NBER WORKING PAPER SERIES

RISK SHARING EXTERNALITIES

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Working Paper 26985
http://www.nber.org/papers/w26985

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2020, Revised February 2023

We thank for useful comments Adrien Auclert, Aniket Baksy, Javier Bianchi, Eduardo Dávila, Sebastian Di Tella, Alessandro Dovis, Mark Gertler, Alberto Martin, Adriano Rampini, Chris Tonetti and seminar participants at Princeton, Yale, Boston University, EIEF, Columbia, Stanford, Boston College, AEA meetings and the SED conference. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.¸

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JEL No. E44,G01,G11

ABSTRACT

Financial crises typically arise because firms and financial institutions choose balance sheets that expose them to aggregate risk. We propose a theory to explain these risk exposures. We study a financial accelerator model where entrepreneurs can issue state-contingent claims to consumers. Even though entrepreneurs could use these contingent claims to hedge negative shocks, we show that they tend not to do so. This is because it is costly to buy insurance against these shocks as consumers are also harmed by them. This effect is self-reinforcing, as the fact that entrepreneurs are unhedged amplifies the negative effects of shocks on consumers' incomes. We show that this feedback can be quantitatively important and lead to inefficiently high risk exposure for entrepreneurs.

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1 Introduction

The exposure of financial institutions to risks from the subprime mortgage market is widely seen as a root cause of the financial crisis of 2008-2009. This exposure created the potential for shocks in the housing market to be heavily amplified, as recognized early on by Greenlaw, Hatzius, Kashyap, and Shin (2008). Why did banks not do more to protect their balance sheet, say by shedding some of their riskier positions or by choosing a safer funding structure? More generally, why were these risks not better spread across the economy?

Spurred by the global financial crisis, economists have developed models in which balance sheet losses of financial institutions can negatively affect firms' hiring and investment decisions—for example, Gertler and Kiyotaki (2010), Jermann and Quadrini (2012), He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014). These contributions provide the framework now commonly used to quantify the importance of financial factors over the business cycle, and to design appropriate policy responses. However, these models sidestep the questions raised above, by assuming that the "specialists"—the agents that represent financial institutions—have limited risk-management tools. In particular, a common assumption in these models is that specialists hold only one risky asset and issue non-state-contingent debt, so that their risk exposure is mechanically linked to their leverage. In this paper, we break this tight link by allowing specialists to issue fully state-contingent debt and study why they choose to be exposed to aggregate risk, whether this exposure is socially efficient, and if not what is the appropriate policy response.

Our paper makes two contributions. First, we offer an explanation of why specialists are exposed to aggregate risk. Our mechanism builds on a general equilibrium effect: when the net worth of specialists falls and the economy experiences a financial crisis, the income of all other agents contracts as well. Due to this feature, insuring these states of the world ex-ante is costly, and this reduces the specialists' incentive to hedge. Second, we show that equilibrium risk-management is sub-optimal from the point of view of social welfare, and study corrective policies. The optimal policy requires taxing differentially debt to be repaid in bad states, and the associated welfare gains cannot be achieved by a simpler, non-state-contingent tax on borrowing.

We develop these arguments in the context of a model with two groups of agents, consumers and entrepreneurs. Entrepreneurs are the specialists, and we can think of them as representing a sector that consolidates financial institutions and the non-financial firms that borrow from them. Entrepreneurs borrow from consumers to finance their purchases of factors of production, capital and labor. The source of risk in the economy is a shock that affects the "quality" of capital held by the entrepreneurs, as in Gertler and Karadi (2011)

and Brunnermeier and Sannikov (2014). Due to limited enforcement, the entrepreneurs face an upper bound on their ability to raise funds from consumers. This implies that reductions in the aggregate net worth of the entrepreneurs can lead to a contraction in economic activity and in the labor income of consumers. This is the general equilibrium effect, or "macro spillover" at the core of our positive and normative results.

The entrepreneurs in our model can issue a full set of state-contingent claims. This assumption is meant to capture a variety of ways in which financial institutions can make their balance sheet less exposed to aggregate shocks, for example, by choosing between debt and equity financing, by choosing debt of different maturities, debt denominated in different currencies, taking derivative positions, etc. By appropriately using state-contingent claims, the entrepreneurs can hedge their net worth against aggregate shocks. For example, they can promise smaller payments to consumers when the economy is hit by a negative shock. This would imply that consumers bear more aggregate risk, and would stabilize entrepreneurs' net worth. A more stable net worth would dampen financial amplification in the economy.

We start by studying the positive implications of the model, focusing on the equilibrium allocation of risk between consumers and entrepreneurs. We show that the elasticity of entrepreneurs' net worth to aggregate shocks depends on two key model ingredients: the strength of the macro spillover described above, and the risk aversion of consumers. The macro spillover implies that states of the world in which the entrepreneurs have low net worth are also states in which the consumers have low labor income. Risk aversion implies that consumers demand a premium for bearing risk in these states of the world. These two ingredients, combined, make it costly for entrepreneurs to hedge.

We first show this result theoretically, in a special case of our model that is analytically tractable. Next, we show that this mechanism can be quantitatively strong and produce a large exposure of entrepreneurs to aggregate risk. Specifically, under plausible calibrations our economy with state-contingent debt produces an elasticity of entrepreneurial net worth to aggregate shocks and a degree of financial amplification that is quantitatively comparable to those obtained in the corresponding economy where entrepreneurs can only issue non-state-contingent debt. These results are not driven by the type of aggregate shocks we consider, as we obtain very similar results when the aggregate shock affects the pledgeability of capital as in Jermann and Quadrini (2012), rather than the capital stock.

The presence of the macro spillover not only hinders risk sharing between consumers and entrepreneurs, but also generates a pecuniary externality that makes the privately optimal portfolio choices of the agents socially inefficient. To understand the source of this externality, consider the problem of consumers. When choosing their financial assets,

they do not understand that any payment received in a given state of the world reduces the net worth of entrepreneurs, and it negatively affects the current and future wages of consumers if the collateral constraint binds. Because consumers fail to internalize these negative spillovers, they overvalue payments received in these states relative to what a social planner would do. So, the interest rate for debt instruments indexed to these states is inefficiently low, and these low interest rates induce entrepreneurs to take on excessive risk.

In the last part of the paper, we study the optimal policy of a social planner that can impose Pigouvian taxes on the state-contingent claims issued by the entrepreneurs. We show that the optimal policy does not tax debt uniformly. Rather, it levies higher taxes on debt instruments indexed to states in which collateral constraints are tighter. These policies are successful in reducing the risk exposure of the entrepreneurs, and the resulting equilibrium features less financial amplification.

We finally contrast the optimal policy with a policy that taxes all debt instruments uniformly. These taxes could reduce the risk exposure of entrepreneurs because they reduce their incentives to issue debt. However, entrepreneurs respond by cutting mostly debt indexed to good states of the world, so their overall risk exposure changes little. That is, these tools are effective in reducing leverage, but they generate an incentive for entrepreneurs to substitute toward riskier types of debt. These substitution effects provide a cautionary tale for macroprudential tools that target leverage uniformly.

Literature. This paper is related to the large literature on the role of financial factors in the amplification and propagation of aggregate shocks. This literature goes back to the seminal contributions of Bernanke and Gertler (1986), Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999), and has been very active following the global financial crisis. The logic of financial amplification in these models builds on two main assumptions: the presence of a financial constraint and incomplete financial markets. The first assumption implies that aggregate shocks affecting the net worth of specialists propagate to the rest of the economy, while the second assumption restricts the ability of the specialists to hedge aggregate shocks ex ante.

Important contributions in this literature show that the assumed incompleteness of financial markets is critical for financial amplification. Krishnamurthy (2003) introduces state-contingent claims in a three period version of Kiyotaki and Moore (1997) and shows that the amplification mechanism disappears, as specialists perfectly hedge their net worth. Di Tella (2017) shows an analogous result in the context of a dynamic model similar to Brunnermeier and Sannikov (2014). Our incomplete hedging results may appear surprising in

light of these contributions. However, as argued above, our results require two ingredients: risk averse consumers and an active macro spillover. One or both of these ingredients are muted in these papers. Other papers that find limited amplification in more quantitative models are Carlstrom, Fuerst, and Paustian (2016), Dmitriev and Hoddenbagh (2017) and Cao and Nie (2017). The mechanism identified in our paper is potentially at work in those models, but—as we discuss in Section 4—their calibrations make it quantitatively weak.

The literature has explored other mechanisms to explain why specialists are exposed to aggregate risk even if they can hedge it. Some papers explore different types of shocks. Di Tella (2017) obtains imperfect hedging in response to shocks to idiosyncratic volatility, while Dávila and Philippon (2017) obtain it in response to shocks to the degree of financial market completeness. Other papers look at alternative models of the financial friction. In particular, Rampini and Viswanathan (2010)'s imperfect hedging result relies on the collateral constraint being always binding and on collateral values being insensitive to the shock, while Asriyan (2018) obtains imperfect hedging by combining information and trading frictions, which leads to distorted state prices. A large literature, including Schneider and Tornell (2004) and Farhi and Tirole (2012), emphasizes the possibility of collective moral hazard, whereby specialists choose to be exposed to aggregate risk, given the expectation of government bailouts when a large enough number of them is in trouble. Finally, a recent literature focuses on neglect of downside risks, coming from deviations from rational expectations, e.g., Bordalo, Gennaioli, and Shleifer (2018) and Farhi and Werning (2020). Our approach emphasizes a simple general equilibrium spillover from specialists to the rest of the economy and we see it as complementary to these other approaches.

Our welfare analysis is related to the large literature on inefficiencies and pecuniary externalities in models with financial market imperfections, going back to Geanakoplos and Polemarchakis (1986) and Kehoe and Levine (1993). The pecuniary externality that matters in our model is "distributive"—using the language introduced by Dávila and Korinek (2018)—and works through wages and labor income. This connects our paper to Caballero and Lorenzoni (2014), Bianchi (2016) and Itskhoki and Moll (2019), although we are the first to explore the implications that this type of pecuniary externality has on risk sharing.

A number of papers study models in which constrained inefficiency takes the form of excessive leverage and derive implications for macroprudential policy, see Bianchi and Mendoza (2018) and reference therein. Unlike many of these papers, we study an environment where the specialists can issue multiple types of debt rather than just a non-contingent bond and study the optimal policy when the planner can tax differently these debt instruments. Lorenzoni (2008) and Korinek (2018) also allow for state-contingent claims and different Pigouvian taxation on these claims. Differently from them, we study optimal policy in an

environment where risk premia are endogenous and we compare it to a more restricted policy that taxes these instruments uniformly. A key insight from our analysis is that, in presence of state-contingency, some simple policies, like a restriction on total leverage, may be ineffective in reducing risk taking or can even backfire and lead to increased risk.

Our emphasis on inefficient risk taking also connects our paper to work by Farhi and Werning (2016) that focuses on the effects of consumers' risk taking on the demand for goods in environments with nominal rigidities. Their analysis shows the possibility of excessive risk taking due to an aggregate demand externality. We emphasize instead the risk taking by the financial and corporate side of the economy, and its transmission through labor demand and the equilibrium volatility of labor income.[1]

Finally, the macro spillover that plays a central role in this paper was also present in our previous work on self-fulfilling currency crises (Bocola and Lorenzoni, 2020). However, the analysis of how that spillover affects amplification and efficiency is novel to this paper.

Layout. The paper is organized as follows. Section 2 introduces the model. Section 3 studies a special case that is analytically tractable. Section 4 presents numerical results for a calibrated version of the model. Section 5 presents the welfare analysis and its implications for macroprudential policies. Section 6 concludes.

2 Model

We consider an economy populated by two groups of agents of equal size: consumers and entrepreneurs. Entrepreneurs accumulate capital, that is used together with labor to produce the final good, and they issue financial claims. Consumers earn labor income and buy financial claims from entrepreneurs. Financial claims are state-contingent promises to repay one unit of consumption in the next period. We now describe the details of the environment and define an equilibrium.

2.1 Environment

Technology and shocks. Time is discrete and indexed by $t = 0, 1, 2 \ldots$. Uncertainty is described by a Markov process that takes finite values in the set $\mathcal{S}$. We denote by s_t the state of the process at time t, and by $s^t = (s_0, s_1, \ldots, s_t)$ the history of states up to period t. The process for s_t is given by the transition matrix $\pi(s_{t+1}|s_t)$.

[1] See also Dávila and Korinek (2018) for a general analysis of the role of state contingent wedges in models of pecuniary externalities.

The capital stock is subject to random depreciation captured by the stochastic parameter u_t. Namely, k_{t-1} units of capital accumulated at the end of time $t-1$ yield $u_t k_{t-1}$ units of capital that can be used in production at time t and a residual stock of $(1-\delta)u_t k_{t-1}$ units of capital after production. The parameter u_t depends on the state of the Markov process according to the function $u_t = u(s_t)$, and is the only exogenous source of uncertainty in the model. The variable u_t is similar to the capital quality shock used in Gertler and Karadi (2011) and Brunnermeier and Sannikov (2014).

Entrepreneurs have exclusive access to the technology that allows capital accumulated in period $t-1$ to be productive in period t, so all capital is held by entrepreneurs in equilibrium. The entrepreneurs use capital and labor services provided by consumers to produce final goods according to the Cobb-Douglas production function:

$$y_t = (u_t k_{t-1})^{\alpha} l_t^{1-\alpha}.$$

The labor market is perfectly competitive, and the wage rate is w_t. We assume that entrepreneurs need to pay a fraction γ of the wage bill before their revenues are realized. This assumption ensures that the financial conditions of entrepreneurs can have a contemporaneous effect on labor demand, see Jermann and Quadrini (2012).

All equilibrium variables are in general functions of the history s^t, but whenever no confusion is possible we leave this dependence implicit in the subscript t.

Preferences. Entrepreneurs have log-preferences over consumption streams $\{c_{e,t}\}$, so they maximize

$$\mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta_e^t \log(c_{e,t}) \right].$$

Consumers have Epstein-Zin preferences, so their utility is defined recursively as

$$V_t = \left\{ (1-\beta) x_t^{1-\rho} + \beta \left[\mathbb{E}_t (V_{t+1}^{1-\sigma}) \right]^{\frac{1-\rho}{1-\sigma}} \right\}^{\frac{1}{1-\rho}},$$

where x_t is given by

$$x_t = c_t - \chi \frac{l_t^{1+\psi}}{1+\psi}.$$

This specification of the consumers' utility eliminates the wealth effect on labor supply as in Greenwood, Hercowitz, and Huffman (1988).

Financial markets and limited commitment. Each period agents trade a full set of one-period state-contingent claims. Let $q(s_{t+1}|s^t)$ be the price at time t of a claim that pays one unit of consumption at $t+1$, conditional on history $s^{t+1} = (s^t, s_{t+1})$. We denote by $a(s^t)$ the claims held by consumers at the beginning of period t. Similarly, $b(s^t)$ denote the claims owed by entrepreneurs at the beginning of the period. Market clearing requires that

$$a(s^t) = b(s^t)$$

for every history s^t.

Entrepreneurs enter period t with $u_t k_{t-1}$ units of capital (in efficiency units) and with debt b_t. Each period t is divided in three stages. In the first stage, entrepreneurs hire workers and issue within-period debt to pay for a fraction γ of their wage bill $w_t l_t$. In the second stage, production takes place, goods are sold, entrepreneurs pay the remaining fraction of the wage bill $(1-\gamma)w_t l_t$ and decide whether to repay their total liabilities $b_t + \gamma w_t l_t$ or to default. If they default, entrepreneurs can hide the firms' profits and a fraction $1-\theta$ of the undepreciated capital stock and start anew with initial wealth

$$y_t - (1-\gamma)w_t l_t + (1-\theta)(1-\delta)u_t k_{t-1}.$$

In the third and last stage, entrepreneurs issue new liabilities $b(s^{t+1})$ and use these resources along with their net worth to buy capital goods.[2]

Notice that we assume that an entrepreneur who defaults is not excluded from financial markets.[3] It follows that the entrepreneur chooses repayment if and only if

$$y_t - w_t l_t - b_t + (1-\delta)u_t k_{t-1} \geq y_t - (1-\gamma)w_t l_t + (1-\theta)(1-\delta)u_t k_{t-1}.$$

Making explicit the dependence on the state of the world, this constraint is equivalent to the state-contingent collateral constraint

$$b(s^t) + \gamma w(s^t) l(s^t) \leq \theta(1-\delta)u(s_t)k(s^{t-1}). \tag{1}$$

[2] If entrepreneurs default we assume that the fraction θ of capital not hidden by the entrepreneurs gets destroyed. Alternative assumptions are possible here, as default only happens off the equilibrium path.

[3] A similar assumption is made in Rampini and Viswanathan (2010) and Cao, Lorenzoni, and Walentin (2019).

