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Using Spatial Factor Analysis to Measure Human Development
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ABSTRACT

We propose a Bayesian factor analysis model as an alternative to the Human Development Index (HDI). Our model provides methodology which can either augment or build additional indices. In addition to addressing potential issues of the HDI, we estimate human development with three auxiliary variables capturing environmental health and sustainability, income inequality, and satellite observed nightlight. We also use our method to build a Millennium Development Goals (MDG) index as an example of constructing a more complex index. We find the "living standard" dimension provides a greater contribution to human development than the official HDI suggests, while the "longevity" dimension provides a lower proportional contribution. Our results also show considerable levels of disagreement relative to the ranks of official HDI. We report the sensitivity of our method to different specifications of spatial correlation, cardinal-to-ordinal data transforms, and data imputation procedures, along with the results of a simulated data exercise.

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1. Introduction

Designed as a ranking system to track global human development, the Human Development Index (HDI) was first introduced in 1990 by the United Nations Development Programme (UNDP) in their now annual series of annual Human Development Reports (HDR's). Prior to the HDI's initial publication, GDP, GDP per capita, and GNP had long served as the primary indicators of development for academics, policymakers, and other interested parties; but each lacked something the UNDP saw as vital to fully understanding global development - the human factor. Defined by the first HDR as, "…the process of enlarging people's choices" (UNDP, 1990), human development is simply any method by which nations expand or strengthen their citizens' access to human capital building resources. Human development theory places emphasis on people being the beneficiaries of development rather than simply a means to an end. Based on this notion, the HDI formulates its national ranks using three key indicators which are believed to reflect a country's human development level: longevity, education, and decency of living standards.[2]

In the decades since its introduction, the HDI has come to serve as the standard for government agencies, private industry professionals, development groups, and academic researchers interested in studying and comparing national levels of human development. During a session in 2006, the National Congress of Indonesian Human Development restated their use of HDI as an economic indicator of development outcomes and the satisfaction of basic human living needs (Fattah and Muji, 2012). The government of Ireland also provides more development aid to countries classified as being "low human development" by the HDI (O'Neill, 2005; Wolff et al., 2011). In private industry, the pharmaceutical company Merck sells drugs at a significant discount to nearly all

[2] For a more detailed account of the rationale behind the design of the first HDI, see Anand and Sen (1994).

countries categorized as "low human development" (Petersen and Rother, 2001; Wolff et al., 2011). Additionally, there have been proposals when designing international climate change policy that each country's HDI ranking should be factored into their reduction obligations for greenhouse gas emissions (Hu, 2009; Wolff et al., 2011). In research, the HDI is widely used as an alternative to traditional economic indicators when evaluating a nation's relative level of human development (Anand and Ravallion, 1993; Easterlin, 2000). Furthermore, the HDI is not only heavily utilized by economists and other social scientists, but a broad range of academic disciplines including the medical research community.[3]

With the HDI's position as a top index now solidified through time and use, it serves as a worthwhile exercise to reevaluate its formulation. When studied critically, the HDI has several technical issues which we seek to address. For example, the three indicators used to calculate the official HDI are assigned deterministic weights relating to the proportional contribution they are *assumed* to provide towards a nation's human development level. This deterministic weighting is not informed by available data, but rather by expert opinion regarding potential effects. Additionally, the HDI does not provide a measure of uncertainty in their rankings; implying that each published list of the official HDI is only one subset of many possible rankings. A considerable number of previous studies have attempted to address these concerns as well as others with methods to correct for deterministic weights across dimensions (Ravallion, 2012), and lack of uncertainty from measurement error, index structure, and formula volatility (Noorbakhsh, 1998; Morse, 2003a; Wolff et al., 2009). Abayomi and Pizarro (2013) utilize a Bayesian framework to

[3] For instance, the relationship between the HDI and health has been studied extensively in topics such as: cancer (Bray et al., 2012), infant and maternal death (Lee et al., 1997), depressive episodes (Cifuentes et al., 2008), kidney cancer incidents and incident-to-mortality rates (Patel et al., 2012), suicide (Shah, 2009), and prevalence of physical inactivity (Dumith et al., 2011).

generate confidence intervals for the HDI with the goal of incorporating uncertainty by first assuming prior distributions for both the underlying data and variable weights, and then examining the posterior replicates. In an even more relevant study to our paper, Hoyland et al. (2012) also adopt a Bayesian factor analysis model; but it differs from our methodology in that they allow for correlations among indicators by first assuming correlations among the factor loadings of the HDI's four manifest variables.

This paper adopts a Bayesian factor analysis model which was initially developed to address many of the same concerns present in the Material Deprivation Index (Hogan and Tchernis, 2004).[4] The model assumes an underlying latent variable, a factor representing the level of human development, which manifests in the observed measures. Theory suggests which observed variables the factor influences, but data inform the degree of relative influence human development has on each variable as opposed to expert opinion. We summarize the results of our model by computing the posterior distribution of ranks for all countries which we then present with confidence intervals. These confidence intervals give a more holistic view of a nation's standing relative to its peers given the inherent uncertainty of the estimation process. To alter the uncertainty of our estimation, we also include measures of spatial correlation and national population. Spatial correlation is often used in the related literature as it allows for the incorporation of potential spillover effects from other factors which are highly correlated with HDI (Eberhardt et al., 2013; Ertur and Koch, 2011; Conley and Ligon, 2002; Keller, 2002).[5] Country populations enter the

[4] The same model has also been used to measure county health rankings for Wisconsin and Texas (Courtemanche et al., 2015).

[5] The spatial dependence of HDI is based on existing empirical and theoretical literatures. Research and development or long-run economic growth, both of which could be correlated with each factor of the index, has the documented potential for international spillovers (Eberhardt et al., 2013; Ertur and Koch, 2011; Conley and Ligon, 2002; Keller, 2002). Additionally, Malczewski (2010) shows that there are statistically significant geographical groups of high and low life expectancies in Poland.

model in a way which reflects the *a priori* assumption that the data of more highly populated nations harbor less uncertainty relative to less populous nations.

Finally, one of the HDI's primary limitations concerns its inability to add or remove variables without fundamentally altering the measure. Given that different sets of variables may capture different dimensions of human development, the official HDI's rigidity hinders its ability to evaluate performance under various theoretical considerations.[6] We illustrate the flexibility of our model to the inclusion of additional data and theory in two ways. First, we include three new variables capturing a nation's level of environmental health and sustainability, income inequality, and satellite observed nightlight. By including each of these three variables, we capture dimensions of human development which current theory believes to be important but the official HDI does not account for. Second, our general method is also easily utilized when trying to construct new indices. To illustrate the process of formulating an entirely new index, we construct an "MDG index" using data from the United Nations Millennium Development Goals (MDG).[7] Since the MDG's primary purpose was to track international progress in human development across time, we can consider it as an alternative measure of human development to the HDI. Furthermore, since the MDG's variables more directly relate to the human development outcomes of developing countries, our index provides valuable information regarding the relative performance of low development level nations which the official HDI's observed variables may not capture. Given the complex and decentralized nature of the MDG's design, a considerable quantity of prior research also attempts

[6] An example of the need to evaluate different specifications of human development can be seen in UNDP (2010), where an inequality adjusted HDI, gender inequality index, and multidimensional poverty index must each be derived separately using the general framework of the official HDI.
[7] Established in 2000, the MDG are a set of eight development goals which the United Nations member countries committed to achieve by the year 2015.

to construct an index summarizing information contained within the MDG's target variables (Alkire and Santos, 2010; De Muro et al., 2011; Abayomi and Pizarro, 2013).

2. Methods

2.1 Methods of the official HDI

Before discussing our methods, we first summarize the methodology used by the UNDP to calculate the official HDI. Since 2010, the HDI has constructed its three development indicators using four manifest variables: life expectancy at birth (LE), mean years of schooling (MYS), expected years of schooling (EYS), and purchasing power-adjusted real Gross National Income (GNI) per capita ($GNIpc$).[8]

First, the development indicators are calculated and normalized using the HDI's four observed variables. The indicators are the Life Expectancy Index (LEI), Education Index (EI), and Income Index (II). Each indicator is measured using the following method:

$$Life\ Expectancy\ Index(LEI) = \frac{LE - 20}{85 - 20} \tag{1}$$

$$\begin{gathered} Education\ Index(EI) = \frac{MYSI + EYSI}{2} \\ MYSI = \frac{MYS}{15}, \qquad EYSI = \frac{EYS}{18} \end{gathered} \tag{2}$$

[8] Following its introduction in 1990, the HDI has seen several alterations to its formulation. Some changes have been minor, but a considerable revision occurred in 2010. Prior to 2010, the four variables used to construct HDI were life expectancy at birth (longevity), adult literacy rate (education), combined educational enrollment (education), and purchasing power-adjusted real GDP per capita (living standard). We refer interested readers to UNDP (2010) for more information regarding the change.

$$Income\ Index\ (II) = \frac{\ln(GNIpc) - \ln(100)}{\ln(75{,}000\ ) - \ln(100)} \tag{3}$$

After calculating the three development indicators, the geometric mean of each country's indicators constitutes its official HDI score using the formula below:

$$HDI = \sqrt[3]{LEI * EI * II}$$

With this method, each HDI score ranges from 0 to 1. Following the designation of each nation's raw HDI score, countries are ranked and categorized into one of the following four development tiers: "very high development" (HDI ≥0.8), "high development" (HDI 0.7-0.8), "medium development" (HDI 0.55-0.7), and "low development" (HDI <0.55).

2.2 Proposed model

The official HDI harbors several technical issues which we seek to address, including: the use of *ad hoc* factor weightings, having no convenient way to include different sets of observed variables, no measure of uncertainty in rankings, no measure of spatial correlation between nations, and no way to account for the effect of country population differences. Alternatively, our hierarchical factor analysis model with spatial correlation address each of these technical issues.

Before adding either spatial correlation or population, our basic factor analysis model is specified as:

$$Y_{ij} = \mu_j + \lambda_j \delta_i + \varepsilon_{ij} \tag{4}$$

where Y_{ij} represents observed manifest variable $j = 1, \dots, J$ of country $i = 1, .., N$; μ_j is the average across countries of manifest variable j; δ_i is a latent factor representing a country's level of human development, and therefore our model based index values; λ_j is the factor loading for variable j, and represents the covariance between the latent development measure, δ_i, and the manifest variable Y_{ij}; and finally $\varepsilon_{ij} \sim N(0, \sigma_j^2)$ is the model's normally distributed idiosyncratic error term.

The model relies on the assumption that ε_{ij}'s be both independently and identically distributed, implying that all manifest variables, Y_{ij}, are correlated with one another only through the nation's latent development factor, δ_i. As spoken to in the initial 1990 Human Development Report (UNDP 1990), each of the variables used to calculate the official HDI are supposedly outcomes which are directly determined by a country's level of human development. If each manifest variable is indeed a reflection of human development, our method is justified in assuming that the shared covariance between a country's manifest variables can be used to estimate their level of human development. The basic factor analysis model also assumes factor scores to be normally distributed, $\delta_i \sim N(0, 1)$.[9]

The next step in developing our full model is incorporating spatial correlation. We use a Conditionally Autoregressive (CAR) model which specifies the relationship between factor scores for both a country, i, and its neighbors. While "neighbors" can be defined in many ways, our primary results use a simple specification based on adjacency in terms of either a land or maritime connection.[10] We define the set of neighbors for country i as $\mathcal{R}_i$, and specify the conditional distribution of the country's factor score in the following way**:**

[9] A potential alternative to the assumption that δ_i be normally distributed involves estimation using a mixture factor analysis model. Such a model is presented in Wall et al. (2012), but the authors find that both the standard and mixture factor analysis models produce minimally biased results when using continuous manifest variables in a simulated data exercise.

[10] We also estimate our model using a different specification of spatial correlation built using trade between countries to identify neighbors in Section 5.

$$\delta_i|\delta_j \sim N\left(\sum_{j\in\mathcal{R}_i} \omega\delta_j\ , \nu\right) \tag{5}$$

where ω measures the degree of spatial correlation, and the conditional variance, ν, captures any residual variation.

The addition of spatial correlation has two attractive properties. First, it intuitively defines the relationship between neighboring countries through the distribution mean of factor scores; implying that the average development level of a country's geographic neighbors partially determines its own level of development. Alternative models could include additional levels of dependence through both the conditional mean and conditional variance, but these are not statistically identified within a factor analysis model.[11] Second, by setting the conditional variance such that $\nu = 1$, our conditional specification results in a simple marginal distribution for the vector of factor scores:

$$\delta \sim \mathrm{N}(0, (I - \omega W)^{-1}) \tag{6}$$

where W is an $N \times N$ "neighbor matrix" such that $W_{ik} = W_{ki} = 1$ if a country k is adjacent to country i in terms of either land or maritime connections, and $W_{ik} = 0$ otherwise. Additionally, $W_{ii} = 0$. It is also important to note that since the covariance matrix of δ is a full matrix under this specification, all countries are correlated with one another even if they do not share a common border. Additionally, given that W determines the covariance matrix for δ, it is required to be symmetric. Even though W is a symmetric matrix, however, the estimated *impacts* of spatial

[11] For a more detailed discussion of this, see Hogan and Tchernis (2004).

correlation for two countries sharing a border are not necessarily the same.[12] For example, the contribution of spatial correlation for each country is partially determined by their total number of neighbors.

