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ARE BOND-FINANCED DEFICITS INFLATIONARY?
A RICARDIAN ANALYSIS

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Abstract

This paper considers the possible theoretical validity of the following "monetarist hypothesis": that a constant, positive government budget deficit can be maintained permanently and without inflation if it is financed by the issue of bonds rather than money. The question is studied in a discrete-time, perfect-foresight version of the competitive equilibrium model of Sidrauski (1967), modified by the inclusion of government bonds as a third asset. It is shown that the monetarist hypothesis is invalid if the deficit is defined exclusive of interest payments, but is valid under the conventional definition. It is also shown that the stock of bonds can grow indefinitely at a rate in excess of the rate of output growth, provided that the difference is less than the rate of time preference. In addition to the main analysis, the paper includes comments on alternative deficit concepts, a brief consideration of data pertaining to the announced budget plans of the Reagan administration, and a new look at a much-studied issue: whether the operation of a Friedman-type constant money growth rule (with non-activist fiscal rules) would be dynamically feasible.

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I. Introduction

Recent developments in U.S. monetary and fiscal affairs have led to a renewal of interest in issues reminiscent of the "Monetarist vs. Keynesian" debates of previous decades. In particular, the Council of Economic Advisers' 1982 forecast of a long string of unusually large federal budget deficits, together with the Federal Reserve's repeated avowals to keep monetary growth rates low, has conferred intense practical interest upon the question of whether bond-financed deficits have significant impact on aggregate demand and, thereby, on price level and/or output magnitudes. The present paper includes a theoretical discussion of that question and a brief consideration of some relevant data pertaining to announced policy plans of the Reagan administration. It also includes comments on alternative deficit concepts and a new look at an issue that has been discussed frequently: whether the operation of a constant-money-growth policy rule of the type recommended by Milton Friedman (1959) (1968) would be dynamically destabilizing.

It should be said at the outset that most of the analysis will be conducted in a model that is distinctly sympathetic to the monetarist position, that is, to the idea that bond-financed deficits are not inflationary. The reason for slanting the analysis in this way is simple: there seems currently to be very little academic support for the monetarist position, so an interesting question is whether an intellectually respectable case for that position can be made. Whether the model in fact conforms to the dictates of respectability is a matter of subjective judgement, but it is at least built upon utility-maximizing behavior of individual agents. It is not, however, here subjected to any empirical testing.

The outline of the paper is as follows. Section II discusses some preliminary notions concerning a possible "Ricardian" rationale for the monetarist position. A Ricardian model is specified in Section III and the main analytical results are obtained. The above-mentioned instability issue is then taken up in Section IV. Following that digression, Section V discusses some points concerning the relationship between inflation and various deficit measures. Next, Section VI compares feasibility conditions derived in Section III with deficit paths implied by announced policy plans of the Reagan administration. Finally, Section VII provides a brief conclusion.

II. Analytical Issues

In order to limit the issues at hand in a manageable fashion, it will be presumed throughout the formal analysis that the economy can be represented by a deterministic, aggregative, flexible-price, equilibrium model. For some issues, such a model might be inadequate or misleading. Our present concern, however, involves the influence on inflation rates of a policy stance maintained over a number of years. For that type of concern, a flexible-price equilibrium model--which presumes that aggregate demand effects are manifested primarily in price level or inflation responses--seems well-suited.

As a matter of terminology--and to sharpen the issues--let us define a monetarist viewpoint as one which asserts that bond-financed deficits have no effect on aggregate demand. More precisely, our monetarist hypothesis is that, for given time paths of the money stock and government spending, it does not matter for aggregate demand whether the necessary revenue is raised by taxation or by bond sales. In other words, bond-financed changes in tax receipts have--according to the monetarist hypothesis--no effects on the price level or on output.

At this point, it perhaps needs to be asked whether there is any explicable reason to believe that the monetarist hypothesis (as defined) might be correct. Discussions in the most well-known references notwithstanding,[1/] the main intellectual support for such a position seems to be provided by the "Ricardian equivalence theorem," which obtains in some models in which infinite-lived agents correctly take account of the effects on future budgets of current budgetary actions. With fixed time paths of government spending and money creation, any bond-financed change in current taxes implies changes in future interest payments to be made by the government. If these are to be financed by lump-sum taxes and if the government and private agents face the same market interest rates, then the change will have no effect on a representative private agent's intertemporal budget constraint. Under such conditions, then, a bond-financed change in taxes will have no effect on the agent's supplies or demands and, consequently, no effect on the price level or output--just as predicted by the monetarist hypothesis.

