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INDUSTRIAL DEMAND FOR ENERGY

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CHAPTER 1

INTRODUCTION AND SUMMARY

In this study we examine the characteristics of industrial demand for energy, which accounts for more than one-fourth of annual energy consumption in the United States. Our research has been focused on four topics:

1. interfuel substitution in two-digit industries;
2. substitution among energy, capital and labor;
3. technical change in energy use; and
4. dynamic structure of energy demand.

Chapter 2 reports the results on interfuel substitution in two-digit industries. Cost share equations derived from transcendental logarithmic (translog) unit cost functions are estimated with cross-section state data for 1971, 1962, and 1958. Results include estimates for each industry of own and cross price elasticities of demand for electric energy, fuel oil, natural gas, and coal.

The model performs well for industries accounting for most of the total consumption of energy in manufacturing in each year. The results indicate considerable variation in energy substitution both across industries and across types of energy. Aggregate manufacturing demand for energy appears to be highly price responsive. Estimated own price elasticities for all types of energy except electric energy are generally substantially greater than unity.

The use of a unit cost function for energy assumes that energy inputs are separable from capital, labor, and other inputs. The separability of energy inputs from capital and labor can be tested with data for 1958. The tests were performed by Jay Ford as part of his dissertation research. As reported in Chapter 3, separability is rejected for four of the eight industries for which it could be tested. The industries for which separability is accepted account for some two-thirds of energy consumption by the group of eight industries. Nevertheless, the rejection of separability for some industries indicates that the interfuel substitution results have to be interpreted with caution.

Estimation with 1958 data of a cost function including capital and labor also makes it possible to calculate estimated cross elasticities of demand between energy inputs and capital and labor. The preliminary results reported in Chapter 3 indicate considerable variation in substitution between different types of energy and non-energy inputs.

The estimated price elasticities of demand reported in Chapters 2 and 3 can be interpreted as long-run elasticities. Short-run elasticities of demand, and the time path of response of demand to price changes, are equally important but have been largely ignored in previous studies. In this study, the model of demand for factors of production developed by Nadiri and Rosen [1969, 1973] is adapted to estimate dynamic demand equations for energy and other inputs for total U.S. manufacturing.

The results are reported in Chapter 4. All estimated short-run elasticities of demand for energy are statistically significant. Estimated long-run elasticities are similar to estimates in other studies. The response to price changes are found to be quite rapid. The response of demand for each input to excess demands for other inputs is also reported. The results indicate that excess demand for energy increases labor stock and capacity utilization and decreases capital stock.

Energy price elasticities measure the response of demand for energy to price changes holding technology constant. In the long-run energy consumption may also be affected by changes in technology, which may in part be induced by changes in prices.

Technical change in energy use in the primary metals industries was examined by John Wills in his Ph.D. dissertation. As discussed in Chapter 5, the results indicate that technical change has occurred through factor augmentation at unequal rates. Statistically significant labor-using and material-saving biases are found. There also appears to have been a small energy-saving bias, but it is not statistically significant.

CHAPTER 2

INTERFUEL SUBSTITUTION IN TWO-DIGIT INDUSTRIES

I. Introduction

Considerable shifts have occurred in the composition of the manufacturing sector's energy consumption in recent years. In 1971 electric energy comprised 15.3% of total purchases of the four major types of energy, fuel oil 14.0%, natural gas 58.2%, and coal 12.5%. The shares in 1958 were electric energy 11.9%, fuel oil 13.5%, natural gas 44.8%, and coal 29.8%. This chapter examines the extent to which shifts in the composition of energy consumption in manufacturing can be explained by changes in relative energy prices.

The characteristics of energy demand can be expected to vary across individual industries. Table 1 provides data on energy consumption by each two-digit manufacturing industry in 1971. Industry shares in total manufacturing energy consumption range from 21.2% for industry 28, chemicals and allied products, to 0.1% for industry 21, tobacco manufacturers. Differences in energy consumption across industries are due both to differences in total output and to differences in the energy intensiveness of production. Energy cost as a percent of value added is also shown in Table 1. By this measure, energy intensiveness varied from 11.3% for industry 29, petroleum and coal products, to 0.8% for industry 21.

The apparent differences in energy consumption across industries indicate that the inter-relationships between the demands for each type of energy should be examined on an industry-by-industry basis. Previous studies of industrial energy demand have generally considered the demand for only one type of energy, usually electric energy,[1] or have provided results only for total manufacturing rather than for individual industries.[2] In this chapter, complete systems of energy demand equations are estimated for each two-digit industry. Duality theory is used to derive the systems of demand equations from flexible cost functions which impose minimal *a priori* restrictions on the estimated elasticities of demand.

Estimates of all own and cross price elasticities of demand are presented for each two-digit industry. The results show wide variation across industries in the characteristics of energy demand. Aggregate elasticities of demand are also calculated and indicate that total manufacturing demand for energy is highly price responsive.

II. The Model

A twice differentiable aggregate production function is assumed to exist at the state level for each two-digit industry,

$$Y = F(E, O, G, C, \underline{X}) \tag{1}$$

where Y is total output, E is electric energy, O is fuel oil, G is natural gas, C is coal, and $\underline{X}$ is a vector of all other inputs. Assuming that the production function is homothetically weakly separable in the energy inputs, it can be written,

$$Y = G[H(E, O, G, C), \underline{X}]$$

where H is an energy input function.

Dual to the energy input function is an energy cost function,

$$W = J(Z, P_E, P_O, P_G, P_C)$$

where W is total cost of energy, Z is aggregate energy input, and P_E, P_O, P_G, and P_C are the prices of electric energy, fuel oil, natural gas and coal respectively. If the energy input function is a positive, nondecreasing, positively linear homogeneous, concave function, then the energy cost function can be written

$$W = Z \cdot V(P_E, P_O, P_G, P_C) \tag{2}$$

where V is a unit cost function satisfying the same regularity conditions, Diewert [1973].

Demand functions can be obtained from the unit cost function using Shepard's lemma,

$$X_i = Z \frac{\partial V}{\partial P_i} \qquad i = E, O, G, C,$$

where X_i is the cost minimizing quantity of energy input i, Diewert [1973]. Thus the characteristics of industrial demand for energy can be examined by specifying an appropriate functional form for the unit cost function and differentiating it to obtain demand functions.

A convenient functional form for the cost function is the transcendental logarithmic (translog),[3]

$$\begin{aligned} \ln V = {} & \alpha_V + \alpha_E \ln P_E + \alpha_O \ln P_O + \alpha_G \ln P_G + \alpha_C \ln P_C + 1/2\gamma_{EE}(\ln P_E)^2 \\ & + \gamma_{EO}\ln P_E \ln P_O + \gamma_{EG}\ln P_E \ln P_G + \gamma_{EC}\ln P_E \ln P_C + 1/2\gamma_{OO}(\ln P_O)^2 \\ & + \gamma_{OG}\ln P_O \ln P_G + \gamma_{OC}\ln p_O \ln P_C + 1/2\gamma_{GG}(\ln P_G)^2 + \gamma_{GC}\ln P_G \ln P_C \\ & + 1/2\gamma_{CC}(\ln P_C)^2. \end{aligned} \tag{3}$$

The translog form provides a second order approximation to an arbitrary twice continuously differentiable unit cost function. The translog unit cost function does not satisfy the regularity conditions globally unless all $\gamma_{ij} = 0$, i.e. unless it collapses into a Cobb-Douglas form. However, the estimated cost function can be tested to determine if the regularity conditions are satisfied in the relevant region.

Demand functions are obtained by logarithmic differentiation of the unit cost function,

$$\frac{\partial \ln V}{\partial \ln P_i} = \frac{\partial V}{\partial P_i} \cdot \frac{P_i}{V} = \alpha_i + \sum_j \gamma_{ij} \ln P_j \qquad i,j = E, O, G, C. \tag{4}$$

By Shepard's lemma, $\partial V/\partial P_i = X_i/Z$. Since the cost function is linear homogeneous in prices, $W = \Sigma P_i X_i$ by Euler's theorem. Therefore, $V = \Sigma P_i X_i/Z$. Substituting in (4),

$$\frac{\partial \ln V}{\partial \ln P_i} = \frac{P_i \cdot X_i/Z}{\Sigma P_i X_i/Z} = \frac{P_i X_i}{\Sigma P_i X_i} = M_i \qquad i = E, O, G, C,$$

where M_i is the cost share for input i. Thus demand functions for energy inputs can be estimated even though aggregate energy input, Z, is not observed.

The system of cost share equations is,

$$M_E = \alpha_E + \gamma_{EE}\ln P_E + \gamma^E_{EO}\ln P_O + \gamma^E_{EG}\ln P_G + \gamma^E_{EC}\ln P_C + u_E \; ,$$

$$M_O = \alpha_O + \gamma^O_{EO}\ln P_E + \gamma_{OO}\ln P_O + \gamma^O_{OG}\ln P_G + \gamma^O_{OC}\ln P_C + u_O \; ,$$

$$M_G = \alpha_G + \gamma^G_{EG}\ln P_E + \gamma^G_{OG}\ln P_O + \gamma_{GG}\ln P_G + \gamma^G_{GC}\ln P_C + u_G \; ,$$

$$M_C = \alpha_C + \gamma^C_{EC}\ln P_E + \gamma^C_{OC}\ln P_O + \gamma^C_{GC}\ln P_G + \gamma_{CC}\ln P_C + u_C \; , \qquad (5)$$

where the additive disturbance terms, u_i, are included to reflect random errors in cost minimizing behavior. Because the cost shares sum to unity at each observation, the parameters must satisfy the following adding-up restrictions,

$$\alpha_E + \alpha_O + \alpha_G + \alpha_C = 1,$$

$$\gamma_{EE} + \gamma^O_{EO} + \gamma^G_{EG} + \gamma^C_{EC} = 0 \; ,$$

$$\gamma^E_{EO} + \gamma_{OO} + \gamma^G_{OG} + \gamma^C_{OC} = 0 \; ,$$

$$\gamma^E_{EG} + \gamma^O_{OG} + \gamma_{GG} + \gamma^C_{GC} = 0 \; ,$$

$$\gamma^E_{EC} + \gamma^O_{OC} + \gamma^G_{GC} + \gamma_{CC} = 0 \; . \qquad (6)$$

Thus only fifteen of the twenty parameters are free and parameter estimates for all four share equations can be derived from the parameter estimates for any three.

Derivation of the share equations from the cost function, (3), implies the following cross-equation equality restrictions on the γ_{ij},[4]

$$\begin{aligned}
&\text{a.}\quad \gamma^{E}_{EO} = \gamma^{O}_{EO}\,, \\
&\text{b.}\quad \gamma^{E}_{EG} = \gamma^{G}_{EG}\,, \\
&\text{c.}\quad \gamma^{E}_{EC} = \gamma^{C}_{EC}\,, \\
&\text{d.}\quad \gamma^{O}_{OG} = \gamma^{G}_{OG}\,, \\
&\text{e.}\quad \gamma^{O}_{OC} = \gamma^{C}_{OC}\,, \\
&\text{f.}\quad \gamma^{G}_{GC} = \gamma^{C}_{GC}\,.
\end{aligned} \tag{7}$$

The cross-equation equality restrictions reduce the number of free parameters to nine. Imposition of these restrictions requires that the equations be estimated simultaneously. Since the cost shares necessarily sum to unity, the sum of the disturbances across the four equations is zero at each observation and the disturbance covariance matrix is singular. Therefore, one equation must be omitted from the system.

The choice of the equation to be omitted is arbitrary. We drop the disturbance term from the equation for M_C and omit this equation from the system. Because γ^{C}_{EC}, γ^{C}_{OC} and γ^{C}_{GC} do not appear in the remaining three equations, an alternative set of cross-equation equality constraints is required for these parameters. Substituting in (7.c), (7.e) and (7.f) from (6),

$$\text{7.c.}'\quad \gamma^{E}_{EC} = -(\gamma_{EE} + \gamma^{O}_{EO} + \gamma^{G}_{EG}),$$

7.e.' $\gamma_{OC}^{O} = -(\gamma_{EO}^{E} + \gamma_{OO} + \gamma_{OG}^{G})$,

7.f.' $\gamma_{GC}^{G} = -(\gamma_{EG}^{E} + \gamma_{OG}^{O} + \gamma_{GG})$.

Solving (7.c') (7.e.') and (7.f.') for γ_{EE}, γ_{OO}, and γ_{GG} and substituting in (5), the system of equations to be estimated is,

$$M_E = \alpha_E + \gamma_{EO}(\ln P_O - \ln P_E) + \gamma_{EG}(\ln P_G - \ln P_E) + \gamma_{EC}(\ln P_C - \ln P_E) + u_E ,$$

$$M_O = \alpha_O + \gamma_{EO}(\ln P_E - \ln P_O) + \gamma_{OG}(\ln P_G - \ln P_O) + \gamma_{OC}(\ln P_C - \ln P_O) + u_O ,$$

$$M_G = \alpha_G + \gamma_{EG}(\ln P_E - \ln P_G) + \gamma_{OG}(\ln P_O - \ln P_G) + \gamma_{GC}(\ln P_C - \ln P_G) + u_G.$$

Estimates of γ_{EE}, γ_{OO}, γ_{GG} and γ_{CC} are calculated from (6).[5]

The vector of disturbance terms, $[u_C\ u_O\ u_G]$, is assumed to be independently and identically normally distributed with mean vector zero and nonsingular covariance matrix Ω. The system of three share equations is estimated with an iterative Zellner efficient procedure, which is equivalent to maximum likelihood estimation.[6] Thus the parameter estimates are invariant to the choice of equation to be omitted from the system.

The equations are estimated with cross-section state data for 1971, 1962, and 1958. The system of cost share equations derived from the four input energy cost function cannot be estimated for all two-digit industries because data on coal consumption are not available for a sufficient number of states for some industries. Restricting the model to electricity, fuel oil and natural gas is appropriate if the production function is weakly separable in these three inputs. The separability of these inputs from coal is tested statistically for those industries for which the four input model is estimated.

Weak separability of the (homogeneous) energy input function in E, O, and G implies weak separability of the unit cost function in P_E, P_O, and P_G. However, the translog approximation of a weakly separable cost function is not necessarily weakly separable. The conditions on the translog unit cost function corresponding to weak separability of the true unit cost function in P_E, P_O, and P_G from P_C are

$$\gamma_{EC} = \theta\alpha_E$$
$$\gamma_{OC} = \theta\alpha_O$$
$$\gamma_{GC} = \theta\alpha_G \ . \qquad (8)$$

Explicit separability of the translog function itself requires the further restriction, $\theta = 0$, in (8).[7]

As noted above, the unrestricted translog unit cost function does not satisfy the regularity conditions globally. Imposition of the equality restrictions on the γ_{ij} together with the adding-up restrictions ensures that the unit cost function is linear homogeneous in the input prices. However, the fitted unit cost function may or may not satisfy the conditions that it be non-decreasing and concave.

The fitted unit cost function is non-decreasing in the input prices if the fitted shares are non-negative, since

$$M_i = \frac{\partial V}{\partial P_i} \cdot \frac{P_i}{V} \qquad i = E, O, G, C$$

and P_i and V are always positive. Concavity of the unit cost function requires that the Hessian matrix be negative semidefinite for each observation. This will be true if the first n - 1 ordered principal minors alternate in sign. The n^{th} order principal minor will be zero due to the imposition of linear homogeneity in input prices. Concavity is checked for each observation by calculating the values of the principal minors. Since it is not determined if the principal minors are statistically significant, this procedure does not constitute a statistical test of concavity.

An appropriate measure of goodness of fit of the estimated equations is the "pseudo-R^2", which states the proportion of generalized variance in the system of equations explained by variation in the right-hand variables.[8] The pseudo-R^2 is calculated as $1 - |r_1|/|r_2|$ where $|r_1|$ is the determinant of the estimated residual moment matrix and $|r_2|$ is the determinant when the coefficients of all right-hand variables are constrained to equal zero. The value of the pseudo-R^2 is invariant to the choice of equation to be omitted from the system.

Estimates of the own and cross-price elasticities of demand are calculated from the estimated cost share equations. The own-price elasticity of demand for energy input i is defined as

$$E_{ii} = \frac{\partial X_i}{\partial P_i} \cdot \frac{P_i}{X_i} .$$

Applying Shepard's lemma to obtain expressions for X_i and $\partial X_i/\partial P_i$ in terms of derivatives of the unit cost function, the own-price elasticity can be rewritten,

$$E_{ii} = \frac{P_i \cdot \partial^2 V/\partial P_i^{\,2}}{\partial V/\partial P_i} = \frac{M_i^{\,2} - M_i + \gamma_{ii}}{M_i} \qquad i = E, O, G, C. \qquad (9)$$

Similarly, the cross-price elasticity of demand for input i with respect to the price of input j is,

$$E_{ij} = \frac{\partial X_i}{\partial P_j} \cdot \frac{P_j}{X_i} = \frac{P_j \partial^2 V/\partial P_i \partial P_j}{\partial V/\partial P_i} = \frac{M_i M_j + \gamma_{ij}}{M_i} \quad i,j = E, O, G, C. \quad (10)$$

Because the elasticities of demand are functions of the cost shares, they will vary across the sample. Rather than report the estimated elasticities for each observation, the elasticities are evaluated at the means of the data and only these values are reported here.[9] The data are scaled so that the means of the prices are equal to unity. Therefore the estimated α_k; k = E, O, G, C, are equal to the fitted cost shares at the means and the formulas for the elasticities at the means are given by (9) and (10) with M_k replaced by α_k.

Since the elasticities at the means are functions only of the estimated parameters, the calculation of their estimated standard errors is considerably simplified. A first order Taylor series approximation to the variance of the estimated elasticities can be computed as

$$S_B' V(B) S_B$$

where S_B is the column vector of first partial derivatives of the elasticities with respect to the parameters α_k and γ_{km} and V(B) is the estimated variance-covariance matrix of the parameter estimates.[10]

III. Empirical Results

The systems of cost share equations are estimated with Census of Manufactures data for 1971, 1962, and 1958. The Census provides data on the quantity consumed and total cost for each type of energy.[11] The price of each type of energy is calculated by dividing cost by quantity consumed.[12]

The model including electric energy, fuel oil, natural gas and coal is estimated for nine industries in 1971, eleven in 1962, and eight in 1958. The industries for which the four input model could be estimated tended to be the major energy users. For example, this group of industries accounted for 67.7% of consumption of total energy and 93.0% of coal consumption in manufacturing in 1971. The model excluding coal is estimated for ten industries in 1971, four in 1962, and four in 1958. There are too few observations to obtain results for the remaining two-digit industries in each year.