For the last step of model development, we introduce population into both the inverse variance of the error terms and factor scores. The intuition behind accounting for population this way is that *a priori* we are less uncertain regarding the amount of noise in the manifest variables and factor scores for countries with larger populations compared to countries with smaller populations.

The final model, in vector notation, is now presented as:

$$
\begin{gathered}
Y|\delta \sim N(\mu + \Lambda\delta, M^{-1} \otimes \Sigma) \\
\delta \sim N(0, M^{-\frac{1}{2}} \boldsymbol{\psi} M^{-\frac{1}{2}})
\end{gathered}
\tag{7}
$$

where Y is the vector of Y_{ij}'s stacked over j and then i ; $\Lambda = I_N \otimes \lambda,$ with I_N as an $N \times N$ identity matrix, $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_J)'$, and $\otimes$ denotes a Kronecker product; Σ is a diagonal matrix with σ_j^2 as the diagonal elements, and 0's as the off-diagonal elements; $\boldsymbol{\psi} = (I - \omega W)^{-1}$; and M is an $N \times N$ matrix with country populations $m_1, m_2, \ldots, m_N$ along the diagonal and 0's elsewhere.

To estimate the model, we must also specify the prior distributions of our parameters. For our main results using the observed variables of official HDI, we specify a set of conjugate non-informative priors which simplify the derivation of the posterior distributions without providing much information. This specification implies that the posterior distributions are informed almost entirely by the data and not the prior distribution assumptions. Alternatively, a strength of our Bayesian model is its ability to incorporate *a priori* theory into the model's estimation through

[12] For a detailed discussion regarding the spatial correlation structure of CAR type models, see Wall (2004).

informative prior distributions. We illustrate the use of priors to incorporate theory in Section 4 by placing an informative prior distribution on the factor loading of our manifest variable representing income inequality. We cover the details of our main result's prior specifications more formally in Appendix A.

Following Hogan and Tchernis (2004), we work with the variance stabilizing square root transformation of the original variables, such that $Y_{ij} = (S_{ij})^{\frac{1}{2}}$, where S_{ij}'s are the HDI's non-transformed variables.[13] This implies that $var(Y_{ij})$ is inversely proportionate to the country's population, m_i (Cressie and Chan, 1989; Hogan and Tchernis, 2004).

Our model is estimated using Markov Chain Monte Carlo (MCMC) methods, specifically the Metropolis-Hastings algorithm within a Gibbs Sampler. The method's primary goal is to produce a summary of the distribution of ranks for each country. In all iterations of the sampler, for which we run 4,000 total iterations after the initial convergence phase of 500 iterations, we rank the draws from the posterior distribution of the factor scores, allowing us to produce samples from the posterior distribution of the countries' ranks. A more detailed description of the estimation process can be found in Appendix A.

From a purely technical standpoint, our model has several advantages over the official HDI. First, our model based ranks are a function of the weighted manifest variables conditional on the observed data. While variable selection is decided by theory, using a data-driven model implies that the data inform the relative contribution of each manifest variable on human development as opposed to expert opinion. Second, we do not constrain our model to a specific set of variables. Instead, different variables can be included or excluded without fundamentally altering the

[13] S_{ij} is already in a "per-capita" form (e.g. GNI per capita, population mean years of schooling, etc.).

estimation process. Third, our model provides a summary of uncertainty through variance in the ranking distributions, giving a more holistic view of relative performance. Fourth, information regarding each country's rank comes from data for both the specific country and any potential spillover effects resulting from spatial correlation across countries. Finally, we incorporate additional information contained in a country's population, leading to lower uncertainty for more populous nations *a priori*. Even though our model provides a flexible structure for the estimation of human development ranks, there are some potential sensitivity issues which we address in Section 5.

Using the methodology outlined in this section, we estimate several sets of human development rankings including: rankings using only data from the official HDI, rankings using the official HDI manifest variables combined with additional variables related to human development according to current theory, and the ranks for our MDG index which uses a comprehensive set of variables found in the MDG data. The next section explains the sources of our data as well as information regarding variable selection.

3. Data

3.1 Data for model based HDI and alternative specifications

For our primary results, we rely on the data used to construct 2010's official HDI.[14] Data for each of the 195 countries are publicly available on the UNDP's website.[15] From the full dataset, we exclude 8 of the 195 countries from our estimation due to missing data as they are also removed

[14] All official HDI scores used in this study are calculated by the authors in order to avoid potential data irregularities between the UNDP's public use data files and the data used to calculate the HDI scores published in UNDP (2010).
[15] The data were downloaded on 07/01/2017 from http://hdr.undp.org/en/data.

from the estimation of official HDI. The four manifest variables used to calculate official HDI are: years of life expectancy at birth, mean years of schooling for adults, expected years of schooling for children, and GNI per capita. The measure of spatial correlation for our primary results uses both land and maritime borders to construct the "neighbor matrix" W.[16] We gather country population measures for 2010 from the World Bank's total population midyear estimates.[17]

To illustrate our method's flexibility in incorporating different sets of variables or structural alterations to reflect theory, we also estimate our model based HDI rankings under a variety of alternative specifications. First, we estimate our model using the official HDI manifest variables along with different combinations of three new variables: the Environmental Performance Index (EPI), Income Quintile Ratios (QR's), and Satellite Observed Light (SOL).[18] The EPI is an index which measures a country's level of environmental health and sustainability using ten different observable variables.[19] The relationship between environmental stewardship and human development is an increasingly pressing topic as discussed in the 2007 HDR (UNDP 2007). QR is a simple representation of a country's income inequality level measured by the ratio of income held by the richest 20% of its population relative to the poorest 20%.[20] Some theoretical justifications for the use of inequality when measuring human development are discussed in the 2010 HDR where the authors also propose an Inequality Adjusted HDI, Gender Inequality Index, and Multidimensional Poverty Index (UNDP 2010). Lastly, SOL represents a country's level of night light which is thought to be related to human development through economic activity and

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[16] Data on shared land and maritime borders are available from multiple sources, i.e. Anderson (2003).
[17] World Bank Total Population Data: http://data.worldbank.org/indicator/SP.POP.TOTL?page=1
[18] We would like to thank an anonymous reviewer for their suggestions regarding potential additions.
[19] Additional information can be found on the EPI website: http://epi.yale.edu. EPI data are collected from the National Aeronautics and Space Administration (NASA) Socioeconomic Data and Applications Center (SEDAC): http://sedac.ciesin.columbia.edu/data/set/epi-environmental-performance-index-pilot-trend-2012/data-download.
[20] Data regarding QR's are collected from the UNDP Human Development Data: http://hdr.undp.org/en/data

electrification (Elvidge et al., 2012; Ghosh et al., 2013).[21] While there does not seem to be adequate consensus in the literature regarding which variables an index should include to "best" capture human development, we believe that combining the official HDI variables, which represent longevity, education, and living standard, with manifest variables representing environmental stewardship, income inequality, and night light covers many dimensions thought to be important in the existing theory.[22] Second, as an alternative to shared land or maritime borders, we estimate our model under a measure of spatial correlation using trade relationships between nations to construct our "neighbor matrix" W, the results of which are discussed in Section 5.[23] Defining spatial correlation based on trade captures potential spillover effects in human development between countries that may or may not share a geographic border. This alternative specification of spatial correlation is also supported by human development theory. For example, UNDP (2010) claims that a large amount of the change in human development during recent years has been determined by the flow of ideas and technology across countries.

3.2 Data for the Millennium Development Goals index

Aside from incorporating new variables into an existing index, our method also applies to the creation of new, and more complex, indices. We illustrate this application by designing a novel index for measuring human development using the United Nation's Millennium Development Goals (MDG). The MDG includes eight broad primary goals with a total of 80 indicator variables

[21] Data regarding SOL are collected from the National Oceanic and Atmospheric Administration (NOAA) National Centers for Environmental Information website: https://ngdc.noaa.gov/eog/download.html.

[22] Any missing data for EPI, QR, or SOL is imputed using the predictive mean matching (PMM) method covered in Section 3.2.

[23] Data regarding country trade are collected from the World Integrated Trade Solution website: http://wits.worldbank.org. Additionally, trade data are missing for five nations (Liechtenstein, Montenegro, Romania, Serbia, and Timor-Leste) so they are excluded from our estimation using trade-based spatial correlation.

used to track their progress. As spoken to in Anand and Sen (1994), different sets of variables may explain differing amounts of variation in human development across the distribution of development levels. Given that the MDG variables focus more heavily on the outcomes of developing countries, our MDG index is also likely to provide valuable insight regarding the relative performance of low-development level countries not captured by the official HDI.

Data for each MDG variable are collected directly from the UNDP.[24] While primary target data are available for 234 countries and comparable areas, there is a considerable quantity of missing observations in the UNDP's dataset. Of the 80 potential MDG indicator variables available to us, we select the 11 which have the least missing data across countries to serve as our MDG index's manifest variables.[25] Considering the substantial number of MDG variables we choose to include in the estimation, our model has an inherent advantage over traditional methods in that we can skip the deterministic assignment of factor weights as they are a direct product of our estimation. Additionally, we can ignore assumptions regarding variable groupings, allowing us to avoid a high quantity of extra correlation parameters. Using our model, correlations between variables, regardless of their dimensions, are fully captured by the spatial correlation structure embedded in the latent factor. Comparing data across time, we also find 2010 to be the year with the most complete collection of data for the greatest number of countries. To help ensure accurate post-analysis comparisons between the HDI and our new MDG index, we also restrict the selection of observations for our MDG data to the same 187 countries ranked by the official HDI.

[24] Millennium Development Goals Indicators: http://mdgs.un.org/unsd/mdg/Data.aspx

[25] The 11 manifest variables are: (1) "maternal morality ratio per 100,000 live births" (MMR), (2) "children under five mortality rate per 1,000 live births" (U5MR), (3) "population undernourished, percentage" (PU), (4) "total net enrolment ratio in primary education, both sexes" (NER), (5) "gender parity index in primary level enrolment" (GPI), (6) "tuberculosis prevalence rate per 100,000 population (mid-point)" (TB), (7) "proportion of the population using improved drinking water sources" (WATER), (8) "people living with HIV, 15-49 years old, percentage" (HIV), (9) "fixed-telephone subscriptions per 100 inhabitants" (TELE), (10) "employment-to-population ratio, both sexes, percentage" (ETP), and (11) "adolescent birth rate, per 1,000 women" (ABR).

After selecting our MDG manifest variables, we impute values for the missing data using two methods. The first method is a naïve imputation process for which we impute missing data for each variable in order from the variable with the highest number of non-missing observations to the variable with the lowest. Table 1 presents summary statistics for the 11 manifest variables both before and after naïve imputation. As our summary statistics show, the number of missing observations among variables differs considerably, but the change in variable means and standard deviations following the naïve imputation is relatively small.

Table 1. Summary Statistics of MDG Indicators

	Before Imputation			After Naïve Imputation		
Variable	**Obs**	**Mean**	**St.D**	**Obs**	**Mean**	**St.D**
TELE	187	18.80	17.66	187	18.80	17.66
TB	186	157.37	190.40	187	156.56	190.21
U5MR	185	38.80	40.77	187	38.45	40.69
WATER	181	86.93	15.53	187	87.09	15.58
MMR	178	176.83	233.37	187	169.70	229.89
PU	162	12.20	10.53	187	12.28	10.53
GPI	149	0.97	0.06	187	0.97	0.06
NER	119	92.41	9.49	187	91.79	9.57
HIV	114	2.38	4.92	187	1.85	4.04
ETP	108	54.77	10.55	187	55.58	11.27
ABR	97	37.62	36.68	187	53.07	48.00

The specific technique used for our naïve imputation is a "univariate imputation using predictive mean matching" (PMM). PMM is a combination of the Ordinary Least Squares (OLS) regression prediction and the nearest-neighbor imputation methods. First, PMM produces linear predictions for all data, missing and observed, using a traditional OLS regression. We then compare predicted values to one another across countries. For each missing observation, the imputed value used is the value of the *non-missing observation* which has the closest predicted value, known as the missing observation's "nearest neighbor." With PMM, we honor existing bounds in the non-imputed portion of the data while also preserving the observed data's distribution (Little, 1988). All PMM imputation procedures are performed using Stata's PMM syntax.

The second imputation method comes from the posterior imputation process embedded in our model. Posterior imputation replaces the naïvely imputed values with observations sampled from the distributions of missing data. This replacement allows us to take the potential uncertainty inherent in missing data into better account during our estimation (Rubin, 1976; Little and Rubin, 2002; Daniels and Hogan, 2008). We address posterior imputation more fully, along with the sensitivity of our results to the choice of imputation process, in Section 5.

4. Results

4.1 Model based ranks vs. official HDI ranks

The rankings of official HDI fail to account for either uncertainty, spatial correlation, or population. Alternatively, we estimate our index ranks in terms of distributions, which provides a measure of uncertainty. Since factor weightings differ between our model based index and the official HDI, there must be some discordance between our posterior mean ranks and the official HDI ranks. We graphically compare the two rankings, including information regarding the 99% confidence interval of the posterior ranks, in Figure 1.