2.2 Competitive equilibrium

In a competitive equilibrium, consumers choose sequences for consumption, labor supply and state-contingent claims to maximize their utility subject to the budget constraint

$$c(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)a(s^{t+1}) = w(s^t)l(s^t) + a(s^t)$$

for each history s^t and a no-Ponzi-game condition. The first order condition for $a(s^{t+1})$ takes the form

$$q(s_{t+1}|s^t) = \beta\pi(s_{t+1}|s_t)\left(\frac{x(s^t)}{x(s^{t+1})}\right)^{\rho}\left(\frac{RW(s^t)}{V(s^{t+1})}\right)^{\sigma-\rho}, \tag{2}$$

where $RW(s^t) = \mathbb{E}_t\left[V(s^t, s_{t+1})^{1-\sigma}\right]^{1/(1-\sigma)}$. Optimal labor supply requires

$$\chi l(s^t)^{\psi} = w(s^t). \tag{3}$$

Entrepreneurs choose sequences for consumption, capital, labor demand and state-contingent claims to maximize their utility subject to the collateral constraints (1) and their budget constraint

$$c_e(s^t) + k(s^t) = n(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)b(s^{t+1}),$$

where $n(s^t)$ is the net worth of the entrepreneurs

$$n(s^t) = y(s^t) - w(s^t)l(s^t) + (1-\delta)u(s_t)k(s^{t-1}) - b(s^t). \tag{4}$$

Denoting by $\mu(s^{t+1})$ the Lagrange multiplier on the collateral constraint in state s^{t+1}, we can write the entrepreneurs' first-order conditions for $b(s^{t+1})$ as

$$q(s_{t+1}|s^t)\frac{1}{c_e(s^t)} = \beta_e\pi(s_{t+1}|s_t)\left(\frac{1}{c_e(s^{t+1})} + \mu(s^{t+1})\right). \tag{5}$$

This condition is a standard intertemporal Euler equation with state-contingent debt and collateral constraints.

Combining equations (2) and (5) we obtain:

$$\beta_e\frac{\frac{1}{c_e(s^{t+1})} + \mu(s^{t+1})}{\frac{1}{c_e(s^t)}} = \beta\left(\frac{x(s^t)}{x(s^{t+1})}\right)^{\rho}\left(\frac{RW(s^t)}{V(s^{t+1})}\right)^{\sigma-\rho}. \tag{6}$$

This is the risk sharing condition that determines the allocation of aggregate risk in this

economy. On the right-hand side, there is the consumers' marginal rate of substitution between consumption at time t and consumption at $t+1$ in state s^{t+1}. On the left-hand side there is a similar expression for entrepreneurs: the marginal value of a unit of resources in state s^{t+1} for entrepreneurs includes both the marginal utility of consumption $1/c_e(s^{t+1})$ and the shadow value of relaxing their collateral constraint $\mu(s^{t+1})$.

The optimality conditions for labor and capital take the following form:

$$\frac{1}{c_e(s^t)}\left[(1-\alpha)[u(s_t)k(s^{t-1})]^{\alpha}l(s^t)^{-\alpha}-w(s^t)\right]=\gamma w_t\mu(s^t) \tag{7}$$

$$\frac{1}{c_e(s^t)}=\mathbb{E}_t\left\{\beta_e\frac{1}{c_e(s^{t+1})}\left[\alpha u(s_{t+1})^{\alpha}\left(\frac{l(s^{t+1})}{k(s^t)}\right)^{1-\alpha}+(1-\delta)u(s_{t+1})\right]\right\}+$$
$$+\beta_e\theta(1-\delta)\mathbb{E}_t[u(s_{t+1})\mu(s^{t+1})]. \tag{8}$$

The first condition shows that there is a wedge between the marginal product of labor and the wage if the collateral constraint is binding, because hiring labor requires some capacity to borrow. The second condition is a standard intertemporal condition for capital accumulation. Relative to a frictionless economy, investing in capital has the additional benefit of relaxing the collateral constraints, which is captured by the last term on the right-hand side.[4]

The advantage of assuming log preferences for entrepreneurs is that their consumption function is linear in net worth, $c_e(s^t)=(1-\beta_e)n(s^t)$, irrespective of whether the collateral constraint is binding or not. This property, proved in Online Appendix B, simplifies the analysis of the equilibrium.

An equilibrium is given by sequences of quantities $\{c(s^t),c_e(s^t),k(s^t),l(s^t),a(s^t),b(s^t)\}$ and prices $\{w(s^t),q(s_{t+1}|s^t)\}$ such that the quantities solve the individual optimization problems above and markets clear.

2.3 Discussion

Before moving on, let us discuss some of the simplifying assumptions we made.

First, our model does not feature endogenous asset prices, as the price of capital is always 1. This mutes a canonical feed-back between asset prices and entrepreneurial net worth, which may lead to inefficiently high levels of risk taking, as shown for example in Lorenzoni (2008). In the current paper, we abstract from this channel in order to isolate

[4]This does not mean that more capital is invested relative to an economy without the collateral constraints because, in equilibrium, the collateral constraints also affect the stochastic discount factor of entrepreneurs.

the novel mechanism that works via the endogeneity of labor income. We do not expect endogenous asset prices to substantially change the mechanism investigated here.

Second, the main driving force in the model is a shock to the quality of capital. In our framework, this shock substitutes for the missing volatility of asset prices and allow us to generate sizable movements in the value of assets held by entrepreneurs. As we discuss in Section 4 and in more details in Online Appendix D, our mechanism does not rely on this specific source of risk, and is still present with different types of aggregate shocks.

Finally, entrepreneurs and consumers are assumed to be distinct agents, a fairly common assumption in the literature. There are different ways to interpret this assumption. One is to view the entrepreneurs as the controlling shareholders of the financial firms they represent and to interpret all equity financing they raise as part of the state-contingent claims issued. The other one is to interpret the entrepreneurs as all the shareholders of these firms, with consumers being barred from holding shares. In the second interpretation, it would be interesting to allow for the possibility of issuing shares to all agents, subject to some friction (as for example in Gertler and Kiyotaki (2010)), something we leave to future work.

3 Equilibrium risk sharing and financial amplification

In this section and the next we characterize the risk sharing problem of consumers and entrepreneurs, and show how it affects the economy's response to aggregate shocks. In particular, we study to what extent the effects of the capital quality shocks are amplified due to the presence of the collateral constraint, and how this "financial amplification" depends on the equilibrium allocation of aggregate risk between consumers and entrepreneurs. In this section we consider a simplified version of the model and focus on analytical results. In the next section we go back to the full model and derive numerical results.

We consider a special case of our economy in which all uncertainty is resolved in one period. The economy starts at date 0 with $u_0 = 1$. At $t = 1$ the capital quality u_1 is drawn from a continuous distribution on the interval $[\underline{u}, \overline{u}]$, with density $f(u_1)$. From $t = 2$ on, the capital quality is deterministic and equal to $u_t = 1$. We make some additional simplifying assumptions: entrepreneurs and consumers have the same discount factor, $\beta_e = \beta$, the elasticity of intertemporal substitution is infinite, $\rho = 0$, there is no working capital requirement, $\gamma = 0$, and labor supply is inelastic at $l_t = 1$.

Given the assumptions above, we can characterize an equilibrium in two steps. First, we study the equilibrium from date 1 on, taking as given the equilibrium level of capital and the contingent bonds chosen by entrepreneurs and consumers at date 0, $\{k_0, b_1(u_1), a_1(u_1)\}$. This part of the analysis is standard. Second, we go back to date 0 and study how these

variables are determined in equilibrium. This is the novel part of our analysis.

3.1 Continuation equilibrium

From date 1 on, the economy follows a deterministic path. Since there is no uncertainty and $\rho = 0$, the interest rate is constant and equal to $1/\beta - 1$. In addition, the absence of working capital requirements means that firms are unconstrained in hiring labor, so wages are equal to the marginal product of labor

$$w_t = (1-\alpha)(u_t k_{t-1})^{\alpha}.$$

The dynamics of k_t and n_t are characterized as follows. For a finite number of periods J, the collateral constraint binds and the dynamics of capital and net worth are determined by the recursion:

$$k_t = \frac{\beta n_t}{1-\beta\theta(1-\delta)}, \qquad n_{t+1} = \alpha k_t^{\alpha} + (1-\delta)(1-\theta)k_t, \tag{9}$$

given an initial condition for net worth at date $t = 1$. The first expression in (9) comes from the fact that entrepreneurs save a fraction β of their wealth and they can lever it at most by the factor $1/[1-\beta\theta(1-\delta)]$. The second expression is obtained by combining the definition of net worth in equation (4), the wage derived above, and the binding collateral constraint (1). After J periods, the collateral constraint is slack, n_t is constant in all following periods, and the capital stock reaches the unconstrained level

$$k^* = \left(\frac{\alpha\beta}{1-(1-\delta)\beta}\right)^{\frac{1}{1-\alpha}}. \tag{10}$$

The number of periods J that the economy spends in the constrained region depends on the value of net worth at date 1,

$$n_1(u_1) = \alpha(u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - b_1(u_1). \tag{11}$$

In the above expression, we use $n_1(u_1)$ to denote the equilibrium relation between net worth and the capital quality shock u_1. If $n_1(u_1)$ is above the threshold $n^* = k^*[1-\beta\theta(1-\delta)]/\beta$, then the entrepreneur has enough resources to finance the unconstrained level of capital k^*. In this case, $J = 0$ and the economy reaches the first-best allocation in period 1. Else, $J > 0$, and the economy evolves according to (9).

Because $\rho = 0$, we can use the consumers' intertemporal budget constraint to compute

consumers' utility at $t = 1$:

$$V_1 = (1-\beta)\left[a_1(u_1) + \sum_{t=0}^{\infty} \beta^t w_{t+1}\right]. \tag{12}$$

For future reference, it is useful to split the present value of labor income in two parts, $w_1 = (1-\alpha)(u_1 k_0)^\alpha$ and $W \equiv \sum_{t=1}^{\infty} \beta^t (1-\alpha) k_t^\alpha$. In equilibrium, W is a function only of the entrepreneurs' net worth n_1. We denote this relation by $W(n_1)$. A higher value of n_1 implies a (weakly) higher path of capital accumulation and, therefore, a (weakly) higher path of wages.

The next lemma summarizes the properties of the continuation equilibrium.

Lemma 1 (Continuation equilibrium)**.** *There is a unique continuation equilibrium that only depends on the state variables $k_0, u_1, b_1(u_1)$ and does not depend on the parameter σ. In the continuation equilibrium, the collateral constraint is binding for a finite number of periods J, with $J = 0$ iff $n_1(u_1) \geq n^*$. The present value of future wages at $t = 1$, $W(n_1(u_1))$, is strictly increasing for $n_1 < n^*$ and constant for $n_1 \geq n^*$.*

3.2 Risk sharing at date 0 and financial amplification

Equation (11) shows that the shape of the function $n_1(u_1)$ depens on the portfolio choices of entrepreneurs a $t = 0$, that is, on $b_1(u_1)$. We now study how $b_1(u_1)$ is determined in equilibrium.

To simplify our discussion, we focus on the special case in which the collateral constraints at date zero, $b_1(u_1) \leq \theta(1-\delta)u_1 k_0$, does not bind in equilibrium, so $\mu_1(u_1) = 0$ for all u_1.[5] The risk sharing condition (6) then takes the form

$$\left(\frac{RW_0}{V_1(u_1)}\right)^\sigma = \frac{n_0}{n_1(u_1)} \qquad \text{for all } u_1. \tag{13}$$

From equation (13) we can derive the equilibrium sensitivity of entrepreneurs' bond issuance and net worth to the capital quality shock, as shown in the next proposition. Let $\omega(u_1)$ denote the period 1 ratio of entrepreneurs' wealth to total wealth in the economy, including the human wealth of consumers, that is,

$$\omega(u_1) \equiv \frac{n_1(u_1)}{n_1(u_1) + a_1(u_1) + w_1(u_1) + W(n_1(u_1))}.$$

[5]This restriction does not imply that the collateral constraint does not bind at $t = 1, 2, \ldots$. Indeed, as we have seen in the analysis of continuation equilibrium, $\mu_t > 0$ for $t = 2, 3, \ldots, J$ when $n_1(u_1) < n^*$.

Proposition 1. *There exists a level of the consumers' coefficient of relative risk aversion $\hat{\sigma} > 0$, such that if $\sigma \in [0, \hat{\sigma}]$ and*

$$n_0 \geq \alpha \left(\bar{u}\hat{k}\right)^{\alpha} + (1-\theta)(1-\delta)\bar{u}\hat{k},$$

where $\hat{k} \equiv \alpha\beta\mathbb{E}\left(u_1^{\alpha}\right) / \left\{1 - (1-\delta)\beta\mathbb{E}\left(u_1^{\alpha}\right)\right\}$, then, in equilibrium, the date 0 collateral constraint does not bind, $\mu_1(u_1) = 0$ for all u_1, and the sensitivities of debt payments and entrepreneurs' net worth to the u_1 shock are:

$$b_1'(u_1) = \alpha^2 u_1^{\alpha-1} k_0^{\alpha} + (1-\delta)k_0 - \frac{\omega}{\omega + (1-\omega)\frac{1}{\sigma} - \omega W'(n_1)} \left(\alpha u_1^{\alpha-1} k_0^{\alpha} + (1-\delta)k_0\right) \quad (14)$$

$$n_1'(u_1) = \frac{\omega}{\omega + (1-\omega)\frac{1}{\sigma} - \omega W'(n_1)} \left(\alpha u_1^{\alpha-1} k_0^{\alpha} + (1-\delta)k_0\right). \quad (15)$$

The proof of this proposition is in Online Appendix A. Equations (14)-(15) provide an expression for the sensitivities of b_1 and n_1 to the shock u_1 in terms of the endogenous quantities k_0, $\omega(u_1)$ and $n_1(u_1)$ (the dependence on u_1 is omitted for readability).

We can use equations (14)-(15) and the results in Lemma 1 to identify the forces that determine financial amplification in this model. As a benchmark, in the first-best case with no collateral constraints we have $k_t = k^*$ for all $t \geq 1$, implying that the shocks to capital quality do not affect the choice of capital by entrepreneurs.[6] Therefore, any positive response of k_1 to the u_1 shock is a form of financial amplification. Figure 1 illustrates the debt payments of entrepreneurs, their net worth and choice of capital as a function of the capital quality shock when consumers are risk neutral (solid lines) and when they are risk averse (dashed lines).

Let us first study the case when consumers are risk neutral, $\sigma = 0$. In this case, we can see from Proposition 1 that

$$b_1'(u_1) = \alpha^2 k_0^{\alpha} u_1^{\alpha-1} + (1-\delta)k_0, \qquad n_1'(u_1) = 0.$$

When consumers are risk neutral, debt payments in equilibrium are structured so that entrepreneurs pay more to consumers when the realization of the capital quality shock is good. This state-contingency in debt payments allows the entrepreneurs to perfectly hedge against the aggregate shock—$n_1(u_1)$ is independent of u_1. Because $n_1'(u_1) = 0$, we know from the characterization of the continuation equilibrium that also k_1 is independent of u_1. Therefore, when consumers are risk neutral, there is no financial amplification in the

[6] This is due to the assumption that consumers have linear preferences after $t = 1$ and the capital quality shock is iid.

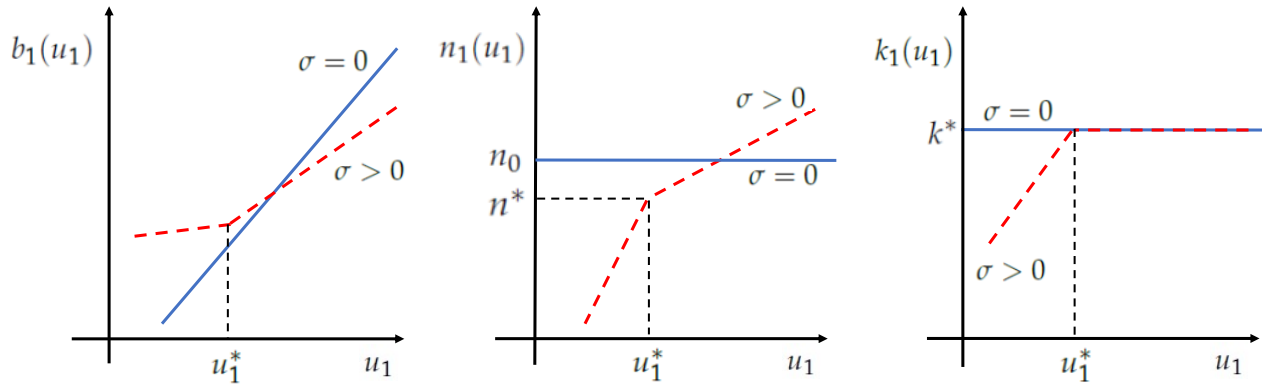

Figure 1: Debt payments, net worth and capital as functions of the capital quality shock u_1

model, in the sense that $k_1'(u_1) = 0$ as in the economy without the collateral constraint.[7] This echoes the baseline result in Krishnamurthy (2003).

When consumers are risk averse ($\sigma > 0$), they demand insurance against low capital quality states because those are states with a low present value of labor income. In equilibrium, this reduces the sensitivity of debt payments to the capital quality shock, and the net worth of entrepreneurs becomes positively related to u_1. If $n_1(u_1) \geq n^*$, k_1 is still independent from u_1. However, a sufficiently negative capital quality shock at $t = 1$ can lead net worth to fall below the threshold n^*, in which case entrepreneurs are constrained and the level of capital falls below its first-best. In the illustration of Figure 1, this occurs for realizations of the capital quality shock below u_1^*, see the dashed line.