Derivation of the cost share equations from the unit cost function implies cross-equation equality restrictions on the γ_{ij}, see (7) above. In order to test whether or not the loss of fit from imposing the equality restrictions is significant, the equations are estimated with and without the restrictions imposed. The results are compared by computing $-2 \log \lambda$, where λ is the ratio of the maximum value of the likelihood function for the restricted equations to the maximum value of the likelihood function for the unrestricted equations. Under the null hypothesis this test statistic is distributed asymptotically as chi-squared with degrees of freedom equal to the number of restrictions being tested.

Test results are shown in Table 2. Because these restrictions are directly implied by derivation of the cost share equations from the cost function, a very small significance level, .001, is used for the tests. The cross-equation equality restrictions are rejected for three industries in 1971, three in 1962, and none in 1958.[12a]

Tests of the separability of electric energy, fuel oil, and natural gas from coal were performed for those industries for which the four input model could be estimated. In every case, both separability and explicit separability were accepted at the .01 level in all years.

Monotonicity of the unit cost function is checked by determining if the fitted values of the cost shares are positive. Of the 942 fitted cost shares in 1971, 938 are positive. Similarly, 708 of 713 are positive in 1962 and 579 of 584 are positive in 1958. Since it is not determined whether or not the negative fitted cost shares are significantly different from zero, this check does not provide a statistical test. However, a statistical test is available at the means of the data where the fitted cost shares are equal to the estimated α_i. With the exception of α_G for industry 26 in 1958, which is positive but insignificant, all α_i are significantly positive in all years. Therefore, monotonicity is accepted at the means of the data.[13]

Concavity of the unit cost function is checked by examining the signs of the principal minors at each observation. The number of observations with principal minors of the incorrect sign are shown in Table 3. Although it is not determined if the principal minors with incorrect signs are statistically significant, the existence of incorrect signs for more than a few percent of the observations is considered to cast doubt on the satisfaction of the concavity condition.[13a]

The performance of the model is questionable with respect to either the regularity conditions or cross-equation equality restrictions for nine industries in 1971, four in 1962, and two in 1958. However, the model performs very well for the remaining industries in each year. The industries for which the model performs well account for most of the industrial consumption of energy in each year. For example, the group of industries for which the model performed well accounted for 81.6% of total energy consumption in manufacturing in 1971. The share of this group in the consumption of each type of energy in 1971 was 76.1% for electric energy, 78.0% for fuel oil, 85.7% for natural gas and 87.6% for coal. For brevity, further results are given only for the industries for which the model performs well.[14]

Parameter estimates and asymptotic standard errors are shown in Tables 4,5, and 6, together with the value of the pseudo-R^2 for each system of equations. Results shown are for the initial regressions, there was no sequential estimation.

The estimates of the α_i parameters are equal to the fitted cost shares at the means of the data. Since the cost shares are equal to the elasticity of the unit cost of energy with respect to the price of each type of energy, $\partial \ln V/\partial \ln P_i$, the estimates of α_i show the responsiveness of the price of aggregate energy to the prices of each type of energy at the means of the data. As shown in Tables 4-6, changes in the price of electric energy have the greatest effect on the price of aggregate energy.

The estimates of the γ_{ij} parameters can be interpreted as estimated share elasticities. The cost share of input i is equal to $\partial \ln V/\partial \ln P_i$. The cross partial derivative

$$\frac{\partial^2 \ln V}{\partial \ln P_i \partial \ln P_j} = \gamma_{ij}$$

can be defined as a constant share elasticity summarizing the response of cost share M_i to a change in $\ln P_j$. Alternatively the share elasticity can be defined as

$$\frac{\partial \ln M_i}{\partial \ln P_j} = \frac{\gamma_{ij}}{M_i} .$$

In the latter case, the estimated share elasticities at the means of the data will be equal to the estimates of γ_{ij}/α_i.

Estimates of the price elasticities of demand at the means of the data are shown in Tables 7, 8 and 9 together with their approximate standard errors. Because the elasticities are derived from unit cost functions for energy, they show the price responsiveness of demand for individual types of energy holding total energy input constant.

Estimates of the own price elasticities are shown in the first four rows of the table. All the own price elasticities have the correct sign in all years. For 1971, 29 of the 37 estimates are significant at the five percent level using a one-tailed test. For 1962, 36 of 40 are significant and for 1958 31 of 35 are significant. There is considerable variation across industries in the estimated own price elasticities. For example, in 1971 the range is -.124 to -1.096 for electric energy, -1.151 to -4.300 for fuel oil, -.425 to -2.134 for natural gas and -.656 to -2.531 for coal.

The estimates of the cross price elasticities also show considerable variation across industries. The relationship between different types of energy appears to be predominantly that of substitutes rather than complements. In 1971, only 18 of the 102 estimates are negative, and only two of the negative estimates are statistically significant at the ten percent level, using a two-tailed test. In 1962, 16 of 108 cross elasticities are negative, of which two are significant. In 1958, 6 of 90 estimated cross elasticities are negative, and none of the negative estimates are statistically significant.

As would be expected, the estimates of the cross price elasticities tend to be smaller in absolute magnitude than the estimates of own price elasticities.[15] However, the results do indicate significant cross price responsiveness of energy demand. For example, of the 84 positive estimates of cross price elasticities in 1971, 54 are significant at the ten percent level.

Aggregate price elasticities for the group of industries for which the model performed well in each year are estimated by constructing weighted averages for individual industries. The weights are each industry's share of total group consumption for the relevant type of energy.[16] Since the model performs well for industries accounting for most of energy consumption in manufacturing in each year, the group elasticities provide reasonable approximations to the aggregate elasticities for total manufacturing.

The weighted average elasticities are shown in Table 10. With the exception of oil, the estimated own price elasticities are quite consistent in each of the three years. The oil own price elasticity is much larger in absolute magnitude in 1971 than in the earlier years. The cross price elasticities of demand for oil with respect to the prices of other types of energy are also considerably larger in 1971 than in earlier years.

Changes in estimated elasticities over time may be due either to changes in the cost shares or changes in the estimated parameters of the unit cost function due to technological change. To test for the consistency of the parameters of the cost function over time, the data for 1971 and 1962 were pooled and F tests were performed for homogeneity of the estimated parameters.[17] The test statistic is

$$\frac{(RSS_{\omega} - (RSS_{\Omega 1} + RSS_{\Omega 2}))/k}{(RSS_{\Omega 1} + RSS_{\Omega 2})/n-2k}$$

where

RSS_{ω} = The residual sum of squares of the joint regression,

$RSS_{\Omega 1}$ = The residual sum of squares of the 1962 regression when run separately,

$RSS_{\Omega 2}$ = The residual sum of squares of the 1971 regression when run separately,

n = Total number of observations in pooled set,

k = Number of parameters estimated.

The test statistic is distributed as F with (k, n-2k) degrees of freedom.

It was possible to compute the test statistic for 13 of the industries for which the model performed well in 1971 and 1962. Test results are shown in Table 12. The null hypothesis of no change in the estimated parameters between the two years is rejected at the .01 level for 8 of the industries. Thus differences in the estimated elasticities of demand appear to reflect changes over time in the parameters of the cost function as well as changes in the shares of each type of energy. Chapter 4 presents further results on technical change in energy use in manufacturing.

As noted above, price elasticities estimated with a unit cost function for energy show the extent of price responsiveness holding aggregate energy input constant. This is clearly not equal to the total price responsiveness since a change in the price of a type of energy will affect the price of aggregate energy and thus will affect total energy input. Treating aggregate energy input as variable, the effect of a change in the price of energy input j on the quantity of energy input i is

$$E_{ij}^{T} = E_{ij} + E_{iH} \cdot E_{Hj}$$

where E_{ij} is the price elasticity holding aggregate energy, H, constant, E_{iH} is the elasticity of demand for energy input i with respect to aggregate energy, and E_{Hj} is the elasticity of demand for aggregate energy with respect to the price of energy input j.

Since the energy input function is assumed to be linear homogeneous, E_{iH} is equal to one. Also,

$$E_{Hj} = \frac{\partial \ln H}{\partial \ln V} \cdot \frac{\partial \ln V}{\partial \ln P_j} = E_{HV} \cdot M_j$$

where E_{HV} is the elasticity of demand for aggregate energy with respect to the price of aggregate energy. Therefore

$$E_{ij}^{T} = E_{ij} + E_{HV} \cdot M_j. \qquad (11)$$

Berndt and Wood [1975] obtained estimates for E_{HV} for each year for total U.S. manufacturing.[18] Their estimates were substituted in (11) to obtain estimates of the elasticities of demand for each type of energy allowing total energy input to vary. The weighted average elasticities for the group of industries are shown in Table 11.[19]

Allowing aggregate energy to vary increases the absolute magnitudes of the own price elasticities and decreases the cross price elasticities. Elasticities involving the price of electric energy are affected the most, since the price of electric energy has the greatest effect on the price of aggregate energy input. The own price elasticity for electric energy in 1971 increases in absolute magnitude to -0.92, which is comparable to estimates obtained in previous studies.[20]

Estimates of Allen partial elasticities of substitution can be calculated from the estimated price elasticities and cost shares. The cross elasticity of substitution is equal to [21]

$$\sigma_{ij} = \frac{1}{M_j} E_{ij} = \frac{M_i M_j + \gamma_{ij}}{M_i M_j} .$$

Thus the cross elasticities of substitution can be interpreted as normalized price elasticities where the normalization is chosen such that the elasticities are invariant to the ordering of the factors. Accordingly, $\sigma_{ij} = \sigma_{ji}$ although, in general, $E_{ij} \neq E_{ji}$. Estimates of the cross elasticities of substitution for 1971 are shown in Table 13 for the industries for which the model performed well.

IV. Concluding Comments

Estimation of demand functions derived from translog unit cost functions provides estimates of elasticities of demand and substitution that are subject to minimal *a priori* restrictions.[22] Disaggregation of the analysis to the two-digit industry level allows for variation across industries in the characteristics of demand for each type of energy.

The model performs well for industries accounting for most of the total consumption of energy in manufacturing in each year. The results indicate considerable variation in energy substitution both across industries and across types of energy. This variation should be taken into account in the formulation of energy policies. Aggregate manufacturing demand for energy appears to be highly price responsive. Estimated own price elasticities for all types of energy except electric energy are generally substantially greater than unity.

Two points should be noted with respect to the interpretation of the estimated elasticities for analysis of public policies toward energy. First, the estimates reflect the long-run effects of prices on energy demand. Short-run effects can be expected to be considerably smaller. Second, the elasticities do not measure the net effects of price changes on consumption of fuel oil, natural gas, and coal. Because these fuels are inputs in the production of electric energy, the net effects of price changes will include changes in the demand for fuels in electric power generation.[23]

CHAPTER 3

SUBSTITUTION AMONG ENERGY, CAPITAL, AND LABOR

Jay Ford

I. Introduction

As discussed in Chapter 2, elasticities of demand for individual types of energy are estimated using a unit cost function for energy. The use of a unit cost function for energy is based on the assumption that energy inputs are separable from all other inputs in the production function. The separability of energy inputs from capital and labor can be tested for 1958 by estimating an expanded cost function including these inputs. Unavailability of adequate data on capital prevents the expanded cost function from being estimated for other years.

Estimation of the cost function including capital and labor also provide estimates of the elasticities of demand for these inputs as well as their cross elasticities with energy inputs. The cross elasticities have considerable interest for policy analysis. For example, the cross elasticities of demand for energy inputs with respect to the price of capital will indicate the effect of investment incentives on demand for each type of energy. Also, since labor input is disaggregated into production and non-production workers, the estimated cross elasticities will indicate whether changes in energy prices have differential effects on employment of different types of labor.[1]

Research on this portion of the study is still in progress. Model formulation and data collection are complete but estimation is not. Therefore the results reported here are preliminary only. Final results will appear in a University of Washington Ph.D. dissertation by Jay Ford.

II. The Model

It is assumed that there exists by state and two-digit manufacturing industry a positive, homogeneous of degree one, concave production function y = (K,B,W,E,O,G,X), where y is gross output, K represents capital services, B and W are the services of production and non-production employees, E, O, and G are electric energy, fuel oil, and natural gas, and X is a vector of remaining inputs. If, in addition, capital, labor, and the energy inputs are separable from X, then it is appropriate to express the production function in the form y = g(h(K,B,W,E,O,G),X), and h will have the same properties as f. These properties guarantee the existence of a unique cost function G, dual to h, for which is selected the transcendental logarithmic form,

$$\ln G = \alpha_0 + \sum_i \alpha_i \ln P_i + \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j, \qquad i,j=K,B,W,E,O,G. \qquad (1)$$

where $\gamma_{ij} = \gamma_{ji}$.

Restrictions on the translog function ensure that the cost function is homogeneous of degree one and the associated demand functions are homogeneous of degree zero. These restrictions are:

(a) $\sum_i \alpha_i = 1$,

(b) $\sum_i \gamma_{ij} = 0$, and

(c) $\sum_j \gamma_{ij} = 0$, $\quad i,j=K,B,W,E,O,G.$

Imposition of these restrictions together with the cross-equation equality restrictions result in the following set of factor share equations:

$$M_K = \alpha_K + \gamma_{KB}(\ln P_B - \ln P_K) + \gamma_{KW}(\ln P_W - \ln P_K) + \gamma_{KE}(\ln P_E - \ln P_K)$$
$$+ \gamma_{KO}(\ln P_O - \ln P_K) + \gamma_{KG}(\ln P_G - \ln P_K)$$

$$M_B = \alpha_B + \gamma_{KB}(\ln P_K - \ln P_B) + \gamma_{BW}(\ln P_W - \ln P_B) + \gamma_{BE}(\ln P_E - \ln P_B) + \gamma_{BO}(\ln P_O - \ln P_B) + \gamma_{BG}(\ln P_G - \ln P_B)$$

$$M_W = \alpha_W + \gamma_{KW}(\ln P_K - \ln P_W) + \gamma_{BW}(\ln P_B - \ln P_W) + \gamma_{WE}(\ln P_E - \ln P_W) + \gamma_{WO}(\ln P_O - \ln P_W) + \gamma_{WG}(\ln P_G - \ln P_W)$$

$$M_E = \alpha_E + \gamma_{KE}(\ln P_K - \ln P_E) + \gamma_{BE}(\ln P_B - \ln P_E) + \gamma_{WE}(\ln P_W - \ln P_E) + \gamma_{EO}(\ln P_O - \ln P_E) + \gamma_{EG}(\ln P_G - \ln P_E)$$

$$M_O = \alpha_O + \gamma_{KO}(\ln P_K - \ln P_O) + \gamma_{BO}(\ln P_B - \ln P_O) + \gamma_{WO}(\ln P_W - \ln P_O) + \gamma_{EO}(\ln P_E - \ln P_O) + \gamma_{OG}(\ln P_G - \ln P_O)$$

$$M_G = \alpha_G + \gamma_{KG}(\ln P_K - \ln P_G) + \gamma_{BG}(\ln P_B - \ln P_G) + \gamma_{WG}(\ln P_W - \ln P_G) + \gamma_{EG}(\ln P_E - \ln P_G) + \gamma_{OG}(\ln P_O - \ln P_G). \quad (3)$$

Because the cost shares must sum to unity any one of the above equations will be a linear combination of the others and may be dropped from the estimation process. The system of five share equations is estimated with an iterative Zellner efficient procedure, which is equivalent to maximum likelihood estimation. Thus the parameter estimates are invariant to the choice of equation to be omitted from the system.

It is natural to think of the conditions necessary for the separability of functions in terms of marginal rates of substitution, however equivalent restrictions can be stated via the equality of certain of the Allen partial elasticities of substitution. In this case the energy inputs are weakly separable from labor and capital if the following equalities hold:

$$ES_{EK} = ES_{OK} = ES_{GK}$$

$$ES_{EB} = ES_{OB} = ES_{GB}$$

$$ES_{EW} = ES_{OW} = ES_{GW} \quad (4)$$

Expressing ES_{ij} in terms of the parameters of the share equations, the first of the conditions in (4) is

$$\frac{M_E M_K + \gamma_{EK}}{M_E M_K} = \frac{M_O M_K + \gamma_{OK}}{M_O M_K} = \frac{M_G M_K + \gamma_{GK}}{M_G M_K}$$

which is equivalent to $\gamma_{EK}/\alpha_G = \gamma_{OK}/\alpha_O = \gamma_{GK}/\alpha_G$ at the means of the data. Thus the substitution of $\theta_K\alpha_i$ for each of these γ's assures the equality of the ES_{iK}, i=E,O,G. In a similar fashion the full set of restrictions of the γ's necessary for the separability of electricity and fuels from labor and capital can be worked out. These are:

$$\begin{array}{lll} \gamma_{EK} = \theta_K\alpha_E & \gamma_{EB} = \theta_B\alpha_E & \gamma_{EW} = \theta_W\alpha_E \\ \gamma_{OK} = \theta_K\alpha_O & \gamma_{OB} = \theta_B\alpha_O & \gamma_{OW} = \theta_W\alpha_O \\ \gamma_{GK} = \theta_K\alpha_G & \gamma_{GB} = \theta_B\alpha_G & \gamma_{GW} = \theta_W\alpha_G . \end{array} \tag{5}$$

The conditions in (5) are substituted into the factor share equations (3) in order to incorporate the type of separability which is being investigated.

The statistical procedure for testing the hypothesis of separability of energy inputs is based on the ratio of the values of the likelihood functions of the restricted and unrestricted sets of equations. Minus twice the logarithm of this ratio has asymptotically a chi-square distribution with degrees of freedom equal to the reduction in the number of estimated parameters in going from the unrestricted to the restricted set of factor share equations. If the hypothesis of weak separability is not rejected, the restriction that $\theta_i = 0$, i=K,B,W, may be imposed in order to test for explicit separability of the translog cost function.