For Figure 1 and subsequent figures of the same design, the dashed grid lines partition the 0%-20% (1^{st}), 20%-40% (2^{nd}), 40%-60% (3^{rd}), 60%-80% (4^{th}), and 80%-100% (5^{th}) quintiles of ranks, respectively. Solid dots show the locations of both posterior mean ranks and official HDI ranks. The solid horizontal lines across each dot represent a 99% confidence interval for a country's posterior rank under our model. The numbers in Figure 1 correspond to individual country identifiers, which are assigned alphabetically and listed in Appendix B.

Figure 1 shows that our model's rankings harbor a considerable level of uncertainty for certain countries, with several confidence intervals reaching across multiple quintiles. Interestingly, this uncertainty persists in various degrees along the entire distribution of ranks as opposed to being constrained to only certain levels of development. As an example, Bhutan, a low-development level country, has a posterior 99% confidence interval of (137, 164), implying that their rank could fall into either the 4^{th} or the 5^{th} quintile of human development. Comparable results are also found for more highly developed nations like Qatar, which has a posterior 99% confidence interval of (15, 56), implying that its rank could fall into either the 1^{st} or 2^{nd} quintile. While Bhutan and Qatar represent more extreme cases, it is not uncommon for the confidence interval of certain nations to reach across quintiles of human development under our model.

The relationship between a country's rank and its level of uncertainty is an inverted U-shape, with levels of uncertainty decreasing for the most and least developed countries. This relationship is likely due to several factors. First, countries ranked at the top (bottom) have the highest (lowest) values for each manifest variable. Second, countries away from the distribution's center often tend to be the most highly populated, implying that they produce a lower degree of uncertainty in our model. Third, these countries are also closer to one another on average geographically, leading to a reduction in uncertainty through spatial smoothing. This result with spatial correlation mirrors the geographical clustering often observed in human development, i.e. having many low-development level countries in sub-Saharan Africa or many high-development level countries in Europe. Finally, the relationship between development and uncertainty is also partially determined by the truncation of variable values from either above or below for the most and least developed countries.

Figure 1 also illustrates the discordance between our model based ranks and those of the official HDI. The greater the distance between solid dots and the 45^{o} line, the greater the disagreement between our model based ranks and the ranks of official HDI. While the results of both models are well correlated, for only eleven countries are the model based and official HDI ranks identical. For 72 countries, the absolute value of the difference between both two ranks is less than five. For 53 countries, however, the absolute value of the difference is larger than 10.

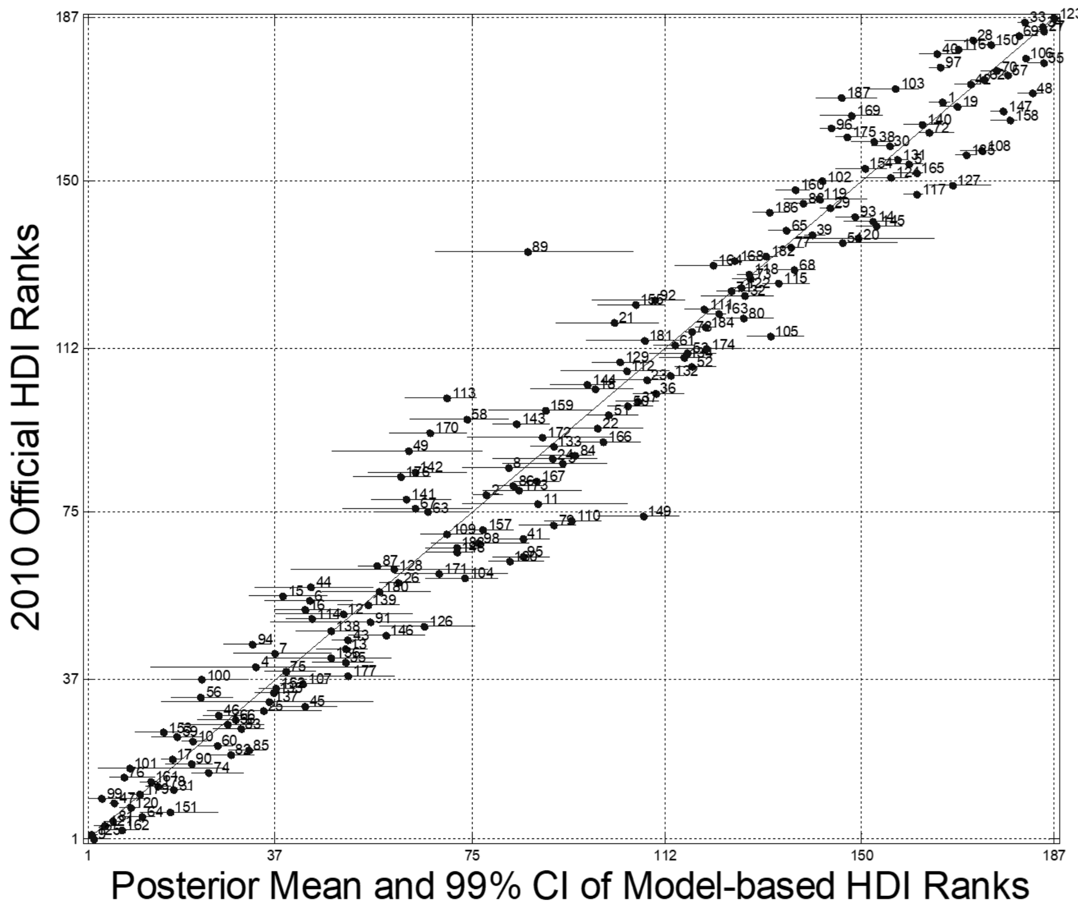

Figure 1. Posterior Mean and 99% CI of Model Based HDI Ranks vs. Official HDI Ranks

4.2 Discordance between model based and official HDI ranks

Table 2(a) shows the ten countries which have the largest differences between their official HDI rankings and their rankings as determined by our model. As an example, Mongolia is ranked 101 using the official HDI but is assigned a posterior mean rank of 70 by our model with a 99% confidence interval of (62, 76). Therefore, Mongolia's posterior confidence interval fails to even cover the range of its official HDI rank. It is reasonable to conclude from our results that the official HDI may underestimate Mongolia's level of human development. Alternatively, Mexico, which has an official HDI rank of 73, has posterior mean rank of 94 in our model with a 99% confidence interval of (89, 100). So, in an opposite pattern to Mongolia, the official HDI may overestimate the human development level of Mexico under the assumptions of our model. Since many of the highly discordant countries shown in Table 2(a) have relatively small populations, we also present the seven nations with large populations (over fifty million) which also have an absolute difference-in-ranks between their model based and official HDI rankings greater than 10 in Table 2(b).

The most plausible reason behind the discordances in rank is the difference in factor weights between the official HDI and our model based index. As we discuss in the following section, our model based index assigns a greater proportional contribution to the "living standard" dimension and a lower proportional contribution to the "longevity" dimension. This difference implies that countries with either outstanding or dismal performance in those two dimensions see a considerable amount of movement between the two models. Furthermore, incorporating population also alters the total level of uncertainty in a country's rank, and therefore the size of the absolute difference in rankings produced by both models. Additionally, the discordance between official HDI and our model may be partially determined by spatial correlation if either positive or negative spillover effects in development are sufficiently influencing each country's distribution of ranks. Since the official HDI is a direct measurement built on observable variables, one could interpret our results under the assumption that the HDI is a "correct" measure. Assuming that the HDI is "correct", we find that the posterior mean ranks of our model and the official HDI correlate well on average. Alternatively, we also evaluate the level of discordance between the official HDI and our model under the opposite assumption using simulated data where our model identifies the correct data generating process in Section 5.

Table 2 (a). Ten Countries with the Largest Differences in Ranks Between Official HDI and Model Based HDI

Country	Ranks		Manifest variable			
	HDI	Model Based[a]	LE	MYS	EYS	GNIpc
Kiribati	134	86 (68, 106)	65.4	7.7	11.9	2556
Seychelles	74	108 (99, 115)	72.6	8.1	13.3	18952
Mongolia	101	70 (62, 76)	67.5	9.8	14.6	7084
Dominica	89	63 (48, 77)	77.4	7.8	12.7	9980
Tonga	93	67 (59, 74)	72.2	10.7	14.4	5103
Zimbabwe	169	146 (141, 153)	49.6	7.3	10.1	1302
Fiji	96	74 (63, 82)	69.3	9.6	14.7	7197
Ukraine	83	61 (52, 67)	69.3	11.3	14.9	7738
Mexico	73	94 (89, 100)	76.1	8.3	12.6	15512
Saint Lucia	84	64 (55, 74)	74.5	9.3	12.9	10416

Note: a. Posterior ranks with 99% confidence intervals in the parenthesis.

Table 2 (b). Countries with Differences in Ranks over 10 and Larger-populations (>50M)

Country	Ranks		Manifest variable			
	HDI	Model Based[a]	LE	MYS	EYS	GNIpc
Bangladesh	141	152 (147, 157)	70.1	4.9	9.4	2652
Congo (DRC)	179	165 (161, 169)	56.9	5.4	8.8	568
Iran	72	91 (84, 95)	74	8.2	13.1	17520
Japan	21	32 (28, 35)	83	11.5	15.1	35343
Mexico	73	94 (89, 100)	76.1	8.3	12.6	15512
Myanmar	147	161 (158, 162)	65	4.1	9.1	3604
Pakistan	149	167 (163, 175)	65.1	4.6	7.5	4460

Note: a. Posterior ranks with 99% confidence intervals in the parenthesis.

4.3 Squared correlation coefficients

Due to differences in methodology, there is no simple way to compare the estimated contributions of each manifest variable on the official HDI's measure of human development or the latent factor in our model. To calculate a general measure of comparability, we follow Ravallion (2012) who suggests calculating the marginal weights of each variable in the official HDI as the partial derivative of the official HDI with respect to each observable variable. Following this approach, we obtain the marginal weights of each variable in the official HDI by regressing standardized HDI scores on standardized manifest variables.

To summarize each variable's contribution to the latent development factor of our model, we apply the method of Hogan and Tchernis (2004) and present normalized "squared correlation coefficients." The squared correlation coefficient of each manifest variable j is specified as:

$$\rho_j^2 = \frac{\lambda_j^2}{\lambda_j^2 + \sigma_j^2}$$

Each squared correlation coefficient corresponds to the proportion of variation in the manifest variable, j, that is explained by the latent human development factor. In Table 3, we compare the normalized marginal weights of each manifest variable from the official HDI to the normalized squared correlation coefficients produced by our model.

Table 3. Comparison of HDI Weights and Normalized Squared Correlation Coefficients ρ^2

Variable	HDI Weights (95% CI)	ρ^2 (95% CI)
Life Expectancy at Birth	0.35 (0.31, 0.39)	0.18 (0.17, 0.20)
Mean Years of Schooling	0.30 (0.26, 0.34)	0.27 (0.27, 0.28)
Expected Years of Schooling	0.28 (0.24, 0.33)	0.29 (0.28, 0.30)
GNI per capita	0.18 (0.16, 0.21)	0.25 (0.25, 0.25)

Concerning our results, we find that the "longevity" dimension offers a smaller contribution to human development than the weights of official HDI would suggest. As Anand and Sen (1994) discuss, differences in the HDI ranks of high development level countries are largely driven by minor changes in relative life expectancy as their values for the other inputs are largely similar. In turn, the increased importance of life expectancy at the higher end of the distribution may inflate the relative weight placed on the "longevity" dimension by official HDI. Our model also attributes a much greater contribution to the "living standard" dimension when compared to official HDI. Additionally, while the official HDI assigns a greater proportional contribution to "mean years of schooling" than "expected years of schooling," our model estimates that the opposite is true. Therefore, under the assumptions of our model, these results suggest that the available data may not support the deterministic weights used to calculate official HDI. If this is indeed the case, the HDI's rankings may bias our understanding of relative human development levels across countries, which in turn could impact both international and national level policies targeting human development. On the other hand, we find that each of the official HDI's manifest variables provides considerable contributions to human development, indicating that the human development theory guiding the variable selection process is supported by both models.

4.4 The most and least developed countries

One of the HDI's primary purposes is identifying countries with both the highest and lowest levels of human development. Distinguishing countries with best practices establishes role models for other nations while identifying the least developed countries has significant economic and policy implications for nations with lower levels of human development. Since comparing relative performance is so important, it again becomes a potential concern that the official HDI offers only a single rank for each country as opposed to a plausible range of values. The lack of uncertainty can be especially detrimental to countries falling just outside the lowest levels of human development, as it may disqualify them from participating in beneficial international assistance programs if their official HDI rank does not meet a program's requirements. Given that our method produces distributions of ranks, we can estimate and assign probabilities for each country to be within the most and least developed groups.

In Figure 2 we present the estimated likelihood of certain countries being among the ten most developed countries using our model along with their official HDI rankings. Of the 187 countries, 17 have non-zero probabilities of being included in our model's "Top 10." Of these 17 countries, seven are not among the "Top 10" according to their official HDI ranks, implying that they may be overlooked when evaluating the successful actions of role model nations. In Figure 3 we present the likelihood of certain countries being among the ten least developed countries using our model along with their official HDI rankings. Of the 13 countries which have non-zero probabilities associated with being included in our model's "Bottom 10," five are not listed among the "Bottom 10" according to official HDI. Mozambique, the Democratic Republic of Congo, and Burundi, all of which are members of the official HDI's "Bottom 10," have zero probability of being in the "Bottom 10" produced by our model. Properly identifying countries with the lowest levels of human development is especially relevant to the policymakers and government officials tasked with making foreign aid distribution decisions regarding at-need nations.