This discussion emphasizes that the degree of financial amplification depends on the equilibrium sensitivity of net worth to u_1. Equation (15) identifies two key determinants of this elasticity: σ and W'. We now discuss the role of these two elements in detail.

The expression in parentheses on the right-hand side of (15) represents the effect of u_1 on the economy's resources. How much of that effect is borne by the entrepreneurs depends on the ratio

$$\frac{\omega}{\omega + (1-\omega)\frac{1}{\sigma} - \omega W'(n_1)}.$$

To interpret this ratio, let us consider separately the cases $n_1(u_1) \geq n^*$ and $n_1(u_1) < n^*$.

If $n_1(u_1) \geq n^*$ then $W'(n_1)$ is zero, and the ratio above is just

$$\frac{\omega}{\omega + (1-\omega)\frac{1}{\sigma}}.$$

[7] The *level* of k_1 can be different from the first-best, because entrepreneurs may still be constrained if $\mathbb{E}[u_1^\alpha] < 1$ and their initial level of net worth n_0 is small enough.

Define the risk tolerance as the inverse of the coefficient of relative risk aversion. Then the risk tolerance of the entrepreneurs is 1—due to log preferences—and the average risk tolerance in the economy, weighted by the agents' wealth shares, is $\omega + (1-\omega)1/\sigma$. Therefore, we obtain the standard result that agents share aggregate risk in proportion to their risk tolerance: the less risk tolerant are consumers, the higher the sensitivity of entrepreneurial net worth to the aggregate shock in equilibrium. See Gârleanu and Panageas (2015) for example.

Equation (15) highlights a second determinant of the equilibrium risk taking behavior of entrepreneurs, which operates only when the collateral constraint in the continuation equilibrium binds, $n_1(u_1) < n^*$. Because $W'(n_1) > 0$ in this constrained region, we can see from equation (15) that the share of the shock borne by entrepreneurs is larger.[8] The intuition for the last result is that a reduction in $n_1(u_1)$ in the constrained region reduces consumers' lifetime labor income, making them more willing to purchase state-contingent claims that pay off in that contingency. In equilibrium, this makes it harder for the entrepreneurs to smooth their net worth in those states of the world, increasing the sensitivity of $n_1(u_1)$ to u_1. In other words, the response of $n_1(u_1)$ increases the background risk perceived by consumers endogenously, making it costlier for the entrepreneurs to insure against the aggregate shock.

The importance of endogenous labor income in the results above can also be seen comparing our model to a different environment with an "AK" technology. With this production function, consumers do not earn labor income and their consumption is only financed by holdings of financial assets. In Online Appendix E we show that such model features no financial amplification relative to the first-best economy even when consumers are risk averse, as long as they have the same CRRA preferences as entrepreneurs. This case is closely related to the no-amplification result in Di Tella (2017), who also considers an economy with an "AK" technology.

4 Quantitative analysis

In this section, we go back to the fully fledged stochastic model and use numerical simulations to evaluate the strenght of financial amplification under plausible calibrations of the model parameters.

We compare our baseline economy with complete markets with two other economies: a *first best* economy, equivalent in all respects to the benchmark with the exception that entrepreneurs do not face the collateral constraints (1); and an *incomplete markets* economy, in

[8] This is the reason why in Figure 1 the relation between $n_1(u_1)$ and u_1 is steeper when $n_1(u_1) < n^*$.

which entrepreneurs can only issue non-state-contingent bonds, so the following additional constraint is present:

$$b(s^t, s_{t+1}) = \bar{b}(s^t) \qquad \forall (s^t, s_{t+1}).$$

In the incomplete markets economy, the limited enforcement friction implies the financial constraints

$$\bar{b}(s^t) + \gamma w(s^{t+1}) l(s^{t+1}) \leq \theta(1-\delta) u(s_{t+1}) k(s^t) \tag{16}$$

for all (s^t, s_{t+1}).

4.1 Calibration

Table 1 reports the model parameters used in our simulations. A period in the model corresponds to a quarter. We set the following parameters to standard values: the capital income share α is 0.33, the depreciation of capital is 2.5%, the discount factor of consumers β is 0.99, and the Frish elasticity of labor supply ψ is 1. In addition, we choose χ so that worked hours are equal to 1 in the deterministic steady state of the model. We further set ρ to 1, so that consumers have a unitary elasticity of intertemporal substitution as entrepreneurs. The parameter γ represents the fraction of the wage bills that needs to be paid in advanced by entrepreneurs. We set it to 0.50, in the mid range of values considered in the literature.[9] Conditional on the above parameters, β_e and θ control the steady state level of the capital to net worth ratio (k_{ss}/n_{ss}) and of the return to capital. We choose β_e and θ so that the former equals 4 and the latter is 50 annualized basis points above the risk-free rate.[10] This gives us β_e equal to 0.984 and θ equal to 0.818. For the consumers' risk aversion, σ, we do not pick a single value, but present numerical results for different values ranging in the interval $[1, 10]$.

We assume that the capital quality shock takes two possible values, $u_t = \{u_H, u_L\}$ with $u_H = 1$. Thus, the calibration of this process consists in choosing values for u_L and for transition probabilities. In line with Gertler and Karadi (2011), we set $u_L = 0.925$ and $P(u_{t+1} = u_L | u_t = u_L) = 0.66$. We further set $P(u_{t+1} = u_H | u_t = u_H) = 0.99$, so that financial crises in the model are rare events. In Online Appendix D, we perform two robustness

[9] For example, Jermann and Quadrini (2012) set this parameter to 1 in their sensitivity analysis, while Bianchi and Mendoza (2018) set it to 0.16. The key results presented in this section survive when using smaller or larger values for γ within this range.

[10] The entrepreneurs in our model consolidate financial and non-financial firms. Using US data, Gertler and Karadi (2011) target an average leverage ratio of 4 for the consolidated financial and non-financial corporate sector. The excess returns to capital that arise in the deterministic steady state reflect deviations from arbitrage induced by the presence of binding collateral constraints. Garleanu and Pedersen (2011) and Bocola (2016) document that these arbitrage rents were sizable during the global financial crisis of 2008-2009, but they typically average few basis points in advanced economies in normal times. We chose 50 basis points to be consistent with this evidence.

Table 1: Model parameters

Parameter	Concept	Value
α	Capital income share	0.330
δ	Capital depreciation	0.025
β	Discount factor, consumers	0.990
ψ	Frisch elasticity	1.000
χ	Disutility of labor	1.980
ρ	Inverse IES, consumers	1.000
γ	Fraction of wages paid in advance	0.500
β_e	Discount factor, entrepreneurs	0.984
θ	Fraction of pleadgeable assets	0.818
u_L	Capital quality in low state	0.925
$\Pr(u' = u_L \mid u = u_L)$	Transition probability	0.660
$\Pr(u' = u_H \mid u = u_H)$	Transition probability	0.990

checks. First, we consider smaller and less persistent capital quality shocks. Second, we study a version of our model where the exogenous shock moves the pleadgeability parameter θ rather than capital quality. In both cases, we find results comparable to those presented in this section.

4.2 Results

In Table 2 we report statistics computed on model simulated data using three different values of the consumers' coefficient of relative risk aversion: $\sigma = 1$, $\sigma = 5$, and $\sigma = 10$. In each case, we report results for the first best economy (FB), the economy with incomplete financial markets (IM), and the baseline economy with state-contingent claims (CM).

For each specification we simulate the model for $T = 200,000$ periods and select the periods in which the capital quality shock switches from H to L between $t-1$ and t. Panel A in the table reports the average percentage change in entrepreneurial net worth when the switch occurs, and the average percentage change in

$$\tilde{n}_t = \theta(1-\delta)u_t k_{t-1} - b_t(u_t),$$

a variable that measures the entrepreneurs' maximum capacity to issue intra-period loans to finance working capital. Both variables are relevant to understand how financial factors

Table 2: Entrepreneurs' balance sheet and financial amplification

	$\sigma = 1$			$\sigma = 5$			$\sigma = 10$		
	FB	IM	CM	FB	IM	CM	FB	IM	CM
Panel A: Quantities									
$\Delta(\log n_t)$		-25.21	-2.96		-25.24	-6.75		-25.30	-16.46
$\Delta(\log \tilde{n}_t)$		-97.92	16.94		-97.47	-14.48		-96.58	-72.29
$\Delta(\log l_t)$	-1.89	-4.80	-1.67	-1.89	-4.57	-6.41	-1.89	-4.11	-11.16
$\Delta(\log i_t)$	-6.45	-16.66	-6.37	-7.60	-17.58	-9.94	-8.99	-19.00	-17.69
$\Delta(\log y_t)$	-3.77	-5.72	-3.63	-3.77	-5.57	-6.80	-3.77	-5.26	-9.99
Panel B: Entrepreneurs' balance sheet									
n_{t-1}		7.89	6.23		7.86	6.20		7.79	6.67
$\tilde{n}_{t-1}$		2.29	0.95		2.29	0.96		2.28	1.56
k_{t-1}/n_{t-1}		3.09	3.94		3.09	3.93		3.08	3.56
$b_{L,t}/b_{H,t}$		1.00	0.91		1.00	0.93		1.00	0.97

Notes: *Each economy is simulated for $T = 200,000$ periods. For each simulation, we select every j such that $u_{j-1} = u_H$ and $u_j = u_L$. We then compute a given statistic x_j and average across j. In panel A the changes in the variables are multiplied by 100, so to obtain percentage changes.*

affect the demand for capital and labor by entrepreneurs. Panel A also reports the average percentage change in labor, investment and output. Panel B reports indicators for the entrepreneurs' balance sheet in the period immediately preceding the L shock: the average net worth, the average value of $\tilde{n}_{t-1}$, the average leverage ratio, and the average ratio between bonds issued in period $t-1$ contingent on the L state realizing at time t and those contingent on the H state, denoted respectively $b_{L,t}$ and $b_{H,t}$. In the incomplete market economy this ratio is always equal to 1 by construction.

Let us start with the case $\sigma = 1$ and look at the differences between the three economies.

In the first best economy, a negative capital quality shock lowers the marginal product of labor, leading to a reduction in labor demand and a fall in hours worked. The direct effect of the shock, coupled with the reduction in labor input, leads to a fall in output. Investment falls because the u_t shock is persistent, so a reduction in u_t reduces the incentives to accumulate capital.

In the incomplete market economy, the shock has larger effects on labor, investment, and output. The differences are due to the financial amplification mechanism. In the incomplete market economy, entrepreneurs issue non-state-contingent claims and face the collateral constraint (16). The first ingredient implies that their balance sheet is exposed to

aggregate risk: a negative capital quality shock reduces the value of the capital held but not the value of entrepreneurs' liabilities. So, both n_t and $\tilde{n}_t$ fall (on average by 25% and 98% respectively). The second ingredient implies that these balance sheet effects depress the demand for capital and labor by entrepreneurs. The combination of these two forces leads to a deeper recession relative to the first best.

When entrepreneurs can issue state-contingent claims, the fall in labor, investment, and output are comparable to those of the first best economy. That is, the financial amplification mechanism is muted. Unlike in the incomplete market case, entrepreneurs can now insure against the capital quality shock by reducing their contingent liabilities in state L. Panel B of Table 2 shows that this is precisely what they do in equilibrium: the ratio $b_{L,t}/b_{H,t}$ is on average 0.91, meaning that entrepreneurs promise to pay less in the L state. This liability structure implies that both n_t and $\tilde{n}_t$ are less affected by the negative capital quality shock, eliminating the first step of the amplification mechanism described above. These results mirror the findings in Carlstrom, Fuerst, and Paustian (2016), Dmitriev and Hoddenbagh (2017) and Cao and Nie (2017). They study financial accelerator models with endogenous labor income and log utility for consumers, and show that in their economies financial amplification is muted when debt contracts can be indexed to aggregate shocks.

The comparison of the three cases (FB, IM, and CM) is very different once we move to the next columns on the right, which correspond to economies with higher consumer risk aversion ($\sigma = 5, 10$).

Table 2 shows that the behavior of the FB and IM economies does not change much once we increase σ, a result related to the findings in Tallarini (2000). However the CM economy behaves very differently: the average ratio $b_{L,t}/b_{H,t}$ increases to 0.93 when $\sigma = 5$ and to 0.97 when $\sigma = 10$. Entrepreneurs use state-contingent debt less to protect their net worth against a negative shock and, as a result, the sensitivity of n_t and $\tilde{n}_t$ to the shock increases. The larger fall in these two variables constrains entrepreneurs' demand of labor and capital, leading to a deeper recession. With $\sigma = 5$ the fall in labor and output in the economy with complete markets is comparable to that of the economy with incomplete markets.[11] Increasing σ further leads to more risk taking by entrepreneurs and to stronger financial amplification.

Figure 2 gives a more complete representation of the dynamics following the shock, plotting impulse response functions (IRFs) for labor, output and investment for different values of consumers' risk aversion. Because for the IM economy the IRFs are virtually

[11] This happens despite the fact that with complete markets the fall in net worth is smaller than with incomplete markets. The reason is that the economy with incomplete markets starts from a higher level of net worth in equilibrium, so the post-shock levels of net worth in the two economies are quantitatively similar.

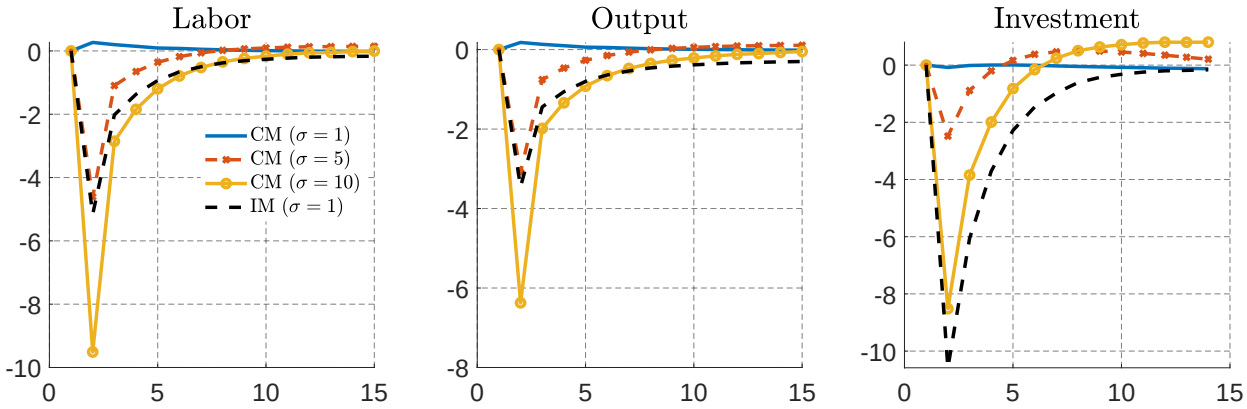

Figure 2: Impulse response functions

Notes: *We compute* $2 \times M$ *simulations of length* T*. We initialize the simulations at* $t = 0$ *setting each state variable at the mean of the ergodic distribution. In the first* M *simulations, we set* $u_1 = u_L$*, in the others we set* $u_1 = u_H$*. The impulse response functions are computed taking the difference in logs between the first and second set of simulations, averaging across* M*. We use* $M = 5,000$ *and* $T = 15$*. The plots show the differences between the impulse response functions of the model considered and the first best impulse response functions.*

identical for the different values of σ, the figure only reports the $\sigma = 1$ case. In addition, to better visualize financial amplification, we plot the difference between the IRFs in the model considered and the IRFs in the first best economy. The CM economy features essentially no financial amplification when $\sigma = 1$. As we increase σ, labor, output and investment respond by more than in the first best. Quantitatively, the responses are comparable to those of the economy with incomplete markets for plausible levels of σ.[12]

Behind the aggregate outcomes plotted in Figure 2 there is the fact that entrepreneurs in the CM economy choose riskier balance sheets when σ is higher, as shown in the top left panel of Figure 3. To provide an interpretation of this result, in Figure 3 we plot three other varibales. In the top right panel we plot the average value of $q(s_{t+1}|s^t)/(\beta\pi(s_{t+1}|s^t))$, for $s_{t+1} = H$ and for $s_{t+1} = L$ in economies with different levels of consumers' risk aversion σ. This ratio measures the price of buying insurance against state s_{t+1} relative to the risk neutral price—a measure of the insurance premium for each state. The remaining panels report the average entrepreneurs' leverage and the average percentage change in net worth after a low capital quality shock.

After a low capital quality shock, consumers' current and future labor incomes decline. For low values of σ, this has a small effect on the insurance premium $q(L|s^t)/(\beta\pi(L|s^t))$

[12]In the CM economy with $\sigma = 1$ the impact responses of labor and output are slightly weaker than in the first best case. The reason for this apparently odd behavior is that, in this calibration of the CM economy, entrepreneurs are on average more constrained in choosing the labor input after the H shock (when they would like to hire more) than after the L shock. So the fall in labor and output when the economy switches from the H to L state is smaller than in the first best.

