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AIMING FOR THE GOAL:
CONTRIBUTION DYNAMICS OF CROWDFUNDING

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ABSTRACT

We study a dynamic contribution game where investors seek private benefits that are offered in exchange for contributions and a single, publicly-minded donor values project success. We show that donor contributions serve as costly signals that encourage socially-productive contributions by investors who face a coordination problem. Investors and the donor prefer different equilibria but all benefit in expectation from the donor's ability to dynamically signal his valuation. We explore various contexts in which our model can be applied and delve empirically into the case of Kickstarter. We calibrate our model and quantify the coordination benefits of dynamic signaling in counterfactuals.

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1 Introduction

Contribution games typically feature positive externalities that result in free-riding and miscoordination. These two forces can lead to underprovision, which has been widely studied in the context of public goods. Although free-riding may be alleviated if investors receive private benefits in exchange for contributions, miscoordination can persist due to uncertainty about future contributions. In this paper, we show how the dynamic interaction between differently motivated participants—a publicly-motivated contributor and privately-motivated investors—can alleviate miscoordination and promote success.

We study a contribution game where randomly arriving investors decide whether to contribute to a project that only succeeds if enough contributions are made before a deadline. If the project succeeds, investors who pledged support receive a fixed, private benefit. A representative, long-lived donor values the public benefits, but his valuation for the project, akin to a budget, is his private information. The donor can contribute throughout the game, and his payoff is equal to his valuation less the total contribution he makes. If the project fails, all contributions are returned, but investors bear an opportunity cost. Therefore, investors contribute only if they believe the project will succeed with a sufficiently high probability. The probability of success is shaped by investor beliefs about the donor's valuation and the commitments made by other investors over time.

Such contribution games arise in many settings. Our leading application is reward-based crowdfunding, where entrepreneurs raise funds via a combination of donations and purchases. Another application is fixed-duration capital campaigns launched by non-profits targeting large leadership donors and small privately-motivated contributors. The forces of our model also manifest in settings in which a government strategically times its interventions (e.g., with subsidies) to encourage investment in a sunrise industry or a multinational firm needs to encourage suppliers to invest in nearby facilities.

Our analysis yields two key new insights. First, we show that the donor can alleviate the coordination risk faced by private investors by signaling his valuation for the project. We derive bounds on the effect of dynamic signaling on the probability of project success and

establish that making the donor's valuation public would result in the lowest probability of success. Second, we show that there exists a trade-off between maximizing the donor's and investors' payoffs. The donor prefers the equilibrium that maximizes the probability of success. However, this equilibrium does not internalize investors' opportunity costs of foregoing the outside option and therefore, investors prefer equilibria with intermediate probabilities of success.

Formally, we first show that a perfect Bayesian equilibrium that maximizes the probability of success can be attained by a Markov equilibrium with an intuitive structure involving a dynamic, state-dependent donation threshold. Donating less than the threshold stops privately-motivated investments irreversibly, causing the project to fail, while donating more squanders funds. Hence, all donors with a sufficiently high valuation donate just above the required threshold. For this reason, we call such equilibria pooling-threshold (PT) equilibria. The threshold decreases after a pledge, but increases in the absence of one.

In the success-maximizing equilibrium, the threshold is such that the donor makes investors just indifferent between contributing and selecting the outside option. We construct this equilibrium by induction on the number of additional investors necessary to achieve the goal if no more donations are made. To establish that it maximizes the probability of success, we use an innovative proof approach that recasts the problem as a dynamic information design problem. We then show that the solution to this relaxed problem is attained by the constructed equilibrium.

We next characterize the unique outcome that minimizes the probability of success in any perfect Bayesian equilibrium. It is also attained by a PT equilibrium, but the donation thresholds are so high that investors contribute even if they assumed that no additional donations will be made. The success-minimizing equilibrium outcomes coincide with a game where the donor's valuation is public information, i.e., a game without signaling.

Campaign participants prefer different equilibria. The donor-preferred equilibrium is the success-maximizing equilibrium. Investors want to reduce their coordination risk associated with forgoing their outside option. They prefer equilibria with probabilities of

success strictly between the success-maximizing and success-minimizing equilibria.

Because our proof approach involves recasting the success-maximization problem as a general information design problem, we can also establish that no other mechanism involving donations, fixed investments, and observable contribution histories can achieve a higher probability of success. For example, the probability of success is lower if the donor is restricted to contribute only at the beginning or at the end of a campaign.

We consider several extensions that highlight the broader applicability of our findings. For example, we show that the success-maximizing equilibrium is still achievable in the presence of multiple donors that know each other's valuations. We also consider an extension where the campaign is no longer all-or-nothing, the goalpost is uncertain, and investments have some scrap value. This extension can represent a government (donor) seeking to encourage investment by private firms in developing industries, or a large, multinational firm establishing a new location, seeking potential suppliers to invest. We highlight that the government/multinational firm can best coordinate firms/suppliers by withholding some resources initially. A single, initial large push to get investors to make commitments, or fully revealing the the level of support of the government/multinational firm, does not maximize the probability of success.

Finally, we apply our framework to reward-based crowdfunding using data from the platform Kickstarter. On this platform, entrepreneurs initiate campaigns to raise funds to launch new products (rewards). The platform allows for both donations and investments—in this case, pledges to buy a product. These campaigns map well to our theoretical model. Our sample covers all campaigns launched between March 2017 and September 2018. A novel aspect of our data is that it distinguishes pledges for rewards from donations at 12-hour frequencies. We document that donations constitute 14% of all funds raised.

Our empirical analysis proceeds in two steps. First, we validate our modeling assumptions and provide descriptive evidence consistent with equilibrium predictions. For example, we show that pledges are consistent with the two distinct contribution incentives of our model. While pledges occur throughout time, donations decline close to zero after

a campaign succeeds as our model predicts. Second, we calibrate our model to the data and conduct counterfactuals. Our results emphasize the crucial role dynamic signaling plays at facilitating coordination as there is a significant gap in the success rate between the success-maximizing (16%) and the success-minimizing equilibria (3%). We also simulate counterfactuals that restrict donations to occur at the start (or end) of the campaign. Although this tends to result in larger initial commitments by the donor, we find that the success rate is even lower than what we estimate under the success-minimizing equilibrium with continuous donations.

1.1 Related Literature

Our setting is related to public good contribution games (Bagnoli and Lipman, 1989; Admati and Perry, 1991; Fershtman and Nitzan, 1991; Varian, 1994), but as in Alaei et al. (2016), participants receive private benefits.¹ At the core of our model is a socially-minded donor who can incentivize contributions, akin to the leadership giver in Andreoni (1998) who can help overcome free-riding. While our setting has no free-riding, we show that signaling by the donor alleviates coordination risk.

Our results on success-maximizing mechanisms complement recent work examining optimal mechanism design in crowdfunding, including Strausz (2017), who finds the optimal mechanism in the presence of moral hazard of investors, Ellman and Hurkens (2019b), who find the optimal mechanism with price discrimination, and Chang (2020), who studies two funding mechanisms when the quality of the project is unknown.²

We also contribute to the literature on dynamic signaling. Unlike classic dynamic signaling environments, including Noldeke and Van Damme (1990) and Swinkels (1999), in our setting investments are welfare enhancing and are not wasteful. Our construction of PT equilibria involves belief thresholds to guarantee participation as common in mod-

¹See also, Marx and Matthews (2000); Campbell et al. (2014); Cvitanic and Georgiadis (2016); Bonatti and Hörner (2011); Sahn (2020). We are also related to work on crowdfunding, including Liu (2018); Chemla and Tinn (2020); Chakraborty et al. (2023); Chakraborty and Swinney (2020); Ellman and Fabi (2022).

²Ellman and Hurkens (2019a) relax the ex-post individual rationality constraint. Belavina et al. (2020) further distinguish between funds misappropriation and performance opacity.

els of reputation, contests, and lemons markets (Bar-Isaac, 2003; Daley and Green, 2012; Gul and Pesendorfer, 2012; Kolb, 2019; Gryglewicz and Kolb, 2022). Similar to Ely and Szydlowski (2020), we solve an information design problem of a principal with private information, but our setting entails multiple agents who face coordination problems themselves. As a result, our donation thresholds depend on a stochastically changing state as in Gryglewicz and Kolb (2022). We adopt an approach of optimizing over beliefs rather than strategies, similar to that used in dynamic information design and mechanism design with limited commitment (e.g., Doval and Ely, 2020; Doval and Skreta, 2022).

Finally, we provide new empirical insights on reward-based crowdfunding. Our findings connect to the economics of innovation (Belleflamme et al., 2014; Lee and Persson, 2016; Sorenson et al., 2016; Grüner and Siemroth, 2019) and complement experimental work on how contributions affect campaign success (Van de Rijt et al., 2014). We quantify the effects of pro-social incentives (Kuppuswamy and Bayus, 2018; Dai and Zhang, 2019), which have also been observed in equity crowdfunding (e.g., Agrawal et al., 2015) and capital campaigns (e.g., List and Lucking-Reiley, 2002).

2 Model of Crowdfunding

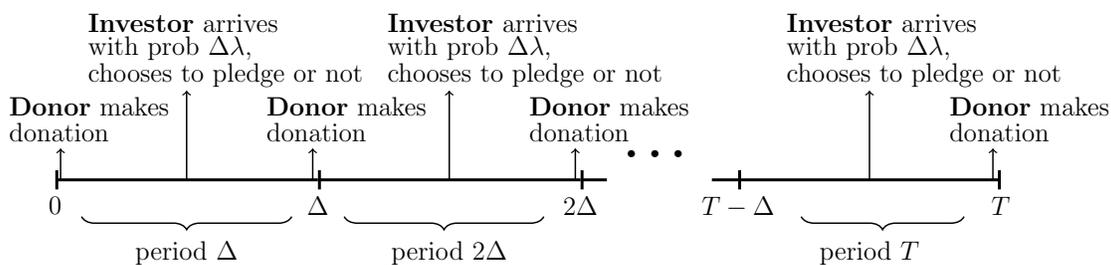
In this section, we present our baseline model of a contribution game. The objective is to raise a goal amount $G > 0$ by a deadline $T > 0$. We say the project succeeds if the goal amount is raised by the deadline. Time is divided into periods of length Δ . Let $\mathbb{T}^\Delta := \{\Delta, 2\Delta, \dots, T\}$ denote the set of periods. For any period $t \in \mathbb{T}^\Delta$, there is a corresponding time remaining $u := T - t \in \mathbb{U}^\Delta \equiv \{T - \Delta, \dots, \Delta, 0\}$.

Players and payoffs. There are two types of contributors, randomly-arriving investors (she), and a single, representative long-lived donor (he). In every period, an investor arrives with probability $\Delta\lambda \in (0, 1)$. Upon arrival, she makes a one-shot decision to either pledge to pay p or to choose an outside option of value $v_0 > 0$. If the project succeeds, all investors who pledged to pay p receive a reward of $v > 0$, resulting in a payoff of $v - p > v_0$. If the

project fails, investors pay nothing and receive a payoff of 0. In the context of reward-based crowdfunding, v is an investor's valuation of a product. The term v_0 can be interpreted as the value of a short-lived investment opportunity.³

The long-lived donor values a successful project at $w \geq 0$. If the project succeeds, the donor's payoff is $w - D_T$, where D_T is the total donor contribution made. If the project fails, donations are returned, and the donor's payoff is 0. Hence, the donor never donates more than w . Investors do not observe w . They only know that it is drawn from a distribution on $[0, \infty)$, with a continuously differentiable and strictly increasing cumulative distribution function (cdf) F_0 . We denote the random variable by an upper-case W .

Figure 1: Timing of the game



Note: The donor can contribute at the beginning of time ($t = 0$). Then, within every period, an investor arrives (or not) and decides to pledge. The donor then decides if/how much to donate. The game ends at $t = T$.

Timing, histories, and strategies. Figure 1 illustrates the timing of the game. Within a period, if an investor arrives, she decides whether to pledge p or not. Then, the donor decides whether and how much to donate. The donor is also allowed to donate at the start of the game ($t = 0$). We denote cumulative pledges and cumulative donations up to and including period t by N_t and D_t , respectively. The final project revenue is equal to $R_T = D_T + N_T p$. Success requires $R_T \geq G$.

The history $h_t^{B,\Delta}$ of an investor who arrives in period t consists of all cumulative pledges and donations up to period $t - \Delta$. We denote the set of period- t investor histories

³It can also be interpreted as an inspection cost, a transaction cost of pledging, or a disappointment cost of not receiving a product if participants are loss-averse.

by $\mathcal{H}_t^{B,\Delta}$.⁴ A period- t donor history $h_t^{D,\Delta}$ additionally includes the period- t pledge, if any. We denote the set of period- t donor histories by $\mathcal{H}_t^{D,\Delta}$. A strategy of a period- t investor is a mapping $\tilde{b}_t^\Delta : \mathcal{H}_t^{B,\Delta} \rightarrow [0, 1]$, where $\tilde{b}_t^\Delta(h_t^{B,\Delta})$ is the probability of an investor purchasing at history $h_t^{B,\Delta}$. We denote the collection of all investor strategies by $\tilde{b}^\Delta := (\tilde{b}_t^\Delta)_t$. The donor strategy is a mapping $\tilde{D}_+^\Delta : \bigcup_{t \in \mathbb{T}^\Delta} \mathcal{H}_t^{D,\Delta} \times [0, \infty) \rightarrow \mathbb{R}$, where $\tilde{D}_+^\Delta(h_t^{D,\Delta}; w) = D_t \geq D_{t-\Delta}$ represents cumulative donations after the donation decision at history $h_t^{D,\Delta}$.⁵ Investor beliefs, $\tilde{F}^\Delta : \bigcup_{t \in \mathbb{T}^\Delta} \mathcal{H}_t^{B,\Delta} \rightarrow [0, \infty)^\mathbb{R}$, map each investor history $h_t^{B,\Delta}$ to a cdf $\tilde{F}^\Delta(\cdot; h_t^{B,\Delta})$.

Solution concept. A perfect Bayesian equilibrium (PBE) is given by an assessment $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, or a tuple of strategies and beliefs, such that

- i) the donor strategy \tilde{D}_+^Δ maximizes the donor's expected payoff at any donor history $h_t^{D,\Delta}$ given investor strategies and beliefs;
- ii) each period- t investor strategy \tilde{b}_t^Δ maximizes the expected payoff of the investor at any history $h_t^{B,\Delta}$, given investor beliefs, and the donor and other investor strategies;
- iii) investor beliefs about W , $\tilde{F}^\Delta(\cdot; h_t^{B,\Delta})$, are derived from all strategies according to Bayes' Rule whenever possible.

An equilibrium outcome is given by a sequence of cumulative pledges and donations. The distribution of investor arrivals, the donor's valuation distribution, and an assessment $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ induce a probability measure \mathbb{P} that govern equilibrium outcomes.

Payoff-relevant state and Markov equilibria. All players' payoffs only depend on the cumulative number of pledges and the cumulative donation amount. Therefore, we define the payoff-relevant state in period t to be

$$\mathbf{x} := (N, D, u) \in \mathbb{X}^\Delta := \mathbb{N} \times [0, \infty) \times \mathbb{U}^\Delta,$$

or equal to $(N_{t-\Delta}, D_{t-\Delta}, T - t)$ for an investor, and $(N_t, D_{t-\Delta}, T - t)$ for a donor.

⁴We assume that investors do not observe arrivals, but only pledges. Arrivals are not payoff relevant, so the Markov equilibria we characterize later are also equilibria in a game where arrivals are observed.

⁵Formally, we allow for mixed strategies. In that case, $\tilde{D}_+^\Delta(h_t^{D,\Delta}; w)$ denotes the random variable that describes the mixed strategy at the corresponding history.

Donor strategies, investor strategies, and investor beliefs are said to be *Markovian* if they only depend on the state, both on and off equilibrium path. These objects are represented by $D^\Delta : \mathbb{X}^\Delta \times [0, \infty) \rightarrow \mathbb{R}$, $b^\Delta : \mathbb{X}^\Delta \rightarrow [0, 1]$, and $F^\Delta : \mathbb{X}^\Delta \rightarrow \mathbb{R}^{\mathbb{R}}$, respectively. We call PBEs in Markovian strategies and beliefs *Markov equilibria*, described by a Markovian assessment $(b^\Delta, D_+^\Delta, F^\Delta)$.

3 Characterization of Equilibrium Outcomes

In this section, we characterize the game's PBE and bound the effects of dynamic signaling on the probability of success. We identify PBEs preferred by the donor and investors, respectively. Finally, we demonstrate that alternate campaign designs with varied donation timings or information about donor valuation cannot increase the probability of success. All proofs are in Appendix A.

3.1 Preliminaries

Given a Markovian assessment $(b^\Delta, D_+^\Delta, F^\Delta)$, let $\pi^\Delta(\mathbf{x})$ denote the induced probability of success from the perspective of the $N + 1$ st investor if she pledges in state $\mathbf{x} = (N, D, u)$. Upon arrival, an investor in state \mathbf{x} is willing to pledge if and only if the expected utility of pledging is greater than the utility of the outside option. That is,

$$\pi^\Delta(\mathbf{x}) \cdot (v - p) \geq v_0. \quad (\text{Investor-PC})$$

The probability of success $\pi^\Delta(\cdot)$ is determined by the assessment $(b^\Delta, D_+^\Delta, F^\Delta)$ and two sources of uncertainty. First, investors are uncertain about the donor's valuation w and update their beliefs based on the observed state \mathbf{x} . Second, there is uncertainty about the number of future investor arrivals. We say a project is active in state \mathbf{x} if beliefs are sufficiently high to incentivize pledging, i.e., $\pi^\Delta(\mathbf{x}) \geq \frac{v_0}{v-p}$.

3.2 Pooling-Threshold Equilibrium Structure

Next, we describe Markov equilibria in which the donor strategies have an intuitive structure. We later establish that equilibrium outcomes that maximize/minimize the probability of success can be attained by such equilibria. In these equilibria, all donors with a sufficiently high valuation donate just above a state-dependent threshold (see Definition 1).

Definition 1. We call a Markovian donor strategy D_+^Δ a *pooling-threshold (PT) strategy* if for any N and u , there is a *donation threshold* $D_*^\Delta(N, u) \geq 0$, with $D_*^\Delta(N, 0) = G - Np$, such that

$$D_+^\Delta(\mathbf{x}; w) = \max\{D, D_*^\Delta(N, u)\}, \forall w \geq D_*^\Delta(N, u),$$

and $D_+^\Delta(\mathbf{x}; w) = \max\{w, D\}$, otherwise.

Given a PT donor strategy, donations serve to signal the donor's valuation. If donations are sufficiently high, investors are optimistic that the project will ultimately succeed. The donation threshold ensures that the next arriving investor will believe that the campaign succeeds with a probability higher than $\frac{v_0}{v-p}$. In equilibrium, investors pledge if and only if cumulative donations exceed the donation threshold of the donor in the preceding period,

$$b^\Delta(N, D, u) = 1 \iff D \geq D_*^\Delta(N, u + \Delta). \quad (\text{PT-investor})$$

Additionally, for the Markov equilibria that we construct, the donation threshold $D_*^\Delta(N, u)$ is decreasing in both N and u . Thus, fewer donations are required to keep the project active when more investors have pledged and more time remains until the deadline. As a result, these equilibria can be supported by investor beliefs that are truncations of the prior distribution F_0 when $D \geq D_*^\Delta(N, u + \Delta)$. PT beliefs are equal to

$$F^\Delta(w; \mathbf{x}) = \begin{cases} \frac{F_0(w) - F_0(D)}{1 - F_0(D)} \cdot \mathbb{1}(w \geq D) & \text{if } D \geq D_*^\Delta(N, u + \Delta) \\ \mathbb{1}(w \geq D) & \text{otherwise} \end{cases}. \quad (\text{PT-belief})$$

As soon as cumulative donations fall below $D_*^\Delta(N, u + \Delta)$, investors believe that the donor

has exhausted his valuation, and $w = D$.⁶ We define a PT assessment in Definition 2.

Definition 2. An assessment $(b^\Delta, D_+^\Delta, F^\Delta)$ is a *pooling-threshold (PT) assessment* if D_+^Δ is a PT strategy with a donation threshold $D_*^\Delta(N, u) \in [0, G - (N+1)p)$ for $u > 0$, if Equations PT-investor and PT-belief are satisfied, and if the following conditions hold for $u > 0$:

- i) Weak monotonicity in u and strong monotonicity in N , i.e.,

$$D_*^\Delta(N, u) \geq D_*^\Delta(N+1, u-\Delta) \geq D_*^\Delta(N+1, u);$$

- ii) Strict monotonicity in N , i.e., $D_*^\Delta(N, u) > D_*^\Delta(N+1, u)$ if $D_*^\Delta(N, u) > 0$;

- iii) No donation threshold after success, i.e., $D_*^\Delta(N, u) = 0$ for $(N+1)p \geq G$.

Condition (i) in Definition 2 imposes that the donation threshold is weakly decreasing in u , but also that if there is a pledge in period $u - \Delta$, then the donation threshold $D_*^\Delta(N+1, u - \Delta)$ does not increase. Monotonicity in N is stronger than the monotonicity in u . Condition (ii) simply imposes strict monotonicity of D_*^Δ in N , and Condition (iii) requires that D_*^Δ drops to zero if investors alone raise the goal amount.

Because the donation threshold is decreasing in N and u , once a project reaches a state in which it is inactive, it can never be active again. This allows us to define cut-off times (CT) such that given realized donor valuation w and the number of additional investors j needed for success, the project will fail unless the next investor arrives before the cut-off time $u = \xi_j^\Delta(w)$. Let $\underline{M}(D) = \lceil \frac{G-D}{p} \rceil$ denote the total number of investors needed for success given total donations D , if no further donations are made. Then, for $j \leq \underline{M}(w)$,

$$\xi_j^\Delta(w) := \min \left\{ u \in \mathbb{U}^\Delta : \pi^\Delta(\underline{M}(w) - j, w, u) \geq \frac{v_0}{v-p} \right\}, \quad (\text{CT})$$

⁶Many other investor beliefs can sustain a PBE in which the donor plays a PT strategy. Technically, the beliefs chosen here violate the “cannot signal what you do not know” condition off equilibrium path as introduced in Fudenberg and Tirole (1991), in the sense that early investor pledges can affect the beliefs of later investors independently of the donor’s actions. We could recover the “cannot signal what you do not know” condition without altering anything qualitatively by imposing that for any off-path history $h_t^{B,\Delta}$ such that there exists a $s \leq t$ with $D_s < D_*^\Delta(N, T-s)$, we have $F(w; h_t^{B,\Delta}) = 1(w \geq \min\{D_s : D_s < D_*^\Delta(N, T-s), s \leq t\})$. Instead of allowing such non-Markovian, off-path beliefs, we choose the Markovian on- and off-path beliefs given in Equation PT-belief for their clean structure.

where we set $\xi_j^\Delta(w) = T$ if $\pi^\Delta(\underline{M}(w) - j, w, u) < \frac{v_0}{v-p}$ for all $u \in \mathbb{U}^\Delta$. Monotonicity of D_*^Δ guarantees that $\pi^\Delta(\underline{M}(w) - j, w, u) \geq \frac{v_0}{v-p}$ for $u \geq \xi_j^\Delta(w)$. Therefore, given state \mathbf{x} with $N = \underline{M}(w) - j$, a donor with valuation w is not able to satisfy Equation Investor-PC if and only if $u < \xi_j^\Delta(w)$. Investors will pledge in equilibrium if and only if they arrive before the specified cutoff times. If they arrive too late, the donor will “run out of funds” because he would need to donate more than w to keep the project active. Ex-ante, there is an information asymmetry between donor and investors about $\xi_j^\Delta(w)$, but once period $\xi_j^\Delta(w)$ is reached, the asymmetry is resolved.

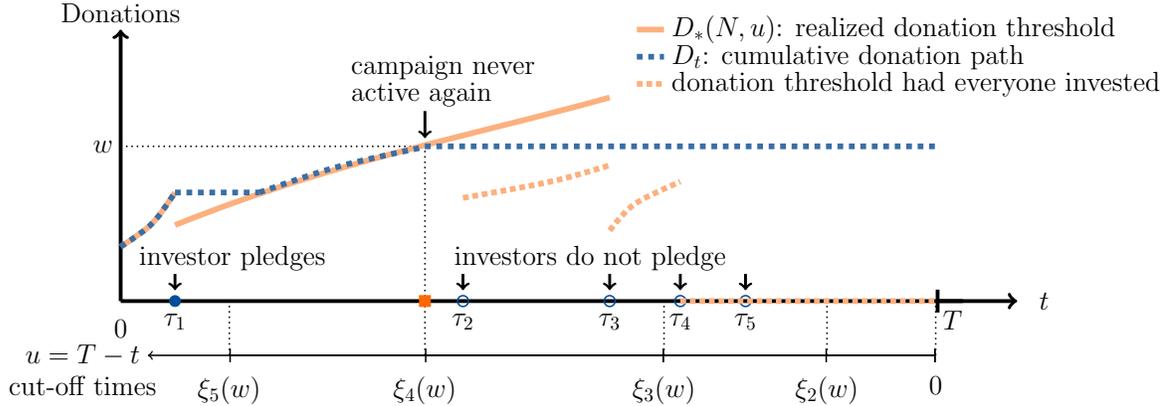
We construct Markov equilibria in which the donor plays a PT strategy, which we refer to as *pooling-threshold (PT) equilibria*. To construct PT equilibria, we define a donation threshold $D_*^\Delta(N, u)$, and show that the equilibrium conditions are satisfied. We show that for any PT assessment, beliefs are consistent with Bayes’ rule and donors are best-responding to the investor strategy. Hence, for any specific PT equilibrium, it only remains to show that investors are best-responding as well.

To provide some intuition, we illustrate a PT equilibrium and a sample equilibrium path as $\Delta \rightarrow 0$ in Figure 2. We have chosen w such that $\underline{M}(w) = 5$ investors are needed for success. The horizontal axis marks time. The τ s mark investor arrival times and $\xi_j(w)$ s denote calculated cut-off times. The N -th investor must arrive with at least $\xi_{\underline{M}(w)-N+1}(w)$ time remaining before the deadline in order to be willing to pledge. If the N -th investor arrives after that instant, the expected utility from pledging for this investor (and subsequent investors) drops below v_0 , and the project fails.

In Figure 2, the donor contributes along the donation threshold (solid orange) through the first investor arrival at τ_1 . He does so to ensure that the first investor pledges. After this investor pledges, the donation threshold drops, and the next arriving investor is more confident that the project will succeed. As a result, donations stop for a while and later resume along the updated donation threshold to keep the project active. In the figure, the second investor arrives after $\xi_4(w)$, and the donor runs out of funds, i.e., $D_t = w$ (dashed blue line). This means that all investors after τ_2 choose the outside option, and the

project fails. The dashed orange lines after $\xi_4(w)$ mark counterfactual donation thresholds if investors had arrived before the relevant cut-off times.

Figure 2: Sample equilibrium path given donor valuation w such that $\underline{M}(w) = 5$



Note: The bottom line shows the cut-off times $\xi_j(w)$ by which the $\bar{M}(w) - j + 1$ -th investor must arrive in order for the project to stay alive. The x-axis marks time and depicts realizations of investor arrivals (by blue dots). Also included are the donation thresholds and cumulative donations (up to w). The second investor does not arrive in time, i.e., $\tau_2 > T - \xi_4(w)$. The donor runs out of funds, and the project fails at $T - \xi_4(w)$. We also include a counterfactual donation threshold had everyone invested.

3.3 Success-Maximizing Equilibrium

We first construct a PBE that maximizes the probability of success. Because the donor is willing to donate up to his valuation if necessary, maximizing the probability of success boils down to providing investors with incentives to pledge whenever possible. In a success-maximizing equilibrium, even relatively low cumulative donations can be a sufficiently strong signal to make investors optimistic enough to pledge. A PT strategy with a minimal donation threshold that can support a PBE can generate such optimistic beliefs. We calculate a minimum donation threshold for any given history and show that at this threshold, investors are exactly indifferent between pledging or not. The donor should never donate more than this threshold because the threshold will increase in the future if investors do not pledge. Therefore, it is prudent to hold back funds to potentially induce later investors to pledge. All donors with a valuation greater than the threshold pool and donate

just enough to meet the threshold. The following proposition summarizes key properties of a success-maximizing equilibrium.

Proposition 1 (Success-Maximizing Equilibrium).

- i) Given any $\Delta > 0$, there exists a success-maximizing PBE that is a PT equilibrium;
- ii) Any sequence of these PT equilibria $\{(b^\Delta, D_+^\Delta, F^\Delta)\}_\Delta$ converges to a unique limit (b, D_+, F) as $\Delta \rightarrow 0$. The donor's limiting PT strategy admits a donation threshold $D_*(N, u) = \underline{D}(N, u)$ where $\underline{D}(N, u) = 0$ if $\pi(N, 0, u) > \frac{v_0}{v-p}$, and otherwise, the next investor is made indifferent such that

$$\pi(N, \underline{D}(N, u), u) = \frac{v_0}{v-p},$$

where $\pi := \lim_{\Delta \rightarrow 0} \pi^\Delta$ (the limit being uniform in D). Given a realized valuation w and cumulative pledges N with $\underline{M}(w) - N = j > 1$, the project fails in the limit at $\xi_j(w) := \lim_{\Delta \rightarrow 0} \xi_j^\Delta(w)$ satisfying

$$\pi(\underline{M}(w) - j, w, \xi_j(w)) = \frac{v_0}{v-p}.$$

If $j \leq 1$, the project never fails.

In the proof, we first construct a PT equilibrium for fixed Δ that satisfies analogous discrete-time properties described in Proposition 1. We then show that this PBE maximizes the probability of success and take the limit as $\Delta \rightarrow 0$.

The construction uses the insights provided in Section 3.2. We cannot directly define a donation threshold for every state such that investors are indifferent between pledging or not pledging because such a condition is endogenous to all participant strategies. Instead, we use an induction in the number of investors needed to reach the goal if no additional donations are made, $j = \underline{M}(D) - N$. Within each induction step in j , we construct equilibrium objects $D_*^\Delta(N, u)$, $\pi^\Delta(N, D, u)$, and $\xi_j^\Delta(w)$.

A success-maximizing PBE is simply one that maximizes investor pledges. The exact donation amounts do not matter, as long as they incentivize pledging. It suffices to consider “reduced histories” that ignore the precise donation amounts and only keep track whether or not a donation keeps the project active. We recast the problem by directly choosing probabilities of reaching each reduced history subject to a martingale constraint and an investor participation constraint. We show that the PBE constructed in the first step achieves the value of this relaxed problem.

3.4 Success-Minimizing Equilibrium—No Signaling Benchmark

In the unique success-minimizing equilibrium, the ability of the donor to signal his value does not facilitate coordination between investors. The donor plays a PT strategy with a donation threshold that is so high that investors are willing to pledge even if the donor stopped contributing. To define this threshold, suppose that w is known to be zero, that is, $F_0(w) = \mathbb{1}(w \geq 0)$ and let $\bar{\xi}_j^\Delta := \xi_j^\Delta(0)$ be the cutoff times from Proposition 1. Furthermore, define $\bar{\xi}_j := \lim_{\Delta \rightarrow 0} \bar{\xi}_j^\Delta$, $\bar{D}^\Delta(N, u + \Delta) := \max\{G - (j-1)p - Np, 0\}$ for $u \in (\bar{\xi}_{j-1}^\Delta, \bar{\xi}_j^\Delta]$, and

$$\bar{D}(N, u) := \max\{G - (j-1)p - Np, 0\} \text{ for } u \in (\bar{\xi}_{j-1}, \bar{\xi}_j] \text{ as } \Delta \rightarrow 0. \quad (\bar{D})$$

This is the maximum donation threshold that can arise in a PBE. Indeed, in any PBE, the donor would not contribute if total revenue already exceeds $G - (j-1)p$ at $u \in (\bar{\xi}_{j-1}^\Delta, \bar{\xi}_j^\Delta]$, since all future investors will pledge anyway. As $\Delta \rightarrow 0$, donors always donate exactly p in order to “compensate” for the absence of an investor arrival.

We show that the PT equilibrium with donation threshold \bar{D} minimizes the probability of success (see Proposition 2). Unlike in the success-maximizing PT equilibrium, now when an investor pledges, she has a strict incentive to do so.

Proposition 2 (Success-minimizing Equilibrium).