The foregoing statement of the Ricardian result presumes that agents have infinite life spans, which is obviously untrue. But, as is well-known, Barro (1974) has demonstrated that an economy of finite-lived agents who care about the utility of their offspring or parents may, under reasonably general conditions, be treated for analytical purposes as one with infinite-lived agents.[2/] Consequently, this feature of the analysis seems acceptable, given the aims of this investigation.[3/]

A second crucial assumption of the Ricardian analysis is that agents are cognizant of effects on their own intertemporal budget constraints of governmental debt issues. But this assumption is merely a particular application of the hypothesis of rational expectations, the merits of which have been detailed extensively elsewhere.[4/]

Other complicating aspects of reality--uncertainty, distribution effects, multiple interest rates--are also ignored in the Ricardian equivalence argument. But the same is true of most policy-oriented theoretical analyses of macroeconomic phenomena. As there is no apparent reason why the issue at hand requires a different type of treatment, it would seem satisfactory to neglect them here, as elsewhere.[5/]

The discussion to this point seems to suggest that bond-financed deficits could be non-inflationary. Each bond issue/tax reduction package has no impact on aggregate demand or the price level, so a sequence of bond-financed tax reductions should apparently have no impact on the inflation rate. But it is possible that the situation is different in the case of a <u>permanent</u> deficit, financed by indefinitely-continuing issuance of bonds. In that case, as the deficit continues the outstanding bond stock continues to grow. So, accordingly, does the interest that must be paid each period on the outstanding stock. If, for example, the magnitude of the real deficit (net of interest payments) is kept at d, the real bond stock b_t will be required to grow according to

$$(1) \qquad b_t = (1+r)b_{t-1} + d(1+r),$$

if the real rate of interest is constant at the value r. Thus the bond stock will in this case grow without bound; if $d > 0$, $b_t \rightarrow \infty$ as $t \rightarrow \infty$.

Barro (1976)(1981) has argued that under these circumstances the Ricardian equivalence argument breaks down. In particular, he suggests that the rate of growth of the bond stock cannot exceed the economy's rate of output growth--here temporarily taken to be zero--because "the value of the outstanding stock of debt at any point in time is bounded by the government's ... present value of future taxing capacity" (1976, p. 343). In a similar vein, Sargent and Wallace (1981) have argued that "if the interest rate on bonds is greater than the economy's growth rate, the real stock of bonds will grow faster than the size of the economy. This cannot go on forever, since the demand for bonds places an upper limit on the stock of bonds relative to the size of the economy" (1981, p.2).[6/]

These arguments do not, however, seem entirely convincing. First, if the bond-issuance policy is permanently maintained, then taxes will never have to be collected so the relevance of inadequate taxing capacity is unclear. And under the Ricardian view, government bonds are not regarded as net wealth to the private sector, so the size of the bond stock also seems to be potentially irrelevant.[7/]

But whether or not the cited arguments by Barro and Sargent-Wallace are subjectively convincing, analysis indicates that the basic idea behind their contention is, as a matter of theory, correct. To develop that analysis is the main objective of the next section. As it turns out, however, the precise statements quoted above--and the implied limitations on bond demands--are not correct.

III. Analysis

Our first task is to specify a maximizing model that incorporates the crucial component of the Ricardian view, namely, infinite-lived agents who correctly take account of the government budget constraint. The model must also be one that accomodates money, bonds, and a physical asset. To that end, let us adopt a discrete-time, perfect-foresight version of the well-known model of Sidrauski (1967), modified to include government bonds.[8/] [9/] In order to keep matters as simple as possible, let us first consider a version with no depreciation or population growth.

Formally, we imagine an economy composed of a large number of similar households, each of which seeks at period t to maximize

$$u(c_t, m_t) + \beta\, u(c_{t+1}, m_{t+1}) + \beta^2\, u(c_{t+2}, m_{t+2}) + \dots \qquad (2)$$

Here c_t is consumption in period t and $m_t = M_t/P_t$, with M_t the household's nominal money stock at the start of t and P_t the price of the (single) consumption good in t. The within-period utility function u is assumed to be well-behaved so that unique, positive values are chosen for c_t and m_{t+1}, $t = 1,2,\dots$. The discount factor β equals $1/(1+\delta)$, with the time-preference parameter δ positive.

Each household has access to a production function that is homogeneous of degree one in its inputs, labor and capital. But since labor is supplied inelastically, this function can be written as $f(k_t)$, where k_t is the stock of capital held at the start of t. The function f is assumed to satisfy the conditions $f' > 0$, $f'' < 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. Thus a unique, positive value will be chosen for k_t in each period. Capital is simply unconsumed output, so its price is the same as that of output and its real rate of return between t and t+1 is $f'(k_{t+1})$.

Each household has the opportunity in t of purchasing government bonds at a money price of Q_t. Each bond is redeemed in t+1 for one unit of money, so the nominal rate of return on bonds between t and t+1 is $R_t = (1-Q_t)/Q_t$. The real rate r_t is then defined by $1+r_t = (1+R_t)/(1+\pi_t)$. Finally, lump-sum transfers net of taxes in the amount v_t are distributed to (or, if negative, collected from) the household in period t. Consequently, the household's budget constraint for period t can be written as

$$f(k_t) + v_t = c_t + (1+\pi_t)\, m_{t+1} - m_t + (1+r_t)^{-1}\, b_{t+1} - b_t + k_{t+1} - k_t \tag{3}$$

where $\pi_t = (P_{t+1} - P_t)/P_t$ is the inflation rate and $b_t = B_t/P_t$, with $B_t \geq 0$ the number of bonds held at the start of t.