III. Data Sources and the Construction of Variables

The Census of Manufactures, 1958 was distinguished by the publication of various data relating to the cost of owning and maintaining capital equipment and structures. This information is reported at infrequent intervals and the 1958 census is the most recent, although much less detailed data collected in

1963 became available in 1971. These data, along with information from which may be inferred the prices of production and non-production workers and energy inputs, are sufficient to estimate the translog cost functions outlined in the previous section.

With respect to the energy inputs the calculation of prices is a simple matter of dividing the reported expenditures on electricity, oil, and gas by the quantity at each observation. These prices are then transformed into cents per kilowatt-hour equivalent. The price of capital services cannot be developed in this straight-forward manner, however, because the transaction normally takes place within the firm. Hence we have adopted with some modification the procedure used by Christensen and Jorgenson [1969], inferring the rental price of capital from the price of investment goods, the rate of return on corporate property, and the rates of taxation, depreciation, and capital gains.

The rental price of capital to industry i in state j is calculated as follows:

$$P_{ij} = \left[\frac{1 - u_{ij}Z_{ij}}{1 - u_{ij}}\right] [q_{t-1}r + q_t w_{ij} - (q_t - q_{t-1})] + q_t d_{ij},$$

where q is the asset price of capital, u is the effective rate of combined Federal and state corporate income taxes, Z is the present value of depreciation deductions for tax purposes on a one dollar investment in producers' durables, w is the rate of depreciation, and d is the effective property tax rate.

As is indicated by the absence of state and industry subscripts on q and r, it is assumed that a single market exists for new investment goods and capital so that then prices are equalized across states and industries. The remaining elements of the rental price of capital are constructed in the following manner:

u_{ij} = the sum of the effective Federal corporate profits tax by industry and the state corporate profits tax by state,

$Z_{ij} = \frac{1}{rL_{ij}} [1 - (\frac{1}{1+r})^{L_{ij}}]$, where r is the discount rate and L_{ij} is an estimate of the lifetimes of depreciable assets used for tax purposes,

w_{ij} = annual depreciation charges in 1957 divided by the gross book value of depreciable assets as of December 31, 1957, and

d_{ij} = property taxes paid during 1957 divided by the gross book value of depreciable assets as of December 31, 1957.

This procedure amounts to the calculation of the annual user cost of one dollar's worth of capital equipment.

The available 1958 employment data by state and two-digit industry consist of total payroll and number of employees and for production workers, the number of workers, man-hours, and total wages. The price of production workers' services is total wages divided by man-hours. The price for non-production workers is based on the residual payroll and number of employees and the assumption that white-collar workers are employed for forty hours per week. The wage rate for both production and non-production workers is augmented to reflect the reported cost of supplementary employee benefits.

These prices will be adjusted to account for interstate differentials in the quality of the labor force. The following labor quality indices, used in Griliches [1967] have been provided by Zvi Griliches:

1. occupational mix of employees by state and industry,
2. median age of employed males,
3. white as fraction of total employed males, and
4. females as a fraction of all employees.

These quality-of-the-labor force variables are intended to serve as proxies for education and training so that nominal wage differentials across states and industries are converted into wage differentials in efficiency units.

IV. Empirical Results

A sufficient number of observations is available for the estimation of the system of cost share equations for eight of the twenty-two digit manufacturing industries. Results available to date are for data that has not been adjusted to account for interstate differentials in the quality of the labor force.

Results of the tests of separability of the energy inputs from capital and labor are shown in Table 1. The null hypothesis of weak separability of energy inputs is not rejected for industries 20, 28, 33, and 34, food and kindred products, chemicals and allied products, primary metal industries, and fabricated metal products, respectively. For these industries, which account for some two-thirds of energy consumption by the group of eight industries, the use of a unit cost function for energy is appropriate. However, the rejection of separability for the remaining four industries indicates that the interfuel substitution results obtained under the assumption of separability have to be interpreted with caution.

Estimated own price elasticities for capital and the two types of labor as well as the cross elasticities of these inputs with the energy inputs are shown in Table 2. All own price elasticities for capital are significant at the five percent level using a one-tailed test. Five of the own price elasticities for blue collar workers and two of the own elasticities for white collar workers are signicant at this level.

The estimated cross price elasticities are generally not statistically significant,[2] but the pattern of results is suggestive. Previous studies have indicated that capital and aggregate energy are complements.[3] However, only nine of the twenty-four cross-elasticities between capital and individual energy inputs are negative. Evidence of complementarity is somewhat greater for non-production workers and energy inputs, with eleven negative cross-elasticities, and is least for production workers, with only five negative cross-elasticities.

TABLE 1

Separability Tests

Industry	Observations	Test Statistic: Energy Separability[a]	Test Statistic: Explicit Energy Separability[b]
20	38	7.38	8.68
26	18	23.62[c]	----
28	22	11.35	21.81[c]
32	20	21.01[c]	----
33	19	10.15	36.78[c]
34	20	10.22	12.56[c]
35	13	23.33[c]	----
37	13	20.78[c]	----

a. Degree of freedom = 6, Critical value = 16.81, significance level = .01

b. Degree of freedom = 3, Critical value = 11.34, significance level = .01

c. The null hypothesis is rejected.

TABLE 2

Estimates of Price Elasticities: 1958

Elasticity	Industry 20	26	28	32
E_{KK}	-.679 (.103)	-1.452 (.482)	-.853 (.173)	-.802 (.141)
E_{BB}	-.260 (.196)	-.647 (.144)	-.508 (.173)	-.442 (.123)
E_{WW}	-.652 (.383)	.371 (.265)	-.543 (.277)	-.272 (.538)
E_{KE}	.048 (.017)	.029 (.056)	.083 (.066)	-.002 (.031)
E_{KO}	.003 (.019)	-.097 (.084)	.032 (.015)	.079 (.035)
E_{KG}	-.013 (.021)	.008 (.038)	.045 (.022)	.134 (.063)
E_{BE}	-.0004 (.016)	.031 (.031)	.123 (.043)	.032 (.030)
E_{BO}	-.026 (.020)	.073 (.043)	.025 (.031)	.010 (.028)
E_{BG}	.019 (.024)	.114 (.021)	.045 (.033)	.043 (.046)
E_{WE}	.016 (.030)	.080 (.068)	.215 (.097)	.033 (.122)
E_{WO}	.056 (.035)	.097 (.089)	-.078 (.045)	-.151 (.097)
E_{WG}	.007 (.047)	-.030 (.073)	.001 (.050)	-.030 (.178)

Figures in parentheses are asymptotic standard errors.

TABLE 2
(Continued)

Estimates of Price Elasticities: 1958

Elasticity	Industry 33	34	35	37
E_{KK}	-1.025 (.368)	-1.085 (.584)	-1.156 (.294)	-2.060 (.566)
E_{BB}	-.996 (.162)	-.419 (.150)	-.358 (.329)	-0.205 (.430)
E_{WW}	-1.302 (.495)	-.183 (.182)	-.304 (.592)	.448 (.933)
E_{KE}	.039 (.085)	-.021 (.018)	.022 (.019)	-.010 (.028)
E_{KO}	.007 (.034)	.029 (.029)	.005 (.008)	-.014 (.019)
E_{KG}	-.011 (.022)	-.014 (.028)	.024 (.007)	-.012 (.005)
E_{BE}	.078 (.046)	.029 (.005)	.056 (.030)	.024 (.022)
E_{BO}	.050 (.026)	.001 (.008)	.019 (.014)	-.026 (.014)
E_{BG}	-.002 (.024)	.025 (.008)	-.005 (.011)	.007 (.004)
E_{WE}	.220 (.091)	-.018 (.016)	-.073 (.055)	-.045 (.051)
E_{WO}	-.057 (.086)	-.013 (.022)	-.029 (.026)	.083 (.032)
E_{WG}	.154 (.071)	-.034 (.023)	.004 (.019)	.008 (.009)

CHAPTER 4

DYNAMIC STRUCTURE OF ENERGY DEMAND

I. Introduction

Energy demand in manufacturing should not be expected to respond instantaneously to changes in energy price. Due to adjustment costs for energy, and other inputs, the full response of energy demand to a change in price may be spread over a number of years. However, previous studies have largely ignored the dynamic structure of energy demand.

In this chapter the model of demand for factors of production developed by Nadiri and Rosen [1969, 1973] is adapted to estimate dynamic demand equations for energy and other inputs. The results include estimates of short-run, long-run and intermediate-run price elasticities of demand. The results also include estimates of the response of demand for each input to temporary excess demands for other inputs. Pairs of inputs are defined as dynamic substitutes if the response of each to excess demand for the other is positive and dynamic complements if the response is negative.

All estimated short-run elasticities of demand for energy are statistically significant. Estimated long-run elasticities are similar to estimates in earlier studies. The responses to price changes are found to be quite rapid, with the full long-run responses being approximately realized within three years after the year in which a price change occurs.

The estimated long-run price elasticities indicate that capital and energy are complements and labor and energy are substitutes in the long run. The estimated responses of demand to excess demands for other inputs indicate that capital and energy are also dynamic complements and labor and energy are dynamic substitutes.

II. The Model

A Cobb-Douglas aggregate production function is assumed,

$$\ell nQ_t = \alpha_0 + \alpha_E \ell nE_t + \alpha_K \ell nK_t + \alpha_U \ell nU_t + \alpha_L \ell nL_t + \alpha_H \ell nH_t \tag{1}$$

where

Q = output,

E = energy input,

K = capital stock,

U = utilization rate of capital,

L = labor stock,

H = utilization rate of labor.

Stocks and utilization rates of capital and labor are entered separately since they are separate objects of choice by the firm. Energy input is measured as a flow. The firm may hold stocks of some types of energy, but most energy is obtained directly from outside the firm. Therefore, it is not meaningful to break down energy input into stock and utilization rate components.

The firm is assumed to minimize costs subject to an output constraint. The firm's costs are

$$C = P_E E + P_K K + P_H (LH) + P_L L$$

where P_E is the price of energy, P_K is the user cost of the capital stock, P_H is the hourly wage rate, and P_L is the user cost of the labor stock. The utilization rate of capital, U, does not appear explicitly in the cost equation but appears implicitly through the effect of U on the depreciation rate and hence on P_K, $P_U = P_K \cdot dP_K/dP_U$.

Solving the static cost minimization problem yields long-run equilibrium demand equations,

$$\begin{bmatrix} \ln E \\ \ln K \\ \ln U \\ \ln L \\ \ln H \end{bmatrix} = \begin{bmatrix} k_E \\ k_K \\ k_U \\ k_L \\ k_H \end{bmatrix} + \begin{bmatrix} \frac{1}{\rho} \\ \frac{1}{\rho} \\ 0 \\ \frac{1}{\rho} \\ 0 \end{bmatrix} [\ln Q] + \begin{bmatrix} \frac{\alpha_E}{\rho}-1 & \frac{\alpha_K-\alpha_U}{\rho} & \frac{\alpha_U}{\rho} & \frac{\alpha_L-\alpha_H}{\rho} & \frac{\alpha_H}{\rho} \\ \frac{\alpha_E}{\rho} & \frac{\alpha_K-\alpha_U}{\rho}-1 & \frac{\alpha_U}{\rho} & \frac{\alpha_L-\alpha_H}{\rho} & \frac{\alpha_H}{\rho} \\ 0 & +1 & -1 & 0 & 0 \\ \frac{\alpha_E}{\rho} & \frac{\alpha_K-\alpha_U}{\rho} & \frac{\alpha_U}{\rho} & \frac{\alpha_L-\alpha_H}{\rho}-1 & \frac{\alpha_H}{\rho} \\ 0 & 0 & 0 & +1 & -1 \end{bmatrix} \begin{bmatrix} \ln P_E \\ \ln P_K \\ \ln P_U \\ \ln P_L \\ \ln P_H \end{bmatrix} \quad (2)$$

where $\rho = \alpha_E + \alpha_K + \alpha_L$. The long-run demand equations have several interesting characteristics. First, the level of output has no effect on utilization rates in the long-run. Second, long-run utilization rates are independent of all cross-price effects except with respect to the price of the corresponding stock variable. Third, the prices of the utilization inputs, P_U and P_H, affect the demand for energy positively.

The long-run demand equations can be written in matrix notation as

$$X^* = k + BQ + CR \quad (3)$$

where all variables are in logarithmic form, X^* is a vector of optimal input demands, k is a vector of intercept terms, B is a vector of scale effects, Q is output, C is a matrix of factor price effects, and R is a vector of factor prices. Since the demand equations are log-linear, the elements of matrix C are equal to long-run price elasticities.

The firm will not be in long-run equilibrium at every point in time due to costs in adjusting inputs to their desired levels. A log-linear adjustment function is assumed

$$\ln X_{it} - \ln X_{it-1} = \sum_j \gamma_{ij}(\ln X_{jt}^* - \ln X_{jt-1}) + \varepsilon_{it} \qquad i,j=E,K,U,L,H, \quad (4)$$

where X_{it} is the quantity of input X_i in period t, X_{jt}^* is the desired or target level of input j in period t and is defined by (3), the γ_{ij} are fixed adjustment coefficients, and the ε_{ij} are random variables with zero means and constant variances. The specification of the adjustment function allows the adjustment of each input to be affected by the level of "excess demand" for all inputs.

Writing (4) in matrix notation,

$$X_t = \Gamma X_t^* + (I - \Gamma)X_{t-1} + \varepsilon_t \tag{5}$$

where all variables are in logarithmic form, X_t is a vector of input quantities in period t, Γ is a matrix of adjustment coefficients, X_t^* is a vector of desired input quantities in period t, I is an identity matrix, and ε_t is a vector of error terms. The equations to be estimated are derived by substituting for X_t^* in (5) from (3),

$$X_t = \Gamma A + \Gamma BQ + \Gamma CR + (I - \Gamma)X_{t-1} + \varepsilon_t \ . \tag{6}$$

The diagonal elements of Γ are own adjustment coefficients and should satisfy the restriction, $0 \leq \gamma_{ii} \leq 1$, for all i. The more variable the factor, the closer to unity will be the own adjustment coefficient. The off diagonal elements, γ_{ij}, indicate the effect on input i of excess demand for input j. The γ_{ij} can be either positive or negative. Assuming that the firm remains on its production function at all times, not all elements in any row of Γ can be of the same sign. Inputs must react positively to excess demand for some inputs and negatively to excess demand for others.[1]

The cross-adjustment coefficients need not be symmetric. Pairs of inputs with identically signed cross-adjustment coefficients can be identified as dynamic substitutes or dynamic complements. If γ_{ij} is positive, excess demand for input j increases the short-run demand for input i. Pairs of inputs for which cross-adjustment coefficients are positive are defined to be dynamic substitutes. If the cross-adjustment coefficients are negative, the inputs are dynamic complements. Due to short-run adjustment costs, inputs may be dynamic complements even though they are substitutes in the long run.

Because the input demand equations are log-linear, the elements of the matrix product ΓC are equal to the short-run price elasticities of demand. From (3), the long-run price elasticities are equal to the elements of matrix C. Given estimates of $(1 - \Gamma)$ and ΓC, the estimated long-run price elasticities are computed from $[I - (I - \Gamma)]^{-1}\Gamma C$.

The estimated response path of input demand to prices can be computed from $\Gamma_k = (I - \Gamma)^k \Gamma C$, see Nadiri and Rosen [1973, p. 75]. Computation of

$$\Gamma_0 + \Gamma_1 + \ldots + \Gamma_n \qquad n = 0, 1, \ldots.$$

provides a matrix of estimated price elasticities showing the extent to which demand responds during a length of time n+1 periods long. Thus in addition to short-run and long-run price elasticities, the results provide estimates of all intermediate-run elasticities.

Dynamic stability of the system of demand equations requires that $(I - \Gamma)^n$ approaches zero as n gets large. Note that $(I - \Gamma) = M'\lambda M$ where M is a matrix of characteristic vectors of $(I - \Gamma)$ and λ is a diagonal matrix of characteristic roots. Thus $(I - \Gamma)^n = M'\lambda^n M$, which approaches zero if the absolute value of each element of λ is less than unity. Therefore dynamic stability can be checked by determining if the absolute values of the characteristic roots of $(I - \Gamma)$ are less than unity. However, this procedure does not provide a statistical test of stability because the sampling distributions of the characteristic roots are unknown.

III. Empirical Results

The equations are estimated with annual data for total manufacturing for 1947-1971. Data availability requires some modifications in the system of equations to be estimated. The prices of labor stock and of capital utilization are omitted due to lack of data. Because data on the utilization rate of capital are not available, the utilization rate of capacity is used as a proxy for this variable. The distinction is important, since the capacity utilization rate reflects the utilization of all inputs, not just capital. Capacity utilization data are from Wharton [1976].

The relevant output variable is the equilibrium level of output perceived by the firm. As measures of this variable, shipments and shipments plus changes in inventories are included in alternative specifications of the demand equations. Data on shipments and inventories are from U.S. Bureau of the Census [1976]. The output variables are deflated by the wholesale price index for manufactured goods from U.S. Department of Labor [1976b].

Data on capital stock, rental price of capital, energy input, and energy price are from Berndt and Wood [1975]. Data on labor stock and the utilization rate of labor are from U.S. Department of Labor [1976a]. The stock of labor is equal to the total number of employees in manufacturing. Average weekly hours of production workers is used as the labor utilization variable. Data on the price of labor utilization, defined as the quality adjusted wage rate, are from Berndt and Wood [1975].

In order to impose homogeneity of degree zero in prices, the demand equations are expressed in terms of price ratios. The equations estimated are then

$$\ell n X_{it} = a_i + b_i \ell n Q_t + c_{1i} \ell n(\frac{P_E}{P_H}) + c_{2i} \ell n(\frac{P_K}{P_H}) + g_{1i} \ell n E_{t-1} + g_{2i} \ell n K_{t-1} + g_{3i} \ell n U_{t-1} + g_{4i} \ell n L_{t-1} + g_{5i} \ell n H_{t-1} + e_{it} \qquad i = E,K,U,L,H. \tag{7}$$

The error terms, ε_{it} , are assumed to have zero means and constant variances. The use of ordinary least squares (OLS) to estimate (7) will result in biased estimates if the error terms are serially dependent. In order to test for first-order serial correlation, the demand equations are estimated by both OLS and a Cochrane-Orcutt [1949] procedure.