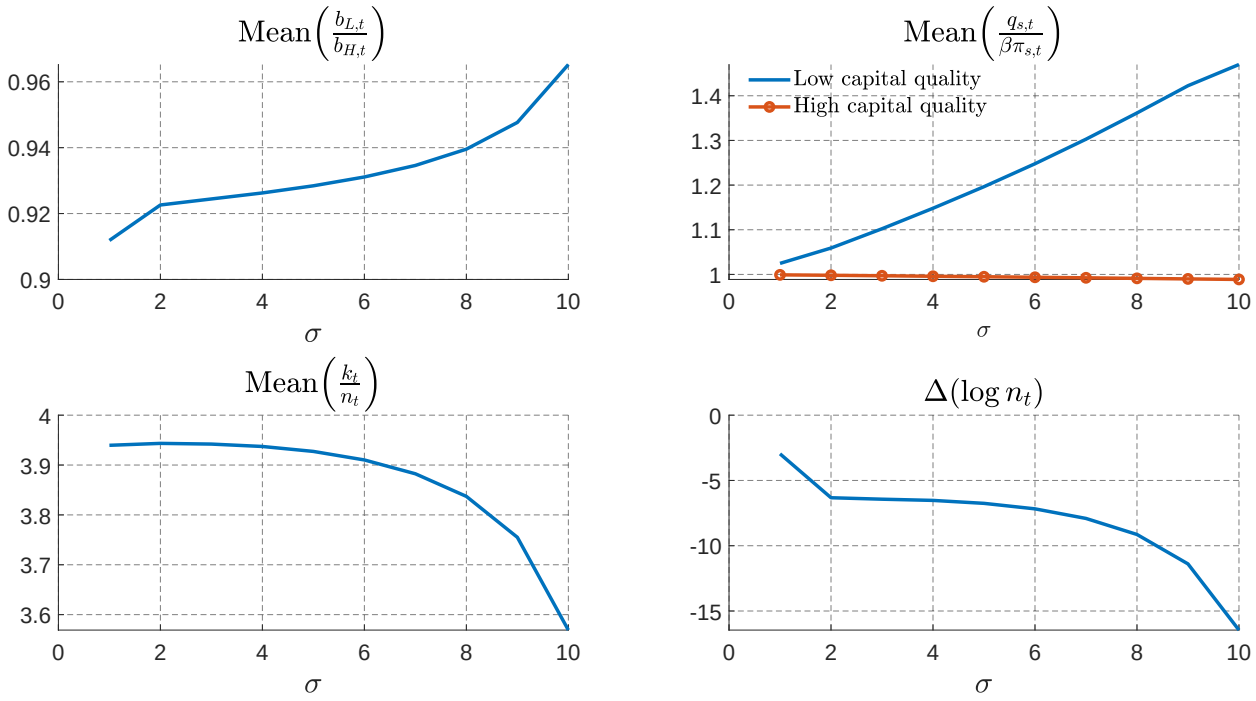

Figure 3: Asset prices and entrepreneurs' balance sheet

Notes: *For each value of σ, we simulate the complete market economy for $T = 200,000$ periods, and we compute average values of $q(s_{t+1}|s^t)/(\beta\pi(s_{t+1}|s^t))$ (panel a), of $b_{L,t}/b_{H,t}$ (panel b), of k_t/n_t (panel c), and of the percentage change in net worth after a negative capital quality shock (panel d).*

which remains close to 1. However, as we increase σ, consumers are less willing to sell insurance against the L state and the premium increases. This incentivizes entrepreneurs to sell more L-contingent debt, so the average $b_{L,t}/b_{H,t}$ ratio increases with σ. As this ratio increases, the entrepreneurs' net worth becomes more sensitive to the capital quality shock and, in general equilibrium, it makes consumers' incomes even more procyclical, reinforcing the process.

The figure shows that there is also a countervailing force at work: as entrepreneurs take on more aggregate risk, they partly adjust by reducing their investment in capital, thus reducing their leverage k_t/n_t, as seen in the bottom-left panel. This force however only partly offsets the mechanism described above.

4.3 Isolating the general equilibrium spillover on labor income

The mechanism just described contains two steps: first, risk averse consumers are willing to pay high premia for insuring a bad realization of the capital quality shock; second, high insurance premia endogenously make consumers' incomes more sensitive to the shock,

Table 3: Quantifying the general equilibrium spillover

	First best	Benchmark	No spillover
Panel A: Quantities			
$\Delta(\log n_t)$		-6.75	-3.24
$\Delta(\log l_t)$	-1.89	-6.41	-1.65
$\Delta(\log i_t)$	-7.44	-9.94	-9.89
$\Delta(\log y_t)$	-3.77	-6.80	-3.61
Panel B: Prices and entrepreneurs' balance sheet			
$\Delta(\log LI_t)$	-3.77	-12.82	-3.29
$q_{L,t}/\pi_{L,t}$		1.20	1.03
$q_{H,t}/\pi_{H,t}$		1.00	1.00
k_{t-1}/n_{t-1}		3.93	3.96
$b_{L,t}/b_{H,t}$		0.93	0.91

Notes: *See Table 2.*

reinforcing the first step.

The results in Table 2 and Figure 2 show that the combined effect of these two steps can be quantitatively relevant. We now attempt a decomposition to evaluate the importance of the second step, that is, to evaluate how much the macro spillover in our model reinforces the direct effect of consumers' risk aversion.

We consider an economy that is identical to that of Section 2, except that consumers earn the counterfactual wage that would arise in the first best economy.[13] Wages still respond to the capital quality shock—as they do in the first best—but they are not affected by the changes in investment and labor demand that are due to the presence of the collateral constraint. By construction, in this economy there is no spillover from entrepreneurs' net worth to consumers' labor income. For brevity, we call it the "no spillover" economy.

Table 3 reports the average response of key variables to the low capital-quality shock in the first best economy, in the benchmark economy with state-contingent claims, and in the economy with no spillover. In all cases, we set $\sigma = 5$. The first two columns reproduce results in Table 2. The third column shows that the amplification mechanism is substantially reduced if we shut down the macro spillover. Net worth falls by 3.2% instead of 6.8% and the responses of labor, investment, and output are comparable to those of the first best economy.

[13]See Online Appendix F for a detailed description of this version of the model.

Panel B of the Table helps understand this result. Absent the spillover, labor income falls by 3.3% after the negative capital quality shock, substantially less than the 12.8% of the benchmark model. Thus, even if consumers are more risk averse than entrepreneurs, they do not bid up as much the price for insuring a low realization of the capital quality shock: $q_{L,t}/(\beta\pi_{L,t})$ is 1.03 in the no spillover economy, compared to 1.20 in our benchmark economy. Given these state prices, entrepreneurs have a better incentive to stabilize their net worth by reducing their contingent debt in the L state.

In summary, to generate quantitatively meaningful financial amplification in our model, we need both consumers to be more risk averse than entrepreneurs and labor income to be sufficiently responsive to entrepreneurs' net worth.

5 Welfare analysis

We now turn to the welfare implications of the model. In Section 5.1 we set up the policy problem of a planner that can tax entrepreneurs' assets and liabilities. We then study the solution to this problem in two steps. In Section 5.2 we characterize analytically the solution to the planner's problem in the special case of Section 3. We show that the laissez-faire competitive equilibrium is inefficient, with entrepreneurs hedging less than what is socially efficient because they do not internalize the stabilizing effects of their risk mitigation strategies on consumers' labor income. In Section 5.3 we go back to the general model—calibrated as in Section 4—and study numerically the optimal policy aimed at correcting this externality. In Section 5.4 we study the relation between the policy prescriptions described here and policy interventions routinely used in practice to deal with financial instability.

5.1 The planner's problem

We start from the laissez-faire equilibrium studied in Section 4 and consider a planner who intervenes for one period only: the planner sets proportional taxes or subsidies on capital purchases and on state-contingent claims issued by entrepreneurs at time t. In addition, the planner can make a lump-sum transfer at date t to redistribute the efficiency gains between consumers and entrepreneurs.

The timing of events within a period is as in the general model, and we assume that the planner intervenes in the third stage of period t: after production has taken place and after entrepreneurs have chosen whether or not to default, at the moment in which they choose their capital investment and trade state-contingent claims with consumers.

Given this timing, the planner cannot relax the collateral constraint in period t, because employment and production have already occurred. The collateral constraint in future periods is also unaffected, because the planner only intervenes for one period. Therefore, all welfare gains are solely due to the planner inducing different choices of capital and state-contingent debt at time t.[14]

Let $\mathbf{s} = [u, K, B]$ be the vector of aggregate state variables in period t. Using the recursive notation of Online Appendix B, we write the entrepreneur's problem as follows

$$\max_{c_e, l, b'(\mathbf{s}), k'} \log(c_e) + \beta_e \mathbb{E}_{\mathbf{s}} \left[V^e \left(b'(\mathbf{s}'), k'; \mathbf{s}' \right) \right],$$

$$n = (uk)^{\alpha} l^{1-\alpha} - w(\mathbf{s}) l + (1-\delta) uk - b$$

$$c_e + \left[1 + \tau_k(\mathbf{s}) \right] k' \leq n + \sum_{\mathbf{s}'} \left[1 - \tau_b \left(\mathbf{s}' | \mathbf{s} \right) \right] q \left(\mathbf{s}' | \mathbf{s} \right) b'(\mathbf{s}') + T_e(\mathbf{s})$$

$$b + \gamma w(\mathbf{s}) l \leq \theta (1-\delta) uk,$$

where $\tau_k(\mathbf{s})$ is a proportional tax on capital, $\tau_b(\mathbf{s}'|\mathbf{s})$ is a tax on the sales of state-contingent claims that pay in state $\mathbf{s}'$, $T_e(\mathbf{s})$ is a lump-sum transfer, and $V^e(.)$ is the value function of entrepreneurs, expressed as function of the individual state variables (b, k) and of the aggregate state $\mathbf{s}$. Because the planner intervenes for only one period, V^e is the laissez-faire equilibrium value function.

Consumers solve the problem

$$\max_{c, l, a'(\mathbf{s}')} (1-\beta) \left(c - \chi \frac{l^{1+\psi}}{1+\psi} \right)^{1-\rho} + \beta \left[\mathbb{E}_{\mathbf{s}} \left(V \left(a'(\mathbf{s}'); \mathbf{s}' \right) \right)^{1-\sigma} \right]^{\frac{1-\rho}{1-\sigma}},$$

$$\sum_{\mathbf{s}'} q \left(\mathbf{s}' | \mathbf{s} \right) a'(\mathbf{s}') + c \leq w(\mathbf{s}) l + a + T_c(\mathbf{s}),$$

where $T_c(\mathbf{s})$ is a lump sum transfer, and $V(.)$ is the laissez-faire equilibrium value function.

A competitive equilibrium with one-period government intervention is given by taxes and transfers, prices, and allocations such that consumers and entrepreneurs solve the optimization problems above, the bond market and capital market clear, $a'(\mathbf{s}') = b'(\mathbf{s}') = B'(\mathbf{s}')$, $k' = K'$, the labor market clears, and the government budget constraint holds.

[14] The main advantage of limiting our analysis to one period interventions is simplicity. In the current formulation, the planner cannot circumvent the collateral constraint, even though Pigouvian taxes at time t are fully enforceable. In a model with multi-period interventions, if taxes are fully enforceable it would be easy for the planner to circumvent the limited enforcement problem—by transferring resources to entrepreneurs when the constraint is binding and redistributing them back to consumers in future periods. Given that, we would need to introduce some form of limited enforcement of tax payments, which would substantially complicate the analysis.

We consider a planner that chooses the policies $\tau_b(\mathbf{s}'|\mathbf{s}), \tau_k(\mathbf{s}), T_c(\mathbf{s}), T_e(\mathbf{s})$ to maximize the utility of the consumers subject to giving entrepreneurs the same utility as in the laissez-faire equilibrium. Because the planner can always implement the laissez-faire allocation by setting zero taxes and transfers, any deviation from such benchmark is, by construction, a Pareto improvement. In Online Appendix G we show that planner's optimum can be characterized by solving the primal problem

$$\begin{aligned} \max_{X, C_e, K', B'(\mathbf{s}')} \quad & \left\{ (1-\beta) X^{1-\rho} + \beta \left[\mathbb{E}_{\mathbf{s}} \left[V\left(B'(\mathbf{s}'); u', B'(\mathbf{s}'), K'\right) \right]^{1-\sigma} \right]^{\frac{1-\rho}{1-\sigma}} \right\}^{\frac{1}{1-\rho}} \\ \text{subject to} \quad & X + C_e + K' \leq (uK)^{\alpha} L(\mathbf{s})^{1-\alpha} + (1-\delta)uK - \chi \frac{L(\mathbf{s})^{1+\psi}}{1+\psi} \\ & \log C_e + \beta_e \mathbb{E}_{\mathbf{s}} \left[V^e\left(B'(\mathbf{s}'), K'; u', B'(\mathbf{s}'), K'\right) \right] \geq V^e(B, K; \mathbf{s}), \end{aligned} \tag{SP}$$

where $L(\mathbf{s})$ is the labor allocation of the laissez-faire equilibrium.[15] The first constraint is the resource constraint and the second constraint ensures that the entrepreneurs are as well off as in the laissez-faire equilibrium.

In order to understand the planner's rationale for intervening, consider the first-order condition with respect to $B'(\mathbf{s}')$. After some manipulations, we obtain

$$\begin{aligned} \beta_e \frac{\frac{1}{C_e(\mathbf{s}')} + \mu(\mathbf{s}')}{\frac{1}{C_e}} - \beta \left[\frac{X}{X(\mathbf{s}')}\right]^{\rho} \left[\frac{RW(\mathbf{s})}{V(\mathbf{s}')}\right]^{\sigma-\rho} &= \beta_e C_e \frac{\partial V^e(\mathbf{s}')}{\partial B'(\mathbf{s}')} + \\ &+ \beta \frac{X^{\rho} [RW(\mathbf{s})]^{\sigma-\rho} V(\mathbf{s}')^{-\sigma}}{1-\beta} \frac{\partial V(\mathbf{s}')}{\partial B'(\mathbf{s}')}, \end{aligned} \tag{17}$$

where $X(\mathbf{s}'), C_e(\mathbf{s}')$ are the individual policy functions at the laissez-faire equilibrium and $\partial V(\mathbf{s}')/\partial B'(\mathbf{s}')$ is a short notation for the partial derivative of $V\left(B'(\mathbf{s}'); u', B'(\mathbf{s}'), K'\right)$ with respect to its third argument (and similarly for $\partial V^e(\mathbf{s}')/\partial B'(\mathbf{s}')$).

The two terms on the left-hand side of (17) are equivalent to the terms in our baseline risk sharing condition (6), which are equalized in every state of nature at the laissez-faire equilibrium. In the planner solution, however, there is a wedge between the two, represented by the terms on the right-hand side of equation (17). Differently from atomistic agents, the planner takes into account that by changing $B'(\mathbf{s}')$ it affects the net worth of entrepreneurs and, thus, the price of state-contingent claims and wages in equilibrium. The impact of these pecuniary externalities on consumers and entrepreneurs welfare are represented by

[15] Because the planner cannot relax the entrepreneurs' collateral constraint, and due to the absence of a wealth effect on consumers' labor supply, the labor allocation in the planner's solution at date t is equivalent to that of the laissez-faire equilibrium.

the partial derivatives of V and V^e with respect to the aggregate state variable $B'(\mathbf{s}')$. As long as the terms on the right-hand side do not cancel out, the planner has incentives to impose taxes or subsidies on state-contingent debt in order to modify the allocation of risk between consumers and entrepreneurs.

5.2 Optimal policy in the simple model

To shed light on how the pecuniary externalities discussed above affect the optimal policy, consider the special case of Section 3. Since the value function of consumers at date $t = 1$ is given by (12), the effect of increasing $B_1(u_1)$ on consumers' welfare is

$$\frac{\partial V_1}{\partial B_1(u_1)} = -(1-\beta)\sum_{t=1}^{\infty}\beta^t\frac{\partial w_{t+1}}{\partial n_1(u_1)} \leq 0. \tag{18}$$

A change in $B_1(u_1)$ affects consumers' welfare through its impact on their lifetime labor income. If the collateral constraint does not bind at u_1, then capital equals k^* in every period after $t = 1$, wages are independent of $B_1(u_1)$, and $\partial V_1/\partial B_1(u_1) = 0$. If the collateral constraint binds at u_1, however, we know from Lemma 1 that capital accumulation depends on entrepreneurial net worth at date $t = 1$. A higher $B_1(u_1)$, by reducing net worth, leads to lower capital and lower wages for a finite number of periods, so $\partial V_1/\partial B_1(u_1) < 0$.

We can follow similar steps and study the impact of an increase in $B_1(u_1)$ on entrepreneurs' welfare. Because entrepreneurs have log-preferences, their consumption is proportional to net worth. Given the effect of wages on net worth from equation (4), we then have

$$\frac{\partial V_1^e}{\partial B_1(u_1)} = \sum_{t=1}^{\infty}\beta^t\frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial n_1(u_1)} \geq 0. \tag{19}$$

Similarly to consumers, a change in $B_1(u_1)$ affects entrepreneurs only through its impact on wages. Differently from consumers, however, an increase in $B_1(u_1)$ has a (weakly) positive spillover for entrepreneurs because it lowers their cost of labor.