- i) Given any $\Delta > 0$, there is a unique success-minimizing PBE that is a PT equilibrium.*

ii) Any sequence of such PT equilibria $\{(b^\Delta, D_+^\Delta, F^\Delta)\}_\Delta$ converges to a unique limit (b, D_+, F) as $\Delta \rightarrow 0$. The donor's limiting PT strategy has a donation threshold $D_*(N, u) = \bar{D}(N, u)$ given by Equation \bar{D} .

Note that if the donation threshold is \bar{D} , then investors contribute as if the donor's valuation is equal to the current level of cumulative donations. As a result, in this equilibrium, there is no signaling to aid coordination. The donor just serves to decrease the effective goal amount from G to $G - w$.

3.5 Donor- and Investor-Preferred Equilibria

Next, we investigate which equilibria are preferred by campaign participants. We show that there is a trade-off between maximizing donor and investor payoffs, but both parties benefit from the coordination arising from the donor signaling his valuation.

First, consider the donor-preferred equilibrium. If the donor could be refunded excess donations after the deadline (see Section 4), then the donor would want to simply maximize the probability of success, and the success-maximizing equilibrium of Proposition 1 is also the donor's preferred equilibrium. When this is not possible, one might conjecture that the donor "over-donates" in the success-maximizing equilibrium and can benefit from reducing donations at the expense of a lower success probability. We show that this is not the case.

Proposition 3 (Donor-Preferred Equilibrium). *The success-maximizing PT equilibrium outcome constructed in Proposition 1 is also optimal for the donor.*

The key step to proving Proposition 3 is to show that if there is a donor-preferred equilibrium, then there must be a donor-preferred equilibrium in which after any history, the donor donates either nothing or just enough to induce the next investor to pledge. The donor's problem then reduces to one in which he chooses probabilities over the same reduced histories (as in the proof of Proposition 1) and minimal donation thresholds that satisfy a donor incentive compatibility constraint. Since early-arriving investors can always be induced to pledge with fewer cumulative donations than later-arriving investors,

the donor-preferred equilibrium coincides with the success-maximizing equilibrium.

Next, we show that investors prefer equilibria with a probability of success strictly between the success-maximizing and success-minimizing equilibrium outcomes.

Proposition 4 (Investor-Preferred Equilibrium). *For sufficiently small Δ , any PT equilibrium preferred by investors yields a probability of success strictly between the success-maximizing one in Proposition 1 and the success-minimizing one in Proposition 2.*

The success-maximizing equilibrium is not investor-preferred because it does not internalize the opportunity cost of pledging. For example, suppose that the first investor arrives late in the game (with u time remaining) and observes total donation $\underline{D}^\Delta(0, u)$. This investor pledges in equilibrium, but if the donor type is such that he is nearly out of funds, then the probability of success is actually low, and pledging would be a mistake ex-post. The investor would be better off in a PT equilibrium with a slightly higher threshold where she could screen out such donor types who are nearly out of funds.

The success-minimizing equilibrium is also not investor-preferred because investors benefit from some dynamic signaling. To understand why, suppose the realized donor valuation is $w = G - p - \epsilon$ for small $\epsilon > 0$. Then, the project succeeds with two investors. The first investor contributes if $(v - p)(1 - (1 - \Delta\lambda)^{u/\Delta}) - v_0 \geq 0$, but total investor surplus is maximized if she instead considered

$$(v - p) \underbrace{(1 - (1 - \Delta\lambda)^{u/\Delta})}_{\text{prob. of at least one more arrival}} - v_0 + \underbrace{(v - p - v_0)\lambda u}_{\text{externality on future investors}} \geq 0.$$

Hence, incentivizing the first investor to contribute beyond $\bar{\xi}_2^\Delta(G - 2p)$ increases overall investor surplus. This can be achieved by lowering the donation threshold $\bar{D}^\Delta(0, u) = G - p$ on $u \in [\bar{u}, \bar{\xi}_2^\Delta(G - 2p))$ by ϵ , where \bar{u} solves the above inequality with equality. We show formally that this lower donation threshold defines a PT equilibrium that investors prefer to the success-minimizing equilibrium.

3.6 Alternative Campaign Designs

We have shown that with incomplete information about the donor valuation, if the donor can dynamically signal his valuation over time, this can maximize the probability of success. A natural question to ask is whether there are campaign designs that outperform this one. The proof of Proposition 1 offers some answers. Restricting the donor’s ability to donate, or altering the information (revealing more or less at different times) about the donor’s valuation, cannot increase the probability of success relative to the success-maximizing equilibrium outcome. This means that a campaign design in which donations are collected in a first stage, and investors engage in a crowdfunding campaign in a second stage, must result in a lower probability of success compared to the success-maximizing equilibrium outcome. The same is true when donations occur only at the deadline to “top up” the campaign. It is possible, however, that these alternative campaign designs result in a higher probability of success than that achieved under the success-minimizing equilibrium. We empirically investigate this possibility in Section 5.

4 Extensions

Next, we discuss some extensions that show that our central insights extend to a broad class of contribution games. Examples for this section can be found in the Online Appendix.

4.1 Relaxing Modeling Assumptions

Our first set of extensions consider relaxing a single assumption at a time. We note any substantive changes to Propositions 1-4.

Multiple Donors: Although our baseline model considers a single donor in order to avoid modeling the free-rider problem, we argue that the success-maximizing equilibrium can still be supported when multiple donors observe each others’ valuations.

To see why, assume that there are two (or any finite number of) long-lived donors with valuations w_1, w_2 such that $w := w_1 + w_2$ is distributed according to F_0 . Donors simultane-

ously decide contributions at the end of each period. The success-maximizing equilibrium can be supported by donor strategies in which the first donor contributes according to the PT strategy of Proposition 1, and the second donor does not contribute until the donation threshold reaches w_1 . Investor beliefs and strategies coincide with those of Proposition 1. However, we note that the presence of free-riding can lower the probability of success in the success-minimizing equilibrium. For example, consider a two-period version of our model with $2p < w_1 < G < w_1 + w_2$. Both donors must contribute for the project to succeed, but it is also an equilibrium for both donors to not contribute.

Time-Varying Arrivals and Outside Options: For expositional reasons, our baseline model assumes a constant arrival rate and outside option. Both assumptions can be relaxed. Our analysis remains unchanged with a time-varying arrival process as long as the arrival rate is smooth in t . A time-dependent outside option is also possible as long as it does not decrease too quickly. Otherwise, an inactive campaign can become active again at a later point in time, and the cutoffs $\xi_j^\Delta(w)$ may not exist. This would complicate the equilibrium analysis significantly.

4.2 Application: Crowdfunding

We capture additional features of crowdfunding with two natural extensions.

Project Creator as the Donor: Our exposition suggests that the campaign creator is distinct from the donor, but all results hold if the donor contributes to his own campaign and excess donations are returned at the deadline. This is because the probability of success and investor outcomes remain unchanged if donations are returned. Consequently, the bounds on the probability of success remain unchanged; however, additional equilibria may arise.

Social Learning about Product Quality: Our framework can accommodate environments in which investors learn about the quality of the product from other investor pledges. This may be relevant when investment occurs prior to production of a product.

To model social learning, we assume that investor valuations are νq , where ν is deterministic and $q \in \{0, 1\}$ is the unknown quality of the project. Each investor receives an

independent signal about quality $s \in \{0, 1\}$ that is equal to 1 with certainty if $q = 1$ and with a probability less than one if $q = 0$. Thus, investors who see $s = 0$ learn with certainty that $q = 0$ and do not pledge. The participation constraint of investors with a positive signal $s = 1$ is

$$\underbrace{\mathbb{P}(N_T p + D_T \geq G | \mathbf{x})}_{\text{probability of success in state } \mathbf{x}} \left(\underbrace{\frac{\mathbb{E}[(vq - p) \cdot \mathbf{1}(N_T p + D_T \geq G) | \mathbf{x}]}{\mathbb{P}(N_T p + D_T \geq G | \mathbf{x})}}_{\text{expected value conditional on success}} - p \right) \geq v_0.$$

This expression shows that with social learning donations can simultaneously increase the probability of success while lowering the expected benefits of contributing. This can make signaling equilibria possible, but harder to sustain. We formalize this intuition in a two-period example in the Online Appendix by showing that a PT equilibrium construction analogous to that in Proposition 1 is possible if there is not too much uncertainty about quality.

4.3 Application: Industrial Policy

Our framework can be applied to settings that are not all-or-nothing. Consider a government interacting with firms in a new sector, such as green energy. Typically, a government encourages investments in a sunrise industry by offering subsidies and other support. Firms receive private benefits for making an investment, but benefit more if a critical mass of firms make complementary investments. We relax three assumptions and showcase how our framework can be used to study industrial policy in this context.

Alternative investor payoffs: To capture the notion of increased returns when contributions are complementary, we assume that if a firm makes an irreversible investment of p , this guarantees a return of at least v_L . If the sunrise industry takes off, i.e., the campaign is successful, investing firms get a higher payoff $v_H > v_L$. This results in an alternative

investor participation constraint of the form

$$v_H \pi(N, D, u) + v_L (1 - \pi(N, D, u)) - p \geq v_0 \iff \pi(N, D, u) \geq \frac{v_0 - v_L + p}{v_H - v_L},$$

which is isomorphic to Equation Investor-PC as long as $\frac{v_0 - v_L + p}{v_H - v_L} \in (0, 1)$. That is, our model immediately extends to campaigns that are not all-or-nothing with this assumption.

Adding uncertainty to the goal: In new industries, there may be uncertainty ex-ante about what defines success, and both the government and contributing firms may update their beliefs on the level of investment required for success. Formally, we assume that all participants face symmetric uncertainty about the goal, given by $\tilde{G} = G - X$, where $G, X \geq 0$, and $X \sim H$ on $[0, G]$. X is unknown to all players until contributions reach \tilde{G} . The government has the same payoff function as in our baseline model. Its valuation, or the fixed budget to spend, is $W \geq 0$ with $W \sim F_0$. The budget includes things, such as political capital, that is not observed to firms.

This game is similar to our baseline model, except that the campaign can succeed early if $D + Np + X \geq G$. We argue that the probability of success of a game where W is announced upfront is lower than the probability of success when W is the government's private information. That is, dynamic signaling is still valuable even if the goal is uncertain. The same argument holds if the government maximizes the probability of success subject to a private budget constraint, which captures the notion of earmarking.

Alternative government payoff: Investments made by the government may have some scrap value if the industry does not take off. Specifically, we consider the government's payoff function to be $(W - \gamma D_T) \mathbf{1}(R_T \geq G) - (1 - \gamma) D_T$, where $\gamma \in [0, 1]$ is the scrap value of investment if the goal is not reached. Note that when $\gamma = 1$, the payoff function coincides with our baseline model.

The ability for government investments to be put to alternative uses if the project fails complicates the solution of the game because the probability of success affects the government's (donor's) decisions. The government may prefer to not contribute if success is unlikely because funds can be put to alternative uses. As a result, the donor-preferred equi-

librium will no longer be success-maximizing. Nonetheless, the qualitative forces remain: The ability for the government to dynamically signal its valuation can increase the probability of success. We illustrate this qualitative force using a two-period example in the Online Appendix. The policy implication is that a government encouraging firms to make investments in a sunrise industry may be better off by not offering an initial big push but rather, providing incremental support as firms make commitments.

Alternative interpretation: The game just described extends to other settings as well. The donor could represent a large multinational firm seeking to expand production in a new location. The firm needs to encourage a potential supplier network to provide components and services. Our model suggests that the multinational firm should dynamically respond to investments made by local complementary businesses.

5 Empirical Application to Reward-based Crowdfunding

We consider an empirical application to reward-based crowdfunding with two objectives. Our first objective is to document empirical patterns that are consistent with our model. Our second objective is to quantify the coordination benefits of dynamic signaling by calibrating our model and performing counterfactuals.

5.1 Reward-Based Crowdfunding Platforms

The core features of our model are similar to what is observed on reward-based crowdfunding platforms, including Kickstarter and Indiegogo. On these platforms, entrepreneurs raise funds to create new products. The entrepreneur specifies a funding goal (G), a funding deadline (T), and prices for rewards (p). Once a campaign goes live, these campaign features cannot be changed. Rewards are exclusive to the participants that pledge support to the campaign. Donating without receiving a reward is also possible.⁷ We refer to in-

⁷In principle, a participant can pledge for a reward and donate simultaneously by entering an amount greater than the reward price, however, we do not observe this possibility without individual-level data.

vestors in this section as “buyers” and retain the term “donor” to refer to an individual who contributes without selecting a reward.

Most platforms use an all-or-nothing model so that transactions are realized if and only if the funding goal is reached by the deadline. Entrepreneurs may offer different reward versions, however, we abstract away from the use of price discrimination. After the campaign period, some rewards eventually become mass-market products, adding (social) value beyond the campaign. Examples of mass-market products that started as crowdfunding campaigns include the Peloton bike and Oculus VR headset (acquired by Meta). In line with our model, we do not examine post-campaign outcomes.

5.2 Data Sample and Summary Analysis

Our empirical analysis uses Kickstarter data for campaigns launched between March 2017 and September 2018. We create a panel by tracking every campaign, every twelve hours, from campaign launch through its deadline. Our data separately identify pledges for rewards from donations.⁸ This allows us to study campaign dynamics and interactions between participant types. In Online Appendix C, we provide additional details on constructing our sample. We also perform 12 sets of robustness exercises, including treating some rewards (e.g., a thank you card or meeting the entrepreneur) as a donation.

We provide summary statistics for the 42,462 campaigns included in our sample in Table 1.⁹ 42% of campaigns succeed. Campaigns with larger goals tend to have longer deadlines (corr. coef. = .14). Unsuccessful campaigns tend to have a larger fraction of revenues coming from donations. Regardless of campaign outcome, donations are a key component of overall contributions, constituting 14% of total revenue.¹⁰

We plot additional campaign outcome summaries in Figure 3. In panel (a), we show

⁸Kuppuswamy and Bayus (2018) identify “family” contributions using last name matching based on self-selected usernames that were visible before 2012.

⁹Table 8 in Online Appendix C contains summary statistics for the largest four categories.

¹⁰We maintain the assumption that rewards are exclusive but nonrivalrous because while campaign creators can assign capacity limits to rewards, these tend to be nonbinding. We find that 92% of rewards do not have a capacity limit, and only 21% of rewards with capacity limits ever sell out.

Table 1: Summary Statistics for the Data Sample

Variable	(All)	Mean (Uns.)	(Suc.)	Median	5th %	95th %
Project Length	33.1	34.7	30.7	30.0	15.0	60.0
Goal (\$)	16366.5	21982.5	8613.9	5113.6	312.0	60247.9
Number of Rewards	7.2	6.0	9.3	6.0	1.0	17.0
Donor Revenue (per 12 hrs.)	19.1	3.3	43.7	0.0	0.0	44.0
Buyer Revenue (per 12 hrs.)	119.5	14.0	283.8	0.0	0.0	391.0
Total Donations / Total Rev. (%)	29.2	33.1	24.6	15.8	0.0	100.0
Number of Campaigns	42462	24624	17838	—	—	—

Note: Summary statistics for the 42,462 campaigns in the sample. Donor and investor contributions are the average over 12-hour periods. Means for all statistics are computed for all campaigns (All), unsuccessful campaigns (Uns.), and successful campaigns (Suc.). Also reported are the 50th, 5th, and 95th percentiles.

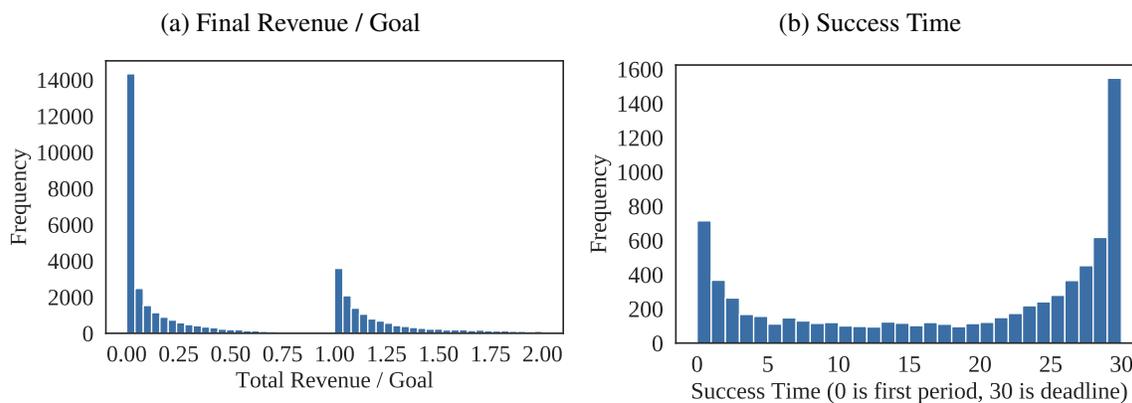
total revenue raised relative to the goal amount (R/G). There is considerable bunching at zero and one meaning that most campaigns either receive little support or raise exactly the goal amount. In panel (b), we plot the time period in which successful campaigns reach their goal. We refer to this time as the *success time*. Success times are bimodal, with many campaigns succeeding both close to the start and close to the deadline.

5.3 Validating Model Assumptions and Theoretical Predictions

Before calibrating our model, we provide descriptive evidence on contributor incentives and contribution dynamics. Recall that in our theoretical model, the donor contributes solely to increase the probability of success and buyers are motivated by obtaining private rewards. This suggests that donations should stop after a campaign reaches success, but buyer activity should continue. We estimate that donations drop by 72% within three days after reaching success, whereas buyer contributions drop by only 37%.

Another salient feature of the data that suggests different contribution incentives is the strong relationship between the relative importance of donations versus purchases and suc-

Figure 3: Frequency Histograms of Campaign Outcomes



Note: (a) Histogram of final campaign revenue (R) over the goal amount (G). The long, thin tail after $R/G = 2$ is not plotted. There is considerable bunching at 0 and 1. (b) Histogram of when 30-day campaigns succeeded. Period $t = 0$ corresponds to the first day of the campaign, and $t = 30$ corresponds to the deadline. Success times are bimodal.

Table 2: Descriptive Statistics for Early-, Middle- and Late- Finishing Campaigns

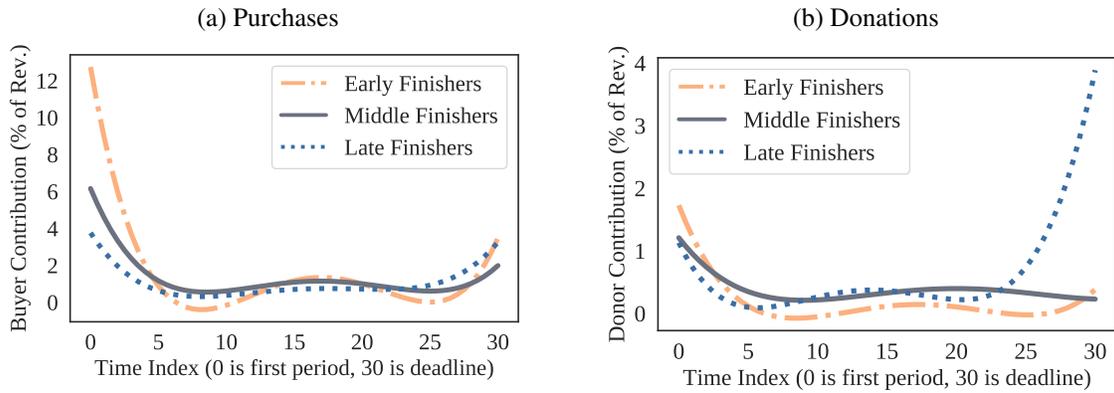
Variable	Early	Middle	Late
Goal (\$)	2000.0	3510.4	5253.6
Number of Rewards	8.0	8.0	8.0
Average Price	90.6	118.7	171.5
R/G	3.1	1.2	1.0
D/R (%)	4.8	14.7	34.3
D/G (%)	17.0	20.9	36.7
Number of Campaigns	1437	3470	2616
Top Categories	Theater	Theater	Theater
	Design	Music	Film & Video
	Games	Design	Music

Note: Summary statistics for successful campaigns partitioned by success time. Only 30-day campaigns are included. Early finishers complete within three days. Late finishers complete in the last three days. The middle category denotes campaigns that complete between 3 and 27 days. R denotes final revenue; G denotes the goal, and D denotes total donations. Medians reported. Donations become increasingly important for campaigns that finish late.

cess times. At the aggregate level, campaigns that succeed close to campaign launch should receive lower total donations than those that complete close to the deadline. To test this hypothesis, we partition the sample according to success times using three groupings: those

with success time in the first three days (*early finishers*), those with success time in days 3-27 (*middle finishers*), and those with success time in the last three days before the deadline (*late finishers*). Table 2 provides summary statistics for these groupings that show stark differences between early- and late-finishing campaigns consistent with participants having different incentives. For example, the relative importance of donations increases in success time: 5%, 14%, 34% for early-, middle-, and late-finishers, respectively. In Figure 4, we plot contribution flows from buyers (panel a) and donors (panel b) for early-, middle- and late-finishing campaigns.¹¹ While all campaigns have an initial spike in purchases at the beginning, the only spike for donations are for campaigns that succeed at the deadline.

Figure 4: Contributions of Buyers and Donors over Time for Successful Campaigns



Note: (a) Fitted values of a polynomial regression of degree 4 of time on percentage of total buyer revenues obtained within the period (B_t/R_T). (b) Fitted values of a polynomial regression of degree 4 of time on percentage of total donor revenues obtained within the period (D_t/R_T). For both figures, the 3 lines correspond to early finishers, middle finishers, and late finishers. Only 30-day campaigns are included. There is a spike in donations for late-finishing campaigns only.

Next, we establish properties on contribution dynamics of pooling-threshold equilibria and provide simple empirical evidence to validate these predictions in the data.

Proposition 5 (Contribution Dynamics in Pooling Threshold Equilibria). *All PT equilibria satisfy the following properties:*

¹¹We provide figures that measure the percentage of campaigns (also weighted by within campaign revenues) and include unsuccessful campaigns in Figure 13 in Online Appendix C.

- i) Campaigns that succeed at the deadline would fail without a donation and raise exactly the goal with high probability. Formally, $\mathbb{P}(N_T p + D_{T-\Delta} < G | \mathcal{S}_T) > 1 - \Delta\lambda$ and $\mathbb{P}(D_T = G - N_T p | \mathcal{S}_T) \geq 1 - \Delta\lambda$, where $\mathcal{S}_t = \{N_{t-\Delta} p + D_{t-\Delta} < G \text{ and } N_t p + D_t \geq G\}$.
- ii) Campaigns that succeed before the deadline succeed due to a buyer pledge. Formally, $\mathbb{P}(D_{\tau-\Delta} + N_\tau p \geq G) = 1$ if $\tau < T$;
- iii) Donations drop to zero after a buyer pledges. Formally, $D_*^\Delta(N, u + \Delta) \geq D_*^\Delta(N + 1, u)$;
- iv) Conditional on failing, campaigns with larger donor valuations fail later. Formally, given donor realizations $w > w'$, if a campaign is unsuccessful for both w and w' , and given the same buyer arrival realization, then the failure time $\iota := \inf\{t \geq 0 \mid \pi(N_t, D_t, T - t) < \frac{v_0}{v-p}\}$ is larger for w than for w' .
- v) In success-minimizing PT equilibria, all donations are at least p .

Empirical Evidence (Proposition 5).

- i) Our model suggests that contributions that cause a campaign to succeed at the deadline must come from donations. This is a consequence of the donor valuing success and the probability of a buyer arrival in the last period being small. We empirically test this by considering all campaigns that succeed at the deadline and subtracting any last-minute donations. We find that 78% of campaigns would have failed if not for last-minute donations.
- ii) A consequence of the PT equilibrium structure is that donations are made to ensure that the next buyer pledges. Hence, a donor will never donate to bring cumulative donations beyond $G - (N + 1)p$. We test this by calculating the percentage of revenue coming from buyers in the period in which a campaign succeeds. We find that the median percentage of revenues from buyers is 91% (see Figure 14-(a) in Online Appendix C for the histogram).
- iii) When beliefs about the probability of success are sufficiently high, donations are not required. As a result, in PT equilibria, the donation threshold drops immediately after

a pledge. To explore how donations respond to contributions over time, we specify dynamic panel regressions where the dependent variable is observed donations. We use lagged values of cumulative revenues over the goal amount relative to the median campaign (within that category, time period) as controls, i.e., $\mathbf{1}[R_{j,t-k}/G_j > \text{median}[R_{t-k}/G]]$, for $k = 1, 2, 3, 4$. We generally find a negative association of donations and progress toward the goal (for all lags), which we report in Table 9 in Online Appendix C.

iv) The positive relationship between the donor’s valuation and when a campaign fails, arises because donors with high valuations pool with donors of lower valuations, i.e., the behavior of donors of high and low valuations look similar, but donors with higher valuations can keep campaigns active for longer. To test this prediction, we infer the failure times of campaigns using a logistic regression, i.e., $\mathbf{1}[\text{reaches success}]_{j,t} = x_{j,t}\beta + \varepsilon_{j,t}$, where the outcome variable is an indicator function of the campaign outcome (success/failure) and $x_{j,t}$ includes campaign state covariates.¹² This simple prediction model produces an out-of-sample prediction accuracy of 91%, which we use to infer the failure time t for each failed campaign. We define this to be the last period in which the probability of success is greater than 10%. We then correlate cumulative donations for failed projects with the estimated failure time. We find that the correlation is positive (corr. coef.=0.15) and significant (t-stat= 10.1), meaning projects with larger donations fail closer to the deadline.¹³

v) The success-minimizing donation threshold requires donations of at least p in order to “compensate” for the absence of a buyer arrival. This motivates a simple analysis to test if campaign dynamics are consistent with success-minimizing equilibria. We calculate the average donation level for each campaign as cumulative donations (excluding the last period), divided by the number of donors observed. We label this average as μ_j^D . We also calculate the quantity-weighted reward price for each campaign, which we denote μ_j^B .

¹²See Figure 15 in Online Appendix C for more details.

¹³Alternatively, we could infer the failure time by marking the last observed donation. However, Kickstarter promotes projects near the deadline, which likely drives some last-minute contributions. This approach implies that campaigns commonly fail at the deadline whereas our prediction model suggests that campaigns fail much earlier.

Finally, we verify if $\mu_j^D \geq \mu_j^B$ which is satisfied in success-minimizing equilibria. We find that only 29% of campaigns satisfy this inequality.

5.4 Empirical Model of Reward-Based Crowdfunding

Next, we calibrate our model to the data. Our model has three sets of unknown parameters: distribution of donor valuations, the arrival process parameters, and investor utilities. We sequentially estimate these unknowns. We outline our approach here and provide additional details in Online Appendix C.

First, we estimate the distribution of donor valuations using maximum likelihood. Note that donor valuations are subject to selection. According to the model, we observe valuation realizations for campaigns that fail, but we do not observe them for campaigns that succeed.¹⁴ We account for selection using the ideas in Heckman (1979). Formally, we aggregate the data to the campaign level (j) and assume that the donor valuations w_j follow a log-linear model $\log(w_j) = x_j^W \beta^W + u_j^{(1)}$. Selection into the sample is governed by

$$s_j = \begin{cases} 1, & z_j^W \gamma^W + u_j^{(2)} > 0 \\ 0, & \text{otherwise} \end{cases},$$

where we assume a bivariate-normal distribution on the unobservables, i.e., $(u_j^{(1)}, u_j^{(2)}) \sim \mathcal{N}(\mathbf{0}, (\tau^2, \tau\mu, 1))$. Hence, the unobserved components are potentially correlated. With these parametric assumptions, the log-likelihood of the donor valuation model is proportional to

$$\begin{aligned} \log(\mathcal{L}^W) \propto & \sum_j s_j \cdot \log \left(\Phi \left(\frac{z_j^W \gamma^W + (\log(w_j) - x_j^W \beta^W) \cdot \mu / \tau}{\sqrt{1 - \mu^2}} \right) \right) \\ & + s_j \cdot \frac{1}{2} \frac{(\log(w_j) - x_j^W \beta^W)^2}{\tau^2} - s_j \cdot \log(\tau) + (1 - s_j) \cdot \log(\Phi(-z_j^W \gamma^W)), \end{aligned}$$

where the superscript W denotes aspects of the model that pertain to the donor, and $\Phi(\cdot)$

¹⁴More precisely, we only observe a lower bound for campaigns that succeed.

denotes the standard normal cdf.

Second, we estimate the arrival process parameters. Without access to Kickstarter search data, buyer arrivals are also only partially observed. We observe arrivals for campaigns that succeed (as all arrivals pledge). However, we do not observe arrivals for campaigns that fail. We follow Terza (1998), who considers Poisson regression subject to sample selection. Let $n_{j,t}$ denote investor arrivals for project j in period t such that

$$\mathbb{E}\left[n_{j,t} \mid x_{j,t}^A, \boldsymbol{\varepsilon}_{j,t}^{(1)}\right] = \lambda_{j,t} = \exp\left(x_{j,t}^A \boldsymbol{\beta}^A + \boldsymbol{\varepsilon}_{j,t}^{(1)}\right).$$

The selection equation is given by

$$a_{j,t} = \begin{cases} 1, & z_{j,t}^A \gamma^A + \boldsymbol{\varepsilon}_{j,t}^{(2)} > 0 \\ 0, & \text{otherwise} \end{cases},$$

where we again assume that the unobservables follow a bivariate-normal distribution, i.e., $(\boldsymbol{\varepsilon}_{j,t}^{(1)}, \boldsymbol{\varepsilon}_{j,t}^{(2)}) \sim \mathcal{N}(\mathbf{0}, (\sigma^2, \sigma\rho, 1))$. With these assumptions, the log-likelihood is equal to

$$\log(\mathcal{L}^A(\boldsymbol{\theta}^A)) = \sum_j \sum_t a_{j,t} \cdot \log\left(\Pr(n_{j,t}, a_{j,t} = 1; x_{j,t}^A, z_{j,t}^A, \boldsymbol{\theta}^A)\right) + (1 - a_{j,t}) \cdot \log\left(\Pr(a_{j,t} = 0; z_{j,t}^A, \boldsymbol{\theta}^A)\right),$$

where

$$\Pr(n_{j,t}, a_{j,t} = 1; x_{j,t}^A, z_{j,t}^A, \boldsymbol{\theta}^A) = \int_{-\infty}^{\infty} \frac{\lambda_{j,t}^{n_{j,t}} \cdot \exp(-\lambda_{j,t})}{n_{j,t}!} \Phi\left(\frac{z_{j,t}^A \gamma^A + \rho \boldsymbol{\varepsilon}^{(1)}}{\sqrt{1 - \rho^2}}\right) \phi(\boldsymbol{\varepsilon}^{(1)}) d\boldsymbol{\varepsilon}^{(1)}, \text{ and}$$

$$\Pr(a_{j,t} = 0; z_{j,t}^A, \boldsymbol{\theta}^A) = \int_{-\infty}^{\infty} \Phi\left(\frac{z_{j,t}^A \gamma^A + \rho \boldsymbol{\varepsilon}^{(1)}}{\sqrt{1 - \rho^2}}\right) \phi(\boldsymbol{\varepsilon}^{(1)}) d\boldsymbol{\varepsilon}^{(1)}.$$

Because the log-likelihood does not have a closed-form solution, we approximate it using Gauss–Hermite quadrature.

Finally, we calibrate investor utilities under the additional assumption that all campaigns follow the success-maximizing equilibrium. We normalize the utility of not pledging ($v_0 = 1$) as our data does not separately identify the value of the outside option versus

choosing to pledge, conditional on the other parameters. We calibrate v_j using method of simulated moments (MSM). More precisely, for a candidate investor utility \tilde{v}_j , we solve for equilibrium objects, $(b(\tilde{v}_j), D_+(\tilde{v}_j), F(\tilde{v}_j))$ using the induction argument of Proposition 1. Implicitly, equilibrium objects depend on $\hat{\lambda}_j, \hat{F}_j, G_j, p_j, T_j, x_j$. After solving the dynamic game, we simulate the probability of success,

$$\Pr(\text{success} | b(\tilde{v}_j), D_+(\tilde{v}_j), F(\tilde{v}_j)) \approx \frac{1}{I} \sum_{i=1}^I \mathbf{1}[\tilde{R}_{T_j}^{(i)} \geq G_j],$$

where \tilde{R} is the final revenue for simulation i . Success depends on strategies, beliefs, arrival realizations, and donor valuation draws. We set $I = 10,000$.

