the reserve requirements bind and $\Phi_y = \phi L$. Finally, we assume that r_x and r_y are relatively large compared to r.

Assumption 3. $(1-\phi)r_x > 2r$ *and* $(1-\phi)r_y > r$.

The condition on r_y is innocuous. If $(1-\phi)r_y$ were less than r, y would not borrow. The condition on r_x deserves some discussion. One might think, at first, that by $r_x > r$, bank x's own project is a better investment than the "interbank investment" of lending to y. Since each investments provide buffer against liquidation of the other, each investment would be non-zero under sufficiently high risk. But by $r_x > r$, I_x would be larger than L. But this simple logic is missing a critical point. Bank y pays interest on the full loan L, not the investment size I_y. At least ϕL is kept by y as reserves, which is a source of short term liquidity for x at the time of shocks. That is, interbank investment has an extra benefit above and beyond its investment value and diversification value. This complicates proofs. For this reason we make a simplifying assumption $(1-\phi)r_x > 2r$ that makes sure there is a pecking order: first priority is the project of bank x, then the interbank investment.

D Proofs

Proof. **(Proof of Lemma 1)**

Here we provide a general proof that allows for public liquidity $m \geq 0$. The proof of Lemma 1 can be obtained by replacing m with 0 below.

It is easy to see that for a given portfolio profile (I_x, L, I_y) and a level of liquidity shortage $\zeta' = (\zeta - m - \Phi_x - \Phi_y)_+$, liquidations induced by the optimal behavior of x at the liquidation stage is given by

- If $\zeta' = 0$, nothing is liquidated.
- If $0 < \zeta' \leq \min\{I_x, I_y\}$, then
  - If $Lr \leq I_x r_x$, then I_y is liquidated.
  - If $Lr > I_x r_x$, then I_x is liquidated.
- If $\min\{I_x, I_y\} < \zeta' \leq \max\{I_x, I_y\}$, then $\max\{I_x, I_y\}$ is liquidated.
- If $\max\{I_x, I_y\} < \zeta'$, then both I_x and I_y are liquidated.

Then I_y does not get liquidated iff one of the following hold:

- $\zeta' = 0$
- $0 < \zeta' \leq \min\{I_x, I_y\}$ and $Lr > I_x r_x$
- $I_y < \zeta' \leq I_x$.

Then the expected profit of y is

$$\begin{aligned}\Pi_y(I_y) &= \left(F_{\zeta'}(0) + 1_{Lr > I_x r_x}\left(F_{\zeta'}(\min\{I_x, I_y\}) - F(0)\right) + 1_{I_x > I_y}\left(F_{\zeta'}(I_x) - F_{\zeta'}(I_y)\right)\right)(I_y r_y - Lr) \\ &= \frac{\alpha}{Z} r_y \times \begin{cases} u_1(I_y) := \left(I_y - \frac{Lr}{r_y}\right)(Z_{\alpha,m} + D - I_y) & \text{if } Lr > I_x r_x \\ u_2(I_y) := \left(I_y - \frac{Lr}{r_y}\right)(Z_{\alpha,m} + D - 2I_y) & \text{on } I_x \geq I_y \text{ if } Lr \leq I_x r_x \\ u_3(I_y) := \left(I_y - \frac{Lr}{r_y}\right)(Z_{\alpha,m} + D - I_x - I_y) & \text{on } I_x \leq I_y \text{ if } Lr \leq I_x r_x \wedge I_x \leq L(1-\phi) \end{cases}\end{aligned}$$

All of u_1, u_2, u_3 are concave quadratics. They are increasing up to their unique unconstrained $\arg\max$ and decreasing afterwards. The unconstrained $\arg\max$ of u_1, u_2, u_3 are given by

$$\begin{aligned} I_1^* &= \frac{1}{2}\left(Z_{\alpha,m} + D + \frac{Lr}{r_y}\right) \\ I_2^* &= \frac{1}{2}\left(\frac{Z_{\alpha,m} + D}{2} + \frac{Lr}{r_y}\right) \\ I_3^* &= \frac{1}{2}\left(Z_{\alpha,m} + D - I_x + \frac{Lr}{r_y}\right) \end{aligned}$$