Given this setup, we can derive optimality conditions for the agent by writing the Lagrangian expression

$$L = \sum_{t=1}^{T} \beta^{t-1} \Big[u(c_t, m_t) + \lambda_t \Big[f(k_t) + v_t - c_t - (1+\pi_t) m_{t+1} + m_t$$
$$- (1+r)^{-1}\, b_{t+1} + b_t - k_{t+1} + k_t \Big] \Big],$$

obtaining Kuhn-Tucker conditions, and then letting $T \to \infty$.[10/] Because of our assumptions that c_t, m_{t+1}, and k_{t+1} will be strictly positive, the conditions associated with those variables can be written as equalities holding for all t = 1,2,.... They are:

(4) $u_1(c_t, m_t) - \lambda_t = 0$

(5) $\beta u_2(c_{t+1}, m_{t+1}) - \lambda_t(1+\pi_t) + \beta\lambda_{t+1} = 0$

(6) $-\lambda_t + \beta\lambda_{t+1}[f'(k_{t+1}) + 1] = 0.$

The condition associated with b_{t+1} must, however, be written in two parts, as follows:

(7a) $-\lambda_t/(1+r_t) + \beta\lambda_{t+1} \leq 0$

(7b) $b_{t+1}[-\lambda_t/(1+r_t) + \beta\lambda_{t+1}] = 0.$

Finally, from the conditions relevant to m_{T+1}, b_{T+1}, and k_{T+1} we obtain--as explained in the appendix--the transversality conditions

(8) $\lim_{T\to\infty} m_{T+1}\beta^{T-1}\lambda T(1+\pi_T) = 0$

(9) $\lim_{T\to\infty} k_{T+1}\beta^{T-1}\lambda T = 0$

(10a) $\lim_{T\to\infty} \beta^{T-1}\lambda T/(1+r_T) \geq 0$

(10b) $\lim_{T\to\infty} b_{T+1}\beta^{T-1}\lambda T/(1+r_T) = 0.$

Conditions (3)-(10) govern the agent's choices of c_t, k_{t+1}, m_{t+1}, b_{t+1}, and λ_t, given initial asset stocks and time paths of prices and transfers.

Before continuing, let us pause to note that, because of our assumptions on u, λ_t will be positive for all t. Also, (6) and (7) together imply that $r_t = f'(k_{t+1})$ whenever $b_{t+1} > 0$; the rates of return on bonds and capital are the same if any bonds are demanded. Furthermore, if it happens that $\lambda_{t+1} = \lambda_t$, as will be the case in a steady state, then each of these rates of return will equal δ (provided that b_{t+1} is positive).

Next we consider the government's budget. Expressing all quantities in per-capita terms and letting g_t denote government purchases of output, we have the identity

$$(11) \qquad M_{t+1} - M_t + Q_t B_{t+1} - B_t = P_t(g_t + v_t)$$

or, in real terms,

$$(12) \qquad (1+\pi_t)\, m_{t+1} - m_t + (1+r_t)^{-1}\, b_{t+1} - b_t = g_t + v_t.$$

The government's choices of time paths for M_t, B_t, g_t, and v_t must conform to (11) and (12).

Given time paths for three of the policy variables, equilibrium values are determined by conditions (3)-(11) and the national income identity:

$$(13) \qquad f(k_t) = c_t + k_{t+1} - k_t + g_t.$$

In particular, if time paths for M_t, g_t, and v_t are selected by the government, conditions (3),(4),(5),(6),(7),(12), and (13) will determine paths for c_t, k_t, b_t, π_t, r_t, λ_t, and m_t.

We now have enough results to demonstrate that the Barro-Sargent-Wallace claim is correct, provided that the deficit is defined exclusive of interest payments on current debt. More specifically, we can show that the model at hand will not support a zero-inflation equilibrium in which a permanently-maintained positive deficit of $g_t + v_t = d$ is financed entirely by bond sales. To do so, we first observe that this monetarist hypothesis implies that $(1+\pi_t)\, m_{t+1} - m_t = 0$ and that steady-state conditions prevail for all variables except b_t so that r_t and λ_t are constant, with $r = \delta > 0$ and $\lambda > 0$.[11/] Then we insert the hypothesized constant values in (12), obtaining

$$(14) \qquad b_{t+1} = (1+r)\, b_t + (1+r)d \qquad\qquad t = 1,2,\ldots.$$

Next we note that the latter implies

$$(15) \qquad b_{t+1} = (1+r)^t\, b_1 + (1+r)d\ [1 + (1+r) + \ldots + (1+r)^{t-1}],$$

which in turn implies

$$(16) \qquad b_{T+1}\, \beta^{T-1}\, \lambda_T/(1+r_T) = \lambda b_1 + \lambda d\ [(1+r) - (1+r)^{1-T}]/r.$$

But it can then readily be seen that the expression in (16) approaches $\lambda b_1 + \lambda d(1+r)/r$ as $T \rightarrow \infty$, which violates condition (10b). Equivalently, but in different words, b_{T+1} grows at the rate r while β^{T-1} decays at the rate $\delta = r$, so their product grows at the rate zero--i.e., does not vanish as T increases. Thus the proposed monetarist path cannot be an equilibrium.[12/]

It should be noted, however, that a constant, maintained deficit can[13/] be financed entirely by bond sales with no resulting inflation if "deficit" is defined--as it typically is--to include current interest payments. To make this argument, let us define the issue value of bonds outstanding at t by $\tilde{B}_t = B_t/(1+R_{t-1})$. Then with $M_{t+1} - M_t = 0$, the government budget identity becomes