We face two challenges in calibrating v_j . First, we do not have repeat observations for a given campaign j , nor do we observe many campaigns with the exact same features, e.g., goal and price. Therefore, it is not possible to calibrate campaign-specific parameters. Instead, we partition the data based on campaigns features. Using this partition, we match the empirical fraction of campaigns that succeed to its model counterpart, i.e., for a given group of campaigns \mathcal{G} , we estimate $v_{\mathcal{G}}$ by solving

$$\min_{v: v - p_{\mathcal{G}} \geq 1} \left(\frac{1}{|\mathcal{G}|} \sum_{j=1}^{|\mathcal{G}|} \mathbf{1}[\text{campaign } j \text{ succeeded}] - \Pr(\text{success} | b(v), D_+(v), F(v)) \right)^2.$$

The second challenge is computational burden. For example, solving the game when G/p is high is difficult. For this reason, we do not calibrate parameters for all campaigns in the sample. We validate our calibration by reporting non-target moments.

5.5 Empirical Specification and Parameter Estimates

For donor valuations, we include fixed effects for the campaign category, an indicator if the project is “loved” by Kickstarter, and the log of the goal amount.¹⁵ We use the log of total pledges plus one, $\log(\sum_t B_{j,t} + 1)$, as the excluded variable that influences selection

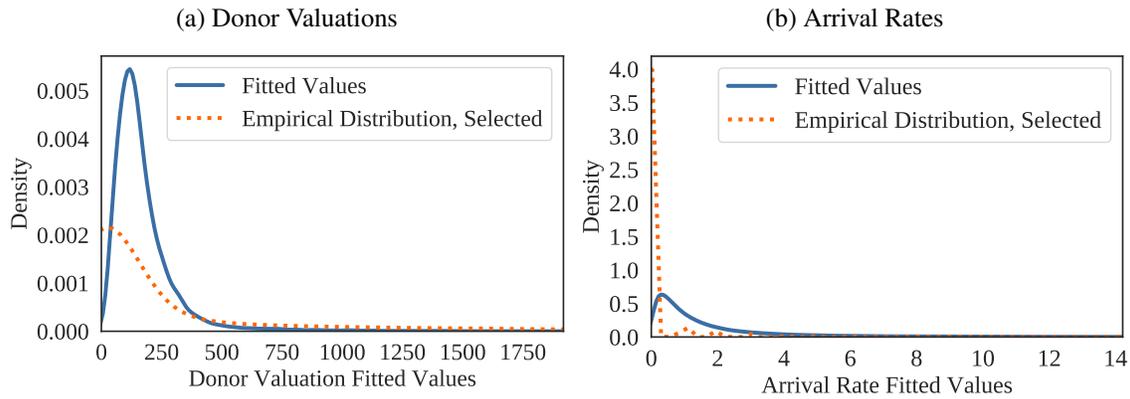
¹⁵For more information on the “loved” indicator, see Figure 16 in Online Appendix C.

but not donor valuations. The intuition is that pledges shift the probability of success but not the distribution of donor valuations. Rather, pledges affect the donor’s strategy.

For investor arrivals, we include fixed effects for the campaign category, a quadratic expansion in time, the “loved” by Kickstarter indicator, and the log of the goal amount. We use cumulative donations divided by the goal as the excluded variable that influences selection but not arrivals. Donations shift the probability of success and investor strategies, but we assume that donations do not directly influence buyers’ arrivals.

Finally, given donor valuation and arrival process parameters, investor utilities are matched to the probability of success. Intuitively, the probability of success is increasing in v : when v approaches $v_0 + p$, the probability that a campaign succeeds goes to zero, and when v increases, the probability of success can approach one.

Figure 5: Parameter Estimates for Arrivals and Donor’s Valuations



Note: (a) Kernel density plot of the fitted values of the donor’s valuation for all categories. The dashed orange line shows the empirical density of the selected sample. (b) Kernel density plot of the fitted values of buyer arrivals for all categories. The dashed orange line shows the empirical density of the selected sample over the range (x-axis) obtained under the model estimates up until the 99th percentile.

We limit our sample by focusing on 30-day campaigns and the first quartile of data in terms of goal amount, by campaign category. We plot fitted values for the donor’s valuation distribution and buyers arrivals in Figure 5. We provide the corresponding parameter estimates in Online Appendix C. In panel (a), we compare the fitted values for the donor’s valuation to the selected sample. Accounting for sample selection shifts the distribution to

the right as we would expect. In panel (b), we compare fitted arrival rates to the selected sample. For readability reasons, we exclude the long-tail of empirical distribution (orange). The mean observed arrival rate is higher than what we find under the fitted values.

In Table 3, we provide estimates for the buyer utility from pledging, a comparison of the data moment to its model counterpart, and three sets of non-targeted moments (total donations, number of pledges, and total donations over the goal amount). We find that for some categories the targeted and all non-targeted moments match well, e.g., fashion. In other categories, it is difficult to match the probability of success given our estimated arrival rates and donor valuation distribution. For example, for the theater category, our empirical estimates understate the variance in observed donations. As a consequence, we understate the probability of success compared to what is observed in the data.

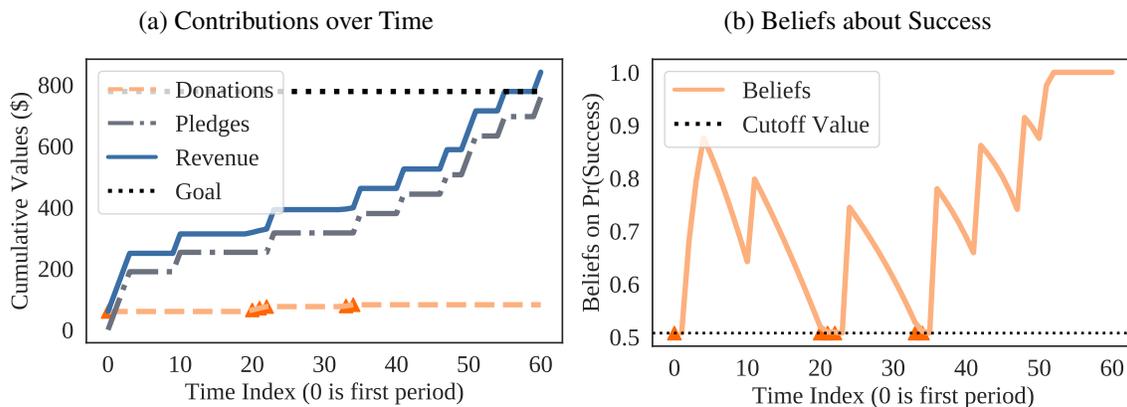
Table 3: Parameter Estimates for Investor Utility

Category	Util. Est.	Data Moment	Model Moment	Non-Targeted Models (Data, Model)					
				Total Donations (\$)	Number of Pledges	Donations / Goal			
Art	62.441	0.215	0.217	41.766	38.336	5.526	2.481	0.100	0.088
Comics	50.154	0.284	0.282	79.629	82.372	11.968	7.367	0.100	0.111
Crafts	42.336	0.130	0.144	45.877	37.575	4.024	3.369	0.093	0.083
Design	85.610	0.147	0.146	117.677	119.509	11.639	4.008	0.078	0.074
Fashion	61.132	0.138	0.138	40.783	43.653	5.619	5.855	0.063	0.055
FilmVideo	95.442	0.289	0.287	192.842	83.385	6.142	4.642	0.175	0.079
Food	124.866	0.185	0.023	233.265	103.505	8.432	8.786	0.113	0.045
Games	46.700	0.091	0.091	61.048	75.007	9.300	5.728	0.054	0.056
Journalism	65.248	0.065	0.066	53.058	49.130	1.419	1.198	0.064	0.063
Music	60.414	0.300	0.295	195.941	63.964	7.281	7.200	0.182	0.065
Photography	50.366	0.183	0.183	96.783	61.013	5.662	9.288	0.109	0.074
Publishing	53.491	0.201	0.199	101.197	77.370	5.727	4.891	0.109	0.088
Technology	112.166	0.084	0.083	156.271	93.723	7.773	3.394	0.058	0.033
Theater	68.239	0.422	0.088	398.868	167.431	8.000	12.835	0.273	0.126

Note: Calibration of buyer utilities assuming the success-maximizing equilibrium and using donor valuation estimated in Table 6 and arrival process estimates in Table 7. We consider each category separately and match moments for the first quartile of campaigns in terms of goal amount. We set $v_0 = 1$. We report our targeted moment (fraction of campaigns that succeed) and non-targeted moments.

In Figure 6, we provide an example equilibrium path. The initial donation spike (orange triangle at $t = 0$) is reminiscent of the role of “seed money” in charitable fund-raising campaigns (Andreoni, 1998). This ensures that beliefs are sufficiently high (panel b). The graphs show that beliefs jump as pledges occur. In the absence of pledges, the donor contributes to keep beliefs above the cutoff value.

Figure 6: Success-Maximizing Equilibrium Example Path



Note: (a) Cumulative donations, pledges, and revenues over time for an example equilibrium path in the Journalism category where $w = 130$. The (orange) triangles mark where donations occur. (b) Buyer beliefs about the probability of success over time. The horizontal (dashed) line corresponds to the cutoff value, $v_0/(v-p)$. The same (orange) triangles mark donation events. The donor ensures that beliefs stay above the cutoff value. In this example, the project reaches success toward the end of the campaign.

5.6 Counterfactual Analysis

Using our parameter estimates, we simulate three counterfactuals: the success-minimizing equilibrium, allowing for a single donation at the beginning of the campaign, and allowing for a single donation at the end of the campaign. The latter two scenarios correspond to the alternative project designs described in Section 3.6.

We report counterfactual results in Table 4, focusing on the probability of success across categories. We find two key results. First, the gap in the probability of success between the success-maximizing and success-minimizing equilibrium tends to be large. We estimate the difference to be 13% on average, across categories. This suggests that coordination benefits from dynamic signaling are high. Second, we find that the probability of success under the alternative project designs tends to be even smaller than that under the success-minimizing equilibrium. This is not obvious ex-ante, as our theory establishes that the probability of success must be lower than that of the success-maximizing equilibrium, but not necessarily lower than that of the success-minimizing equilibrium. To demonstrate the implications of this finding, we provide initial donation values under the success-maximizing equilibrium

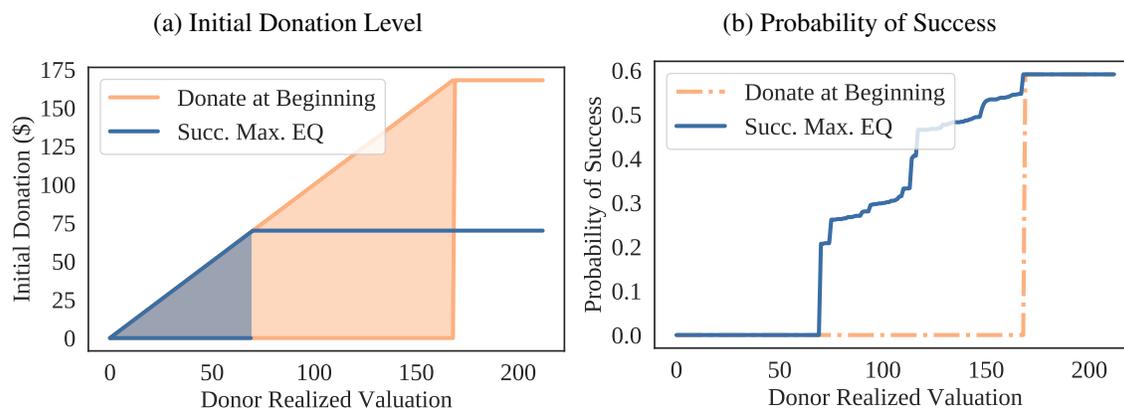
Table 4: Counterfactual Results

Category	Probability of Success (%)			
	Success-Maximizing	Success-Minimizing	Donate at Beginning	Donate at End
Art	21.750	0.381	0.312	0.000
Comics	28.160	0.605	0.564	0.000
Crafts	14.440	13.260	12.448	8.950
Design	14.590	0.312	0.301	0.000
Fashion	13.840	9.267	8.280	12.440
FilmVideo	28.730	6.768	6.391	0.000
Food	2.270	1.089	0.884	0.860
Games	9.120	0.750	0.644	0.000
Journalism	6.550	1.592	1.472	0.000
Music	29.550	0.864	0.768	0.000
Photography	18.310	6.631	5.341	8.210
Publishing	19.880	0.487	0.460	0.000
Technology	8.290	0.040	0.037	0.000
Theater	8.830	4.102	2.778	3.190

Note: Counterfactual estimates of the percentage of campaigns that reach the goal. We consider the success-maximizing equilibrium (assumed in estimation), success-minimizing equilibrium, and two alternative campaign designs where the donor can only contribute at the beginning or end of the campaign.

and the when the donor only contributes at the beginning of time across W (x-axis) in Figure 7. Panel (a) shows that initial donation levels when donations occur once are higher than under the success-maximizing case, but this does not lead to a higher probability of success (panel b). Allowing the donor to provide incremental support to the campaign is the best way to achieve success.

Figure 7: Counterfactual Equilibrium Initial Donation Levels and Success



Note: (a) Initial donation amount (y-axis) given realized donor valuation (x-axis) for Publishing. The shaded regions correspond to situations where there is multiplicity in initial donations because all campaigns fail. (b) Probability of success (y-axis) given realized valuation (x-axis). The two lines correspond to restricting the donor to just contribute at the start of the campaign (orange) and the success-maximizing equilibrium (blue).

6 Conclusion

We introduce a dynamic contribution game in which randomly arriving investors can contribute to obtain an uncertain private benefit, and a long-lived donor who values the public benefits donates, while seeking to minimize his total contributions required for project success. Participants face two forms of uncertainty: uncertainty in arrivals and uncertainty in the donor's valuation. We show that allowing the donor to dynamically signal his valuation by strategically timing contributions, benefits all participants by facilitating coordination. Indeed, the success-minimizing equilibrium corresponds to outcomes where dynamic signaling is not possible. The success-maximizing equilibrium maximizes ex-ante donor payoffs, but exacerbates uncertainty borne by investors. Investors prefer equilibria strictly in-between the success-maximizing and the success-minimizing equilibria. Our model extends to many economic environments where investment is socially-productive. We present an empirical application to reward-based crowdfunding using data from Kickstarter. Our model calibrations show that restricting the donor's ability to dynamically signal his valuation would greatly reduce the percentage of campaigns that succeed.

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Appendix

A Proofs

A.1 General properties of PT assessments and PT equilibria

A.1.1 Properties of PT assessments

In this section, we present some properties of PT assessments and the induced probability of success $\pi^\Delta(N, D, u)$ that we will use for the construction of PT equilibria.

Lemma 1. *Consider a PT assessment with donation threshold $D_*^\Delta(N, u)$. If the campaign reaches a state (N, D, u) with $D < D_*^\Delta(N, u + \Delta)$, it has failed with probability one.*

Proof. Assume that a state $(N_t, D_t, T - (t + \Delta))$ with $D_t < D_*^\Delta(N_t, T - t)$ is reached. Then $D_t = w$, because the donor is playing a PT strategy and $w < D_*^\Delta(N_t, T - t')$ for all $t' \geq t$ by Condition i) in Definition 2 of PT assessments. Thus, $N_{t'} = N_t$ for all $t' > t$, given the investor strategy in Equation PT-investor. All in all, $(N_{t'}, D_{t'}) = (N_t, w)$ for all $t' > t$, where $N_t p + w < N_t p + D_*^\Delta(N_t, T - t) < N_t p + G - (N_t + 1)p < G$. This concludes the proof. ■

Lemma 1 implies that beliefs in a PT assessment are consistent and that the induced probability of success π^Δ can be written in a recursive manner as we show in Lemma 2. We also derive some other properties of π^Δ . For the proof, we use that for a PT assessment, cumulative donations at time t must satisfy

$$D_t = \max_{t' \leq t} \min\{D_*^\Delta(N_{t'}, T - t'), w\}. \quad (1)$$

Lemma 2. *A PT assessment $(b^\Delta, D_+^\Delta, F^\Delta)$ with donation threshold $D_*^\Delta(N, u)$ satisfies the following properties:*

- i) *Beliefs F^Δ are consistent with the strategies b^Δ, D_+^Δ ;*
- ii) *The induced probability $\pi^\Delta(N, D, u)$ satisfies the following:*

- $N + 1 \geq \underline{M}(D)$ if and only if $\pi^\Delta(N, D, u) = 1$;
- If $N + 1 < \underline{M}(D)$ and $D \geq D_*^\Delta(N, u + \Delta)$, then $\pi^\Delta(N, D, 0) = \frac{1 - F_0(G - p(N + 1))}{1 - F_0(D)}$, and for $u > 0$,

$$\begin{aligned} \pi^\Delta(N, D, u) = & \mathbb{E}^{F_0} \left[\sum_{i=1}^{\frac{u}{\Delta}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \right. \\ & \pi^\Delta \left(N + 1, \max \{ D, D_*^\Delta(N + 1, u - (i - 1)\Delta) \}, u - i\Delta \right) \\ & \mathbb{1} \left(W \geq D_*^\Delta(N + 1, u - (i - 1)\Delta) \right) \\ & \left. + (1 - \Delta\lambda)^{u/\Delta} \mathbb{1} (W \geq G - (N + 1)p) \mid W \geq D \right]; \end{aligned}$$

- If $N + 1 < \underline{M}(D)$ and $D < D_*^\Delta(N, u + \Delta)$, $\pi^\Delta(N, D, 0) = 0$, and for $u > 0$,

$$\pi^\Delta(N, D, u) = \mathbb{P} \left(D \geq \max_{\substack{N < N' \leq \underline{M}(D) \\ \tau_{N'-N}^u < T}} D_*^\Delta(N', T - \tau_{N'-N}^u) \right), \quad (2)$$

where τ_n^u is the time of the n -th arrival after time $t = T - u$.¹⁶

- iii) $\pi^\Delta(N, D, u)$ is continuous and strictly increasing in D for $G - (N + 1)p \geq D \geq D_*^\Delta(N, u + \Delta)$, and $\pi^\Delta(N, D, u)$ is weakly increasing in D otherwise;
- iv) $\pi^\Delta(N, D, u) \leq \pi^\Delta(N + 1, D, u - \Delta) \leq \pi^\Delta(N + 1, D, u)$, and $\pi^\Delta(N, D, u)$ is strictly increasing in N, u , if $0 < \pi^\Delta(N, D, u) < 1$.

Proof. i) Consider an investor in an on-path state (N, D, u) . By (1) this state is reached with zero probability by donors with $w < D$, and if $D < D_*^\Delta(N, u + \Delta)$, then $D = w$. Further, if $D \geq D_*^\Delta(N, u + \Delta)$, any donor with $w \geq D$ must have followed the same donation strategy on any equilibrium path history that led to (N, D, u) . Hence, by Bayes' rule, the distribution of donor types in a state (N, D, u) is a truncation of F_0 at D .

¹⁶Note that $\pi^\Delta(N, D, u)$ is defined even if the corresponding purchase is not consistent with the investor strategy. If $D < D_*^\Delta(N, u + \Delta)$ and the investor pledges, this deviation is not observed by an investor in period $u' < u$. Thus, she pledges if $D \geq D_*^\Delta(N + 1, u' + \Delta)$. The probability is with respect to the random arrival time $\tau_{N'-N}^u$.

ii) For $N + 1 \geq \underline{M}(D)$, $\pi^\Delta(N, D, u) = 1$ as the goal is reached if the $(N + 1)$ th investor pledges. For $N + 1 < \underline{M}(D)$, absent additional donations, at least one more investor must arrive to reach the goal G after the $(N + 1)$ th investor pledges because $D_*^\Delta(N, u) < G - (N + 1)p$, so $\pi^\Delta(N, D, u) < 1$. The probability of success must satisfy the following recursive property: First, $\pi^\Delta(N, D, 0) = \frac{1 - F_0(G - (N + 1)p)}{1 - F_0(D)} \mathbb{1}(D \geq D_*^\Delta(N, \Delta))$, given F^Δ defined in Equation PT-belief. For $u > 0$ and $D \geq D_*^\Delta(N, u + \Delta)$,

$$\begin{aligned} \pi^\Delta(N, D, u) = & \mathbb{E}^{F_0} \left[\sum_{i=1}^{\frac{u}{\Delta}} \underbrace{(1 - \Delta\lambda)^{i-1} \Delta\lambda}_{\substack{\text{next investor arrives} \\ \text{at } u - i\Delta}} \underbrace{\pi^\Delta(N + 1, \max\{D, D_*^\Delta(N + 1, u - (i - 1)\Delta)\}, u - i\Delta)}_{\substack{\text{probability of success if the } N + 2\text{nd investor pledges} \\ \text{wealth exceeds donation threshold}}} \right. \\ & \left. + \underbrace{(1 - \Delta\lambda)^{u/\Delta}}_{\substack{\text{no investor} \\ \text{beliefs are} \\ \text{truncation of} \\ F_0 \text{ at } D}} \mathbb{1}(W \geq G - (N + 1)p) \right], \end{aligned}$$

because by Lemma 1 the campaign fails with probability one if $W < D_*^\Delta(N + 1, u - (i - 1)\Delta)$.

For $D < D_*^\Delta(N, u + \Delta)$, the investor believes that $W = D$ with probability one. Hence, in the last period ($u = 0$), the campaign cannot succeed since $D_*^\Delta(N, u + \Delta) < G - (N + 1)p$ even if the $N + 1$ th investor pledges. If $u > 0$ and the $N + 1$ th investor pledges, then a subsequent investor arriving in state (N', D, u') with $N' \geq N + 1$ and $u' < u$ pledges if $D \geq D_*^\Delta(N', u' + \Delta)$.

iii) We first show that $\pi^\Delta(N, D, u)$ is strictly increasing and continuous in D for $D_*^\Delta(N, u + \Delta) \leq D \leq G - (N + 1)p$ by induction in u .

Induction start ($u = 0$): $\pi^\Delta(N, D, 0) = \frac{1 - F_0(G - (N + 1)p)}{1 - F_0(D)} \mathbb{1}(D \geq D_*^\Delta(N, \Delta))$ is continuous and strictly increasing in D for $D_*^\Delta(N, \Delta) \leq D \leq G - (N + 1)p$.

Induction hypothesis for u : $\pi^\Delta(N, D, u)$ is continuous and strictly increasing in D for $D_*^\Delta(N, u + \Delta) \leq D \leq G - (N + 1)p$.

Induction step ($u \rightsquigarrow u + \Delta$): For $D_*^\Delta(N, u + 2\Delta) \leq D \leq G - (N + 1)p$ we have by ii)

$$\begin{aligned} \pi^\Delta(N, D, u + \Delta) = & \\ & \frac{\sum_{i=1}^{\frac{u+\Delta}{\Delta}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \pi^\Delta(N + 1, \max\{D, D_*^\Delta(N + 1, u + \Delta - (i-1)\Delta)\}, u + \Delta - i\Delta)}{1 - F_0(\max\{D, D_*^\Delta(N + 1, u + \Delta - (i-1)\Delta)\})} + (1 - \Delta\lambda)^{u/\Delta} \frac{1 - F_0(G - (N + 1)p)}{1 - F_0(D)}, \end{aligned}$$

which is continuous in D by the induction hypothesis because $D_*^\Delta(N + 1, u + \Delta - (i-1)\Delta) \leq \max\{D, D_*^\Delta(N + 1, u + \Delta - (i-1)\Delta)\} \leq G - (N + 1)p$ and also strictly increasing because $\frac{1 - F_0(\max\{D, D_*^\Delta(N + 1, u + \Delta - (i-1)\Delta)\})}{1 - F_0(D)}$ is equal to 1 if $D \geq D_*^\Delta(N + 1, u + \Delta - (i-1)\Delta)$ and $\frac{1}{1 - F_0(D)}$ is strictly increasing in D .

Finally, if $D > G - (N + 1)p$, then $\pi^\Delta(N, D, u) = 1$, and if $D < D_*^\Delta(N, u + \Delta)$, then it follows that $\pi^\Delta(N, D, u)$ is weakly increasing in D directly from (2).

iv) By Condition i) in Definition 2 of PT assessments, $D_*^\Delta(N, u) \geq D_*^\Delta(N + 1, u - \Delta) \geq D_*^\Delta(N + 1, u)$. Hence, a donor w , who can incentivize the next investor to pledge in a state (N, D, u) , can incentivize the next investor to pledge in state $(N + 1, D, u - \Delta)$ in the next period. Thus, more future investors are incentivized to pledge after state $(N + 1, D, u - \Delta)$ than after (N, D, u) , so $\pi^\Delta(N + 1, D, u - \Delta) \geq \pi^\Delta(N, D, u)$. Similarly, a donor w , who can incentivize the next investor to pledge in a state $(N + 1, D, u - \Delta)$, can incentivize the next investor to pledge in state $(N + 1, D, u)$ in the period before. Thus, more future investors are incentivized to pledge after state $(N + 1, D, u)$ than after $(N + 1, D, u - \Delta)$, so $\pi^\Delta(N + 1, D, u) \geq \pi^\Delta(N + 1, D, u - \Delta)$.

Next, we show by induction in N that if $0 < \pi^\Delta(N, D, u) < 1$, then $\pi^\Delta(N + 1, D, u) > \pi^\Delta(N, D, u)$. To this end, note that for $N + 1 < \underline{M}(D)$ and $D \geq D_*^\Delta(N, u + \Delta)$ we can write by ii) for $u > 0$

$$\begin{aligned} \pi^\Delta(N, D, u) = & \mathbb{E}\left[\left(\Delta\lambda\pi^\Delta(N + 1, \max\{D, D_*^\Delta(N + 1, u)\}, u - \Delta) + \right. \right. \\ & \left. \left. (1 - \Delta\lambda)\pi^\Delta(N, \max\{D, D_*^\Delta(N + 1, u)\}, u - \Delta)\right) \right. \\ & \left. \mathbb{1}(W \geq D_*^\Delta(N + 1, u)) \Big| W \geq D\right] \end{aligned}$$

because if no investor arrives in period $u - \Delta$, then the probability of success is as if the investor in period u arrived a period later, but with a new donation threshold, i.e., it is $\pi^\Delta(N, \max\{D, D_*^\Delta(N+1, u)\}, u - \Delta)$.

Induction start ($N = \underline{M}(D) - 1$): $\pi^\Delta(N+1, D, u) = 1 > \pi^\Delta(N, D, u)$.

Induction hypothesis for $N < \underline{M}(D) - 1$: Assume $\pi^\Delta(N+1, D, u) > \pi^\Delta(N, D, u)$ if $0 < \pi^\Delta(N, D, u) < 1$.

Induction step ($N \rightsquigarrow N-1$): Let $0 < \pi^\Delta(N-1, D, u) < 1$. If $D \geq D_*^\Delta(N, u + \Delta)$, then

$$\begin{aligned}
\pi^\Delta(N, D, u) &= \mathbb{E}\left[\underbrace{(\Delta\lambda \pi^\Delta(N+1, \max\{D, D_*^\Delta(N+1, u)\}, u - \Delta))}_{> \pi^\Delta(N, D, u - \Delta) \text{ by induction hypothesis and monotonicity in } D} + \right. \\
&\quad \left. \underbrace{(1 - \Delta\lambda) \pi^\Delta(N, \max\{D, D_*^\Delta(N, u)\}, u - \Delta)}_{> \pi^\Delta(N, D, u - \Delta) \text{ by monotonicity in } D} \mathbb{1}(W \geq D_*^\Delta(N+1, u)) \Big| W \geq D\right] \quad (3) \\
&> \pi^\Delta(N, D, u - \Delta) \mathbb{P}(W \geq \underbrace{D_*^\Delta(N+1, u)}_{< D_*^\Delta(N, u)} | W \geq D) \geq \pi^\Delta(N-1, D, u)
\end{aligned}$$

because $\mathbb{P}(W \geq D_*^\Delta(N, u) | W \geq D) = 1$ for $D \geq D_*^\Delta(N, u)$. If $D < D_*^\Delta(N, u + \Delta)$, then for $\pi^\Delta(N-1, D, u) > 0$,

$$\begin{aligned}
\pi^\Delta(N, D, u) &= \mathbb{P}(D \geq \underbrace{\max_{\substack{N < N' \leq \underline{M}(D) \\ \tau_{N'-N}^u < T}} D_*^\Delta(N', T - \tau_{N'-N}^u)}_{> \pi^\Delta(N-1, D, u)}) \\
&< \max_{\substack{N-1 < N' \leq \underline{M}(D) \\ \tau_{N'-N+1}^u < T}} D_*^\Delta(N', T - \tau_{N'-N+1}^u)
\end{aligned}$$

Finally, we consider strict monotonicity in u . Consider $N+1 < \underline{M}(D)$. If $D \geq D_*^\Delta(N, u + \Delta)$, then (3) implies $\pi^\Delta(N, D, u) > \pi^\Delta(N, D, u - \Delta)$, where we use the strict monotonicity of π^Δ in N . If $D < D_*^\Delta(N, u + \Delta)$, then since $\tau_{N'-N}^{u-\Delta}$ and $\tau_{N'-N}^u + 1$ are equally

distributed by the Markov property, and since $D_*^\Delta(N, u)$ is decreasing u for $D_*^\Delta(N, u) > 0$,

$$\mathbb{P}(D \geq \max_{\substack{N < N' \leq \underline{M}(D) \\ \tau_{N'-N}^{u-\Delta} < T}} D_*^\Delta(N', T - \tau_{N'-N}^u)) > \mathbb{P}(D \geq \max_{\substack{N < N' \leq \underline{M}(D) \\ \tau_{N'-N}^u < T}} D_*^\Delta(N', T - \tau_{N'-N}^{u-\Delta})).$$

Hence, $\pi^\Delta(N, D, u) > \pi^\Delta(N, D, u - \Delta)$ as long as $\pi^\Delta(N, D, u) \in (0, 1)$. ■

For the construction of the donation thresholds, it is useful to consider the auxiliary probability of success in a state (N, D, u) if the investor believed that donor wealth is distributed according to F_0 truncated at D for all D :

$$\begin{aligned} \tilde{\pi}^\Delta(N, D, u) := & \sum_{i=1}^{\frac{u}{\Delta}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \frac{1 - F_0(\max\{D, D_*^\Delta(N+1, u - (i-1)\Delta\})}{1 - F_0(D)} \\ & \pi^\Delta(N+1, \max\{D, D_*^\Delta(N+1, u - (i-1)\Delta\}), u - i\Delta) \\ & + (1 - \Delta\lambda)^{u/\Delta} \frac{1 - F_0(G - (N+1)p)}{1 - F_0(D)}. \end{aligned} \quad (4)$$

The following is a corollary of Lemma 2. We use it in the proof of Proposition 1 to define the donation threshold $\underline{D}(N, u)$.

Corollary 1. *The auxiliary probability of success $\tilde{\pi}^\Delta(N, D, u)$ is continuous and (strictly) increasing in D (as long as $\tilde{\pi}^\Delta(N, D, u) \in (0, 1)$).*

Finally, Lemma 3 shows that the donor strategy specified in any PT assessment is a best response to the specified investor strategy.

Lemma 3. *For any PT assessment with donation threshold $D_*^\Delta(N, u)$, the donor PT strategy is a best response to the investor strategy.*

Proof. We argue by backwards induction in t .

Induction start ($t = T$): First, consider histories in the last period $h_T^{D, \Delta}$ with cumulative contributions N_T and $D_{T-\Delta}$. Ignoring the constraint imposed by previous donations, the donor would want to donate $\min\{w, G - N_T p\}$, since he would want to give just enough for the campaign to succeed without exceeding his valuation. However, the donor cannot

take out funds. Thus, a cumulative donation of $\max\{D_{T-\Delta}, \min\{w, G - N_T p\}\}$ is a best response. Hence, in all histories that correspond to a state $(N, D, 0)$, a Markov strategy of $\tilde{D}_+^\Delta(h_T^{D,\Delta}; w) = D_+^\Delta(N_T, D_{T-\Delta}, 0; w) = \max\{D_{T-\Delta}, \min\{w, G - N_T p\}\}$ is optimal.

Induction hypothesis for $s \geq t$: Next, we assume that for all $s \geq t$ and all $h_s^{D,\Delta}$ with corresponding cumulative contributions N_s and $D_{s-\Delta}$, the donor payoff is maximized by $\tilde{D}_+^\Delta(h_s^{D,\Delta}; w) = D_+^\Delta(N_s, D_{s-\Delta}, T-s; w) = \max\{D_{s-\Delta}, \min\{w, D_*^\Delta(N_s, T-s)\}\}$.