The OLS and Cochrane-Orcutt (CO) results are compared using the F-statistic,

$$F(1,\ n\text{-}k\text{-}1) = \frac{SSR_{OLS} - SSR_{CO}}{SSR_{CO}/n\text{-}k\text{-}1}\ ,$$

where n is the number of observations, k is the number of coefficients, SSR is the sum of squared residuals and the subscripts indicate the estimation procedure. This test is equivalent to a test of the null hypothesis that the first order serial correlation coefficient is equal to zero. The null hypothesis is rejected at the .10 level for four of the five demand equations. Therefore, the results obtained using the Cochrane-Orcutt procedure are reported here.

The estimated parameters in the equation using shipments plus changes in inventories as the measure of output are very similar to those obtained using shipments. Since the estimated standard errors are somewhat smaller when shipments are used, the results here are for this specification. Inclusion of a time variable to allow for trends in equilbrium output has little effect on the estimated parameters but does cause problems of collinearity. Therefore, the results reported here are for the equations excluding time.

Estimated parameters are shown in Table 1 together with their t-statistics. The short-run elasticities with respect to output are significant at the five percent level in all equations except the one for energy. The effect of output on input demand is largest for capacity utilization and next largest for labor stock. The insignificant effect of output on energy demand indicates that short-run forecasts of energy demand do not depend critically on predicted output.

The relative price of capital is significant at the five percent level in three equations and at the ten percent level in one more. However, the relative price of energy is significant only in the energy equation. The coefficients of the relative prices of capital and energy are equal to the estimated short-run elasticities with respect to these prices. The short-run elasticity with respect to the wage rate is equal to the negative of the sum of the capital and energy price elasticities. The elasticity with respect to the wage rate is significant only in the energy equation.

The estimated coefficients of the lagged endogenous variables are equal to the estimated elements of the matrix $(I - \Gamma)$. The elements of the matrix Γ are shown in Table 2. All own-adjustment coefficients are positive as required. The own-adjustment coefficient for labor stock is greater than unity but not significantly so.[2]

The own-adjustment coefficient for labor stock is largest, followed by the coefficients for average hours and energy. The magnitudes of these own-adjustment coefficients indicate that the corresponding inputs are truly variable. The own-adjustment coefficient for capital stock is considerably smaller, as would be expected. The own-adjustment coefficient for utilization is smallest of all. This apparently incongruous result may be due to the use of capacity utilization rather than capital utilization for this variable.

Seven of the twenty cross-adjustment coefficients are significant at the five percent level. The coefficients of capital stock and average hours worked are both highly significant in the energy equation, indicating that excess demand for these inputs has a significant effect on energy demand. The coefficient of energy is significant only in the capacity utilization equation.

The cross-adjustment coefficients need not be symmetric. Where the signs of the cross-adjustment coefficients for a pair of inputs are identical, it is possible to identify the inputs as dynamic substitutes or dynamic complements. The results indicate that energy and capital stock are dynamic complements, while capacity utilization and labor stock are dynamic substitutes for energy. Thus, excess demand for energy increases labor stock and capacity utilization and decreases capital stock. These results provide tentative information on the important policy question of the effect of temporary energy shortages on demand for other inputs.

All three short-run price elasticities for energy are significant at the .025 level. The time path of the price elasticities is shown in Table 3. The elasticities through year zero are equal to the estimated short-run elasticities. The estimated short-run own price elasticity is -0.28 and the estimated short-run cross elasticities with respect to the hourly wage rate and the user cost of capital are 0.34 and -0.10 respectively.

The last column of Table 3 shows the estimated long-run price elasticities. The results indicate that energy and labor are substitutes and energy and capital are complements in the long run. The estimated long-run own price elasticity is -0.42 and the estimated long-run cross elasticities with respect to the hourly wage rate and the user cost of capital are 0.57 and -0.15 respectively.

The estimated intermediate-run elasticities shown in Table 3 indicate quite rapid response to price changes. The cumulative elasticities through year three are approximately equal to the long-run elasticities. Two of the elasticities are actually slightly larger in absolute value than the long-run elasticities, indicating a small degree of over-shooting in the intermediate-run response to price changes.

Dynamic stability of the system of demand equations is checked by calculating the characteristic roots of $(I - \Gamma)$. The characteristic roots for the matrix used in calculating the elasticities shown in Table 3 are $1.0019 \pm 0.0435\, i$, $0.0425 \pm 0.0275\, i$, and 0.0652. The condition that all characteristic roots be less than unity is not satisfied. However, the departure from the conditions for stability is small and its statistical significance cannot be determined. The existence of complex roots is consistent with the non-monotonic behavior of the intermediate-run elasticities shown in Table 3.

IV. Concluding Comments

The use of a dynamic model of demand for inputs provides new information on the characteristics of energy demand in U.S. manufacturing. The estimated cross-adjustment coefficients indicate that energy is a dynamic complement of capital stock and a dynamic substitute for both labor stock and capacity utilization. Estimated short-run price elasticities of demand for energy are found to be statistically significant. The response of demand to price changes is quite rapid, with the full long-run response being approximately realized within three years after the year in which a price change occurs.

The estimated long-run price elasticities are similar to those reported in studies using static demand models. Berndt and Wood's [1975] study of U.S. manufacturing and Fuss' [1977] study of Canadian manufacturing also find that energy and capital are complements and energy and labor are substitutes in the long-run. The estimated long-run own price elasticity of -0.42 in the present study is comparable to the value of -0.47 obtained by Berndt and Wood for 1959, the mid-year of the sample period used here.[3]

Table 1

Estimated Parameters
(t-statistics in parentheses)

Independent Variables	Dependent Variables E	K	U	L	H
Constant	-4.699 (-2.779)	-0.614 (-0.354)	4.125 (1.301)	2.479 (0.858)	4.008 (4.447)
Output	0.095 (1.100)	0.142 (2.655)	1.011 (10.389)	0.619 (6.930)	0.174 (6.261)
P_E/P_H	-0.279 (-2.250)	-0.100 (-0.926)	0.162 (0.861)	0.015 (0.089)	0.038 (0.669)
P_K/P_H	-0.098 (-2.287)	-0.046 (-1.458)	0.116 (2.186)	0.117 (2.420)	0.032 (1.944)
E_{t-1}	0.175 (1.734)	0.094 (1.512)	-0.233 (-2.210)	-0.152 (-1.583)	-0.027 (-0.825)
K_{t-1}	0.547 (4.846)	0.531 (5.052)	-0.245 (-1.345)	-0.009 (-0.057)	-0.034 (-0.630)
U_{t-1}	-0.038 (-0.291)	0.163 (1.121)	0.675 (2.361)	0.178 (0.685)	0.038 (0.514)
L_{t-1}	-0.204 (-1.095)	0.342 (2.168)	-0.817 (-3.083)	-0.152 (-0.629)	-0.198 (-2.394)
H_{t-1}	1.634 (3.334)	-1.195 (-3.072)	-0.756 (-0.970)	0.796 (1.124)	0.014 (0.071)
$\bar{R}^2$	0.998	0.990	0.905	0.836	0.818

Table 2

Adjustment Coefficients

Independent Variables	Dependent Variables K	U	L	H	E
K_{t-1}	0.469	0.245	0.009	0.034	-0.547
U_{t-1}	-0.163	0.325	-0.178	-0.038	0.038
L_{t-1}	-0.342	0.817	1.152	0.198	0.204
H_{t-1}	1.195	0.756	-0.796	0.986	-1.634
E_{t-1}	-0.094	0.233	0.152	0.027	0.825

Table 3

Time Path of Energy Price Elasticities

Independent Variable	Cumulative Elasticity Through Year: 0	1	2	3	4	5	Long-run Elasticity
P_E	-0.279	-0.330	-0.393	-0.437	-0.444	-0.437	-0.415
P_H	0.337	0.406	0.514	0.566	0.563	0.550	0.568
P_K	-0.098	-0.116	-0.162	-0.170	-0.160	-0.155	-0.153

TABLE 1

Energy Consumption by Two-Digit Industries, 1971

Industry		Energy Consumption[a]	Percent of Total	Energy Intensiveness[b]
20	Food and kindred products	300.6	7.81%	2.59%
21	Tobacco manufacturers	5.5	0.14	0.76
22	Textile mill products	106.5	2.77	4.01
23	Apparel, other textile products	19.6	0.51	1.00
24	Lumber and wood products	68.4	1.78	4.32
25	Furniture and fixtures	17.8	0.46	1.54
26	Paper and allied products	385.4	10.02	7.47
27	Printing and publishing	30.1	0.78	0.98
28	Chemicals and allied products	814.2	21.16	5.67
29	Petroleum and coal products	466.9	12.13	11.33
30	Rubber and plastics, n.e.c.	66.3	1.72	2.79
31	Leather and leather products	9.8	0.25	1.43
32	Stone, clay, and glass products	382.3	9.93	7.75
33	Primary metal industries	716.7	18.62	10.00
34	Fabricated metal products	102.7	2.66	2.01
35	Machinery, except electrical	107.6	2.79	1.52
36	Electrical equipment and supplies	80.1	2.08	1.37
37	Transportation equipment	114.2	2.96	1.41
38	Instruments and related products	20.1	0.52	1.01
39	Miscellaneous manufacturing industries	18.4	0.47	1.52
	Total	3,847.1	100.00%	

a. Billions of kilowatt-hours equivalent.

b. Energy cost as percent of value added.

TABLE 2

Tests of Cross-Equation Equality Restrictions

Industry	1971 Test Statistic	1971 Critical Value[a]	1962 Test Statistic	1962 Critical Value[a]	1958 Test Statistic	1958 Critical Value[a]
20	23.1[b]	22.4	24.0[b]	22.4	17.7	22.4
22	17.1[b]	16.3	19.7	22.4	19.5	22.4
23	11.1	16.3	5.3	16.3	n.e.	
24	4.0	16.3	9.4	16.3	6.7	16.3
25	13.7	16.3	13.9	22.4	n.e.	
26	4.1	22.4	7.4	22.4	21.7	22.4
27	1.0	16.3	12.6	16.3	n.e.	
28	4.9	22.4	18.5	22.4	13.6	16.3
29	11.3	16.3	8.6	16.3	n.e.	
30	17.0	22.4	27.9[b]	22.4	8.6	16.3
31	58.4[b]	16.3	n.e.		n.e.	
32	10.5	22.4	10.5	22.4	15.1	22.4
33	12.4	22.4	14.3	22.4	4.4	16.3
34	21.7	22.4	54.9[b]	22.4	18.0	22.4
35	13.6	22.4	22.1	22.4	15.5	22.4
36	12.3	16.3	21.0	22.4	10.8	16.3
37	1.1	22.4	n.e.		7.7	16.3
38	3.5	16.3	n.e.		n.e.	
39	0.3	16.3	n.e.		n.e.	

n.e. not estimated

[a]Significance level = .001

[b]The null hypothesis is rejected.

TABLE 3

Signs of Principal Minors

Industry	Number of Observations with Incorrect Signs 1971	1962	1958
20	24	0	0
22	0	0	6
23	0	0	n.e.
24	10	0	0
25	9	0	n.e.
26	0	2	0
27	10	0	n.e.
28	0	6	0
29	0	0	n.e.
30	0	10	2
31	1	n.e.	n.e.
32	0	0	0
33	1	0	0
34	0	10	1
35	0	0	0
36	0	0	0
37	9	n.e.	0
38	9	n.e.	n.e.
39	3	n.e.	n.e.

n.e. - not estimated

TABLE 4

Parameter Estimates: 1971

Parameter	Industry 23	26	28	29	30
α_E	.856 (.017)	.414 (.022)	.518 (.021)	.371 (.041)	.683 (.018)
α_O	.065 (.009)	.206 (.031)	.107 (.016)	.175 (.034)	.093 (.016)
α_G	.079 (.013)	.200 (.023)	.210 (.020)	.454 (.050)	.162 (.013)
α_C	n.e.	.181 (.029)	.165 (.025)	n.e.	.062 (.016)
γ_{EE}	- .033 (.077)	.158 (.127)	- .104 (.087)	- .149 (.116)	.132 (.111)
γ_{EO}	.073 (.044)	- .006 (.113)	.161 (.058)	.174 (.092)	- .120 (.061)
γ_{EG}	- .070 (.052)	- .093 (.081)	.048 (.063)	- .024 (.080)	- .037 (.087)
γ_{EC}	n.e.	- .060 (.079)	- .105 (.074)	n.e.	.025 (.065)
γ_{OO}	- .104 (.040)	- .302 (.148)	- .282 (.077)	- .228 (.095)	- .022 (.078)
γ_{OG}	.031 (.028)	.206 (.148)	.123 (.058)	.055 (.064)	.101 (.060)
γ_{OC}	n.e.	.103 (.087)	- .002 (.064)	n.e.	.041 (.053)
γ_{GG}	.040 (.046)	- .266 (.098)	- .248 (.083)	- .030 (.098)	- .098 (.084)
γ_{GC}	n.e.	.153 (.076)	.077 (.070)	n.e.	.033 (.046)
γ_{CC}	n.e.	- .197 (.101)	.030 (.106)	n.e.	- .099 (.060)
Number of Observations	13	13	23	17	9
Pseudo R^2	.309	.603	.565	.216	.799

Figures in parentheses are asymptotic standard errors.

n.e. - not estimated

TABLE 4
(Continued)

Parameter Estimates: 1971

Parameter	Industry 32	33	34	35	36
α_E	.354 (.013)	.561 (.020)	.662 (.014)	.668 (.008)	.744 (.012)
α_O	.083 (.015)	.120 (.021)	.078 (.026)	.061 (.008)	.078 (.011)
α_G	.390 (.024)	.269 (.020)	.232 (.019)	.230 (.009)	.178 (.016)
α_C	.173 (.022)	.050 (.013)	.027 (.006)	.041 (.005)	n.e.
γ_{EE}	.118 (.091)	- .219 (.073)	- .503 (.238)	- .308 (.074)	- .012 (.055)
γ_{EO}	- .061 (.056)	.098 (.054)	.231 (.133)	.088 (.063)	.001 (.033)
γ_{EG}	- .018 (.064)	.100 (.059)	.191 (.148)	.153 (.045)	.011 (.048)
γ_{EC}	- .040 (.081)	.020 (.042)	.079 (.067)	.067 (.022)	n.e.
γ_{OO}	- .214 (.072)	- .224 (.076)	- .248 (.179)	- .204 (.069)	- .074 (.035)
γ_{OG}	.146 (.070)	.036 (.063)	.018 (.127)	.081 (.045)	.073 (.037)
γ_{OC}	.129 (.073)	.090 (.041)	- .002 (.052)	.035 (.022)	n.e.
γ_{GG}	- .280 (.123)	- .056 (.079)	- .160 (.156)	- .173 (.049)	- .083 (.062)
γ_{GC}	.153 (.100)	- .081 (.040)	- .049 (.041)	- .060 (.019)	n.e.
γ_{CC}	- .242 (.136)	- .029 (.043)	- .028 (.027)	- .041 (.013)	n.e.
Number of Observations	18	15	8	9	24
Pseudo R^2	.539	.638	.413	.912	.184

Figures in parentheses are asymptotic standard errors.

n.e. - not estimated.

TABLE 5

Parameter Estimates: 1962

Parameter	Industry 22	23	24	25	26	27
α_E	.635 (.022)	.787 (.026)	.677 (.020)	.657 (.017)	.377 (.032)	.784 (.010)
α_O	.170 (.047)	.123 (.022)	.220 (.025)	.106 (.017)	.254 (.037)	.089 (.014)
α_G	.085 (.026)	.090 (.020)	.103 (.021)	.108 (.024)	.053 (.045)	.127 (.020)
α_C	.110 (.035)	n.e.	n.e.	.129 (.015)	.316 (.059)	n.e.
γ_{EE}	- .032 (.044)	- .025 (.133)	- .074 (.067)	- .952 (.330)	.062 (.082)	.074 (.040)
γ_{EO}	.007 (.008)	.009 (.013)	- .045 (.013)	.049 (.015)	.002 (.012)	.004 (.010)
γ_{EG}	- .055 (.040)	.024 (.135)	.118 (.075)	.937 (.349)	.031 (.080)	- .078 (.036)
γ_{EC}	.081 (.046)	n.e.	n.e.	- .034 (.069)	- .094 (.076)	n.e.
γ_{OO}	.019 (.015)	.013 (.011)	.053 (.015)	.016 (.013)	.044 (.012)	- .011 (.013)
γ_{OG}	- .020 (.009)	- .014 (.010)	- .008 (.014)	- .056 (.020)	- .031 (.017)	.007 (.017)
γ_{OC}	- .006 (.011)	n.e.	n.e.	- .009 (.011)	- .016 (.020)	n.e.
γ_{GG}	.025 (.064)	- .010 (.138)	- .110 (.084)	- .926 (.389)	- .177 (.120)	.071 (.039)
γ_{GC}	.051 (.065)	n.e.	n.e.	.038 (.095)	.177 (.103)	n.e.
γ_{CC}	- .125 (.086)	n.e.	n.e.	.004 (.047)	- .067 (.133)	n.e.
Pseudo R^2	.64	.21	.62	.65	.70	.31
Number of Observations	11	10	11	8	18	9

Figures in parentheses are asymptotic standard errors.
n.e. - not estimated

TABLE 5
(Continued)

Parameter Estimates: 1962

Parameter	29	32	33	35	36
α_E	.295 (.082)	.318 (.015)	.478 (.023)	.593 (.022)	.689 (.008)
α_O	.232 (.082)	.092 (.023)	.173 (.029)	.168 (.027)	.139 (.016)
α_G	.474 (.048)	.393 (.044)	.286 (.018)	.157 (.022)	.124 (.014)
α_C	n.e.	.197 (.030)	.062 (.010)	.082 (.017)	.048 (.010)
γ_{EE}	- .514 (.309)	.096 (.052)	- .202 (.043)	- .184 (.068)	- .306 (.013)
γ_{EO}	- .047 (.027)	.003 (.009)	- .018 (.110)	- .020 (.014)	.011 (.006)
γ_{EG}	.647 (.116)	- .074 (.050)	.178 (.032)	.073 (.049)	.146 (.025)
γ_{EC}	n.e.	- .025 (.049)	.042 (.019)	.131 (.056)	.149 (.021)
γ_{OO}	.030 (.026)	.011 (.014)	.010 (.013)	.022 (.016)	.034 (.011)
γ_{OG}	.018 (.019)	- .026 (.026)	.003 (.008)	- .008 (.014)	- .023 (.011)
γ_{OC}	n.e.	.012 (.018)	.005 (.004)	.005 (.011)	- .023 (.007)
γ_{GG}	- .666 (.130)	- .182 (.132)	- .178 (.028)	- .001 (.059)	- .138 (.085)
γ_{GC}	n.e.	.282 (.104)	- .004 (.014)	- .066 (.047)	.014 (.060)
γ_{CC}	n.e.	- .269 (.100)	- .043 (.019)	- .070 (.067)	- .141 (.047)
Pseudo R^2	.80	.53	.81	.34	.99
Number of Observations	9	17	14	13	9