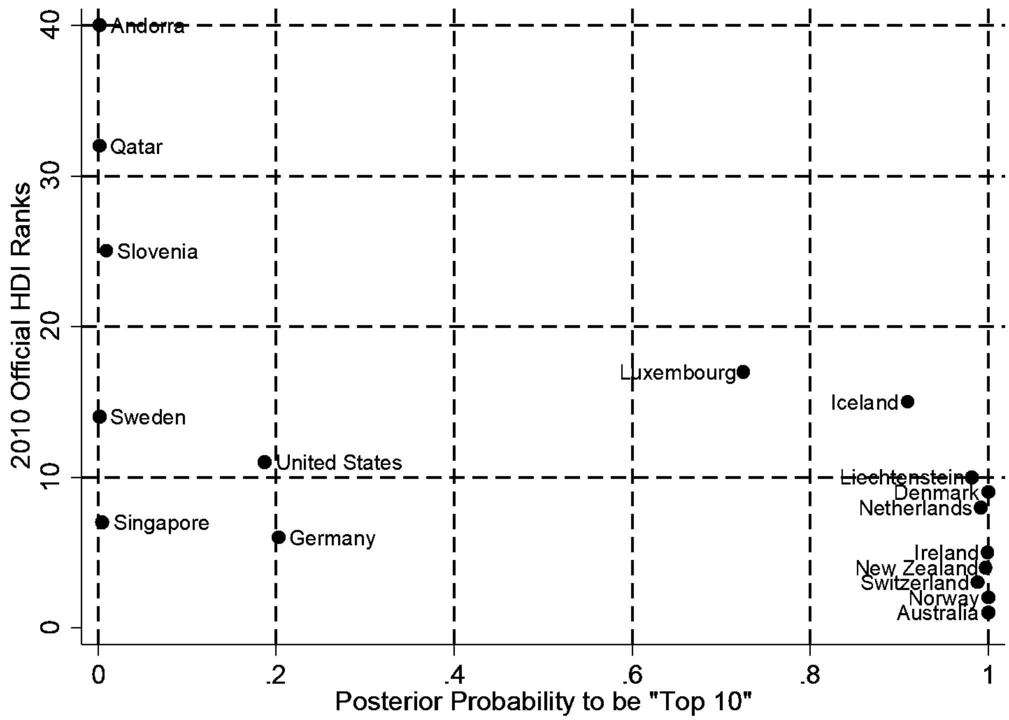

Figure 2. The Probability to be Model Based "Top 10" vs. Official HDI Ranks

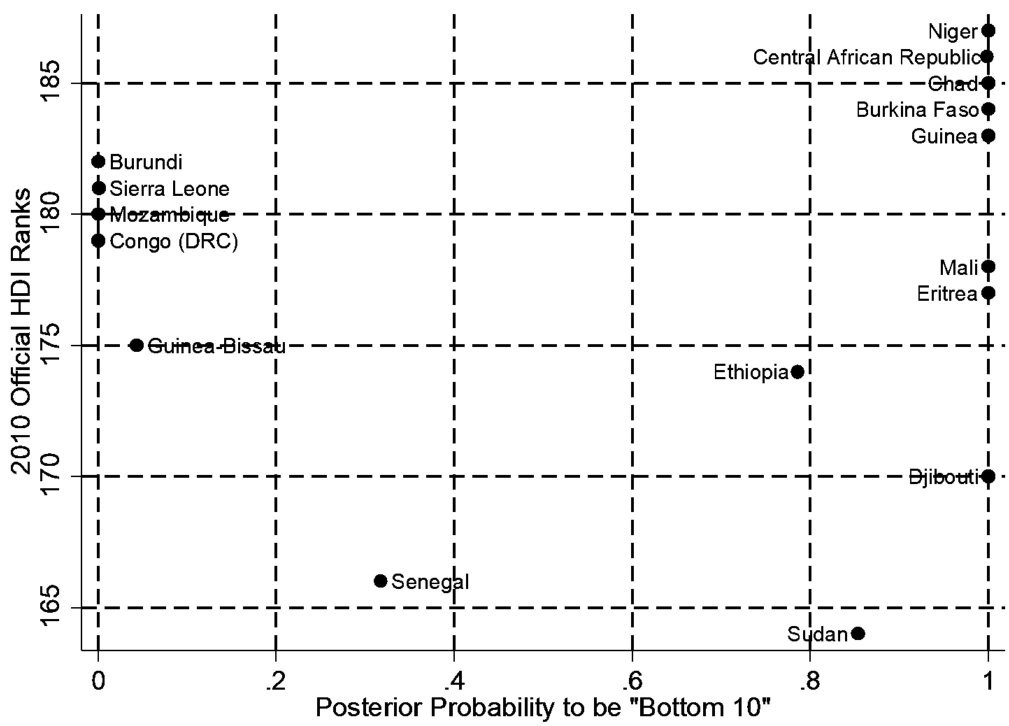

Figure 3. The Probability to be Model Based "Bottom 10" vs. Official HDI Ranks

4.5 Alternative variables specifications

Given the ease with which variables can be interchanged in our model, we now present estimations of human development using the four manifest variables of official HDI in combination with three additional variables: The Environmental Performance Index (EPI), Income Quintile Ratios (QR's), and Satellite Observed Light (SOL). These three new variables are meant to represent dimensions of human development that current theory believes to be relevant but the official HDI may not capture.[26] More specifically, EPI accounts for a nation's level of environmental health and sustainability, QR represents income inequality, and SOL provides an objective measure of night light. In combination with the official HDI's manifest variables, we first estimate our model with each new variable added separately, and then combined within a single model. As an additional way of incorporating human development theory into our model, we also place a negative and informative prior distribution on the factor loading of QR. Under our Bayesian specification, having a negative and informative prior on the relationship between QR and human development captures the *a priori* theoretical assumption that higher income inequality should negatively reflect a country's level of human development. While estimation with either the informative or non-informative prior produces a negative factor loading for QR, with the informative prior distribution of $\lambda_{QR} \sim N(-10{,}0.1)$, we pull the posterior mean and standard deviation of our estimated factor loading to -0.24 and 0.06 respectively, compared to the posterior mean and standard deviation of -0.04 and 0.04 which we find when using a conjugate non-informative prior.[27] Prior distributions on the factor loadings of EPI and SOL remain non-informative, and both are estimated to positively

[26] See UNDP (2007), UNDP (2010), Elvidge et al. (2012), and Ghosh et al. (2013) and the studies they discuss for information regarding the relationship between environmental stewardship, inequality, and night light with human development, respectively.
[27] A more extreme approach to incorporating prior theoretical beliefs into our model is restricting parameters to fall only within a certain range of values, but we do not illustrate this in our study.

reflect human development, implying that better environmental stewardship and more satellite observed night light correspond to higher levels of human development.

Comparisons between human development rankings using the official HDI and our model with EPI, QR, and SOL, are shown in Figures C.1(a), C.1(b), and C.1(c) of Appendix C, respectively. Figure C.2 of Appendix C compares the official HDI ranks to the results of our model when including the four HDI manifest variables and all three alternative variables simultaneously. As Figures C.1 and C.2 show, there is relatively little visible movement in the mean ranks or confidence intervals for each country between model specifications.

Table C.1 of Appendix C provides a comparison of each variable's weight under the official HDI and the normalized squared correlation coefficients produced by our model across all four alternative specifications. As the squared correlation coefficients show, our model estimates that the four manifest variables used by the official HDI capture the greatest proportion of variation in human development across alternative specifications. When added separately, EPI accounts for roughly 9% of the variation in human development, while QR and SOL are both estimated to account for 4%. Interestingly, the squared correlation coefficient on LE varies with the addition of EPI, but not with QR or SOL. This relationship seems intuitive when considering that both LE and EPI are meant to directly capture aspects of health, while QR and SOL are less likely to do so. Including QR in the model leads to a decrease in the squared correlation coefficients of both education variables, but not GNIpc. This change implies that the variation in human development captured by QR (but not GNIpc) is potentially related to the relative contribution of education when income inequality varies. Including SOL decreases the effect of MYS, EYS, and GNIpc when added into the model separately, supporting the assumption that night light may represent features related to the shared relationship between electrification, education, and econonomic activity. When included simultaneously, our model estimates that EPI, QR, and SOL account for roughly 16% of the total variation in human development, while LE, MYS, EYS, and GNIpc account for the remaining 84%. While 16% is a nontrivial share, the four manifest variables used to calculate official HDI identify the majority of a country's human development level. Therefore, the issue regarding human development's measurement may be one of estimation method more so than variable selection. Comparing the level of discordance between the results of our base model and those of our model including EPI, QR, and SOL shows that only one country (Kiribati) has an absolute difference in rank greater than five.

4.6 Results for MDG index

We now present the results of our model estimated using manifest variables from the Millenium Development Goals (MDG). Initially, we construct our MDG index using a naïve imputation process to estimate any missing data. We also estimate the index using posterior imputation, the results of which are disscussed in Section 5. In Figure 4, we compare the posterior mean ranks of our MDG index with the ranks of official HDI using the naïvely imputed data. Figure 4 shows a positive association between the ranks of our "MDG index" and those of the official HDI which we would expect even between measures of human development using different variables.[28]

Because the MDG index includes both a greater number of variables and variables which the official HDI does not use, it naturally produces a higher level of discordance with the official HDI compared to our main results or those obtained from our alternative specifications built around the four HDI manifest variables. More specifically, the sum of absolute differences between our model

[28] The correlation between the posterior mean ranks of our MDG index and the ranks of official HDI is roughly 0.95.

based HDI ranks and the ranks of official HDI is 1,439, while the sum of absolute differences between the ranks of our MDG index and the official HDI is 2,448.[29] Referencing the top-right corner of Figure 4 for a visual example of the discordance between the two indices, Equatorial Guinea, Congo, Zambia, Kenya, and Swaziland, none of which fall into the lowest development quintile of official HDI, are all located in the lowest development quintile of our MDG index. Therefore, the official HDI may overestimate the development levels of these countries under the assumptions of our MDG index. Given that the MDG's variables focus more on developing countries, a driving factor of this discordance may be information regarding the relative performance of low development level countries across dimensions which are captured by our MDG index but not by the HDI's manifest variables. Looking to the bottom-left corner of Figure 4, Brunei, Qatar, and the United States are all ranked outside of the most developed quintile of our posterior MDG ranks while they are included in the most developed quintile of the official HDI. Therefore, it is possible that the official HDI overestimates the development level of these countries given our findings. We also estimate the total level of uncertainty in ranks produced by our MDG index to be lower than our estimations using the HDI manifest variables which is most likely the result of including a greater number of total variables in the model.[30]

Table 4 presents the normalized squared correlation coefficients for the variables of our MDG index and the sign of their factor loadings. Our MDG index suggests that maternal mortality (MMR) and child mortality (U5MR) account for the greatest shares of variation in human development. The relative contributions of MMR and U5MR are both in line with the assumption that an untimely death represents the worst-case human development scenario for individuals

[29] The average absolute difference in ranks between the official HDI and MDG index is 13.1.
[30] For example, the average standard deviation of ranks for our MDG index is 1.58 compared to an average standard deviation of 2.25 for our model based HDI.

under a human outcomes focused theory of development. Alternatively, the adult HIV rate (HIV) and employment-to-population ratio (ETP) account for the smallest amounts of variation in human development according to our MDG index.