Induction step ($t \rightsquigarrow t-\Delta$): Consider an arbitrary donor strategy \tilde{D}_+^Δ where for all $s \geq t$, $\tilde{D}_+^\Delta(h_s^{D,\Delta}; w) = D_+^\Delta(N_s, D_{s-\Delta}, T-s; w) = \max\{D_{s-\Delta}, \min\{w, D_*^\Delta(N_s, T-s)\}\}$. Consider an on-path history $h_{t-\Delta}^{D,\Delta}$ with corresponding cumulative contributions $N_{t-\Delta}, D_{t-2\Delta}$ and a donor valuation $w \geq \max\{D_{t-2\Delta}, D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))\}$ such that

$$\tilde{D}_+^\Delta(h_{t-\Delta}^{D,\Delta}; w) < D_*^\Delta(N_{t-\Delta}, T-(t-\Delta)).$$

According to the PT assessment, if an investor arrives in period t , the investor does not pledge. Since $D_*^\Delta(N_{t-\Delta}, T-(t-\Delta)) < D_*^\Delta(N_{t-\Delta}, u)$ for all $u < T-(t-\Delta)$, the donor needs to donate at least $D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))$ in order to make a future investor pledge and to prevent the campaign from failing. Furthermore, $D_*^\Delta(N_{t-\Delta}, u) > D_*^\Delta(N', u)$ for all $N' > N_{t-\Delta}$. Hence, a donor with valuation w is strictly better off by donating $D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))$ after history $h_{t-\Delta}^{D,\Delta}$, so an optimal donor strategy must be to give at least $D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))$. Similarly, monotonicity of D_*^Δ in N, u implies that it cannot be optimal that the donor gives more than $\max\{D_{t-2\Delta}, D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))\}$. If $w < \max\{D_{t-2\Delta}, D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))\}$, the campaign succeeds with probability zero as cumulative donations are below w . Thus, a best-response donor strategy is given by

$$\tilde{D}_+^\Delta(h_{t-\Delta}^{D,\Delta}; w) = \max\{D_{t-2\Delta}, \min\{w, D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))\}\}.$$

■

A.1.2 Properties of PT equilibria

Recall that a PT equilibrium is a PT assessment $(b^\Delta, D_+^\Delta, F^\Delta)$ such that given the induced probability of success $\pi^\Delta(\mathbf{x})$ we have buyer optimality: $\pi^\Delta(\mathbf{x}) > \frac{v_0}{v-p} \Rightarrow b^\Delta(\mathbf{x}) = 1$ and $\pi^\Delta(\mathbf{x}) < \frac{v_0}{v-p} \Rightarrow b^\Delta(\mathbf{x}) = 0$. Donor-optimality is guaranteed automatically by Lemma 3. The buyer optimality condition allows us to define cutoff times $\xi_j^\Delta(w)$ as in Equation CT for each j, w with $j \leq \underline{M}(w)$. We can show that $\xi_j^\Delta(w)$ is monotone in j .

Lemma 4. *In any PT equilibrium, the cutoff time $\xi_j^\Delta(w)$ is strictly increasing in j .*

Proof. By Lemma 2 iv), we have for $j' > j$, that if $\pi^\Delta(\underline{M}(w) - j', w, u) \geq \frac{v_0}{v-p}$, then $\pi^\Delta(\underline{M}(w) - j, w, u - \Delta) \geq \pi^\Delta(\underline{M}(w) - j', w, u) \geq \frac{v_0}{v-p}$, so

$$\pi^\Delta(\underline{M}(w) - j, w, \xi_{j'}^\Delta(w) - \Delta) \geq \pi^\Delta(\underline{M}(w) - j', w, \xi_{j'}^\Delta(w)) \geq \frac{v_0}{v-p}.$$

Hence, $\xi_j^\Delta(w) \leq \xi_{j'}^\Delta(w) - \Delta < \xi_{j'}^\Delta(w)$. ■

As a result, in a PT equilibrium, after $\xi_j^\Delta(w)$ is reached, no buyer pledges, i.e.,

$$\begin{cases} \pi^\Delta(\underline{M}(w) - j, w, u) \geq \frac{v_0}{v-p} & \text{for } u \geq \xi_j^\Delta(w) \\ \pi^\Delta(\underline{M}(w) - j, w, u) < \frac{v_0}{v-p} & \text{for } u < \xi_j^\Delta(w) \end{cases} \quad (5)$$

$\xi_j^\Delta(w) > \xi_{j'}^\Delta(w)$ for any $j > j'$ if $\xi_j^\Delta(w) > 0$. Furthermore, $w \geq D_*^\Delta(N, u + \Delta) \Leftrightarrow u \geq \xi_{\underline{M}(w)-N}^\Delta(w)$. This allows us to re-write the probability of success in a different way. For $N < \underline{M}(D) - 1$ and $u > 0$, the probability of success is given by

$$\begin{aligned} \pi^\Delta(N, D, u) = & \mathbb{E}^{F_0} \left[\sum_{i=1}^{\max\{(u - \xi_{\underline{M}(w)-(N+1)}^\Delta(w))/\Delta, 0\}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \right. \\ & \pi^\Delta(N + 1, \max\{D, D_*^\Delta(N + 1, u - \Delta(i - 1))\}, u - \Delta i) \\ & \left. + (1 - \Delta\lambda)^{u/\Delta} \mathbb{1}(W \geq G - (N + 1)p) \Big| W \geq D \right], \end{aligned} \quad (6)$$

if $D \geq D_*^\Delta(N, u + \Delta)$. If $D < D_*^\Delta(N, u + \Delta)$, $\pi^\Delta(N, D, u) < \frac{v_0}{v-p}$.

A.2 Proof of Proposition 1 (Success-Maximizing Equilibrium)

In Subsection A.2.1, we first construct a PT equilibrium. Subsection A.2.2 states that the limit of these equilibria as $\Delta \rightarrow 0$ exists and is as specified in Proposition 1. The limit is formally derived in the Online Appendix. Finally, in Subsection A.2.3, we show that for any $\Delta > 0$, the constructed equilibrium maximizes the probability of success and that the outcomes of any sequence of success-maximizing PBE converge to the same limit.

A.2.1 Construction of a PT equilibrium

The following lemma specifies a PT equilibrium with a donation threshold that makes the next investor just indifferent between pledging and not.

Lemma 5 (Success-maximizing equilibrium). *Given any $\Delta > 0$, there exists a PT equilibrium $(b^\Delta, D_+^\Delta, F^\Delta)$ with donation threshold $\underline{D}^\Delta(N, u)$ and induced probability of success $\pi^\Delta(\mathbf{x})$, $\mathbf{x} \in \mathbb{X}^\Delta$ such that for $u > 0$*

$$\begin{cases} \underline{D}^\Delta(N, u) = 0 & \text{if } \pi^\Delta(N, 0, u - \Delta) > \frac{v_0}{v-p}, \\ \pi^\Delta(N, \underline{D}^\Delta(N, u), u - \Delta) = \frac{v_0}{v-p} & \text{if } \pi^\Delta(N, 0, u - \Delta) \leq \frac{v_0}{v-p}. \end{cases}$$

We denote the corresponding probability of success from the investor's perspective in state N, D, u if the investor contributes by $\underline{\pi}(N, D, u)$.

Proof. We construct the equilibrium strategies and beliefs for every state (N, D, u) by induction in $j = \underline{M}(D) - N$. In order to define the donation threshold $\underline{D}(N, u)$ such that investors are indifferent between pledging and not, we need to know the probability of success $\pi^\Delta(N, D, u)$ induced by the assessment for arbitrary D . We tackle this issue by constructing a sequence of PT assessments $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$ for $j = 1, \dots, M_0 = \underline{M}(0)$ such that $(b_{M_0}^\Delta, D_{+,M_0}^\Delta, F_{M_0}^\Delta)$ is a PBE and satisfies the properties in Lemma 5. We start with an arbitrary PT assessment $(b_1^\Delta, D_{+,1}^\Delta, F_1^\Delta)$. The induction hypothesis assumes that for each $1 \leq j' \leq j - 1$ there is a PT assessment $(b_{j'}^\Delta, D_{+,j'}^\Delta, F_{j'}^\Delta)$ such that in states (N, D, u) with $\underline{M}(D) - N \leq j'$ investor strategies are optimal, i.e., in the continuation games after such

states, the assessment specifies a PBE. Donor strategies are automatically optimal in a PT assessment by Lemma 3. Then, in the induction step $j-1 \rightsquigarrow j$, we construct a PT assessment $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$ such that for states (N, D, u) with $\underline{M}(D) - N \leq j$, investor strategies are optimal, and

$$\left. \begin{aligned} b_j^\Delta(N, D, u) &= b_{j-1}^\Delta(N, D, u), \\ D_{+,j}^\Delta(N, D, u) &= D_{+,j-1}^\Delta(N, D, u), \\ F_j^\Delta(N, D, u) &= F_{j-1}^\Delta(N, D, u), \end{aligned} \right\} \text{ for all states } (N, D, u) \text{ with } \underline{M}(D) - N \leq j-1,$$

which implies that for the corresponding probabilities of success we have

$$\pi_j^\Delta(N, D, u) = \pi_{j-1}^\Delta(N, D, u) \text{ for } \underline{M}(D) - N \leq j-1.$$

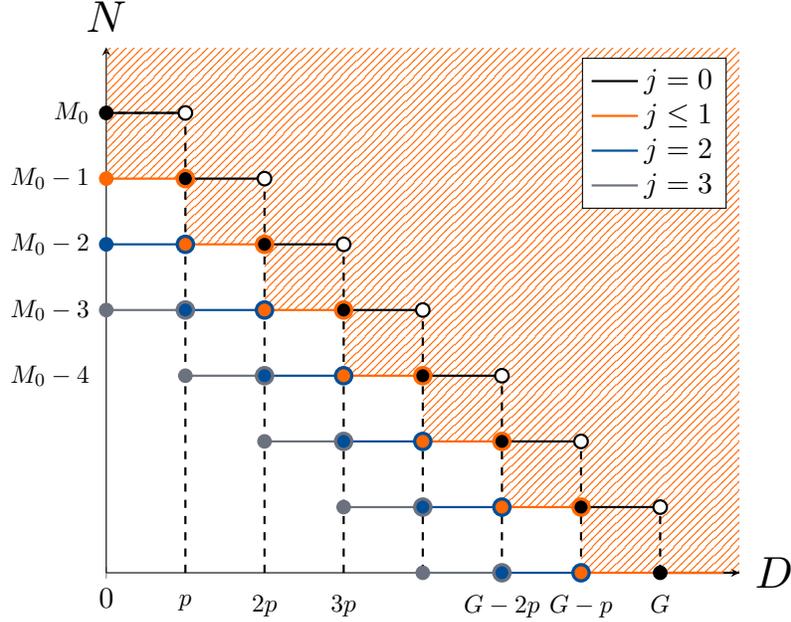
Figure 8 depicts pairs of (N, D) such that $j = \underline{M}(D) - N$ for $j = 0, 2, 3$ and the shaded region including the orange line captures all $j \leq 1$, which is our induction start for the equilibrium construction. The induction ends at $j = M_0$, when the entire state space is covered. Importantly, if the game is in state (N, D, u) , then N and D only increase in the continuation game, i.e., j is decreasing over time.

While we denote by $D_{*,j}^\Delta(N, u)$ the donation threshold corresponding to $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$, we also construct $\xi_j^\Delta(\cdot)$ and parts of the threshold function $\underline{D}^\Delta(N, u)$ in each step. In particular, in step j , we define $\underline{D}^\Delta(N, u)$ for (N, u) such that $N = M_0 - j$, or such that $N < M_0 - j$ and $u \leq \xi_j^\Delta(G - (N + j)p)$. After the last step ($j = M_0$), $\underline{D}^\Delta(N, u)$ is defined for all N and u and $\underline{D}^\Delta(N, u) = D_{*,M_0}^\Delta(N, u)$. Figure 9 illustrates this construction schematically. For a cleaner illustration that avoids drawing step functions, we assume $\Delta \rightarrow 0$ in this figure.

Finally, Table 5 summarizes the relevant notation.

(a) Induction start ($j \leq 1 \Leftrightarrow D \geq G - (N + 1)p$): We set $(b_1^\Delta, D_{+,1}^\Delta, F_1^\Delta)$ to be an arbitrary PT assessment (which trivially exists). Further, for $j \leq 1$, we set $\xi_j^\Delta(w) := 0$ for all w which is consistent with Equation CT. We also set $\underline{D}(N, u) := 0$ for $N \geq M_0 - 1$. Finally, consider states (N, D, u) with $\underline{M}(D) - N \leq 1$. The probability of success is $\pi_1^\Delta(N, D, u) = 1$, so it

Figure 8: Schematic illustration of induction in $j = \underline{M}(D) - N$



Note: The figure depicts pairs of (N, D) such that $j = \underline{M}(D) - N$ for $j = 0, j \leq 1, j = 2, j = 3$. The induction start considers states $j \leq 1$ and each $j > 1$ corresponds to one induction step.

Table 5: List of Notation

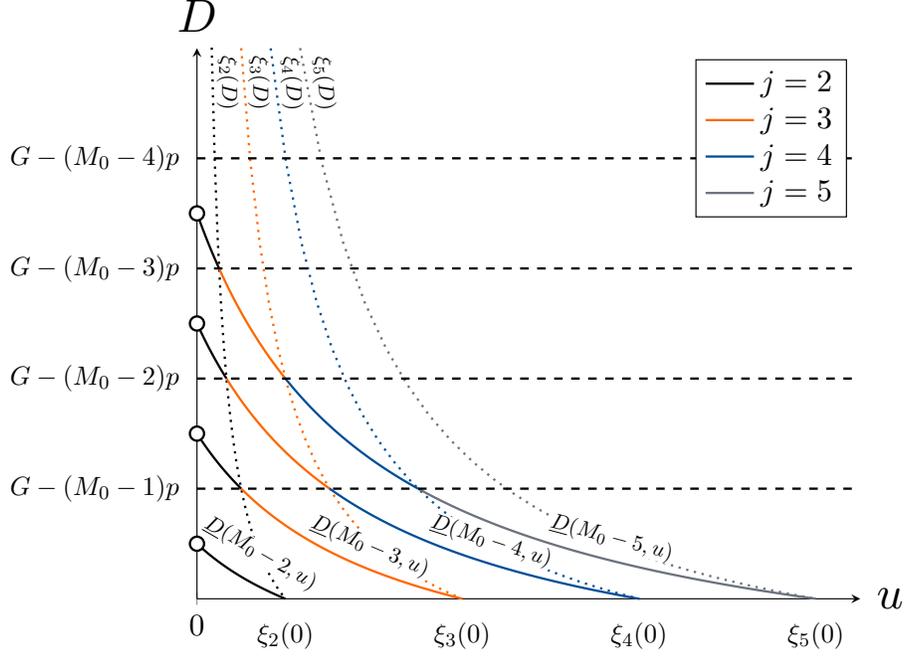
Notation	Description
$(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$	assessment in the j -th induction step
$\xi_j^\Delta(w)$	time threshold defined for all w in the j -th induction step
$D_{*,j}^\Delta(N, u)$	donation threshold corresponding to $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$
$\underline{D}^\Delta(N, u)$	donation threshold that is defined inductively for $N = M_0 - j$, and $N < M_0 - j$ and $u \leq \xi_j^\Delta(G - (N + j)p)$

is a best response for investors to pledge. Trivially, $\pi_1^\Delta(N, D, u)$ is weakly increasing in N, D, u for $D \geq G - (N + 1)p$.

(b) Induction hypothesis ($j' \leq j - 1$): For the induction hypothesis, we suppose that we have constructed PT assessments $(b_{j'}^\Delta, D_{+,j'}^\Delta, F_{j'}^\Delta)$ with a donation threshold $D_{*,j'}^\Delta(N, u)$ for $j' = 1, \dots, j - 1$ with the following properties:

- i) **Time threshold $\xi_{j'}^\Delta(w)$:** For $w < G - (j' - 1)p$, we define $\xi_{j'}^\Delta(w)$ by (5). For $w \geq$

Figure 9: Schematic illustration of construction of $\underline{D}^\Delta(N, u)$ and $\xi_j(D)$ (for small Δ)



Note: The figure depicts the donation thresholds $\underline{D}(M_0 - j, u)$ as a function of u in the limit $\Delta \rightarrow 0$. In step j , the portion between $\xi_{j-1}(G - (N + j - 1)p)$ and $\xi_j(G - (N + j)p)$ of each $\underline{D}(N, u)$ is constructed.

$G - (j' - 1)p$, we set $\xi_{j'}^\Delta(w) = 0$. $\xi_{j'}^\Delta(w) > \xi_{j'-1}^\Delta(w)$ if $\xi_{j'}^\Delta(w) > 0$.

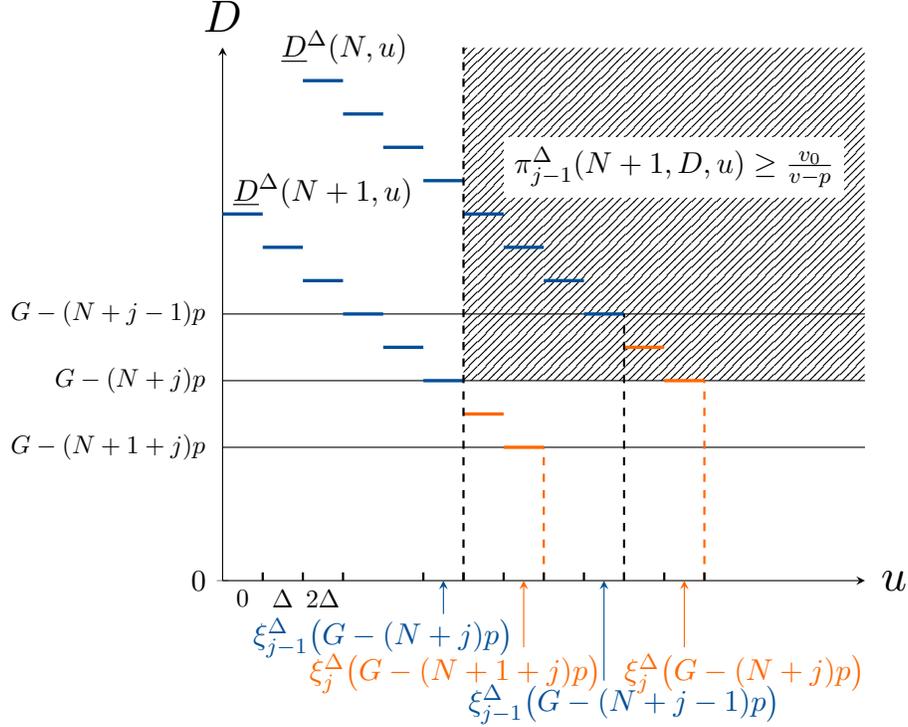
ii) **Donation threshold $\underline{D}^\Delta(N, u)$:** $\underline{D}^\Delta(N, u)$ is defined for (N, u) such that either $N \geq M_0 - (j - 1)$, or such that $N < M_0 - (j - 1)$ and $u \leq \xi_{j-1}^\Delta(G - (N + j - 1)p)$. For (N, u) with $N \leq M_0 - j'$ and $u \leq \xi_{j'}^\Delta(G - (N + j')p)$

$$\pi_{j-1}^\Delta(N, \underline{D}^\Delta(N, u), u - \Delta) = \frac{v_0}{v - p}. \quad (7)$$

Note that in that case, $\underline{D}^\Delta(N, u) < G - (N + 1)p$. For $N = M_0 - j'$, $u > \xi_{j'}^\Delta(0)$, $\underline{D}^\Delta(N, u) = 0$. $\underline{D}^\Delta(N, u)$ is strictly decreasing in N, u when it satisfies (7).

In Figure 10, the blue step functions represent the portion of \underline{D}^Δ at N and $N + 1$ that are defined in the induction hypothesis, and black dotted lines show the corresponding

Figure 10: Schematic illustration of construction of $\underline{D}^\Delta(N, u)$ for N and $N + 1$



Note: The figure depicts the donation thresholds for cumulative purchases N and $N + 1$ with $N < M_0 - j$. In step $j - 1$ only the blue portion of \underline{D}^Δ is constructed, while in step j the orange portion is added. For example, we construct $\underline{D}^\Delta(N + 1, u)$ for $u \leq \xi_{j-1}^\Delta(G - (N + 1 + j - 1)p)$ in step $j - 1$ and extend it to $u \leq \xi_{j-1}^\Delta(G - (N + 1 + j)p)$ in step j . With $D \geq G - (N + j)p$, and $N + 1$ purchases, the campaign is active until $\xi_{j-1}^\Delta(G - (N + j)p) + \Delta$ or longer, even if no additional donations are being made (shaded area). For such states, strategies of the assessment $(b_{j-1}^\Delta, D_{+,j-1}^\Delta, F_{j-1}^\Delta)$ are not optimal and π_{j-1}^Δ might not be increasing and continuous in D . We only assume $\pi_{j-1}^\Delta \geq 0$. Hence, the donation threshold cannot be constructed for $(N + 1, u)$ with $u > \xi_{j-1}^\Delta(G - (N + j)p)$ in step $j - 1$.

$$\xi_{j-1}^\Delta(G - (N + j - 1)) \text{ and } \xi_{j-1}^\Delta(G - (N + 1 + j - 1)p) = \xi_{j-1}^\Delta(G - (N + j)p).$$

iii) **PT assessment:** $(b_{j'}^\Delta, D_{+,j'}^\Delta, F_{j'}^\Delta)$ are PT assessments (as in Definition 2) with donation thresholds $D_{*,j'}^\Delta(N, u)$ satisfying

$$D_{*,j'}^\Delta(N, u) = \underline{D}^\Delta(N, u) \quad \text{for } u \leq \xi_{j'}^\Delta(G - (N + j')p), \text{ and}$$

$$\text{for } N = M_0 - j', u > \xi_{j'}^\Delta(0).$$

iv) **Probability of success:** For all $N \geq \underline{M}(D) - j'$, $\pi_{j'}^\Delta(N, D, u)$ satisfies (6) if $D \geq D_{*,j'}^\Delta(N, u + \Delta)$ and $\pi_{j'}^\Delta(N, D, u) < \frac{v_0}{v-p}$ if $D < D_{*,j'}^\Delta(N, u + \Delta)$.

Note that by monotonicity of $\pi_{j'}^\Delta(N, D, u)$ in N, u (Lemma 2 iii),

$$\pi_{j'}^\Delta(N, D, u) \geq \frac{v_0}{v-p} \quad \text{for } u > \xi_{j'}^\Delta(G - (N + j')p), D \geq G - (N + j')p.$$

This is illustrated in Figure 10 in the shaded area. Similarly, it implies that $u \leq \xi_{j'}^\Delta(D) \Leftrightarrow D < D_{*,j'}^\Delta(\underline{M}(D) - j', u + \Delta) = \underline{D}^\Delta(\underline{M}(D) - j', u + \Delta)$.

v) **Best response:** For the PT assessments $(b_{j'}^\Delta, D_{+,j'}^\Delta, F_{j'}^\Delta)$, investors best respond by pledging if and only if $D \geq D_{*,j'}^\Delta(N, u + \Delta)$ in states all (N, D, u) with $N \geq \underline{M}(D) - j$.

(c) **Induction step** ($j - 1 \rightsquigarrow j$, $j \geq 2$): In this step, we assume the induction hypothesis (b) and construct a PT assessment $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$ such that the same statements are true for states (N, D, u) with $N = \underline{M}(D) - j$, i.e., $G - (N + j)p \leq D < G - (N + (j - 1))p$.

i) **Time threshold** $\xi_j^\Delta(w)$: First, note that for $w \geq G - (N + j)p$, there is a $j' \leq j - 1$ such that $\underline{M}(w) - j' = N + 1$. Then, we know by the induction hypothesis that

$$\left\{ \begin{array}{ll} \text{for } u' < \xi_{j'}^\Delta(w): & w < D_{*,j'-1}^\Delta(N + 1, u') = \underline{D}^\Delta(N + 1, u') \\ \text{for } \xi_{j'}^\Delta(w) \leq u' \leq \xi_{j-1}^\Delta(G - (N + j)p): & w \geq \underline{D}^\Delta(N + 1, u') = D_{*,j-1}^\Delta(N + 1, u') \\ \text{for } u' > \xi_{j-1}^\Delta(G - (N + j)p) & w \geq \underline{D}^\Delta(N + 1, \xi_{j-1}^\Delta(G - (N + j)p)) \\ & > D_{*,j-1}^\Delta(N + 1, u') \end{array} \right.$$

Hence, $w \geq D_{*,j-1}^\Delta(N + 1, u') \Leftrightarrow u' \geq \xi_{j'}^\Delta(w)$. Therefore, letting $\tilde{\pi}_{j-1}^\Delta(N, D, u)$ be the auxiliary probability corresponding to the assessment $(b_{j-1}^\Delta, D_{+,j-1}^\Delta, F_{j-1}^\Delta)$ as defined in Equation 4, we can write

$$\begin{aligned} \tilde{\pi}_{j-1}^\Delta(N, D, u) = & \mathbb{E}^{F_0} \left[\sum_{i=1}^{\max\{(u - \xi_{\underline{M}(W) - (N+1)}^\Delta(W))/\Delta, 0\}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \right. \\ & \pi_{j-1}^\Delta(N + 1, \max\{D, D_{*,j-1}^\Delta(N + 1, u - \Delta(i - 1))\}, u - \Delta i) \\ & \left. + (1 - \Delta\lambda)^{u/\Delta} \mathbb{1}(W \geq G - (N + 1)p) \mathbb{1}(W \geq D) \right]. \end{aligned}$$

Next, note that the above also implies that for $u - i\Delta < \xi_{j-1}^\Delta(D)$, then $D < D_{*,j-1}^\Delta(N+1, u - (i-1)\Delta) = \underline{D}^\Delta(N+1, u - (i-1)\Delta)$ and for $u - i\Delta \geq \xi_{j-1}^\Delta(D)$, $D \geq D_{*,j-1}^\Delta(N+1, u - (i-1)\Delta)$. Hence, for $j = \underline{M}(D) - N$

$$\begin{aligned} \tilde{\pi}_{j-1}^\Delta(N, D, u) = & \mathbb{E}^{F_0} \left[\sum_{i=1}^{\max\{(u - \xi_{\underline{M}(D)-(N+1)}^\Delta(W))/\Delta, 0\}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \right. \\ & \left. \left(\pi_{j-1}^\Delta(N+1, D, u - \Delta i) \mathbb{1}(u - \Delta i \geq \xi_{j-1}^\Delta(D)) + \frac{v_0}{v-p} \mathbb{1}(u - \Delta i < \xi_{j-1}^\Delta(D)) \right) \right. \\ & \left. + (1 - \Delta\lambda)^{u/\Delta} \mathbb{1}(W \geq G - (N+1)p) \mid W \geq D \right]. \end{aligned}$$

Note that this expression only depends on $\xi_{j'}^\Delta(\cdot)$, $j' \leq j-1$, and $\pi_{j-1}^\Delta(N+1, D, u')$ where $\underline{M}(D) - (N+1) \leq j-1$, which are defined in the induction hypothesis. Since $\pi_{j-1}(N, D, u)$ is strictly increasing in u and $\pi_{j-1}^\Delta(N+1, D, u - \Delta i) \geq \frac{v_0}{v-p}$ for $u - \Delta i \geq \xi_{j-1}^\Delta(D)$, $\tilde{\pi}_{j-1}(N, D, u) < 1$ is strictly increasing in u . Hence, for any $j \leq \underline{M}(D)$ there is a unique

$$\xi_j^\Delta(D) = \operatorname{argmin} \left\{ u \mid \tilde{\pi}_{j-1}^\Delta(\underline{M}(D) - j, D, u) \geq \frac{v_0}{v-p} \right\}$$

Recall that $\pi_{j-1}^\Delta(\underline{M}(D) - (j-1), D, u) = \tilde{\pi}_{j-1}^\Delta(\underline{M}(D) - (j-1), D, u)$ for $D \geq D_{*,j-1}^\Delta(\underline{M}(D) - (j-1), u + \Delta)$, and by the induction hypothesis, $\pi_{j-1}^\Delta(\underline{M}(D) - (j-1), D, u) < \frac{v_0}{v-p}$ and $\tilde{\pi}_{j-1}^\Delta(\underline{M}(D) - (j-1), D, u) < \frac{v_0}{v-p}$ for $D < D_{*,j-1}^\Delta(\underline{M}(D) - (j-1), u + \Delta)$. Hence,

$$\begin{cases} \tilde{\pi}_{j-1}^\Delta(\underline{M}(D) - (j-1), D, u) \geq \frac{v_0}{v-p} \text{ for } u \geq \xi_{j-1}^\Delta(D) \\ \tilde{\pi}_{j-1}^\Delta(\underline{M}(D) - (j-1), D, u) < \frac{v_0}{v-p} \text{ for } u < \xi_{j-1}^\Delta(D) \end{cases},$$

and we have $\xi_j^\Delta(w) > \xi_{j-1}^\Delta(w)$ if $\xi_j^\Delta(w) > 0$.

ii) **Donation threshold $\underline{D}^\Delta(N, u)$:** Since $(b_{j-1}^\Delta, D_{+,j-1}^\Delta, F_{j-1}^\Delta)$ is a PT assessment by the induction hypothesis, $\tilde{\pi}_{j-1}^\Delta(N, D, u)$ is strictly increasing in D by Corollary 1. For $N \geq M_0 - j$ and $u < \xi_j^\Delta(G - (N+j)p)$, we define $\underline{D}^\Delta(N, u + \Delta)$ to be the unique value satisfying

$$\tilde{\pi}_{j-1}^\Delta(N, \underline{D}^\Delta(N, u + \Delta), u) = \frac{v_0}{v-p},$$

which must also be satisfied for $u < \xi_{j'}(G - (N + j')p)$, $N \geq M_0 - j'$, $j' \leq j - 1$ by the induction hypothesis ii). Since $\tilde{\pi}_{j-1}^\Delta$ is increasing in N, D and u , \underline{D}^Δ is decreasing in N and u . Further, for $N = M_0 - j$, we set $\underline{D}^\Delta(N, u + \Delta) = 0$ for $u > \xi_j^\Delta(0)$.

iii) **PT assessment:** We set

$$D_{*,j}^\Delta(N, u) := \underline{D}^\Delta(N, u) \quad \text{for } u \leq \xi_j^\Delta(G - (N + j)p), \text{ and} \\ \text{for } N = M_0 - j, u > \xi_j^\Delta(0),$$

and otherwise, define $D_{*,j}^\Delta(N, u)$ arbitrarily so that it is overall decreasing in N and u . This defines a PT assessment $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$. Note that $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta) = (b_{j-1}^\Delta, D_{+,j-1}^\Delta, F_{j-1}^\Delta)$ for states (N, D, u) with $\underline{M}(D) - N \leq j - 1$ because for all such states $D_{*,j-1}^\Delta(N, u) = D_{*,j}^\Delta(N, u)$.

iv) **Probability of success:** The corresponding probability of success has the following properties:

- $\pi_j^\Delta(N, D, u) = \pi_{j-1}^\Delta(N, D, u)$ for $\underline{M}(D) - N \leq j - 1$ by definition of the corresponding donation thresholds because $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta) = (b_{j-1}^\Delta, D_{+,j-1}^\Delta, F_{j-1}^\Delta)$ for these states and all states (N', D', u') with $N' \geq N$, $D' \geq D$ that can be reached in a continuation game, as they satisfy $\underline{M}(D') - N' \leq j - 1$.
- For $D \geq D_{*,j}^\Delta(N, u + \Delta)$, $\pi_j^\Delta(N, D, u) = \tilde{\pi}_{j-1}^\Delta(N, D, u)$ by Lemma 2 ii), and for $D < D_{*,j}^\Delta(N, u + \Delta)$, $\pi_j^\Delta(N, D, u) < \frac{v_0}{v-p}$ and $\tilde{\pi}_{j-1}^\Delta(N, D, u) < \frac{v_0}{v-p}$ by monotonicity of the probabilities in D . Hence, $\xi_j^\Delta(D)$ satisfies (5). Further, this implies that $\pi_j^\Delta(N, D, u)$ is strictly increasing in u for $D \geq D_{*,j}^\Delta(N, u + \Delta)$, $N + 1 < \underline{M}(D)$. Otherwise, $\pi_j^\Delta(N, D, u) = 1$ or $\pi_j^\Delta(N, D, u)$ is given by (2) which is strictly increasing in u or equal to zero.

v) **Best response:** It is immediate from the construction and because π_j^Δ is increasing in D , that for all (N, D, u) with $N \geq \underline{M}(D) - j$, $\pi_j^\Delta(N, D, u) \geq \frac{v_0}{v-p}$ if and only if $D \geq D_{*,j}^\Delta(N, u + \Delta)$. ■

A.2.2 Taking the continuous time limit

The following lemma implies Proposition 1 ii):

Lemma 6 (Success-maximizing equilibrium limit). *i) The point-wise limit of the donation threshold $\underline{D}(N, u) := \lim_{\Delta \rightarrow 0} \underline{D}^\Delta(N, \lceil \frac{u}{\Delta} \rceil \Delta)$ exists, where $\lceil \frac{u}{\Delta} \rceil \Delta$ is the smallest multiple of Δ that is larger than u . Further, for any $\mathbf{x} = (N, D, u)$ the following point-wise limits exist:*

$$\begin{aligned} b(\mathbf{x}) &:= \lim_{\Delta \rightarrow 0} b^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta), & D_+(\mathbf{x}; w) &:= \lim_{\Delta \rightarrow 0} D_+^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta; w), \\ \xi_j(w) &:= \lim_{\Delta \rightarrow 0} \xi_j^\Delta(w), & F(w; \mathbf{x}) &:= \lim_{\Delta \rightarrow 0} F^\Delta\left(w; (N, D, \lceil \frac{u}{\Delta} \rceil \Delta)\right) \end{aligned} \quad (8)$$

Finally,

$$\pi(N, D, u) := \lim_{\Delta \rightarrow 0} \pi^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta) \text{ uniform in } u \text{ and } D. \quad (9)$$

ii) Proposition 1 ii) holds for this limit.