The above discussion shows that the pecuniary externalities triggered by an increase in $B_1(u_1)$ hurt consumers and help entrepreneurs, so their overall effects on the optimal policy are in principle ambiguous. However, we can show that the negative effect on consumers dominates in the states in which the constraint is binding.

Substituting (18) and (19) on the right-hand side of equation (17) we obtain

$$\beta \frac{n_0}{n_1(u_1)} \sum_{t=1}^{\infty} \beta^t \frac{n_1(u_1)}{n_{t+1}} \frac{\partial w_{t+1}}{\partial n_1(u_1)} - \beta \left(\frac{RW_0}{V_1(u_1)} \right)^{\sigma} \sum_{t=1}^{\infty} \beta^t \frac{\partial w_{t+1}}{\partial n_1(u_1)}. \tag{20}$$

Let us evaluate this expression at the laissez-faire allocation studied in Section 3. Using the risk sharing condition (13), the sign of (20) is equal to the sign of

$$\sum_{t=1}^{\infty} \beta^t \left[\frac{n_1(u_1)}{n_{t+1}} - 1 \right] \frac{\partial w_{t+1}}{\partial n_1(u_1)}.$$

If the collateral constraint binds at u_1, we know from Section 3.1 that entrepreneurs' net worth increases over time, $n_t < n_{t+1}$, for a finite number of periods. So, in those states the expression in (20) is negative: the reduction in consumers' welfare is larger than the increase in entrepreneurs' welfare.

The derivations above suggest that, starting at the laissez-faire allocation, the planner has a motive to reduce entrepreneurs' debt payments in states where the constraint binds. The intuition is that reducing debt payments causes two reallocations in resources: the first, internalized by private agents, is a direct reallocation from consumers to entrepreneurs at $t = 1$; the second, not internalized, is a reallocation from entrepreneurs to consumers, caused by the general equilibrium increase in wages in periods $t = 2, 3, 4, ...$ Because the entrepreneurs are constrained at date 1, they value resources relatively more at $t = 1$ than in future periods, so the combined effects of these reallocations is to increase social welfare.

A planner who internalizes the general equilibrium effects above can achieve the social optimum using the taxes characterized in the following proposition.

Proposition 2. *In the special case of Section 3, the taxes on state-contingent claims and capital that implement the planner's optimum are*

$$\tau_b(u_1) = \frac{\sum_{t=1}^{\infty} \beta^t \left[1 - \frac{n_1(u_1)}{n_{t+1}} \left(\frac{1}{1+\mu_1(u_1)(1-\beta)n_1(u_1)} \right) \right] \frac{\partial w_{t+1}}{\partial n_1(u_1)}}{1 - \left(\frac{1}{1+\mu_1(u_1)(1-\beta)n_1(u_1)} \right) \sum_{t=1}^{\infty} \beta^t \frac{n_1(u_1)}{n_{t+1}} \frac{\partial w_{t+1}}{\partial n_1(u_1)}} \geq 0 \tag{21}$$

$$\tau_k = \beta \mathbb{E} \left\{ \frac{n_0}{n_1(u_1)} \left[\sum_{t=0}^{\infty} \beta^t \left[\frac{n_1(u_1)}{n_{t+1}} - \frac{1 + \mu_1(u_1)(1-\beta)n_1(u_1)}{1 - \tau_b(u_1)} \right] \frac{\partial w_{t+1}}{\partial k_0} \right] \right\} \leq 0. \tag{22}$$

Proposition 2 provides expressions for the optimal taxes as a function of the planner's

allocation and of the continuation equilibrium characterized in Lemma 1.[16] Given the properties of the continuation equilibrium, we can characterize key properties of these taxes. The optimal tax on state-contingent claims, given by equation (21), is zero if the collateral constraint does not bind in state u_1, and it is positive otherwise. This reflects the planner's motive, discussed above, to reduce entrepreneurs' debt payments when the collateral constraint binds. To the extent that $n_1(u_1)$ is increasing in u_1 in the planner's allocation, Proposition 2 also implies that the planner levies taxes toward state-contingent claims that pay in low capital quality states.

The proposition also derives the optimal tax on capital, given by (22). In the proof of the proposition, presented in Online Appendix A, we show that τ_k is strictly negative when the collateral constraint at date $t = 1$ binds with positive probability. The planner's motive for subsidizing capital is closely related to that of taxing debt, as higher capital at date $t = 1$ triggers the same pecuniary externality of a reduction in entrepreneurs' debt payments we studied earlier.

5.3 Numerical analysis

We now go back to the full model, calibrated as in Section 4, to give a quantitative assessment of the optimal taxes and of their effects on the equilibrium allocation. In addition, we compare the optimal policy to a blunter policy that taxes borrowing equally in all states of the world. Specifically, we impose an additional constraint on the planner's problem, requiring $\tau_b(\mathbf{s}'|\mathbf{s})$ to be constant in $\mathbf{s}'$. The latter policy is equivalent to a simple leverage constraint of the type usually studied in existing models of macroprudential policy.

We solve the planner's problem numerically and report the response of the economy to a negative capital quality shock in Table 4. Specifically, we simulate the economy for many periods, select all the periods in which the shock switches from u_H to u_L between $t-1$ and t, and report statistics regarding the entrepreneurs' balance sheet and the behavior of macroeconomic variables, assuming that the planner intervened at $t-1$. We compare four different cases: the first best (FB), the laissez-faire equilibrium (LF), the equilibrium under optimal policy (PL), and the equilibrium under the constrained policy (PL-c). For this illustration, we set σ to 10.

First, let us consider the behavior of quantities in Panel A. Under the optimal policy financial amplification is substantially reduced: the fall in labor and output in column PL are smaller than in column LF and closer to the FB case. In addition, comparing columns PL and PL-c shows that different tax rates on different state-contingent claims are critical

[16] Note that here we are not restricting the entrepreneurs to be unconstrained at date $t = 0$, so the Lagrange multiplier $\mu_1(u_1)$ can be positive in some states.

Table 4: Optimal policy

	FB	LF	PL	PL-c
Panel A: Quantities				
$\Delta(\log n_t)$		-16.46	-16.11	-16.38
$\Delta(\log \tilde{n}_t)$		-72.29	-54.12	-72.33
$\Delta(\log l_t)$	-1.89	-11.16	-2.36	-11.16
$\Delta(\log i_t)$	-8.99	-17.69	-19.40	-21.78
$\Delta(\log y_t)$	-3.77	-9.99	-4.06	-9.95
Panel B: Taxes				
$1-\tau_b(u_L)$		1.00	0.80	1.00
$1-\tau_b(u_H)$		1.00	1.00	1.00
$1+\tau_k$		1.00	0.98	0.98

Notes: *See note to Table 2.*

for this result: a planner restricted to impose a uniform tax on state-contingent claims does not dampen financial amplification.

Panels B reports the average taxes set by the planner. The results for the PL economy are consistent with the analytical derivations of the simple model: the planner subsidizes capital accumulation and imposes a tax on bonds that pay in low capital quality states.[17] Quantitatively, the subsidy on capital is 2% on average, while the planner levies a tax of 19% on sales of L-contingent bonds and a zero tax on H-contingent bonds. These taxes induce the entrepreneurs to reduce their reliance on the L-contingent debt, which explains why their balance sheet is less exposed to the negative capital quality shock at date t, and why financial amplification is muted.

Turning to the PL-c economy, we can see that when the restricted planner chooses an optimal tax on debt close to zero on average. Consistently, balance sheets and aggregate effects in the PL-c economy are similar to those in the LF economy.

The result of a near zero tax in the PL-c economy may appear surprising in light of several papers in the literature that report sizable optimal debt taxes in similar models with non-state-contingent debt. To better understand this result, Figure 4 reports entrepreneurs' debt in the PL-c economy when the planner varies τ_b. The left panel shows that as τ_b in-

[17] Incidentally, the subsidy on capital explains why investment falls more in the planner solution than in the laissez-faire equilibrium in panel A. Since the planner can only intervene at $t-1$, the investment subsidy is only present at date $t-1$, driving down investment between $t-1$ and t. In the laissez-faire equilibrium this policy effect is absent.

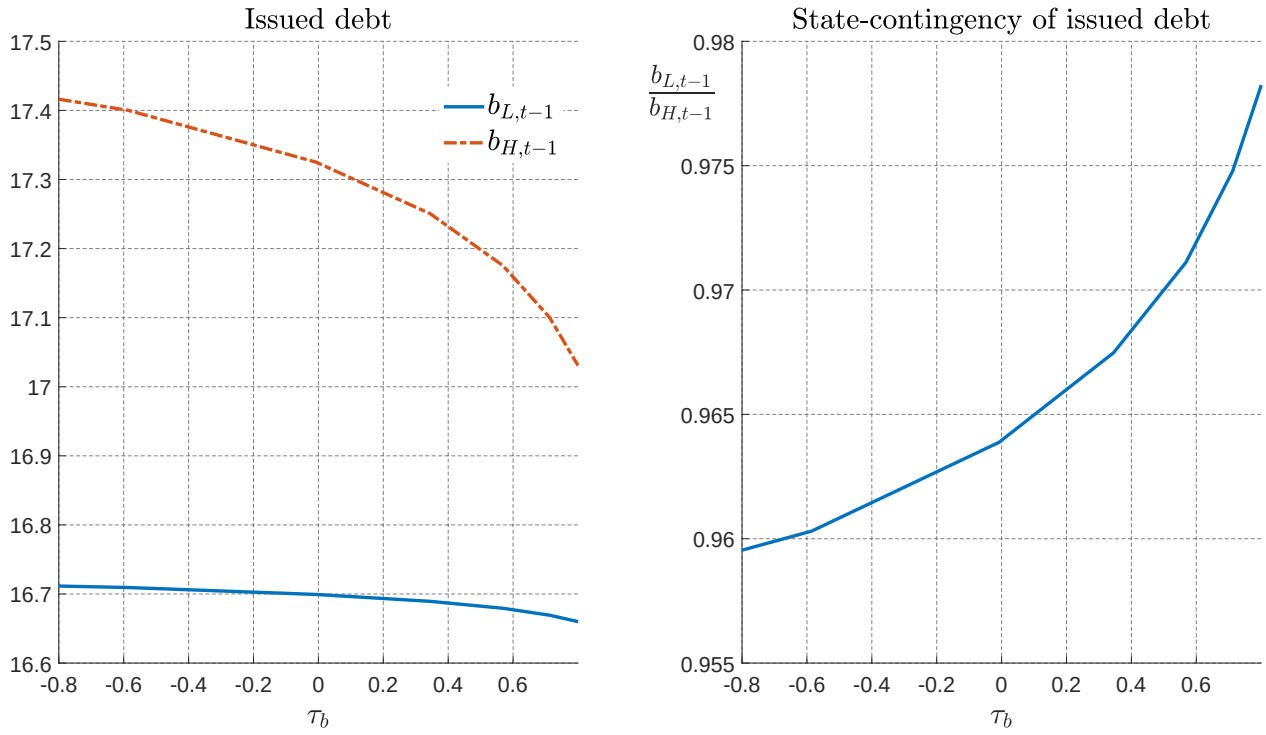

Figure 4: Tax on debt, leverage and risk taking

Notes: *The left panel reports the equilibrium levels of $b_{L,t-1}$ and $b_{H,t-1}$ when varying τ_b. When constructing the figure, we set $u_{t-1} = 1$ and set the other state variables at $t-1$ at the ergodic mean. In addition, the tax on capital and the transfers are set so that the level of capital remains at its optimal level in the constrained planner problem and the entrepreneur achieves the same utility as in the laissez-faire competitive equilibrium. The right panel is constructed in a similar fashion.*

creases entrepreneurs reduce their contingent debt in both states of the world, but much more in state H, so the ratio of L-contingent to H-contingent debt increases (right panel). Thus, a uniform tax on borrowing is not particularly effective in reducing the risk taking of entrepreneurs, because entrepreneurs respond by reducing the degree of state-contingency in future debt payments. Because of this feature, the planner chooses essentially not to engage in macroprudential policy. In models where debt is not state-contingent, the private sector cannot respond to the tax by altering the degree of state-contingency of debt payments, so a uniform tax on debt is more effective in curbing risk-taking incentives.

We summarize the discussion above in two observations. First, when borrowers have means to adjust the state-contingency of their liabilities, the welfare benefits of a uniform tax on leverage may be overstated. Second, the ability of regulation to reduce financial amplification and improve welfare rests crucially on the ability to discourage the riskier forms of borrowing with targeted instruments.

5.4 Bailouts and financial regulation

We now discuss the connection between the welfare analysis above and policy interventions routinely used to deal with financial instability. In particular, we discuss bailouts and capital adequacy ratios.

Suppose we start at the laissez-faire equilibrium and, at date t, consumers and entrepreneurs have exchanged the state-contingent claims

$$a'(\mathbf{s}') = A'(\mathbf{s}') = b'(\mathbf{s}') = B'(\mathbf{s}').$$

Suppose the government *unexpectedly* introduces state-contingent transfers $T'(\mathbf{s}')$ at date $t+1$, so the consumers receive $A'(\mathbf{s}') - T'(\mathbf{s}')$ and the entrepreneurs' net worth increases by $T'(\mathbf{s}')$ and suppose these transfers are positive after low capital quality shocks and negative after high ones. We can interpret these transfers as bailouts to entrepreneurs in states of the world in which they are distressed, compensated by a levy in good states. It is possible to proceed as in the analysis above and construct examples in which the transfers $T'(\mathbf{s}')$ lead to a Pareto improvement.[18]

What is the problem with the policy above? If consumers and entrepreneurs anticipate that the policy will be in place, the contingent bailouts turn out to be completely neutral. To be precise, suppose that the expected present value of the transfer $\sum_{\mathbf{s}'} q(\mathbf{s}'|\mathbf{s})\, T'(\mathbf{s}')$ is zero at the laissez-faire state prices. Then, there exists an equilibrium in which the values of $b'(\mathbf{s}') - T'(\mathbf{s}')$ are identical to the values of $b'(\mathbf{s}')$ at the original laissez-faire equilibrium. In other words, the entrepreneurs completely undo the transfers, by taking additional risky debt in states of the world in which they expect to receive a bailout.[19] The fact that agents have access to perfect state-contingent markets means that they are more flexible in taking advantage of ex post government help. In this framework, this leads to an extreme form of moral hazard as anticipated bailouts are essentially useless.

Turning to capital requirements, an alternative to the Pigouvian taxes introduced in 5.1 is to introduce, at $t = 0$, restrictions to the issuance of debt in proportion to the assets held

[18] This does not requires the government to have superior capacity to enforce payments, as we can build examples in which the transfers always respect the entrepreneurs' no default constraint

$$b'(\mathbf{s}') - T'(\mathbf{s}') + \gamma w(\mathbf{s}')l' \leq \theta(1-\delta)u'k'.$$

[19] If the present value of the transfer is not zero, the effect of the policy is not neutral but is equivalent to a single ex ante, non state-contigent transfer.

by the entrepreneurs, imposing the constraint

$$\sum_{\mathbf{s}'} \omega\left(\mathbf{s}'|\mathbf{s}\right) q\left(\mathbf{s}'|\mathbf{s}\right) b'(\mathbf{s}') \leq k'. \tag{23}$$

In the expression above, $\omega\left(\mathbf{s}'|\mathbf{s}\right)$ are risk weights applied to each state-contingent claim traded. In Online Appendix G we show that the optimal policy can be equivalently implemented by imposing constraint (23) on entrepreneurs, with the appropriate set of risk weights $\omega\left(\mathbf{s}'|\mathbf{s}\right)$, and using a tax on capital.

It is important to notice that the presence of different risk weights $\omega\left(\mathbf{s}'|\mathbf{s}\right)$ plays an essential role. If $\omega\left(\mathbf{s}'|\mathbf{s}\right)$ was constant across states, the intervention would be analogous to a uniform tax on debt, which, as we saw in the previous subsection, is a poor substitute for a state-contingent tax. In practice, risk weights are more usually applied on the asset side of the balance sheet. Our framework provides a macro-prudential argument for using risk weights on the liability side.[20]

6 Conclusion

In this paper we have asked why financial institutions tend to be exposed to aggregate risk despite the availability of several instruments to hedge this exposure. To answer this question, we have used a canonical financial accelerator model in which agents trade fully state-contingent claims. We have obtained two main results.

First, we showed that entrepreneurs may not hedge negative aggregate shocks in equilibrium because insuring these states can be too costly for them. We have isolated the importance of two factors for this result: the general equilibrium spillover of entrepreneurs' net worth on consumers' labor income and the risk aversion of consumers. Under plausible calibrations of our model, these two effects are strong enough to make the productive sector as exposed to aggregate risk as it would be in a corresponding economy where only a non-state-contingent bond can be used for risk-management. These results show that it is feasible to introduce risk-management considerations in this class of models without compromising their ability to generate financial amplification.

Second, we showed that the resulting competitive equilibrium is constrained inefficient and it features too much exposure of entrepreneurs to aggregate risk. In the optimal policy, a planner reduces this exposure by taxing only certain debt instruments, specifically those whose payments are indexed to the negative aggregate shocks. On the contrary, uniform

[20] This connects the analysis here to papers that suggest imposing regulatory constraints based on the sensitivity of balance sheets to correlated shocks, as in Adrian and Brunnermeier (2016).

taxes on all debt instruments, despite reducing overall leverage, are not effective in limiting the entrepreneurs' risk exposure because they incentivize a substitution toward riskier debt instruments. More generally, our results emphasize that macroprudential policies targeted toward certain debt instruments can be substantially more effective than policies that discourage leverage *tout court*—a common prescription of the incomplete market models used in the literature.