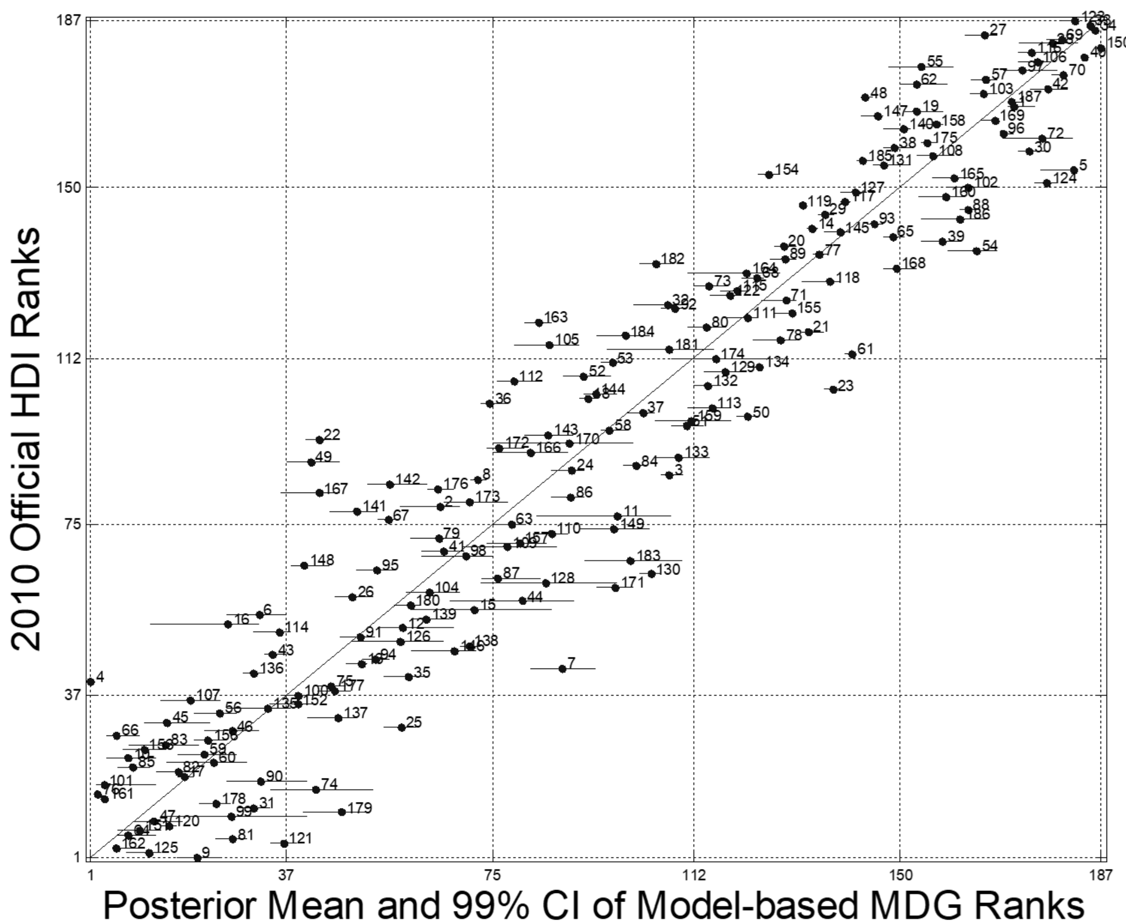

Figure 4. Posterior Mean and 99% CI of Model Based MDG Ranks Using PMM vs. Official HDI Ranks

Table 4. MDG Normalized Squared Correlation Coefficients ρ^2 and Signs of Factor Loadings λ Using PMM

Variable	ρ^2 (95%CI) With Naïve Imputation	Sign of λ
TELE	0.15 (0.14, 0.16)	+
TB	0.11 (0.10, 0.11)	-
U5MR	0.17 (0.16, 0.20)	-
WATER	0.11 (0.10, 0.11)	+
MMR	0.17 (0.15, 0.19)	-
PU	0.08 (0.08, 0.09)	-
GPI	0.03 (0.01, 0.04)	+
NER	0.06 (0.05, 0.07)	+
HIV	0.02 (0.01, 0.03)	-
ETP	0.00 (0.00, 0.01)	+
ABR	0.10 (0.09, 0.10)	-

5. Sensitivity Analysis

In this section, we explore the sensitivity of our model's results across four separate dimensions. First, we examine the change in our results using an alternative specification of spatial correlation based on trade rather than geographical boundaries. Second, we evaluate the roles of ordinality and cardinality in our model by estimating human development using the ranks, rather than raw values, of each country's observed outcomes.[31] Third, we evaluate the sensitivity of our model to imputation procedure by comparing the results of our MDG index under both PMM and posterior imputation. Finally, we evaluate the performance of both our model and the official HDI in a simulated data exercise where our model is assumed to capture the correct data generating process.

[31] We would again like to thank an anonymous reviewer for suggesting this test of our model.

5.1 Estimation with an alternative spatial correlation structure

Spatial correlation plays a significant role in our model as it allows for the estimation of spillover effects in human development across countries. We estimate the results presented in Section 4 under a framework of spatial correlation where countries are considered "neighbors" if they share a common land or maritime border. This method of defining neighbors captures the geographic clustering of similar development level countries observed in the data (i.e. many low-development level countries in sub-Saharan Africa or many very-high development level countries in Europe). On the other hand, since the transfer of ideas and technologies related to human development is not restricted to countries sharing a common geographical border, a logical alternative is a spatial correlation framework based on trade flows between countries.

To evaluate the sensitivity of our model to changes in spatial correlation structure, we re-estimate our primary results using an alternative "neighbor matrix" W such that for two countries i and j, $W_{ij} = W_{ji} = \frac{Exports_{ij}}{GDP_i + GDP_j}$, where $Exports_{ij}$ is the sum of total exports shared between both countries, and GDP_i and GDP_j are country i and j's total GDP, respectively. If i and j are not trading partners, $W_{ij} = W_{ji} = 0$. Finally, $W_{ii} = 0$. Figure C.3 of Appendix C shows the correspondence between the ranks of official HDI and the ranks of our model under this trade-based framework of spatial correlation. As Figure C.3 illustrates, the estimated ranks of our model using trade-based spatial correlation correlate well with those of official HDI. Comparing the results across models more closely, the average absolute deviation between the ranks of official HDI and our trade-based model is 6.56.

Table C.2 of Appendix C compares the normalized squared correlation coefficients of our trade-based model with those of our primary results. As the table shows, both specifications of spatial correlation produce almost identical estimations of the covariance in human development explained by each manifest variable. Comparing both specifications' posterior means and confidence intervals of the model's other parameters shows that the only non-trivial difference comes from the spatial correlation parameter, ω, which has a mean posterior value of 0.126 in our primary model and 4.25 in our trade-based specification. Of course, variation in ω between models is expected when altering the underlying spatial correlation structure. Given these results, we conclude that our model is generally robust to spatial correlation specifications regarding how countries are related with one another.

5.2 Estimation using ranks of manifest variables

Given that the outcome we are most interested in estimating is the human development ranks for each country, the roles of cardinality and ordinality in our model are of particular importance. One dimension by which we can examine this is to compare the results of our model using the cardinal (raw) values of each manifest variable to the results we find using ordinal (rank) values. By converting manifest variable values into ranks, each country's performance is evaluated only with regards to their relative standing rather than the magnitude to which their manifest variable values differ. Naturally, the use of ranks preserves order, but it also limits the effect of outlier countries in variables like GNIpc which harbor high degrees of variation.

Figure C.4 of Appendix C presents the correspondence between the ranks of official HDI and our primary model using rank values of each manifest variable. Compared to the results of our model using the raw manifest variables shown in Figure 1, the most obvious change between the raw and ranked models' correspondences with official HDI is in the top-right-hand corner for the lowest development level countries. Comparing the raw and rank manifest variable model rankings with official HDI more formally, however, we find the average absolute deviation in ranks to be almost identical, at 7.72 and 7.79, respectively.[32] Alternatively, while the average levels of absolute discordance between models are nearly identical, some countries with outlier values for their manifest variables see considerable changes in their posterior ranks. This result implies that using rankings of manifest variables may abstract away from potentially valuable information coming from cardinal relationships in the data. Table C.3 of Appendix C presents the normalized squared correlation coefficients of our model when using raw manifest variable values and rank manifest variable values. Comparing the raw and rank models, Table C.3 shows that MYS and GNIpc see the most notable change in their estimated contribution to human development. Given that the amount of discordance between both model specifications and the official HDI change only trivially, however, we conclude that the rankings of our model are generally robust to the ordinal transformation of manifest variables.

5.3 Results using posterior imputation

Following the naïve imputation method used to predict the missing MDG data, we next formulate our MDG index using the posterior imputation process built into our model. As spoken to in previous sections, a substantial quantity of data is missing for the MDG index manifest variables, implying that they must be imputed before estimation. For our main results, we use these imputed values as data without accounting for the inherent uncertainty of the imputation process. As an alternative, we now incorporate the imputation of missing data into the estimation algorithm.

[32] The standard deviation for the absolute deviations of both models are also nearly identical, at 7.04 for the raw model and 7.11 for the rank model.

Unlike our naïve imputation method, posterior imputation draws from a posterior distribution of missing values during each iteration of the sampler. We present the results of our MDG index under posterior imputation graphically in Figure C.5 of Appendix C.

While the posterior mean rank for most countries remains stable, the uncertainty of rankings following posterior imputation appears much larger for some countries when compared to the uncertainty of the naïve imputation results. More specifically, the more missing data a country has, the more uncertainty it will show following posterior imputation. This relationship leads countries like Liechtenstein and Hong Kong to have extreme confidence intervals compared to the average. Additionally, higher levels of missing data increase the magnitude of separation between a country's naïve and posterior imputation mean ranks.

Formally measuring the amount of discordance between our model under the two imputation processes with the official HDI, we see an increase in the sum of squared differences in rank from 51,410 to 64,890 using posterior imputation, a change of roughly 26%. While the sum of squared differences increases considerably following posterior imputation, the sum of absolute differences remains relatively unchanged (a 3% increase from 2,444 to 2,522).[33] This result implies that several outlier countries see a considerable change in rank between the two imputation methods while the general discordance changes a comparably small amount for countries with less missing data.

Table C.4 of Appendix C shows the normalized squared correlation coefficients of our MDG index under both naïve and posterior imputation. Our model still estimates that MMR and U5MR account for the greatest proportion of covariance in human development, while NER, HIV, and

[33] The average of absolute differences in rank increases from 13.07 to 13.49, and the standard deviation of absolute differences between ranks increases from 10.2 to 12.9.

ABR see the most significant amount of relative change between both models. Since posterior imputation extrapolates the non-missing relationship between a manifest variable and the latent factor onto the missing data, changes in the magnitude of each squared correlation coefficient reflect the observed effect's strength. For example, the increased effect of HIV under posterior imputation suggests that HIV is highly reflective of human development in the non-missing portion of our data. Using non-Bayesian methods to estimate an MDG index would force practitioners to rely on naïve imputation and potentially miss the prominent level of contribution variables like HIV expresses in the non-missing data.

5.4 Simulated data exercise

Since both our model and the official HDI are inherently incomplete models of human development, it is important that we better understand the relative capabilities of each approach. One way to evaluate the performance of both models is using a simulation where we can directly specify the true underlying relationship of the data. More specifically, we use the posterior mean parameter values estimated with our model under real data to simulate a set of data for each of the official HDI's four manifest variables using the assumed data generating process of our model. The outcome of our simulation is a set of artificial data for all 187 countries which we know matches the assumptions of our model.

Comparing the results of official HDI to our model using simulated data shows that the level of average absolute deviation between ranks increases by nearly a factor of three.[34] Furthermore,

[34] The average absolute deviation in ranks between the official HDI and our model goes from 7.7 using real data to 22.0 using simulated data.

our model can estimate each data generating parameter to within one standard deviation of the posterior mean. We present the data generating parameter values and the estimated posterior means and standard deviations in Table C.4 of Appendix C. Given that the official HDI is a direct measure as opposed to a model which assumes an underlying data generating process, one can interpret our results using real data under the assumption that the official HDI is the "correct" measure of human development. Under this assumption, the results discussed in Section 4 show that the posterior mean ranks of our model are well correlated with the ranks of official HDI. Under the opposite assumption that our model is correct, the simulated data exercise shows that the official HDI is not able to achieve a similar level of agreement using data for which we know our model identifies the true data generating process. This result suggests that our model is more flexible when estimating human development using data generated from different sources relative to the official HDI.

6. Conclusion

In this paper, we propose a Bayesian factor analysis model which serves as both an alternative approach to calculating the UNDP's Human Development Index and a general methodology which can be used to either augment existing indices or build new ones. We address several technical issues of the official HDI in the following ways. First, our model produces data-driven weights for each manifest variable's contribution to the latent factor of human development. Informing weights with observed data stands in contrast to the *ad hoc* factor weights used to calculate the ranks of official HDI. Second, our model estimates its ranks in terms of distributions, allowing for a measure of uncertainty which is absent from the official HDI. This measure of uncertainty provides a more holistic view of relative performance across countries. Finally, we adjust the uncertainty in ranks by incorporating a measure of spatial correlation between countries while also including country populations in our estimation. These additions improve the precision of our rank distributions and allow for the estimation of spillover effects in human development.

Using our model to estimate human development with the same observed variables as the official HDI, we find that the “living standard” dimension provides a greater proportional contribution to human development than it is assigned by the official HDI, while the “longevity” dimension provides a lower proportional contribution. The results of our model also show considerable levels of disagreement when compared to the ranks of the official HDI. Under our model, it is is not uncommon for the confidence intervals of country ranks to cover more than one quintile of human development level. Therefore, a country’s relative performance according to the rankings of our model may vary considerably when compared to its relative performance according to the official HDI.

Aside from its technical advantages, we show the flexibility of our methodology by estimating human development with three additional variables not used in the official HDI and by creating a novel MDG index using data from the Millennium Development Goals. As our alternative specifications illustrate, sets of variables can easily be added or removed from our model without fundamentally restructuring its estimation. This stands in contrast to the HDI’s rigidity with respect to variable selection which makes the addition or removal of information impractical. We find that EPI, QR, and SOL explain roughly 16% of the total variation in human development when estimated along with the official HDI manifest variables. Under the assumptions of our model, this result implies that the alternative variables account for variation across dimensions of human development not captured by the official HDI. Therefore, our model supports the use of alternative human development indices such as the Inequality Adjusted HDI, Gender Inequality Index, and Multidimensional Poverty Index proposed by the 2010 HDR (UNDP, 2010). As opposed to the official HDI and its related indices, however, our model provides a convenient framework for measuring an index using different manifest variables. Additionally, even with the complicated structure of the MDG's indicator variables, we show that our approach is suited to constructing the desired index. The MDG index not only exemplifies the adaptive nature of our methodology, but also provides a blueprint which researchers can follow to build indices that may have previously seemed too complex. The results of our MDG index suggest that mother and child mortality outcomes explain the greatest proportions of covariance in human development. This finding is supported by the assumption that early death represents one of the most severe and adverse outcomes for countries under the human-centered theory of development that measures like the HDI are meant to represent. Future studies of human development may wish to examine the effect of these mortality measures by incorporating them directly in models of development as opposed to relying on the official HDI's life expectancy measure.