The proof of this lemma is in the Online Appendix.

A.2.3 Optimality of constructed equilibrium

Proof Outline: Next, we show that the equilibrium constructed in Section A.2.1 maximizes the probability of success and that for any success-maximizing sequence of PT equilibria, the outcome converges point-wise to the same limit as specified in Proposition 1. The proof proceeds in four steps. In **Step 1**, we formulate a relaxed version of the success maximization problem. In **Step 2**, we solve the relaxed problem. In **Step 3** we show that the outcome of the solution is attained by the equilibrium constructed Section A.2.1. In **Step 4** we show convergence as $\Delta \rightarrow 0$.

The key idea of the proof stems from the observation that the donor will always donate enough to reach the goal at the deadline if needed and feasible. Hence, to maximize the probability of success, the exact amount the donor donates during the campaign before the deadline is not important as long as investors keep pledging. To find the PBE outcomes that maximize the probability of success, we consider reduced histories that ignore donation

amounts and only keep track of whether a donation incentivizes the next potential investor to pledge or not. This idea allows us to recast the success maximization problem into one in which we choose probabilities of reaching these reduced histories, rather than choosing over the set of PBEs.

Proof:

Step 1: The relaxed success-maximization problem

Consider a particular assessment $(\tilde{D}_+^\Delta, \tilde{b}^\Delta, \tilde{F}^\Delta)$. Given this assessment, any investor history $h_t^{B,\Delta} = \prod_{s \in \mathbb{T}^\Delta, s \leq t} (N_{s-\Delta}, D_{s-\Delta})$ corresponds to a reduced investor history

$$\tilde{h}_t^B := \prod_{s \in \mathbb{T}^\Delta, s \leq t} (N_{s-\Delta}, b_{s-\Delta}), \quad \text{where} \quad b_{s-\Delta} := \tilde{b}^\Delta \left(\prod_{s' \in \mathbb{T}^\Delta, s' \leq s} (N_{s'-\Delta}, D_{s'-\Delta}) \right),$$

so that instead of the donation $D_{s-\Delta}$, the history records the probability $b_{s-\Delta} \in [0, 1]$ with which an investor arriving in period s pledges on observing cumulative donation amount $D_{s-\Delta}$, and the entire history of donations and pledges. We omit the Δ -superscripts for the reduced histories to simplify notation. Let $\mathcal{R}_{\tilde{b}^\Delta}$ be the mapping so that

$$\mathcal{R}_{\tilde{b}^\Delta} : h_t^{B,\Delta} \mapsto \tilde{h}_t^B$$

as defined above. We will use this mapping in the proof of Proposition 3.

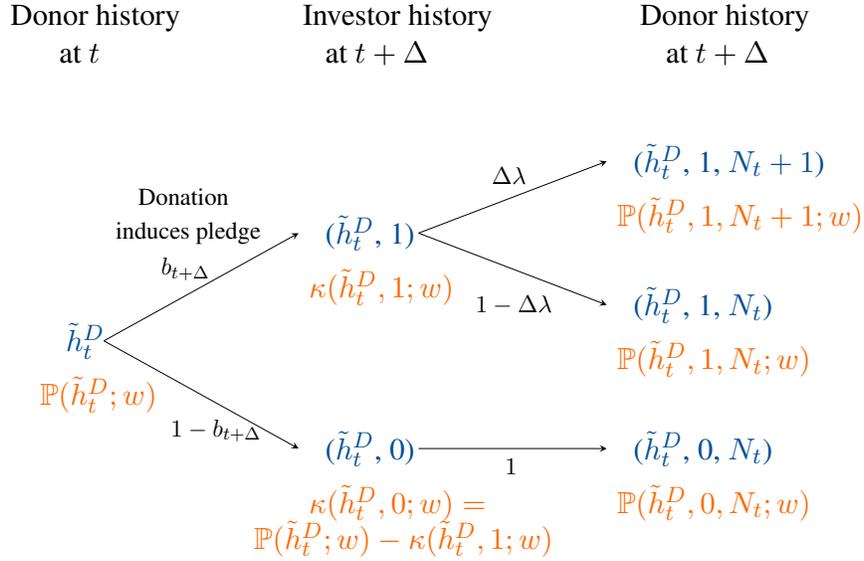
In a platform-optimal equilibrium, the investor always pledges when she is indifferent between pledging and not pledging, so henceforth we assume $b_{s-\Delta} \in \{0, 1\}$. Let the set of such reduced investor histories in period t be $\tilde{\mathcal{H}}_t^B$. Further, let us denote the corresponding set of reduced donor histories in period t by

$$\tilde{\mathcal{H}}_t^D := \left\{ \tilde{h}_t^D = (\tilde{h}_t^B, N_t) \mid \tilde{h}_t^B \in \tilde{\mathcal{H}}_t^B, N_t \in \{N_{t-\Delta}, N_{t-\Delta} + 1\} \right\}.$$

The assessment, the arrival process and distributions of donor valuation define a probability measure \mathbb{P} on the space of outcomes $\prod_{t \in \mathbb{T}^\Delta} (N_t, D_t)$ and hence on $\tilde{\mathcal{H}}_t^B$ and $\tilde{\mathcal{H}}_t^D$. Given this probability space, we define the following probabilities:

- i) $\kappa(\tilde{h}_t^B; w)$ is the probability that $\tilde{h}_t^B \in \tilde{\mathcal{H}}_t^B$ is reached if the donor's valuation is w ;
- ii) $\mathbb{P}(\tilde{h}_t^D; w)$ is the probability that $\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D$ is reached if the donor's valuation is w .

Figure 11: Transitions between reduced histories



Note: The blue brackets represent reduced histories and the orange expressions below the probability of reaching the corresponding reduced history.

Note that this implies that for each w and $t \in \mathbb{T}^\Delta$, we have

$$\sum_{\tilde{h}_t^B \in \tilde{\mathcal{H}}_t^B} \kappa(\tilde{h}_t^B; w) = \sum_{\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D} \mathbb{P}(\tilde{h}_t^D; w) = 1 \text{ and } \mathbb{P}(\tilde{h}_t^D; w) = \kappa(\tilde{h}_t^D, 1; w) + \kappa(\tilde{h}_t^D, 0; w),$$

and in particular

$$\kappa(\tilde{h}_t^D, 1; w) \leq \mathbb{P}(\tilde{h}_t^D; w) \text{ for all } \tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D. \quad (\mathbb{P})$$

Further, the following inter-temporal link between reduced histories must hold,

$$\left. \begin{aligned} \mathbb{P}(\tilde{h}_t^D, 1, N_t + 1; w) &= \Delta\lambda \kappa(\tilde{h}_t^D, 1; w) \\ \mathbb{P}(\tilde{h}_t^D, 1, N_t; w) &= (1 - \Delta\lambda) \kappa(\tilde{h}_t^D, 1; w) \\ \mathbb{P}(\tilde{h}_t^D, 0, N_t; w) &= \mathbb{P}(\tilde{h}_t^D; w) - \kappa(\tilde{h}_t^D, 1; w) \end{aligned} \right\} \text{ for all } \tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D. \quad (\mathbb{P} - t)$$

The reduced histories and probabilities are illustrated in Figure 11. The probabilities of reaching investor histories after which an investor pledges uniquely determines all other probabilities, so we define

$$\tilde{\mathcal{H}}_t^1 := \left\{ \tilde{h}_t^B = (\tilde{h}_{t-\Delta}^D, 1) \mid \tilde{h}_{t-\Delta}^D \in \tilde{\mathcal{H}}_{t-\Delta}^D \right\} \subset \tilde{\mathcal{H}}_t^B.$$

Formally, $\mathbb{P}(0; w) = 1$ and the sequence $\kappa_\Delta(0; w) := ((\kappa(\tilde{h}_t^B; w))_{\tilde{h}_t^B \in \tilde{\mathcal{H}}_t^1})_{t \geq \Delta}$ uniquely define $((\mathbb{P}(\tilde{h}_t^D; w))_{\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D})_{t \geq 0}$ and $((\kappa(\tilde{h}_t^D; 0; w))_{\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D})_{t \geq 0}$. Thus, $(\kappa_\Delta(0; w))_{w \in [0, \infty)}$ determines the outcome of the game and will be the choice variable in the relaxed problem. In order to be able to formulate investor IC constraints after reaching an arbitrary donor history $\tilde{h}_{t-\Delta}^D$, we define *continuation donor histories* at times $t' \geq t$ by

$$\tilde{\mathcal{H}}_{t'}^D(\tilde{h}_{t-\Delta}^D) := \{ \tilde{h}_{t'}^D \in \tilde{\mathcal{H}}_{t'}^D : \text{the first entries of } \tilde{h}_{t'}^D \text{ are } \tilde{h}_{t-\Delta}^D \}.$$

The problem to maximize the probability of success can be written as

$$\begin{aligned} \max_{(\kappa_\Delta(0; w))_{w \in [0, \infty)}} \quad & \sum_{\tilde{h}_{T-\Delta}^D \in \tilde{\mathcal{H}}_{T-\Delta}^D} \Delta \lambda \mathbb{E}^{F_0} [\kappa(\tilde{h}_{T-\Delta}^D, 1; W) \mathbb{1}(G - (N_{T-\Delta} + 1)p \leq W)] + \\ & (1 - \Delta \lambda) \mathbb{E}^{F_0} [\kappa(\tilde{h}_{T-\Delta}^D, 1; W) \mathbb{1}(G - N_{T-\Delta}p \leq W)] + \\ & \mathbb{E}^{F_0} [(\mathbb{P}(\tilde{h}_{T-\Delta}^D; W) - \kappa(\tilde{h}_{T-\Delta}^D, 1; W)) \mathbb{1}(G - N_{T-\Delta}p \leq W)], \end{aligned}$$

subject to $\mathbb{P}(0; w) = 1$, Equation \mathbb{P} , Equation $\mathbb{P} - t$, and for all $\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D$, $t \in \mathbb{T}^\Delta$, $N_t \in \mathbb{N}$, $w \in [0, \infty)$,

$$\frac{\int \overbrace{q_{t+\Delta}(\tilde{h}_t^D, 1, N_{t-\Delta} + 1; W)}^{\text{prob. of success if period-}t \text{ investor pledges}} dF_0(W)}{\int \kappa(\tilde{h}_t^D, 1; W) dF_0(W)} \geq \frac{v_0}{v - p}, \quad (\text{Investor IC})$$

where the unconditional probability of success if a period- t investor pledges after history

\tilde{h}_t^D is given by

$$q_{t+\Delta}(\tilde{h}_{t+\Delta}^D; w) = \sum_{\tilde{h}_{t-\Delta}^D \in \tilde{\mathcal{H}}_{T-\Delta}^D(\tilde{h}_{t+\Delta}^D)} \Delta\lambda \kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}(G - (N_{T-\Delta} + 1)p \leq w) + \\ (1 - \Delta\lambda) \kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}(G - N_{T-\Delta}p \leq w) + \\ (\mathbb{P}(\tilde{h}_{T-\Delta}^D; w) - \kappa(\tilde{h}_{T-\Delta}^D, 1; w)) \mathbb{1}(G - N_{T-\Delta}p \leq w)].$$

This is a relaxed problem because the vectors $(\kappa_\Delta(0; w))_{w \in [0, \infty)}$ that satisfy the above constraints do not necessarily correspond to a PBE. Further, we are ignoring donor incentives by considering reduced histories.

Finally, note that for a PT equilibrium, it must be that for any investor history $(\tilde{h}_{t-\Delta}^D, 1) \in \tilde{\mathcal{H}}_t^1$ there exists $\tilde{D}^*((\mathbb{P}(\tilde{h}_{t-\Delta}^D; w))_w) \geq 0$ such that

$$\kappa(\tilde{h}_{t-\Delta}^D, 1; w) = \begin{cases} \mathbb{P}(\tilde{h}_{t-\Delta}^D; w) & \text{for } w \geq \tilde{D}^*((\mathbb{P}(\tilde{h}_{t-\Delta}^D; w))_w) \\ 0 & \text{otherwise} \end{cases}. \quad (\text{PT-}\kappa)$$

Step 2: Solution to the relaxed problem

In the following, we show any solution satisfies Equation PT- κ . Such κ_Δ with $\tilde{D}^*((\mathbb{P}(\tilde{h}_{t-\Delta}^D; w))_w) = \underline{W}((\mathbb{P}(\tilde{h}_{t-\Delta}^D; w))_w)$ where

$$\underline{W}((\mathbb{P}(\tilde{h}_{t-\Delta}^D; w))_w) := \min \{ \underline{w} \mid (\text{Investor IC}) \text{ is satisfied for } \kappa(\tilde{h}_{t-\Delta}^D, 1; w) = \mathbb{P}(\tilde{h}_{t-\Delta}^D; w) \mathbb{1}(w \geq \underline{w}) \} \quad (\underline{W})$$

is always a solution. We set $\underline{W}((\mathbb{P}(\tilde{h}_t^D; w))_w) = \infty$ if the set on the right-hand side is empty. Further, to establish uniqueness in the limit $\Delta \rightarrow 0$, we show that for any solution satisfying Equation PT- κ it must be that

$$\tilde{D}^*((\mathbb{P}(\tilde{h}_{t-\Delta}^D; w))_w) \in \left[\underline{W}((\mathbb{P}(\tilde{h}_{t-\Delta}^D; w))_w), \max \left\{ G - (N_{t-\Delta} + \frac{T-(t-\Delta)}{\Delta})p, \underline{W}((\mathbb{P}(\tilde{h}_{t-\Delta}^D; w))_w) \right\} \right], \quad (\tilde{D}^*\text{-Region})$$

where $G - \left(N_{t-\Delta} + \frac{T-(t-\Delta)}{\Delta}\right)p$ is the amount that the donor needs to donate even if a investor arrives and pledges in every future period. Note that as $\Delta \rightarrow 0$, $G - \left(N_t + \frac{T-t}{\Delta}\right)p \rightarrow -\infty$.

We show that the solution must satisfy Equation PT- κ with (\tilde{D}^* -Region) by contradiction. Consider an arbitrary solution κ_Δ^* and corresponding \mathbb{P}^* such that there is at least one history in which it does not satisfy Equation PT- κ with (\tilde{D}^* -Region). Consider the latest period \bar{t} in time after which Equation PT- κ with (\tilde{D}^* -Region) is satisfied for all histories, and consider a period $\bar{t} - \Delta$ history $\tilde{h}_{\bar{t}-\Delta}^D$ such that $\kappa(\tilde{h}_{\bar{t}-\Delta}^D, 1; w)$ does not satisfy Equation PT- κ with (\tilde{D}^* -Region). Then, the probability of success conditional on reaching history $\tilde{h}_{\bar{t}}^B = (\tilde{h}_{\bar{t}-\Delta}^D, 1)$ given by $\frac{q_t(\tilde{h}_{\bar{t}-\Delta}^D, 1, N_{\bar{t}-\Delta+1}; w)}{\kappa(\tilde{h}_{\bar{t}-\Delta}^D, 1; w)}$ is increasing in w and independent of the choice of $\kappa(\tilde{h}_{\bar{t}-\Delta}^D, 1; w)$. Let

$$c(\tilde{h}_{\bar{t}-\Delta}^D) := \int \kappa^*(\tilde{h}_{\bar{t}-\Delta}^D, 1; W) dF_0(W).$$

We now construct a κ'_Δ such that the objective function is higher than with κ_Δ^* , while keeping $\int \kappa'(\tilde{h}_{\bar{t}-\Delta}^D, 1; W) dF_0(W) \leq c(\tilde{h}_{\bar{t}-\Delta}^D)$ in all histories. To this end, let $\underline{W}_c(\tilde{h}_{\bar{t}-\Delta}^D)$ be the uniquely defined by¹⁷

$$\int_{\underline{W}_c(\tilde{h}_{\bar{t}-\Delta}^D)}^{\infty} \mathbb{P}^*(\tilde{h}_{\bar{t}-\Delta}^D; W) dF_0(W) = c(\tilde{h}_{\bar{t}-\Delta}^D).$$

Since $\frac{q_t(\tilde{h}_{\bar{t}-\Delta}^D, 1, N_{\bar{t}-\Delta+1}; w)}{\kappa(\tilde{h}_{\bar{t}-\Delta}^D, 1; w)}$ is increasing in w , $\kappa(\tilde{h}_{\bar{t}-\Delta}^D, 1; w) = \mathbb{P}^*(\tilde{h}_{\bar{t}-\Delta}^D; w)\mathbb{1}(w \geq \underline{W}_c(\tilde{h}_{\bar{t}-\Delta}^D))$ satisfies Equation Investor IC. We set $\kappa'(\hat{h}_t^D; w) := \kappa^*(\hat{h}_t^D; w)$ for all histories \hat{h}_t^D at $t < \bar{t} - \Delta$ and all histories $\hat{h}_t^D \notin \tilde{\mathcal{H}}_t^1(\tilde{h}_{\bar{t}-\Delta}^D)$, $t \geq \bar{t} - \Delta$. Further, let

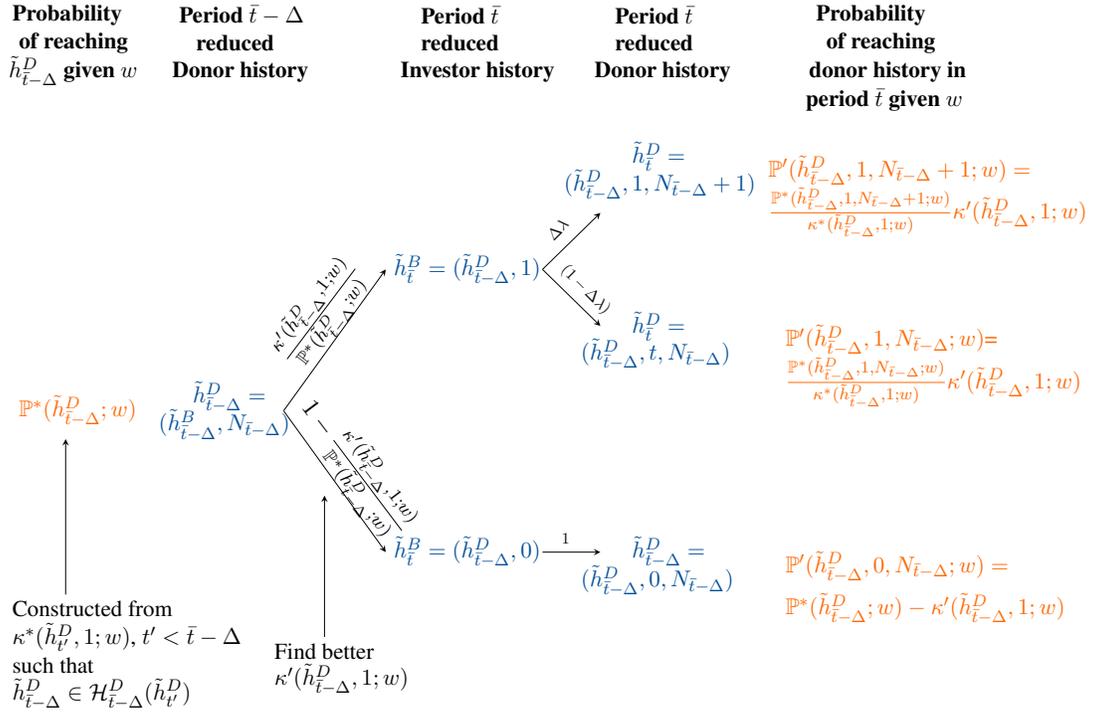
$$\kappa'(\tilde{h}_{\bar{t}-\Delta}^D, 1; w) := \begin{cases} \mathbb{P}^*(\tilde{h}_{\bar{t}-\Delta}^D; w) & \text{for } w \geq \underline{W}_c(\tilde{h}_{\bar{t}-\Delta}^D) \\ 0 & \text{otherwise,} \end{cases}$$

and for histories $\hat{h}_t \in \tilde{\mathcal{H}}_t^1(\tilde{h}_{\bar{t}-\Delta}^D)$ where $t > \bar{t} - \Delta$, we set $\kappa'(\hat{h}_t^D; w) := \mathbb{P}'(\hat{h}_t^D; w) \frac{\kappa^*(\hat{h}_t^D; w)}{\mathbb{P}^*(\hat{h}_t^D; w)}$ so that all constraints remain satisfied and the transition probabilities remain unchanged.

¹⁷Uniqueness follows because for all $t \geq \bar{t}$, $\kappa(\hat{h}_t^D, 1; w)$ satisfies Equation PT- κ .

Figure 12 illustrates the transitions. In the objective function, this κ' achieves states with

Figure 12: Schematic illustration of transition probabilities



Note: The blue brackets represent reduced histories and the orange expressions the probability of reaching the corresponding reduced histories given a realized w .

higher $N_{T-\Delta}$ more frequently, so κ' yields strictly higher profits than κ^* . Thus, any solution κ_Δ^* must satisfy Equation PT- κ with $(\tilde{D}^*$ -Region) almost surely.

Step 3: Implementation by equilibrium

Finally, we show that the optimal solution is achieved by the PBE constructed in Proposition 1. To this end, it is useful to write the probability of success for donor type w after a

history $\tilde{h}_{t-\Delta}^D$ recursively as a function of $\kappa_t(\tilde{h}_{t-\Delta}^D; w)$ and $\mathbb{P}(\tilde{h}_{t-\Delta}^D; w) > 0$:

$$\begin{aligned}
& \Pi_{t-\Delta}(\kappa_t(\tilde{h}_{t-\Delta}^D; w), \mathbb{P}(\tilde{h}_{t-\Delta}^D; w); w) = \\
& \underbrace{\Delta\lambda}_{\text{arrival}} \underbrace{\frac{\kappa(\tilde{h}_{t-\Delta}^D, 1; w)}{\mathbb{P}(\tilde{h}_{t-\Delta}^D; w)}}_{\text{investor pledges}} \Pi_t(\kappa_{t+\Delta}(\tilde{h}_{t-\Delta}^D, 1, N_{t-\Delta} + 1; w), \mathbb{P}(\tilde{h}_{t-\Delta}^D, 1, N_{t-\Delta} + 1; w); w) \\
& + \underbrace{(1-\Delta\lambda)}_{\text{no arrival}} \underbrace{\frac{\kappa(\tilde{h}_{t-\Delta}^D, 1; w)}{\mathbb{P}(\tilde{h}_{t-\Delta}^D; w)}}_{\text{investor pledges}} \Pi_t(\kappa_{t+\Delta}(\tilde{h}_{t-\Delta}^D, 1, N_{t-\Delta}; w), \mathbb{P}(\tilde{h}_{t-\Delta}^D, 1, N_{t-\Delta}; w); w) \quad (\text{W-II}) \\
& + \underbrace{\left(1 - \frac{\kappa(\tilde{h}_{t-\Delta}^D, 1; w)}{\mathbb{P}(\tilde{h}_{t-\Delta}^D; w)}\right)}_{\text{investor does not pledge}} \Pi_t(\kappa_{t+\Delta}(\tilde{h}_{t-\Delta}^D, 0, N_{t-\Delta}; w), \mathbb{P}(\tilde{h}_{t-\Delta}^D, 0, N_{t-\Delta}; w); w),
\end{aligned}$$

and for $\mathbb{P}(\tilde{h}_{t-\Delta}^D; w) = 0$, we set $\Pi_{t-\Delta}(\kappa_t(\tilde{h}_{t-\Delta}^D; w), \mathbb{P}(\tilde{h}_{t-\Delta}^D; w); w) = 0$ without loss. Then, we can write the Investor IC constraint as follows:

$$\frac{\int \underbrace{\kappa(\tilde{h}_t^B; W)}_{\text{prob. of reaching } \tilde{\mathbf{h}}_t^B} \underbrace{\Pi_t(\kappa_{t+\Delta}(\tilde{h}_t^B, N_{t-\Delta} + 1; W), \mathbb{P}(\tilde{h}_t^B; W); W)}_{\text{prob. of success if period-}t \text{ investor pledges}} dF_0(W)}{\int \kappa(\tilde{h}_t^B; W) dF_0(W)} \geq \frac{v_0}{v-p}. \quad (\text{Investor IC'})$$

Consider the PT equilibrium $(D_+^\Delta, b^\Delta, (F^\Delta(\cdot|\mathbf{x}))_x)$ from the proof of Proposition 1. This assessment induces a probability measure \mathbb{P} on outcomes and a corresponding systems of probabilities $\kappa(\tilde{h}_t^D, 1; w)$ and $\mathbb{P}(\tilde{h}_t^D; w)$ over reduced histories, as defined in the **Step 1**. Consider any on-path investor history in the last period $h_T^{B,\Delta} = \prod_{s \in \mathbb{T}^\Delta, s \leq T} (N_{s-\Delta}, D_{s-\Delta})$. The PBE specifies that investors pledge if and only if the probability of success is at least $\frac{v_0}{v-p}$. In addition, in the preceding period, unless success is already guaranteed, donors with $w \geq \underline{D}^\Delta(N_{T-\Delta}, \Delta)$ donate $\max\{D_{T-2\Delta}, \underline{D}^\Delta(N_{T-\Delta}, \Delta)\}$. This makes the next investor just indifferent between pledging and not pledging if such a donation amount exists and $\underline{D}^\Delta(N_{T-\Delta}, \Delta) = \overline{W}$ otherwise.

Therefore, for any on-path history $h_{T-\Delta}^{D,\Delta} = \left(\prod_{s \in \mathbb{T}^\Delta, s \leq T-\Delta} (N_{T-\Delta}, D_{s-\Delta}), N_{T-\Delta} \right)$, the induced probabilities over reduced histories satisfy

$$\kappa(\tilde{h}_{T-\Delta}^D, \mathbf{1}; w) = \mathbb{P}(\tilde{h}_{T-\Delta}^D; w) \text{ if and only if } w \geq \underline{D}^\Delta(N_{T-\Delta}, \Delta).$$

Notice that since $\underline{D}^\Delta(N_{T-\Delta}, \Delta)$ is calculated using the indifference condition for investors, $\pi^\Delta(N, D, u)$ is increasing in D , and F^Δ is a truncation given by Equation PT-belief, this $\underline{D}^\Delta(N_{T-\Delta}, \Delta)$ is exactly $\underline{W}((\mathbb{P}(\tilde{h}_{T-\Delta}^D; w))_w, N_{T-\Delta})$ defined in Equation \underline{W} in the solution to the relaxed problem when we write the expression for the indifference condition as in Equation Investor IC'. Analogous arguments apply to any history $h_t^{D,\Delta} = \left(\prod_{s \in \mathbb{T}^\Delta, s \leq t} (N_{s-\Delta}, D_{s-\Delta}), N_t \right)$. Therefore, the PBE assessment from the proof of Proposition 1 induces exactly $(\kappa_\Delta^*(0; w))_w$ and it achieves the optimum in the relaxed problem. Hence, $(\kappa_\Delta^*(0; w))_w$ is platform-optimal in the full class of PBEs.

Step 4: Uniqueness of limits

We have shown in **Step 2** that solutions to the reduced problem satisfy Equation PT- κ with Equation \tilde{D}^* -Region. Now, for a given t if Δ is sufficiently small, then $G - \left(N + \frac{T-t}{\Delta}\right)p < 0$, so any sequence of outcomes converges point-wise to the equilibrium outcome attained by the Markov equilibrium constructed in **Step 1**.

A.3 Proof of Proposition 2 (Success-Minimizing Equilibrium)

We first show in Section A.3.1, we characterize a PT equilibrium for each Δ . In Section A.3.2, we show that the limit of these equilibria as $\Delta \rightarrow 0$ exists, and is as specified in Proposition 2. Section A.3.3 establishes that this PBE minimizes the probability of success.

A.3.1 Characterization of PT equilibrium

Lemma 7 (Success-minimizing equilibrium). *Given any $\Delta > 0$, a PT assessment $(b^\Delta, D_+^\Delta, F^\Delta)$ with donation threshold $\overline{D}^\Delta(N, u) \in [0, G - (N + 1)p)$ constitutes a PT equilibrium.*

We denote the corresponding probability of success from the investor's perspective in state N, D, u if the investor contributes by $\bar{\pi}(N, D, u)$.

Proof. Note that the donation threshold is well-defined in Section 3.4 (unlike in the construction of the success-maximizing equilibrium): $\bar{D}^\Delta(N, u + \Delta) := \max\{G - (j - 1)p - Np, 0\}$ for $u \in (\bar{\xi}_{j-1}^\Delta, \bar{\xi}_j^\Delta]$. This defines strategies and beliefs of the PT assessment. It is immediate that $\bar{D}^\Delta(N, u)$ is strictly decreasing in N and u as long as $\bar{D}^\Delta(N, u) > 0$, weakly decreasing otherwise, $\bar{D}^\Delta(N, u) \in [0, G - (N + 1)p)$, and $\bar{D}^\Delta(N, u) = 0$ for $(N + 1)p \geq G$. It only remains to show that the investor strategies are optimal in every state (N, D, u) , since the donor is best responding by Lemma 3. We show this by induction in $j = \underline{M}(D) - N$ and for each j by backward-induction in u .

(a) Induction start ($j \leq 1 \Leftrightarrow D \geq G - (N + 1)p$): For $N \geq \underline{M}(D) - 1$, the campaign is either already successful, or an investor can complete the campaign. Hence $\pi^\Delta(N, D, u) = 1$ and $b^\Delta(N, D, u) = 1$ for all $u \in \mathbb{U}^\Delta$, and $D \in [0, \bar{W}]$ in any equilibrium. Note that $\bar{\xi}_1^\Delta = 0$ and $D_+^\Delta(N, D, u; w) = D$.

(b) Induction hypothesis ($j' \leq j - 1$): Assume that we have shown that the above strategy profiles are best responses for investors for all states (N, D, u) with $N = \underline{M}(D) - j'$ with $j' \leq j - 1$.

(c) Induction step ($j - 1 \rightsquigarrow j, j \geq 2$): Consider an investor in state (N, D, u) with $N = \underline{M}(D) - j$. If $D < \bar{D}^\Delta(N, u + \Delta)$, then $u < \bar{\xi}_j^\Delta$, and the belief system dictates that an investor assigns a probability of success equal to

$$\pi^\Delta(\underline{M}(D) - j, D, u) = \mathbb{P}(\tau_1^u \leq T - \bar{\xi}_{j-1}^\Delta, \dots, \tau_{j-2}^u \leq T - \bar{\xi}_2^\Delta, \tau_{j-1}^u \leq T) < \frac{v_0}{v - p},$$

where τ_i^u is the arrival time of the i -th investor after period u . The inequality follows directly from the definition of $\bar{\xi}_j^\Delta$. Hence, $b^\Delta(\underline{M}(D) - j, D, u) = 0$ is optimal for the investor.