Figures in parentheses are asymptotic standard errors.

n.e. - not estimated

TABLE 6

Parameter Estimates: 1958

Parameter	Industry 20	24	26	28	32
α_E	.498 (.015)	.729 (.028)	.333 (.025)	.632 (.024)	.314 (.010)
α_O	.162 (.016)	.165 (.031)	.138 (.044)	.143 (.018)	.149 (.025)
α_G	.190 (.016)	.106 (.014)	.047 (.012)	.225 (.024)	.282 (.039)
α_C	.150 (.017)	n.e.	.483 (.053)	n.e.	.255 (.023)
γ_{EE}	- .062 (.058)	- .119 (.036)	- .057 (.063)	- .153 (.049)	- .037 (.031)
γ_{EO}	.071 (.058)	- .015 (.021)	.036 (.057)	.067 (.029)	.054 (.021)
γ_{EG}	- .018 (.045)	.134 (.032)	.094 (.026)	.106 (.044)	.063 (.034)
γ_{EC}	.009 (.040)	n.e.	- .074 (.063)	n.e.	- .079 (.033)
γ_{OO}	- .261 (.048)	.001 (.024)	- .061 (.091)	- .213 (.018)	- .001 (.056)
γ_{OG}	.103 (.042)	.014 (.010)	- .003 (.029)	.146 (.047)	- .085 (.067)
γ_{OC}	.086 (.037)	n.e.	.029 (.095)	n.e.	.033 (.051)
γ_{GG}	- .096 (.061)	- .148 (.034)	- .105 (.024)	- .252 (.067)	- .211 (.123)
γ_{GC}	.011 (.039)	n.e.	.014 (.028)	n.e.	.234 (.080)
γ_{CC}	- .106 (.048)	n.e.	.032 (.120)	n.e.	- .187 (.091)
Pseudo R^2	.56	.72	.83	.69	.53
Number of Observations	29	7	10	22	17

Figures in parentheses are asymptotic standard errors.

n.e. - not estimated

TABLE 6
(Continued)

Parameter Estimates: 1958

Parameter	Industry 33	34	35	36	37
α_E	.550 (.023)	.646 (.012)	.707 (.036)	.813 (.016)	.724 (.025)
α_O	.234 (.030)	.146 (.014)	.081 (.010)	.090 (.009)	.149 (.029)
α_G	.217 (.021)	.145 (.013)	.109 (.010)	.097 (.011)	.127 (.008)
α_C	n.e.	.062 (.008)	.103 (.024)	n.e.	n.e.
γ_{EE}	- .227 (.048)	.087 (.073)	- .135 (.210)	- .160 (.110)	- .114 (.081)
γ_{EO}	.115 (.050)	- .032 (.043)	- .029 (.072)	.127 (.060)	.070 (.075)
γ_{EG}	.112 (.041)	- .042 (.320)	.127 (.062)	.033 (.068)	.043 (.040)
γ_{EC}	n.e.	- .013 (.457)	.037 (.114)	n.e.	n.e.
γ_{OO}	- .179 (.078)	.075 (.051)	- .013 (.036)	- .185 (.043)	- .126 (.086)
γ_{OG}	.064 (.062)	- .041 (.372)	.050 (.024)	.058 (.033)	.056 (.023)
γ_{OC}	n.e.	- .003 (.030)	- .009 (.032)	n.e.	n.e.
γ_{GG}	- .176 (.072)	.061 (.036)	- .259 (.029)	- .091 (.051)	- .099 (.030)
γ_{GC}	n.e.	.023 (.022)	.082 (.033)	n.e.	n.e.
γ_{CC}	n.e.	- .007 (.038)	- .110 (.077)	n.e.	n.e.
Pseudo R^2	.57	.33	.94	.74	.61
Number of Observations	20	11	8	11	13

Figures in parentheses are asymptotic standard errors.

n.e. - not estimated.

TABLE 7

Estimates of Price Elasticities, Total Energy Constant: 1971

Elasticity	Industry 23	26	28	29	30
E_{EE}	- .148 (.090)	- .203 (.307)	- .684 (.171)	-1.031 (.317)	- .124 (.162)
E_{OO}	-2.528 (.658)	-2.262 (.759)	-3.513 (.791)	-2.132 (.610)	-1.151 (.835)
E_{GG}	- .425 (.582)	-2.134 (.519)	-1.972 (.421)	- .613 (.213)	-1.439 (.522)
E_{CC}	n.e.	-1.907 (.601)	- .656 (.640)	n.e.	-2.531 (1.092)
E_{EO}	.150 (.053)	.191 (.273)	.418 (.116)	.643 (.252)	- .082 (.091)
E_{EG}	- .003 (.061)	- .024 (.198)	.303 (.124)	.388 (.214)	.109 (.127)
E_{EC}	n.e.	.037 (.190)	- .037 (.144)	n.e.	.979 (.096)
E_{OE}	1.976 (.717)	.383 (.546)	2.015 (.577)	1.364 (.573)	- .608 (.693)
E_{OG}	.552 (.433)	1.198 (.518)	1.350 (.561)	.766 (.366)	1.255 (.669)
E_{OC}	n.e.	.680 (.428)	.148 (.592)	n.e.	.504 (.568)
E_{GE}	- .029 (.665)	- .050 (.410)	.748 (.302)	.318 (.183)	.458 (.532)
E_{GO}	.454 (.362)	1.237 (.526)	.691 (.289)	.295 (.151)	.716 (.382)
E_{GC}	n.e.	.948 (.386)	.533 (.339)	n.e.	.265 (.284)
E_{CE}	n.e.	.084 (.434)	- .117 (.454)	n.e.	1.081 (1.068)
E_{CO}	n.e.	.775 (.492)	.096 (.386)	n.e.	.754 (.884)
E_{CG}	n.e.	1.047 (.440)	.676 (.430)	n.e.	.695 (.749)

n.e. - not estimated

TABLE 7
(Continued)

Estimates of Price Elasticities, Total Energy Constant: 1971

Elasticity	Industry 32	33	34	35	36
E_{EE}	- .312 (.257)	- .829 (.136)	-1.096 (.358)	- .793 (.112)	- .272 (.075)
E_{OO}	-3.497 (.986)	-2.748 (.671)	-4.113 (2.510)	-4.300 (1.181)	-1.870 (.468)
E_{GG}	-1.329 (.326)	- .936 (.297)	-1.457 (.678)	-1.522 (.218)	-1.292 (.355)
E_{CC}	-2.223 (.811)	-1.529 (.895)	-2.004 (1.041)	-1.968 (.350)	n.e.
E_{EO}	- .088 (.159)	.295 (.102)	.428 (.202)	.193 (.096)	.079 (.045)
E_{EG}	.340 (.182)	.449 (.108)	.520 (.224)	.460 (.068)	.192 (.067)
E_{EC}	.060 (.230)	.086 (.076)	.147 (.101)	.141 (.033)	n.e.
E_{OE}	- .377 (.687)	1.379 (.470)	3.645 (1.984)	2.123 (1.053)	.758 (.420)
E_{OG}	2.143 (.902)	.571 (.518)	.469 (1.626)	1.560 (.757)	1.111 (.499)
E_{OC}	1.731 (.919)	.798 (.354)	- .001 (.667)	.617 (.356)	n.e.
E_{GE}	.309 (.165)	.933 (.224)	1.483 (.644)	1.333 (.197)	.804 (.272)
E_{GO}	.456 (.184)	.254 (.237)	.157 (.547)	.411 (.218)	.488 (.214)
E_{GC}	.564 (.260)	- .251 (.154)	- .183 (.178)	- .221 (.084)	n.e.
E_{CE}	.124 (.471)	.966 (.854)	3.554 (2.585)	2.298 (.580)	n.e.
E_{CO}	.829 (.432)	1.922 (.996)	- .004 (1.891)	.912 (.555)	n.e.
E_{CG}	1.270 (.590)	-1.359 (.934)	-1.546 (1.552)	-1.242 (.507)	n.e.

n.e. - not estimated

TABLE 8

Estimates of Price Elasticities, Total Energy Constant: 1962

Elasticity	Industry 22	23	24	25	26	27
E_{EE}	- .416 (.076)	- .245 (.172)	- .432 (.109)	-1.793 (.492)	- .459 (.223)	- .121 (.049)
E_{OO}	- .718 (.103)	- .769 (.094)	- .538 (.066)	- .746 (.122)	- .571 (.057)	-1.030 (.156)
E_{GG}	- .625 (.762)	-1.022 (1.524)	-1.966 (.741)	-9.418 (4.309)	-4.303 (2.446)	- .314 (.314)
E_{CC}	-2.024 (.805)	n.e.	n.e.	- .843 (.360)	- .895 (.450)	n.e.
E_{EO}	.181 (.043)	.124 (.023)	.154 (.028)	.180 (.028)	.259 (.048)	.094 (.020)
E_{EG}	- .002 (.069)	.120 (.175)	.278 (.122)	1.535 (.524)	.134 (.230)	.028 (.048)
E_{EC}	.237 (.084)	n.e.	n.e.	.078 (.105)	.066 (.208)	n.e.
E_{OE}	.677 (.067)	.794 (.126)	.473 (.070)	1.113 (.176)	.384 (.076)	.826 (.118)
E_{OG}	- .035 (.064)	- .025 (.099)	.065 (.079)	- .415 (.210)	- .068 (.107)	.204 (.201)
E_{OC}	.076 (.090)	n.e.	n.e.	.047 (.110)	.255 (.127)	n.e.
E_{GE}	- .015 (.520)	1.056 (1.498)	1.826 (.663)	9.340 (4.024)	.960 (1.461)	.171 (.291)
E_{GO}	- .069 (.144)	- .035 (.140)	.140 (.147)	- .409 (.254)	- .326 (.781)	.143 (.132)
E_{GC}	.709 (.816)	n.e.	n.e.	.486 (1.885)	3.669 (2.848)	n.e.
E_{CE}	1.364 (.456)	n.e.	n.e.	.396 (.534)	.078 (.246)	n.e.
E_{CO}	.117 (.104)	n.e.	n.e.	.039 (.088)	.205 (.070)	n.e.
E_{CG}	.543 (.592)	n.e.	n.e.	.403 (.732)	.612 (.397)	n.e.

n.e. - not estimated

TABLE 8
(Continued)

Estimates of Price Elasticities, Total Energy Constant: 1962

Elasticity	Industry 29	32	33	35	36
E_{EE}	-2.738 (.791)	- .380 (.162)	- .944 (.091)	- .719 (.124)	- .756 (.022)
E_{OO}	- .641 (.139)	- .788 (.143)	- .770 (.090)	- .699 (.097)	- .615 (.076)
E_{GG}	-1.931 (.212)	-1.069 (.327)	-1.334 (.117)	- .845 (.365)	-1.986 (.684)
E_{CC}	n.e.	-2.171 (.557)	-1.634 (.302)	-1.764 (.833)	-3.890 (1.200)
E_{EO}	.071 (.099)	.102 (.040)	.135 (.030)	.135 (.032)	.156 (.015)
E_{EG}	2.667 (.845)	.161 (.166)	.658 (.068)	.282 (.090)	.336 (.040)
E_{EC}	n.e.	.118 (.161)	.151 (.040)	.303 (.099)	.264 (.031)
E_{OE}	.091 (.144)	.352 (.108)	.373 (.072)	.476 (.093)	.771 (.051)
E_{OG}	.550 (.132)	.107 (.290)	.305 (.059)	.112 (.096)	- .039 (.084)
E_{OC}	n.e.	.328 (.219)	.092 (.035)	.111 (.075)	- .116 (.060)
E_{GE}	1.662 (.197)	.130 (.128)	1.100 (.130)	1.061 (.306)	1.867 (.242)
E_{GO}	.269 (.008)	.025 (.065)	.184 (.036)	.120 (.087)	- .044 (.097)
E_{GC}	n.e.	.914 (.273)	.049 (.051)	- .335 (.304)	.164 (.482)
E_{CE}	n.e.	.191 (.251)	1.155 (.316)	2.178 (.729)	3.804 (.801)
E_{CO}	n.e.	.153 (.089)	.254 (.060)	.226 (.124)	- .340 (.221)
E_{CG}	n.e.	1.827 (.616)	.225 (.228)	- .640 (.553)	.425 (1.267)

n.e. = not estimated

TABLE 9

Estimates of Price Elasticities, Total Energy Constant: 1958

Elasticity	Industry 20	24	26	28	32
E_{EE}	-.626 (.116)	-.435 (.051)	-.837 (.190)	-.641 (.081)	-.804 (.098)
E_{OO}	-2.449 (.342)	-.827 (.162)	-1.308 (.693)	-2.345 (.373)	-.861 (.365)
E_{GG}	-1.316 (.324)	-2.288 (.359)	-3.189 (.761)	-1.895 (.322)	-1.466 (.445)
E_{CC}	-1.554 (.347)	n.e.	-.452 (.254)	n.e.	-1.480 (.365)
E_{EO}	.304 (.079)	.144 (.030)	.246 (.173)	.249 (.053)	.320 (.072)
E_{EG}	.154 (.091)	.290 (.046)	.331 (.083)	.393 (.079)	.482 (.120)
E_{EC}	.168 (.080)	n.e.	.261 (.192)	n.e.	.002 (.108)
E_{OE}	.938 (.235)	.639 (.134)	.594 (.427)	1.099 (.210)	.674 (.150)
E_{OG}	.828 (.267)	.188 (.078)	.022 (.214)	1.246 (.363)	-.290 (.450)
E_{OC}	.623 (.229)	n.e.	.692 (.703)	n.e.	.476 (.345)
E_{GE}	.403 (.238)	1.996 (.354)	2.353 (.778)	1.102 (.200)	.536 (.125)
E_{GO}	.705 (.228)	.292 (.091)	.064 (.629)	.793 (.223)	-.153 (.240)
E_{GC}	.208 (.204)	n.e.	.771 (.581)	n.e.	1.083 (.307)
E_{CE}	.557 (.267)	n.e.	.180 (.131)	n.e.	.002 (.132)
E_{CO}	.735 (.258)	n.e.	.197 (.203)	n.e.	.278 (.198)
E_{CG}	.263 (.258)	n.e.	.075 (.058)	n.e.	1.199 (.334)

n.e. - not estimated

TABLE 9
(Continued)

Estimates of Price Elasticities, Total Energy Constant: 1958

Elasticity	Industry 33	34	35	36	37
E_{EE}	- .864 (.103)	- .219 (.112)	- .484 (.291)	- .383 (.135)	- .433 (.112)
E_{OO}	-1.534 (.345)	- .336 (.358)	-1.075 (.463)	-2.979 (.527)	-1.699 (.631)
E_{GG}	-1.598 (.341)	- .441 (.244)	-3.266 (.368)	-1.841 (.543)	-1.651 (.247)
E_{CC}	n.e.	-1.054 (.601)	-1.967 (.822)	n.e.	n.e.
E_{EO}	.443 (.103)	.097 (.067)	.309 (.100)	.246 (.074)	.246 (.106)
E_{EG}	.420 (.082)	.080 (.050)	.289 (.086)	.138 (.084)	.187 (.054)
E_{EC}	n.e.	.042 (.070)	.155 (.159)	n.e.	n.e.
E_{OE}	1.042 (.217)	.429 (.293)	.347 (.893)	2.230 (.688)	1.798 (.530)
E_{OG}	.492 (.268)	- .136 (.258)	.734 (.288)	.749 (.365)	.501 (.181)
E_{OC}	n.e.	.042 (.202)	- .005 (.394)	n.e.	n.e.
E_{GE}	1.066 (.192)	.356 (.220)	1.874 (.610)	1.151 (.706)	1.065 (.317)
E_{GO}	.531 (.291)	- .136 (.255)	.542 (.213)	.690 (.340)	.587 (.189)
E_{GC}	n.e.	.221 (.152)	.850 (.301)	n.e.	n.e.
E_{CE}	n.e.	.438 (.731)	1.068 (1.131)	n.e.	n.e.
E_{CO}	n.e.	.099 (.492)	- .004 (.309)	n.e.	n.e.
E_{CG}	n.e.	.517 (.351)	.903 (.352)	n.e.	n.e.

n.e. - not estimated

TABLE 10

Estimated Aggregate Elasticities
Total Energy Input Constant

Elasticity	1971	1962	1958
E_{EE}	- .66	- .87	- .67
E_{OO}	-2.75	- .70	-1.63
E_{GG}	-1.32	-1.75	-1.76
E_{CC}	-1.46	-1.62	-1.51
E_{EO}	.30	.14	.33
E_{EG}	.34	.59	.35
E_{EC}	.09	.13	.03
E_{OE}	1.27	.41	.89
E_{OG}	1.12	.15	.46
E_{OC}	.69	.14	.40
E_{GE}	.43	.95	1.01
E_{GO}	.50	.12	.42
E_{GC}	.32	.53	.32
E_{CE}	.31	.71	.30
E_{CO}	.74	.18	.42
E_{CG}	.40	.72	.79

TABLE 11

Estimated Aggregate Elasticities

Total Energy Input Variable

Elasticity	1971	1962	1958
E_{EE}	- .92	-1.12	- .97
E_{OO}	-2.82	- .77	-1.72
E_{GG}	-1.47	-1.91	-1.87
E_{CC}	-1.52	-1.71	-1.61
E_{EO}	.23	.08	.21
E_{EG}	.20	.46	.25
E_{EC}	.04	.10	.00
E_{OE}	.74	.18	.63
E_{OG}	1.03	.03	.37
E_{OC}	.63	.07	.20
E_{GE}	.35	.51	.75
E_{GO}	.44	.12	.34
E_{GC}	.25	.60	.26
E_{CE}	.07	.90	.09
E_{CO}	.69	.05	.34
E_{CG}	.28	.48	.67

TABLE 12

Tests for Homogeneity of Parameters in 1971 and 1962

Industry	Degrees of Freedom	Test Statistic	Critical Value[a]
20	9,158	5.94[b]	2.41
23	5,59	0.76	3.34
24	5,74	1.71	3.30
26	9,126	4.94[b]	2.56
27	5,59	1.22	3.34
28	9,150	3.43[b]	2.41
29	5,68	3.11	3.32
30	9,98	5.56[b]	2.62
32	9,122	4.64[b]	2.56
33	9,98	3.05[b]	2.62
34	5,116	14.26[b]	3.18
35	9,70	0.38	2.70
36	5,107	4.16[b]	3.22

[a]Significance level = .01

[b]The null hypothesis of homogeneity is rejected.