We also evaluate the sensitivity of our model across several dimensions. First, we estimate human development ranks using an alternative specification of spatial correlation built on trade rather than geographical borders. On average, we find that our estimates remain stable across specifications, implying that our model is robust to different definitions of spatial correlation. Second, to evaluate the role of cardinality and ordinality in our model, we estimate human development using the rankings of each manifest variable as opposed to their raw values. The squared correlation coefficients of our model change across specifications, but the average absolute deviation in ranks between the official HDI and our model remains nearly constant. Alternatively, some outlier countries see relatively significant changes in their rank using the rank model, implying that we may be losing information captured by differences in magnitude when not using raw variable values. Third, to account for the inherent uncertainty of imputation in our estimation of the MDG index, we compare the results of our model using both naïve and posterior imputation. We find that posterior imputation leads to an increase in the discordance between our ranks and the ranks of official HDI for some countries with substantial amounts of missing data, but minimal movement in the discordance on average. Finally, since both the official HDI and our model are incomplete measures of human development, we perform a simulated data exercise where our model assumes the correct data generating process. In our simulated exercise, the average absolute difference between the ranks produced by the official HDI and our model increase by nearly a factor of three compared to results when using real data. This result implies that while the rankings of our model are very close on average to those of the official HDI under the assumption that the HDI is "correct" when using real data, the official HDI is not able to do the same in a simulation where the assumption is reversed.

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Appendix A: Gibbs Sampler Algorithm

Following Hogan and Tchernis (2004), our hierarchical factor analysis model is as follows:

$$Y|\delta \sim N(\mu + \Lambda\delta, M^{-1} \otimes \Sigma)$$

$$\delta \sim N(0, M^{-\frac{1}{2}}\boldsymbol{\psi}M^{-\frac{1}{2}})$$

where:

$\mu = [\mu_1, \mu_2, \mu_3, \mu_4]'$;

$\Lambda = I_N \otimes \lambda$, with $\lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4]'$;

$\boldsymbol{\psi} = (I - \omega W)^{-1}$;

$\Sigma = diag(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2)$ with all the off-diagonal elements equal to 0.

Therefore, the parameters to estimate are λ, δ, μ, Σ and ω.

Step 1: Sample elements of λ.

Let 1_N be an $N \times 1$ vector with all elements equal to 1. Therefore, for each $\lambda_j, j = 1,2,3,4$, let the estimation equation be $Y_j - 1_N{}'\mu_j = \lambda_j\delta + \varepsilon_j$, where Y_j is the $N \times 1$ vector of manifest variable Y_{ij}, and $\varepsilon_j \sim N(0, \sigma_j^2/M)$. Let the prior distribution be $\lambda_j \sim N(a, A)$, where $a = 0, A = 1000$. Hence, the posterior of λ_j is drawn from conditional distribution $N(b, B)$, where:

$$B = \left(1/A + \delta' M\delta/\sigma_j^2\right)^{-1}$$

$$b = B[a/A + \delta' M\left(Y_j - 1_N\mu_j\right)/\sigma_j^2]$$

As factor loadings, λ_j's, are restricted to be positive.

Step 2: Sample δ.

Let the estimation equation be $Y - \mu \otimes 1_N = \Lambda\delta + \varepsilon$, where Y is the $NJ \times 1$ vector of manifest variable Y_{ij}, and $\varepsilon \sim N(0, M^{-1} \otimes \Sigma)$. Given that the prior distribution is $\delta \sim \mathrm{N}(0, M^{-\frac{1}{2}}\boldsymbol{\psi}M^{-\frac{1}{2}})$.

Hence, the posterior of δ is drawn from conditional distribution $N(d, D)$, where:

$$D = \left[\left(M^{-\frac{1}{2}}\boldsymbol{\psi}M^{-\frac{1}{2}}\right)^{-1} + \Lambda'(M^{-1} \otimes \boldsymbol{\Sigma})^{-\mathbf{1}}\Lambda\right]^{-1}$$

$$d = D[\Lambda'(M^{-1} \otimes \Sigma)^{-\mathbf{1}}(Y - \mu \otimes 1_N)]$$

Step 3: Sample elements of μ.

For each $\mu_j, j = 1,2,3,4$, let the estimation equation be $Y_j - \lambda_j\delta = 1_N\mu_j + \varepsilon_j$. Let the prior distribution be $\mu_j \sim N(c, C)$, where $c = 0, C = 1000$.

Hence, the posterior of μ_j is drawn from conditional distribution $N(e, E)$, where:

$$E = \left(1/C + 1_N{}'M1_N/\sigma_j^2\right)^{-1}$$

$$e = E[c/C + 1_N{}'M\left(Y_j - \lambda_j\delta\right)/\sigma_j^2]$$

Step 4: Sample elements of Σ.

For each $\sigma_j^2, j = 1,2,3,4$, let the estimation equation be $Y_j = \lambda_j\delta + 1_N\mu_j + \varepsilon_j$. Let the prior distribution be $\sigma_j^2 \sim IG(\alpha_0, \beta_0)$, where $\alpha_0 = 0.001, \beta_0 = 0.001$.

Hence, the posterior of σ_j^2 is drawn from conditional distribution $IG(\alpha_1, \beta_1)$, where:

$$\alpha_1 = \alpha_0 + \frac{N}{2}$$

$$\beta_1 = \left(Y_j - \lambda_j\delta - 1_N\mu_j\right)' M\left(Y_j - \lambda_j\delta - 1_N\mu_j\right) + \beta_0$$

Step 5: Sample ω using a Metropolis-Hasting algorithm.

Let the prior distribution of ω be $\pi(\omega) = N(0{,}1000)I(\xi_1^{-1} < \omega < \xi_N^{-1})$, where ξ_1 and ξ_N denote the minimum and maximum eigenvalues of the spatial correlation matrix W. Hence, the target density of ω is $f(\delta|\boldsymbol{\psi}(\omega))\pi(\omega)$, where $f(\delta|\boldsymbol{\psi}(\omega))$ is the kernel of the distribution of δ conditional on $\boldsymbol{\psi} = (I - \omega W)^{-1}$. Let the proposal density be $q(\omega'|\omega) \sim N(\omega, \rho^2)$, so that the candidate ω' is drawn from a random walk equation: $\omega' = \omega + \epsilon$, where $\epsilon \sim N(0, \rho^2)$, and ρ^2 is a tuning parameter. The generated ω is also restricted to the domain $\xi_1^{-1} < \omega < \xi_N^{-1}$.

Therefore, ω' is accepted with probability:

$$\min\{1, \frac{f\left(\delta\middle|\boldsymbol{\psi}(\omega')\right)\pi(\omega')q(\omega|\omega')}{f\left(\delta\middle|\boldsymbol{\psi}(\omega)\right)\pi(\omega)q(\omega'|\omega)}\}$$

Appendix B: Numbering of Countries

#	Country
1	Afghanistan
2	Albania
3	Algeria
4	Andorra
5	Angola
6	Antigua and Barbuda
7	Argentina
8	Armenia
9	Australia
10	Austria
11	Azerbaijan
12	Bahamas
13	Bahrain
14	Bangladesh
15	Barbados
16	Belarus
17	Belgium
18	Belize
19	Benin
20	Bhutan
21	Bolivia (Plurinational State of)
22	Bosnia and Herzegovina
23	Botswana
24	Brazil
25	Brunei Darussalam
26	Bulgaria
27	Burkina Faso
28	Burundi
29	Cambodia
30	Cameroon
31	Canada
32	Cape Verde
33	Central African Republic
34	Chad
35	Chile
36	China
37	Colombia
38	Comoros
39	Congo
40	Congo (Democratic Republic of the)
41	Costa Rica
42	Croatia
43	Cuba
44	Cyprus
45	Czech Republic
46	Côte d'Ivoire
47	Denmark
48	Djibouti
49	Dominica
50	Dominican Republic
51	Ecuador
52	Egypt
53	El Salvador
54	Equatorial Guinea
55	Eritrea
56	Estonia
57	Ethiopia
58	Fiji
59	Finland
60	France
61	Gabon
62	Gambia
63	Georgia
64	Germany
65	Ghana
66	Greece
67	Grenada
68	Guatemala
69	Guinea
70	Guinea-Bissau
71	Guyana

72	Haiti	112	Moldova (Republic of)
73	Honduras	113	Mongolia
74	Hong Kong	114	Montenegro
75	Hungary	115	Morocco
76	Iceland	116	Mozambique
77	India	117	Myanmar
78	Indonesia	118	Namibia
79	Iran (Islamic Republic of)	119	Nepal
80	Iraq	120	Netherlands
81	Ireland	121	New Zealand
82	Israel	122	Nicaragua
83	Italy	123	Niger
84	Jamaica	124	Nigeria
85	Japan	125	Norway
86	Jordan	126	Oman
87	Kazakhstan	127	Pakistan
88	Kenya	128	Palau
89	Kiribati	129	Palestine, State of
90	Korea (Republic of)	130	Panama
91	Kuwait	131	Papua New Guinea
92	Kyrgyzstan	132	Paraguay
93	Lao People's Democratic Republic	133	Peru
94	Latvia	134	Philippines
95	Lebanon	135	Poland
96	Lesotho	136	Portugal
97	Liberia	137	Qatar
98	Libya	138	Romania
99	Liechtenstein	139	Russian Federation
100	Lithuania	140	Rwanda
101	Luxembourg	141	Saint Kitts and Nevis
102	Madagascar	142	Saint Lucia
103	Malawi	143	Saint Vincent and the Grenadines
104	Malaysia	144	Samoa
105	Maldives	145	Sao Tome and Principe
106	Mali	146	Saudi Arabia
107	Malta	147	Senegal
108	Mauritania	148	Serbia
109	Mauritius	149	Seychelles
110	Mexico	150	Sierra Leone
111	Micronesia (Federated States of)	151	Singapore

152	Slovakia
153	Slovenia
154	Solomon Islands
155	South Africa
156	Spain
157	Sri Lanka
158	Sudan
159	Suriname
160	Swaziland
161	Sweden
162	Switzerland
163	Syrian Arab Republic
164	Tajikistan
165	Tanzania (United Republic of)
166	Thailand
167	Macedonia (FYROM)
168	Timor-Leste
169	Togo
170	Tonga
171	Trinidad and Tobago
172	Tunisia
173	Turkey
174	Turkmenistan
175	Uganda
176	Ukraine
177	United Arab Emirates
178	United Kingdom
179	United States
180	Uruguay
181	Uzbekistan
182	Vanuatu
183	Venezuela (Bolivarian Republic of)
184	Viet Nam
185	Yemen
186	Zambia
187	Zimbabwe

Appendix C: Ancillary Figures and Tables

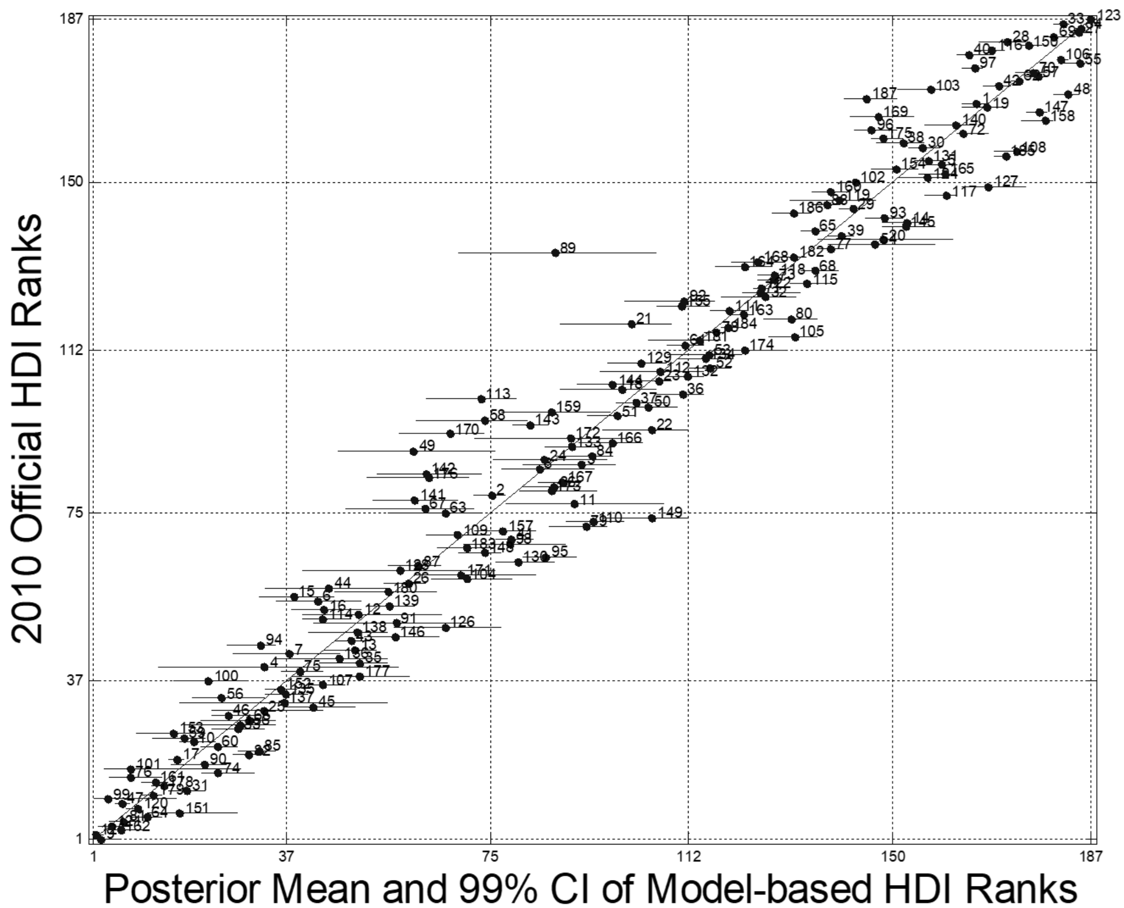

Figure C.1(a). Posterior Mean and 99% CI of Model Based HDI Ranks with EPI vs. Official HDI Ranks