If $D \geq \bar{D}^\Delta(N, u + \Delta)$, then $u \geq \bar{\xi}_j^\Delta$; by the induction hypothesis, we have

$$\begin{aligned} \pi^\Delta(N, D, u) &= \mathbb{E}^{F_0} \left[\sum_{i=1}^{\max\{(u - \bar{\xi}_{M(W) - (N+1)}^\Delta)/\Delta, 0\}} (1 - \Delta\lambda)^{i-1} \Delta\lambda \right. \\ &\quad \left. \pi^\Delta(N+1, \max\{D, \bar{D}^\Delta(N+1, u - \Delta(i-1))\}, u - \Delta i) \right. \\ &\quad \left. + (1 - \Delta\lambda)^{u/\Delta} \mathbb{1}(W \geq G - (N+1)p) \mid W \geq D \right] \\ &> \mathbb{P}(\tau_1^u \leq T - \bar{\xi}_{j-1}^\Delta, \dots, \tau_{j-2}^u \leq T - \bar{\xi}_2^\Delta, \tau_{j-1}^u \leq T) \geq \frac{v_0}{v-p}, \end{aligned}$$

where the last inequality follows because $u \geq \bar{\xi}_j^\Delta$ and the definition of $\bar{\xi}_j^\Delta$ via Proposition 1.

Hence, indeed $b^\Delta(M(D) - j, D, u) = 1$. ■

A.3.2 Taking the continuous time limit

We know from Proposition 1 that the point-wise limits $\bar{\xi}_j := \lim_{\Delta \rightarrow 0} \bar{\xi}_j^\Delta$ and

$$\bar{D}(N, u) := \lim_{\Delta \rightarrow 0} \bar{D}^\Delta(N, \lceil \frac{u}{\Delta} \rceil \Delta) = \max\{G - (j-1)p - Np, 0\} \text{ for } u \in (\bar{\xi}_j, \bar{\xi}_{j-1}]$$

exist. This implies that the point-wise limits $D_+(N, D, u; w) := \lim_{\Delta \rightarrow 0} D_+^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta; w)$,

$b(N, D, u) = \lim_{\Delta \rightarrow 0} b^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta)$, and $F(w; (N, D, u)) = \lim_{\Delta \rightarrow 0} F^\Delta(w; (N, D, \lceil \frac{u}{\Delta} \rceil \Delta))$ exist.

This concludes the proof of Proposition 2 ii).

A.3.3 Minimization of probability of success

Next, we show that the equilibrium just constructed minimizes the probability of success in the class of PBE. To this end, we consider an arbitrary PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$. We show

by backward induction in t that for any investor history $h_t^{B,\Delta} = \prod_{s \in \mathbb{T}^\Delta, s \leq t} (N_{s-\Delta}, D_{s-\Delta})$ an equilibrium investor history must satisfy

$$D_{t-\Delta} > \bar{D}^\Delta(N_{t-\Delta}, T - (t - \Delta)) \Rightarrow \tilde{b}^\Delta(h_t^{B,\Delta}) = 1. \quad (10)$$

(a) **Induction start** ($t = T$): $\bar{D}^\Delta(N, \Delta) = G - (N - 1)p$, so Equation 10 is satisfied for any PBE.

(b) **Induction hypothesis** ($s \geq t$): Assume that Equation 10 is satisfied for any history $h_s^{B, \Delta}$ with $s \geq t$.

(c) **Induction step** ($t \rightsquigarrow t - \Delta$): For an arbitrary history $h_{t-\Delta}^{B, \Delta}$, from an investor's perspective in period $t - \Delta$, the probability of success after a contribution is bounded from below by $\bar{\pi}(N_{t-2\Delta}, D_{t-2\Delta}, T - (t - \Delta))$ by the induction hypothesis. Thus, the investor must contribute if $\bar{\pi}(N_{t-2\Delta}, D_{t-2\Delta}, T - (t - \Delta)) \geq \frac{v_0}{v-p}$. Since for the constructed PT equilibrium,

$$D > \bar{D}^\Delta(N, T - 2t) \Rightarrow \bar{\pi}(N, D, T - (t - \Delta)) \geq \frac{v_0}{v-p},$$

we have $D_{t-2\Delta} > \bar{D}^\Delta(N_{t-2\Delta}, T - (t - 2\Delta)) \Rightarrow \tilde{b}^\Delta(h_{t-\Delta}^{B, \Delta}) = 1$.

Finally, if Equation 10 is satisfied, then the probability of success in the PBE must be at least as in the constructed PT equilibrium, since investors contribute whenever they contribute in the PT equilibrium and the donor contributes up to his wealth at the deadline in any PBE whenever necessary for success.

A.4 Proof of Proposition 3 (Donor-Preferred Equilibrium)

Proof Outline: Given any assessment, we use the same class of reduced histories and systems of probabilities $\kappa(\tilde{h}_t^B; w)$ and $\mathbb{P}(\tilde{h}_t^D, N_t; w)$, as in the proof of Proposition 1. Just as in the equilibrium that maximizes the probability of success, in a donor-preferred equilibrium, the investor always pledges when she is indifferent between pledging and not pledging, so we can assume that $b_s \in \{0, 1\}$ for all histories. The induced probability measure \mathbb{P} allows us to define $(\kappa_\Delta(0; w))_w$, which determines the outcome of the game, except for the donation amount.

The proof proceeds in four steps. **Step 1** establishes that donor-preferred equilibrium outcomes can be attained by PBE in a smaller class of assessments. In **Step 2**, we formulate a relaxed donor problem (analogously to Proposition 1). In **Step 3**, we solve the donor's

problem and show that the success-maximizing solution also corresponds to a solution of the donor's problem. We also prove that all solutions that are PT equilibria converge to the same limit as $\Delta \rightarrow 0$. Finally, in **Step 4**, we verify that the donor strategy constructed in **Step 3** of the proof of Proposition 1 is consistent with the donor-preferred solution.

Proof:

Step 1: Limiting the class of assessments

To find a donor-preferred equilibrium, we first show (in Lemmata 8 and 9 below) that donor-preferred equilibrium outcomes can be attained by PBE in a smaller class of assessments. First, at histories at which investors are induced to pledge, all donor types that donate positive amounts make the same cumulative donation. Second, if a donor does not incentivize pledging, he donates nothing. Within the class of assessments satisfying these two properties, the mapping from reduced histories to donations becomes unique, a fact we use when we formulate the donor's maximization problem.

Lemma 8. *For any donor-preferred PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, there exists a donor-preferred PBE $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ such that*

- i) *both assessments generate the same probability measures $(\kappa_\Delta(\mathbf{0}; w))_w$,*
- ii) *for each $h_t^{D,\Delta}$, there exists a $D_*(h_t^{D,\Delta}) \in \mathbb{R}$ such that*

$$\begin{aligned} \hat{D}_+^\Delta(h_t^{D,\Delta}; w) &= \begin{cases} \tilde{D}_+(h_t^{D,\Delta}; w) & \text{if } \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+(h_t^{D,\Delta}; w)) = 0 \\ D_*(h_t^{D,\Delta}) & \text{if } \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+(h_t^{D,\Delta}; w)) = 1 \end{cases}, \text{ and} \\ \hat{b}^\Delta(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) &= \begin{cases} 1 & \text{if } D_{t-\Delta} = D_*(h_{t-\Delta}^{D,\Delta}) \\ 0 & \text{otherwise} \end{cases}. \end{aligned} \quad (11)$$

Proof of Lemma 8. Given a donor-preferred PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, define

$$D_*(h_t^{D,\Delta}) := \inf \{ \tilde{D}_+^\Delta(h_t^{D,\Delta}; w) \mid \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+^\Delta(h_t^{D,\Delta}; w)) = 1 \},$$

which is the smallest donation amount that incentivizes pledging at a history $h_t^{D,\Delta}$. Donating this amount is feasible for all donor types $w \geq D_*(h_t^{D,\Delta})$. Moreover, it is consistent with play on equilibrium path. In particular, donating this amount is feasible for all types that incentivize pledging after $h_t^{D,\Delta}$ in $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$.

Then, define a new assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ where \hat{b}^Δ and \hat{D}_+^Δ are given by Equation 11. On equilibrium path, $\hat{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ is derived by Bayes' rule. Off path, if $D_{t-\Delta} > D_*(h_{t-\Delta}^{D,\Delta})$, then let $\hat{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ be such that it is optimal for the investor not to pledge (e.g. $\hat{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) = \mathbb{1}(w = 0)$), and let $\hat{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) = \tilde{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ otherwise.

Note that the strategies are such that $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ and $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ result in the same probability measures $(\kappa_\Delta(0; w))_w$, i.e., the same purchasing outcome after any realization of arrivals and donor type. The donation amount with $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ is by definition weakly lower after any arrival and donor type realization. Hence, if $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ is a PBE, then it must be donor-preferred. It remains to be shown that $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ is a PBE.

First, consider donor incentives. Given a PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, a donor type w with $\tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+(h_t^{D,\Delta}; w)) = 0$ does not find it profitable to incentivize pledging after a history $h_t^{D,\Delta}$. Pledging can be incentivized by donations of at least $D_*(h_{t-\Delta}^{D,\Delta})$. Hence, also with assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$, deviating to incentivize pledging cannot be profitable. For a donor type w with $\tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+(h_t^{D,\Delta}; w)) = 1$, it is optimal to donate in the PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$. Given the assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$, the donor can donate weakly less and still incentivize pledging, but the donor has a larger set of feasible donations in any future period. Thus, no donor type has an incentive to deviate given the assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$.

Next, consider investor incentives. Investors at a history $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ where $D_{t-\Delta} < D_*(h_{t-\Delta}^{D,\Delta})$ have identical beliefs about donor types in both assessments, and the purchasing outcome is also identical as argued above. Hence, the probability of success is the same across assessments and an investor with such a history must prefer not to pledge given the assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ because $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ is a PBE. Investors at a history $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$, where $D_{t-\Delta} = D_*(h_{t-\Delta}^{D,\Delta})$, believe that they face donor types that they would face if they

played a PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, and if they were at any of the histories $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ after which an investor pledges. Hence, investors must prefer to pledge at a history $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$, where $D_{t-\Delta} = D_*(h_{t-\Delta}^{D,\Delta})$, given the assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$. A history $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ with $D_{t-\Delta} > D_*(h_{t-\Delta}^{D,\Delta})$ is now off equilibrium path for assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$, and we assumed that \hat{F} is such that the investor does not wish to pledge in this case.

It follows that $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ is a PBE.

Hence, to find a donor-preferred equilibrium, it suffices to restrict attention to assessments $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ such that for any $h_t^{D,\Delta}$, there exists a $D_*(h_t^{D,\Delta}) \in \mathbb{R}$ with

$$\tilde{D}_+^\Delta(h_t^{D,\Delta}; w) = D_*(h_t^{D,\Delta}), \text{ whenever } \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+^\Delta(h_t^{D,\Delta}; w)) = 1, \quad (12)$$

and \tilde{b}^Δ as is defined in Equation 11. Indeed, the success-maximizing equilibrium constructed in Proposition 1 is in this class.

Lemma 9. *For any donor-preferred PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ for which the donor strategy satisfies Equation 12 and investor strategy Equation 11, there exists a donor-preferred PBE $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ so that*

- i) both assessments generate the same probability measures $(\kappa_\Delta(0; w))_w$,
- ii) $\hat{b}^\Delta = \tilde{b}^\Delta$ and for each $h_t^{D,\Delta}$,

$$\hat{D}_+^\Delta(h_t^{D,\Delta}; w) = \begin{cases} D_{t-\Delta} & \text{if } \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+^\Delta(h_t^{D,\Delta}; w)) = 0 \\ \tilde{D}_+^\Delta(h_t^{D,\Delta}; w) & \text{if } \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+^\Delta(h_t^{D,\Delta}; w)) = 1 \end{cases}. \quad (13)$$

Proof of Lemma 9. Given the donor-preferred PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ satisfying Equation 12, let $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ be given by Equation 13, $\hat{b}^\Delta = \tilde{b}^\Delta$, and $\tilde{F}^\Delta(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$, so that it is consistent with Bayes' rule on equilibrium path, and $\tilde{F}^\Delta(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) = \hat{F}^\Delta(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ off path. Then, it follows immediately that the two assessments generate the same outcomes and hence, the same probability measures $(\kappa_\Delta(0; w))_w$. It remains

to show that $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ constitutes a PBE. The donor does not have a profitable deviation after histories after which the investor is incentivized to pledge as the donor plays exactly the same strategy as in $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$. Whenever the donor does not incentivize pledging, the donor cannot have a profitable deviation since incentivizing pledging is not profitable for $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, and moreover, $\hat{D}_+^\Delta(h_t^{D,\Delta}; w) = D_{t-\Delta} \leq \tilde{D}_+^\Delta(h_t^{D,\Delta}; w)$ implies that every donor type w has a weakly larger set of feasible donations in the future under \hat{D}_+^Δ than under \tilde{D}_+^Δ . Each investor is also best-responding as she pledges after the same histories in both assessments, and whenever she does not pledge, her belief is a mixture of beliefs after histories after which she did not pledge in $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$.

Hence, in the following, we restrict attention to assessments $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ that satisfy Equation 13 and Equation 12. The donor strategy in such assessments only depends on the reduced history $\mathcal{R}_{\tilde{b}^\Delta}(h_t^{D,\Delta})$, so we can define $\mathcal{D}(\tilde{h}_t^{D,\Delta}) := D_*(h_t^{D,\Delta})$ for $\tilde{h}_t^{D,\Delta} = \mathcal{R}_{\tilde{b}^\Delta}(h_t^{D,\Delta})$. Indeed, the platform-optimal equilibrium from Proposition 1 satisfies Equation 13.

Step 2: Relaxed donor problem.

Consider an arbitrary assessment $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ that satisfies Equation 13. Recall that, analogously to Proposition 1, we can define reduced histories, systems of probabilities $\kappa(\tilde{h}_t^B; w)$, $\mathbb{P}(\tilde{h}_t^D, N_t; w)$, the mapping $\mathcal{R}_{\tilde{b}^\Delta}$ that maps general histories to the corresponding reduced history, and $\mathcal{D}(\tilde{h}_t^{D,\Delta})$ the corresponding donation threshold for reduced history $\tilde{h}_t^{D,\Delta}$. In order to formulate the donor's payoff, we write for $t' \leq t$ that $\tilde{h}_{t'}^D \subseteq \tilde{h}_t^D$ if $\tilde{h}_{t'}^D$ is a sub-history that leads to \tilde{h}_t^D . Then, let

$$\bar{\mathcal{D}}(\tilde{h}_t^D) := \max_{\substack{\tilde{h}_{t'}^D \subseteq \tilde{h}_t^D, \\ t' \leq t, b_{t'}=1}} \mathcal{D}(\tilde{h}_{t'}^D)$$

be the cumulative donations after period t if the donor follows a donation strategy as specified in Equation 13, so that he donates in all periods t' in which the reduced history \tilde{h}_t^D dictates that $b_{t'} = 1$.

The donor's problem can be written as

$$\begin{aligned} \max_{\substack{(\kappa_\Delta(0;w))_w, \\ (\mathcal{D}(\tilde{h}_t^D))_{\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D, t \in \mathbb{T}^\Delta}} \quad & \sum_{\tilde{h}_{T-\Delta}^D \in \tilde{\mathcal{H}}_{T-\Delta}^D} \Delta\lambda \mathbb{E}^{F_0} [\kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}(G - (N_{T-\Delta} + 1)p \leq W)(W - \tilde{\mathcal{D}}(\tilde{h}_{T-\Delta}^D))] + \\ & (1 - \Delta\lambda) \mathbb{E}^{F_0} [\kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}(G - N_{T-\Delta}p \leq W)(W - \tilde{\mathcal{D}}(\tilde{h}_{T-\Delta}^D))] + \\ & \mathbb{E}^{F_0} [(\mathbb{P}(\tilde{h}_{T-\Delta}^D; w) - \kappa(\tilde{h}_{T-\Delta}^D, 1; w)) \mathbb{1}(G - N_{T-\Delta}p \leq W)(W - \tilde{\mathcal{D}}(\tilde{h}_{T-2\Delta}^D))], \end{aligned}$$

subject to $\mathbb{P}(0; w) = 1$, Equation \mathbb{P} , Equation $\mathbb{P} - t$, and for all $\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D, t \in \mathbb{T}^\Delta, N_t \in \mathbb{N}, w \in [0, \infty)$ Equation Investor IC, and given

$$\begin{aligned} d_t(\tilde{h}_t^D; w) := & \sum_{\tilde{h}_{T-\Delta}^D \in \tilde{\mathcal{H}}_{T-\Delta}^D(\tilde{h}_t^D)} \Delta\lambda \kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}(G - (N_{T-\Delta} + 1)p \leq w)(w - \tilde{\mathcal{D}}(\tilde{h}_{T-\Delta}^D)) + \\ & (1 - \Delta\lambda) \kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}(G - N_{T-\Delta}p \leq w)(w - \tilde{\mathcal{D}}(\tilde{h}_{T-\Delta}^D)) + \\ & (\mathbb{P}(\tilde{h}_{T-\Delta}^D; w) - \kappa(\tilde{h}_{T-\Delta}^D, 1; w)) \mathbb{1}(G - N_{T-\Delta}p \leq w)(w - \tilde{\mathcal{D}}(\tilde{h}_{T-2\Delta}^D)), \end{aligned}$$

we can formulate a donor incentive compatibility constraint for all $\tilde{h}_{t-\Delta}^D \in \tilde{\mathcal{H}}_{t-\Delta}^D$

$$\begin{aligned} & d_t(\tilde{h}_{t-\Delta}^D, 0, N_{t-\Delta}; w) < \Delta\lambda d_t(\tilde{h}_{t-\Delta}^D, 1, N_{t-\Delta} + 1; w) + (1 - \Delta\lambda) d_t(\tilde{h}_{t-\Delta}^D, 1, N_{t-\Delta}; w) \\ \Rightarrow & \kappa(\tilde{h}_{t-\Delta}^D, 1; w) = \mathbb{P}(\tilde{h}_{t-\Delta}^D; w). \end{aligned} \tag{Donor IC}$$

This donor IC constraint puts a lower bound on donations as it imposes that the donor must donate whenever it is optimal to do so, but does not impose that the donor does not donate if it is optimal not to donate. Hence, this donor problem is a relaxed maximization problem.

We denote a solution to the above problem by κ_Δ^{**} and $((\mathcal{D}_{**}(\tilde{h}_t^D, 1))_{\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D})_{t \geq 0}$. Recall that the solution that we presented to the platform's relaxed problem was denoted κ_Δ^* .

Step 3: Solution to the relaxed problem

Next, we show the following two statements:

- i) Any solution of this relaxed problem must satisfy in Equation PT- κ with Equation \tilde{D}^* -Region;

ii) κ_Δ as in Equation PT- κ with $\tilde{D}^*((\mathbb{P}(\tilde{h}_t^D; w))_w, N_t) = \underline{W}((\mathbb{P}(\tilde{h}_t^D; w))_w)$ is a solution.

Given these two statements, it follows immediately that in the limit as $\Delta \rightarrow 0$, the outcome is unique by the proof of Proposition 1.

Analogously to the proof of Proposition 1, we show that the solution must satisfy Equation PT- κ with $(\tilde{D}^*$ -Region) by contradiction. Consider an arbitrary solution κ_Δ^{**} , corresponding \mathbb{P}^{**} and $((\mathcal{D}_{**}(\tilde{h}_t^D, 1))_{\tilde{h}_t^D \in \tilde{\mathcal{H}}_t^D})_{t \geq 0}$ that does not satisfy Equation PT- κ with $(\tilde{D}^*$ -Region). Consider the latest period \bar{t} in time after which Equation PT- κ with $(\tilde{D}^*$ -Region) is satisfied for all histories, and consider a period $\bar{t} - \Delta$ history $\tilde{h}_{\bar{t}-\Delta}^D$ such that $\kappa^{**}(\tilde{h}_{\bar{t}-\Delta}^D, 1; w)$ does not satisfy Equation PT- κ with $(\tilde{D}^*$ -Region). Then, the probability of success conditional on reaching history $\tilde{h}_{\bar{t}}^B = (\tilde{h}_{\bar{t}-\Delta}^D, 1)$ given by $\frac{q_t(\tilde{h}_{\bar{t}-\Delta}^D, 1, N_{\bar{t}-\Delta+1}; w)}{\kappa(\tilde{h}_{\bar{t}-\Delta}^D, 1; w)}$ is increasing in w and independent of the choice of $\kappa(\tilde{h}_{\bar{t}-\Delta}^D, 1; w)$. We can also again define $c(\tilde{h}_{\bar{t}-\Delta}^D) := \int \kappa^{**}(\tilde{h}_{\bar{t}-\Delta}^D, 1; W) dF_0(W)$. Note that by Equation Donor IC, it must be for $t \geq \bar{t}$, $\tilde{D}^{**}(\tilde{h}_t^D) = \underline{W}((\mathbb{P}(\tilde{h}_t^D; w))_w)$. Further, by Equation Donor IC,

$$\begin{aligned} \tilde{D}^{**}(\tilde{h}_{\bar{t}}^D) &= \min\{\tilde{D}^*(\tilde{h}_{\bar{t}}^D) \mid d_{\bar{t}+\Delta}(\tilde{h}_{\bar{t}}^D, 0, N_{\bar{t}}; w) \geq \\ &\quad \Delta \lambda d_{\bar{t}+\Delta}(\tilde{h}_{\bar{t}}^D, 1, N_{\bar{t}} + 1; w) + (1 - \Delta \lambda) d_{\bar{t}}(\tilde{h}_{\bar{t}}^D, 1, N_{\bar{t}}; w) \\ &\quad \text{for all } w \text{ such that } \kappa(\tilde{h}_{\bar{t}}^D, 1; w) < \mathbb{P}(\tilde{h}_{\bar{t}}^D; w)\}. \end{aligned}$$

We now construct a κ'_Δ such that the donor's objective function is higher than with κ_Δ^* , while keeping $\int \kappa'(\tilde{h}_{\bar{t}-\Delta}^D, 1; W) dF_0(W) \leq c(\tilde{h}_{\bar{t}-\Delta}^D)$ in all histories. Analogously to Proposition 1, we can uniquely define $\underline{W}_c(\tilde{h}_{\bar{t}-\Delta}^D)$ by

$$\int_{\underline{W}_c(\tilde{h}_{\bar{t}-\Delta}^D)}^{\infty} \mathbb{P}^{**}(\tilde{h}_{\bar{t}-\Delta}^D; W) dF_0(W) = c(\tilde{h}_{\bar{t}-\Delta}^D).$$

Since $\frac{q_t(\tilde{h}_{\bar{t}-\Delta}^D, 1, N_{\bar{t}-\Delta+1}; w)}{\kappa(\tilde{h}_{\bar{t}-\Delta}^D, 1; w)}$ is increasing in w , $\kappa(\tilde{h}_{\bar{t}}^D, 1; w) = \mathbb{P}^{**}(\tilde{h}_{\bar{t}}^D; w) \mathbb{1}(w \geq \underline{W}_c(\tilde{h}_{\bar{t}-\Delta}^D))$ satisfies Equation Investor IC. We set $\kappa'(\tilde{h}_t^D; w) := \kappa^{**}(\tilde{h}_t^D; w)$ for all histories \tilde{h}_t^D at $t <$

$\bar{t} - \Delta$ and all histories $\hat{h}_t^D \notin \tilde{\mathcal{H}}_t^1(\tilde{h}_{t-\Delta}^D)$, $t \geq \bar{t} - \Delta$. Further, let

$$\kappa'(\tilde{h}_{t-\Delta}^D, \mathbf{1}; w) := \begin{cases} \mathbb{P}^{**}(\hat{h}_{t-\Delta}^D; w) & \text{for } w \geq \underline{W}_c(\tilde{h}_{t-\Delta}^D) \\ 0 & \text{otherwise,} \end{cases}$$

and for histories $\hat{h}_t \in \tilde{\mathcal{H}}_t^1(\tilde{h}_{t-\Delta}^D)$ where $t > \bar{t} - \Delta$, we set $\kappa'(\hat{h}_t^D; w) := \mathbb{P}'(\hat{h}_t^D; w) \frac{\kappa^*(\hat{h}_t^D; w)}{\mathbb{P}^*(\hat{h}_t^D; w)}$ so that all constraints remain satisfied and the transition probabilities remain unchanged.

Further, the lowest donation amount by Equation Donor IC is then

$$\tilde{D}'(\tilde{h}_t^D) = \underline{W}_c(\tilde{h}_{t-\Delta}^D).$$

In the objective function, this κ' achieves states with higher $N_{T-\Delta}$ more frequently and $\tilde{D}'(\tilde{h}_t^D) < \tilde{D}^{**}(\tilde{h}_t^D)$, so κ' yields strictly higher donor payoffs than κ^{**} . Thus, any solution κ_Δ^* must satisfy Equation PT- κ with $(\tilde{D}^*$ -Region) almost surely.

Step 4: Implementation by equilibrium

We have already shown in Proposition 1 that $(\kappa_\Delta^*(0; w))_w$ is induced by the constructed assessment and established that the wealth threshold $\underline{D}^\Delta(N, u)$ corresponds to $\underline{W}(\tilde{h}_t^D)$ if there is a history $h_t^{D,\Delta}$ with $\mathcal{R}_{b\Delta}(h_t^{D,\Delta}) = \tilde{h}_t^D$ and $u = T - t$, $N_t = N$. This concludes the proof.

A.5 Proof of Proposition 4 (Investor-Preferred Equilibrium)

Finding an equilibrium that maximizes the sum of investor surplus is a complex problem since each investor's decision has externalities both on past investors who have pledged already, and future investors. For sufficiently small period length Δ , we separately construct a PT equilibrium yielding higher investor surplus than the success-maximizing equilibrium and one yielding higher surplus than the success-minimizing equilibrium.

We start with the construction of a PT equilibrium with higher investor surplus than the success-minimizing equilibrium for a general contribution game. First, note that if the

realized donor valuation was known to be $w \in [G - 2p, G - p)$, then the campaign requires exactly two investor pledges to succeed. Since the second investor can always lead the campaign to succeed, the first investor pledges if and only if $(v - p)(1 - (1 - \Delta\lambda)^{u/\Delta}) = v_0$. Conditional on such a W , investor surplus is maximized if the first investor pledges if

$$(v - p) \underbrace{(1 - (1 - \Delta\lambda)^{u/\Delta})}_{\text{prob. of at least one more arrival}} - v_0 + \underbrace{(v - p - v_0)\lambda u}_{\text{externality on future investors}} \geq 0 \iff \frac{(1 - \Delta\lambda)^{u/\Delta}}{1 + \lambda u} \leq 1 - \frac{v_0}{v - p},$$

because the expected number of arrivals time u is $\frac{u}{\Delta}\Delta\lambda$. Denote the smallest $u \in \mathbb{U}^\Delta$ such that the above inequality is satisfied \bar{u} , i.e., the inequality is equivalent to $u \geq \bar{u}$ (noting that $\frac{(1 - \Delta\lambda)^{u/\Delta}}{1 + \lambda u}$ is decreasing in u). Note that $\bar{\xi}_2^\Delta(G - 2p) > \bar{u}$ because $\bar{\xi}_2^\Delta(G - 2p)$ solves $(v - p)(1 - (1 - \Delta\lambda)^{u/\Delta}) = v_0$. We define a donation threshold \bar{D}_ϵ^Δ as follows:

- $\bar{D}_\epsilon^\Delta(N, u) := \bar{D}^\Delta(N, u)$ for $N > 0$, and for $N = 0$ with $u \in [0, \bar{u}) \cup [\bar{\xi}_2^\Delta(G - 2p), \infty)$,
- $\bar{D}_\epsilon^\Delta(0, u) := \bar{D}^\Delta(0, u) - \epsilon = G - p - \epsilon$ for $u \in [\bar{u}, \bar{\xi}_2^\Delta(G - 2p))$.

Consider a sufficiently small $\Delta > 0$. Then, the PT assessment with donation threshold $\bar{D}_\epsilon^\Delta(N, u)$ for small $\epsilon > 0$ still defines an equilibrium: All investors' incentives to pledge except the ones for a first investor arriving at $u \in [\bar{u}, \bar{\xi}_2^\Delta(G - 2p))$ do not change. If the first investor arrives at $u \in [\bar{u}, \bar{\xi}_2^\Delta(G - 2p))$, and the donor has wealth $W \geq G - p - \epsilon$, then the donor can contribute $G - p - \epsilon = \bar{D}_\epsilon^\Delta(0, u) - \epsilon$ and incentivize the investor to pledge. Indeed the probability of success is simply a truncation of F_0 at $G - p - \epsilon$ which is close to 1 for small ϵ , so

$$(1 - (1 - \Delta\lambda)^{u/\Delta}) + (1 - \Delta\lambda)^{u/\Delta} \frac{1 - F_0(G - p)}{1 - F_0(G - p - \epsilon)} \geq v_0.$$

If the donor has valuation $W < G - p - \epsilon$, then the first investor does not want to contribute as she knows that $W < G - p$, by definition of $\bar{\xi}_2^\Delta(G - p) > \bar{\xi}_2^\Delta(G - 2p)$. Furthermore, by definition of \bar{u} , this PT equilibrium makes investors collectively better off.

Next, we construct a PT equilibrium with higher investor surplus than the success-maximizing equilibrium. We define a donation threshold $\underline{D}_{\epsilon, \delta}^{\Delta}$ for small $\epsilon > 0$, $\delta > \Delta$ as follows:

- $\underline{D}_{\epsilon, \delta}^{\Delta}(N, u) := \underline{D}^{\Delta}(N, u)$ for $N > 0$ and $(N, u) = (0, u)$ with $u \geq \delta$, and
- $\underline{D}_{\epsilon, \delta}^{\Delta}(0, u) := \underline{D}^{\Delta}(0, u) + \epsilon$ for $u < \delta$.

This defines a PT equilibrium because the incentive to pledge only changes if the first investor arrives in $[0, \delta)$ and if the donor valuation is in $W \in [\underline{D}^{\Delta}(0, u), \underline{D}_{\epsilon, \delta}^{\Delta}(0, u) + \epsilon)$. The probability of success in the success-maximizing equilibrium satisfies

$$\underline{\pi}^{\Delta}(0, \underline{D}^{\Delta}(0, u), u) = \frac{v_0}{v - p},$$

so if investors knew $W \in [\underline{D}_{\epsilon, \delta}^{\Delta}(0, u), \underline{D}^{\Delta}(0, u) + \epsilon)$, then the probability of success is smaller than $\frac{v_0}{v - p}$ for sufficiently small ϵ , so it is optimal for the investor not to pledge. If $W \geq \underline{D}_{\epsilon, \delta}^{\Delta}(0, u) + \epsilon$, the donor can keep incentivizing investors to pledge in states $(0, u)$, $u < \delta$. Furthermore, the equilibrium outcome of this PT equilibrium yields higher investor surplus than the success-maximizing equilibrium since if $W \in [\underline{D}_{\epsilon, \delta}^{\Delta}(0, u), \underline{D}^{\Delta}(0, u) + \epsilon)$, $N = 0$ and $u < \delta$, then contributing creates collective investor surplus of less than

$$(v - p)(1 - (1 - \Delta\lambda)^{\delta/\Delta}) + (v - p)\lambda\delta \xrightarrow{\Delta \rightarrow 0} (v - p)(1 - e^{-\lambda\delta} + \lambda\delta)$$

and not contributing a surplus of $v_0(1 + \lambda\delta)$. Hence, for δ sufficiently small (and Δ sufficiently small), there is a PT equilibrium with higher investor surplus than the surplus-maximizing equilibrium.

A.6 Alternative Campaign Designs

Recall the relaxed problem in the optimality proof of Proposition 1. The control variables are simply probabilities of reaching reduced histories given realized w that ignore donation

amounts and donor incentives. The objective is to maximize the probability of success subject to investor participation. As a result, this relaxed problem can also be viewed as a constrained information or mechanism design problem that maximizes the probability of success.

Formally, let an allocation be a sequence $(a_t)_{t \in \mathbb{T}} \in [0, 1]^{\mathbb{T}^\Delta}$ that determines whether a period- t investor (if she arrives) takes the outside option ($a_t = 0$) or stays in the game ($a_t = 1$), and a variable \bar{a} that determines whether the project is successful. An allocation is feasible if, given the realized arrival process A_t , $\bar{a} = 1 \Leftrightarrow \sum_t (A_t - A_{t-\Delta}) a_t p + D_T \geq G$.

Let us first assume that the mechanism designer knows the donor's type. Given her beliefs, an investor in period t can decide whether to participate in the mechanism or not. And, beliefs are formed based on the chosen probabilities of reaching reduced histories. Note that we cannot allow for transfers between investors. Then, it follows that the relaxed problem in the proof of Proposition 1 corresponds to an information design problem where the designer chooses an optimal dynamic signal structure representing the information released about W over time. This shows that, for example, revealing the donor's valuation prior to the campaign is not profit maximizing.