TABLE 13

Estimates of Allen Elasticities of Substitution: 1971

	Industry				
	23	26	28	29	30
σ_{EO}	2.308 (.830)	.927 (1.319)	3.894 (1.117)	3.680 (1.546)	- .890 (1.016)
σ_{EG}	- .034 (.777)	- .122 (.991)	1.446 (.581)	.855 (.471)	.670 (.779)
σ_{EC}	n.e.	.203 (1.050)	- .226 (.877)	n.e.	1.582 (1.558)
σ_{OG}	6.980 (5.574)	6.003 (2.515)	6.436 (2.723)	1.687 (.797)	7.736 (4.053)
σ_{OC}	n.e.	3.764 (2.379)	.898 (3.588)	n.e.	8.151 (9.374)
σ_{GC}	n.e.	5.244 (2.202)	3.223 (2.040)	n.e.	4.285 (4.627)

	32	33	34	35	36
σ_{EO}	-1.064 (1.942)	2.459 (.839)	5.488 (2.968)	3.177 (1.581)	1.019 (.564)
σ_{EG}	.872 (.466)	1.664 (.392)	2.239 (.969)	1.995 (.293)	1.080 (.365)
σ_{EC}	.349 (1.329)	1.723 (1.518)	5.367 (3.893)	3.440 (.854)	n.e.
σ_{OG}	5.496 (2.268)	2.119 (1.933)	2.020 (6.998)	6.774 (3.273)	6.252 (2.739)
σ_{OC}	9.992 (5.385)	16.019 (8.225)	- .051 (24.293)	15.044 (8.793)	n.e.
σ_{GC}	3.258 (1.496)	-5.041 (3.513)	-6.652 (6.837)	-5.391 (2.277)	n.e.

Figures in parentheses are asymptotic standard errors.

n.e. - not estimated.

TABLE A.1

Parameter Estimates: Other Industries, 1971

Parameter	Industry 20	22	24	25	27
α_E	.521 (.014)	.653 (.020)	.716 (.022)	.764 (.030)	.802 (.015)
α_O	.127 (.017)	.202 (.022)	.119 (.015)	.079 (.013)	.067 (.010)
α_G	.276 (.018)	.145 (.010)	.165 (.026)	.157 (.019)	.131 (.012)
α_C	.077 (.012)	n.e.	n.e.	n.e.	n.e.
γ_{EE}	.208 (.088)	- .043 (.074)	.196 (.088)	- .288 (.165)	.151 (.085)
γ_{EO}	- .123 (.065)	.153 (.057)	- .088 (.045)	- .079 (.070)	- .083 (.046)
γ_{EG}	- .223 (.066)	- .110 (.046)	- .107 (.083)	.367 (.108)	- .067 (.056)
γ_{EC}	.138 (.042)	n.e.	n.e.	n.e.	n.e.
γ_{OO}	- .043 (.067)	- .309 (.062)	- .058 (.041)	.109 (.037)	.014 (.038)
γ_{OG}	.209 (.058)	.156 (.028)	.146 (.048)	- .030 (.046)	.069 (.029)
γ_{OC}	- .043 (.037)	n.e.	n.e.	n.e.	n.e.
γ_{GG}	.059 (.079)	- .046 (.042)	- .039 (.100)	- .338 (.082)	- .002 (.046)
γ_{GC}	- .044 (.045)	n.e.	n.e.	n.e.	n.e.
γ_{CC}	- .050 (.038)	n.e.	n.e.	n.e.	n.e.
Number of Observations	24	16	17	9	14

Figures in parentheses are asymptotic standard errors

n.e. - not estimated

TABLE A.1
(Continued)

Parameter Estimates: Other Industries, 1971

Parameter	Industry 31	37	38	39
α_E	.805 (.040)	.657 (.006)	.704 (.022)	.665 (.028)
α_O	.111 (.023)	.085 (.018)	.139 (.019)	.156 (.017)
α_G	.084 (.035)	.166 (.012)	.157 (.013)	.179 (.020)
α_C	n.e.	.092 (.009)	n.e.	n.e.
γ_{EE}	.106 (.037)	- .057 (.053)	.260 (.107)	.168 (.197)
γ_{EO}	- .055 (.023)	.108 (.088)	- .019 (.068)	- .138 (.134)
γ_{EG}	- .051 (.032)	- .114 (.072)	- .241 (.073)	- .030 (.102)
γ_{EC}	n.e.	.064 (.029)	n.e.	n.e.
γ_{OO}	- .075 (.050)	- .630 (.256)	- .132 (.061)	- .059 (.112)
γ_{OG}	.130 (.043)	.612 (.198)	.151 (.041)	.198 (.057)
γ_{OC}	n.e.	- .089 (.077)	n.e.	n.e.
γ_{GG}	- .079 (.047)	- .485 (.176)	.090 (.069)	- .168 (.075)
γ_{GC}	n.e.	- .012 (.059)	n.e.	n.e.
γ_{CC}	n.e.	.037 (.046)	n.e.	n.e.
Number of Observations	6	9	9	15

Figures in parentheses are asymptotic standard errors.

n.e. - not estimated

TABLE A.2

Estimates of Price Elasticities: Other Industries, 1971

Elasticity	20	22	24	25	27
E_{EE}	- .079 (.169)	- .413 (.114)	- .011 (.124)	- .613 (.253)	- .011 (.106)
E_{OO}	-1.211 (.523)	-2.328 (.355)	-1.370 (.348)	.458 (.588)	- .725 (.508)
E_{GG}	- .511 (.288)	-1.172 (.293)	-1.072 (.613)	-2.994 (.718)	- .832 (.352)
E_{CC}	-1.574 (.518)	n.e.	n.e.	n.e.	n.e.
E_{EO}	- .109 (.125)	.437 (.092)	- .005 (.065)	- .025 (.107)	- .036 (.058)
E_{EG}	- .152 (.126)	- .024 (.071)	.015 (.118)	.638 (.166)	.047 (.069)
E_{EC}	.341 (.080)	n.e.	n.e.	n.e.	n.e.
E_{OE}	- .450 (.526)	1.410 (.297)	- .029 (.344)	- .240 (1.040)	- .430 (.690)
E_{OG}	1.925 (.513)	.918 (.164)	1.398 (.437)	- .218 (.693)	1.155 (.428)
E_{OC}	- .264 (.295)	n.e.	n.e.	n.e.	n.e.
E_{GE}	- .289 (.240)	- .108 (.322)	.066 (.510)	3.103 (.910)	.286 (.423)
E_{GO}	.884 (.223)	1.281 (.215)	1.005 (.334)	- .109 (.347)	.596 (.219)
E_{GC}	- .085 (.165)	n.e.	n.e.	n.e.	n.e.
E_{CE}	2.314 (.626)	n.e.	n.e.	n.e.	n.e.
E_{CO}	- .436 (.479)	n.e.	n.e.	n.e.	n.e.
E_{CG}	- .304 (.593)	n.e.	n.e.	n.e.	n.e.

n.e. - not estimated

TABLE A.2
(Continued)

Estimates of Price Elasticities: Other Industries, 1971

Elasticity	Industry 31	37	38	39
E_{EE}	- .063 (.069)	- .430 (.081)	.074 (.153)	- .082 (.294)
E_{OO}	-1.568 (.418)	-8.336 (3.407)	-1.806 (.467)	-1.224 (.718)
E_{GG}	-1.848 (.638)	-3.753 (1.079)	- .268 (.440)	-1.759 (.457)
E_{CC}	n.e.	- .504 (.499)	n.e.	n.e.
E_{EO}	.042 (.041)	.249 (.135)	.112 (.098)	- .052 (.200)
E_{EG}	.021 (.063)	- .008 (.110)	- .185 (.104)	.134 (.152)
E_{EC}	n.e.	.189 (.045)	n.e.	n.e.
E_{OE}	.308 (.261)	1.923 (1.073)	.566 (.492)	- .220 (.854)
E_{OG}	1.259 (.425)	7.365 (2.803)	1.240 (.323)	1.444 (.372)
E_{OC}	n.e.	- .952 (.944)	n.e.	n.e.
E_{GE}	.197 (.530)	- .032 (.436)	.831 (.475)	.497 (.572)
E_{GO}	1.651 (.894)	3.768 (1.226)	1.099 (.267)	1.261 (.342)
E_{GC}	n.e.	.016 (.355)	n.e.	n.e.
E_{CE}	n.e.	1.356 (.326)	n.e.	n.e.
E_{CO}	n.e.	- .382 (.848)	n.e.	n.e.
E_{CG}	n.e.	.030 (.644)	n.e.	n.e.

n.e. - not estimated

CHAPTER 5

TECHNICAL CHANGE IN ENERGY USE

John M. Wills

I. Introduction

Over time man's stock of knowledge about potential production processes has increased. When additions to that stock of knowledge are translated into actual changes in production technique we say that technical change has occurred. It may be that a technical change is "biased;" that is, it results not only in an increase in the amount of output obtainable from a given bundle of inputs, but also alters the resource mix which minimizes the cost of output. It is the purpose of this chapter to seek evidence of biased technical changes in energy and other inputs in the U.S. primary metals industry after World War II.

In the theory of the firm without technical change, the production function is assumed not to change. The great bulk of econometric work in this area, being concerned with other issues than technical change, has also followed this path. But it is the essence of technical change that the production function does change over time. For empirical work the notion that a production function might simply change in any unrestricted fashion is too general to be useful. There are, however a variety of ways in which some amount of "flexibility" can be built into estimated production functions.

It is useful, first of all, to require that the functional form and the para- parameter values be unchanged over time. Otherwise the estimating problems, especially with time series data, are overwhelming. Indeed, in the general case it is impossible to simultaneously estimate both production parameters and technical change measures from the same data unless additional restrictions are imposed.

One such restriction which is sufficient to identify both the production function and technical change, but which permits some flexibility, is that technical change occurs at a constant rate (or at least with a constant rate of change) over the time period. This can be accomplished by entering "time" in the production function symmetrically as an input, so that the change in output per unit of time, inputs held constant, is a constant. This is one procedure followed here.

This can be done in such a fashion as to restrict all technical change to being a magnification of output possible from given inputs, or it can permit changes in technique which will imply changes in the input mix for any set of prices. In the former case technical change is "neutral," in the latter it is "biased."

The conventional definition of bias was first proposed by Hicks [1935] in the context of a two-factor production function. In that case the following three definitions are equivalent:

$$\text{sign}\,\frac{d(\frac{dL}{dK})}{dt} = \text{sign}\,\frac{dM_L}{dt} = \text{sign}\,\frac{d(\frac{f_K}{f_L})}{dt} \lesseqgtr 0 \quad \begin{array}{|l}\text{Labor saving}\\ \text{neutral}\\ \text{Labor using}\end{array}$$

where K and L are capital and labor, respectively, M_L is the cost share of labor, and f_i(i=K,L) is the marginal product of the i^{th} factor. All of the above differentials were to be evaluated at constant factor prices. In the general n-factor case, however, the first and last of the above versions will yield n-1 measures of the bias of technical change. Only version two yields a single number; on that basis it is the definition used in this chapter.

One convenient characterization of biased technical change is that it is factor-augmenting. Technical change is factor-augmenting if the production function can be written:

$$Q_t = f(A_{1_t} X_{1_t} ,\ldots, A_{n_t} X_{n_t})$$

where the t-subscript refers to time. The parameters of the production function remain unchanged; all technical change can be considered to change the values of the A_i, the "augmentation coefficients." The X's now measure input quantities in natural units (e.g., manhours) and the A_iX_i measure input quantities in "efficiency units." So technical change has the effect of increasing the effective input which can be derived from the natural units purchased in the marketplace. This method can, obviously, be used to permit biased technical change, since the various augmentation coefficients can change at different rates.

As discussed below, the hypothesis of factor augmenting technical change is not rejected by the data. The hypothesis that technical change has been neutral is rejected indicating that factors have been augmented at different rates. The results indicate significant labor-using and material-saving biases. There also appears to have been a small energy-saving bias, but it is not statistically significant.

II. Econometric Specification

In Part A below we show how time-series data can be used to test for biased technical change and for factor augmenting biased technical change (agsinst alternative hypotheses of neutral technical change or no technical change). Estimates of the rates of factor augmentation and of the Hicks factor-use biases are also derived. In Part B we show how cross-section and time series data can be used together to test for factor augmentation. Also, two cross-sections on the same data at different points in time can provide additional evidence on technical change.

A. The Time-Series Model

Production technology can be represented by a production function:

$$Q = f([X])$$

where Q is output and [X] a vector of inputs.[1] For simplicity of presentation the vector [X] is assumed to exhaust the input set. When this is not the case the principal problem arising is that of defining the output of a "sub-production" function and insuring that the empirical conditions for its validity are met. This is discussed where relevant below.

If production exhibits factor augmenting technical change the corresponding production function is

$$Q = f([AX]).$$

In all empirical work we choose to represent the production function by the translog functional form. The translog production function (without technical change) can be written

$$\ln Q = \alpha_0 + \sum_i \alpha_i \ln X_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln X_i \ln X_j \tag{1}$$

where $\gamma_{ij} \equiv \gamma_{ji}$, Q is the quantity of output and the X_i's are the quantities of inputs. Equation (1) can be interpreted as a production function in its own right, or as a second order Taylor's series approximation to an arbitrary twice differentiable functional form.

Differentiating with respect to $\ln X_i$

$$\frac{\partial \ln Q}{\partial \ln X_i} = \frac{\partial Q}{\partial X_i} \cdot \frac{X_i}{Q} = \alpha_i + \sum_j \gamma_{ij} \ln X_j \;, \qquad i=1\ldots n \;. \tag{2}$$

Given competitive input markets $\frac{\partial Q}{\partial X_i} = W_i$ since each factor will be paid the value of its marginal product. If production exhibits constant returns to scale then

$$\sum_i \frac{W_i X_i}{Q} = 1$$

so the left hand side of (2) represents the share of total cost accuring to factor i, called M_i. (It also happens to equal the output elasticity with respect to factor i.)

All we need to know about the production function can be gotten from estimating the system of share equations (3)

$$M_i = \alpha_i + \sum_j \gamma_{ij} \ln X_j + u_i \qquad i = 1,\ldots n - 1 \qquad (3)$$

where an additive disturbance term has been appended to each equation. Because the shares necessarily sum to one, (and $\gamma_{ij} = \gamma_{ji}$) any one of the n equations is a linear combination of the n - 1 others. This implies the following restrictions on the parameters:

$$\sum_i \alpha_i = 1$$
$$\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0 \qquad (4)$$

Because the equations represent shares in total cost, we should expect the errors to exhibit joint covariance. Therefore the equations are estimated with a Zellner efficient procedure (joint GLS). The sum of the disturbances across the share equations is zero at each observation and the disturbance covariance matrix is singular. Therefore, we eliminate one equation and then solve for the parameters of that one equation via the restrictions (4). Ordinarily the Zellner estimate is sensitive to the choice of the equation to be omitted, but this undesirable result is avoided by adopting an iterative Zellner efficient procedure.

Especially in the production function it may be the case that the input quantities, being chosen by the entrepreneur along with output rates, are in fact correlated with the error vector. Under this condition the (iterative) three-stage least squares (3SLS) estimator is appropriate, provided we can identify the pre-determined variables of the system.

Generally it is not possible to use any time series to identify both production parameters and technical change. There are several ways to achieve identification. One way is by using a priori information obtained perhaps, from supplementary cross-section data. This is discussed further below, in Part B.

Here we identify technical change by assuming it occurs at a constant rate, in the manner of Berndt and Wood [1975a]. In

$$Q = f([X], t)$$

let t be treated symmetrically as a factor. Then the translog function is:

$$\ln Q = \alpha_0 + \sum_i \alpha_i \ln X_i + \tfrac{1}{2}\sum_i \sum_j \gamma_{ij} \ln X_i \ln X_j + \alpha_T T + \sum_i \gamma_{iT} \ln X_i T + \tfrac{1}{2}\gamma_{TT} T^2 .$$

Differentiating with respect to $\ln X_i$ and T we have the system of cost shares plus an equation for $\frac{\partial \ln Q}{\partial t}$, the rate of change of total output when input quantities are fixed:

$$M_i = \alpha_i + \sum_j \gamma_{ij} \ln X_j + \gamma_{iT} T \qquad i = 1,\ldots, n-1$$

$$\frac{\partial \ln Q}{\partial t} = \alpha_T + \sum_i \gamma_{iT} \ln X_i + \gamma_{TT} T \tag{5}$$

where, in addition to restrictions (4),

$$\sum_i \gamma_{iT} = 0 . \tag{6}$$

Following Berndt and Wood, $\frac{\partial \ln Q}{\partial t} \equiv \frac{\dot{Q}}{Q}\Big|_{\overline{X}}$ is measured as an index of "total factor productivity." The computation of that index is explained in the Data Appendix.

A specialization is to treat technical change as factor augmenting. Factor augmenting technical change (at a constant rate) implies that

$$Q_t = f([A_t X_t])$$

where

$$A_{i_t} = A_{i_o} e^{\lambda_i T}$$

and λ_i is the constant rate of augmentation for factor i. Then the production function becomes

$$Q_t = f([X_t e^{\lambda_i T}]) \ .$$

Expanding and differentiating we arrive at production function share equations and a productivity equation identical in form to (5) but with the following additional restrictions implied:

$$\alpha_T = \sum_i \alpha_i \lambda_i \tag{7}$$

$$\gamma_{iT} = \sum_j \gamma_{ij} \lambda_j$$

$$\gamma_{TT} = \sum_i \gamma_{iT} = \sum_i \sum_j \gamma_{ij} \lambda_i \lambda_j$$

Of course, restrictions (4) and (6) still apply.