These policy prescriptions are obtained in an environment where a full set of state-contingent claims is available. In future research on macroprudential policy, it may be useful to consider models in between the two extremes of no state contingency and full state contingency, to capture more realistically the set of risk-management tools available to financial institutions.

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Online Appendix to "Risk Sharing Externalities"

by Luigi Bocola and Guido Lorenzoni

A Proofs

Proof of Proposition 1

Proof. We divide the proof of this proposition in two parts. First, we establish that if the collateral constraint does not bind at date 0, then the equilibrium sensitivities of entrepreneurial debt payments and net worth to the capital quality shocks are given by (14) and (15). Second, we show that the restrictions on the primitives in the statement of the proposition guarantee that the collateral constraint does not bind at date 0.

Starting with the first part, we can use the expression for V_1 in equation (12) and the market clearing condition $a_1(u_1) = b_1(u_1)$ to write the equilibrium risk sharing condition as

$$\left[\frac{RW_0}{(1-\beta)[b_1(u_1)+w(u_1)+W(n_1(u_1))]}\right]^{\sigma} = \frac{n_0}{n_1(u_1)} \qquad \forall\, u_1.$$

From the definition of net worth in (11), we have that $b_0(u_1) + w(u_1) = (u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - n_1(u_1)$. Substituting this in the above expression and rearranging terms, we obtain

$$n_1(u_1) = \xi\left[(u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - n_1(u_1) + W(n_1(u_1))\right]^{\sigma},$$

where $\xi = n_0 / RW_0^{\sigma} > 0$ is a constant, independent of u_1. Differentiating with respect to u_1 we obtain

$$n_1'(u_1) = \sigma \frac{[\alpha u_1^{\alpha-1} k_0^{\alpha} + (1-\delta)k_0 - n_1'(u_1) + W'(n_1(u_1))n_1'(u_1)]n_1(u_1)}{(u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - n_1(u_1) + W(n_1(u_1))}. \tag{A.1}$$

Using again the definition of net worth in (11) and the market clearing condition for bonds, we have $(u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - n_1(u_1) + W(n_1(u_1)) = a_1(u_1) + w(u_1) + W(n_1(u_1))$. Substituting this expression in the denominator of (A.1) and using the definition of ω in the text, we can rewrite equation (A.1) as

$$n_1'(u_1) = \sigma[\alpha u_1^{\alpha-1} k_0^{\alpha} + (1-\delta)k_0 - n_1'(u_1) + W'(n_1(u_1))n_1'(u_1)]\frac{\omega}{1-\omega}.$$

Collecting on the left hand side the $n_1'(u_1)$ terms, we have

$$n_1'(u_1)\left[\frac{1}{\sigma}\frac{(1-\omega)}{\omega}+1-W'(n_1(u_1))\right]=\alpha u_1^{\alpha-1}k_0^{\alpha}+(1-\delta)k_0,$$

and solving for $n_1'(u_1)$ gives equation (15).

The expression for $b_1'(u_1)$ is obtained by differentiating equation (11) with respect to u_1 and using equation (15) to substitute for $n_1'(u_1)$.

The second part of the proof shows that the conditions of the proposition are sufficient to guarantee that the collateral constraint does not bind at date 0. Let's assume first that $\sigma=0$. In that case, the unconstrained level of capital at date 0 equals

$$k_0=\hat{k}\equiv\frac{\alpha\beta\mathbb{E}\left(u_1^{\alpha}\right)}{1-(1-\delta)\beta\mathbb{E}\left(u_1^{\alpha}\right)}.$$

In addition, from the risk sharing condition (13) we know that $n_1(u_1)=n_0$ for all u_1 when $\sigma=0$. From the definition of $n_1(u_1)$ in equation (11) we then have

$$b_1(u_1)=\alpha(u_1\hat{k})^{\alpha}+(1-\delta)u_1\hat{k}-n_0.$$

The collateral constraint does not to bind at date 0 if $b_1(u_1)<\theta(1-\delta)u_1\hat{k}$ for all $u_1\in[\underline{u},\overline{u}]$. Using the above expression for $b_1(u_1)$, we can rewrite these conditions as

$$n_0>\alpha(u_1\hat{k})^{\alpha}+(1-\theta)(1-\delta)u_1\hat{k}\qquad\forall\,u_1\in[\underline{u},\overline{u}]$$

Because the right hand side of the above expression increases in u_1, the condition on n_0 in the statement of the proposition guarantees that the above is satisfied for all $u_1\in[\underline{u},\overline{u}]$.

Let's now consider the case with $\sigma>0$, and let k_0 be the unconstrained choice of capital by entrepreneurs. If the collateral constraint does not bind at date 0, $b'(u_1)$ is given by equation (14). Using that expression, we have that

$$\begin{aligned}\frac{\partial}{\partial u_1}[b_1(u_1)-\theta(1-\delta)u_1k_0] &= \alpha^2u_1^{\alpha-1}k_0^{\alpha}+(1-\theta)(1-\delta)k_0-\\ &- \frac{\sigma\omega}{\sigma\omega(1-W'(n_1))+(1-\omega)}\left(\alpha u_1^{\alpha-1}k_0^{\alpha}+(1-\delta)k_0\right),\end{aligned}$$

which, for a σ small enough, is positive for every u_1. So, for σ small enough, we have

$$b_1(\overline{u})<\theta(1-\delta)\overline{u}k_0 \tag{A.2}$$

is a sufficient condition for $b_1(u_1) < \theta(1-\delta)u_1 k_0$ for all $\in [\underline{u}, \overline{u}]$.

We now show that the condition on n_0 in the statement of the proposition guarantees that the inequality (A.2) is satisfied. Because $V_1(u_1)$ increases in u_1, we have that $V_1(\overline{u}) \geq RW_0$. From the risk sharing condition (13) it follows that $n_1(\overline{u}) \geq n_0$. So, from the definition of $n_1(u_1)$ we have that

$$b_1(\overline{u}) \leq [\alpha(\overline{u}k_0)^\alpha + (1-\delta)\overline{u}k_0] - n_0.$$

Because $k_0 \leq \hat{k}$ when $\sigma > 0$, we have that

$$n_0 > \alpha(\bar{u}\hat{k})^\alpha + (1-\theta)(1-\delta)\bar{u}\hat{k}$$

guarantees that the inequality (A.2) is satisfied. So, for σ small enough, the condition on n_0 in the statement of the proposition guarantees that $b_1(u_1) < \theta(1-\delta)u_1 k_0$ for all $u_1 \in [\underline{u}, \overline{u}]$.

□

Proof of Proposition 2

Proof. Let us start by solving for the optimal tax on state-contingent claims. Substituting equations (18) and (19) in (17), we have that the planner's allocation needs to satisfy the following condition for every u_1

$$\begin{aligned}\left[\frac{RW_0}{V_1(u_1)}\right]^\sigma - c_{e,0}\left[\frac{1}{c_{e,1}(u_1)} + \mu_1(u_1)\right] &= \left[\frac{RW_0}{V_1(u_1)}\right]^\sigma \sum_{t=1}^{\infty}\beta^t \frac{\partial w_{t+1}}{\partial n_1(u_1)} - \\ &- c_{e,0}\sum_{t=1}^{\infty}\beta^t \frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial n_1(u_1)}. \end{aligned} \tag{A.3}$$

From the consumers' and entrepreneurs' problem, we also know that in any competitive equilibrium with taxes the following condition must hold for every u_1

$$\left[\frac{RW_0}{V_1(u_1)}\right]^\sigma [1-\tau_b(u_1)] = c_{e,0}\left[\frac{1}{c_{e,1}(u_1)} + \mu_1(u_1)\right] \tag{A.4}$$

Substituting equation (A.4) in the left and right-hand side of (A.3) and simplifying, we have that

$$\tau_b(u_1) = \sum_{t=1}^{\infty}\beta^t \frac{\partial w_{t+1}}{\partial n_1(u_1)} - \frac{[1-\tau_b(u_1)]}{\frac{1}{c_{e,1}(u_1)} + \mu_1(u_1)}\sum_{t=1}^{\infty}\beta^t \frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial n_1(u_1)}. \tag{A.5}$$

Given that $c_{e,1}(u_1) = (1-\beta)n_1(u_1)$ in the continuation equilibrium, we can use equation

(A.5) to obtain an expression for $\tau_b(u_1)$,

$$\tau_b(u_1) = \frac{\sum_{t=1}^{\infty} \beta^t \left[1 - \frac{n_1(u_1)}{n_{t+1}} \left(\frac{1}{1+\mu_1(u_1)(1-\beta)n_1(u_1)}\right)\right] \frac{\partial w_{t+1}}{\partial n_1(u_1)}}{1 - \left(\frac{1}{1+\mu_1(u_1)(1-\beta)n_1(u_1)}\right) \sum_{t=1}^{\infty} \beta^t \frac{n_1(u_1)}{n_{t+1}} \frac{\partial w_{t+1}}{\partial n_1(u_1)}}. \tag{A.6}$$

We use the properties of the continuation equilibrium in Lemma 1 to sign $\tau_b(u_1)$. Specifically, we know that $\partial w_{t+1}/\partial n_1(u_1) \geq 0$, with strict inequality if the collateral constraint binds at u_1. In addition, we know that in the continuation equilibrium $n_1 \leq n_j$ for all $j > 1$, with strict inequality if the collateral constraint binds at $j-1$. Given that $\mu_1(u_1) \geq 0$, these properties guarante that $\tau_b(u_1) \geq 0$, with strict inequality if the collateral constraint binds at u_1.

We follow a similar approach to solve for the tax on capital. From the primal problem we know that the planner's allocation must satisfy the following condition

$$\begin{aligned}\beta c_{e,0} \mathbb{E}\left\{\frac{1}{c_{e,1}(u_1)}\left[\alpha u_1^{\alpha} k_0^{\alpha-1} \quad + \quad (1-\delta)u_1\right] + \theta\delta\mu_1(u_1)u_1\right\} = 1 + \beta c_{e,0}\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t \frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial k_0}\right] - \\ - \quad \beta\mathbb{E}\left[\left[\frac{RW_0}{V_1(u_1)}\right]^{\sigma}\sum_{t=0}^{\infty}\beta^t\frac{\partial w_{t+1}}{\partial k_0}\right].\end{aligned} \tag{A.7}$$

In addition, the entrepreneurs' optimality condition for capital implies that in any competitive equilibrium with taxes the following condition holds

$$\beta c_{e,0}\mathbb{E}\left\{\frac{1}{c_{e,1}(u_1)}\left[\alpha u_1^{\alpha}k_0^{\alpha-1} + (1-\delta)u_1\right] + \theta\delta\mu_1(u_1)u_1\right\} = 1 + \tau_k. \tag{A.8}$$

Inspecting equations (A.7) and (A.9), we can see that the optimal tax on capital must be

$$\tau_k = \beta c_{e,0}\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial k_0}\right] - \beta\mathbb{E}\left[\left[\frac{RW_0}{V_1(u_1)}\right]^{\sigma}\sum_{t=0}^{\infty}\beta^t\frac{\partial w_{t+1}}{\partial k_0}\right]. \tag{A.9}$$

Substituting for $[RW_0/V_1(u_1)]^{\sigma}$ in the above expression using equation (A.4) and rearranging terms, we obtain the expression in the main text

$$\tau_k = \beta\mathbb{E}\left\{\frac{n_0}{n_1(u_1)}\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{n_1(u_1)}{n_{t+1}} - \frac{1+\mu_1(u_1)(1-\beta)n_1(u_1)}{1-\tau_b(u_1)}\right]\frac{\partial w_{t+1}}{\partial k_0}\right]\right\}. \tag{A.10}$$

Again, we can use the properties of the continuation equilibrium to sign τ_k. First, we

have that $\partial w_{t+1}/\partial k_0 \geq 0$. Second, the term

$$\left[\frac{n_1(u_1)}{n_{t+1}} - \frac{1+\mu_1(u_1)(1-\beta)n_1(u_1)}{1-\tau_b(u_1)}\right]$$

is necessarely non-negative, and it is strictly negative if the collateral constraint binds at u_1. It follows that $\tau_k \leq 0$, with strict inequality if the collateral constraint binds with positive probability at date 1. □

B Recursive equilibrium

The aggregate state vector is $\mathbf{s} = [u, K, B]$, where K denotes the aggregate capital stock and B the total claims entrepreneurs need to pay to consumers. The state follows the law of motion $\mathbf{\Gamma}(.)$ with transition matrix $\pi(\mathbf{s}'|\mathbf{s})$ and both consumers and entrepreneurs observe this. By a slight abuse of notation, $w(\mathbf{s})$ denotes the wage as a function of the state $\mathbf{s}$ and $q(\mathbf{s}'|\mathbf{s})$ the price of an Arrow-Debreu security that pays one unit of consumption next period if the state is $\mathbf{s}'$.

The representative consumer's problem is then

$$V(a;\mathbf{s}) = \max_{c,l,a'(\mathbf{s}')} \left\{(1-\beta)\left(c - \chi\frac{l^{1+\psi}}{1+\psi}\right)^{1-\rho} + \beta\left[\mathbb{E}_{\mathbf{s}}\left(V(a'(\mathbf{s}');\mathbf{s}')^{1-\sigma}\right)\right]^{\frac{1-\rho}{1-\sigma}}\right\}^{\frac{1}{1-\rho}} \quad \text{s.t.}$$

$$c + \sum_{\mathbf{s}'} q(\mathbf{s}'|\mathbf{s})a'(\mathbf{s}') \leq w(\mathbf{s})l + a.$$

The representative entrepreneur's problem is

$$\begin{aligned} V^e(b,k;\mathbf{s}) &= \max_{c_e,k',l,b'(\mathbf{s}')} \left\{\log(c_e) + \beta_e \mathbb{E}_{\mathbf{S}}\left[V^e(b'(\mathbf{s}'),k';\mathbf{s}')\right]\right\} \quad \text{s.t.} \\ n &= (uk)^\alpha l^{1-\alpha} - w(\mathbf{s})l + (1-\delta)uk - b \\ c_e + k' &\leq n + \sum_{\mathbf{s}'} q(\mathbf{s}'|\mathbf{s})b'(\mathbf{s}') \\ b + \gamma w(\mathbf{s})l &\leq \theta(1-\delta)uk. \end{aligned}$$

We can now define a recursive competitive equilibrium.

Definition A-1. *A recursive competitive equilibrium is given by value functions and policy functions for consumers and for entrepreneurs and pricing functions* $\{q(\mathbf{s}'|.), w(.)\}$ *such that* ***(i)*** *consumers' and entrepreneurs' policies solve their decision problems taking prices as given;* ***(ii)*** *the labor*

market and the markets for contingent claims clear; ***(ii)*** *the law of motion* $\mathbf{\Gamma}(.)$ *is consistent with agents' optimization.*

The following result simplifies the numerical computation of the equilibrium

Lemma A-1. *The consumption function of entrepreneurs' is linear in net worth,*

$$c_e(b,k;\mathbf{s}) = (1-\beta_e)n(\mathbf{s}).$$

Proof. Consider the problem of entrepreneurs, and suppose that we know the optimal policy for the ratios $\tilde{b} = b/(uk)$ and $\tilde{l} = l/(uk)$. Under the optimal policy, these ratios must satisfy the collateral constraint in the entrepreneur's problem. So, given $\tilde{b}'(\mathbf{s}')$ and $\tilde{l}(\mathbf{s})$, we can solve for the optimal consumption/investment problem of the entrepreneur by solving

$$\begin{aligned} V^e(k;\mathbf{s}) &= \max_{c_e,k'} \left\{\log(c_e) + \beta_e \mathbb{E}_{\mathbf{S}}\left[V^e(b'(\mathbf{s}'),k';\mathbf{s}')\right]\right\} \quad \text{s.t.} \\ c_e + \left[1 - \sum_{\mathbf{s}'} q(\mathbf{s}'|\mathbf{s}) b'(\mathbf{s}') u' \tilde{b}'(\mathbf{s}')\right] k' &\leq \left[\tilde{l}(\mathbf{s})^{1-\alpha} - w(\mathbf{s})\tilde{l}(\mathbf{s}) + (1-\delta) - \tilde{b}\right] uk, \end{aligned}$$

This is an optimal saving problem with log utility and a single asset that pays the stochastic return

$$\frac{\left[\tilde{l}(\mathbf{s}')^{1-\alpha} - w(\mathbf{s}')\tilde{l}(\mathbf{s}') + (1-\delta) - \tilde{b}'(\mathbf{s}')\right] u'}{\left[1 - \sum_{\mathbf{s}'} q(\mathbf{s}'|\mathbf{s}) b'(\mathbf{s}') u' \tilde{b}'(\mathbf{s}')\right]}.$$

Following standard arguments, we can then show that the optimal consumption policy satisfies

$$c_e = (1-\beta_e)n(\mathbf{s}),$$

where

$$n(\mathbf{s}) = \left[\tilde{l}(\mathbf{s})^{1-\alpha} - w(\mathbf{s})\tilde{l}(\mathbf{s}) + (1-\delta) - \tilde{b}\right] uk.$$

□

C Numerical solution

We solve for a recursive equilibrium using a global solution algorithm that approximate the policy function for capital, the value function of households and the following functional

$$h(\mathbf{s}'|\mathbf{s}) = \frac{q(\mathbf{s}'|\mathbf{s})}{\pi(\mathbf{s}'|\mathbf{s})\left(C(\mathbf{s}) - \chi\frac{1}{1+\eta}L(\mathbf{s})^{1+\eta}\right)^{\rho}}. \tag{A.11}$$

at a given set of points in the state space. For this purpose, it will be useful to define the state vector as $\mathbf{s} = [u, K, \tilde{N}]$ where the variable $\tilde{N}$ is defined by

$$\tilde{N} \equiv \theta(1-\delta)uK - B. \tag{A.12}$$

Because of the collateral constraints, $\tilde{N} \geq 0$.