Alternatively, we can consider a mechanism design problem, assuming that the donor's valuation is private information. Consider direct mechanisms where the donor sends a message $m \in [0, \infty)$ about his type. An investor in period t can decide whether to participate in the mechanism or not. Then, a direct, donor-incentivizing mechanism is given by a message strategy of the donor, a participation strategy of investors, an allocation mapping that maps messages and participation decisions to feasible allocations, and a donor transfer $D \in [0, \infty)$. Again, we cannot allow for transfers between investors. Then, it follows that the relaxed problem in the proof of Proposition 1 is a relaxed problem of the mechanism design problem that finds the success-maximizing, donor-incentivizing mechanism. This shows that, for example, allowing the donor to only donate before or after the crowdfunding stage is not profit maximizing.

A.7 Parameter Estimates

Table 6: Donor Valuation Parameter Estimates

Parameter	Valuation Equation		Selection Equation	
	Param Est.	Std. Error	Param Est.	Std. Error
Corr. Coef.	μ	0.708	0.005	
Std. Dev. Unobs.	τ	2.293	0.003	1
Comics		0.603	0.037	0.181
Crafts		0.047	0.031	0.352
Dance		0.843	0.083	-1.023
Design		0.704	0.028	0.735
Fashion		0.029	0.025	0.511
Film & Video		0.501	0.027	-0.587
Food		0.396	0.029	-0.097
Games		0.302	0.025	0.648
Intercept		-0.880	0.035	-0.640
Journalism		-0.036	0.057	-0.541
Love		1.433	0.050	-0.358
Music		0.302	0.027	-0.754
Photography		0.256	0.042	0.243
Publishing		0.451	0.025	-0.174
Technology		0.293	0.025	0.601
Theater		0.997	0.072	-0.931
log(G)		0.324	0.004	0.952
log(B)				-0.993
Number of Parameters.		37		
Number of Obs.		13,863		
Log Likelihood		-17,321.86		

Note: Parameter estimates of donor valuations accounting for sample selection. We assume a bivariate normal distribution on the unobserved components. Standard errors computed using 1,000 bootstrap samples.

Table 7: Investor Arrival Process Parameter Estimates

Parameter		Arrival Equation		Selection Equation	
		Param Est.	Std. Error	Param Est.	Std. Error
Corr. Coef.	$\operatorname{atanh}(\rho)$	-0.016	0.001		
Std. Dev. Unobs.	$\log(\sigma)$	0.057	0.000		
Comics		0.653	0.002	0.248	0.003
Crafts		0.050	0.005	-0.531	0.004
Dance		-0.200	0.004	0.476	0.005
Design		0.101	0.003	-0.131	0.003
Fashion		-0.110	0.002	-0.214	0.002
Film & Video		-0.196	0.002	0.044	0.002
Food		-0.123	0.002	-0.168	0.003
Games		0.359	0.003	-0.453	0.003
Intercept		-6.155	0.004	-0.847	0.003
Journalism		-0.034	0.005	-0.347	0.006
Love		0.295	0.001	1.244	0.002
Music		0.114	0.002	0.367	0.002
Photography		0.106	0.003	-0.013	0.004
Publishing		0.116	0.002	-0.057	0.002
Technology		-0.279	0.003	-0.528	0.003
Theater		-0.016	0.003	0.556	0.004
Time-Linear		-0.190	0.000	0.031	0.000
Time-Quadratic		0.007	0.000	-0.000	0.000
$\log(G)$		0.730	0.001	-0.117	0.000
$\log(D)$				9.474	0.014
Corr. Coef.	ρ	-0.016	0.001		
Std. Dev. Unobs.	σ	1.057	0.034	1	—
Number of Parameters.		41			
Number of Obs.		845,643			
Log Likelihood		-385,554.946			

Note: Parameter estimates of arrival process accounting for sample selection. We assume a bivariate normal distribution on the unobserved components. Standard errors computed using 1,000 bootstrap samples.

Aiming for the Goal: Online Appendix

B Additional Proofs and Examples

B.1 Proof of Lemma 6

(a) Induction start ($j \leq 1 \Leftrightarrow D \geq G - (N + 1)p$): For $j \leq 1$ and $\mathbf{x} = (N, D, u)$ with $\underline{M}(D) - N \leq 1$, it is immediate that the point-wise limits in (8) exist and are given by

$$\begin{aligned} b(\mathbf{x}) &:= \lim_{\Delta \rightarrow 0} b^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta) \equiv 1 & D_+(\mathbf{x}; w) &:= \lim_{\Delta \rightarrow 0} D_+^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta; w) = D \\ \xi_j(w) &:= \lim_{\Delta \rightarrow 0} \xi_j^\Delta(w) \equiv 0 & F(w; \mathbf{x}) &:= \lim_{\Delta \rightarrow 0} F^\Delta\left(w; (N, D, \lceil \frac{u}{\Delta} \rceil \Delta)\right) = \frac{F_0(w) - F_0(D)}{1 - F_0(D)} \mathbb{1}(w \geq D), \end{aligned}$$

where $\lceil \frac{u}{\Delta} \rceil \Delta$ is the smallest multiple of Δ that is larger than u . Further, $\pi(\mathbf{x}) := \lim_{\Delta \rightarrow 0} \pi^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta) = 1$ uniformly in $D \geq G - (N + 1)p$ and u .

(b) Induction hypothesis ($j - 1$): We assume that the point-wise limits (8) exist for all $\mathbf{x} = (N, D, u)$ with $N \geq \underline{M}(D) - (j - 1)$ and $j' \leq j - 1$, where for $w < G - p$:

$$\pi(\underline{M}(w) - j', w, \xi_{j'}(w)) = \frac{v_0}{v - p}.$$

Further, assume that the point-wise limit $\underline{D}(N, u) := \lim_{\Delta \rightarrow 0} \underline{D}^\Delta(N, \lceil \frac{u}{\Delta} \rceil \Delta)$ exists for $u \leq \xi_{j-1}(G - (N + j - 1)p)$. If $\pi(N, 0, u) < \frac{v_0}{v - p}$, then \underline{D} is strictly decreasing in N and u , and

$$\pi(N, \underline{D}(N, u), u) = \frac{v_0}{v - p}.$$

Further, the uniform limit in $D \geq G - (N + j - 1)p$ and u , $\pi(N, D, u) := \lim_{\Delta \rightarrow 0} \pi^\Delta(N, D, u)$, exists and is equal to

$$\mathbb{E}^{F_0} \left[\int_0^{\max\{u - \xi_{\underline{M}(W) - (N+1)}(W)\}} \lambda e^{-\lambda s} \pi(N + 1, \max\{D, \underline{D}(N + 1, u - s)\}, u - s) ds \Big| W \geq D \right]. \quad (14)$$

Finally, $\pi(N, D, u)$ is strictly increasing in N, D, u .

(c) Induction step ($j-1 \rightsquigarrow j, j \geq 2$): Consider a state (N, D, u) with $N \geq \underline{M}(D) - j$, i.e.,

$$G - (N + j)p \leq D.$$

L.1) *Uniform convergence (in D and u) of $\tilde{\pi}^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta)$ for $D \geq G - (N + j)p$:*

Recall that the auxiliary probability of success is given by

$$\begin{aligned} & \lim_{\Delta \rightarrow 0} \tilde{\pi}^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta) = \\ & \lim_{\Delta \rightarrow 0} \mathbb{E}^{F_0} \left[\sum_{i=1}^{\max\{\lceil \frac{u}{\Delta} \rceil \Delta - \xi_{\underline{M}(W) - (N+1)}^\Delta(W), 0\} / \Delta} (1 - \Delta\lambda)^{i-1} \Delta\lambda \right. \\ & \left. \left(\pi^\Delta(N + 1, D, \Delta(\lceil \frac{u}{\Delta} \rceil - i)) \mathbb{1}(\Delta(\lceil \frac{u}{\Delta} \rceil - i) \geq \xi_{j'-1}^\Delta(D)) + \frac{v_0}{v-p} \mathbb{1}(\Delta(\lceil \frac{u}{\Delta} \rceil - i) < \xi_{j'-1}^\Delta(D)) \right) \right. \\ & \left. + (1 - \Delta\lambda)^{u/\Delta} \mathbb{1}(W \geq G - (N + 1)p) \middle| W \geq D \right], \end{aligned}$$

where $j' := \underline{M}(D) - N \leq j$. The uniform convergence of $\pi^\Delta(N + 1, D, u')$ in D (by the induction hypothesis) and the Arzelà-Ascoli Theorem imply that the family of functions $D \mapsto \pi^\Delta(N + 1, D, u)$ is equicontinuous with respect to Δ . Hence, we may replace π^Δ by π . Finally, because $\lim_{\Delta \rightarrow 0} \xi_{j'-1}^\Delta(w) = \xi_{j'-1}(w)$, the dominated convergence theorem allows us to conclude that

$$\begin{aligned} \tilde{\pi}_j(N, D, u) &:= \lim_{\Delta \rightarrow 0} \tilde{\pi}_j^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta) = \mathbb{E}^{F_0} \left[\int_0^{\max\{u - \xi_{\underline{M}(W) - (N+1)}(W), 0\}} \lambda e^{-\lambda s} \right. \\ & \left. \left(\pi(N + 1, D, u - s) \mathbb{1}(u - s \geq \xi_{j'-1}(D)) + \frac{v_0}{v-p} \mathbb{1}(u - s < \xi_{j'-1}(D)) \right) ds \right. \\ & \left. + e^{-\lambda u} \mathbb{1}(W \geq G - (N + 1)p) \middle| W \geq D \right]. \end{aligned}$$

Note that $\tilde{\pi}^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta)$ indeed converges uniformly in $D \geq G - (N + j)p$ for fixed u because the sum is bounded by one, F_0 is (uniformly) continuous on $[0, G]$, and

$F_0(G) < 1$. Then, since

$$\pi(\underbrace{M(D) - (j' - 1)}_{N+1}, D, \xi_{j'-1}(D)) = \frac{v_0}{v-p},$$

for $u' < \xi_{j'-1}(D)$, $D < \underline{D}(N+1, u')$ $\pi(N+1, \underline{D}(N+1, u'), u') = \frac{v_0}{v-p}$ and for $u' \geq \xi_{j'-1}(D)$, $D \geq \underline{D}(N+1, u')$. Hence, we have:

$$\begin{aligned} \tilde{\pi}(N, D, u) &:= \lim_{\Delta \rightarrow 0} \tilde{\pi}^\Delta(N, D, \lfloor \frac{u}{\Delta} \rfloor \Delta) = \mathbb{E}^{F_0} \left[\int_0^{\max\{u - \xi_{\underline{M}(W) - (N+1)}(W), 0\}} \lambda e^{-\lambda s} \right. \\ &\left. \pi(N+1, \max\{D, \underline{D}(N+1, u-s)\}, u-s) ds + e^{-\lambda u} \mathbb{1}(W \geq G - (N+1)p) \Big| W \geq D \right]. \end{aligned} \quad (15)$$

L.2) *Continuity and strict monotonicity of $\tilde{\pi}$ in $D \geq G - (N+j)p$ and u* : First, $\tilde{\pi}(N, D, u)$ is continuous in D and u because $\tilde{\pi}_j(N+1, D, u)$ is continuous in D and u , $\underline{D}(N+1, u)$ is continuous in u by the induction hypothesis and because F_0 is continuous.

Furthermore, $\tilde{\pi}_j(N, D, u)$ is strictly increasing in $D \geq G - (N+j)p$ because $\tilde{\pi}(N+1, D, u)$ is weakly increasing in D by the induction hypothesis and $\frac{1}{1-F_0(D)}$ is strictly increasing.

Now the integrand is strictly positive as long as $u > \xi_{\underline{M}(w) - (N+1)}(w)$. Hence, $\tilde{\pi}(N, D, u)$ is strictly increasing in $u > \xi_{\underline{M}(w) - (N+1)}(w)$ because $\tilde{\pi}_j(N+1, D, u)$ is weakly increasing in u by the induction hypothesis and because $u - \xi_{\underline{M}(w) - (N+1)}(w)$ is strictly increasing in u .

L.3) *Point-wise convergence of $\underline{D}^\Delta(N, \lfloor \frac{u}{\Delta} \rfloor \Delta)$ and $D_{+,j}^\Delta(N, D, \lfloor \frac{u}{\Delta} \rfloor \Delta; W)$* : First, note that if $\tilde{\pi}_j^\Delta(N, 0, \lfloor \frac{u}{\Delta} \rfloor \Delta) \geq \frac{v_0}{v-p}$ then $\tilde{\pi}_j(N, 0, u) \geq \frac{v_0}{v-p}$ and hence, $\underline{D}(N, u) := \lim_{\Delta \rightarrow 0} \underline{D}^\Delta(N, \lfloor \frac{u}{\Delta} \rfloor \Delta) = 0$. If $\tilde{\pi}_j^\Delta(N, 0, u) < \frac{v_0}{v-p}$, then $\tilde{\pi}(N, 0, u) \leq \frac{v_0}{v-p}$. Then, since $\tilde{\pi}(N, D, u)$ is continuous and strictly increasing in D , there is a unique solution $D'(N, u)$ to

$$\tilde{\pi}_j(N, D'(N, u), u) = \frac{v_0}{v-p}.$$

Since $\tilde{\pi}^\Delta(N, D, \lfloor \frac{u}{\Delta} \rfloor \Delta)$ converges uniformly, we have $\underline{D}(N, u) := \lim_{\Delta \rightarrow 0} \underline{D}^\Delta(N, \lfloor \frac{u}{\Delta} \rfloor \Delta) =$

$D'(N, u)$. It follows immediately that for all $u > 0$,

$$\begin{aligned} D_+(N, D, u; w) &:= \lim_{\Delta \rightarrow 0} D_{+,j}^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta; w) \\ &= \lim_{\Delta \rightarrow 0} \min \{ \max \{ D, \underline{D}^\Delta(N, \lceil \frac{u}{\Delta} \rceil \Delta) \}, w \} \\ &= \min \{ \max \{ D, \underline{D}(N, u) \}, w \}. \end{aligned}$$

L.4) *Point-wise convergence of $b_j^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta)$* : Note that $b_j^\Delta(N, D, \lceil \frac{u}{\Delta} \rceil \Delta) = 1$ if $D \geq \underline{D}^\Delta(N, (\lceil \frac{u}{\Delta} \rceil \Delta + 1)\Delta)$ and $b_j^\Delta(N, D, (\lceil \frac{u}{\Delta} \rceil \Delta + 1)\Delta) = 0$ otherwise. Since $\lim_{\Delta \rightarrow 0} \underline{D}^\Delta(N, (\lceil \frac{u}{\Delta} \rceil \Delta + 1)\Delta) = \underline{D}(N, u)$, $b_j^\Delta(N, D, u)$ converges point-wise to

$$\lim_{\Delta \rightarrow 0} b_j^\Delta(N, D, u) = \begin{cases} 1 & \text{if } D \geq \underline{D}(\underline{M}(D) - (j-1), u) \\ 0 & \text{if } D < \underline{D}(\underline{M}(D) - (j-1), u) \end{cases}.$$

L.5) *Point-wise convergence of $\xi_j(w)$ and $\pi(\underline{M}(w) - j', w, \xi_{j'}(w)) = \frac{v_0}{v-p}$* : If $\hat{\pi}^\Delta(\underline{M}(w) - j, w, 0) \geq \frac{v_0}{v-p}$, then it follows immediately that $\xi_j^\Delta(w) = 0$. If $\hat{\pi}^\Delta(\underline{M}(w) - j, w, 0) < \frac{v_0}{v-p}$, it follows that $\xi_j^\Delta(w) > 0$ and

$$\begin{cases} \hat{\pi}^\Delta(\underline{M}(w) - j, W, \xi_j^\Delta(w)) \geq \frac{v_0}{v-p} \\ \hat{\pi}^\Delta(\underline{M}(w) - j, W, \xi_j^\Delta(w) - \Delta) < \frac{v_0}{v-p}. \end{cases}$$

Furthermore, since $\hat{\pi}(\underline{M}(w) - j, w, u)$ is continuous and strictly increasing in u for $u \geq \xi_{j-1}(\overline{W})$ and weakly increasing for $u < \xi_{j-1}(\overline{W})$, there is a unique solution $\xi'(w)$ to

$$\hat{\pi}(\underline{M}(w) - j, W, \xi'(w)) = \frac{v_0}{v-p}.$$

Hence, as $\Delta \rightarrow 0$, it must be that $\lim_{\Delta \rightarrow 0} \xi_j^\Delta(w) = \xi'(w)$.

L.6) *Point-wise convergence of $F^\Delta(w; (\underline{M}(D) - j, D, \lceil \frac{u}{\Delta} \rceil \Delta))$* : It follows immediately

from point-wise convergence of $\underline{D}^\Delta(\underline{M}(D) - j, \lceil \frac{u}{\Delta} \rceil \Delta)$ that

$$F(w; (\underline{M}(D) - j, D, u)) := \lim_{\Delta \rightarrow 0} F^\Delta \left(w; (\underline{M}(D) - j, D, \lceil \frac{u}{\Delta} \rceil \Delta) \right) \\ = \begin{cases} \frac{F_0(w) - F_0(D)}{1 - F_0(D)} \mathbb{1}(w \geq D) & \text{if } D \geq \underline{D}(\underline{M}(D) - j, u) \\ \mathbb{1}(w \geq D) & \text{otherwise} \end{cases}.$$

L.7) $\pi(N, D, u)$ is strictly increasing in N , D , and u , as long as $G - (N + 1)p > D \geq \underline{D}(N, u)$: By Definition 2, $\underline{D}(N, u) \geq \underline{D}(N + 1, u - \Delta) \geq \underline{D}(N + 1, u)$ and $\underline{D}(N, u) \geq \underline{D}(N = 1, u)$. An analogous argument to Lemma 2 iii) and iv) implies monotonicity in N, D, u .

L.8) $\underline{D}(N, u)$ is strictly decreasing in N and u , as long as $\pi(N, 0, u) < \frac{v_0}{v-p}$: Strict monotonicity of $\underline{D}(N, u)$ in N and u follows from the strict monotonicity properties in N, D , and u of $\tilde{\pi}(N, D, u)$ and because $\tilde{\pi}(N, \underline{D}(N, u), u) = \frac{v_0}{v-p}$ for $\pi(N, 0, u) < \frac{v_0}{v-p}$.

L.9) $\xi_j(w)$ is strictly increasing in j as long as $\xi_j(w) > 0$.

Since $\pi(N + 1, w, \xi_{j-1}(w)) = \frac{v_0}{v-p}$ and $\pi(N, D, u)$ is strictly increasing in N , $\xi_j(w) > \xi_{j-1}(w)$.

B.2 Application: Crowdfunding

A widely-mentioned benefit of crowdfunding is that it enables potential investors to learn about product quality from the behavior of other investors. In this section, we illustrate how social learning interacts with the signaling incentive of the donor by presenting a 2-period example. We highlight two insights. First, in the presence of social learning the donor is less effective in solving the coordination problem. Second, our analysis is robust to some amount of social learning.

Let $q \in \{0, 1\}$ denote the unknown quality of the product. All players (the donor and investors) share the prior that $q = 1$ with probability $\mu_0 \in (0, 1)$. We view q as the inherent quality of the product or an unknown common value component of demand. In order to

keep the example simple, we assume that the quality of the product only affects investors' payoffs but not the donor's payoff. Investors value a product of quality q at $v(q) = v \cdot q$. So, if an investor pledges, she gets payoff $vq - p$ if the campaign is successful and zero otherwise. If she does not pledge, she receives the outside option v_0 . As before, the donor values a successful campaign at $w \sim F_0$. He receives a payoff $w - D_T$ if the campaign succeeds, and zero otherwise. In the following, we set $v = 3$, $p = 1$, $v_0 = 1$ and $1 - F_0(0.5) = 0.3$. For simplicity let us define $\phi := \frac{\mu_0}{1 - \mu_0}$.

In every period $t = 1, 2$, an investor arrives with probability $\Delta\lambda = 0.9$. On arrival, each investor privately observes a signal $s \in \{0, 1\}$. For simplicity, we consider a "bad news" signal process: An investor who receives a bad signal $s = 0$ knows with certainty that quality is low ($q = 0$). Specifically, we set $\Pr(s = 1|q = 1) = 1$ and $\Pr(s = 1|q = 0) = 0.5$.

First, we consider $G = 1.5$, so that the campaign is successful if at least two investors pledge or if one investor pledges and the donor valuation w is greater or equal to 0.5.¹⁸ An investor in period $t = 2$ can socially learn only if the period-1 investor's strategy is to pledge if $s = 1$ and not to pledge if $s = 0$. In that case, the posterior belief of a period-2 investor if period-1 investor has pledged is by Bayes' rule

$$\mu_2(1) = \frac{\mu_0}{\mu_0 + (1 - \mu_0) \cdot 0.25} = \frac{4}{4 + \phi^{-1}}$$

and if no pledge occurred in period 1, it is

$$\mu_2(0) = \frac{\mu_0 \cdot 0.1}{\mu_0 \cdot 0.1 + (1 - \mu_0)(0.1 + 0.9 \cdot 0.5) \cdot 0.5} = \frac{4}{4 + 11\phi^{-1}}.$$

Let us assume that ϕ is such that $3\mu_2(1) \geq p + v_0 = 2$, so that the second investor with a positive signal always pledges after a pledge in the first period. Let us further assume that $3\mu_2(0) < 2$, so that the second investor never pledges if no pledge occurred in the first period. Hence, $2 \geq \phi^{-1} > 2/11$.

¹⁸The campaign can also succeed if no investor pledges and the donor valuation exceeds the goal amount, but this case is irrelevant for strategic pledging incentives of investors.

In the first period, an investor with a positive signal pledges if she believes that given cumulative donations D , the donor valuations are distributed according to $w \sim F(\cdot|D)$ if

$$\frac{4}{4+2\phi^{-1}}(0.9+0.1(1-F(0.5|D)))(3-1) - \frac{2\phi^{-1}}{4+2\phi^{-1}}(0.45+0.55(1-F(0.5|D))) \geq 1$$

The left-hand side is decreasing in $1-F(0.5|D)$ if $\phi^{-1} > 8/11$. Furthermore, let

$$\phi^{-1} \leq \frac{2((0.9+0.1(1-F_0(0.5))2-1)}{0.45+0.55(1-F_0(0.5))+1} \approx 1.07 \quad (16)$$

to make it worthwhile for the investor to pledge absent donations. Hence, e.g., for $\phi^{-1} = 0.8 > 8/11$, in the success-maximizing equilibrium the donor optimally donates nothing until the deadline. The campaign succeeds if either $w > 1.5$ or if two investors with a high signal realization arrive.

If the scope of social learning is small, e.g. $\phi^{-1} = 0.5 < 8/11$, then a PT equilibrium in which the donor donates just enough to make the next investor buy exists. Hence, our analysis is robust to some amount of social learning.

The example highlights several new forces: First, increasing donations is less effective in increasing the probability of success because it also increases the probability of buying the product when the quality is actually low. Second, the benefit from pledging might even be decreasing in cumulative donations.

B.3 Application: Industrial Policy

We first consider the setting in which the government's payoff is given by $(W - D_T)\mathbb{1}(R_T \geq G)$. Because there is uncertainty about the goal, participants only know if the goal is reached once $D + Np + X \geq G$. We can construct a PT equilibrium analogously to the success-maximizing PT equilibrium in Proposition 1 such that the donation threshold makes investors just indifferent between pledging and not. All expressions are analogous to

where the inductive definition of the probability of success contains an additional integral:

$$\begin{aligned} \tilde{\pi}_{j-1}^\Delta(N, D, u) = & \\ \mathbb{E}^{F_0} \left[\int \sum_{i=1}^{\max\{u - \xi_{M(W)-(N+1)}^\Delta(W)/\Delta, 0\}} & (1 - \Delta\lambda)^{i-1} \Delta\lambda (\mathbb{1}(D_{*,j-1}^\Delta(N+1, u - \Delta(i-1)) + Np \geq G + X) + \\ \pi_{j-1}^\Delta(N+1, \max\{D, D_{*,j-1}^\Delta(N+1, u - \Delta(i-1))\}, u - \Delta i) & \mathbb{1}(D_{*,j-1}^\Delta(N+1, u - \Delta(i-1)) + Np < G + X)) \\ + (1 - \Delta\lambda)^{u/\Delta} \mathbb{1}(W + X \geq G - (N+1)p) dH(X) & \Big| W \geq D \Big]. \end{aligned}$$

Similarly, we can construct a PT equilibrium analogous to Proposition 2 using cutoff times ξ that correspond to a game without a donor, but an uncertain goal. In this equilibrium, investors pledge even if they believed there were no additional donations. Hence, this equilibrium coincides with one, in which investors know W . By construction, the donation thresholds for this equilibrium are higher for any state (N, u) than the threshold for the equilibrium that corresponds to Proposition 1. Thus, the probability of success in this equilibrium yields a higher probability of success than the equilibrium in which there was no signaling and W was made public. The same equilibria could be supported if the donor was maximizing the probability of success subject to a budget constraint.

Next, we consider alternative donor payoffs $(W - \gamma D_T) \mathbb{1}(R_T \geq G) - (1 - \gamma) D_T$, where $\gamma \in [0, 1]$ is the scrap value of investment if the goal is not reached. Solving a fully dynamic game is beyond the scope of this paper, but we can highlight that there is value of signaling using a two-period example.

We assume $t = 1, 2$ and that G is uniformly distributed on $\{2, 3\}$. We also assume that the arrival rate of investors to be $\lambda = 1$, and each investment requires a contribution of $p = 1$. We assume a cutoff probability of 0.5 (e.g., $v = 2$, $v_0 = 0.7$, so $\frac{v_0}{v-p} = 0.5$) and W to be uniformly distributed on $\{0, 2\}$.

If W is announced ex-ante, then investors recognize that the donor will commit up to W at $t = 2$ to ensure success. Thus, investors effectively face a goal of $G - W$. If $W = 2$, one investment suffices for success, so investors always contribute. Conversely, if $W = 0$, the second-period investor never contributes, as the probability of success is at most $0.5 < 0.7$.

Consequently, the first-period investor also refrains from contributing even if $G = 2$.

If W remains private information, we can construct an equilibrium such that the project succeeds if the realized donor valuation is $W = 0$ and the realized goal is $G = 2$. If an investment occurred in period 1, then the probability of success for an investor at $t = 2$ is $0.5 + 0.5 \cdot 0.5 = 0.75 \geq 0.7$, so she invests. A period-1 investor also contributes, with a calculated success probability of $0.5 + 0.5 \cdot 0.5 \cdot 0.9 = .725 \geq 0.7$. An optimal strategy for the donor is to only contribute at $t = 2$, and only if it facilitates success. Thus, the project succeeds if $G = 2$ and $W = 0$. Thus, the probability of success is greater when W is undisclosed.

B.4 Proof of Proposition 5 (Donation Dynamics of PT Equilibria)

i) **Claim:** $\mathbb{P}(N_T p + D_{T-\Delta} < G | \mathcal{S}_T) > 1 - \Delta\lambda$ and $\mathbb{P}(D_T = G - N_T p | \mathcal{S}_T) \geq 1 - \Delta\lambda$.

If the campaign has not succeeded by the beginning of the last period, then $D_{T-\Delta} + N_{T-\Delta} p < G$. Then, it can only be that $N_T p + D_{T-\Delta} \geq G$ if a consumer arrives in the last period which occurs with probability $\Delta\lambda$. Even if a consumer arrives, the campaign remains unsuccessful without a donation. If $N_T p + D_{T-\Delta} < G$, then the donor donates exactly such that $D_T = G - N_T p$ if his valuation w is large enough. If w is smaller, the campaign fails.

ii) **Claim:** $\mathbb{P}(D_{\tau-\Delta} < G - N_{\tau} p) = 1$ if $\tau < T$.

In any PT equilibrium with donation threshold D_*^Δ , the donor never donates more than $\max\{D, D_*^\Delta(N, u)\}$ at $u > \Delta$, where $D_*^\Delta(N, u) < G - (N + 1)p$. Thus, if the campaign succeeds for $u > \Delta$, it must be due to a purchase.

iii) **Claim:** $D_*^\Delta(N, u + \Delta) \geq D_*^\Delta(N + 1, u)$

This is simply Condition i) in Definition 2 of PT assessments.

iv) **Claim:** Given donor realizations $w > w'$, if a campaign is unsuccessful for both w and w' , then the failure time ι is larger for w than for w' .

We can write the failure time of a campaign in a PT equilibrium as $t = \min_j \{ \tau_j \geq 0 \mid W < D_*^\Delta(j, T - \tau_j) \}$. Hence, it follows immediately that a donor with valuation w fails later than a donor with valuation w' .

v) **Claim:** In success-minimizing PT equilibria, all donations are at least p .

This follows immediately from the definition of the success-minimizing threshold $\overline{D}^\Delta(N, u)$ in Section 3.4.

C Data Appendix

C.1 Data Construction

We directly observe pledge counts for each reward level (buyer pledges) as well as total revenues. Total revenues are inclusive of donations and shipping costs—on Kickstarter, shipping costs are included in the progress towards the goal but are not included in the prices for rewards. This means that we observe both left-hand-side variables individually in the following equation,

$$\text{Total Revenue}_t - \text{Buyer Revenue}_t = \text{Donor Revenue}_t + \text{Shipping Costs}_t,$$

but we only observe the sum of the right-hand-side variables. In order to recover donations, we need an estimate of shipping costs.

We collect shipping costs for every campaign-reward-country combination and then assign a shipping cost to every observed pledge. Since donations are positive contributions to campaigns, we also incorporate the constraint $\text{Shipping Costs}_t \leq \text{Total Revenue}_t - \text{Investor Revenue}_t$.

In total, we collect over 516,000 shipping quotes. The most frequently observed shipping options are free shipping, single-rate shipping, or worldwide shipping with region-specific or country-specific prices. We complete our analyses under three shipping-cost assignments: (i) least-expensive shipping, (ii) assuming all buyers are located in the United States, and (iii) most-expensive shipping. Specifications (i) and (iii) provide lower and upper bounds on the importance of donations. We use (ii) as our main specification because most campaigns originate in the U.S.

We define a buyer to be an individual who pledges for any reward, however, some rewards may better be classified as a donation. For example, if the lowest reward is a thank-you card, but the main reward is a novel product, the lowest reward may be better treated as a donation. Another example may be the existence of an expensive option that

includes the main reward but also allows the buyer to meet with the entrepreneur. We repeat all of our analyses treating the most-expensive, the least-expensive rewards, or both the least- and most-expensive rewards as donations.

For our empirical analysis, we use the following cleaning criteria:

- i) Some entrepreneurs request that buyers pledge in excess of the posted price if they are interested in obtaining additional product features—called “add-ons” or “optional buys.” Other campaigns have “stretch goals,” which means that the entrepreneur informally adjusts the goal, and if met, adjusts the final product. Unfortunately, we do not have access to individual-level data to measure the prominence of buyers contributing in excess of the goal. We attempt to minimize the presence of these contributions by removing any campaigns that include words related to add-ons, optional buys, and stretch goals in the HTML pages.
- ii) We winsorize the sample by dropping the bottom 0.5% and the top 0.5% of campaigns in terms of the goal amount. This removes campaigns with low \$1 goals and campaigns with several million dollar goals (one in the billions). These extreme values impact some means, such as average goal, but medians are unchanged.
- iii) We drop campaigns that were removed by the creator, campaigns under copyright dispute, and campaigns with optional add-ons.