Then $I_y^* = \arg\max \Pi_y(I_y)$ in these three regions are given by

$$\Pi_y(I_y^*) = \frac{\alpha}{Z}\begin{cases} u_1\left(\min\left\{L(1-\phi), I_1^*\right\}\right) & \text{if } Lr > I_x r_x \\ u_2\left(\min\left\{L(1-\phi), I_x, I_2^*\right\}\right) & \text{on } I_x \geq I_y \text{ if } Lr \leq I_x r_x \\ u_3\left(\max\left\{I_x, \min\left\{L(1-\phi), I_3^*\right\}\right\}\right) & \text{on } I_x \leq I_y \text{ if } Lr \leq I_x r_x \wedge I_x \leq L(1-\phi) \end{cases}$$

By Assumption 2, we have $L(1-\phi) \leq I_3^*$. Also clearly $I_3^* \leq I_1^*$. Then

$$\min\left\{L(1-\phi), I_1^*\right\} = \max\left\{I_x, \min\left\{L(1-\phi), I_3^*\right\}\right\} = L(1-\phi)$$

Then

$$\Pi_y(I_y^*) = \frac{\alpha}{Z} \begin{cases} u_1\left(L(1-\phi)\right) & \text{if } Lr > I_x r_x \\ u_2\left(\min\left\{L(1-\phi), I_2^*\right\}\right) & \text{on } I_x \geq I_y \text{ if } Lr \leq I_x r_x \,\wedge\, I_x \geq L(1-\phi) \\ u_2\left(\min\left\{I_x, I_2^*\right\}\right) & \text{on } I_x \geq I_y \text{ if } Lr \leq I_x r_x \wedge I_x \leq L(1-\phi) \\ u_3\left(L(1-\phi)\right) & \text{on } I_x \leq I_y \text{ if } Lr \leq I_x r_x \,\wedge\, I_x \leq L(1-\phi) \end{cases}$$

For the first case [if $Lr > I_x r_x$], $I_y^* = L(1-\phi)$. For the second case [on $I_x \geq I_y$ if $Lr \leq I_x r_x \wedge I_x \geq L(1-\phi)$] note that $I_x \geq L(1-\phi)$ implies both $I_x \geq I_y$ and $Lr \leq I_x r_x$. So this case can be restated as simply [$I_x \geq L(1-\phi)$]. For the third and fourth cases jointly, we compare $u_2\left(\min\left\{I_x, I_2^*\right\}\right)$ and $u_3\left(L(1-\phi)\right)$ under [$Lr \leq I_x r_x \wedge I_x \leq L(1-\phi)$]. Note that

$$u_2(I_2^*) = \frac{1}{2}\left(\frac{Z_{\alpha,m}+D}{2} - \frac{Lr}{r_y}\right)^2$$
$$u_3(L(1-\phi)) = \left(L(1-\phi) - \frac{Lr}{r_y}\right)\left(Z_{\alpha,m} + D - I_x - L(1-\phi)\right)$$

Suppose $I_2^* < L(1-\phi)$. Then we have $\frac{1}{2}\left(\frac{Z_{\alpha,m}+D}{2} + \frac{Lr}{r_y}\right) < L(1-\phi)$, and so $\frac{1}{2}\left(\frac{Z_{\alpha,m}+D}{2} - \frac{Lr}{r_y}\right) < L(1-\phi) - \frac{Lr}{r_y}$. Also $\left(\frac{Z_{\alpha,m}+D}{2} - \frac{Lr}{r_y}\right) < (Z_{\alpha,m} + D - I_x - L(1-\phi))$. Thus, $u_2\left(\min\left\{I_x, I_2^*\right\}\right) \leq u_2(I_2^*) < u_3(L(1-\phi))$. Now suppose $I_2^* \geq L(1-\phi)$. Then by $I_x \leq L(1-\phi)$ we have $I_2^* \geq I_x$. Then $u_2\left(\min\left\{I_x, I_2^*\right\}\right) = u_2(I_x)$. Recall that $u_2(I_x) = u_3(I_x)$, $L(1-\phi) \leq I_3^*$, and u_3 is increasing up to I_3^*. Then we have $I_x \leq L(1-\phi) \leq I_3^*$ and $u_3(I_x) \leq u_3(L(1-\phi)) \leq u_3(I_3^*)$. Combining these we have $u_2\left(\min\left\{I_x, I_2^*\right\}\right) = u_2(I_x) = u_3(I_x) \leq u_3(L(1-\phi))$. So in general, $u_2\left(\min\left\{I_x, I_2^*\right\}\right) \leq u_3(L(1-\phi))$ and $I_y^* = L(1-\phi)$ in the union of third and fourth cases, i.e. [$Lr \leq I_x r_x \wedge I_x \leq L(1-\phi)$]. Therefore,

$$I_y^* = \begin{cases} \min\left\{L(1-\phi), I_2^*\right\} & \text{if } I_x > L(1-\phi) \\ L(1-\phi) & \text{otherwise} \end{cases}$$