(17) $\tilde{B}_{t+1} - \tilde{B}_t = P_t(g_t + v_t) + \tilde{B}_t R_{t-1}.$

Now assume that policy keeps the real value of the right-hand side of (17) constant at $\tilde{d}$, so that

(18) $\tilde{B}_{t+1} - \tilde{B}_t = \tilde{d}\ P_t.$

Next we conjecture that, with $\tilde{d}$, g, and M held constant, the price level will also be constant so that (18) becomes

(19) $b_{t+1} - b_t = \tilde{d}(1+r).$

But in this case we have

(20) $b_{t+1} = b_1 + \tilde{d}[1 + 1 + \ldots + 1^{t-1}]\ (1+r) = b_1 + \tilde{d}\ t(1+r),$

so that

(21) $b_{T+1}\ \beta^{T-1}\ \lambda_T/(1+r_T) = [\ \lambda b_1 + \lambda\tilde{d}\ T(1+r)]/(1+r)^T,$

which does approach zero as $T \rightarrow \infty$.

To verify that a non-inflationary steady state is also consistent with the other relevant conditions, we argue as follows. Given values for $\tilde{d}$ and g, steady-state values of k, m, c, and λ are determined by (4), (5), (6), and (13). With $b_t > 0$ for all t, then, equation (7a) holds as an equality and determines r. Conditions (8) and (9) are satisfied with constant values for m, λ, and k since $\beta^{T-1} \rightarrow 0$ as $T \rightarrow \infty$. Finally, with constant m and $\pi = 0$, $v_t = (1+r)^{-1}\ b_{t+1} - b_t - g$ by (12), so (3) reduces to (13).

The policy rule in the last example is one which makes b_t grow at a rate that decreases over time, approaching zero in the limit. Let us next consider an example in which b_t grows at a constant, positive rate that is numerically smaller than δ. For simplicity, let us suppose that the rate is $\delta/2$. From our previous discussions it is clear that in this case condition (10b) is not violated. Furthermore, it can be readily verified that conditions (3)-(10a) are all satisfied by constant values of c, k, m, r, λ, and $\pi = 0$. The behavior of v_t in this case satisfies

$$\text{(22)} \qquad g + v_t = (1+r)^{-1} (1+\delta/2)\, b_t - b_t$$
$$= \left[(1+\delta/2)(1+\delta)^{-1} - 1\right] b_t = -(\delta/2)(1+\delta)^{-1}\, b_t$$

so $d_t = g + v_t$ is negative and decays at the rate $\delta/2$. The alternative (conventional) deficit measure $\tilde{d}_t = d_t + \delta b_t$ is positive, however, and grows at the rate $\delta/2$. Thus we see that the conventionally-defined deficit can--in the Ricardian/monetarist Sidrauski model--grow forever without causing inflation. Furthermore, we see that the real stock of bonds can grow forever at a rate exceeding zero--which is, in this instance, the growth rate of the economy.

It appears, furthermore, that the foregoing conclusions are not necessarily affected by population growth. Specifically, suppose that as in Sidrauski (1967) the size of each household grows at the rate n and that the utility function remains as in (2) with c_t and m_t now measuring per-capita values. Then, with all other quantities also expressed in per-capita terms, the household budget constraint becomes

$$(3') \qquad f(k_t) + v_t = c_t + (1+n)(1+\pi_t)\, m_{t+1} - m_t$$
$$+ (1+n)(1+r_t)^{-1}\, b_{t+1} - b_t + (1+n)\, k_{t+1} - k_t.$$

One relevant effect is that the counterpart of (7) now implies that $(1+n)(1+r_t)^{-1} = (1+\delta)^{-1}$ if $b_t > 0$ and $\lambda_t = \lambda_{t+1}$. Another is that condition (10b) is replaced with

$$(10b') \qquad \lim_{T \to \infty} b_{T+1}\, \beta^{T-1}\, \lambda_T(1+n)/(1+r_T) = 0.$$

In addition, the government budget identity becomes, in per-capita terms,

$$(12') \qquad (1+n)(1+\pi_t)m_{t+1} - m_t + (1+n)(1+r_t)^{-1}\, b_{t+1} - b_t = g_t + v_t.$$

Consequently, if the per-capita magnitudes g_t, v_t, and M_t are held constant over time--so that the aggregate deficit d and money stock each grow at the rate n in every period--the following equation will govern the behavior of b_t under the monetarist hypothesis:[14/]

$$(14') \qquad b_{t+1} = (1+r)(1+n)^{-1}\, b_t + (d-nm)(1+r)(1+n)^{-1}.$$

Thus the per-capita bond stock grows at the rate $(1+r)(1+n)^{-1} -1$. But since 1+r equals $(1+n)(1+\delta)$ in the hypothesized steady state, b_{T+1} then grows at the rate δ which equals the rate at which β^{T-1} contracts. So the product $b_{T+1}\, \beta^{T-1}$ grows at the rate zero. This violates (10b) as in our first example with n = 0. But it _just_ violates (10b), so it is clear that the second and third examples, in which the per-capita bond stock grows at a diminishing rate that approaches zero and at a constant rate less than δ (respectively), will not violate (10b').

Allowing proportional depreciation of capital would leave the crucial relationship between δ and the growth rate of bonds unchanged. The marginal product of capital would exceed r_t by the amount of the depreciation rate, but the steady-state condition $1+r = (1+\delta)(1+n)$ would continue to hold and it is the relationship between r and δ that governs the relative growth rates of b_{T+1} and β^{T-1}.