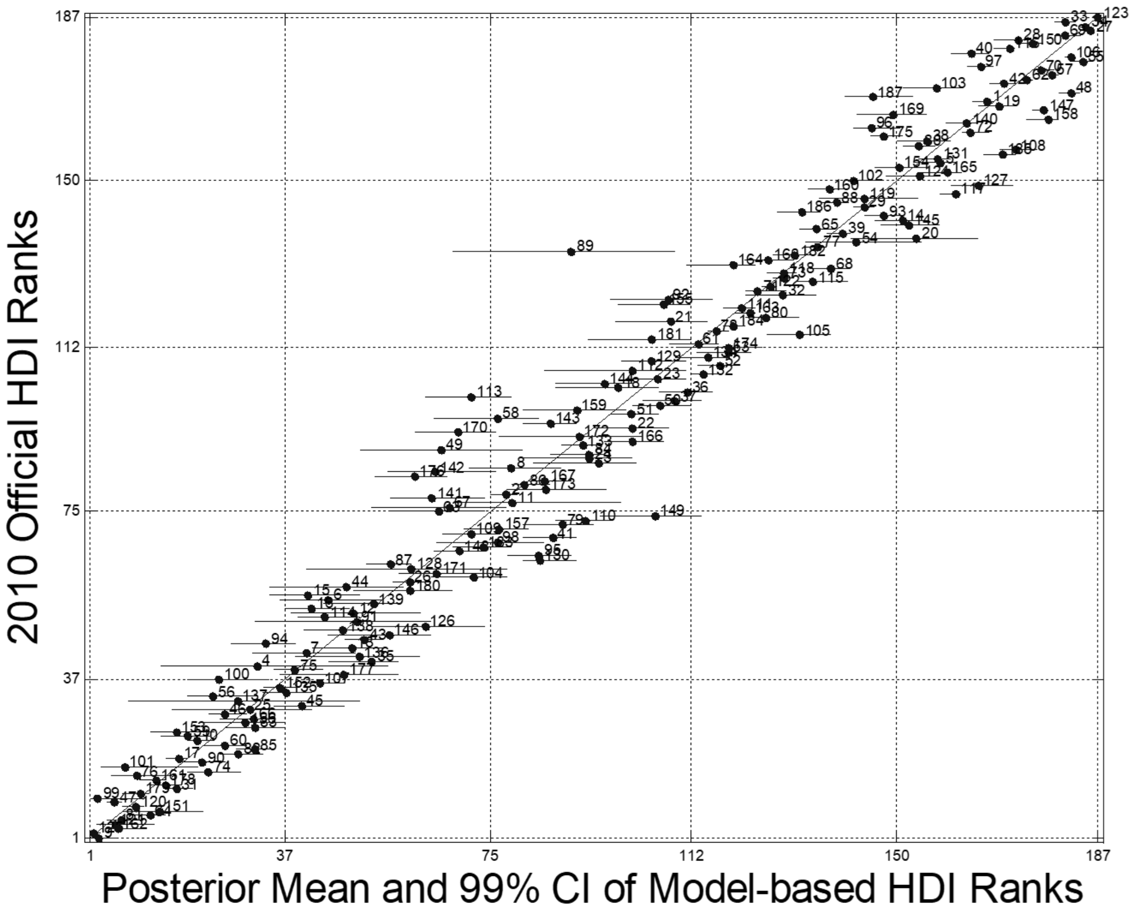

Figure C.1(b). Posterior Mean and 99% CI of Model Based HDI Ranks with QR vs. Official HDI Ranks

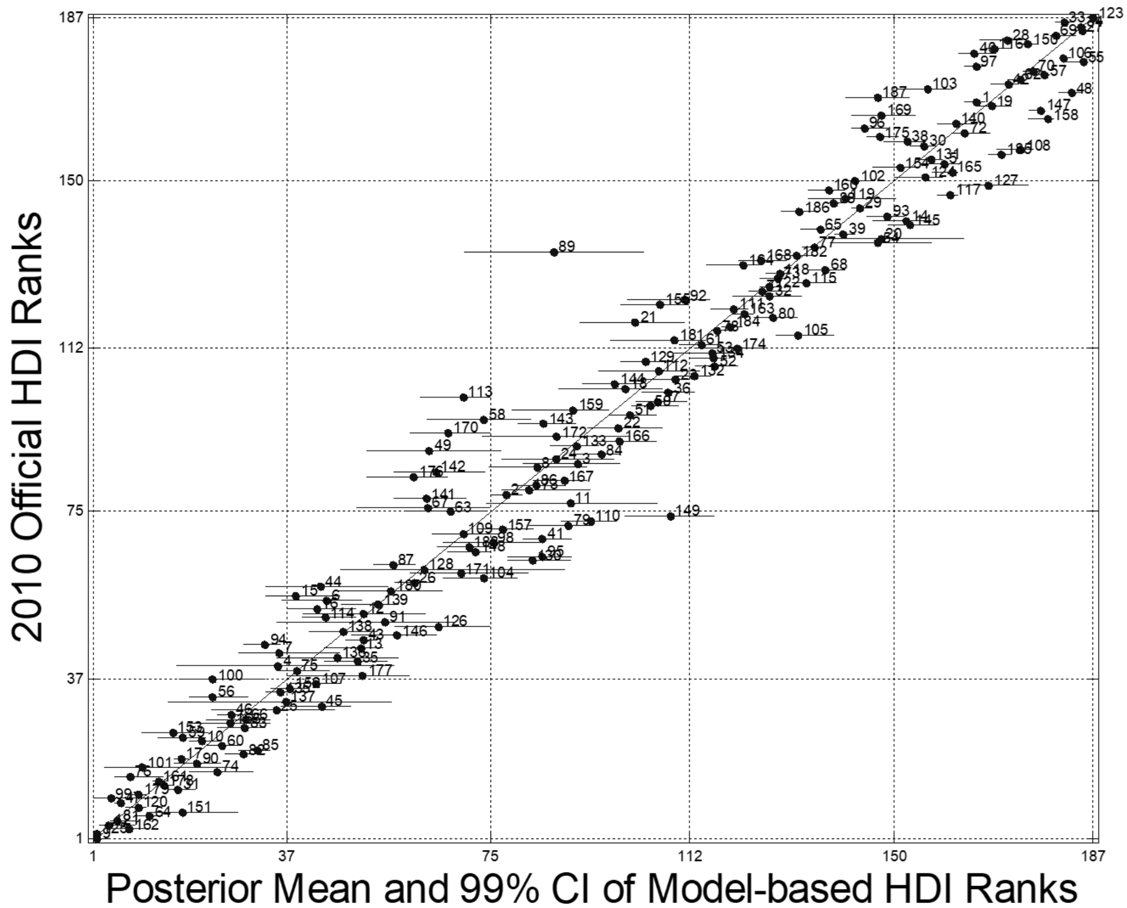

Figure C.1(c). Posterior Mean and 99% CI of Model Based HDI Ranks with SOL vs. Official HDI Ranks

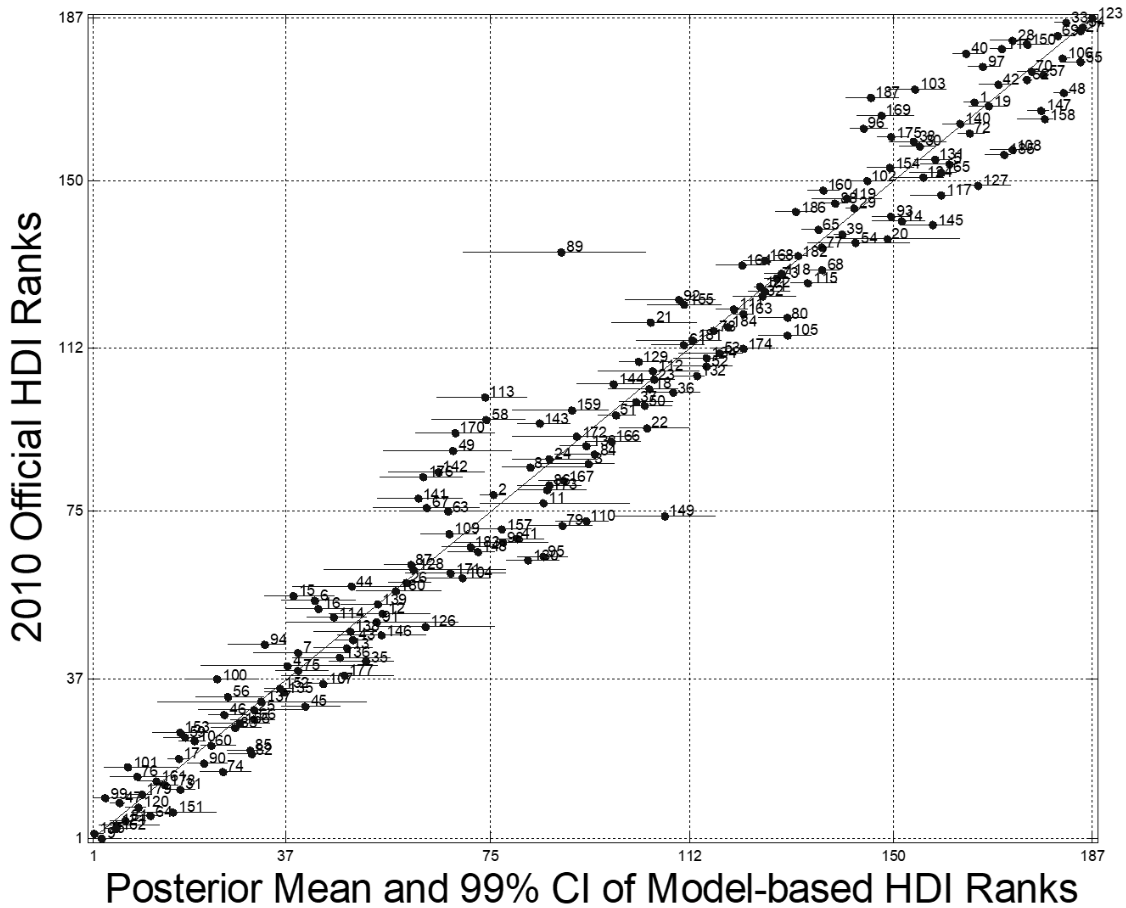

Figure C.2. Posterior Mean and 99% CI of Model Based HDI Ranks with EPI, QR, and SOL vs. Official HDI Ranks