Under $I_x > L(1-\phi)$ and Assumption 2, we have $L(1-\phi) \leq I_2^*$ and so $I_y^* = L(1-\phi)$.

Next consider the optimal portfolio of x. Let (I_x, L) be optimal and suppose that $I_x < L(1-\phi)$. Then the expected profit of x is

$$\begin{aligned} \Pi_x = & F_{\zeta'}(0)\left(I_x r_x + Lr\right) \\ & + \left(F_{\zeta'}(I_x) - F_{\zeta'}(0)\right)\max\left\{Lr, I_x r_x\right\} \\ & + \left(F_{\zeta'}(L(1-\phi)) - F_{\zeta'}(I_x)\right) I_x r_x \end{aligned}$$

$$\underset{(I_x,L)}{\propto} \left(Z_{\alpha,m} + D - I_x - L(1-\phi)\right)\left(I_x r_x + Lr\right) + I_x \max\left\{Lr, I_x r_x\right\} + \left(L(1-\phi) - I_x\right) I_x r_x$$

By $I_x < L(1-\phi)$, right partial derivative w.r.t. I_x must be negative and left partial derivative w.r.t. L must be positive. If $I_x r_x \neq Lr$, these derivatives are given by the following: The F.O.C. w.r.t. I_x is

$$\begin{aligned}
0 \geq& - \left(I_x r_x + Lr\right) + r_x\left(Z_{\alpha,m} + D - I_x - L(1-\phi)\right) \\
&+ \begin{cases} Lr & \text{if } Lr > I_x r_x \\ 2I_x r_x & \text{if } Lr < I_x r_x \end{cases} \\
&+ \left(L(1-\phi) - 2I_x\right) r_x \\
=& r_x\left(Z_{\alpha,m} + D - 2I_x\right) - \begin{cases} 2I_x r_x & \text{if } Lr > I_x r_x \\ Lr & \text{if } Lr < I_x r_x \end{cases} \\
\implies& Z_{\alpha,m} + D \leq 2I_x + \begin{cases} 2I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases}
\end{aligned}$$

The F.O.C. w.r.t. to L is

$$\begin{aligned}
0 \leq& - (1-\phi)\left(I_x r_x + Lr\right) + r\left(Z_{\alpha,m} + D - I_x - L(1-\phi)\right) \\
&+ \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases} \\
&+ (1-\phi) I_x r_x \\
=& r\left(Z_{\alpha,m} + D - I_x - 2L(1-\phi)\right) \\
&+ \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases} \\
\implies& Z_{\alpha,m} + D \geq I_x + 2L(1-\phi) - \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases}
\end{aligned}$$

Combining the two, we get

$$2I_x + \begin{cases} 2I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases}$$

$$\geq I_x + 2L(1-\phi) - \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases}$$

$$\implies 0 \leq I_x - 2L(1-\phi) + \begin{cases} 3I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases}$$

Under $Lr < I_x r_x$, we get

$$0 \leq I_x - 2L(1-\phi) + \frac{Lr}{r_x} < I_x - 2L(1-\phi) + I_x < 0$$

So we must have $Lr > I_x r_x$. Then $2I_x \geq L(1-\phi)$. But then $2I_x r_x \geq L(1-\phi)r_x > 2Lr$, by Assumption 3. Hence $I_x r_x > Lr$. This is a contradiction.