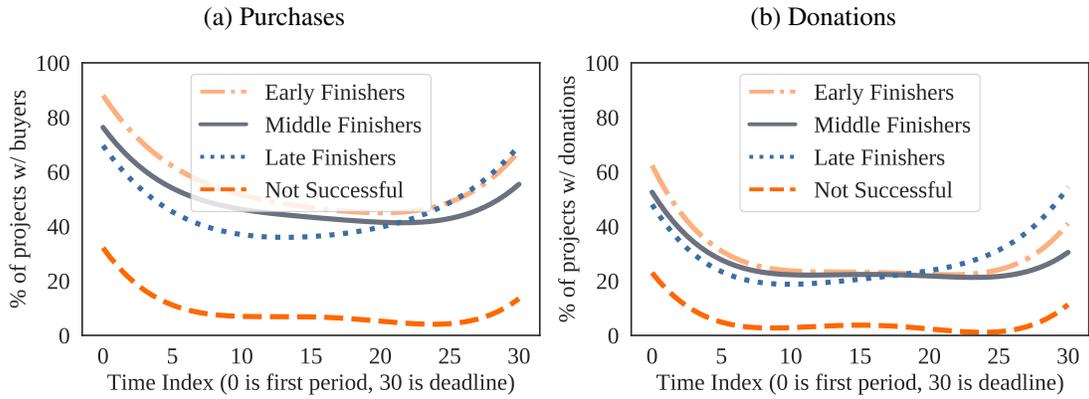
C.2 Additional Tables and Figures

Table 8: Top Category Summary Statistics

	Design	Film & Video	Music	Technology
Project Length	33.8 (10.9)	33.0 (12.5)	33.1 (12.3)	35.9 (12.2)
Goal (\$)	19185.6 (32655.5)	18003.5 (44140.1)	9849.1 (27223.1)	36482.4 (61063.7)
Number of Rewards	8.1 (5.2)	7.4 (5.6)	7.1 (5.8)	6.2 (4.6)
Donor Revenue (per period)	31.1 (525.8)	26.2 (286.2)	17.8 (160.6)	17.9 (238.7)
Buyer Revenue (per period)	320.9 (2112.4)	55.9 (488.7)	50.1 (360.6)	213.2 (1881.7)
Percentage Donations at Deadline	17.3 (25.1)	40.9 (32.3)	37.8 (31.5)	29.5 (36.2)
Percentage Donations of Goal	18.5 (52.8)	22.6 (51.6)	24.5 (103.0)	8.3 (28.6)
Percentage Successful	49.1	44.0	54.0	24.6
Number of Campaigns	4819	5176	4328	4804

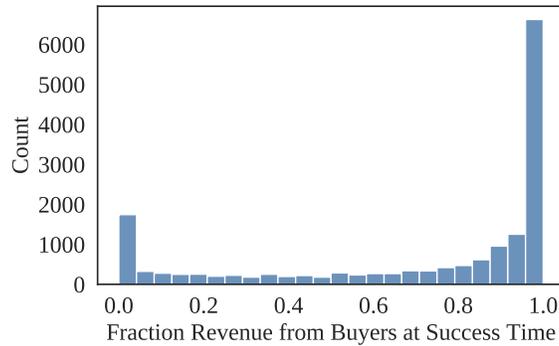
Note: Summary statistics for the top four Kickstarter categories, based on the number of campaigns within a category. Standard deviation reported in parentheses. Donor and buyer revenue are reported as 12-hour averages. Rows involving percentages use final campaign outcomes.

Figure 13: Percentage of Campaigns that Receive Purchases/Donations over Time



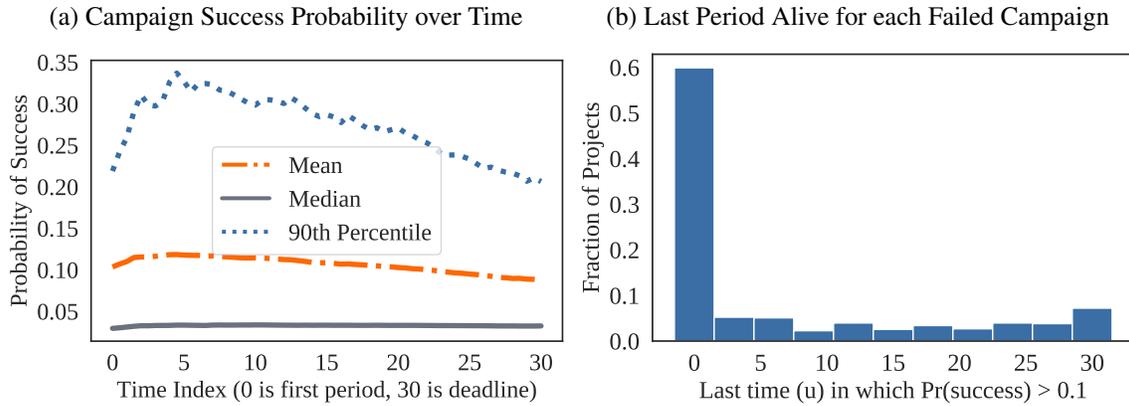
Note: These figures show the percentage of campaigns that have purchases or donations over time, for 30 day campaigns. 30 denotes the campaign deadline. Four lines are shown: early finishers (succeed within 3 days), middle finishers (succeed between 3 and 27 days), late finishers (succeed in the last 3 days) and unsuccessful campaigns. The lines are fitted values of polynomial regressions.

Figure 14: Percentage of Rev. from Buyers at Success Time



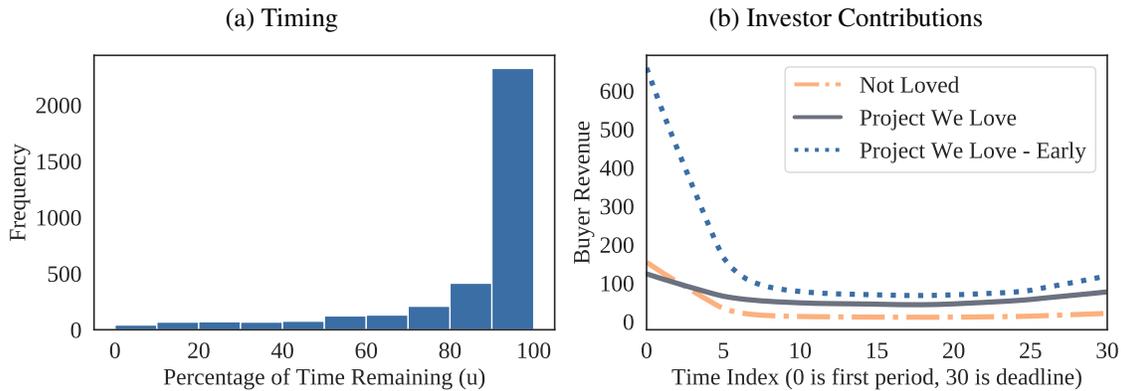
Note: Histogram of the fraction of revenue from buyers in the period in which a campaign succeeds. Selected campaigns finish at least one day before the deadline.

Figure 15: Logistic Regression: Probability of Success for Campaigns that Eventually Fail



Note: (a) Fitted values of a logistic regression of campaign success over time for campaigns that eventually fail. Controls are fraction of total revenues over the goal amount (or 1 if greater) interacted with time. Plotted is the mean project, the median campaign, and the 90th percentile of projects. The results suggest that more than half of projects have lower probability of success at the start. (b) A histogram of the last time a campaign had a probability of success greater than 10%. Time rounded to three-day bins.

Figure 16: Projects We Love, Timing and Investor Contributions



Note: Projects We Love is a designation assigned to campaigns by Kickstarter staff. These campaigns may be featured on the site homepage as well as advertised in emails. The left panel (a) presents a histogram of when the designation is applied, as a function of time remaining in the campaign. The right panel (b) presents average buyer revenue for three scenarios: (1) campaigns that never receive the designation, (2) campaigns that receive the designation after 10% of time has elapsed and (3) campaigns that receive the designation within the first 10% of time.

Table 9: Dynamic Donation Regressions

	(1)	(2)	(3)	(4)
Lag1 Above Median	10.327 (2.415)	11.437 (2.415)	-9.401 (2.452)	-8.200 (2.452)
Lag2 Above Median	-2.147 (3.001)	-1.888 (3.000)	-6.387 (2.937)	-6.017 (2.936)
Lag3 Above Median	-0.304 (2.875)	-0.084 (2.874)	-4.471 (2.814)	-4.136 (2.812)
Lag4 Above Median	3.428 (2.205)	4.110 (2.205)	-9.886 (2.210)	-8.853 (2.211)
Time Fixed Effects	—	✓	—	✓
Campaign Fixed Effects	—	—	✓	✓

Note: Results of panel data regressions where the dependent variable is $D_{j,t}$ and the independent variables are $\mathbf{1}[R_{j,t-k}/G_j > \text{median}[R_{t-k}/G]]$, for $k = 1, 2, 3, 4$. That is, these variables mark is a campaign's cumulative revenue over the goal amount is above the median. We calculate the median at the time, category level for 30-day campaigns that eventually succeed. We run regressions for 30-day campaigns that have not yet reached success. Therefore, the interpretation of the model is donations as a function of whether or not a given campaign is above the median in terms of reaching the goal. The number of observations in each regression is 907,183.

C.3 Bounding Donations

Our definition of a donation comes from contributors entering an amount in the donation box, or from contributors paying more than the reward price. However, some rewards may be interpreted as donations. Examples include a low-priced rewards that approximate a thank-you, or an expensive reward that includes the product but also includes special recognition. The bias is in only one direction: we are possibly understating the magnitude of donations. This is not a problem, per se, but we investigate how it affects our results.

Given the number of projects and buckets per project, manually assigning a reward or part of a reward as a donation is infeasible. There are over 500,000 rewards in the data. Instead, we perform the following analyses. First, we assume the least expensive bucket represents a donation. Next, we assume the most expensive bucket represents a donation. Finally, we assume both the least and most expensive buckets constitute donations.

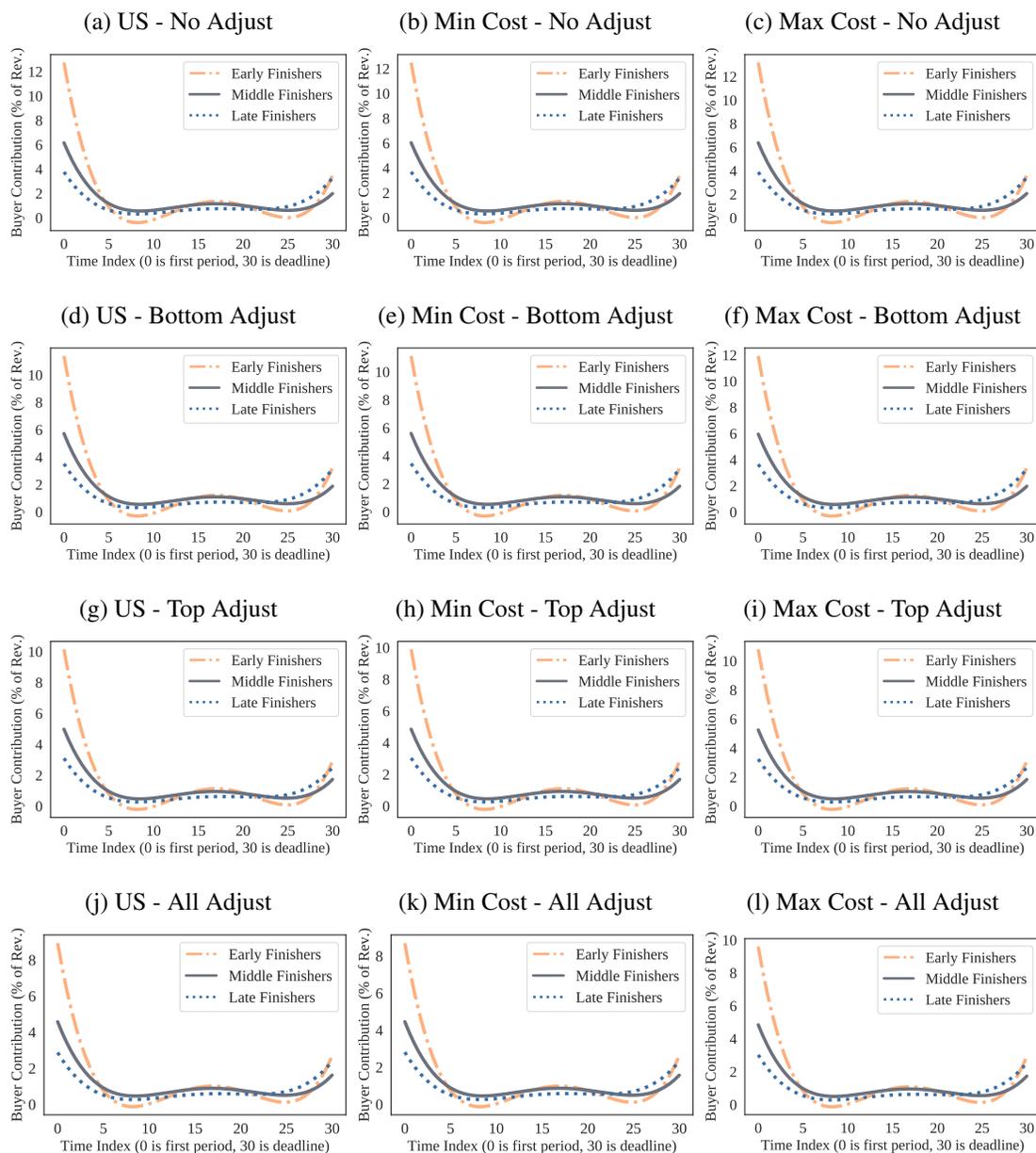
We also conduct robustness to our calculation of shipping costs. This is important because donations are determined after subtracting off shipping costs. If we understate shipping costs, we overstate donations. We reprocess all the data assuming all purchases are made from the country with the lowest, and then most expensive, shipping costs.

In Figure 17 and Figure 18, we plot average contributions divided by total campaign revenue over time for pledges and donations respectively under 12 scenarios. In Figure 19 and Figure 20, we plot the percentage of campaigns that see pledges and donations over time, respectively, under 12 scenarios. The labeling in the figures uses the legend in Table 10.

Table 10: Robustness Analysis Figure Legend

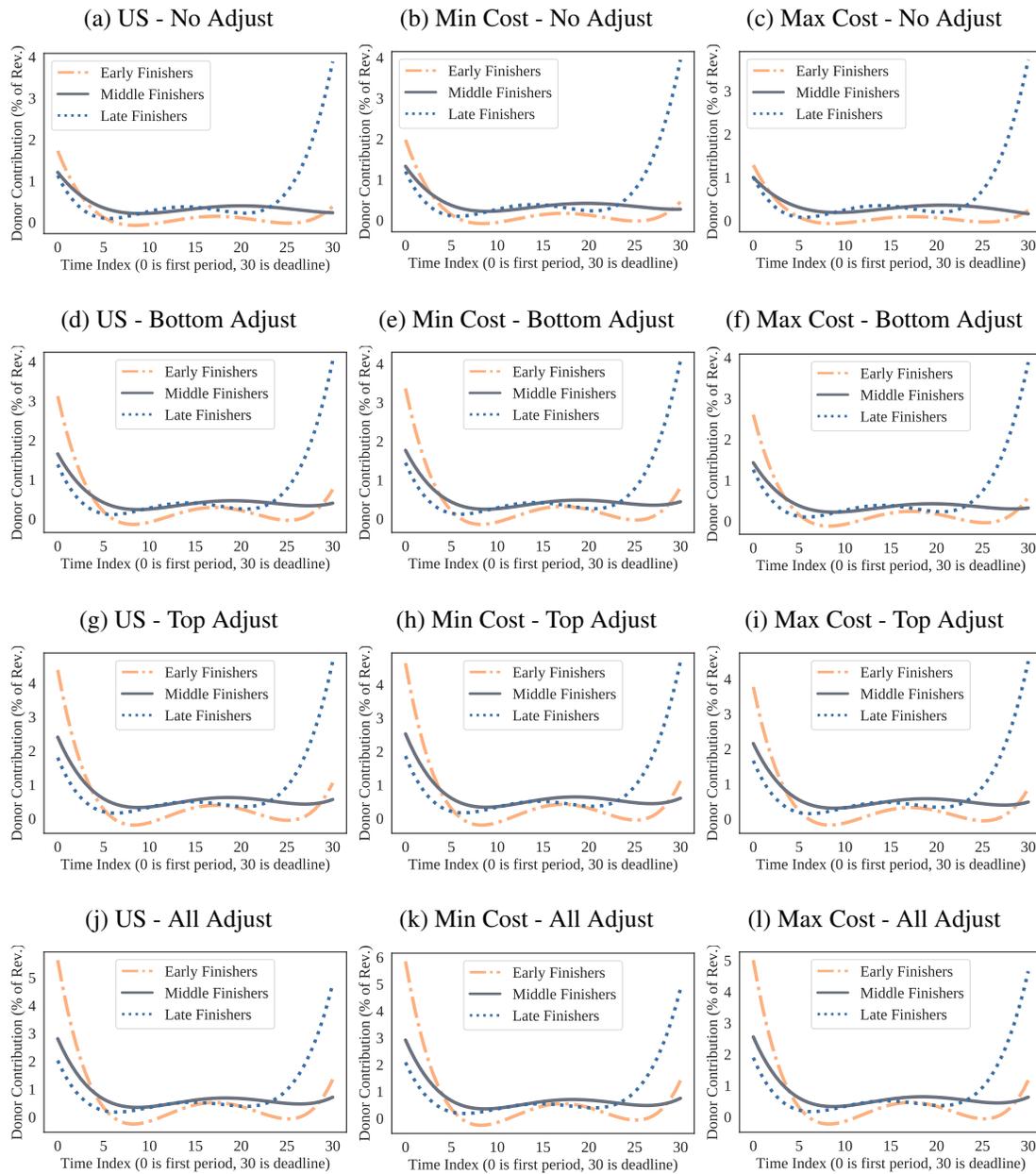
US	Assumes all rewards shipped under USA shipping costs
Min Cost	Assumes all rewards shipped to least expensive country
Max Cost	Assumes all rewards shipped to most expensive country
No Adjust	Treats all rewards as products
Bottom Adjust	Treats the least expensive reward as a donation
Top Adjust	Treats the most expensive reward as a donation
All Adjust	Treats both the least and most expensive reward as a donation

Figure 17: Robustness: Buyer Contributions over Time



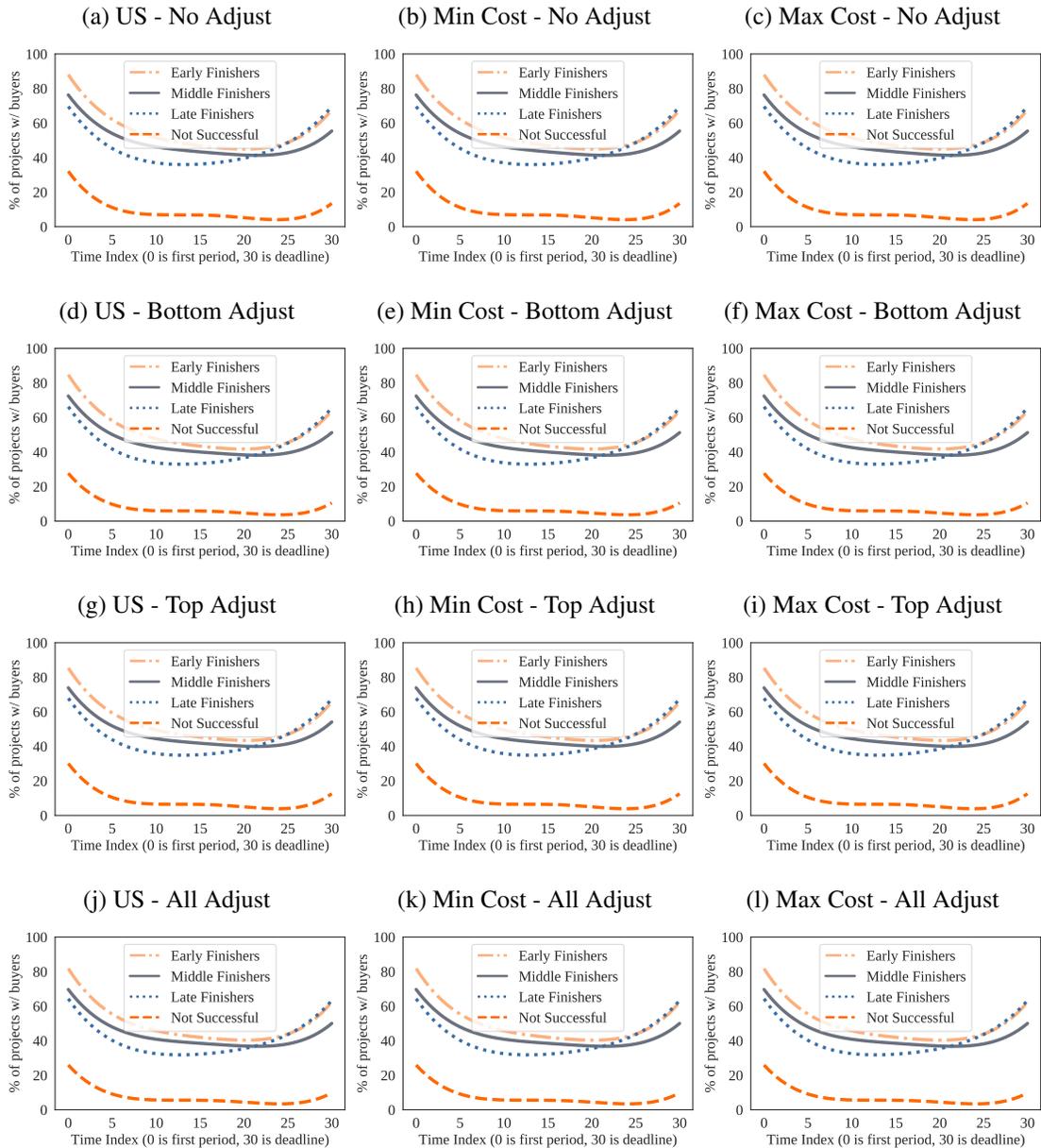
Note: These figures show average buyer contributions over time (over total campaign revenue) for different assumptions on shipping costs and what constitutes a donation. See Table 10 for label descriptions.

Figure 18: Robustness: Donor Contributions over Time



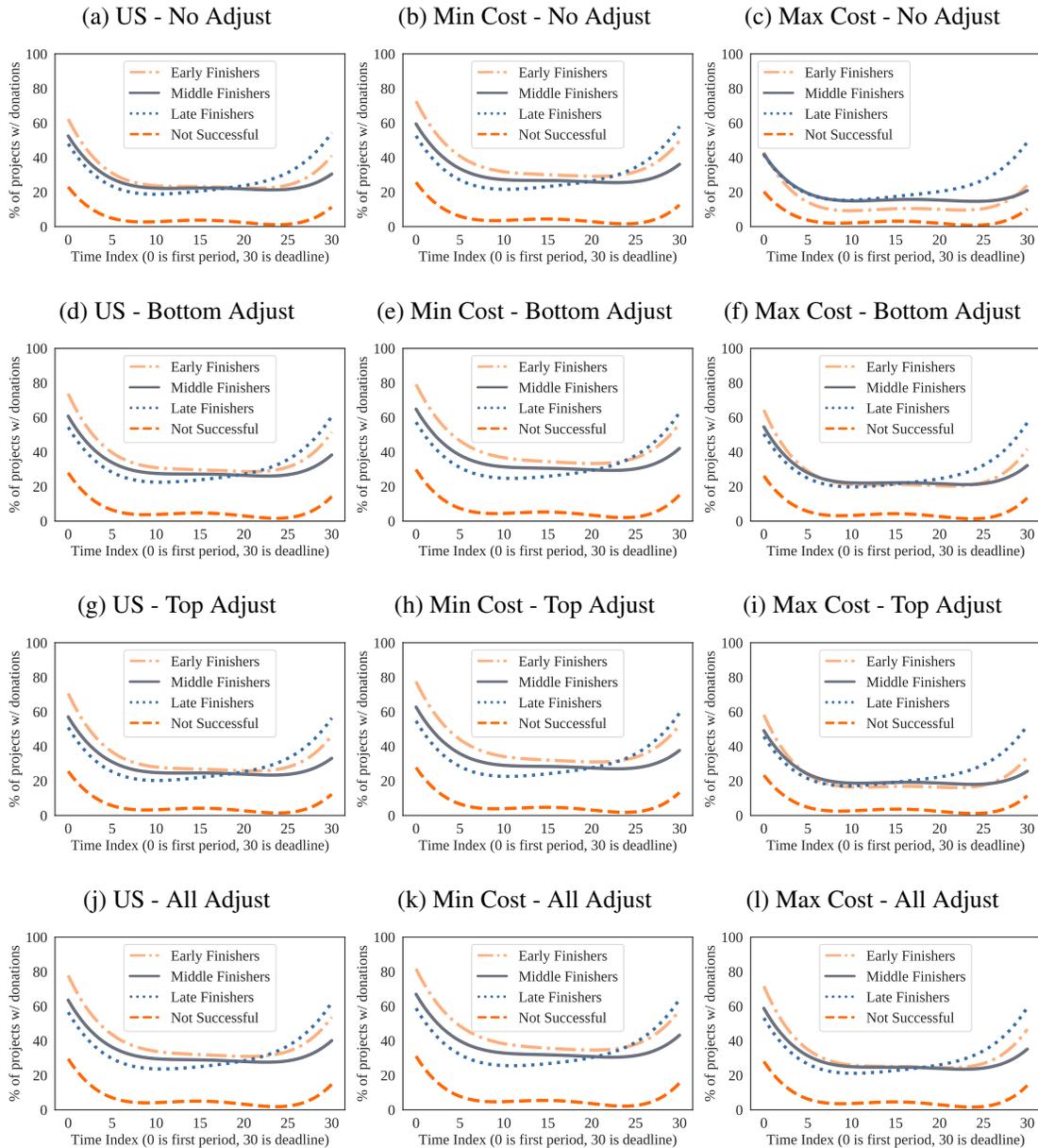
Note: These figures show average donor contributions over time (over total campaign revenue) for different assumptions on shipping costs and what constitutes a donation. Table 10 for label descriptions.

Figure 19: Robustness: Percentage of Projects that Receive Purchases over Time



Note: These figures show the percentage of campaigns that receive pledges over time for different assumptions on shipping costs and what constitutes a donation. Table 10 for label descriptions.

Figure 20: Robustness: Percentage of Projects that Receive Donations over Time



Note: These figures show the percentage of campaigns that receive donor contributions over time for different assumptions on shipping costs and what constitutes a donation. Table 10 for label descriptions.

Table 11: Robustness: Donation Revenue per Period

Shipping	Donation Adjust	<u>Mean</u>			Median	5th %	95th %
		(All)	(Uns.)	(Suc.)			
US	No Adjust	19.1	3.3	43.7	0.0	0.0	44.0
Min Cost	No Adjust	20.7	3.5	47.4	0.0	0.0	50.0
Max Cost	No Adjust	14.9	2.9	33.5	0.0	0.0	15.0
US	Bottom Adjust	28.3	4.5	65.5	0.0	0.0	62.0
Min Cost	Bottom Adjust	29.8	4.7	69.0	0.0	0.0	72.0
Max Cost	Bottom Adjust	23.9	4.1	54.6	0.0	0.0	39.0
US	Top Adjust	33.6	5.6	77.1	0.0	0.0	75.0
Min Cost	Top Adjust	35.1	5.8	80.8	0.0	0.0	85.2
Max Cost	Top Adjust	28.1	5.2	63.7	0.0	0.0	46.0
US	All Adjust	42.1	6.7	97.2	0.0	0.0	100.0
Min Cost	All Adjust	43.5	6.9	100.5	0.0	0.0	100.0
Max Cost	All Adjust	36.9	6.3	84.5	0.0	0.0	70.0

Note: Summary statistics for the 42,462 campaigns in the sample. Means for all statistics are computed for all campaigns (All), unsuccessful campaigns (Uns.), and successful campaigns (Suc.). Also reported are the 50th, 5th, and 95th percentiles.

Table 12: Robustness: Pledge Revenue per Period

Shipping	Donation Adjust	(All)	<u>Mean</u> (Uns.)	(Suc.)	Median	5th %	95th %
US	No Adjust	119.5	14.0	283.8	0.0	0.0	391.0
Min Cost	No Adjust	117.7	13.8	279.8	0.0	0.0	384.0
Max Cost	No Adjust	123.9	14.4	294.6	0.0	0.0	408.5
US	Bottom Adjust	113.2	13.0	269.3	0.0	0.0	368.2
Min Cost	Bottom Adjust	111.5	12.8	265.4	0.0	0.0	360.0
Max Cost	Bottom Adjust	118.4	13.5	282.0	0.0	0.0	389.0
US	Top Adjust	107.7	11.9	257.0	0.0	0.0	347.1
Min Cost	Top Adjust	105.9	11.7	252.9	0.0	0.0	339.4
Max Cost	Top Adjust	113.6	12.4	271.4	0.0	0.0	369.5
US	All Adjust	101.8	11.0	243.4	0.0	0.0	325.0
Min Cost	All Adjust	100.1	10.8	239.5	0.0	0.0	316.6
Max Cost	All Adjust	108.0	11.5	258.4	0.0	0.0	348.0

Note: Summary statistics for the 42,462 campaigns in the sample. Means for all statistics are computed for all campaigns (All), unsuccessful campaigns (Uns.), and successful campaigns (Suc.). Also reported are the 50th, 5th, and 95th percentiles.

Table 13: Robustness: Percentage of Revenues from Donations

Shipping	Donation Adjust	<u>Mean</u>			Median	5th %	95th %
		(All)	(Uns.)	(Suc.)			
US	No Adjust	29.2	33.1	24.6	15.8	0.0	100.0
Min Cost	No Adjust	30.5	34.3	26.1	17.9	0.0	100.0
Max Cost	No Adjust	26.9	31.5	21.5	11.4	0.0	100.0
US	Bottom Adjust	36.9	43.3	29.4	25.5	0.0	100.0
Min Cost	Bottom Adjust	38.2	44.4	30.9	27.1	0.3	100.0
Max Cost	Bottom Adjust	34.5	41.7	26.1	21.9	0.0	100.0
US	Top Adjust	41.1	43.8	38.0	33.7	0.0	100.0
Min Cost	Top Adjust	42.5	44.9	39.6	35.4	0.0	100.0
Max Cost	Top Adjust	38.4	42.0	34.2	29.8	0.0	100.0
US	All Adjust	47.8	52.3	42.4	43.5	0.0	100.0
Min Cost	All Adjust	49.0	53.3	43.8	44.8	1.0	100.0
Max Cost	All Adjust	45.1	50.6	38.6	40.3	0.0	100.0

Note: Summary statistics for the 42,462 campaigns in the sample. Means for all statistics are computed for all campaigns (All), unsuccessful campaigns (Uns.), and successful campaigns (Suc.). Also reported are the 50th, 5th, and 95th percentiles.

C.4 Equilibrium Construction using Kickstarter Data

Estimation Sample: We incorporate five selection criteria when estimating the model:

- i) Select projects that have a deadline of 30 days;
- ii) Select projects in which the maximum number of pledges per period is below the 95th percentile;
- iii) Select projects in which the maximum amount of donations per period is below the 95th percentile;
- iv) Select projects in which the maximum ending revenue over the goal amount is below the 75th percentile;
- v) After implementing (i)-(iv), select the first quartile of projects by goal amount for each category.

Estimation of Donor Valuations: We estimate the selection model (Heckman, 1979) in a single step.

Estimation of Arrival Process: We estimate the selection model (Terza, 1998) in a single step. We do not estimate the variance-covariance parameters directly. Instead, we estimate the variance as $\log(\sigma)$. We estimate the covariance term as $\tanh(\rho)$. We approximate the integrals in the log-likelihood using Gauss–Hermite quadrature with 25 integration points.

Calibration of Investor Utility: We calibrate the model using method of simulated moments (MSM). To calculate the success-maximizing equilibrium, we implement the dual induction argument of Proposition 3.3 with the following adjustments:

- i) We discretize donations in increments of \$1. The donor’s strategy is defined over the estimated log-normal distribution up to the 99.9th percentile;
- ii) We adjust the length of a time interval so that arrival rates are less than 1, i.e., we define $\lambda^{\max} = \lceil \max_{t=0, \dots, T} \hat{\lambda}_t \rceil$, and then define the length of a period to be $1/\lambda^{\max}$ for all t . Estimated arrival rates are multiplied by $1/\lambda^{\max}$;

iii) The goal is set to be $\lceil G \rceil$ for each category and goal quartile. Donation thresholds, time cutoffs, and beliefs are defined up to $\lceil G \rceil$;

We use derivative-free search over the parameter space, initially selecting candidate solutions between $\nu_0 + p$ and $\nu_0 + p + 1$. Our method searches over larger values of ν if the objective is minimized beyond $\nu_0 + p + 1$. We also account for potential flat spots in the objective function. For example, the probability of success may be equal to zero over a range of potential solutions because beliefs are sufficiently low. We use a stopping criterion of $1e-6$ in our procedure.

We do not report results for the dance category as the valuation distribution and arrival process parameters are such that the objective is flat over all potential ν .