From the cases considered, then, we reach the following conclusions regarding bond-financed deficits in a Ricardian/monetarist economy:

(i) A permanent per-capita deficit cannot be financed solely with bonds if the deficit is defined exclusive of interest payments.

(ii) A permanent per-capita deficit can be financed solely with bonds, and without inflation, if the deficit is defined inclusive of interest payments.

(iii) It is feasible to maintain a positive growth rate of real bonds per capita, but this growth rate must be smaller than the rate of time preference.

If output grows only as a result of population growth, the per-capita growth rates in these three statements can be interpreted as aggregate growth rates measured relative to aggregate output growth.[15]

IV. Instability with a Friedman-Type Policy Rule

There exists a sizeable body of literature concerning potential "instability" under a policy regime in which taxes are collected according to a proportional (or progressive) schedule, rather than in a lump-sum fashion. In particular, authors including Blinder and Solow (1976), Christ (1979), Scarth (1980), Turnovsky (1977) and others have argued that the macroeconomic system will exhibit dynamic instability due to explosive bond growth if the government adopts a policy regime of the type championed by Milton Friedman (1968): one that makes the money stock grow at a constant rate and prohibits endogeneous responses of government spending or income tax schedules. This difficulty arises, it should be emphasized, even when these schedules are designed to yield a balanced budget in the steady state. From the analysis of Section III it appears that explosive behavior of the bond stock would indeed prevent the attainment of equilibrium, so an examination of the issue seems warranted.

For this analysis we shall retain the model of Section III, but now assume that (proportional) taxes are levied at the rate τ on production and interest received from the government. In per-capita terms, real taxes during period t are then $\tau[f(k_t) + r_{t-1} b_t]$.[16/] For simplicity, let the transfer component of v_t be zero. Then the per-capita government budget identity becomes

$$(23) \qquad (1+n)(1+\pi_t)m_{t+1} - m_t + (1+n)(1+r_t)^{-1} b_{t+1} - b_t$$

$$= g_t - \tau[f(k_t) + r_{t-1} b_t].$$

Now we impose the Friedman policy rule by requiring M_t and g_t to be constant over time, and we hypothesize the existence of a steady state with constant values of r_t, k_t, and P_t. Under those assumptions, (23) reduces to

$$(24) \qquad (1+n)(1+r)^{-1} b_{t+1} - b_t = [g-\tau f(k)] - \tau r b_t.$$

It is also assumed that τ is set at a magnitude that permits b_t to remain constant at some chosen value. But if b_t ever departs from that value, its behavior will be described by

$$(25) \qquad b_{t+1} = (1+r)(1-\tau r)(1+n)^{-1} b_t + \text{constant}$$

so that dynamic stability prevails only if $(1+r)(1-\tau r) < (1+n)$. 17/

To see whether that condition would obtain in the monetarist steady state, we next examine the household's choice problem under the revised budget constraint, which is

$$(26) \qquad (1-\tau)f(k_t) = c_t + (1+n)(1+\pi_t)m_{t+1} - m_t$$
$$+ (1+n)(1+r_t)^{-1} b_{t+1} - b_t + \tau r_{t-1} b_t + (1+n) k_{t+1} - k_t.$$

The optimality conditions that are the counterparts of (6) and (7a) are now

$$(27) \qquad -(1+n)\lambda_t + \beta \lambda_{t+1} [(1-\tau) f'(k_{t+1}) + 1] = 0$$

$$(28) \qquad -(1+n)(1+r_t)^{-1} \lambda_t + \beta \lambda_{t+1} (1-\tau r_t) \leq 0.$$

Using the strict equality in the latter, we then see that a steady state with positive bond holdings implies

$$(29) \qquad (1+r)(1-\tau r) = (1+n)(1+\delta).$$

Consequently, we see that $(1+r)(1-\tau r) > (1+n)$ so that (25) is dynamically unstable. If b_t ever departs from the constant value associated with τ, it will grow at the rate δ, just as in the first example of Section III. And the relevant transversality condition is again

$$\text{(10b')} \quad \lim_{T \to \infty} b_{T+1} \beta^{T-1} \lambda_T (1+n)/(1+r_T) = 0$$

so that path is not an equilibrium. Thus the analysis implies that the Friedman rule is unsatisfactory in the following sense: any departure of b_t from its intended constant value will place the system on an infeasible path.18/

The foregoing argument contradicts the conjecture in McCallum (1981, p.137), where it is suggested that the presence of growth and taxes would tend to produce stability in the behavior of bonds per unit of output. In that paper an equation similar to (25) is utilized, but it is suggested that r would tend to equal n in the steady state. Here we see that suggestion to be incorrect: in the present model, the after-tax real rate of interest is approximately equal to the rate of growth, n, plus the rate of time preference, δ.

Consequently, the suggestion in the last paragraph of McCallum (1981) assumes a heightened importance. In order to implement a non-discretionary and non-activist set of monetary and fiscal policy rules, as is often recommended by Friedman and other monetarists, it would seem to be preferable to set the growth rate of B_t, rather than M_t, at some constant value. With a fixed tax schedule and a fixed path for g_t, cyclically-induced deficits and surpluses would then be automatically financed by changes in the stock of money. From the perspective of dynamic stability, the "monetary and fiscal framework" originally proposed by Milton Friedman (1948) appears to be superior to the one promoted in his later writings (1959) (1968).