Let $\mathcal{S} = \{\mathbf{s}_i\}_{i=1}^{N_S}$ be a set of points in the state space, and let $\{K_i', V_i, h_{Li}, h_{Hi}\}$ be an initial guess at a point $\mathbf{s}_i$ for the next period capital stock, the consumers' value function and for $h(\mathbf{s}'|\mathbf{s})$ evaluated, respectively, at $u' = u_L$ and $u' = u_H$. Our algorithm consists in updating this guess using the equilibrium conditions of the model untill a convergence criterion is met.

In what follows, we first outline the details of the algorithm. We next explain how to use the equilibrium conditions of the model to update the initial guess $\{K_i', V_i, h_{Li}, h_{Hi}\}$.

Numerical algorithm. Our algorithm to find a competitive equilibrium is based on the following three steps:

> **Step 0: Defining the grid.** First, let $\mathcal{U} = [u_L, u_H]$. Set upper and lower bounds on the state variables $(K, \tilde{N})$, and construct for each of these a set of points $\mathcal{K} = [K_1, \ldots, K_{N_K}]$, $\tilde{\mathcal{N}} = [\tilde{N}_1, \ldots, \tilde{N}_{N_{\tilde{N}}}]$. The grid $\mathcal{S}$ is constructed by taking the Cartesian product of $\mathcal{U}, \mathcal{K}, \tilde{\mathcal{N}}$.
>
> **Step 1: Equilibrium conditions at the candidate solution.** Start with a guess at each collocation point $\{K_i', V_i, h_{Li}, h_{Hi}\}$. Use the equilibrium conditions of the model to update the guess $\{\hat{K}_i', \hat{V}_i, \hat{h}_{Li}, \hat{h}_{Hi}\}$.
>
> **Step 2: Iteration.** Compute the Euclidean distance between the initial and updated guess at every collocation point, and let r to be the maximum distance. If $r \leq 10^{-6}$, stop the algorithm. If not, update the guess and repeat Step 1-2. □

The specifics for the algorithm are as follows. The upper bound on K is 15% above its value in a deterministic steady state while the lower bound is 200% below this value. The

points for $\tilde{N}$ are between $[0, 5]$. We let $N_K = 61$ and $N_{\tilde{N}} = 41$. So, we have a total of 5002 collocation points. The initial guess for $\{K_i', V_i, h_{Li}, h_{Hi}\}$ is obtained from the solution of the first-best economy. After every iteration, the new guess for variable x_i is

$$x_i = \alpha x_i + (1 - \alpha)\hat{x}_i,$$

where $\alpha = 0.8$ and $\hat{x}_i$ is computed as described above.

Updating the initial guess. In this section we detail how we use the equilibrium conditions of the model to generate an update for $\{K_i', V_i, h_{Li}, h_{Hi}\}$.

We update K_i' using the Euler equation for capital, which we report below for convenience

$$1 = \beta_e \sum_s \pi_{si} \left\{ \frac{N_i}{N_{si}'} \left[\alpha u_s^{\alpha} (\hat{K}_i')^{\alpha - 1} (L_{si}')^{1-\alpha} + (1 - \delta) u_s \right] \right\} + \\ + (1 - \beta_e) N_i \theta (1 - \delta) \sum_s \pi_{si} u_s \mu_{si}'. \tag{A.13}$$

In the above notation, π_{si} is the conditional probability of $u' = u_s$ given that today we are in state $\mathbf{s}_i$ and $\hat{K}_i$ is the updated choice of capital.

In order to obtain $\hat{K}_i$ from this expression we need to compute $(N_i, N_{si}', L_{si}', \mu_{si}')$ given $\hat{K}_i$ at each $\mathbf{s}_i$ and for each realization of the of the capital quality shock tomorrow $u' = u_s$. Below we explain how we can obtain these variables using the equilibrium conditions of the model.

To compute N_i, we can first compute aggregate labor at $\mathbf{s}_i$ using the expression

$$L_i \quad = \quad \min \left\{ \left[\frac{(1 - \alpha)(u_i K_i)^{\alpha}}{\chi} \right]^{\frac{1}{\alpha + \eta}}, \left(\frac{\tilde{N}_i}{\chi \gamma} \right)^{\frac{1}{1 + \eta}} \right\}. \tag{A.14}$$

The first expression in the "min" is aggregate labor when the collateral constraint does not bind, and it is obtained by equating (3) and (7) when the collateral constraint does not bind. The second expression in the "min" gives the maximum amount of labor that the entrepreneur can finance, and it follows from the definition of $\tilde{N}$, the collateral constraint (1), and the fact that $W_i L_i$ equals $\chi L_i^{1+\psi}$ from the consumers' optimal labor supply (3).

Given this expression for L_i, we have that aggregate net worth at a collocation point $\mathbf{s}_i$ is

$$N_i = (u_i K_i)^{\alpha} L_i^{1-\alpha} - \chi L_i^{1+\psi} + (1 - \delta)(1 - \theta) u_i K_i + \tilde{N}_i, \tag{A.15}$$

where we have used the definition of $\tilde{N}$.

The values for $(N'_{si}, L'_{si}, \mu'_{si})$ depends on the value of $\hat{K}'$ and on whether the collateral constraints bind or not at $u' = u_s$. Let us first guess that the constraints do not bind. Setting $\mu'_{si} = 0$, we can compute $\{N'_{si}, L'_{si}\}$ for $s = \{L, H\}$ using the risk sharing conditions (5) and the optimality conditions for labor

$$\begin{aligned} N'_{si} &= \frac{\beta_e N_i}{\left(C_i(\hat{K}'_i) - \chi \frac{1}{1+\eta} L_i^{1+\eta}\right)^{\rho} h_{si}} \\ L'_{si} &= \left[\frac{(1-\alpha)(u_s \hat{K}'_i)^{\alpha}}{\chi}\right]^{\frac{1}{\alpha+\eta}}. \end{aligned}$$

These expressions use the values for $\{L_i, N_i\}$ obtained earlier, the initial guess for h_{si} and the value of $C_i(\hat{K}'_i)$ computed from the resource constraint using the fact that entrepreneurs' consumption is linear in net worth

$$C_i(\hat{K}'_i) = (u_i K_i)^{\alpha} L_i^{1-\alpha} + (1-\delta) u_i K_i - (1-\beta_e) N_i - \hat{K}'_i. \tag{A.16}$$

To verify that the collateral constraint does not bind, we can compute B'_{si} from the definition of net worth,

$$B'_{si} = (u_s \hat{K}'_i)^{\alpha} (L'_{si})^{1-\alpha} - \chi (L'_{si})^{1+\psi} + (1-\delta) u_s \hat{K}'_i - N'_{si},$$

and our guess is verified if $B'_{si} \leq \theta(1-\delta) u_s \hat{K}'_i - \gamma\chi (L'_{si})^{1+\psi}$.

If $B'_{si} > \theta(1-\delta) u_s \hat{K}'_i - \gamma\chi (L'_{si})^{1+\psi}$ for some s, we need to solve for the next period constrained allocation conditional on $u' = u_s$. This is done by solving a fixed-point problem in N'_{si}. To do so, we can write

$$(1-\beta_e) N'_{si} \mu'_{si} = \frac{\left(C_i(\hat{K}'_i) - \chi \frac{1}{1+\eta} L_i^{1+\eta}\right)^{\rho} h_{si} N'_{si}}{\beta_e N_i} - 1 \tag{A.17}$$

using equation (5). Using the demand and supply of labor, we can express L'_{si} as a function of N'_{si} and of the initial guess for capital K'_i,

$$L'(N'_{si}) = \left[\frac{(1-\alpha)(u_s \hat{K}'_i)^{\alpha}}{\chi\left[1 + \gamma\left(\frac{\left(C_i(\hat{K}'_i) - \chi \frac{1}{1+\eta} L_i^{1+\eta}\right)^{\rho} h_{si} N'_{si}}{\beta_e N_i} - 1\right)\right]}\right]^{\frac{1}{\alpha+\eta}}. \tag{A.18}$$

The fixed-point problem consists in choosing N'_{si} so that the following equation is satisfied

$$N'_{si} = (u_s \hat{K}'_i)^{\alpha} L'(N'_{si})^{1-\alpha} - (1-\gamma)\chi L'(N'_{si})^{1+\psi} + (1-\theta)(1-\delta)u_s \hat{K}'_i.$$

Once we have found N'_{si}, we can compute (μ'_{si}, L'_{si}) using equations (A.17) and (A.18).

After having computed $(N_i, N'_{si}, L'_{si}, \mu'_{si})$ for each $\hat{K}'_i$, we can then choose $\hat{K}'_i$ that satisfies equation (A.13). This will be our updated value for $\hat{K}'_i$.

Note that as a by-product of this previous step we also obtain an expression for B'_{si} at the updated value of $\hat{K}'_i$, and so we can compute $\tilde{N}'_{si}$ using equation (A.12).

After having updated $\hat{K}'_i$, we can easily obtain an update for $\{V_i, q_{Li}, q_{Hi}\}$. The update for the value function of consumers at $\mathbf{s}_i$, $\tilde{V}_i$, is then given by

$$\hat{V}_i = \left\{ (1-\beta)\left(C_i(\hat{K}'_i) - \chi \frac{L_i^{1+\psi}}{1+\psi} \right)^{1-\rho} + \beta \left[\sum \pi_{si} (V'_{si})^{1-\sigma} \right]^{\frac{1-\rho}{1-\sigma}} \right\}^{\frac{1}{1-\rho}},$$

where V'_{si}, the value function at the point $(u_s, \hat{K}'_i, \tilde{N}'_{si})$, is obtained by interpolating the initial guess V_i.

The update for $\{h_{Li}, h_{Hi}\}$ is obtained similarly using equation (A.11) and the definition of the state prices in the main text. This step requires interpolating the initial guess for capital in order to obtain C'_{si}.

D Sensitivity analysis

In this section we present a sensitivity analysis. Specifically, we consider two exercises. First, we consider a different calibration of the capital quality shock characterized by smaller and less persistent shocks. Second, we study a version of the model in which the only source of risk is a shock to the pleadgeability of capital as in Jermann and Quadrini (2012).

D.1 Sensitivity to the calibration of the capital quality shock

In our baseline calibration of the model we have set $u_L = 0.925$ and $P(u_{t+1} = L | u_t = L) = 0.66$. We now study the sensitivity of our results with respect to the size and persistence of the capital quality shocks. Specifically, we consider two alternative calibrations. In the first, we keep all parameters at their baseline level, but set $u_L = 0.95$. We refer to this as the "small shock" calibration. In the second, which we label "low persistence", we set

$P(u_{t+1} = L|u_t = L) = 0.33$ while keeping all the other parameters at their baseline level. Table A-1 reports the same moments of Table 2 in the paper for the baseline calibration and for these two alternative calibrations. In all these illustrations, we set $\sigma = 10$.

Table A-1: Sensitivity Analysis

	Baseline			Small shock			Low persistence		
	FB	IM	CM	FB	IM	CM	FB	IM	CM
Panel A: Quantities									
$\Delta(\log n_t)$		-25.30	-16.46		-16.72	-6.43		-25.55	-18.13
$\Delta(\log \tilde{n}_t)$		-96.58	-72.29		-77.78	-30.63		-97.25	-78.36
$\Delta(\log l_t)$	-1.89	-4.11	-11.16	-1.24	-6.70	-9.73	-1.91	-3.83	-10.36
$\Delta(\log i_t)$	-8.99	-19.00	-17.69	-5.32	-11.76	-10.54	-2.48	-19.26	-17.75
$\Delta(\log y_t)$	-3.77	-5.26	-9.99	-2.48	-6.14	-8.17	-3.82	-5.10	-9.48
Panel B: Entrepreneurs' balance sheet									
n_{t-1}		7.79	6.67		7.43	6.44		8.00	6.95
$\tilde{n}_{t-1}$		2.28	1.56		1.83	1.08		2.34	1.70
k_{t-1}/n_{t-1}		3.08	3.56		3.37	3.88		3.09	3.52
$b_{L,t}/b_{H,t}$		1.00	0.97		1.00	0.96		1.00	0.97

Notes: *See the note to Table 2 in the paper.*

Let us study first the "small shock" calibration. Starting from the FB economy, we can see that the response of labor, output and investment is smaller with a smaller capital quality shock. Interestingly, this does not happen in the IM economy. As we reduce the size of the shock, the response of output and labor slightly increases compared to the baseline. This is a manifestation of what Brunnermeier and Sannikov (2014) refer to as the "volatility paradox"—the observation that financial amplification may increase when reducing the volatility of aggregate shocks due to the endogenous response of entrepreneurs' leverage. In the CM economy, instead, this volatility paradox is not operative. In the small shock calibration, consumers are more willing to offer protection to entrepreneurs than in the baseline because labor income falls by less; so, entrepreneurs in equilibrium increase their leverage mostly by increasing debt that pays when $u_{t+1} = u_H$. That is, reducing the size of the shock tends to reduce financial amplification in the CM economy, and move it closer to the frictionless case. It is worth pointing out that in our numerical experiment this effect is not strong enough, and the behavior of the CM economy is still much closer to that of the IM economy than that of the FB. So, our core result from Section 4 is preserved in this

robustness exercise.

Turning to the second experiment, we can see from Table A-1 that a reduction in the persistence of the low capital quality state does not alter much the behavior of the CM or IM economy. Indeed, the statistics regarding macroeconomic aggregates and the entrepreneurs' balance sheet in the "low persistence" calibration are virtually identical to those in the baseline.

D.2 A shock to the pleadgeability of capital

We now ask whether the mechanism in this paper also operates for other types of shocks. In particular, we look at a shock that is also widely used in models of financial amplification: a shock to the financial constraint.

Following Jermann and Quadrini (2012), we consider shocks to the parameter θ that determines the fraction of the capital stock that can be pledged in financial contracts. We assume that θ can take two values, $\theta(s_t) \in \{\theta_L, \theta_H\}$, with transition matrix $\pi(s_t|s_{t-1})$. We parametrize the transition probabilities following Jermann and Quadrini (2012), and consider a symmetric Markov chain with $\Pr(\theta' = \theta_s|\theta = \theta_s) = 0.97$. We set $\theta_H = 0.82$, as in the previous sections, while θ_L is set to 0.72.[21] All remaining parameters are the same as in Table 1, with the exception that capital quality $u_t = 1$ for all t.

A shock to θ_t, unlike a shock to u_t, has no effects on the first-best allocation. Therefore, we focus on comparing the complete market and the incomplete market economies. Figure A-1 reports impulse response functions to a negative shock to θ_t. Let us start with the incomplete market economy, with $\sigma = 1$.[22] A fall in θ_t does not directly affect the balance sheet of the entrepreneur: neither the value of the assets, nor debt payments change. Thus, on impact, net worth does not change. However, the shock affects the borrowing capacity of the entrepreneur leading to a contemporaneous fall in the demand of capital and to a fall in the demand of labor in future periods. The net worth of entrepreneurs increases in the periods following the shock, because of the higher profits made by the entrepreneurs when the collateral constraint binds. The increase in net worth mitigates the reduction in θ_t, so the effects of the shock on output essentially go away after the first two periods.

Consider now the economy with state-contingent claims. When $\sigma = 0$, entrepreneurs use contingent claims to partly insure against the shock to θ_t: entrepreneurial net worth increases on impact after the shock because contingent debt payments are lower. The in-

[21] Jermann and Quadrini (2012) construct a time series for θ_t using US data. A fall of 0.1 is in line with the fall observed in this time series during the Great Recession.

[22] As in the case of u shocks, the responses to θ shocks in the incomplete market economy are minimally affected by σ.