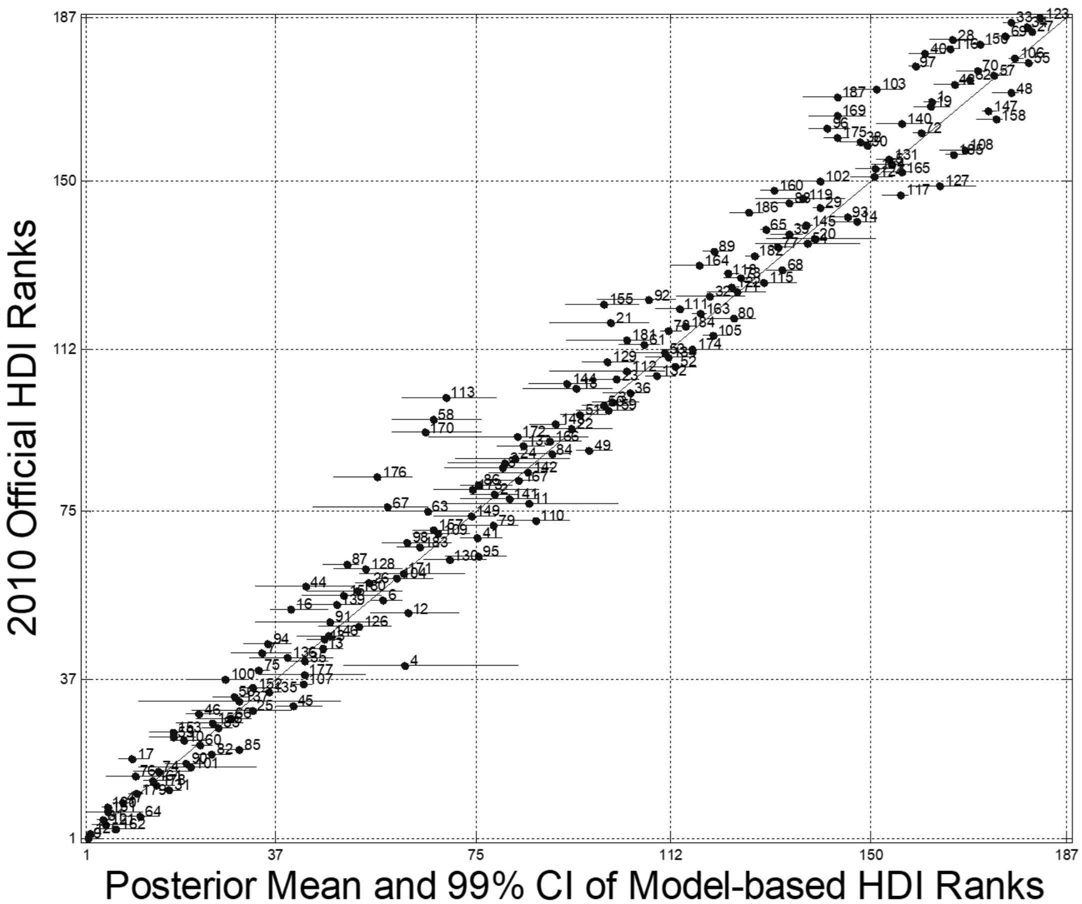

Figure C.3. Posterior Mean and 99% CI of Model Based HDI Ranks with Trade-Based Spatial Correlation vs. Official HDI Ranks

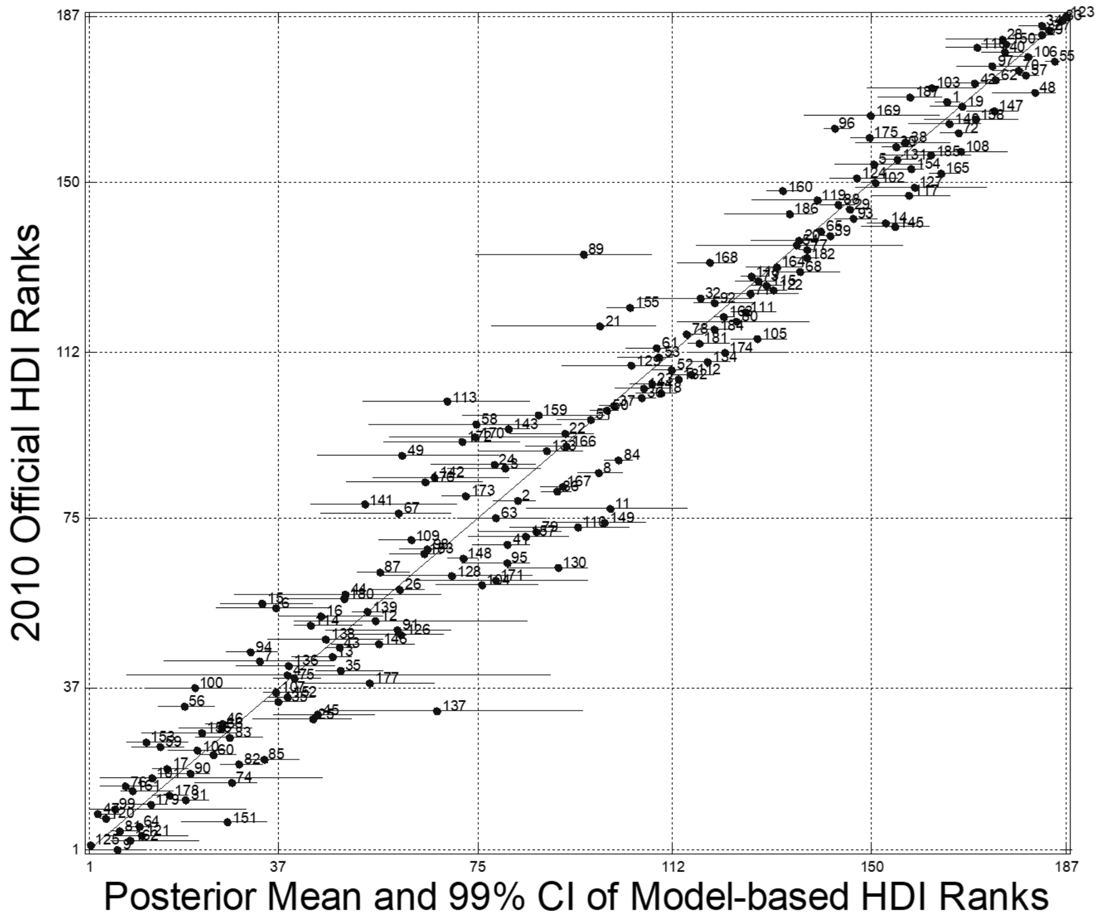

Figure C.4. Posterior Mean and 99% CI of Model Based HDI Ranks with Ranked Manifest Variables vs. Official HDI Ranks

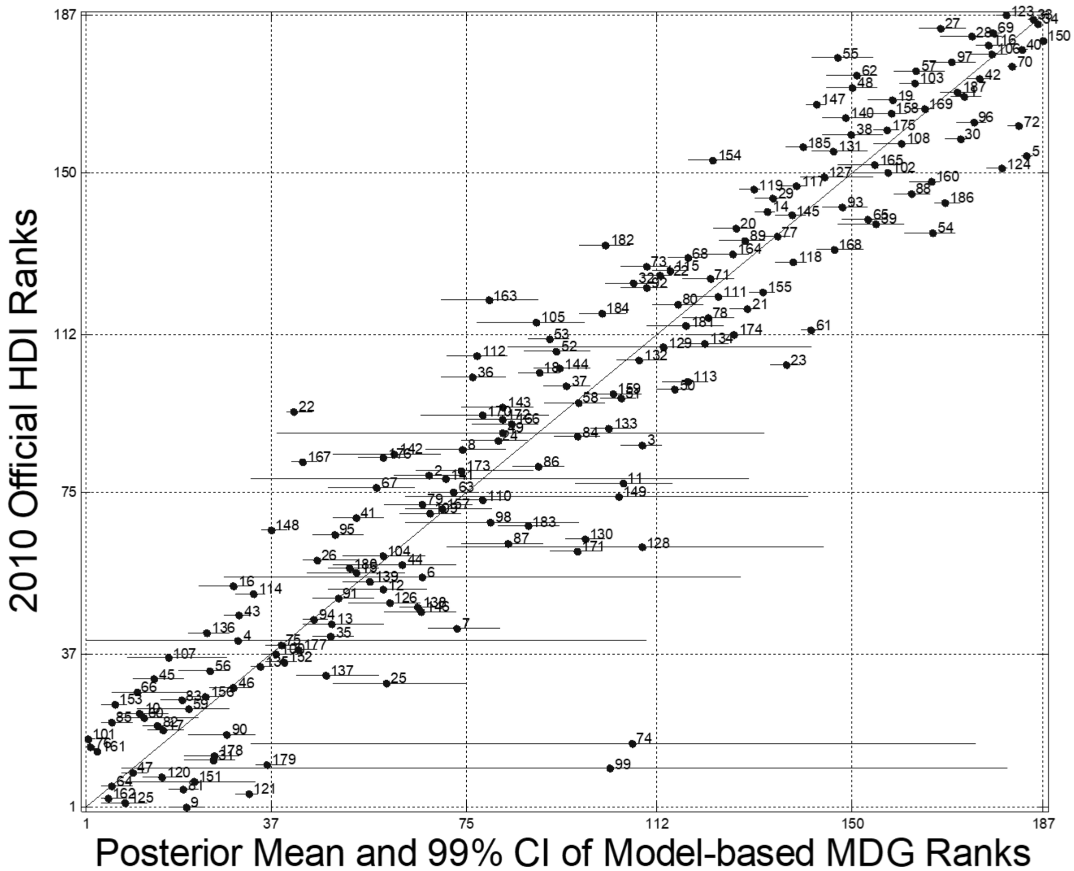

Figure C.5. Posterior Mean and 99% CI of Model Based MDG Ranks Using Posterior Imputation vs. Official HDI Ranks

Table C.1. Comparison of HDI Weights and Normalized Squared Correlations ρ^2 Adding Other Manifest Variables

Variable	HDI Weights (95% CI)	ρ^2(95% CI)				
		Base Model	Added Separately			Added Together
LE	0.35 (0.31, 0.39)	0.18 (0.17, 0.20)	0.17 (0.15 ,0.18)	0.18 (0.17 ,0.19)	0.18 (0.16 ,0.19)	0.16 (0.15 ,0.16)
MYS	0.30 (0.26, 0.34)	0.27 (0.27, 0.28)	0.25 (0.24 ,0.25)	0.26 (0.25 ,0.27)	0.26 (0.25 ,0.27)	0.22 (0.21 ,0.25)
EYS	0.28 (0.24, 0.33)	0.29 (0.28, 0.30)	0.26 (0.25 ,0.28)	0.27 (0.25 ,0.29)	0.28 (0.26 ,0.30)	0.24 (0.21 ,0.26)
GNIpc	0.18 (0.16, 0.21)	0.25 (0.25, 0.25)	0.23 (0.23 ,0.24)	0.25 (0.24 ,0.25)	0.24 (0.24 ,0.25)	0.22 (0.20 ,0.23)
EPI			0.09 (0.07 ,0.11)			0.09 (0.07 ,0.11)
QR				0.04 (0.02 ,0.07)		0.04 (0.02 ,0.06)
SOL					0.04 (0.02 ,0.06)	0.04 (0.02 ,0.06)

Table C.2. Normalized Squared Correlations ρ^2 of Model Based HDI Ranks with Trade-based Spatial Correlation vs. Model Based HDI Ranks with Geographically-Based Spatial Correlation

	ρ^2 (95%CI)	
Variable	**Geographically-Based**	**Trade-Based**
LE	0.18 (0.17, 0.20)	0.18 (0.16, 0.20)
MYS	0.27 (0.27, 0.28)	0.27 (0.27, 0.28)
EYS	0.29 (0.28, 0.30)	0.29 (0.28, 0.30)
GNIpc	0.25 (0.25, 0.25)	0.26 (0.26, 0.26)

Table C.3. Normalized Squared Correlations ρ^2 of Model Based HDI Ranks Using Raw Manifest Variable Values vs. Model Based HDI Ranks Using Ranked Manifest Variable Values

	ρ^2 (95%CI)	
Variable	**Raw Values**	**Ranked Values**
LE	0.18 (0.17, 0.20)	0.17 (0.15, 0.18)
MYS	0.27 (0.27, 0.28)	0.23 (0.23, 0.23)
EYS	0.29 (0.28, 0.30)	0.31 (0.31, 0.33)
GNIpc	0.25 (0.25, 0.25)	0.29 (0.28, 0.29)

Table C.4. MDG Normalized Squared Correlation ρ^2 and Signs of Factor Loadings λ Using PMM vs. Posterior Imputation

Variable	ρ^2 (95%CI)		Sign of λ
	With PMM Imputation	With Posterior Imputation	
TELE	0.15 (0.14, 0.16)	0.17 (0.15, 0.19)	+
TB	0.11 (0.10, 0.11)	0.12 (0.11, 0.12)	-
U5MR	0.17 (0.16, 0.20)	0.20 (0.16, 0.23)	-
WATER	0.11 (0.10, 0.11)	0.11 (0.11, 0.11)	+
MMR	0.17 (0.15, 0.19)	0.19 (0.16, 0.22)	-
PU	0.08 (0.08, 0.09)	0.09 (0.07, 0.09)	-
GPI	0.03 (0.01, 0.04)	0.02 (0.00, 0.03)	+
NER	0.06 (0.05, 0.07)	0.01 (0.00, 0.03)	+
HIV	0.02 (0.01, 0.03)	0.07 (0.05, 0.07)	-
ETP	0.00 (0.00, 0.01)	0.01 (0.00, 0.02)	+
ABR	0.10 (0.09, 0.10)	0.03 (0.00, 0.06)	-

Table C.5. Data Generating Parameter Values vs. Estimated Posterior Parameter Means and Standard Deviations

Parameter	True and Estimated Values	
	Data Generating Values	Posterior Mean (Posterior St. D)
μ_{LE}	0.84	0.83 (0.003)
μ_{MYS}	2.66	2.65 (0.04)
μ_{EYS}	3.49	3.48 (0.03)
μ_{GNIpc}	1.01	0.98 (0.04)
λ_{LE}	0.19	0.17 (0.02)
λ_{MYS}	2.54	2.35 (0.16)
λ_{EYS}	1.85	1.86 (0.12)
λ_{GNIpc}	2.52	2.54 (0.18)
σ^2_{LE}	0.03	0.03 (0.004)
σ^2_{MYS}	1.26	1.39 (0.19)
σ^2_{EYS}	0.41	0.31 (0.08)
σ^2_{GNIpc}	1.76	2.03 (0.26)
ω	0.127	0.127 (0.006)