The observation of the last paragraph, that Friedman originally proposed a constant bond-growth rule, leads one naturally to ask what led to his change of position. The only explanation I have been able to find in Friedman's writings is as follows:

> I have become increasingly persuaded that the [1948] proposal is more sophisticated and complex than is necessary, that a much simpler rule would also produce highly satisfactory results and would have two great advantages: first, its simplicity would facilitate the public understanding and backing that is necessary if the rule is to provide an effective barrier to opportunistic "tinkering"; second, it would largely separate the monetary problem from the fiscal and hence would require less far-reaching reform over a narrower area. (Friedman, 1959, p.90)

From this I would infer that Friedman's change in position resulted not from economic considerations but from a belief that it would be easier, politically, to achieve adoption of the money-stock rule in the United States. In particular, less cooperation between the monetary and fiscal authorities would be required. But now that Friedman (1982, pp. 114-118) has concluded that the incentive structure facing Federal Reserve officials is not conducive to effective monetary control, and that improved performance probably requires termination of the independence of the Fed, this rationale for the second-best rule seems less satisfactory.

V. Measures of the Deficit

Previous sections have mentioned two possible definitions of the real deficit, $d_t = g_t + v_t$ and the more conventional $\tilde{d}_t = g_t + v_t + \tilde{b}_t R_{t-1}$. (Here we ignore population growth.) Recently, a number of writers[19/] have suggested that a more appropriate concept would be the change in the real value of the government's liabilities. Thus the suggested definition is

$$\begin{aligned} (30) \quad d_t^* &= m_{t+1} - m_t + \tilde{b}_{t+1} - \tilde{b}_t \\ &= g_t + v_t - \tilde{b}_t R_{t-1} - \pi_t(m_{t+1} + \tilde{b}_{t+1}), \end{aligned}$$

which subtracts from the conventional measure the capital gains to the government (losses to the public) that result from inflation. This concept incorporates an entirely sensible adjustment, one that helps to produce a measure that more accurately reflects real resource flows between the government and the private sector. There are, however, two points that need to be made concerning the measure d_t^*.

First, if one is discussing a Ricardian economy it seems inappropriate to include the bond component $\tilde{b}_{t+1} - \tilde{b}_t$. As the discussion in Sections II and III indicates, changes in b or $\tilde{b}$ unaccompanied by changes in M or g have no effects on private supplies or demands in this type of economy. Such changes do not, therefore, give rise to resource transfers to or from the government. And from the balance sheet point of view, changes in b that have no effect on private wealth--as the Ricardian result is often expressed--should not be regarded as altering the liabilities of the government. In a Ricardian economy, therefore, a better measure of the real deficit would seem to be

(31) $d_t^{**} = m_{t+1} - m_t$,

the change between periods t and t+1 in the real value of the stock of (high-powered) money. The difference in treatment between bonds and money implied by this definition is appropriate because changes in real money holdings do, in contrast with those for bonds, give rise to supply-demand adjustments and resource transfers. In the Sidrauski setup these come about because m_t is an argument of u. More fundamentally, the idea is that transaction cost economies provided by holdings of the medium of exchange alter the consumption/leisure possibilities faced by individual agents.

The second point to be made applies to d_t^{**} as well as d_t^*--indeed, it is most transparent in an economy in which there are no bonds.. There the point is that d_t^{**}, i.e., the period-t change in real money balances $m_{t+1} - m_t$, is not a useful measure of the inflationary impact of the period's fiscal/monetary actions. It is the percentage change in nominal money balances (corrected for output growth, of course) that measures the inflationary potential of those actions. As is familiar from discussions of hyperinflationary experiences, real money balances will generally be falling over time during periods of increasing inflation rates, even though the inflation is entirely due to increasing rates of growth of nominal money stocks. Alternatively, the measure d_t^{**} will obviously equal zero in an inflationary steady-state, whatever the rate of inflation.

Turning to economies with money and bonds, we first note that if the economy is Ricardian, d_t^{**} is preferred to d_t^* and the argument is just as above. If, on the other hand, the economy is not Ricardian, the inflationary impact of a given percentage change in M+B will depend upon the mix of money and bonds. Thus no single measure will be fully adequate. If one nevertheless seeks a single measure, a natural point of reference is provided by the case of equiproportionate issues of M and B, in which case the ratio of M to B does not change. In that case the point is that d_t^* is not a good measure of the inflationary impact of the M-plus-B issue, even if scaled and corrected for output growth. The basic reason is the same as for the no-bonds economy; it is the growth of M+B, not m+b, that is relevant for the generation of inflation.

VI. Reagan Administration Plans

Let us now briefly attempt to determine whether the results of Section III imply that the monetary and fiscal policy plans of the Reagan administration--as of early 1982--are inconsistent, in the sense of requiring an infeasible path for the stock of government bonds.