So we must have $I_x r_x = Lr$. This implies that $I_x \neq 0$. Then the right partial derivative of the profit w.r.t. I_x must be negative and left partial derivative of the profit w.r.t. I_x must be positive. In particular, the right derivative is

$$r_x \left(Z_{\alpha,m} + D - 2I_x\right) - Lr$$

and the left derivative is

$$r_x \left(Z_{\alpha,m} + D - 2I_x\right) - 2I_x r_x$$

Then the left derivative is smaller than the right derivative. Contradiction. So the optimal portfolio satisfies $I_x \geq L(1-\phi) = L - \Phi_y$. By $(1-\phi)r_x > r$ this further implies that $I_x r_x > Lr$. □

Proof. **(Proof of Proposition 1 and Proposition 2)**

Proposition 1 is simply a corollary of Proposition 2, obtained by replacing m with 0, so we provide the proof for Proposition 2.

By the proof of Lemma 1 above, which allows for $m \geq 0$, we have $\Phi_y = L\phi$ and $I_x > L(1-\phi)$. Then the ex-post profit for bank x is given by

$$\pi_x = \begin{cases} I_x r_x + Lr & if\, 0 \leq \zeta \leq m + \Phi_x + L\phi \\ I_x r_x & if\, m + \Phi_x + L\phi < \zeta \leq m + \Phi_x + L \\ Lr & if\, m + \Phi_x + L < \zeta \leq m + \Phi_x + L\phi + I_x \\ 0 & if\, m + \Phi_x + L\phi + I_x < \zeta \end{cases}$$

The expected profit is

$$\Pi_x \underset{(I_x,L)}{\propto} r_x \left(Z_{\alpha,m} + D - I_x\right) I_x + 2(1-\phi) r \left(\frac{Z_{\alpha,m}+D}{2(1-\phi)} - L\right) L$$

The unconstraint maximizer is

$$L = \frac{Z_{\alpha,m}+D}{4(1-\phi)}, \quad I_x = \frac{Z_{\alpha,m}+D}{2}$$

At these values, $L, I_x \geq 0$ and $I_x \geq L(1-\phi)$ hold. The remaining constraint is

$$D \geq L + I_x \iff \alpha \geq \frac{Z}{Z + D\left(\frac{1-2\phi}{3-2\phi}\right) - m}$$

(Note that this lower bound is less than $\overline{\alpha}$ for $m = 0$ if $\frac{r}{r_y} \geq \frac{2(1-2\phi)(1-\phi)}{3-2\phi}$, which makes this region of parameters is non-empty for $m = 0$. This guarantees that the following regions are also non-empty for $m = 0$. As m grows, it is natural that some regions become obsolete in the pecking order.)

Next consider $\alpha < \frac{Z}{Z+D\left(\frac{1-2\phi}{3-2\phi}\right)-m}$ $(D\frac{1-2\phi}{3-2\phi} < Z_{\alpha,m})$. The constraint $I_x + L \leq D$ binds. Under constraint $I_x = D - L \in [0, D]$, the FOC gives

$$\frac{d\Pi_x}{dL} = 0 \implies I_x = \frac{D\left(4(1-\phi)r + r_x - r\right) + Z_{\alpha,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}$$

As $r_x > r$ we have $L \leq D$ and $I_x \geq 0$. On the other hand

$$L \geq 0 \iff \alpha \geq \frac{Z}{Z + D\frac{r_x+r}{r_x-r} - m}$$

This also ensures $I_x \leq D$. The last constraint $I_x \geq L(1-\phi)$ holds trivially.

Finally, under $\alpha < \frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-m}$ $(D\frac{r_x+r}{r_x-r} < Z_{\alpha,m})$, we have $L = 0$ and $I_x = D$.

Summarizing these:

1. If $\overline{\alpha} \geq \alpha > \frac{Z}{Z+D\left(\frac{1-2\phi}{3-2\phi}\right)-m}$ $(D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m})$,

$$I_x = \frac{D + Z_{\alpha,m}}{2}, \quad L = \frac{D + Z_{\alpha,m}}{4\left(1-\phi\right)}, \quad \Phi_x = D - I_x - L > 0$$

2. If $\frac{Z}{Z+D\left(\frac{1-2\phi}{3-2\phi}\right)-m} > \alpha > \frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-m}$ $(D\frac{r_x+r}{r_x-r} > Z_{\alpha,m} > D\frac{1-2\phi}{3-2\phi})$ then

$$I_x = \frac{D\left(4(1-\phi)r + r_x - r\right) + Z_{\alpha,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}$$

$$L = \frac{D\left(r_x + r\right) - Z_{\alpha,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}, \quad \Phi_x = 0$$