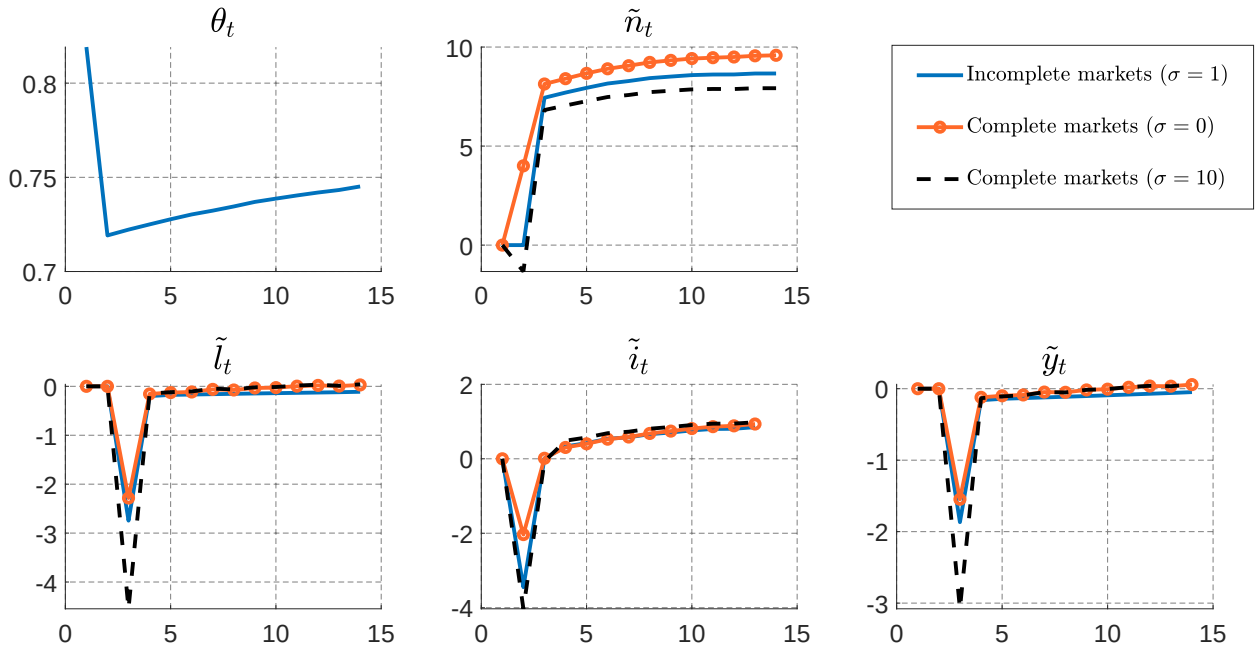

Figure A-1: Impulse response functions: θ shocks

Notes: *See the note to Figure 2 for the calculation of impulse response functions. Net-worth, labor, investment, and output are in percentage deviations from pre-shock values.*

crease in net worth on impact immediately dampens the contraction in θ_t, so the reduction in investment and labor are less pronounced relative to the incomplete market economy. As in the model with capital-quality shocks, the combination of complete markets and low consumers' risk aversion dampens financial amplification.[23]

As we increase σ, consumers are less willing to bear risk and the degree of financial amplification in the economy increases. With $\sigma = 10$ (dotted line in the figure), net worth actually falls after the θ_t: consumers are willing to pay a premium for hedging the fall in their future labor income when $\theta_t = \theta_L$, and entrepreneurs provide this insurance by issuing more debt that pays in those states of the world. The associated fall in net worth implies a stronger decline in the demand of labor and capital, larger even than in the economy with incomplete markets.

Summing up, the mechanism emphasized in our paper seems to play a relevant role also in models with shocks to the collateral constraint.

[23] It is worth noting that the economy still features a substantial degree of financial amplification even with risk neutral consumers. Unlike for the capital quality shock, a shock to θ_t does not have a negative effect on entrepreneurs' net worth, so entrepreneurs have weaker incentives to insure against it. This is related to results in Di Tella (2017) and Dávila and Philippon (2017), that show that the nature of the shock matters for hedging incentives.

E The "AK" economy

We consider two modifications to the model of Section 2. First, we assume that the production function is linear in capital,

$$y_t = u_t k_{t-1},$$

with u_t being an iid stochastic process. Because labor is not a factor of production, consumers do not earn labor income and the collateral constraint is as in equation (1) with $\gamma = 0$. Second, we assume that consumers and entrepreneurs have the same CRRA preferences,

$$u(c(s^t)) = \frac{c(s^t)^{1-\sigma}}{1-\sigma}.$$

Let us denote

$$A_t = u_t[1 + (1-\delta)].$$

We make the following restrictions on the distribution of A_t,

$$\beta \mathbb{E}\left[A_{t+1}^{1-\sigma}\right] \leq 1, \qquad \left\{\theta\beta + (1-\theta)\beta_e \mathbb{E}\left[A_{t+1}^{1-\sigma}\right]\right\}^{\frac{1}{\sigma}} \frac{\beta}{\theta\beta + (1-\theta)\beta_e} \leq 1. \tag{A.19}$$

These assumptions are satisfied for any distribution of u when $\sigma = 1$.

The next proposition characterizes the competitive equilibrium of this economy.

Proposition A-1. *Suppose that the restrictions in (A.19) are satisfied. Then, there is a stationary equilibrium in which*

$$b_{t+1} = \theta A_t k_t \qquad k_t = \kappa A_t k_{t-1} \qquad c_{e,t} = h A_t k_{t-1} \qquad c_t = f A_t k_{t-1}. \tag{A.20}$$

Proof. To prove the proposition, we verify that the allocation in (A.20) satisfies the necessary and sufficient conditions for an equilibrium for some (κ, h, f).

First, given the allocation in (A.20), we can write the resource constraint as

$$f + \kappa + h = 1.$$

Thus, we need to show that there exists $\kappa > 0$ and $h > 0$ that satisfy the optimality conditions of consumers and entrepreneurs, and such that $\kappa + h < 1$.

Given the allocation in (A.20) the growth rate of consumption of entrepreneurs and consumers is given by κA_{t+1}. Because $\beta > \beta_e$, we have that the risk sharing conditions are

satisfied with the collateral constraint binding in every state of the world,

$$\mu_{t+1} c_{e,t} = (\beta - \beta_e)(A_{t+1}\kappa)^{-\sigma}.$$

Using this expression, we can write the Euler equation for capital, (8), as follows

$$1 = \mathbb{E}_t \left\{ \left[\theta\beta \left(A_{t+1}\kappa\right)^{-\sigma} + (1-\theta)\beta_e \left(A_{t+1}\kappa\right)^{-\sigma} \right] A_{t+1} \right\}.$$

Solving for κ, we obtain,

$$\kappa^{\sigma} = (\theta\beta + (1-\theta)\beta_e)\mathbb{E}[A_{t+1}^{1-\sigma}] < 1 \tag{A.21}$$

The budget constraint of entrepreneurs can be written as

$$k_t + c_{e,t} = (1-\theta)A_t k_{t-1} + \mathbb{E}_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \theta A_{t+1} k_t \right].$$

Substituting the allocation in (A.20) in the above equation and rearranging terms we obtain

$$h = (1-\theta)\left(1 - \kappa \frac{\beta_e}{\theta\beta + (1-\theta)\beta_e}\right) < 1. \tag{A.22}$$

Given the restrictions in (A.19), we can easily verify from equations (A.21) and (A.22) that $\kappa + h < 1$. □

Given the allocation in (A.20), we can obtain the impulse response function of capital to a percentage increase in A_t,

$$\frac{\partial \mathbb{E}_t[\log k_{t+j}]}{\partial \log A_t} = 1 \quad \forall j \geq 0.$$

It is straightforward to verify that this is the same impulse response function that would arise in a version of the model without the collateral constraint. In other words, the "AK" economy with state-contingent claims features no financial amplification relative to the first-best economy.

F Model without general equilibrium spillover

In this section we explain how we eliminate the general equilibrium spillover on wages. In the first best economy, labor income can be written as

$$LI(u, K^{fb}) = \chi \left[\frac{(1-\alpha)(uK^{fb})^{\alpha}}{\chi} \right]^{\frac{1+\eta}{\alpha+\eta}}. \tag{A.23}$$

To eliminate the general equilibrium spillover of entrepreneurs' net worth on consumers' labor income we proceed as follows. We assume that consumers solve the same decision problem as in Section 2, with the exception that their wages are the ones of the first best. That is, consumers supply labor as if they were working for firms operating in an economy without the collateral constraints (1), and their labor income is given by (A.23).

Letting the aggregate state vector be $\mathbf{s} = [u, \tilde{N}, K, K^{fb}]$, we can write the consumers' problem as

$$V(a; \mathbf{s}) = \max_{c, a'_{\mathbf{s}'}} \left\{ (1-\beta) \left(c - \frac{LI(u, K^{fb})}{1+\psi} \right)^{1-\rho} + \beta \left[\mathbb{E}_{\mathbf{s}} \left(V(a'_{\mathbf{s}'}; \mathbf{s}')^{1-\sigma} \right) \right]^{\frac{1-\rho}{1-\sigma}} \right\}^{\frac{1}{1-\rho}} \quad \text{s.t.}$$

$$c + \sum_{\mathbf{s}'} q(\mathbf{s}'|\mathbf{s}) a'_{\mathbf{s}'} \leq LI(u, K^{fb}) + a,$$

where we have substituted for the optimal labor supply of consumers if they were to face the wage process of the first best economy.

The entrepreneurs, instead, solve the same decision problem as before. One way of interpreting this extension is that there are two consumers in the economy: hand to mouth consumers that work for entrepreneurs and supply labor optimally, and Ricardian consumers that work for a sector that does not face the collateral constraints (1) and that trade contingent claims with entrepreneurs.

In equilibrium, we require that *i)* the supply of labor by the hand to mouth consumers to equal the demand of labor by entrepreneurs; and *ii)* the supply of bonds by entrepreneurs equal the demand of bonds by the Ricardian consumers. Note that the resource constraint in this economy will not be satisfied because the labor income earned by Ricardian consumers differ from the payments for labor services by entrepreneurs.

To solve numerically for an equilibrium, we proceed in two steps. In the first step, we solve the equivalent first best economy and obtain the policy function $K'^{fb}(u, K^{fb})$. This policy function is relevant because it allows to forecasts the future labor income of

Ricardian consumers according to equation (A.23). In the second step, we solve for the decision problem of consumers and entrepreneurs and make sure that the entrepreneurs' labor market and the market for contingent claims clear.

G Optimal Policy

This section consists of three parts. First, we show that we can characterize the best competitive equilibrium with taxes by studying the planner's primal problem defined in Section 5.1. Second, we show that we can equivalently implement the planner's allocation using a capital-adequacy ratio with risk-weighting on entrepreneurs' liabilities. Third, we describe the algorithm that we use to numerically solve the planner's primal problem.

The primal problem and the competitive equilibrium with taxes Fix a point $\mathbf{s} = [u, K, B]$ in the state space and consider the problem (SP). We want to prove that the problem of choosing optimally the tax vector $\{\tau_b(\mathbf{s}'|\mathbf{s}), \tau_k(\mathbf{s}), T_c(\mathbf{s}), T_e(\mathbf{s})\}$ in a competitive equilibrium with taxes is equivalent to the planning problem (SP).

First, because any competitive equilibrium with taxes must satisfy the resource constraint, the solution to problem (SP) delivers an upper bound to the consumers' utility in the best competitive equilibrium with taxes. Therefore, to complete our argument, we just need to show that the solution to problem (SP) can be implemented as a competitive equilibrium with taxes.

Let $\{X, C_e, K', B'(\mathbf{s}')\}$ be the allocation that solves (SP). Using the policy functions at the laissez-faire competitive equilibrium and the states $K', B'(\mathbf{s}')$ we can compute the next period values of $X'(\mathbf{s}'), V'(\mathbf{s}')$ for any realization of the state $\mathbf{s}'$. The optimality condition of consumers in the competitive equilibrium with taxes implies that the price of state-contingent claims must satisfy

$$q(\mathbf{s}'|\mathbf{s}) = \beta\pi(s'|s)\left(\frac{X}{X'(\mathbf{s}')}\right)^{\rho}\left(\frac{RW(\mathbf{s})}{V'(\mathbf{s}')}\right)^{\sigma-\rho}.$$

Let us consider next entrepreneurs. Using the policy functions of the competitive equilibrium, we can calculate the future labor effort $L'(\mathbf{s}')$, the future net worth $N'(\mathbf{s}')$, and the future value of the Lagrange multiplier $\mu'(\mathbf{s}')$. The optimality conditions of the entrepreneur then take the form

$$\left[1-\tau_b(\mathbf{s}'|\mathbf{s})\right] q(\mathbf{s}'|\mathbf{s})\frac{1}{C_e} = \beta_e\pi(s'|s)\left[\frac{1}{(1-\beta_e)N'(\mathbf{s}')} + \mu'(\mathbf{s}')\right] \tag{A.24}$$

$$[1+\tau_k(\mathbf{s})]\frac{1}{C_e} = \beta_e \mathbb{E}\left\{\frac{1}{(1-\beta_e)N'(\mathbf{s}')}\left[\alpha\left(u'\right)^{\alpha}\left(K'\right)^{\alpha-1}(L'(\mathbf{s}'))^{1-\alpha}+(1-\delta)u'\right]\right\}+ \\ +\beta_e\theta(1-\delta)\mathbb{E}\left[u'\mu'(\mathbf{s}')\right]. \tag{A.25}$$

These conditions can be solved to obtain $\tau_b(\mathbf{s}'|\mathbf{s})$ and $\tau_k(\mathbf{s})$. The transfers $\{T_b(\mathbf{s}), T_e(\mathbf{s})\}$ can then be chosen so that the budget constraints of entrepreneurs, consumers and the government are satisfied. Therefore, given the optimal allocation from problem (SP), there exists a vector of taxes and a competitive equilibrium with taxes that support the same allocation.

Capital requirements We now show that the solution to the planner's primal problem can be equivalently implemented using capital requirements, a tax on capital and lump sum transfers.

Consider the entrepreneur's problem of Section 5.1, replacing the second constraint with:

$$c_e + \left(1+\tau_k'\right)k' \leq n + \sum_{\mathbf{s}'} q\left(\mathbf{s}'|\mathbf{s}\right)b'(\mathbf{s}') + T_e'(\mathbf{s}),$$

and introducing the additional constriant

$$\sum_{\mathbf{s}'} \omega\left(\mathbf{s}'|\mathbf{s}\right)q\left(\mathbf{s}'|\mathbf{s}\right)b'(\mathbf{s}') \leq k'.$$

The entrepreneur's optimality conditions can then be written as:

$$\frac{1}{c_e}\left(1+\tau_k'\right) - \nu = \beta_e \mathbb{E}_{\mathbf{s}}\left[\frac{\partial V^e}{\partial k'}\right],$$

$$\frac{1}{c_e}q\left(\mathbf{s}'|\mathbf{s}\right) - \nu\omega\left(\mathbf{s}'|\mathbf{s}\right)q\left(\mathbf{s}'|\mathbf{s}\right) + \beta\pi\left(s'|s\right)\frac{\partial V}{\partial b'} = 0,$$

where ν is the Lagrange multiplier on the capital requirement constraint. Comparing these optimality conditions with those from the entrepreneur's problem in the equilibrium with Pigouvian taxes, we can show that the same allocation can be implemented in the economy with a capital requirement. In particular, the mapping between the two cases is obtained by letting the value of ν be given by

$$\nu = \frac{1}{c_e}\sum_{\mathbf{s}'}\tau_b\left(\mathbf{s}'|\mathbf{s}\right)q\left(\mathbf{s}'|\mathbf{s}\right)\frac{b'(\mathbf{s}')}{k'},$$

by letting the risk weights be

$$\omega\left(\mathbf{s}'|\mathbf{s}\right) = \frac{1}{\nu}\frac{1}{c_e}\tau_b\left(\mathbf{s}'|\mathbf{s}\right),$$

and by setting the tax on capital equal to

$$\tau_k' = \tau_k + \nu c_e,$$

where τ_k is the tax on capital in the equilibrium with Pigouvian taxes.

Notice that the risk weights are proportional to the state-contingent Pigouvian taxes on debt, proving the assertion in the text that the case of constant risk weights is equivalent to the case of constant Pigouvian taxes.

Solving the planner's primal problem numerically. The numerical algorithm to solve for the planner's primal problem builds on the algorithm in Section C of this Appendix. Let's fix a point $\mathbf{s}_i$ in the state space, and let V_i^e be the value of entrepreneurs in the competitive equilibrium with taxes at $\mathbf{s}_i$. Construct a grid of feasible values for $[K', \tilde{N}_H', \tilde{N}_L']$. For each point in the grid, we perform the following steps in order to evaluate the objective function:

i Given a point in the grid, $[K', \tilde{N}_H', \tilde{N}_L']$, compute the continuation values of entrepreneurs by interpolating the value function of the competitive equilibrium V^e. Choose C_e so that entrepreneurs get as much utility as in the competitive equilibrium without taxes, V_i^e.

ii Given $[K', \tilde{N}_H', \tilde{N}_L']$ and C_e, compute C using the resource constraint.

iii Given $[K', \tilde{N}_H', \tilde{N}_L']$, use consumers' value function of the competitive equilibrium V to interpolate their continuation values at $[K', \tilde{N}_H', \tilde{N}_L']$.

Given these three steps, we can evaluate the welfare of consumers at any point in the grid. We then choose the point that maximizes consumers' welfare. We repeat this procedure for all the points in the state space $\{\mathbf{s}_i\}$.

Once we obtain the solution to the primal problem, we can compute the optimal taxes at any point in the state space using equations (A.24), (A.25) and the entrepreneurs' and consumers' budget constraints.