Factor augmenting technical change implies, therefore, testable parametric restrictions. A logical next step would be to test for neutrality of technical change. Hicks neutral technical change implies that $\lambda_i = \lambda_j$, all i, j. Substituted into (7) this implies that

$$\gamma_{iT} = \gamma_{jT} = 0, \qquad \text{all } i, j.$$

and

$$\gamma_{TT} = 0$$

and again we can test for this.

The least general hypothesis of all is that there has been no technical change whatsoever; i.e. the $\lambda_i = \lambda_j = 0$. In this case we are left with the systems of equations (3).

To summarize, the nested hypotheses are:

non-neutral, non-factor-augmenting technical change

non-neutral, factor augmenting technical change

neutral factor augmenting technical change

no technical change

These tests are based on the log of the ratio of likelihood functions. See Theil [1971].

The bias of technical change is measured by the γ_{iT} terms. This is a constant, independent of prices. Hence the possibility that the bias of technical change depends upon where along the isoquant it is measured is ruled out in this estimating procedure.

B. Cross-Section Model

An alternative to assuming constant rates of technical change is to permit the bias or factor augmentation coefficients to adopt different values in each time period. In $Q_t = f([A_t X_t])$ the A_t's are not directly observable. If we already know the parameters of the production function, we can estimate their implicit values, but we cannot use any single time series to estimate both the parameters <u>and</u> the values of the A_t. In this section we will impose information from cross-section estimates on time series data to derive non-constant rates of factor augmentation.[2]

Return to the production function with factor augmenting technical change, but not now constrained to a constant rate:

$$\ln Q = \alpha_0 + \sum_i \alpha_i \ln(A_i X_i) + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln(A_i X_i) \ln(A_j X_j) .$$

Differentiate with respect to $\ln(A_i X_i)$:

$$\frac{\partial \ln Q}{\partial \ln(A_i X_i)} = \alpha_i + \sum_j \gamma_{ij} \ln(A_j X_j)$$

$$\frac{\partial Q}{\partial A_i X_i} \quad \frac{A_i X_i}{Q} = \alpha_i + \sum_j \gamma_{ij} \ln X_j + \sum_j \gamma_{ij} \ln A_j$$

but

$$\frac{\partial Q}{\partial A_i X_i} = \frac{w_i}{A_i}$$, from the first order conditions for profit maximization, so

$$M_i = \alpha_i + \sum_j \gamma_{ij} \ln X_j + \sum_j \gamma_{ij} \ln A_j$$

or, in matrix notation

$$[M] = [\alpha] + \Gamma[\ln X + \ln A]$$

hence

$$[\ln A] = \Gamma^{-1}[M - \alpha] - [\ln X]$$

so if we know the Γ and α matrices we can solve for lnA,[3] or for changes in lnA:

$$[d\ln A] = \Gamma^{-1}[dM] - [d\ln X]. \tag{8}$$

The bias is the change in M which occurred as a result of technical change. Call this dM*. From above

$$[dM] = \Gamma[d\ln X + d\ln A]$$

and the effect of technical change is

$$[dM^*] = \Gamma[d\ln A]$$

and from (8)

$$[dM^*] = \Gamma[\Gamma^{-1}[dM] - [d\ln X]\,]$$

$$[dM^*] = dM - \Gamma[d\ln X]. \tag{9}$$

and again, knowing the parameters we can estimate the biases.

A practical problem is that our estimates of [dlnA] are likely to have large variances, since they are rather complicated functions of the parameter estimates. Our [dlnA] estimates are functions of random variables and hence are random variables themselves. We know that $\hat{\Gamma}$ is consistent for Γ, therefore Γ^{-1} is consistent for Γ^{-1}. Therefore our estimates will at least be consistent. The biases, being simpler functions, should exhibit smaller standard errors.

Our parameter estimates are derived from cross-section data where variation in input use depends only on variation in prices, and not on technical change. An initial adjustment for labor quality differentials can be made. Then labor input over time should be measured to account for this.

If non-neutral factor augmenting technical change has occurred over some time period then production function estimates which ignore it will appear to be unstable. For example, suppose the production function

$$Q_t = X_t B + u_t$$

is estimated at two separate time periods from cross-section data. If the true production function is

$$Q_t = (A_t X_t) B + u_t$$

and if the ratio of the A_i's changes over the time period then the $\hat{B}$ estimates derived will be biased. This will increase the probability of wrongfully rejecting the hypothesis that the parameters are unchanged. If we adjust the observed quantities of inputs in the later sample by estimated factor augmentation, however, we should derive parameter estimates closer to those of the initial sample. This can be tested by a Chow test.

III. Empirical Results

In Part A we use time-series data to estimate rates of factor augmentation and factor-use biases and come up with significant estimates of each. We reject models of neutral technical change in favor of biased technical change; and accept factor augmentation. In Part B we estimate two cross section models and compare them. On the basis of this comparison we accept the hypothesis of an unchanged production structure, both in natural and in efficiency units. In Part C we compare the time series and cross section results. The results of the "modified" time series model are similar to those of the full time series model, and also similar to the 1963 cross section results. Again we reject neutral technical change. Data sources are discussed in the Appendix.

A. The Time Series Results

The specifications of the nested hypotheses to be tested are (in order of decreasing generality):

1. Biased technical change
2. Biased factor augmenting technical change
3. Hicks-neutral technical change
4. No technical change.

Table 1 presents the coefficient estimates and summary statistics for specifications 1-3. The test statistic is equal to minus twice the difference in the log-likelihood values for the null and alternative hypotheses, and is distributed χ^2 with degrees of freedom equal to the number of restrictions imposed, see Theil [1971, p. 396].

Table 2 gives the results of the tests. The null hypothesis of "No technical change" amounts to deleting an equation from the system and so requires a complicated correction of the estimated likelihood function value. But this is also equivalent to a simple parametric restriction ($\alpha_T = 0$) which we test with a t-test.[6]

The evidence is strongly for accepting factor augmentation. Factor augmentation is a relatively weak restriction of non-neutral technical change. It imposes five independent parameter restrictions but introduces four new variables; net the number of restrictions is one. Indeed, the sum of squared residuals actually falls (slightly) under the restrictions. (This is possible since the parameter restrictions are non-linear.)

The estimated rates of factor augmentation are as follows: (t-values in parentheses)

LK =	-.0005	(.09)
LL =	.0194	(7.61)
LE =	-.0014	(.74)
LM =	-.0041	(2.74)

These results and tests are all based on the joint GLS estimates of the production function. To the extent that there is any difference when a 3SLS procedure is used to account for possible simultaneity, the differences do not affect the implications for technical change. Since the joint GLS estimates generally provided better fit we use these estimates in the tests.

Two of the augmentation rates are not significantly different from zero, but labor appears to have been augmented at just less than 2% per year, on average. Materials inputs have been negatively augmented at approximately -4/10 of 1% per year. One may conjecture that this is the result of a decline in the quality of some inputs.

We strongly reject the hypothesis that technical change is neutral. This makes the final test of "No technical change" vs. "Neutral technical change" a bit meaningless, but it is worth pointing out that it is possible to accept the hypothesis of "No technical change" if the alternative is "Neutral technical change" rather than "Biased technical change." What this means is that shifts in the shape of the isoquants are a relatively more important feature of technical change than are simple "scale-contractions" of the isoquants toward the origin.

There is a significant labor-using, material-saving bias, measured by the γ_{iT} terms. The share of labor in total cost increased, on average, by just less than ½ of 1 percentage point every year, holding prices constant. The cost share of materials inputs decreased at approximately the same rate. (As seen in Table 1 these estimates are virtually identical whether or not factor augmentation is imposed.) There is evidence of a borderline energy-saving trend; but if so, the trend is weak. The capital bias is not significantly different from zero.

Imposing factor augmentation does curiously affect the coefficient of Time in the TFP equation. This coefficient represents the "acceleration" of technical change; that is, TFP itself represents $\frac{\partial Q}{\partial T}$ and so $\gamma_{TT} = \frac{\partial^2 Q}{\partial T^2}$. When factor augmentation is not imposed the rate of acceleration of technical change is .003, but with a large standard error. When factor augmentation is imposed, the rate declines to .0001 but the standard error falls dramatically and the latter estimate is easily significantly different from zero.

Concavity of the production function

Estimated production functions should display the same characteristics as theoretical production functions. In particular, they should be monotonically increasing; that is, an increase in any input quantity should, ceteris paribus, increase output. Also, they should display convex isoquants; that is, the production function should be at least quasi-concave.

Some functional forms such as Cobb-Douglas automatically impose these features on estimated production functions. There may be values of the γ_{ij}, however, for which the translog production function is not concave or not monotonic. If the fitted cost shares are positive the production function is monotonic and if the bordered Hessian of second partial derivatives of the production function is negative definite (principal minors alternate in sign) then the production function is quasi-concave, which is necessary for convex isoquants.

All fitted cost shares were positive, so we conclude that our production function is monotonic. The last eighteen of the twenty-four years, however, display a non-concavity in some dimension. That is, the fitted bordered Hessian is not negative definite when evaluated at these observations. It is not possible to derive estimated standard errors for the fitted values of the principal minors, so we cannot test directly for the significance of this result.

We can, however, impose concavity on the production function at the means of the data, though not at each observation. Lau [1974] shows the parametric method for imposing concavity; the restrictions are quite complex.

Given the complexity of the non-linear parametric restrictions, it is not too surprising that we experienced some difficulty in estimating the equations under this constraint. In particular, different starting values converged to different estimates; often to local maxima which did not satisfy the convergence criteria of the computer program used. Here we report the estimates that correspond to the smallest residual sum of squares of all the attempts. These estimates <u>did</u> satisfy the convergence criteria. They are reported in Table 3 below. The unconstrained estimates are reproduced for convenience in comparison.

We do not report the standard errors here; these reported α and γ estimates are non-linear functions of the parameters actually directly estimated; which themselves entered the estimating equations non-linearly. Calculation of standard errors here involves twice successively applying a Taylor's approximation, and it was felt that these numbers would mean little. The constrained estimates, however, are generally not significantly different from zero although almost all of the unconstrained estimates are significant.

The constrained model fits the data much more poorly than the unconstrained model; we reject concavity on the basis of a likelihood ratio test. Because of the poor fit it was judged that the tests of the nested hypotheses would be better carried out with the unconstrained model.

Despite the fact that the concavity restriction causes significant changes in the coefficient estimates in general, the implications with respect to technical change are not too different. Both models indicate a labor-using, material saving bias of the same order of magnitude; and weaker effects on energy and capital inputs.

The issue of concavity of the production function remains open. The data clearly reject concavity, at least with respect to some input(s). And a production function which is not concave implies some bizarre results; i.e., violent input quantity changes upon small changes in relative input prices. But yet we achieve good fit and "reasonable" parameter estimates from the unconstrained model. Since convex isoquants are properties of firms and we are here estimating an industry production function, there may be a problem of aggregation involved. Given that the rest of our unconstrained empirical estimates seem reasonably good, we leave the concavity problem unsolved.

B. Cross Section Results

Because of the unavailability of a cross-section materials price index, the cross section production models assume that the production function is separable in these inputs; that is, that the production function

$$Q = Q(K,L,E,M)$$

can be written

$$Q = Q^*(K,L,E) + H(M).$$

This is in principle testable, but the same unavailability of data that requires the assumption forbids the test.

The two years of cross sections are 1963 and 1958. They were not generally similar years; the former was an expansionary year and the latter a moderate recession. Klein and Summers [1966] measure the annual rate of capacity utilization in the primary metals industry in 1963 and 1958 as 80.45% and 68.75% respectively. Since the regression model assumes no lag in input adjustment, we adjusted capital input by multiplying the index by the above capacity utilization rates for one of the sets of pooled estimates.

Also, because we construct our energy quantity input by "translog aggregation" of electricity, oil, natural gas, and coal, we must discard any observation for which we lack data on a single one of these. This reduces the sample size to fourteen in 1963 and ten in 1958, but this is clearly a better measure of energy input than to aggregate by BTU content.

The mean values for each of the three cost shares for each year are:

	1963	1958
MK	.471	.341
ML	.464	.575
ME	.063	.084

In Table 4 we present the coefficient estimates and summary statistics for each cross section separately. In Table 5, we present the results of three sets of pooled estimates: 1. The simple 1963 and 1958 data, unadjusted for capacity utilization differences, 2. The same, but with a capital use adjustment, and 3. The 1958 data pooled with the 1963 data which has been "augmented" by applying the rates of factor augmentation for capital, labor, and energy derived in Part A, above.

Our first test is a test for constancy of the production structure over the five years. The procedure is a standard Chow test. We consider two specifications of the null hypothesis:

1. The production function is unchanged in natural input units,
2. The production function is unchanged in efficiency input units, but is changed in terms of natural units.

The first corresponds to the pooled estimates where all inputs are unadjusted for technical change, the second to the pooled estimates where the 1963 data was augmented. For the former we use the results from the estimates made when the capacity utilization rate adjustment was made, since these were generally better results, i.e., lower residual sum of squares and more significant parameter estimates. The test results are presented in Table 6.

We accept the notion that the production structure is unchanged, both in natural units and in efficiency units. Indeed, augmenting the latter cross section by the augmentation coefficients derived from the time series model lowers the explanatory power of the joint regression, though only negligibly.

<u>Deriving annual estimates of technical change</u>

We use the 1963 parameter estimates to derive year-by-year estimates of the annual rate of change in the augmentation coefficients of capital, labor, and energy inputs. This is achieved through the use of formula (8) from Part B of Section II, where $d \ln A_i = (\dot{A}_i/A_i)$.

Note that estimating this series requires deleting one row and column from the matrix of estimated γ_{ij} coefficients. Because of sampling variation the derived series will be sensitive to this choice, so the results presented below are for simple averages of the series derived from each possible set of coefficient estimates; capital and energy, capital and labor, and labor and energy. We also estimate the annual Hicks factor-use bias in a similar manner, but using formula (9) of Section II.

The entire series are not presented here. The derived numbers displayed considerable variation, though in most years the estimated rate of change in any augmentation coefficient was less than 10%. The estimated average rates of factor augmentation for the three inputs is presented below.

Average Augmentation Rates, Cross Section Data

LK	-.0024
LL	.0230
LE	-.0810

and for the Hicks biases:

dMK* = -.0049

dML* = .0028

dME* = -.0032

The cross section and time series results will be compared in Part C.

C. Time Series and Cross Section Comparison

To compare time series and cross section regression estimates directly, we must re-estimate the time series model using the (Capital, Labor, Energy) specification rather than the full, four-input model. Once we have done this, we no longer estimate the Total Factor Productivity equation with the cost share equations. We can still enter time into the production function to allow for technical change, though some information is lost since not all of the parameter restrictions of the full model can be imposed. Still, our estimators are consistent.

We present the parameter estimates and summary statistics in Table 7 for two specifications of the times series K, L, E model, with and without technical change. The estimating equations here are:

1. without technical change:

$$M_i = \alpha_i + \sum_j \gamma_{ij} \log X_j + u_i \qquad i = K, L, E$$

and

2. with technical change:

$$M_i = \alpha_i + \sum_j \gamma_{ij} \log X_j + \gamma_{iT} T + u_i \qquad i = K, L, E$$

where, as before, the α's and γ's are parameters and the X's are input quantities.

Here again, as in the complete time series model, we have the annoying failure of the conditions for convex isoquants at some of the observations. And again, on the basis of a log-likelihood test, we reject the hypothesis of no technical change in favor of non-neutral technical change. And the derived biases are similar to those of the full time series model, being labor using and energy saving. There is a slight capital saving bias in the modified time series model that does not exist in the full model.

The estimated coefficients of this time series model appear roughly comparable to those of the 1963 cross section, as can be seen by comparing Table 7 with Table 4. The coefficient estimates are always of the same sign, for example and of roughly comparable relative magnitudes.

With respect to technical change we derive roughly the same implications from both the time series and the cross section data. The estimated constant rates of factor augmentation from the time series are compared to the average rates from the cross section below:

Factor Augmentation Rates

	Time Series	Cross Section
Capital	-.0005	-.0024
Labor	.0194	.0230
Energy	-.0014	-.0810

The one difference is the time series gives an insignificantly negative estimate for the rate of energy augmentation; in the cross section it is negative and of a much larger magnitude in absolute value.

Also we compare the average Hicks factor use bias rates below:[5]

Hicks Bias Rates

	Time Series	Cross Section
Capital	-.0034	-.0049
Labor	.0044	.0028
Energy	-.0010	-.0032

Both samples imply the same conclusion: Technical change has been labor using and perhaps slightly capital and energy saving over the time period.

TABLE 1

Coefficient Estimates and Summary Statistics

(t-statistics in parentheses)

	Biased Technical Change		Factor-Augmenting Biased Technical Change		Hicks-neutral Technical Change	
α_K	.184	(19.95)	.184	(20.06)	.188	(79.03)
α_L	.151	(20.76)	.152	(21.12)	.208	(87.59)
α_E	.051	(26.70)	.051	(25.78)	.048	(134.31)
α_M	.614	(71.11)	.614	(78.87)	.557	(233.71)
γ_{KK}	.087	(4.71)	.086	(4.69)	.110	(10.05)
γ_{KL}	0	(0)	.001	(.09)	.023	(2.28)
γ_{KE}	-.023	(7.74)	-.023	(7.76)	-.026	(9.98)
γ_{KM}	-.064	(4.11)	-.064	(4.13)	-.108	(6.67)
γ_{LL}	.172	(10.12)	.173	(10.11)	.014	(.96)
γ_{LE}	-.012	(2.65)	-.013	(2.85)	-.005	(1.70)
γ_{LM}	-.161	(9.55)	-.161	(9.63)	-.032	(1.65)
γ_{EE}	.055	(22.20)	.056	(22.01)	.056	(21.95)
γ_{EM}	-.021	(3.96)	-.020	(3.78)	-.025	(5.16)
γ_{MM}	.246	(10.46)	.246	(10.44)	.165	(5.08)
α_T	.0008	(.05)	.0003	(.52)	-.003	(.39)
γ_{KT}	.0003	(.42)	.0003	(.43)		
γ_{LT}	.0041	(7.91)	.0040	(7.95)		
γ_{ET}	-.0002	(1.70)	-.0002	(1.69)		
γ_{MT}	-.0042	(6.64)	-.004	(6.82)		
γ_{TT}	-.0003	(.26)	.0001	(5.93)		
λ_K			-.0005	(.09)		
λ_L			.0194	(7.61)		
λ_E			-.0014	(.74)		
λ_M			-.0041	(2.74)		
Log-likelihood	188.136		188.417		167.001	
R^2 M_K	.8302		.8300		.8029	
R^2 M_L	.8344		.8354		.2689	
R^2 M_E	.9902		.9902		.9902	
R^2 M_M	.7722		.7734		.6259	
R^2 TFP	.0064		.0516		.0000	
Total RSS	.0395813		.0379953		.043537	

TABLE 2

Tests of Hypotheses

1. H_n: Factor augmenting biased technical change

   H_a: Biased technical change

   # restrictions = 1

   $X_c^2 = 3.841$ ($\alpha = .05$)

   $\hat{X}^2 = -.562$

   Conclusion: Accept H_n.