3. If $\frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-m} > \alpha$ $(Z_{\alpha,m} > D\frac{r_x+r}{r_x-r})$, then

$$I_x = D, \quad L = 0, \quad \Phi_x = 0.$$

□

Proof. **(Proof of Proposition 3)** Note that there is some inconsequential multiplicity in the amount of ex-post short term borrowing. As the short-term borrowing is risk-free in the model, for simplicity, we have assumed away interest on it. For robustness, we assume the smallest amount of short-term borrowing to meet the shock takes place. If $\zeta < \Phi_x$, there is no need for short-term borrowing. For $\Phi_x < \zeta \leq \Phi_x + \Phi_y + m$, y can lend the shortage $\zeta - \Phi_x$ to x to avoid liquidations. If $\zeta > \Phi_x + \Phi_y + m$, liquidation is inevitable. If $\Phi_x + L + m > \zeta > \Phi_x + \Phi_y + m$, x liquidates L. This gives L extra liquidity to x on top of its reserves Φ_x. Bank x can still borrow m from y in this case. But if $\zeta < L + \Phi_x$, x does not need to borrow from y. Only when $\zeta > L + \Phi_x$, there is borrowing from y at the amount of shortage $\zeta - L - \Phi_x$. Therefore, when $\Phi_x + L + m > \zeta > \max\{\Phi_x + \Phi_y + m, L + \Phi_x\}$, there is $\zeta - L - \Phi_x$ borrowing. Continuing with the same logic, we find that the ex-post amount of short-term borrowing by x from y under m is given by

$$b = \begin{cases} \zeta - \Phi_x & \text{if } \Phi_x < \zeta \leq \Phi_x + \Phi_y + m \\ \zeta - L - \Phi_x & \text{if } \Phi_x + \max\{L, \Phi_y + m\} < \zeta \leq \Phi_x + L + m \\ \zeta - I_x - \Phi_x & \text{if } \Phi_x + \max\{I_x, L + m\} < \zeta \leq \Phi_x + I_x + \Phi_y + m \\ 0 & \text{otherwise} \end{cases}$$

The expectation of this w.r.t. ζ is

$$B = \frac{\alpha}{Z}\left(2(m + \Phi_y)^2 + m^2 - \max\{0, m + \Phi_y - L\}^2 - \max\{0, m + L - I_x\}^2\right)$$

Under $D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m}$, this is

$$B = \frac{\alpha}{2Z}\left(2(m+\Phi_y)^2 + m^2 - \max\left\{0, m-(1-\phi)L\right\}^2 - \max\left\{0, m-(1-2\phi)L\right\}^2\right)$$

Note that $D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m}$ implies $L = \frac{D+Z_{\alpha,m}}{4(1-\phi)} > \frac{Z_{\alpha,m}}{1-2\phi} > \frac{m}{1-2\phi}$. So $B = \frac{\alpha}{2Z}\left(2(m+\Phi_y)^2 + m^2\right)$ which is increasing in m.

For the case of $D\frac{1-2\phi}{3-2\phi} < Z_{\alpha,m}$, note that B is continuous in m. Also, the negative terms $\max\left\{0, m+\Phi_y - L\right\}$ and $\max\left\{0, m+L-I_x\right\}$ are increasing in m. So if

$$2(m+\Phi_y)^2 + m^2 - (m+\Phi_y-L)^2 - (m+L-I_x)^2$$

is increasing in m, then B is increasing in m. The derivative of this expression w.r.t. m is 2 times

$$2(m+\Phi_y)\left(1+\phi\frac{dL}{dm}\right) + m - (m+\Phi_y-L)\left(1-(1-\phi)\frac{dL}{dm}\right) - (m+L-I_x)\left(1+\frac{dL}{dm}-\frac{dI_x}{dm}\right)$$

Under $Z_{\alpha,m} > D\frac{r_x+r}{r_x-r}$ this is

$$2(m+\Phi_y) + m - (m+\Phi_y-L) - (m+L-I_x) = m+\Phi_y+I_x > 0$$

Under $D\frac{r_x+r}{r_x-r} > Z_{\alpha,m} > D\frac{1-2\phi}{3-2\phi}$ this is

$$\begin{aligned} &2(m+\Phi_y)\left(1+\phi\frac{dL}{dm}\right) + m - (m+\Phi_y-L)\left(1-(1-\phi)\frac{dL}{dm}\right) - (m+2L-D)\left(1+2\frac{dL}{dm}\right) \\ >&D\left(1-\frac{r_x-r}{r_x+2(1-\phi)r}\right) + L\left(\frac{(5-2\phi)(r_x-r)}{2(r_x+2(1-\phi)r)} - (1-\phi)\right) > 0 \end{aligned}$$

Thus, B is continuous and increasing.