The values of the planned federal budget[20] deficits for 1982-87 announced by the Council of Economic Advisers (1982, p.98) are as follows:

Fiscal Year	Federal Budget $ billion	Deficit % of GNP
1982	118.3	3.8
1983	107.2	3.1
1984	97.2	2.6
1985	82.8	2.0
1986	77.0	1.7
1987	62.5	1.3

Here we see that, although the deficit magnitudes are large by historic standards, they decrease each year as a fraction of GNP. Thus the deficits as forecast by the CEA do not imply monetary-fiscal inconsistency, even if per-capita money growth is taken to be negligible.

But of course these forecasts are sensitively dependent upon the forecast values of output and inflation, and the CEA's forecast values for those variables are highly optimistic. In the following table, the output growth and inflation values forecast by the CEA are compared with "assumptions" used by the Congressional Budget Office (CBO) in preparing its "Baseline Budget Projections" (1982, p.6):

Year	Output Growth, % CEA	CBO	CPI Inflation, % CEA	CBO
1982	3.0	-0.1	6.6	7.5
1983	5.2	4.4	5.1	6.9
1984	4.9	3.6	4.7	6.9
1985	4.6	3.5	4.6	6.4
1986	4.3	3.5	4.6	6.0
1987	4.3	3.5	4.4	5.7

This difference in assumptions about macroeconomic performance leads to large discrepancies in deficit forecasts, even with similar assumptions regarding spending rules and tax schedules. The CBO's baseline projection for deficits is as follows:

Fiscal Year	Federal Budget Deficit $ billion	% of GNP
1982	109	3.6
1983	157	4.6
1984	188	5.0
1985	208	5.0
1986	234	5.1
1987	248	5.0

Here the deficit does not fall, as time passes, as a percent of GNP. Furthermore, the CBO describes alternative projections based on relatively "optimistic" and "pessimistic" assumptions regarding output and inflation. Under the latter alternative, the deficit increases steadily in relation to GNP, reaching 7.4% in 1987 (CBO, pp.15-16).

From the foregoing discussion it seems clear that it is difficult to be confident about the inconsistency possibility, one way or the other. And of course the relevant transversality conditions in our model apply only in the limit so, strictly speaking, six years of forecasts can not be conclusive whatever the magnitudes. Perhaps the best way to think of the issue, consequently, is as follows. The tax legislation embodied by the Economic Recovery Tax Act of 1981 is scheduled to introduce indexing in 1985, thereby eliminating "bracket creep" resulting from inflation. In addition, progressivity was markedly reduced. As a first approximation, then, one might view the relevant tax structure for the period beginning in 1985 as one in which a constant fraction of real output is collected by the Federal government in taxes. The crucial issue, then, is whether government spending will be similarly curtailed. If, instead, spending rises as a fraction of output, inconsistency would result.

VII. Conclusions

To a certain extent, the analytical results described above provide support for the monetarist position. In particular, we have seen that it is possible to construct a utility-maximizing model in which bond-financed deficits are not inflationary if the bond stock does not grow too rapidly. Furthermore, the model's restriction on bond growth permits a permanently-maintained, positive per-capita deficit, provided that the deficit is measured (as is conventional) as inclusive of interest payments. And the model places no upper bound on the ratio of bonds to output.

On the other hand, it is also true that the model places strict limits on the extent of bond sales, and these limits imply that a constant non-inflationary per-capita deficit is not feasible if the deficit is measured exclusive of interest payments. These limits obtain, moreover, in a model that is designed to be highly sympathetic to the monetarist position. Thus the results are not unreservedly supportive of the monetarist position.[21/]

In addition, it is shown that when a zero per-capita money stock growth rule is combined with a constant per-capita level of government purchases and a constant proportional tax schedule, the implied behavior of the bond stock is dynamically infeasible. If this Friedman-type policy rule is considered "monetarist," then an additional unsupportive result is obtained. In any event, the analysis suggests that Friedman's original proposal (1948)--in which bond growth is held constant and money issued or returned over the cycle to satisfy the government budget identity--is superior to the one promoted in his more recent writings.

Appendix

The object here is to describe in more detail the derivation of the optimality conditions for the individual household. We take the most general conditions described in the body of the paper, those in which the household's population grows at the rate n and taxes are imposed on income. Thus the objective function is (2) and the budget constraint (26). We begin with a version in which the household has a planning horizon of T periods. Thus the relevant Lagrangian expression is

$$(A-1)\quad L = \sum_{t=1}^{T} \beta^{t-1} \Big[u(c_t, m_t) + \lambda_t[(1-\tau)\ f(k_t) - c_t - (1+n)(1+\pi_t)m_{t+1} + m_t - (1+n)(1+r_t)^{-1}\ b_{t+1} + b_t - \tau r_{t-1} b_t - (1+n)k_{t+1} + k_t]\Big].$$

It is assumed that u and f have properties such that the Kuhn-Tucker conditions are necessary and sufficient for a maximum. These are:

$$(A-2)\quad u_1(c_t, m_t) - \lambda_t = 0 \qquad t = 1,\ldots,T$$

$$(A-3)\quad \beta u_2(c_{t+1}, m_{t+1}) - \lambda_t(1+n)(1+\pi_t) + \beta\lambda_{t+1} = 0 \qquad t = 1,\ldots,T-1$$

$$(A-4)\quad -(1+n)\lambda_t + \beta\lambda_{t+1}\Big[(1-\tau)f'(k_{t+1}) + 1\Big] = 0 \qquad t = 1,\ldots,T-1$$

$$(A-5a)\quad -(1+n)(1+r_t)^{-1}\ \lambda_t + \beta\lambda_{t+1}\ (1-\tau r_t) \leq 0 \qquad t = 1,\ldots,T-1$$

$$(A-5b)\quad b_{t+1}\Big[-(1+n)(1+r_t)^{-1}\ \lambda_t + \beta\lambda_{t+1}\ (1-\tau r_t)\Big] = 0 \qquad t = 1,\ldots,T-1$$