2. H_n: Hicks neutral technical change

   H_a: Factor augmenting biased change

   # restrictions = 3

   $X_c^2 = 9.488$ ($\alpha = .05$)

   $\hat{X}^2 = 42.832$

   Conclusion: Reject H_n.

3. H_n: No technical change

   H_a: Hicks neutral technical change

   # restrictions = 1

   $t_c = 1.99$ ($\alpha = .05$)

   $\hat{t} = .39$

   Conclusion: Accept H_n.

TABLE 3

	Concavity imposed	Unconstrained
α_K	.188	.184
α_L	.146	.151
α_E	.053	.051
α_M	.613	.614
γ_{KK}	.152	.087
γ_{KL}	-.027	0
γ_{KE}	-.010	-.023
γ_{KM}	-.115	-.064
γ_{LL}	.129	.172
γ_{LE}	-.006	-.012
γ_{LM}	-.096	-.161
γ_{EE}	.145	.055
γ_{EM}	-.129	-.021
γ_{MM}	.340	.246
α_T	.0008	.0008
γ_{KT}	0	.0003
γ_{CT}	.0044	.0041
γ_{CE}	-.0004	-.0002
γ_{MT}	-.0040	-.0042
γ_{TT}	.0003	-.0003

TABLE 4

Regression Results for the Two Cross Sections

		1963		1958	
	α_K	.505	(22.26)	.341	(13.94)
	α_L	.435	(21.18)	.571	(26.14)
	α_E	.060	(21.75)	.232	(2.60)
	γ_{KK}	.199	(3.64)	.008	(.127)
	γ_{KL}	-.163	(3.32)	-.040	(.73)
	γ_{KE}	-.035	(4.61)	.032	(1.98)
	γ_{LL}	.177	(3.99)	.127	(2.37)
	γ_{LE}	-.014	(2.41)	-.087	(4.37)
	γ_{EE}	.049	(11.80)	.054	(3.05)
	MK	.4432		.1418	
R^2	ML	.5371		.4067	
	ME	.8179		.4774	
	Log-like	32.5164		17.9108	
	RSS	.1525797		.0811044	
	# obs./d.f.	14/23		10/15	
	# obs. with wrong signed principal minor	5		3	
	# obs. with wrong signed predicted cost share	0		0	

t-statistics in parentheses.

TABLE 5

Regression Results for the Joint Estimates

		Unadjusted data		Adjusted for capacity utilization		Adjusted for factor and capacity utilization	
	α_K	.442	(22.43)	.486	(18.02)	.445	(22.45)
	α_L	.483	(28.40)	.441	(19.31)	.481	(28.20)
	α_E	.075	(13.51)	.072	(8.44)	.074	(13.21)
	γ_{KK}	.133	(2.30)	.151	(3.26)	.147	(2.98)
	γ_{KL}	-.141	(3.40)	.147	(3.93)	.145	(3.61)
	γ_{KE}	.007	(.46)	-.004	(.25)	-.002	(.13)
	γ_{LL}	.159	(4.09)	.157	(4.76)	.159	(4.56)
	γ_{LE}	-.018	(1.64)	.010	(1.00)	-.015	(1.38)
	γ_{EE}	.011	(1.48)	.014	(1.31)	.017	(1.60)
	MK	.3901		.4218		.3895	
R^2	ML	.5059		.5195		.5094	
	ME	-.0072		.0076		.0072	
	Log-like	8.20294		8.24759		8.64104	
	RSS	.3333997		.3199693		.3324256	
	obs./d.f.	24/67		24/67		24/67	
	# obs. with wrong signed principal minor	0		0		0	
	# obs. with wrong signed predicted cost share	0		0		0	

t-statistics in parentheses.

TABLE 6

Chow Tests

H_n: No structural change in production when inputs are measured in natural units.

H_a: $\tilde{}H_n$

$\hat{F} = 2.81$

$F_c(5,38) = 3.54$ $(\alpha = .05)$

Conclusion: Do not reject H_n.

H_n: No structural change in production when inputs are measured in efficiency units.

H_a: $\tilde{}H_n$

$\hat{F} = 3.21$

$F_c(5,38) = 3.54$ $(\alpha = .05)$

Conclusion: Do not reject H_n.

TABLE 7

Time Series K,L,E Model Estimates

		without tech. change		with tech. change	
	α_K	.421	(100.18)	.470	(22.17)
	α_L	.470	(113.32)	.407	(20.75)
	α_E	.108	(108.50)	.123	(24.24)
	γ_{KK}	.121	(10.21)	.189	(6.30)
	γ_{KL}	-.026	(2.20)	-.115	(4.10)
	γ_{KE}	-.095	(32.22)	-.074	(10.13)
	γ_{LL}	.058	(3.74)	.172	(5.89)
	γ_{LE}	-.032	(3.67)	-.058	(5.85)
	γ_{EE}	.127	(17.33)	.132	(22.31)
	γ_{KT}	-		-.003	(2.30)
	γ_{LT}	-		.004	(3.25)
	γ_{ET}	-		-.001	(2.92)
	MK	.8359		.8273	
R^2	ML	.2151		.4791	
	ME	.9868		.9891	
	Log-like	78.8486		87.1849	
	RSS	.0218424		.0158268	
	# obs. with wrong signed princ. minor	0		18	
	# obs. with wrong signed pred. share	0		0	
	# obs./d.f.	25/70		25/68	

t-statistics in parentheses.

APPENDIX

DATA SOURCES

The data is for the primary metals industry; Standard Industrial Classification (SIC) number 33. Included are the manufacturer of pig iron and iron products, all types of steel, aluminum, zinc, copper, and various other metals accounting for a small portion of output.

Part A: Time Series Data

There are two special problems in construction of the data: 1) Construction of a total factor productivity (TFP) index, and 2) construction of a series for capital input and the rental price of capital.

1) The index of total factor productivity is usually a Divisia index of output less a Divisia index of input. According to Jorgenson and Griliches [1967],

$$TFP = \dot{Q}/Q - \sum_i W_i(\dot{X}_i X_i)$$

where

$$W_i = \frac{P_i X_i}{\Sigma P_i X_i}$$

Q = output

P_i = input price

X_i = quantity of the i^{th} input

Because of lack of data on inter-industry flows we cannot use a net output measure of TFP. The data is readily available for a value added based measure but as Star [1971] notes, if the material inputs do not enter the production function in approximately fixed proportions, their neglect will bias the total factor productivity measure. Our use of gross output data for a two-digit industry group implies homogeneous industries within the group. This is slightly different than assuming an aggregate production function for the industry group.

The total factor productivity index is readily calculated from data on input quantities and output quantity; or on the corresponding prices. The capital price and quantity indices are discussed in detail below. Otherwise data is as follows:

L: An index of production and non-production employees, from the Annual Survey of Manufactures (ASM). Following Griliches [1967], we calculate labor inputs in production worker man-hour equivalents. The average wage rate of production workers is calculated as indicated below. The difference between total payroll and production worker payroll is then converted into production worker man-hour equivalents by dividing by the average wage of production workers.

P_L: An average wage equal to production worker wage bill divided by number of production worker man hours, both from the Annual Survey of Manufacturers. Note that P_L ignores non-payroll labor costs: social security, pensions, insurance, etc. These data are not available, though at this high a level of aggregation it would almost certainly be highly correlated with the variable used.

E: Nominal expenditure on all energy inputs (from the ASM) divided by a nominal price index for all energy inputs (from the Bureau of Labor Statistics). This measure will be incorrect if industry purchases of energy inputs are not divided among the inputs in proportion to the inputs' weights in the aggregate price index, but no alternative is available.

P_E: The Bureau of Labor Statistics (BLS) nominal energy price index.

Q: Value of Shipments in current dollars adjusted for changes in the value of inventories of final goods (both from the ASM), divided by the BLS nominal price index for the industry group output. (The BLS index actually includes some metal products not classified in SIC-33, but their weight is trivial.) Value of Shipments estimates are available only for 1953 on. Gross Output for 1947-52 is estimated on the basis of the Office of Business Economics index of industrial production for SIC-33 (published in Business Stastics) which covers the entire period.

P_Q: The BLS industry-group price index.

ρ: Moody's index of AAA Corporate Bond yields.

Material Inputs: Expenditure on materials inputs (in current dollars) is calculated as Gross Output less the sum of total payroll, expenditure on energy, and expenditure on capital services (all in current dollars). This is divided into price and quantity components by dividing by a nominal price index for inputs.

This price index is constructed as a weighted average of three published BLS indices from the Handbook of Labor Statistics: 1. non-food, non-fuel crude materials, 2. intermediate materials and components for manufacturing, and 3. industry output price (since there are significant inter-industry flows). The weights are derived from the 1963 Input-Output tables for the United States, according to the source of direct requirements per dollar of Gross Output.

Capital:

We require three pieces of information: The quantity (physical units) of a stock of capital actually "used up" in the production process in a given time period; the corresponding price per unit of service flow which firms can be considered to charge themselves (the rental price), and the share of capital in total cost. The last is equal to the product of the first two, and is also identically equal to total property income associated with any producing unit.

The procedure for separating the value of capital services into price and quantity components is as follows:

1. The first step is construction of an index for the capital stock. The perpetual inventory formula is

$$K_t = I_t + (1-u)K_{t-1}$$

where u is the rate of depreciation. Note that this is a physical relationship, not a value relationship. Beginning with a benchmark K_t we add annual net investment deflated by a nominal price index.

2. The capital stock index (K) is used, along with an asset price index (q), the rate of depreciation (u), the value of capital services (Y_K) (property income), and the effective tax rate (tx) to compute the rate of return (ρ) on capital:

$$\rho = \rho(K, q, u, Y_K, tx).$$

The exact form of this function depends upon the pattern of depreciation and of taxation; see Christensen and Jorgenson [1969] for this and other details of the procedure.

4. Finally, since the value of capital services equals $p_K \cdot K$, which is total property income, we can compute an index of the flow of services as total property income divided by our rental price.

We use Christensen and Jorgenson's [1969] corporate Capital Price Index. This series ends in 1968. We update it through 1971 with the help of Berndt and Wood's [1975b] capital input price index.

Total property income is calculated as value added less total payroll (both from the ASM).

Part B: Cross-section data

Since it is not possible to construct a cross-section price index for non-energy intermediate inputs we estimate only a three-factor K, L, E model rather than the four-factor K, L, E, M time series model.

Total expenditure on capital services is measured as value added less total payroll. We assume the flow of services is proportional to the stock, and use gross book value as a proxy for input quantity. Griliches [1967] considers this as well as several other measures involving insurance payments, rental payments, property taxes, and depreciation but notes that the simple correlation between all the measures is greater than or equal to .99.

L and P_L are measured as in the time series section. The quantity index for energy inputs is constructed in a special fashion. We define a separate "sub-production function" for energy inputs:

Q = Q(capital, labor, "energy")

where energy = F(electricity, oil, natural gas, coal). We assume that the energy function is translog, so

$$\ln(\text{energy}) = \alpha_0 + \sum_i \alpha_i \ln X_i + \tfrac{1}{2} \sum_i \sum_j \alpha_{ij} \log X_i \log X_j$$

where

X_1 = electricity X_3 = natural gas

X_2 = fuel oil X_4 = coal

Estimating these α's and γ's exactly as we do in the body of the chapter, we can "predict" ln(energy) up to a constant, which is sufficient for our purposes. This translog aggregation is permissible if the four fuel inputs are separable from the basic production function.

The unit of observation is the state. All cross-section data are from the appropriate Censuses of Manufactures.

FOOTNOTES

Chapter 2

[1]See Anderson [1971], Fisher and Kaysen [1962], Halvorsen [1976] and Mount, Chapman and Tyrell [1973].

[2]Berndt and Wood [1975b] consider the demand for aggregate energy and other inputs by total U.S. manufacturing. Berndt and Jorgenson [1973] and Fuss [1977] consider the demand for individual types of energy by total U.S. and total Canadian manufacturing respectively.

[3]The translog form was introduced by Christensen, Jorgenson and Lau [1971] and [1973].

[4]Note that the adding-up restrictions together with the cross-equation equality restrictions on the γ_{ij} impose linear homogeneity in prices on the cost function.

[5]Note that the parameter estimates obtained from estimation of the cost share equations can be used in (3) to compute the unit cost of aggregate energy. Alternatively, the unit cost of aggregate energy can be indexed using superlative index numbers, see Diewert [1976].

[6]See Zellner [1962] and Oberhofer and Kmenta [1974].

[7]Jorgenson and Lau [1975] develop these restrictions in the context of translog utility functions. The test for implicit separability is exact only at the point of approximation. However, the test for explicit separability is invariant to the scaling of the price variables.

[8]See Berndt [1977].

[9]Not having the elasticities of demand constrained to be constant is one of the major advantages of the use of a flexible functional form for the unit cost function.

[10]See Kmenta [1971, pp. 443-444].

Chapter 2 (Continued)

[11]Consumption of residual and distillate oil is reported separately. The quantity of fuel oil is computed by weighting the number of barrels of each type of oil by kilowatt hours equivalent factors in U.S. Bureau of the Census [1973]. The use of prices as weights provided very similar results.

[12]Thus the price data are equal to average prices. The use of declining rate schedules for electric energy and natural gas results in a divergence between marginal and average prices for these inputs, but data on marginal prices are not available. See Halvorsen [1975, 1976] for further discussion.

[12a]Use of a .01 significance level would result in rejection of the cross-equation equality restrictions for an additional four industries in 1971 and five in 1962 and 1958.

[13]The monotonicity test at the means of the data can be interpreted as a local test at the point of expansion, see Jorgenson and Lau [1975].

[13a]A procedure proposed by Lau [1974] provides a statistical test of concavity but was not used due to computational difficulties.

[14]Results for 1971 for the industries for which the model did not perform well are given in Appendix Tables A.1 and A.2.

[15]The cross price elasticities should generally be smaller because the sum of the own and cross price elasticities is zero and most of the cross price elasticities are positive.

[16]For elasticities involving coal, the weights are the shares of each industry in consumption by the industries for which the four-input model performed well.

[17]Data for 1958 were not included in the pooled regressions because 1958 was a recession year and therefore not fully comparable with 1971 and 1962.

[18]Fuss [1977] obtained a virtually identical result for E_{HV} for total Canadian manufacturing for 1971.

[19]Since the estimates of E_{HV} are obtained holding manufacturing output constant, these elasticities do not reflect induced output effects on energy demand.

Chapter 2 (Continued)

[20]Halvorsen [1976] obtained an estimate of -1.24 for E^T_{EE} with cross-section state data for total industrial demand for electric energy in 1969. The estimate of E^T_{EG} in that study, 0.23, is very close to the estimate of 0.20 obtained in this study for 1971.

[21]See Allen [1966] and Uzawa [1962].

[22]Because the Cobb-Douglas functional form is a special case of the translog form, it is possible to test whether the more restrictive Cobb-Douglas form is appropriate for the cost function. The Cobb-Douglas form was rejected for all industries at the .05 level except industry 34 in 1958, industries 22, 23, 25, 27, and 35 in 1962, and industries 23, 29, 34, and 36 in 1971.

[23]Estimates of the elasticities of demand for fuel oil, natural gas, and coal in electric power generation are reported in Atkinson and Halvorsen [1976].

Chapter 3

[1]The results will also provide further information on the existence of a consistent aggregate of production and non-production workers, see Cook [1968] and Berndt and Christensen [1974].

[2]Sixteen of the seventy-two estimated cross elasticities are significant at the ten percent level using a two-tailed test.

[3]See Berndt and Wood [1975b] and Fuss [1977].

Chapter 4

[1]The constraint that the firm is on its production function also implies singularity of $(I - \Gamma)$, see Nadiri and Rosen [1973, pp. 32-33]. The restrictions are not imposed on the system of equations.

[2]In calculating intermediate- and long-run elasticities, the value of the labor own-adjustment coefficient is set equal to one.

[3]Fuss [1977] reports as a representative result for Canada an estimate of -0.49 at the means of the data for Ontario.

Chapter 5

[1] We could also investigate technical change through its effects on the cost function which is dual to the production function. This would have the econometric advantage of permitting a parametric test for constant-returns-to-scale, rather than imposing it as we do here. But it would also have the disadvantage of requiring us to maintain that the factor augmentation values were exogenous, since if they are endogenous they will not show up in the dual cost function. Some empirical work was done using a translog cost function; the results were generally poorer than with the production function.

[2] This is the procedure used by Sato [1970], and by Binswanger [1973] who uses a translog cost function.

[3] We are looking at the reduced system of n - 1 share equations here; for the full system Γ is singular so Γ^{-1} does not exist.

[4] It should be noted that there is some question about the proper estimation of the standard errors of the parameter estimates. The issue has to do with the proper number of degrees of freedom for the model. The standard errors and t-statistics here conform to Theil's [1971] and should be interpreted as asymptotic standard errors and t-statistics.

[5] In principle the cross section bias estimates should sum to zero, as do those of the time series. This constraint is difficult to impose given the way in which these numbers were calculated. See above, Part B.

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