□

Proof. **(Proof of Proposition 4)**

Now suppose that m is independently drawn from distribution F_m with support $[0, \overline{m}]$ and mean m^*. Assume $\overline{m} < Z - D$.

In principle, stochastic m could complicate the algebra dramatically. But, as the shocks can always be larger than the shocks the all results regarding portfolios still hold with m^* instead of m. In order to formalize this, go back to the liquidations induced by the optimal behavior of x after the shock, as outlined in the proof of Lemma 1. The last region of the shock where both project are liquidated is given by $\max\{I_x, I_y\} < \zeta' = \zeta - m - \Phi_x - \Phi_y$. This is, $\zeta > \max\{I_x, I_y\} + m + \Phi_x + \Phi_y$. By $\overline{m} < Z - D$,

$$\max\{I_x, I_y\} + m + \Phi_x + \Phi_y < \max\{I_x, I_y\} + Z - D + \Phi_x + \Phi_y < Z$$

Therefore, there is positive probability that both project get liquidated regardless of the portfolio. So all regions of shocks in the cases for liquidations have positive probability. Then the expected payoffs are given by

$$\begin{aligned}
\frac{Z}{\alpha}\mathbb{E}_m\left[\Pi_x\right] &= \left(Z_{\alpha,0} + m^* + D - I_x - I_y\right)\left(I_x r_x + Lr\right) + \min\{I_x, I_y\}\max\left\{Lr, I_x r_x\right\} \\
&\quad + \left(\max\{I_x, I_y\} - \min\{I_x, I_y\}\right)\min\{I_x, I_y\} r_{\arg\min_z I_z} \\
\frac{Z}{\alpha}\mathbb{E}_m\left[\Pi_x\right] &= \left(I_y r_y - Lr\right)\Big[\left(Z_{\alpha,0} + m^* + D - I_x - I_y\right) + \mathbf{1}_{I_x r_x < Lr}\min\{I_x, I_y\} \\
&\quad + \mathbf{1}_{I_x < L}\left(\max\{I_x, I_y\} - \min\{I_x, I_y\}\right)\Big]
\end{aligned}$$

So the solution is identical, just by replacing m with m^* now.

For closed form results we suppose that m is 0 w.p. β and $U[0, \frac{2m^*}{1-\beta}]$ w.p. $1-\beta$ where $m^* < \frac{1-\beta}{2}(Z - D)$. Note that this has mean m^*.

We first consider the event that all funded projects get liquidated, which we call systemic risk. This is, $\zeta' > I_x$. (Under $D\frac{r_x+r}{r_x-r} \geq Z_{\alpha,m^*}$ y's project is indeed funded. Otherwise, the only funded project is x's.) Systemic risk is

$$\begin{aligned}
&\frac{\alpha}{Z}\left(Z - D + (1-\phi)L - m^*\right) \\
&\underset{(m^*,\beta)}{\propto} - m^* + (1-\phi)\begin{cases} \frac{D + Z_{\alpha,m^*}}{4(1-\phi)} & \text{if } D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m^*} \\ \frac{D(r_x+r) - (r_x-r)Z_{\alpha,m^*}}{2(r_x+2(1-\phi)r)} & \text{if } D\frac{r_x+r}{r_x-r} > Z_{\alpha,m^*} > D\frac{1-2\phi}{3-2\phi} \\ 0 & \text{if } Z_{\alpha,m^*} > D\frac{r_x+r}{r_x-r} \end{cases}
\end{aligned}$$

The first term $-m^*$ is the direct effect of the availability of public liquidity. This has a natural effect of reducing the risk of liquidations. The second term after the bracket is the equilibrium effect of public liquidity. The availability of public liquidity influences the availability of private liquidity in the system through the portfolio choices, in particular, through L. The equilibrium effect increases in m^* up to $D\frac{1-2\phi}{3-2\phi} - Z_{\alpha,0}$ and decreases afterwards. The net effect is always to reduce systemic risk.