Here the first three are written as equalities because the household will choose positive magnitudes for c_t, m_{t+1}, and k_{t+1} for $t = 1,\ldots,T-1$. The optimal value for some b_t's may be zero, however, so the two-part condition is required. Also relevant are values of m_{T+1}, k_{T+1}, and b_{T+1}. The household would like each of these to be negative (and large in absolute value) but non-negativity requirements pertain. Thus we have the two-part conditions:

$$\text{(A-6)} \quad -\beta^{T-1} \lambda_T(1+\pi_T)(1+n) \leq 0, \qquad m_{T+1} \beta^{T-1} \lambda_T(1+\pi_T)(1+n) = 0$$

$$\text{(A-7)} \quad -\beta^{T-1} \lambda_T(1+n) \leq 0, \qquad k_{T+1} \beta^{T-1} \lambda_T(1+n) = 0$$

$$\text{(A-8)} \quad -\beta^{T-1} \lambda_T(1+r_T)^{-1} (1+n) \leq 0, \qquad b_{T+1} \beta^{T-1} \lambda_T(1+r_T)^{-1}(1+n) = 0.$$

Finally, to obtain conditions for the infinite-horizon assumption we let $T \rightarrow \infty$. In this case (A-2)-(A-5) become applicable for all $t = 1,2,...$ while (A-6)-(A-8) hold in the limit. In the body, the first parts of (A-6) and (A-7) are ignored, since with $\lambda_T > 0$ they will be satisfied automatically. By setting $n = 0$ and $\tau = 0$, the above conditions collapse to those presented in (4)-(10). Also applicable, of course, are the budget constraints relevant for each $t = 1,2,....$

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Footnotes

1. Many of there are included in Gordon (1974) and Stein (1976). The models typically assume that consumption or private expenditure depends directly upon private wealth, with government debt included as a component of the latter. See, for example, Brunner and Meltzer (1976, p. 72). In these models, then, a bond-financed tax reduction directly increases aggregate demand.
2. This statement presumes that intergenerational transfers are operative. A discussion of circumstances under which Barro's result is inapplicable is provided by Drazen (1978)
3. Recall that the analysis is intended to determine whether a case can reasonably be made for the monetarist position.
4. My own arguments appear in McCallum (1980).
5. Barro (1974) (1981) argues that neglect of these complicating features does not serve to distort the results in a predictable direction.
6. This assumption, it should be noted, plays an important role in the Sargent-Wallace analysis.
7. Irrelevance of continued bond growth is assumed, without any utility-maximizing justification, in McCallum (1978).
8. The Sidrauski model is, of course, one in which bonds and money can easily co-exist because real money balances appear as an argument of the household's within-period utility function. The rationale for this appearance --which has been severely criticized by Bryant and Wallace (1980) and others--is that transaction costs are reduced by money balances, so that more preferred bundles of consumption and leisure can be obtained. For a rather lengthy discussion of related issues, see McCallum (1982).

9. A notable feature of the Sidrauski model is the invariance of the steady-state capital-labor ratio to expected inflation rates. This invariance (or superneutrality) does not survive minor modifications, such as making utility dependent upon leisure. It is my impression that the superneutrality property is not crucial for the issues under discussion here; the superneutral version of the model has been adopted for simplicity.

10. The limiting procedure is a modification of one described by Sargent (1979, pp. 333-335).

11. In this experiment (and those that follow) it is assumed that steady-state values for k and m prevail in period 1. Since the crucial aspects of the analysis involve limiting conditions, this simplification seems adequate for the issues at hand.

12. It is true, as Gray and Salant (1981) have recently emphasized, that there are some problems for which transversality conditions such as (10b) are not necessary for optimality. That the proposed path cannot be an equilibrium in the case at hand can nevertheless be verified by observing that the representative household could, in any period, reduce its bond holdings to zero and obtain extra consumption in that period without reducing consumption in any other period (and without altering any value of m_t).

13. According to our model.

14. Under that hypothesis, the inflation rate will be zero.

15. Preliminary investigations with technical progress indicates that the results remain valid in cases in which steady-state growth is possible. That the use of income taxes, instead of head taxes, does not overturn the results is implied by the analysis of the next section.

16. Given our definition of the interest rate, $r_{t-1}b_t$ is an approximation to interest received in period t.

17. It is assumed that $\tau > 0$ and $1-\tau r > 0$.

18. The analysis also establishes that the presence of income taxes is consistent with the results of Section III, as claimed in footnote 14.

19. These include Barro (1981), Siegel (1979), and Tobin (1980), among others.

20. Here and in the following tables, both budget and off-budget items are included in the reported deficits while the conventional deficit concept, $\tilde{d}_t$, is employed.

21. Robert Barro has emphasized that the model pertains to a closed economy; the results might not obtain if migration were possible.