Next consider contagion risk, the probability that the project of y gets liquidated. This event is the union of $\zeta' > I_x$ (systemic risk) and $0 < \zeta' \leq L(1-\phi)$, "only-contagion." The probability of only-contagion is $\frac{\alpha}{Z}L(1-\phi)$. This is increasing in m^* for $m^* < D\frac{1-2\phi}{3-2\phi} - Z_{\alpha,0}$ and decreasing afterwards m^*. We have already calculated systemic risk. Then contagion risk is

$$\frac{\alpha}{Z}\left(Z - D + 2(1-\phi)L - m^*\right) \underset{(m^*,\beta)}{\propto} 2(1-\phi)L - m^*$$

This is always decreasing in m^*.

Now consider direct risk, the probability that the project of x gets liquidated. This event is given by $L(1-\phi) < \zeta'$. The part $I_x < \zeta'$ is the systemic risk. The part of $L(1-\phi) < \zeta' \leq I_x$ is "only-direct-risk." Only-direct-risk is given by

$$\begin{aligned}&\frac{\alpha}{Z}\left(I_x - L(1-\phi)\right)\\ \underset{(m^*,\beta)}{\propto}&\begin{cases}\frac{D+Z_{\alpha,m^*}}{4} & \text{if } D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m^*}\\ \frac{D(3(1-\phi)r - r + \phi r_x) + (2-\phi)(r_x - r)Z_{\alpha,m^*}}{2(r_x+2(1-\phi)r)} & \text{if } D\frac{r_x+r}{r_x-r} > Z_{\alpha,m^*} > D\frac{1-2\phi}{3-2\phi}\\ D & \text{if } Z_{\alpha,m^*} > D\frac{r_x+r}{r_x-r}\end{cases}\end{aligned}$$

This is always increasing in m^*. The public liquidity always increases the only-direct-risk. This is perhaps particularly relevant for the Great Depression. The combined direct-risk to x is

$$\begin{aligned}&\frac{\alpha}{Z}\left(Z - D + (1-\phi)L - m^* + I_x - L(1-\phi)\right)\\ \underset{(m^*,\beta)}{\propto}& -m^* + \begin{cases}\frac{D+Z_{\alpha,m^*}}{2} & \text{if } D\frac{1-2\phi}{3-2\phi} \geq m^* + Z_{\alpha,0}\\ \frac{D(4(1-\phi)r + r_x - r) + Z_{\alpha,m^*}(r_x - r)}{2(r_x+2(1-\phi)r)} & \text{if } D\frac{r_x+r}{r_x-r} > m^* + Z_{\alpha,0} > D\frac{1-2\phi}{3-2\phi}\\ D & \text{if } m^* + Z_{\alpha,0} > D\frac{r_x+r}{r_x-r}\end{cases}\end{aligned}$$

This is always decreasing in m^*. The public liquidity always reduces the direct-risk to x.

Finally, we consider vulnerability, that is, the risks conditional on $m = 0$. Systemic vulnerability is given by

$$\frac{\alpha}{Z}\left(Z - D + (1-\phi)L\right)$$

This is increasing in m^* for small m^* and decreasing for large m^*. Contagion vulnerability is

$$\frac{\alpha}{Z}\left(Z - D + 2(1-\phi)L\right)$$

This is also increasing in m^* for small m^* and decreasing for large m^*. Direct vulnerability is

$$\frac{\alpha}{Z}\left(I_x - L(1-\phi)\right)$$

This is always increasing in m^*.

□

Proof. **(Proof of Proposition 5)**

Now there is θ probability that x_i gets a shock. Then Proposition 1 goes through by replacing α with θ. Note that $\frac{Z}{Z+D\left(\frac{1-2\phi}{3-2\phi}\right)-m} > \frac{1}{2} > \theta$ so we do not have the region in which $\Phi_{x_i} > 0$.

Now suppose that the the core banks can borrow each others reserves. We assume $Z > 2D+m$ so that the shock can always be larger than the total cash in the system and we can avoid corner cases. For the pair i, the cash reserves of y_i act as an addition to m. Also note that x_i and x_j do not keep reserves and so we do not need to worry about x_i short-term lending to y_i and y_i intermediating this to y_j. Thus, for x_i, the best response is given by

$$L_i = \left(\frac{D\left(r_x + r\right) - \left(Z_{\theta,m} + L_j\phi\right)\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}\right)_+$$

The symmetric equilibrium is given by

$$L = \left(\frac{D\left(r_x + r\right) - \left(Z_{\theta,m} + L\phi\right)\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}\right)_+$$

1. If $\frac{1}{2} > \theta > \frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-m}$ $\left(D\frac{r_x+r}{r_x-r} > Z_{\theta,m}\right)$ then

$$L_{x_i} = \frac{D\left(r_x + r\right) - Z_{\theta,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right) + \phi(r_x - r)}, \quad I_x = D - L$$

2. If $\frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-m} > \theta$, $\left(D\frac{r_x+r}{r_x-r} < Z_{\theta,m}\right)$ then

$$L_{x_i} = 0, \;\; I_{x_i} = D$$

□

Proof. **(Proof of Lemma 2)**

For $D\frac{r_x+r}{r_x-r} < Z_{\theta,m}$, $L_C = L_D = 0$. There is no network. So consider the region $D\frac{r_x+r}{r_x-r} > Z_{\theta,m}$. From the earlier analysis we know that if both banks connect to their regional correspondents, in equilibrium,

$$\frac{Z}{\alpha}\Pi_{x_i}^{C} = \left(Z_{\theta,m} + D - I_x\right) I_x r_x + \left(Z_{\theta,m} + D - 2(1-\phi)L\right) Lr$$

where

$$L_C = \frac{D\left(r_x + r\right) - Z_{\theta,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}$$

If both regions connect to NY, in equilibrium,

$$\frac{Z}{\alpha}\Pi_{x_i}^{N} = \left(Z_{\theta,m} + \phi L + D - I_x\right) I_x r_x + \left(Z_{\theta,m} + \phi L + D - 2(1-\phi)L\right) Lr - c(L)$$

where

$$L_N = \frac{D\left(r_x + r\right) - Z_{\theta,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right) + \phi(r_x - r)}$$

Note

$$\frac{d\left(\frac{Z}{\alpha}\Pi_{x_i}^{C}\right)}{dm} = I_{x,C} r_x + L_C r$$

and

$$\frac{d\left(\frac{Z}{\alpha}\Pi_{x_i}^{N}\right)}{dm} = \left(I_{x,N} r_x + L_N r\right)\left(1 + \phi\frac{dL_N}{dm}\right)$$

Denote $A = 2\left(r_x + 2(1-\phi)r\right)$ and $B = D\left(r_x + r\right) - Z_{\theta,m}\left(r_x - r\right)$. Then

$$\begin{aligned}
\frac{Z}{\alpha}\frac{d\left(\Pi_{x_i}^{C}\right)}{dm} &= Dr_x - (r_x - r)\frac{B}{A} \\
\frac{Z}{\alpha}\frac{d\left(\Pi_{x_i}^{N}\right)}{dm} &= \left(Dr_x - (r_x - r)\frac{B}{A + \phi(r_x - r)}\right)\left(\frac{A}{A + \phi(r_x - r)}\right) \\
\frac{Z}{\alpha}\left(\frac{d\left(\Pi_{x_i}^{C}\right)}{dm} - \frac{d\left(\Pi_{x_i}^{N}\right)}{dm}\right) &= Dr_x - (r_x - r)\frac{B}{A} \\
&> \frac{Dr(r_x - r)\phi}{A + \phi(r_x - r)} > 0
\end{aligned}$$

□

Proof. **(Proof of Proposition 6)**

Since the difference in the derivative is bounded away from zero, as m grows, $\Pi_{x_i}^{C}$ exceeds $\Pi_{x_i}^{N}$ eventually. The switching point m_c depends on the fixed cost c as well. If the cost c is very large, the stable network is regional for all m. In this case, $m_c = D\frac{r_x+r}{r_x-r} - Z\frac{1-\theta}{\theta}$. If c is very small, the stable network is central for all m. Then $m_c = 0$. In between as c grows, m_c grows from 0 to $D\frac{r_x+r}{r_x-r} - Z\frac{1-\theta}{\theta}$.

□